# Metasurfaces for control of electromagnetic waves 

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List of Acronyms and Abbreviations

EM - electromagnetic
MM - Metamaterials
FSS - Frequency Selective Surfaces
RF - Radar Frequencies
MS - Metasurface
SRR - Split-ring resonator
OAM - orbital angular momentum
OA - optical axis
LSP - localized surface plasmon
DNG - Double Negative Material
DPS - Double Positive Material
LH - Left-handed material
CST/MWS - CST Microwave studio
PB phase shift - Pancharatnam-Berry phase shift

## Пєрí入пчך









 $\varepsilon \varphi \alpha \rho \mu о ү \eta ́ ~ \sigma т \eta ~ \sigma т \rho \varepsilon ́ \psi \eta ~ \eta ́ ~ т \eta v ~ \varepsilon \sigma т i ́ a \sigma \eta ~ т о u ~ \varphi \omega т o ́ \varsigma / т \omega v ~ \eta \lambda \varepsilon к т \rho о \mu \alpha ү v \eta т ו к \omega ́ v ~ к u \mu a ́ t \omega v . ~ O ı ~$








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#### Abstract

Metamaterials are a man-made form of matter, manufactured with the purpose of achieving non-conventional electromagnetic (EM) properties. The manipulation of electromagnetic radiation achieved with the metametarials can be used in a multiple of applications, from telecommunications to imaging and detection. Control of EM wave propagation in metamaterials is caused, in most cases, by elements (meta-atoms) with resonance at wavelengths much larger than their dimensions. The goal of my Thesis focuses on meta-surfaces (i.e. metamaterials with thickness much smaller than wavelength) suitable for application on steering or focusing of light/electromagnetic waves. These reflect-arrays consist of identical metallic antennas which introduce abrupt phase changes on the reflected wave. These changes are of geometric origin and by rotating the antennas properly (in respect to the polarization plane of the incident wave), they can reach values from 0 to $2 \pi$. The aim is to investigate and identify the optimal shape/geometry of the antennas to achieve steering or focusing of the reflected wave.

In the current thesis we discuss first the basic characteristics of meta-surfaces and geometrical phase, called Pancharatnam-Berry phase. The numerical results of the work were made with the use of the software Microwave Studio from Computer Simulation Technology (CST). In our numerical study we mainly focus on reflection amplitude of the cross-polarization state and the respected phase response for the selected shapes of seven antennas. For each structure, under circularly polarized light, the amplitude, phase and direction of the cross-polarized reflected wave are investigated. In addition, by arranging the antennas in a suitable manner to achieve focusing from the meta-surface, we demonstrate focusing of the reflected electromagnetic wave, which resembles a Bessel-like beam.


## Chapter 1: Introduction

Manipulation of the electromagnetic (EM) waves, in general, has been one of the fascinating quests in the field of electrodynamics. Since the late 1940, many researchers have been investigating the electromagnetic properties of man-made structures and their possible applications. The great interest on artificial materials can be attributed to the fact that the permittivity or permeability of such a material can be engineered at will. They can take values less than unity and even negative, simultaneously, which is unattainable with well-known materials in nature, commonly used in microwaves and optics applications [1],[2]. The most interesting feature of the man-made structures is that their properties arise from structures' geometry rather than chemically interacting atoms. Moreover, the structural unit cell of the artificial materials, also known as metamaterials (MMs), is much smaller than the wavelength of operation ( $\lambda \gg \mathrm{a}$ ), where the ' $\lambda$ ' is the free space wavelength and 'a' is the size of the unit cell. This subwavelength characteristic is the feature that make possible effective properties to be defined for the MMs and it distinguishes them from the more common and found in nature periodic photonic crystals (where $\lambda \sim a$ ).

The recent appearance of precise fabrication techniques made possible for the researchers to realize meta-surfaces (metamaterials less than a wavelength thick). They are designed to reflect, transmit or absorb EM waves in peculiar and unusual in many cases ways. The principle of operation of theses metasurfaces is different from the transformation optics used for the MMs for shaping a wavefront. The MMs rely on gradual phase shift accumulated over distances of multiple wavelengths, where the shaping light beams mechanism is applied. In contrast, the metasurfaces (MS) introduce abrupt phase changes, which provide new degrees of freedom for EM wave manipulation; thus they enable wavefront manipulation in subwavelength scale, providing a control over the phase, the amplitude and the polarization of an impinging wave.

### 1.1 Background on metamaterials and metasurfaces

The term "meta-" on the words "metamaterials" and "metasurfaces" is of Greek origin (meaning "beyond") and is referred to materials with electromagnetic properties beyond those of materials, found in nature [2][3]. Evolving from a theoretical concept proposed several decades ago, the associated research area, currently called metamaterials, is now rapidly developing. In addition, with the fast development of nano-fabrication techniques the MMs are not limited any more to the microwave region; more complex structures are created, which are applicable at higher frequencies; therefore a wider range of material properties becomes possible. The start was made by Veselago (1968), when proposed the possibility of negative refractive index materials [4], by combing negative permittivity and negative permeability structures [1][3]. The MMs with refractive index reaching negative values exhibit extraordinary electromagnetic properties, such as the inversion of Snell's law or of the Doppler shifts. These novel material properties imply potential of enhancing the performance of the conventional optical components.

It took nearly thirty years, before it was possible to fabricate suitable elements to achieve negative permeability and the theory to be verified. A milestone has been set in 1999 by J. Pendry, who proposed a nonmagnetic particle with strong magnetic resonance [3]. The particle was a split-ring resonator (SRR), which had dimensions smaller than the wavelength. Using SSRs, researchers were able to engineer artificial subwavelength structures that can generate dia-magnetic effects at the microscopic level and achieve an effective bulk negative permeability. It was claimed that, a superlens made of materials with simultaneously negative values of permittivity and permeability may overcome the diffraction limit in optics and provide perfect imaging [3]. Based on this work [3][5], Smith in 2000 demonstrated a double negative material (DNG) [6][7], which exhibited a combined negative permeability and permittivity. It consisted of SRRs providing the negative permeability, combined with a system of straight wires providing negative permittivity. The negative refractive index of such a medium was experimentally demonstrated in 2001 by Shelby [8], who gave a great enhancement to metamaterials research.

As can be concluded from the above, the MMs are considered as periodic arrays with unit cell dimensions sufficiently smaller than the wavelength of operation. The unit cell contains selected inclusions with specific geometry made of certain materials and providing (electric or magnetic) resonances. Essentially, MMs may be macroscopically treated as homogeneous bulk media characterized by a set of effective parameters. These bulk EM properties of MMs are directly related to the characteristics of their constituting inclusions, such as polarization change and resonance frequency.


Figure 1: Classification of materials based on the values of permittivity and permeability. The ENG (epsilon negative) media are materials with permittivity having negative values, as it respectively with MNG (mu negative) media where the permeability is negative. The DPS (double positive) term defines most of known natural materials, which have permittivity and permeability both positive, while the DNG (double negative) term, i.e. both permittivity and permeability negative refers to a region unattainable with natural materials, attainable though with metamaterials.

The diagram in Fig. 1 presents the materials classified by the values of their permittivity and permeability. Generally, materials are defined by their permittivity $\varepsilon$ and permeability $\mu$, which are complex quantities in the frequency domain:

$$
\begin{aligned}
& \varepsilon=\varepsilon_{r} \varepsilon_{o}=\varepsilon_{o}\left(\varepsilon_{r}^{\prime}-j \varepsilon_{r}^{\prime \prime}\right) \\
& \mu=\mu_{r} \mu_{o}=\mu_{o}\left(\mu_{r}^{\prime}-j \mu_{r}^{\prime \prime}\right)
\end{aligned}
$$

The real part of the relative permittivity $\left(\varepsilon_{r}\right)$ or permeability $\left(\mu_{r}\right)$ is denoted with a single superscript accent mark, in comparison with the imaginary part which has two. The vacuum value of permittivity or permeability is denoted with a subscript zero mark. $J$ is the imaginary unit. Effective permittivity and permeability are the two main parameters that determine all the MMs response to incident EM waves. The majority of the natural materials have positive permittivity and permeability. Some materials have negative permittivity or permeability, but no materials have both parameters negative, simultaneously [2]. That becomes possible only with MMs. Among the unique EM properties of double negative (i.e. of both permittivity and permeability negative) metamaterials is that the wavevector has a direction opposite to the power flow given by the Poynting vector. In general, the electromagnetic properties of a MM could be engineered in arbitraries ways; thus they can find many applications in optics, such as to enhance imaging resolution, or to improving sensing technologies such as Radar devices. Indeed, the MMs can be used in particle accelerators, satellites, radars in naval ships or airplanes devices.

Important subcategory of the MMs are the ultra-thin metamaterials, also known as metasurfaces. The metasurfaces (MS) are the topic of this Master Thesis, especially metasurfaces composed of metallic antennas (as meta-atoms) with subwavelength size and spatially varying phase response. The convenience of the fabrication of ultrathin metallic shaped antennas made possible to engineer the appropriate geometrical parameters in order to exhibit tunable optical properties for manipulation of phase, amplitude and polarization of EM waves [9]. The selection of a certain type/shape of antennas introduces an abrupt phase change of the wave impinging on metasurfaces, allowing us to realize extraordinary light manipulation in subwavelength scale.

The discussion of the possibility to replace conventional optical components with metasurface-based ones revealed three distinctive features. Firstly, there is the independent control on the electric and magnetic field components. This is engineered by properly selected the scatterers comprising of the metasurface. Secondly, the arrays of scatterers accomplish the modulation of the wavefront in a shorter distance than a wavelength. This feature of metasurfaces has been reported in both reflection and transmission. Finally, the metasurfaces based on antennas engineer the spatial distribution of amplitude, phase and polarization response with subwavelength resolution, achieve to transfer the full incident's power to a single beam [10].

The advantage of using resonant metallic elements to construct the metasurface provides ability for shaping wavefronts of EM waves almost at will. Metallic ultra-thin shaped antennas on silicon substrate are reported to produce phase jumps (in subwavelength scale) on travelling waves and control the scattered wave phase over the range of 0 -to- $2 \pi$ [9]. To achieve spatially varying phase response on a metasurface, as is required for getting desired wave control, the shaped antennas (with subwavelength separation) are chosen to form a supercell comprising of antennas of different, properly selected geometrical parameters, in order to produce difference on the actual optical path.


Figure 2: The color vectors represent the light rays from a source point A (in air medium) propagating through an inclined surface to point B (in the blue area/ higher refractive index media). The optical paths of the two rays have transevrse distance on the glass surface equal to $d x$.

The changes of the optical path can be understood by the 'ray tracing' method, which considers the light beam as continuous rays. The vectors in Figure 2 follow the Fermat's principle of least optical path and propagate on an inclined surface, where they must bend in different angles toward reaching point B . A more realistic situation is the pioneer work from Harvard's group [9], which examined an array of eight dissimilar Vshaped antennas to create a constant gradient of phase shift in the plane of the antennas. In that situation the gradient was created by the different phase response of the antennas, which is engineered by adjusting properly the V-shape arm length and angle. Nonetheless, the Huygens-Fresnel principle is applied on the structure's scatterers, where the (later) scattered waves are not restricted/contained necessarily on the same plane as the incident wave, thus they are named anomalously refracted or reflected waves. These metasurfaces create a linear gradient from the phase response of the planar antennas, which works as beams steerer.

The introduction of an abrupt phase change (phase discontinuity) by a metasurface provides the ability to control the amplitude, phase and polarization of incident EM waves. People have used a variety of antenna structures to provide a $2 \pi$ phase coverage on the plane of metasurface, which is needed for a full control of the wavefront [10]. Such metasurfaces can replace conventional optical components, since they exhibit the minimal diffraction aberrations, own to the subwavelength size of antennas and their distances. Such characteristics inspire new devices, e.g. achromatic lens (type of lens which corrects the colors aberrations) and multicolor hologram.

The topic of thesis is focused on metasurfaces based on Pancharatnam-Berry (PB) phase. In our case, the change of the phase response of an antenna is created by rotating the antenna with respect to its Optical Axis (OA). Applying a generalized form of Snell's law is in this case necessary to describe the supercells phase profile. Creating a supercell with linear phase gradient along it, we demonstrate anomalous reflection and wave steering. Secondary, Axicon-like structures are constructed with two linear gradients, which are mirror image of each other along the axis. The formed structure is built with the purpose of use for focusing application.

### 1.2 Thesis outline

The importance of the metasurfaces and their applicability, as it has been described previously, are the reason which led me to focus my research on them. Furthermore, the aim of thesis is the investigation of different shapes of antennas as meta-atoms of reflectarray metasurfaces with linear phase gradient (on the transverse direction) constructed by exploiting the PB phase. The theoretical background for the operation of such antennas is presented and analyzed in three parts in chapter 2. The first part is the law of refraction and reflection, which is a generalized form of the Snell's law. The second part analyzes the PB phase and demonstrates the use of antennas for creating phase gradients. The third part is focused on Axicon devices and the generation of Bessel beams. In chapter 3 the theoretical tools used are described. The numerical results for a single antenna on the substrate are presented and the respected phase shift of the reflected wave. Chapter 4 presents the numerical results of a linear phase gradient created by a supercell of 6 antennas individually rotated with respect to the optical axis (OA). The results include plots of the electric field to show the beam steering capabilities of the structure. Finally, chapter 5 shows structures operating as a reflective Axicon, which is created by two mirror symmetric linear phase gradient systems.

## Chapter 2: Theoretical background

This chapter presents the theoretical background relevant to metasurface reflect-arrays formed from shaped antennas with subwavelength size and separation, and spatially varying phase response. The generalized form of Snell's law is applied in this case, which is presented in the first two sections. Afterward, the third section is focused on the adiabatic phase change named Pancharatnam-Berry phase shift, created by in-plane rotation of the antennas. The final section presents a summary of the principle for the Axicon structure operation and its application for generation of Bessel beams.

### 2.1 Metasurfaces as optical components

The conventional optical components, like lenses, prisms and wave-plates, are used for altering the wavefront (manipulation of the phase, polarization or amplitude) through propagation over distances of multiple wavelengths. The propagation through mediums of specific refractive index is implemented for engineer a control over the optical path of light. The variation of the optical path is described in Figure 3, where the incident plane wave propagate through the grey medium of certain refractive index. The thickness of the medium is unequal, therefore the wave will be reflected not as a plane wave, but with a different wavefront, related to the paths travelled in the medium.


Figure 3: The figure shows a medium of the same refractive index with geometric parameters able to vary the optical path of a plane wave shaping the wave front. The line behind medium represent a mirror. Therefore, an incident plane wave will be reflected in such way portrait by the black rectangular line.

It was proposed later to make a metamaterial with ability to cause wavefront manipulation at subwavelength scale. The realization of this idea needs an array of subwavelength scatterers with subwavelength separation and a spatially varying phase response in the scatterers plane. The first property clearly indicates, that metallic optical antennas on a dielectric substrate, giving rise to localized surface plasmons, can be used as subwavelength apertures. The spatially varying phase response needs an array of scatterers with a linear gradient in the phase of either the transmitted or reflected wave,
aimed to vary the phase over the range of 0 -to- $2 \pi$. Thus, the transmitted or reflected wave is possible to propagate at an arbitrary, designed direction.


Figure 4: The trajectories of reflected and transmitted waves after the interaction of an incident wave with a metasurface. Designing properly the metasurface one can create a gradient in the phase jump ( $\Phi$ ) of the reflected or transmitted wave on the metasurface plane. In this case, reflected and/or transmitted waves could propagate out-of-plane of incidence, at angles on a range 0 -to- $2 \pi$. [10]

As is reported in [10][11], the scatterers operate as a phased array introducing a phase change to the scattered waves, which is related to the metasurfaces' gradient. The phased array can be engineered to exhibit different behavior, aiming applications such as focusing or defocusing, or beam steering, or wave-plates [12][13].

### 2.2 Generalized laws of refraction and reflection

The conventional law of Snell's is not sufficient in the occurrence of the phase jumps exhibiting a gradient on transverse directions (i.e. on the interface). In this case modifications of Snell's law for refraction and reflection should be introduced, which are as follows (see Fig.4) [10].

$$
\begin{gather*}
n_{i} \sin \theta_{i}-n_{t} \sin \theta_{t}=\frac{1}{k_{o}} \frac{d \Phi}{d x}, \quad n_{t} \cos \theta_{t} \sin \varphi_{t}=\frac{1}{n_{t} k_{o}} \frac{d \Phi}{d y}  \tag{1}\\
\sin \theta_{r}-\sin \theta_{i}=\frac{1}{n_{i} k_{o}} \frac{d \Phi}{d x}, \quad \cos \theta_{r} \sin \varphi_{r}=\frac{1}{n_{r} k_{o}} \frac{d \Phi}{d y}
\end{gather*}
$$

where $n_{i}, n_{t}$ and $n_{r}$ are the refractive index of the incident, transmitted and reflected wave, respectively. The angles of incidence, refraction and reflection are represented by $\theta_{\mathrm{i}}, \theta_{\mathrm{t}}$ and $\theta_{\mathrm{r}}$. The ' $\Phi$ ' on the differential represents the phase of the refracted and reflected wave. Also the free space wave-vector is represented by ko. The above equations are
predicting the direction of the refracted and reflected waves, which are depended both on the optical properties of the surrounding media and on the intrinsic phase response of the artificial interface connecting them.

The metasurface operation follows the principle of Huygens-Fresnel, which exploits the interference of the scattered waves on the refraction and on the reflection. The principle of Huygens-Fresnel states that every point of disturbance on the plane wave's wavefront is a center of a secondary source with spherical wavefront; then by mutual interfering the wavelets could reconstruct a prime plane wave [13]. Figure 5 illustrates the wavefront shaping by a plane interface and a metasurface.


Figure 5: The propagation of a plane wave on a homogeneous structure (left panel) and a metasurface (right panel) is dictated by the Huygens-Fresnel principle. The plane wave from the metasurface is steered by the phase gradient and change the wavefront compared to the incident plane wave. This is applied both on reflection and on transmission [14].

Metasurfaces are functionally similar to diffractive optical components: both technologies shape light beams for different applications by using a planar architecture. However, metasurfaces rely on a different toolset of design structures from those used in diffractive optics. These new structures provide such unprecedented flexibility, which leads to complete control of the phase, amplitude and polarization response of the wave. This control could be extended to a certain spectral range by suitable engineering of the inclusions elements in the unit cell, therefore is not limited near the resonance wavelength. These advantageous features go well beyond the capabilities of conventional diffractive optical components, which do typically have little control of chromatic dispersion. It is likely that new functionalities will emerge from flat optics based on metasurfaces, not possible with conventional diffractive optics with aberrations such as astigmatism or coma [15].

To achieve a spatially dependent phase variation by a metasurface one approach suggested is to use the geometrical phase of the antennas [10]. This robust approach uses antennas with identical geometrical properties and needs the polarization of incident wave to be circular. In general, the electromagnetic radiation carries not only energy but also momentum. A circularly polarized light beam transfers both spin and orbital angular momentum, which propagate along a specific axis. The spin momentum is associated with the polarization conversion, whereas the spatial distribution is linked with orbital angular momentum arise from the phase profile of the metasurface [16][20]. The spin-toorbital conversion, showed in inhomogeneous mediums, is defined as geometrical phase, named Pancharatnam-Berry phase. The next section discusses this subject, where the geometrical phase is analyzed.

### 2.3 Pancharatnam-Berry phase

The Pancharatnam-Berry (PB) phase is a well-known geometrical phase associated with polarization states of light. The manipulation of these polarization states uses the space variant (transverse inhomogeneous) subwavelength aperture. For the latter, the phase modification is acquired by rotation of the subwavelength elements, which compels the light beams to undergo an adiabatic phase shift converting the circular polarization into an azimuthal form [18][19]. The PB phase is a completely different approach from the variation of optical path, which is associated with the dispersion engineering of antenna's resonance [17] [21]. The space variant polarization modulation is created by anisotropic, subwavelength scatterers with identical geometric parameters but spatially varying orientations. In fact this idea comes from the pioneer work of Hasman and co-workers [20,21], which presented manipulation of the wavefront through control over the polarization states by using subwavelength gratings. In our case, the suitable element for this geometrical phase are metallic shaped antennas of subwavelength size.

To illustrate the PB phase associated with the rotation of a subwavelength scatterer illuminated by a circularly polarized wave, we will employ the Jones reflection matrix. In general, the Jones-reflection- matrix of an anisotropic scatterer can be written as [22]

$$
\hat{r}=\left(\begin{array}{ll}
r_{x x} & r_{x y}  \tag{3}\\
r_{y x} & r_{y y}
\end{array}\right)
$$

where $r_{x x} r_{y y}, r_{x y}$ and $r_{y x}$ are the reflection coefficients for linearly polarized incident light. The first subscript in the coefficients $r$ denotes the polarization of the reflected wave and the second the polarization of the incident wave. The reflection matrix $\hat{r}$ can be expressed also in circular polarization bases. The new matrix $\hat{r}_{\text {circ }}$, which is obtained by a similarity transformation, with transformation matrix $\hat{\Lambda}=\left(\begin{array}{cc}1 & 1 \\ i & -i\end{array}\right)$ (the matrix of the circular polarizion eigenvectors), i.e. $\hat{r}_{\text {circ }}=\hat{\Lambda}^{-1} \hat{r} \hat{\Lambda}$ is expressed as

$$
\hat{r}_{\text {circ }}=\left(\begin{array}{ll}
r_{R R} & r_{R L} \\
r_{L R} & r_{L L}
\end{array}\right)
$$

where the subscripts $R$ and $L$ denote right and left circularly polarized wave. Rotating the scatterer by an angle $\theta$, the new reflection matrix is obtained as $\hat{r}(\alpha)=\hat{R}^{-1}(\theta) \hat{\mathrm{r}}_{\text {circ }} \hat{R}(\theta)$, where the rotation matrix $R(\theta)$ is

$$
\hat{R}(\theta)=\left(\begin{array}{cc}
e^{i \theta} & 0  \tag{4}\\
0 & e^{-i \theta}
\end{array}\right)
$$

with ' $\theta$ ' the rotation angle. Performing the calculation we can find $\hat{r}(\theta)$. An incident wave of right/left circular polarization is denoted as 'EInc' and the reflected light from the anisotropic scatterer is written as $E_{R / L}$. The general relation of this semi analytical method can then be written in the general form [12]:

$$
\begin{align*}
& \binom{E_{R}}{E_{L}}=\hat{r}(\theta) \vec{E}_{i n c}=\left(\begin{array}{cc}
r_{R R} & e^{2 i \theta} r_{R L} \\
e^{-2 i \theta} r_{L R} & r_{L L}
\end{array}\right) E_{i n c}  \tag{5}\\
& \binom{R_{L}}{R_{R}}=\frac{1}{2}\left[r_{L L}\binom{1}{0}+\left(e^{i 2 \theta}\right) r_{R L}\binom{0}{1}\right] \tag{6}
\end{align*}
$$

The equation (6) gives the electric field of the reflected wave in formalized form of the Jones matrix for a left-handed incident wave. The first term represents circularly polarized reflected waves with the same handedness as the incident light. Additionally, the second term represents circularly polarized reflected waves with opposite handedness and an additional Pancharatnam-Berry phase factor. The second term can be selected in experiments by using a quarter-wave plate and a polarizer. Its phase can cover the entire $2 \pi$ range if the anisotropic scatterers rotation angles range from 0-to-m.

Based on this principle, a phase-gradient metasurface has been demonstrated to steer light into different directions depending on the handedness of the incident circular polarization [9][10][14]. The unit cell of the metasurface consists of series of aperture antennas with an incremental angle of rotation between adjacent antennas, with the total rotation angle ranging from 0 to $\pi$ within the unit cell. Similar aperture antennas have been used to create a planar lens, which either functions as a focusing or diverging lens depending on the handedness of the incident circular polarization [26]. A major advantage of the approach based on the Pancharatnam-Berry phase is ultra-broadband performance for any antenna's geometry. Moreover, the wavefront is not distorted by the antenna materials dispersion, because the magnitude of the phase shift is a function of the orientation angle and the sign determined by the handedness of the incident circularly polarized (CP) wave. The operating bandwidth is limited on the long- wavelength side by the reduced scattering efficiency and on the short-wavelength side by the requirement that the wavelength has to be at least several times larger than the spacing between scatterers [23].

The PB phase shift is this kind of adiabatic phase where the resonance of the anisotropic subwavelength scattererer with high amplitude can be manipulated with the orientation of its optical axis rather than its geometrical parameters. Thereby, antenna arrays can be introduced, which by a rotation with respect to the optical axis (OA) engineer a phase profile with radial symmetry [24]. Not only circularly polarized (CP) light is eligible for PB metasurface; a linearly polarized incident wave can produce CP waves of opposite handedness and an opposite phase delay [18]. Metasurfaces based on PB phase have been employed in many metasurfaces-related applications, with most prominent the creation of lenses, creating either a linear or parabolic phase profile [12]. Example is the U-shaped aperture, where different phase profile leads to light bending or
lens application. Another type of geometrical phase metasurfaces is made by nanorods in parabolic phase profile, which are presented for vortex beam applications [13].

### 2.4 Axicon and generation of Bessel Beams

Bessel beam is a kind of non-diffractive beams, which can propagate at large distances without losing intensity [25][26]. Their amplitude presents azimuthal symmetry and is described by Bessel functions of $1^{\text {st }}$ order. Bessel beams are used in a variety of applications, including optical trapping, tweezing, drilling of high-precision holes, and controlling the propagation of ultra-short pulses in dispersive media. The most common component to create Bessel beams is the Axicon, a conical lens that can be used as a refractive or diffractive optic or as a digital hologram written to a spatial light modulator. The reflective Axicon has been investigated to demonstrate that the shape of an image is unchanged on the image plane no matter its distance from the source [27].

## Chapter 3: Mathematical method and simulation tool

This chapter focuses on providing the necessary data for the construction of the reflectarrays which are investigated in the chapter 4 and 5 . Firstly, it presents the numerical tool used in the calculation, which is the CST Microwave Studio. Then it presents results for the reflection coefficient for antennas of different shapes, and investigates how reflection amplitude and phase change by rotation of antennas by angle $\theta$, as discussed in connection with Eq. (4)-(6).

### 3.1 Build-up model with Microwave studio

The CST Microwave Studio ${ }^{\text {TM }}$ was chosen as our simulation tool, as it is able to provide fast, efficient results and analysis of the EM components. The calculations apply the Maxwell equations on our structure of interest, which is shown schematically in Figure 6. After the model has been constructed, a fully automatic meshing procedure is applied before the simulation solver is started [28]. Our calculations apply the Frequency domain solver of CST (employing the Finite Elements Method) on the unit cell of metasurface under investigation with periodic boundary conditions on the plane perpendicular to that of the propagation (i.e. along directions $x$ - and $y$ - axes - see Fig.6). The metasurfaces investigated are composed of different shape ultra-thin antennas on a plastic substrate with a backside metallic sheet, which works as a mirror (see Fig. 6). The incident wave is circularly polarized and we calculate the reflected wave of the opposite polarization.


Figure 6: The schematics of the structure under investigation is shown. The yellow color indicates the metal, while blue indicates dielectric. The vectors of reflectance and incidence are appeared and the colors represent the polarization state of the waves. The orange is the reflected wave with opposite handedness polarization while the light blue is for the same polarization as the incident wave. There is no transmission in this model.

The construction of the desired structure and the selection of the materials is accomplished by the options of the software. An example of a structure studied is shown in Fig. 7 (the unit cell is shown). The blue-green color in Figure 7 is the substrate of plastic material with dimension of $\mathrm{D}_{\mathrm{x}}=\mathrm{D}_{\mathrm{y}}=10 \mathrm{~mm}$ and a thickness of 3.1 mm . The yellow color defines the metallic component of the model, which is on top of the substrate with thickness of $36 \mu \mathrm{~m}$. The length of the metallic straight wire in the simulation is $d=8 \mathrm{~mm}$ and its width is 1 mm .


Figure 7: The view of the straight wire antenna on top of a plastic substrate of thickness 3.1 mm , backed with a metal mirror (not shown). The picture shows the structure unit cell designed from CST Microwave Studio ${ }^{\text {TM }}$.

### 3.2 Single antenna unit cells examined in this thesis

This section is focused on investigating the structure shown in Fig. 6, where different shapes of front antennas are employed. The aim is to find large cross-polarized reflectance with constant amplitude and phase linearly dependent on the antenna rotation angle, covering a range from 0 to $2 \pi$. The antenna shapes investigated are: a) straight wire, b) zeta, c) symmetric and antisymmetric bow-ties, d) rectangular center split-ring and e) rectangular double split-ring. All these antennas were placed on a substrate of thickness either 3.1 or 5.1 mm and are backed with a metal mirror, which results (among other things) to vanishing of the transmission.

### 3.2.1: Straight wire

This section is focused on the straight wire antenna behavior. The unit cells' geometrical parameters are defined as $\mathrm{D}_{\mathrm{x}}=\mathrm{D}_{\mathrm{y}}=10 \mathrm{~mm}$ with substrate thickness 3.1 mm . The substrate is backed with a metallic sheet of thickness equal to $36 \mu \mathrm{~m}$. This is a square-shaped sheet, which, as is also mentioned earlier, mainly functions as mirror, suppressing the transmission and contributing to the reflection. The antenna geometrical parameters are shown in Fig. 8.


Figure 8: Front view of the straight wire antenna unit cell.

The results of reflection amplitude and phase response for an array of straight wires with a backplane mirror are shown below. The numerical results concern the lefthanded circularly polarized reflected wave created by a right-handed incident wave. The different curves correspond to different rotation angles of the straight wire in respect to $x$ axis (rotation in $x-y$ plane).


Figure 9: The unit cell of the straight wire model. At the bottom-left side the numerical results of the reflection amplitude for lefthanded circularly polarized generated by a right-handed incident wave. Theta is the angle of rotation between the metallic antenna and the $x$-axis (for the structure shown theta=0). The bottom-right graph shows the reflection phases for the same cases as is the left side.

The information shown in the above diagrams is exported to investigate the relation of the phase shift with the rotation angle 'theta'. Results showing the reflection phase versus theta for different (representative) frequencies are presented in Fig.10. They show that the rotation angle of metallic element is in linear relation with the phase shift, which for rotation angle from 0 to $\pi$ spans a range from 0 to $2 \pi$. (As it is discussed at previous texts [28] and in the previous chapter, the phase angle associated with the antenna rotation is twice the rotation angle.)

As can be seen in Fig. 9 and 10, for the straight wire antenna we have very large and very broadband cross-polarized reflection associated with linear reflection phase vs antenna rotation angle response, and the phase spans the range 0 to $2 \pi$. This means that the straight wire antenna system has all the features desired to construct a beam forming (e.g. axicon) or a beam steering structure.


Figure 10: The diagrams present the relation between phase variations of reflected wave for the straight-wire antenna system (see caption of Fig. 9) with respect to rotation angle for different frequencies. For the frequencies selected the variation is linear.

### 3.2.2: Zeta shaped antenna

This is a similar structure to the straight wire, as it can be considered as a union of three equal straight wires. The single element of Zeta shaped antenna is simulated with periodic boundary condition on the frequency range 4 to 19 GHz . The results for the crosspolarized reflection amplitude and phase response for the different rotation angles in respect to the x-axis are shown in Figs. 11 and 12.


Figure 11: The unit cell of the zeta shaped antenna model. At the left side the numerical results of the reflection amplitude for left -handed circularly polarized generated by right-handed incidence wave. Theta is the angle of rotation between the metallic
antenna and the $x$-axis (for the structure shown $\vartheta=0$ ). The right graph shows the reflection phases for the same cases as is the left side.

The reflection amplitude is higher at the frequencies of $10-16 \mathrm{GHz}$. The investigation is continued for the relation between rotation angle and phase response (see Fig. 12).


Figure 12: The diagrams present the relation of the phase response of the reflected wave for the zeta-shaped antenna (see caption of Fig. 11) with respect to rotation angle theta for the selected frequencies. For the frequencies picked the variation is linear.

The results of Figs. 11 and 12 again show broadband regions of large amplitude (e.g. $12-16 \mathrm{GHz}$ ) and phase linear with the rotation angle, spanning a range 0 to $2 \pi$. These regions though correspond to wavelengths comparable to the structure unit cell, where diffraction may impede the beam steering and beam forming capabilities of the structure.

### 3.2.3: Trapezoid shaped antenna

Another design of scatterer is the trapezoid, which is very similar to a triangle shaped antenna. The antenna is placed on top of the plastic substrate and has thickness values of 36 micrometers $(\mu \mathrm{m})$. The bottom base of the trapezoid is 8.3 mm and the top is 3.2 mm (see Fig 13-top). The length of the side is 5.08 mm . The essential restriction of this structure is eligibility for full rotation around the unit cell center (in the $x-y$ plane).

As can be seen in Fig. 13, the trapezoid is exhibiting high cross-polarized reflection amplitude, unchanged by the rotation, at $6-8 \mathrm{GHz}$. This response is quite narrowband compared to the straight wire and zeta shaped antenna. Moreover, the reflection phase (see Fig. 13 and 14) versus rotation angle lacks the perfect linear response observed in the two previous antenna designs.

Therefore, the trapezoid shaped antenna is inefficient for the purpose of use in metasurfaces.


Figure 13:. The unit cell of the metallic trapezoid shaped antenna model. At the left side the numerical results of the reflection amplitude for left -handed circularly polarized wave generated by right-handed incident wave. Theta is the angle of rotation between the metallic antenna and the x-axis (for the structure shown theta=0). The right graph shows the reflection phases for the same cases as in the left side.


Figure 14:. The diagrams present the relation of the reflected wave phase response for the trapezoid-shaped antenna (see caption of Fig. 13) with respect to rotation angle theta for different frequencies. For the frequencies shown the variation is not totally linear.

### 3.2.4: Bow-tie shaped antenna

The bow-tie shaped antenna is a very well investigated antenna design, especially in the context of plasmonics. The implementation as ultrathin antenna, for unravelling the PB phase shift is investigated here for 2 shapes, one symmetric (see Fig 15 and 16) and another with one side more blunted than the other (see Fig. 18).


Figure 15: Blue print of the bow-tie antenna (yellow) on dielectric substrate (light blue).
The symmetric bow-tie's numerical results are shown at Figure 16 for the reflection coefficient amplitude and phase response, as the antenna is rotated around the unit cell center in the $x-y$ plane.


Figure 16: The front view of the bow-tie antenna in orthogonal position (i.e. theta $=900$ ). The bottom-left panel shows the crosspolarized reflection coefficient over the range of $4-\mathrm{to}-17 \mathrm{GHz}$ for different rotation angles, theta. The right-bottom panel shows the reflection phase response for a selection of rotation angles.

To analyze further the reflection phase vs rotation angle relation, we plot this relation for four different frequencies (see Fig. 17). As can be seen in Fig.17, we have linear relation covering range from 0 to $2 \pi$. The frequency band through where the linear relation is associated with high transmission is quite narrow.


Figure 17: The relation between the reflection phase (see caption of Fig. 15) of the bow-tie element versus the rotation angle for selected frequencies shows a linear phase response.

### 3.2.5: Asymmetric bow-tie shaped antenna

Another shape of the bow-tie antenna is the asymmetrical design with one of the side lengths to be larger than the other. The investigated antenna has the right side length $\mathrm{I}_{\mathrm{R}}=$ 5.3 mm and the left side length $\mathrm{I}=4.33 \mathrm{~mm}$ (see Fig. 18). The base of this bow-tie is 6.86 mm , making able the antenna rotation within the dimension $10 \times 10 \mathrm{~mm}$ unit cell. This design exhibits high cross-polarized reflection amplitude for a broader band in comparison with the symmetric design, as can be seen comparing Figs. 16 and 18.

For the investigation of the reflection phase response with rotation angle the following frequencies are selected: $6.19,8.53,9.805$ and 16 GHz . The results are shown in Fig. 19, demonstrating a linear phase response covering a $2 \pi$ range.


Figure 18: The unit cell of the asymmetric bow-tie antenna (yellow color) on a plastic substrate (light blue color) is shown on the top-left panel (for the structure shown $\vartheta=0$ ). The amplitude of the cross-polarized reflected wave over the selected frequency range (bottom-left panel) and its phase response (bottom-right panel) for different rotation angles. At the bottom-left panel is shown the numerical results of the reflection amplitude for left -handed circularly polarized wave generated by a right-handed incidence wave, for different rotation angles (theta) between the metallic antenna and the $x$-axis. Bottom-right panel shows the corresponding to the left-panel phase responses.


Figure 19: The diagrams show the relation of the reflection phase response for the asymmetric bow-tie antenna (see caption of Fig. 18) with respect to rotation angle for different frequencies. For the frequencies selected the variation is linear.

### 3.2.6: Rectangular center split-ring - Hammer

Here we investigate the antenna design shown in Fig. 20. It resembles a classic split-ring resonator [29]. The start of this investigation is by simulating the structure (calculating the reflection) for many different rotation angles with respect to the structure optical axis parallel to the $z$-direction.


Figure 19: Blueprint figure of the rectangular center split-ring resonator (yellow color) on top of the dielectric substrate (light blue color).

The rectangular center split-ring (electrical equivalent of split-ring resonator) is of $8 \times 5 \mathrm{~mm}$ dimension on the plane (see Fig. 20), the metal width is 1 mm and thickness 36 $\mu \mathrm{m}$. The plastic thickness in the simulation is 3.1 mm - the corresponding results are shown at Fig. 21 - and 5.1 mm - the corresponding results are shown in Fig. 23. The simulation is applied to all the selected rotation angles with respect to the $z$-optical axis of the geometry.


Figure 20: Perspective view of the structure's unit cell (for the structure shown theta=0.). At the left-bottom side of the panel the numerical results of the reflection amplitude for left -handed circularly polarized wave generated by right-handed incident wave. The angle of rotation, theta, for the structure shown is theta=0. The right-bottom graph shows the reflection phase responses for the same cases as in the left graph.

The results of the reflection coefficient for the cross-polarized wave are shown at the bottom panels of figure 21 for board thickness 3.1 mm . The outcome shows high amplitude for the frequencies $4-14 \mathrm{GHz}$ and phase response appropriate for the construction of a supercell with linear phase gradient.


Figure 21: The diagrams present the relation between phase shifts of reflected wave (see caption of Fig. 21) with respect to rotation angle for different frequencies. For the frequencies picked the variation is linear.

To illustrate the linear phase with rotation angle response we plot in Fig. 22 the phase response relation with rotation angle for different frequencies relevant for the construction of a supercell for a beam steering and manipulation (see Chapter 4 and 5).

Another simulation as the above was performed for the same shape antenna but on a substrate thicker by 2 mm , i.e. the substrate thickness now is 5.1 mm (the rest of the geometrical characteristics of the unit cell remain the same). The corresponding results for the cross-polarized reflection amplitude and phase for different rotation angles of the structure are shown in Fig. 23.

The reflection coefficient now appears to have a broad peak between $4-6 \mathrm{GHz}$ and two narrower ones, around 10 GHz and 12 GHz . The phase response at the peaks seem to have linear dependence on the rotation angle, something that is verified further in Fig. 24 , where the reflection phase versus rotation angle is shown for selected frequencies.

A further investigation of the current antenna will be presented in the next chapter, where the antenna will be employed for a construction of a reflect-array presenting a linear phase gradient.


Figure 22: The structure of the rectangular center split-ring antenna is shown on the top right panel for theta= $0^{\circ}$ for the case of the substrate thickness of 5.1 mm . The cross-polarized reflection amplitude (bottom-left graph) and phase response (bottom-right graph) for left-handed circularly polarized wave generated by right-handed incident wave for different rotation angles (theta) are also shown.


Figure 23: The four diagrams show the relation of the reflected wave phase response versus rotation angle for the structure of Fig. 23, for different frequencies. For the frequencies selected the variation is linear.

### 3.2.7: Rectangular double split-ring

Here we present results for the last antenna model targeting high cross-polarized reflection amplitude and linear phase response which is investigated on this thesis. It is a split-ring with similar dimensions with that of the previous section, but with two splits. A picture of the antenna with associated geometrical parameters is shown in Fig. 25. The Frequency Solver is applied over the model for 6 angles of rotation with respect to the zaxis (or the optical axis). The angles are 00, 30o, 60o, 90o, 120o, 150o. The results for the cross-polarized reflection amplitude and phase are shown in Fig. 25.


Figure 24: The unit cell of the rectangular double split-ring antenna (yellow color) on substrate thickness (light blue) of value 3.1 mm . At the bottom-left side the numerical results of the reflection amplitude for left -handed circularly polarized wave generated by right-handed incidence wave are shown. Theta is the angle of rotation between the metallic antenna and the $x$-axis (for the structure shown $\vartheta=0$ ). The bottom-right graph shows the reflection phases for the same cases as is the left side.

Further step is to search for frequencies with linear phase relation with rotation angle and simultaneously high amplitude of opposite handedness polarization. In Fig. 25 we observe high cross-polarized reflection amplitude from 7-14 GHz, which remains almost unchanged as we change the antenna rotation angle.

Observing the phase versus rotation angle results for all the range of frequencies from $4.5-16.5 \mathrm{GHz}$ (see Fig. 25, bottom-right graph) we see that the rectangular double split-ring is exhibiting linear relation between phase shift and rotation angle. This is verified in Fig. 26 for selected frequencies.

As a final step we investigated the potential of a thicker structure to increase the frequency band of high reflection coefficient and linear phase response. The results for a structure of board thickness 5.1 mm are shown in Figs. 27 and 28. The results of Fig. 27 and 28 show a broad-band high cross-polarized reflection response, observed at lower frequencies, and associated with a linear phase vs rotation angle dependence. Moreover, the reflection amplitude remains almost unchanged by the structure rotation and the phase spans almost a $2 \pi$ range by rotating the antenna.


Figure 25: The diagrams present the relation between phase of reflected wave and rotation angle for the structure described in connection with Fig. 25, for different frequencies. For the frequencies selected the variation is linear.


Figure 26: The unit cell of the rectangular double split-ring antenna (yellow color) on substrate (light blue) of thickness 5.1 mm. At the bottom-left side the numerical results of the reflection amplitude for left -handed circularly polarized wave generated by right-handed incidence wave are shown. Theta defines the rotation angle between the metallic antenna and the x-axis (for the structure shown $\vartheta=0)$. The bottom-right graph shows the reflection phase for the same cases as is the left side.


Figure 27: The diagrams present the relation between phase responses of reflected wave and rotation angle for the structure discussed in connection with Fig. 27 for different frequencies. For the frequencies selected the relation is linear.

All the reflection characteristics of the rectangular double split-ring structure indicate that the structure is appropriate for the construction of beam shaping and steering device; as we will show also in the next chapters.

For the seven shaped thin antennas discussed above we calculated the rotation angles required to construct a reflect array of anomalous reflection and an axicon. The corresponding calculations and the results are presented in chapter 4 and chapter 5 .

## Chapter 4: Metasurface for linear phase gradient

The current chapter is focused on the demonstration of anomalous reflection using a linear phase gradient from the antenna designs studied in the previous chapter. For each design we create a structure (reflect-array) of unit cell made by six elements, which have 0 -to-m rotation, necessary to produce reflection phase control over the range 0 -to- $2 \pi$. The selected elements of the unit cell are arranged following the diagrams of phase response relation with the rotation angle, which are shown in the previous chapter.

### 4.1 Straight wire design

The first antenna design employed is the straight wire design. The corresponding six element unit cell (supercell) is shown in Fig. 29, where one can see six equally spaced elements on the plastic substrate, terminated by a back reflector (not visible in the figure).


Figure 28: Front view of six elements of straight wire unit cell forming the reflect array. The wires (yellow color) are placed on a plastic substrate (light blue color) of thickness 3.1 mm and permittivity 4.2, backed by back-reflector. The geometrical parameters of the wires are shown in Fig. 8.


Figure 29: The graph shows the cross-polarized reflection coefficient over the range of selected frequencies for right-handed circularly polarized incident wave on the structure shown in Fig.29. It is shown small cross-polarized reflection. The substrate thickness value is 3.1 mm

The range of frequencies analyzed is from $4-\mathrm{to}-19 \mathrm{GHz}$, where the cross-polarized reflection of the supercell has been calculated, for circularly polarized waves. The results are shown in Fig. 30.

As can be seen in Fig. 30, the supercell shows a very small reflection with narrow band peaks. The highest reflection is at 6.85 GHz . The field at the frequency 6.85 GHz is shown in the figure below (Fig. 30), for a normally incident circularly polarized wave.


Figure 30: Plot of the electric field amplitude (contour bar) in the x-z plane crossing the center of the unit cell in the $y$-direction, at frequency 6.85 GHz . The incident wave is right-handed circularly polarized. It appears a reflection angle which is described by the generalized Snell's law. The black arrow show the direction of the incident field and red arrow the reflected field direction.

In the graph of the electric field (Fig. 31), the angle of reflection is approximatly $\Theta_{\mathrm{r}}=$ 38.35 degrees, as it is calculated by the generalized form of Snell's law. We will revisit the straight wire in a relation with the axicon model in the following chapter, for further research.

Under the principle of PB phase shift the geometrical parameters of the antenna must remain the same for all array's elements. A feature that can change is the substrate thickness. In Figure 32 the results of the cross-polarized reflection are presented for different substrate thickness values.


Figure 31: The diagram make a comparison of the cross-polarized reflection coefficient for the structure of Fig. 29, for two substrate thickness values, 3.1 mm and 5.1 mm .

As it can be seen in Fig. 32 the supercell of the straight wire reflect-array is not capable to provide strong radiation for cross-polarized reflected waves.

### 4.2 Bow-tie antenna

### 4.2.1: Symmetric bow-tie design

The next structure creates a linear phase gradient from the bow-tie antennas, with geometrical properties as discussed in chapter three. The construction of bow-tie based supercell starts by replicating the bow-ties and the substrates and the properly rotating the bow-ties. The supercell of the elements is shown in figure 33, which leads to a linear phase gradient.


Figure 32: Front view of the supercell of bow-tie antennas operating as beam steerer. The geometrical parameters of the antennas are mentions in section 3.2.4. The bow-tie antennas (yellow color) are placed on a plastic substrate of thickness 3.1 mm and permittivity is 4.2 (light blue color), backed by back-reflector.

It should be mentioned that the bow-tie supercell doesn't show high crosspolarization reflection amplitude over any range of frequencies for any substrate thickness. This is confirmed in Fig. 34 where we show the cross-polarized reflection for two different substrate thickness values.


Figure 33: The graph of cross-polarized reflection coefficient for the bow-tie antenna supercell of Fig. 33, for two substrate thickness values, 3.1 mm and 5.1 mm .

Moreover, the PB phase shift is not following the effect observed in the V -shaped antennas [9], where the neighbor currents coupling enhance the anomalous phenomena. This is shown in figure 35, which compares the structure of Fig. 33 with one other model with vertical neighbor distance (along y) increased by 2 mm . The graph shows a broad peak at frequencies near 16 GHz for the supercell of the newer structure, showing that reducing the coupling favor the cross-polarized reflection.


Figure 34: The diagram shows the cross-polarized reflection amplitude for two bow-tie structures. The red dots represent a structure of rectagular unit cell with vertical (y) dimension increased by 2 mm compared to black dots. The value of substrate thickness is 3.1 mm for both examined structures.

### 4.2.2: Asymmetric bow-tie design

This section is focused on the asymmetric bow-tie supercell, which is constructed by the bow-tie design discussed in the section 3.2.5 of the previous chapter. As discussed in the previous chapter, the single element exhibits a broad frequency range with high cross polarization reflection amplitude, over all rotation angles. Figure 36 is a front view of the bow-tie supercell, showing the orientation of each element. Following the calculations of the previous chapter, the arrangements start from the element with the smallest phase shift, followed by elements rotated as to produce a linear phase gradient.


Figure 35: The front view of the asymmetric bow-tie shaped antenna supercell showing a linear cross-polarized reflection phase gradient. The geometrical parameters of the antennas are presented in the previous chapter (see section 3.2.5).

The numerical results of the cross-polarized reflection for the supercell shown in the following Figure 37 and the fields, which are exported from CST for the visual
representation of the reflection angle are shown in Figure 38, for frequency 10.3 GHz and substrate thickness 3.1 mm .


Figure 36: The reflection coefficient results for the structure of Figure 36 over the selected range of frequencies. The diagram makes a comparison of the cross-polarized reflection coefficient for two substrate thickness values, 3.1 mm and 5.1 mm .


Figure 37: The electric field amplitude in the x-z plane at the frequency of 10.3 GHz for the asymmetric bow-tie based reflect-array. The incident wave is right-handed circular polarized. The black arrow shows the direction of the incident field.

In Fig. 38, the electric field cannot reveal the reflection angle of the cross-polarized wave, partly due to the fact that there is also significant co-polarized reflection component, and also because the plot includes also the incident wave.

### 4.3 Rectangular center split-ring

This section investigates the array of rectangular center split-rings, which were investigated in section 3.2.6 of the previous chapter. The supercell of the structure is constructed by six geometrical identical antennas with rotation step angle 300 and is shown in Figure 39. The cross-polarized reflection coefficient for the array of Fig. 39 for two different values of substrate thickness is shown in Figure 40.


Figure 38: Front view of the six elements of rectangular center split-ring supercell cell forming the reflect-array. The antennas (yellow color) are placed on a plastic substrate (light blue color) of thickness 3.1 mm and permittivity 4.2, backed by back-reflector. The geometrical parameters of the antennas are shown in Fig. 20.


Figure 39: The diagram makes a comparison of the cross-polarized reflection amplitude for the structure of Fig. 39, for two substrate thickness values, 3.1 mm (black dots) and 5.1 mm (red dots).

As can be seen in Fig. 40, the increase of the substrate thickness leads to no improvement of the reflection amplitude. Following the results of the above graph, we plot in Fig. 41 the electric field amplitude at frequency 14.2 GHz for substrate thickness 3.1 mm.


Figure 40: Plot of the electric field amplitude (contour bar) in the $x$-z plane crossing the center of the unit cell in the $y$-direction at 14.2 GHz for the supercell of the rectangular center split-ring structure shown in Fig. 39. It appears a reflection angle of 17.420,
which is described by the generalized Snell's law. The black arrow shows the direction of the incident field and red arrow the reflected field direction. The substrate thickness value is 3.1 and in the picture shows two supercells of six antennas each.

From Fig. 41 it can be seen an anomalously reflected wave at an angle of 17.42 degrees from the incident wave axis (normal incidence). This result is consistent with what is obtained by applying the generalized Snell' law in the case of this array.

### 4.4 The rectangular double split-ring

The last reflect-array discussed in this chapter is a reflect-array made of double rectangular split-ring antennas like the ones presented in section 3.2.7. We constructed the reflect-array by placing the six antennas with a rotation angle step of 30 degrees. The resulting supercell is shown in Fig. 42. The phase shift of the cross-polarized reflected wave by the reflect-array supercell has linear relation with the rotation angle, therefore its reflection angle should be found from the generalized form of Snell's law (eq. 2). To identify the frequencies where we have large cross-polarized reflection by the reflectarray, we calculated the reflection from the supercell. The result is shown in Fig. 43.


Figure 41: Front view of the supercell of rectangular double split-ring antennas (yellow color); the rotation angles, from the left, are $00,300,60 \mathrm{o}, 90 \mathrm{o}, 120 \mathrm{o}, 150 \mathrm{o}$. The antennas are placed on a plastic substrate (light blue color) of thickness 3.1 mm and permittivity 4.2, backed by a back-reflector. The geometrical parameters of the wires are shown in Fig. 25 (see section 3.2.7).


Figure 42: The rectangular double split-ring antennas (yellow color) are placed on a plastic board (light blue color) of thickness 3.1 mm and permittivity 4.2, backed by metallic back-reflector. The right graph shows the cross-polarized reflection for the supercell of six elements (black symbols), shown on top-left graph, and the expanded supercell, shown on bottom-left graph, which has an added-space in the $x$-direction made only by board and back reflectors (red symbols).

In fact, Figure 43 shows the cross-polarized reflection for two different structures, the supercell of which are shown on the left side. The second structure supercell is created by the first one (the one of Fig. 42) with the addition of extra space along the horizontal ( $x$ ) direction (this extra space is to reduce the coupling between neighboring unit cells in the horizontal direction). One can see that the second structure presents narrower reflection and overall minimal change in the reflection amplitude compared to the first (original) one.

Therefore, for the demonstration of anomalous reflection we employ the regular supercell, i.e. the one shown in Fig. 42. For this supercell Figures $44-46$ show the electric field amplitude for three different frequencies, 5 GHz (Fig. 44), 6.8 GHz (Fig. 45) and 9.8 GHz (Fig. 46), revealing the anomalous reflection of a normally incident wave. The reflection angle in each case is in agreement with the angle which can be calculated from the generalized Snell's law, eq. (2).


Figure 43: Contour plot of the electric field amplitude for the supercell of Fig. 42 at the frequency of 5 GHz for nomally incident left-handed cicularly polarized wave. The field is shown at a x-z plane of the system. The system depicted here consists of 2 supercells along the $x$-direction (for better field visualization; the supercell is marked with black outlined ). The reflection angle derived from eq. (2) is 58.2 degrees.


Figure 44: Same as Fig. 44 for frequency of 6.8 GHz . The reflection angle for the cross-polarized wave (marked with the red arrow) is 38.68 degrees. The black arrow is the incident wave, which is nomally normally incident left-handed cicularly polarized wave. The field is shown at a x-z plane of the system.


Figure 45: Contour plot of the electric field amplitude for the supercell in Fig. 42 at the frequency of 9.8 GHz . The cross-polarized reflection angle here is 25.7 degrees and is marked with the red arrow, while the black arrow marks the direction of the incident wave (nomally incident, left-handed cicularly polarized wave). The field is shown at a x-z plane of the system. The system depicted here consisted of 2 supercells along the $x$-direction (for better field visualization; the supercell is marked with black outline).

The final issue that we wanted to examine here is how the cross-polarized reflectance depends on the substrate thickness of the antennas. For that we calculated the reflectance for two different substrate thickness values, 5.1 mm and 3.1 mm . The results are shown in Fig. 47.


Figure 46: Cross-polarized reflection coefficient for structure of Fig. 42, for two substrate thickness values: 3.1 mm and 5.1 mm .
From Fig. 47 it can be seen that the reflection coefficient is decreased by increasing the values of thickness of the substrate. It should be mentioned that for the thicker substrate the position of the antennas with different rotation angles is different from the one shown in Fig. 42 . Fig. 48 shows the new arrangement in the supercell, where orientation angles in respect to the $x$-axes are (from left to right) $0 \mathrm{o}, 120 \mathrm{o}, 150 \mathrm{o}, 90 \mathrm{o}, 30 \mathrm{o}$ and 600 .


Figure 47: Front view of the supercell of rectangular double split-ring antennas (yellow color) with rotation angles (from left to right) $00,1200,1500,900,300$ and 600 . The antennas are placed on a plastic substrate (light blue color) of thickness 5.1 mm and permittivity of 4.2 backed by a back-reflector. The geometrical parameters of the antennas are shown in Fig. 27 (see section 3.2.7).

The electric field amplitude for the thicker substrate case and for two frequencies associated with high cross-polarized reflection amplitudes, 5.8 GHz and 8.7 GHz , are shown in Figs. 49 and 50, respectively. The colorbar is representing the intensity of the electric field in units of Volt per meter.


Figure 48: The electric field for the supercell of Fig. 48 (substrate thickness value 5.1 mm ) at the frequency of 5.8 GHz , in the $\mathrm{x}-\mathrm{z}$ plane crossing the center of the unit cell in the $y$-direction. The black arrow shows the direction of the incident field and the red arrow the reflected field direction. The reflection angle derived from eq. (2) is 47.11 degrees. Here the field is depticted in two structure supercells along x-direction.

In Fig. 49, the reflection angle of the electric field plot is 49.11 degrees, which is predicted also from the generalized law of reflection (eq. 2). In Fig. 50 the cross-polarized reflected wave is propagating at an angle of 29.25 degrees.

With the rectangular double split-ring antenna we concluded the study of the reflect-arrays showing linear phase gradient that are suitable for beam steering. In the next chapter we combined two mirror-image linear gradient supercells to create axicons for Bessel beam generation.


Figure 49: The electric field for the structure of Fig. 48 (substrate thickness value 5.1 mm ) at the frequency of 8.7 GHz , in the x-z plane crossing the center of the unit cell in the $y$-direction. The black arrow shows the direction of the incident field and red arrow the reflected cross-polarized field direction. It appears a reflection angle ( $\Theta r$ r) which is described by the generalized Snell's law and its value is 29.25 degrees. Here the field is depticted in one structure supercell along $x$-direction.

## Chapter 5: Metasurface for the axicon-like model

This chapter presents the investigation of axicons constructed by two linear phase gradients, one being the mirror image of the other in the $y-z$ plane. The starting element at each axicon supercell is the antenna with the exhibited lower level phase shift. The supercells are constructed appropriately in order to present the ability of focusing the electromagnetic wave, and to convert a plane wave into a Bessel-like beam (only in the $x-z$ plane). In fact, the Bessel-like beam is formed by the cross-polarized reflected wave. It is expected that the axicon provided by the reflective surface produces an on-axis sharp image regardless the distance of the image plane [27].

### 5.1 Straight wire

The first antenna employed for the construction of the axicon reflect-array is the straight wire. The front view of this straight wire-based reflect-array supercell is presented in Figure 51, with each element having different rotation angle around to its z optical axis (OA). The supercell is made to have almost twice the x-size of the previous supercell, i.e. it is created by two linear phase gradients. They are placed mirror-symmetrically in respect to the y-z plane and are used to focus the incident EM waves only in the x-z plane (the supercell is repeated along y-direction, resulting to translational symmetry along y).


Figure 50: The supercell of the reflect-array created by straight wire antennas (yellow color) and targeting axicon operation in the $x-z$ plane. The geometric parameters of the antennas are presented in Fig. 8. The antennas are placed on a dielectric board (light blue color) of the permittivity is 4.2, which is terminated by a back-reflector.

Fig. 52 shows the cross-polarized reflection amplitude for the axicon structure of Fig. 51 for two different substrate thickness values, 3.1 mm and 5.1 mm . For substrate thickness 5.1 mm one can see a high peak near the frequency 9.4 GHz , where the reflection exceeds 0.6 . The cross-polarized fields for the structure are exhibiting a focusing, showing a wave behavior analog to a Bessel function. This focusing and beam collimation for three different frequencies is demonstrated in Figs. 53, 54 and 55.


Figure 51: Cross-polarized reflectance for the axicon structure of Fig.51, for two different dielectric board thickness values, 3.1 mm (black symbols) and 5.1 mm (red symbols). The incident field is left-handed circularly polarized.


Figure 52: Electric field amplitude (color) for the axicon structure of Fig. 51 at frequency of 9.3 GHz (for a board thickness 3.1 mm ). The field shows a focusing at a distance of few millimeters from the reflect-array surface.


Figure 53: Electric field amplitude for the axicon structure of Fig. 51 at frequency 12.4 GHz (for a board thickness 3.1 mm ). The field shows a focusing at a distance of few millimeters from the axicon reflect array.


Figure 54: Electric field amplitude (color) for the axicon structure of Fig. 51 at frequency 8.9 GHz (for a board thickness 3.1 mm). The field shows a focusing at a distance of few millimeters from the reflect array.

### 5.2 Bow-tie shape

### 5.2.1: Symmetric bow-tie shape

The next axicon design which we examined is composed of bow-tie antennas, like the one discussed in section 4.2. It is in fact the design of Fig. 33 combined with its mirror image along the x-direction. The design is shown in Fig. 56.


Figure 55: Front view of the supercell created by bow-tie antennas (yellow color) and targeting axicon operation in the x-z plane. The geometric parameters of the antenna are presented in Fig. 15. The antennas are placed on a dielectric board (light blue color) with permittivity 4.2, which is backed by a back-reflector.

The cross-polarized reflectance for this axicon design is shown in Fig. 57 for two different substrate thickness values, 3.1 mm and 5.1 mm .


Figure 56: The cross-polarized reflectance for the axicon-like structure of Fig.56, for two different dielectric board thickness values, 3.1 mm (black symbols) and 5.1 mm (red symbols).

Comparing Fig. 57 with the corresponding results for linear gradient (i.e. half axicon), Fig. 34, one can see different resonances and in general increase of reflection coefficient. The most probable reason for these effects is a constructive interference of the waves reflected by the two mirror-imaged phase gradients composing the axicon.

### 5.2.2: Asymmetric bow-tie shape

Investigating the axicon made of the asymmetric bow-tie antennas we found a broad frequency of high amplitude cross-polarize reflection. The front view of this asymmetric bow-tie axicon design (its unit cell) is shown in Figure 58 with axis view the $x-y$ plane


Figure 57: The supercell of the reflect-array created by asymmetric bow-tie antennas (yellow color) and targeting axicon operation in the $x$-z plane. The geometric parameters of the antenna are presented in Fig. 18. The antennas are placed on a dielectric board (light blue color) of permittivity 4.2, which is terminated by a back-reflector.


Figure 58: The cross-polarized reflection amplitude for the axicon-like structure of Fig.58, for two different dielectric substrate thickness values, 3.1 mm (black symbols) and 5.1 mm (red symbols).

In Fig. 59 we show the cross-polarized reflectance for the asymmetric bow-tie axicon for substrate thickness 3.1 mm and 5.1 mm , and in Fig 60 we compare the crosspolarized reflectance for symmetric and asymmetric bow-tie axicon for the thicker substrate case. In Fig. 60 it can be seen that the asymmetric bow-tie axicon design is more efficient in a broader range of frequencies than the symmetric bow-tie one. In fact, the asymmetric bow-tie design produces a shift on the peaks in reflection compared to the symmetric shape, which operates better in higher frequencies.


Figure 59: The comparison of the cross-polarized reflection amplitude for the axicon-like structures of Fig. 56 and 58, for substrate thickness 5.1 mm (the graphs are copied from Figs. 57 (red symbols) and 59 (black symbols).

### 5.3 Rectangular center split-ring

Next axicon design, which is analyzed here is the one based on rectangular center splitrings like the ones discussed in section 4.3 of the previous chapter. The design is presented at the Fig. 61, comprising elements with rotation angle from 0-to-m. As in the previous cases the design here is also constructed combing two linear phase gradient structures (in the x-direction), one the mirror image of the other


Figure 60: The axicon-like supercell of the reflect-array created by rectangular center split-ring antennas (yellow color) and targeting axicon operation in the x-z plane. The geometric parameters of the antenna are presented in Fig. 20. The antennas are placed on a dielectric board (light blue color) of permittivity 4.2, which is terminated by a back-reflector.

The cross-polarized reflection coefficient for the structure of Fig. 61 with different substrate thickness values is presented in Fig. 62, showing high amplitude cross polarization in a broad range of frequencies.


Figure 61: The cross-polarized reflection amplitude for the axicon-like structure of Fig.61, for two different dielectric substrate thickness values, 3.1 mm (black symbols) and 5.1 mm (red symbols).

In Figs. 63, 64 and 65 we show the electric field amplitude for selected frequencies associated with high amplitude cross polarization reflection.


Figure 62: Electric field amplitude for the axicon structures (two supercells are shown) of Fig. 61 at frequency of 7 GHz (for a board thickness 3.1 mm ). The field shows a focusing at a distance of few millimeters from the reflect-array surface

As can be seen in Fig. 63 the electric field plot shows a focusing behavior. The focusing appears at 20 mm away from the structure, with a rather wide focal point. A similar focusing behavior can be observed also in Fig. 64.


Figure 63: Electric field amplitude for the axicon structure of Fig. 61 at frequency of 6.4 GHz (for a board thickness 3.1 mm ). The field shows a focusing at a distance of few millimeters from the surface of the reflect-array.

While Figs. 63 and 64 concern a structure with board thickness 3.1 mm for Fig. 65 the substrate thickness value is 5.1 mm .


Figure 64: Electric field amplitude for the axicon structure of Fig. 61 at frequency of 8.5 GHz (for a board thickness 5.1 mm ). The field shows a focusing at a distance of few millimeters from the reflect-array.

### 5.4 Rectangular double split-ring

The last structure examined in this thesis is the axicon made of the rectangular double split-rings discussed in section 4.4. This axicon supercell is shown in Fig. 65.


Figure 65: The supercell of the reflect-array created by rectangular double split-ring antennas (yellow color) and targeting axicon operation in the x-z plane. The geometric parameters of the antenna are presented in Fig. 20. The antennas are placed on a dielectric substrate (light blue color) of permittivity is 4.2, which is backed by a back-reflector.

The simulated cross-polarized reflection coefficient for this axicon structure is presented in Fig. 66, over the range of frequencies $4-$-to- 12 GHz , for two different board thickness values.


Figure 66: The cross-polarized reflection amplitude for the axicon-like structure of Fig.65, for two different dielectric board thickness values, 3.1 mm (red symbols) and 5.1 mm (black symbols).

The electric field in a xz plane at frequency 5 GHz for the rectangular double splitring axicon is shown in Fig. 67. The field magnitude is reaching values of ten to the firth $\left(10^{5}\right)$. The focusing appears at a distance around 20 mm from the structure surface.


Figure 67: Electric field amplitude for the axicon structure of Fig. 65 at frequency of 5 GHz (for a board thickness 5.1 mm ). The field shows a focusing at a distance of few millimeters from the reflect-array.


Figure 68: Electric field amplitude for the axicon structure of Fig. 65 for a board thickness 5.1 mm for the frequencies of 6.4 GHz (top left), at 5 GHz (right top) and at 7.8 GHz (buttom figure). The field shows a focusing appearance at a distance of few millimeters from the reflect-array.

Figure 68 compares the 5 GHz field plot with field plots at two different frequencies, 6.2 GHz and 7.8 GHz , for the rectangular double split-ring resonator-based reflect-array axicon. As can be seen in Fig. 68, at 6.2 and 7.8 GHz the focal point of the produced Bessel-like beam is clearer that that observed for 5 GHz .

## Concluding remarks

This thesis investigates the behavior of GHz metasurfaces made by metallic antennas of different shapes: straight-wire, zeta-shaped, trapezoid-shaped, bow-tie-shaped, and split-ring-like. We focused on the potential of those antennas to create linear phase gradient metasurfaces (exploiting the Patcharatnam-Berry phase shift) leading to anomalous reflection and axicon-like operation.

We started by investigating metasurfaces made of a single antenna per unit cell. The antenna was placed on a plastic board terminated by a metal back reflector. We examined the amplitude and phase of the cross-polarized reflected wave (for circularly polarized incident wave) as we rotate the antenna. The target was to achieve high crosspolarized reflectance at low frequencies, with reflection amplitude remaining constant and phase linearly changed by the antenna rotation. The best performance regarding large cross-polarized reflection amplitude was found for the straight-wire antenna, while the lowest operation frequency was found for the antennas of split-ring-like shapes.

Furthermore, we examined systems made of six antennas per unit cell (supercell), with the successive antennas relatively rotated as to create a linear gradient in the phase of the cross-polarized reflected wave. The target was to achieve controllable anomalous reflection for normally incident circularly polarized waves, according to the generalized Snell's laws of reflection and refraction. In the six-antenna unit cell systems, due to the interference between the relatively rotated antennas in the unit cell, the crosspolarized reflection amplitude does not show the very large values and broadband behavior observed in the single antenna unit cell systems. We found though frequency regions of high cross-polarized reflection amplitude in almost all the systems studied and we exploited them to demonstrate anomalous reflection, with reflection angle in agreement with the generalized Snell's law of reflection. Among the different shapes of antennas studied, the best performance, regarding broadband response, was found for the split-ring-like shapes.

Joining two identical supercells with linear phase gradient, with one being the mirror-image of the other, it results to another supercell operating as one-dimensional axicon-like device. Such structure presents the ability to create reflected waves of the form of non-dispersive Bessel beams (in the normal-incidence plane). Using this approach and employing our antennas, we examined the possibility of efficient Bessellike beam generation. Among the different antenna shapes studied, the most efficient Bessel-like beam generation was obtained again for the split-ring-like shapes, while the other shapes gave small or narrow-band cross-polarized reflectance. The superior performance of the split-ring-like shapes has its origin to the more subwavelength response of this type of antennas (compared to the other shapes studied).

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