Bulk viscosity in effective holographic models with asymptotically exponential potentials

Master Thesis

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Abstract

Numerical and holographic models for strongly coupled systems indicated a rise in the vicinity of the confinement – deconfinement phase transition of the quark gluon plasma. We investigate the behaviour of bulk viscosity in holographic models that are 4+1 dimensional Einstein – dilaton gravity theories with potentials that are asymptotically exponential in the IR. We find that in such models where the phase transition is continuous, the bulk viscosity / entropy density ratio has a maximum just above a critical temperature and remains finite.

Acknowledgements

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Contents

Chapter 1

Introduction

The bulk / boundary correspondence [1] is a useful tool for doing calculations in strongly coupled theories where one relates string theory in a higher dimensional space to a quantum field theory defined on its boundary. The approaches to the correspondence can be classified as bottom-up or top-down. Bottom-up holographic models are approaches to the bulk / boundary correspondence that employ an effective theory to represent the dynamics of the string theory living in the bulk. While in the original top-down approach [1], and many other models inspired by it, one starts building the model from the string theory side constructing brane embeddings etc. such that the boundary theory has the desired features, in bottom up models one does not worry about the brane embeddings that are not necessarily simple for realistic models. Instead, one chooses the dynamical bulk fields and their effective potentials suitably so that the bulk theory captures the properties of the boundary theory. First examples of such bottom-up models are intended for QCD and they are known as AdS/QCD. These models [2–4], with the advantage of being phenomenologically tunable, successfully match QCD experimental data.

Improved Holographic QCD (IHQCD) [5], is another application of the bottom-up approach to QCD. It is a 5 dimensional Einstein - dilaton gravity theory with the dilaton potential appropriately chosen to match the β function of the boundary theory to QCD. Application of this type of models to other strongly coupled systems such as condensed matter systems is also possible. For d dimensional strongly coupled systems a minimal bottom up model without any gauge fields would be a $d+1$ dimensional theory of gravity with a scalar that can be the dilaton or another scalar coming from the bulk string theory.

Holographic models of systems at finite temperature are typically constructed with bulk geometries involving a black hole and their thermodynamics are mainly determined by the near horizon asymptotics of the background. In bottom-up models with an effective potential, the relevant asymptotics of the scalar effective potential are therefore directly related to the thermodynamical properties of the theory. This direct relation allows a classification of potentials with respect to their thermodynamical properties.

For strongly coupled plasmas, especially for QGP, an interesting quantity is the bulk viscosity. One can talk about two viscosities for a fluid, namely bulk and shear viscosities. They are defined in the following way. The general viscous stress tensor of a three dimensional fluid has the following form:

$$\sigma_{ij} = \eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\partial\cdot v\right) + \zeta\delta_{ij}\partial\cdot v \qquad (1.0.1)$$

The coefficient η is the shear viscosity while ζ is the bulk viscosity. It has been shown in [6] that the shear viscosity η in strongly coupled theories with black hole - AdS gravity duals is universal and equal to $\frac{1}{4\pi}$. Note that the coefficient of η in (1.0.1) is traceless. Bulk viscosity vanishes for incompressible fluids or in a conformal field theory, which is approximately the case for high temperature QCD. The gold collision experiments at RHIC showed that the QGP behaves like very close to a perfect fluid [7–10]. However at lower temperatures, just above the QCD phase transition, a rise in bulk viscosity is expected.

In QGP at temperatures $T \gtrsim T_c$, and any other strongly coupled fluid, transport coefficients as well as other quantities cannot be evaluated in perturbation theory. Therefore one needs to consider non perturbative methods such as numerical simulations or holography. The authors of [11] and [12] calculated the bulk viscosity from the low energy theorems of QCD supported by lattice data. In [12] it has been argued that the bulk viscosity should diverge at QCD confinement – deconfinement phase transition.

On the other hand, an holographic method for the calculation of transport coefficients has been developed in [13] and applied to theories with 4+1 dimensional Einstein - dilaton gravity duals. There, it has been shown that in a QCD-like holographic model the bulk viscosity rises near the phase transition but remains finite. Following the holographic method of [13], in [14] it was found out that the bulk viscosity rises just above the critical temperature in

IHQCD as well. In another holographic computations for the bulk viscosity which is for the the $\mathcal{N} = 2*$ plasma [15] and the cascading plasma [16] the bulk viscosity has been observed to rise near critical temperature and remain finite. This was also the case in [13] for a QCD-like bottom-up model. However in the gauge theory dual of exotic black holes the bulk viscosity diverges [17].

In this work we calculate the bulk viscosity in effective dilaton potentials $V(\Phi)$ with exponential large Φ asymptotics: $\sim e^{a\Phi}$. Such cases are generic in supergravity and they allow power solutions that have simple scalings. Among the theories we consider and have a phase transition, one can identify two types according to the bulk geometry at the critical temperature.

In the first type that corresponds to the marginal $a = \frac{4}{3}$ theories the relative free energy becomes zero as the black hole size shrinks to zero. In the second type ($a > \frac{5}{3}$), the geometry with zero relative free energy has finite black hole size. This is similar to IHQCD where the potential is of the form $\approx e^{\frac{4}{3}\Phi} \times$ powers of Φ and the phase transition occurs at some finite T other than the minimum temperature. For both types of theories, the bulk viscosity has a maximum just above the critical temperature and does not diverge.

The organisation of the text is as follows. Firstly, we review 4+1 dimensional Einstein - dilaton gravity and its equations of motion in section 2. We present the technique of scalar variables that is used throughout the text. We conclude this section by giving formulae for thermodynamical quantities expressed in terms of the scalar variables.

In section 3 we state the class of potentials we are interested in. We give the asymptotic behaviour of the geometry and thermodynamical quantities and make a classification based on the asymptotics of the temperature as the black hole horizon goes far away from the boundary. For a marginal subclass where the temperature asymptotes to a constant, the thermodynamical quantities depend on the subleading terms of the potential [18].

Finally, in chapter 4 we present results for bulk viscosity in the backgrounds reviewed in section 3. We review the analytical computation of the bulk viscosity in backgrounds where $V(\Phi) = e^{a\Phi}$ along with a numerical computation for a possible AdS completion of it. We conclude by a study of bulk viscosity concentrated on the marginal subclass of IR asymptotics.

Chapter 2

Black hole and thermal gas solutions in Einstein – dilaton gravity

The 5d dimensional effective holographic models we consider are gravitational theories with an action of the following form:

$$\mathcal{S} = -M_p^3 N_c^2 \int d^5x \sqrt{g} \left[\mathcal{R} - \frac{4}{3} (\partial\Phi)^2 + V(\Phi). \right] + \mathcal{S}_{GH} \tag{2.0.1}$$

In our notation M_p is is the five dimensional Planck mass, N_c is the number of colours of the gauge theory living on the boundary, $\mathcal{R}$ is the scalar curvature of the bulk and Φ is the dilaton. $\mathcal{S}_{GH}$ is the Gibbons-Hawking boundary term given by

$$\mathcal{S} = M_p^3 N_c^2 \int_{\partial M} d^4x \sqrt{h} \mathcal{K} \tag{2.0.2}$$

where h is the induced metric on the boundary of the 5d space-time and $\mathcal{K}$ is its extrinsic curvature. In this section we will review the equations of motion of this theory and the thermodynamics for a relatively a certain class of dilaton potentials.

2.1 Equations of motion and the bulk geometry

We will study two types of solutions with distinct metric ansatz. In a particular coordinate system, called conformal coordinates as in [19], the first type of solutions are of the form

$$\mathrm{BH}: \quad ds^2 = e^{2A}\left[e^{-g}dr^2 - e^g dt^2 + dx_m dx^m\right] \tag{2.1.1}$$

and they are referred to as BH solutions due to the (possible) existence of an horizon where the function e^g vanishes.

For the second type, we set $g = 0$ everywhere. Such solutions are called the thermal gas (TG) solutions and their metric ansatz reads:

$$\mathrm{TG}: \quad ds^2 = e^{2A}\left[dr^2 - dt^2 + dx_m dx^m\right] \tag{2.1.2}$$

The equations of motion for the BH metric functions are [5]

$$\frac{\ddot{f}}{\dot{f}} + 3\frac{\dot{b}}{b} = 0, \quad 6\frac{\dot{b}^2}{b^2} + 3\frac{\ddot{b}}{b} + 3\frac{\dot{b}}{b}\frac{\dot{f}}{f} = \frac{b^2}{f}V \tag{2.1.3}$$

where:

$$b = e^A, \qquad f = e^g. \tag{2.1.4}$$

The dilaton equation of motion is:

$$\ddot{\Phi} + 4\dot{A}\dot{\Phi} + \frac{3}{8}e^{2A}\frac{dV}{d\Phi} = 0 \tag{2.1.5}$$

The dots represent derivation with respect to the radial coordinate r.

The coupled system of equations (2.1.3) and (2.1.5) can be translated into equations for two scalar variables defined as:

$$X(\Phi) = \frac{1}{3}\frac{\dot{\Phi}}{\dot{A}} \qquad Y(\Phi) = \frac{1}{4}\frac{\dot{g}}{\dot{A}} \tag{2.1.6}$$

and they obey the following first order differential equations:

$$\frac{dX}{d\Phi} = -\frac{4}{3}\left(1 - X^2 + Y\right)\left(1 + \frac{3}{8X}\frac{d\log V(\Phi)}{d\Phi}\right) \tag{2.1.7a}$$

$$\frac{dY}{d\Phi} = -\frac{4}{3}\left(1 - X^2 + Y\right)\frac{Y}{X}. \tag{2.1.7b}$$

Specifying the value of Φ at the horizon

$$\Phi_h := \Phi(r_h) \tag{2.1.8}$$

and demanding regularity for X completely fixes X and Y. Once X and Y are given as the solution to (2.1.7), the physical integration constants of equations (2.1.3)-(2.1.5) are fixed. When working with these variables it is more transparent that one can use the value of the dilaton as the radial coordinate itself, as long as it is a monotonic function of the radial coordinate.

One can then invert (2.1.6) and find the metric functions in terms of X and Y:

$$A(\Phi) = \frac{1}{3}\int^{\Phi} \frac{d\tilde{\Phi}}{X(\tilde{\Phi})}, \qquad g(\Phi) = \frac{4}{3}\int^{\Phi} d\tilde{\Phi}\frac{Y(\tilde{\Phi})}{X(\tilde{\Phi})} \tag{2.1.9}$$

The potentials we will deal with have an exponential large Φ leading term making the logarithmic derivative of them asymptote to a *positive* constant a in this region. Therefore a solution of (2.1.7) has X asymptoting to a negative constant

$$X(\Phi) \approx -\frac{3a}{8}, \qquad \Phi \gg 1. \tag{2.1.10}$$

with

$$a = \lim_{\Phi\to\infty} \frac{d\log V}{d\Phi}. \tag{2.1.11}$$

Moreover, the aforementioned boundary conditions for X and Y lead to divergent asymptotic behaviour for Y near the horizon Φ_h:

$$Y(\Phi) = \frac{9V'}{32V}\frac{1}{\Phi_h - \Phi} + \mathcal{O}(1). \tag{2.1.12}$$

With the assumption that X (Y) is a negative (positive) definite monotonic function of Φ asymptoting to zero as $\Phi \to -\infty$, equations (2.1.9) tell that the metric scaling factor e^{2A} diverges monotonically with smaller Φ (UV limit) and the blackness function e^g indeed vanishes at Φ_h. We will verify this for the potentials we study in the following section.

TG solutions are in a sense a special case of BH solutions with f set to 1. It can be seen from equation (2.1.6) that for these backgrounds $Y = 0, \quad \forall \Phi$. The asymptotics (2.1.10) still satisfy (2.1.7).

2.2 Computation of thermodynamical quantities

Here we will only review the expressions for the thermodynamic quantities for the solutions such as temperature, entropy free energy, etc. in terms of the scalar variables. For their derivations see: [19].

The entropy of the BH solutions is given by the area of the horizon. In the $X-Y$ notation this is given by the following equation

$$S = 4\pi M_p^3 N_c^2 V_3 e^{3A(\Phi_h)} = 4\pi\sigma \exp\left[\int^{\Phi_h} d\tilde{\Phi}\frac{1}{X(\tilde{\Phi})}\right] \tag{2.2.1}$$

where we have used equation (2.1.9) and defined $\sigma = M_p^3 N_c^2 V_3$. V_3 is the volume of the periodical spatial directions . The temperature is given by the derivative of the blackness function $f = e^g$ evaluated at the horizon:

$$T = \frac{\left|\dot{f}(r_h)\right|}{4\pi}. \tag{2.2.2}$$

Using equation (7.38) of [19] and the expression for the entropy, the temperature is given in terms of X as [1]:

$$T = \frac{\ell}{12\pi}V(\Phi_h)\exp\left[\frac{1}{3}\int^{\Phi_h} d\tilde{\Phi}\left(\frac{1}{X(\tilde{\Phi})} + 4X(\tilde{\Phi})\right)\right] \tag{2.2.3}$$

The free energy of a solution is the on shell action evaluated at it. This quantity is divergent and it is regularised by taking the difference of the on shell actions of two backgrounds. It can be represented as an integral using

[1] The author thanks Umut Gürsoy who pointed out this.

the first law of thermodynamics.

$$F = F_{BH} - F_{TG} = -\int S dT = \int_{\Phi_h}^{\infty} S(\tilde{\Phi}_h) \frac{dT}{d\tilde{\Phi}_h} d\tilde{\Phi}_h \tag{2.2.4}$$

Other quantities can be computed using thermodynamical equations.

Chapter 3

Potentials with exponential leading term in the IR

Effective holographic models of 4 dimensional strongly coupled plasmas ([5], [20]) as well as lower dimensional effective holographic models of condensed matter systems [21] have exponential IR asymptotics for the scalar potential.

$$V \sim e^{a\Phi}, \quad \Phi \gg 1 \tag{3.0.1}$$

Where and why is this a good approximation

3.1 Liouville potentials and AdS completion

For potentials that are given as an exponential everywhere [1]

$$V = e^{a\Phi}, \qquad \forall\Phi \tag{3.1.1}$$

the solution of (2.1.7) can be found analytically as:

$$X = -\frac{3a}{8}, \qquad Y = \frac{\frac{9a^2}{64} - 1}{1 - \exp\left[-\frac{9a^2-64}{18a}\left(\Phi_h - \Phi\right)\right]} \tag{3.1.2}$$

[1]Such backgrounds have been studied by Chamblin and Reall [22].

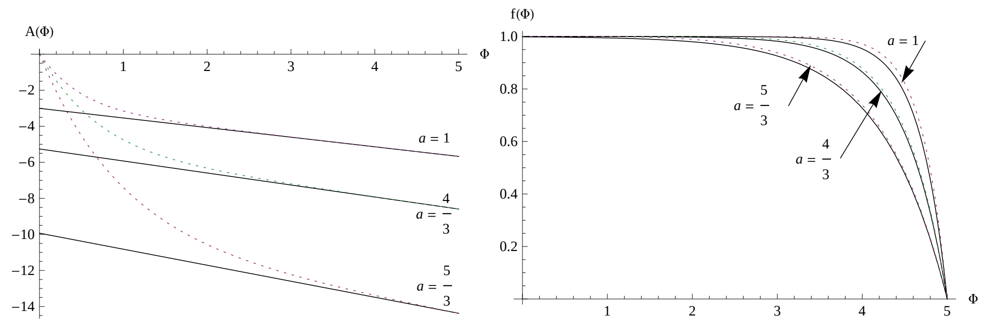


Figure 3.1: *Metric functions A (left), f (right) for* $\Phi_h = 5$ *and* $a \in \left\{1, \frac{4}{3}, \frac{5}{3}\right\}$. *Solid curves are the solutions for exponential potentials and the dotted curves are numerical solutions for potentials with AdS completion.*

Equation (3.1.2) implies via equation (2.1.9) the following solutions for the metric functions:

$$A = A_0 - \frac{8}{3a}\left(\Phi - \Phi_0\right) \qquad g = \log\left[1 - \exp\left[\frac{9a^2 - 64}{18a}\left(\Phi_h - \Phi\right)\right]\right] \tag{3.1.3}$$

where Φ_0 is a UV cut-off value for the dilaton and Φ_h is its value at the horizon.

While the potential (3.1.1) is a good approximation in the IR, it needs completion in the UV to allow solutions with asymptotically AdS boundary. To do so, one can include a term that asymptotes to a constant as $\Phi \to -\infty$ and is negligible as $\Phi \to +\infty$. For instance

$$V(\Phi) = 1 + e^{a\Phi} \tag{3.1.4}$$

will give an asymptotically AdS background. For comparison we plot numerical solutions of the Einstein equations for the potential (3.1.4) along with the analytical solutions for (3.1.1) in figure 3.1.

3.2 Thermodynamics

The parameter a does not result in major qualitative changes in the bulk geometry as long as $a < \frac{8}{3}$ [2]. However this is not the case for the thermodynamics of the solutions, in particular for the temperature. The behaviour of the temperature for smaller black holes (that is $\Phi_h \to \infty$) has critical dependence on a. This limit is of interest because there the free energy will go to zero implying a transition between the BH and TG phases.

Temperature

From equation (2.2.3) we can see the large Φ_h behaviour of the temperature depending on the IR asymptotics of the dilaton potential. It is determined by which of the two exponential factors are dominant. There is a critical parameter a where these factors cancel each other and the temperature goes to a constant. Thus the possibilities can be classified as follows:

1. $a = \frac{4}{3}$: This is the critical value of a. In this case

$$X(\Phi) \approx -\frac{1}{2}, \qquad \text{for} \quad 1 \ll \Phi \tag{3.2.1}$$

and the two factors on the RHS of equation (2.2.3) become inversely proportional to each other. The temperature asymptotes to a constant as $\Phi_h \to \infty$.

2. $a > \frac{4}{3}$: In this case the potential dominates the second factor. The potential goes like $e^{a\Phi}$ while the integral depends on the asymptotic value of X given in equation (2.1.10). Since

$$a > \frac{1}{3}\left(\frac{8}{3a} + \frac{3a}{2}\right), \qquad \forall a > \frac{5}{3} \tag{3.2.2}$$

the temperature diverges as $\Phi_h \to \infty$.

3. $a < \frac{4}{3}$: As the opposite of case 2, the temperature goes to zero as the black hole shrinks.

[2] As pointed out in [19], for $a > \frac{8}{3}$ the singularity is not of a good type according to the classification of [23].

The temperature for potentials (3.1.1) along with the asymptotically AdS case (3.1.4) is plotted in figure 3.2.

Entropy

The entropy vanishes in the limit $\Phi_h \to \infty$ independently of the choice of $a < \frac{8}{3}$. This can be seen from equation (2.2.1) by noting that X asymptotes to a negative constant. Figure 3.2 demonstrates the dependence of the entropy on the temperature for three sample parameters a.

Free Energy

In the limit $\Phi_h \to \infty$ also the free energy goes to zero as equation (2.2.4) suggests. The integrand in (2.2.4) is regular while the limits of integration go to zero. As we claimed in the previous section, for solutions with a local minimum of the temperature ($a > \frac{4}{3}$ with AdS completion), there exists a finite critical temperature where the free energy goes to zero. However this temperature corresponds to a black hole with a finite value of Φ_h in contrast to the critical $a = \frac{4}{3}$ case where the only phase transition happens as $\Phi_h \to \infty$. In critical backgrounds this only phase transition happens at a finite critical temperature

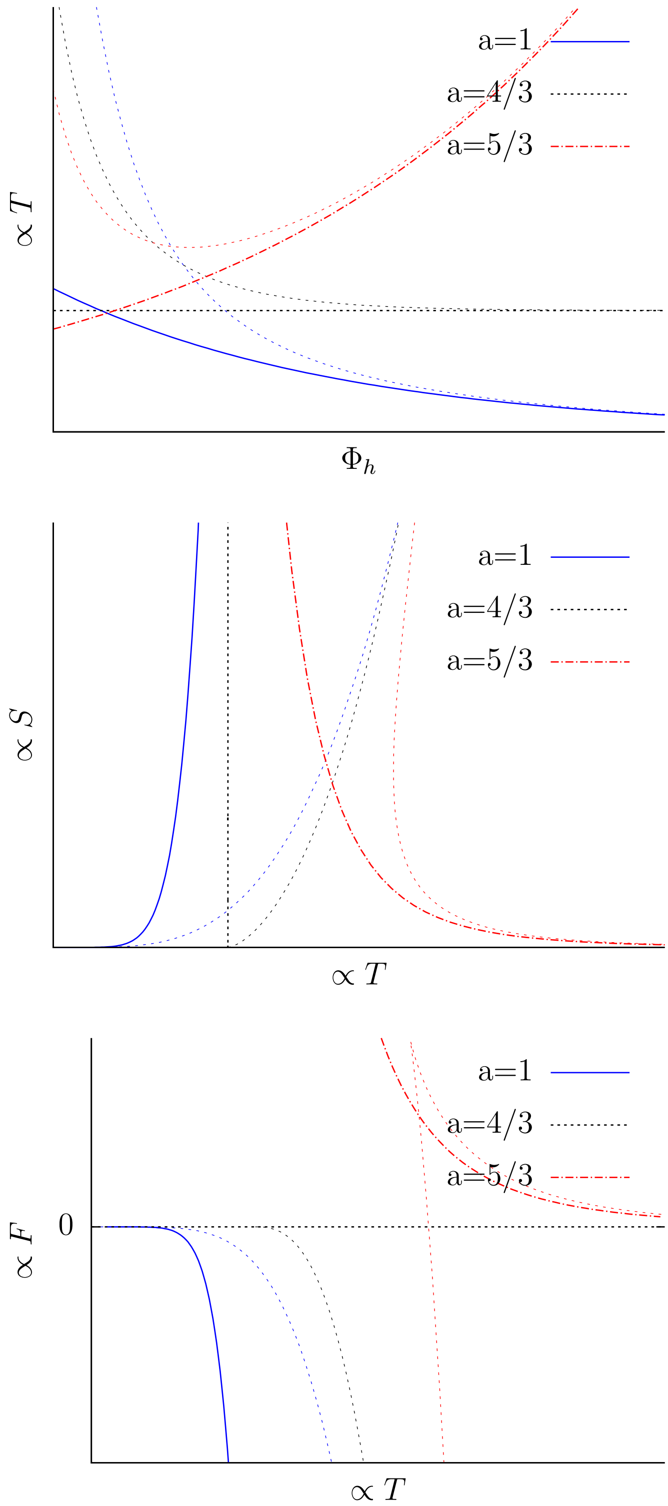
$$T_c = \lim_{\Phi_h \to \infty} T(\Phi_h), \qquad \text{for} \quad a = \frac{4}{3} \tag{3.2.3}$$

In the marginal case $a = \frac{4}{3}$ without any corrections to exponential potential, the temperature is a constant and the free energy is zero for any Φ_h. However in realistic cases there may be corrections to the potential that can be factorised like:

$$V \approx e^{\frac{4}{3}\Phi}\left(1 + e^{-k\Phi}\right) \tag{3.2.4}$$

and the large Φ_h behaviour of the thermodynamical quantities will be determined by the subleading term in 3.2.4 [18][3]. Let us begin by analysing the temperature for a more precise demonstration of this claim. The temperature

[3]The potential (3.1.4) that we considered to demonstrate the AdS completion also has a subleading IR term of the form of (3.2.4)



a)Temperature:

b)Entropy:

c)Free Energy:

Figure 3.2: *Comparison of analytical results for an exponential potential* $V = e^{a\Phi}$ *with numerical computations for a potential like* $V = 1 + e^{a\Phi}$.

is given by equation (2.2.3) and for large Φ_h we can write

$$T \propto e^{\frac{4}{3}\Phi_h} \left(1 + e^{-k\Phi_h}\ \right) e^{-\frac{4}{3}\Phi_h} = 1 + e^{-k\Phi_h} \tag{3.2.5}$$

or

$$t \sim e^{-k\Phi_h} \tag{3.2.6}$$

with definition of the deviation t from the critical temperature as:

$$t = \frac{T - T_c}{T_c} \tag{3.2.7}$$

From the large Φ_h behaviour of 2.2.1 and using (3.2.5) we calculate the scaling of entropy near the critical temperature:

$$S \sim t^{n-1}, \qquad n = \frac{2}{k} + 1 \tag{3.2.8}$$

Similarly, from (2.2.4) we find that the free energy will scale as

$$F \sim t^n \tag{3.2.9}$$

The above listed scalings of the temperature, entropy and free energy imply an n'th order continuous phase transition at T_c..

Chapter 4

The bulk viscosity

Bulk viscosity of strongly coupled systems can easily be obtained by holographic techniques. In linear response theory, transport coefficients are related to the imaginary part of the Fourier transform of the retarded energy momentum tensor correlator by the Kubo's formula

$$\zeta = -\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}G_R\left(\omega,0\right). \tag{4.0.1}$$

The retarded correlator of the energy-momentum tensor can be calculated holographically from the fluctuations of the metric as they become a source for the energy-momentum tensor of the boundary theory. As seen from their definitions in the stress tensor (1.0.1), the bulk viscosity is related to the spatial diagonal fluctuations while the off diagonal spatial compinents are related the shear viscosities.

For the kinds of effective holographic theories we consider, a prescription for the calculation of the bulk viscosity has been given in [13] which uses the calculation of the Minkwoski space corelator of [24]. In the notation of [14] the result for the bulk viscosity / entropy density ratio is:

$$\frac{\zeta}{s} = \frac{3}{32\pi}\left(\frac{V'\left(\Phi_h\right)}{V(\Phi_h)}\right)^2 |c_b|^2 \tag{4.0.2}$$

where c_b is the coefficient of the near horizon asymptotics of the fluctuations:

$$h_{11} \approx c_b \left(\Phi_h - \Phi\right)^{-i\tilde{\omega}}, \qquad \tilde{\omega} = \frac{\omega}{4\pi T}. \tag{4.0.3}$$

Again in the notation of [14], the equation that the fluctuations of the diagonal metric component satisfy, is written like:

$$h''_{11} = c(\Phi)h'_{11} + d(\Phi)h_{11} \tag{4.0.4}$$

with

$$c(\Phi) = \frac{1 - X^2 + Y}{X}\left(\frac{8}{3} + \frac{3}{2X}\frac{V'}{V}\right) \tag{4.0.5a}$$

$$d(\Phi) = -\frac{16Y}{9X^2}\left(1 - X^2 + Y\right)\left(1 + \frac{3}{8X}\frac{V'}{V}\right) - \left(\frac{\omega Y}{3\pi T X}\right)^2 e^{-2\int_{\Phi}^{\Phi_h}\frac{1}{X}}. \tag{4.0.5b}$$

In holographic calculations ([13], [14]) and in lattice simulations [25] there is evidence for a rise in the bulk viscosity in the vicinity of a phase transition. In section 4.2 we find similar behaviour also for backgrounds with marginal IR asymptotics which exhibit a phase transition in the $\Phi_h \to \infty$ limit at a finite temperature.

4.1 Analytic solution for exponential potential

For exponential potentials it is possible to compute the bulk viscosity analytically. This is most conveniently done by matching the low frequency expansion of the fluctuations to the zero frequency solution of (4.0.4). A full solution can be obtained in the $\omega = 0$ case. Then, imposing regularity at the horizon and unit normalisation at the boundary fixes h_{11}:

$$h_{11} = 1 \tag{4.1.1}$$

for all r. We leave the details of this calculation to appendix A

We remind that this result is compatible with the "adiabatic" approximation of [19] which states that $c_b \approx 1$ where the logarithmic derivative can be approximated by a constant. For exponential potentials the logarithmic derivative is constant everywhere.

4.2 Bulk viscosity in the presence of IR corrections

In section 3 we mentioned how the backgrounds can be classified by the leading IR behaviour of the dilaton potential and showed the existence of a marginal type of asymptotics. We also have seen that some non-trivial information regarding thermodynamical quantities are determined by the subleading terms. In this section we shall compute bulk viscosity for potentials with such corrections and see the emergence of a non-trivial dependence on temperature.

As the subleading terms make the fluctuation equation complicated enough, it is required to solve the fluctuation equation (4.0.4) numerically with the following boundary conditions: in falling waves at the horizon and unit normalisation of h_{11} at the boundary, as mentioned before. Firstly we solve the fluctuation equation with the potential (3.1.4).

Again, we find three distinct behaviours depending on the parameter a as we found for the temperature:

1. $a = \frac{4}{3}$: In this marginal case, the bulk viscosity has a maximum at a temperature T_{max} near the phase transition and for temperatures above T_{max} it decreases to zero monotonically as the temperature is increased.
2. $a > \frac{4}{3}$: In this case the bulk viscosity has a maximum near the confinement - deconfinement phase transition. It goes to a constant with the increasing temperature (on the big black hole branch which is thermodynamically preferable)
3. $a < \frac{4}{3}$: In this case there is neither a phase transition, nor a maximum in the bulk viscosity. The bulk viscosity goes to a constant with decreasing temperatures and vanishes with increasing temperatures.

The three cases are plotted in figure 4.1. In every case the bulk viscosity is finite.

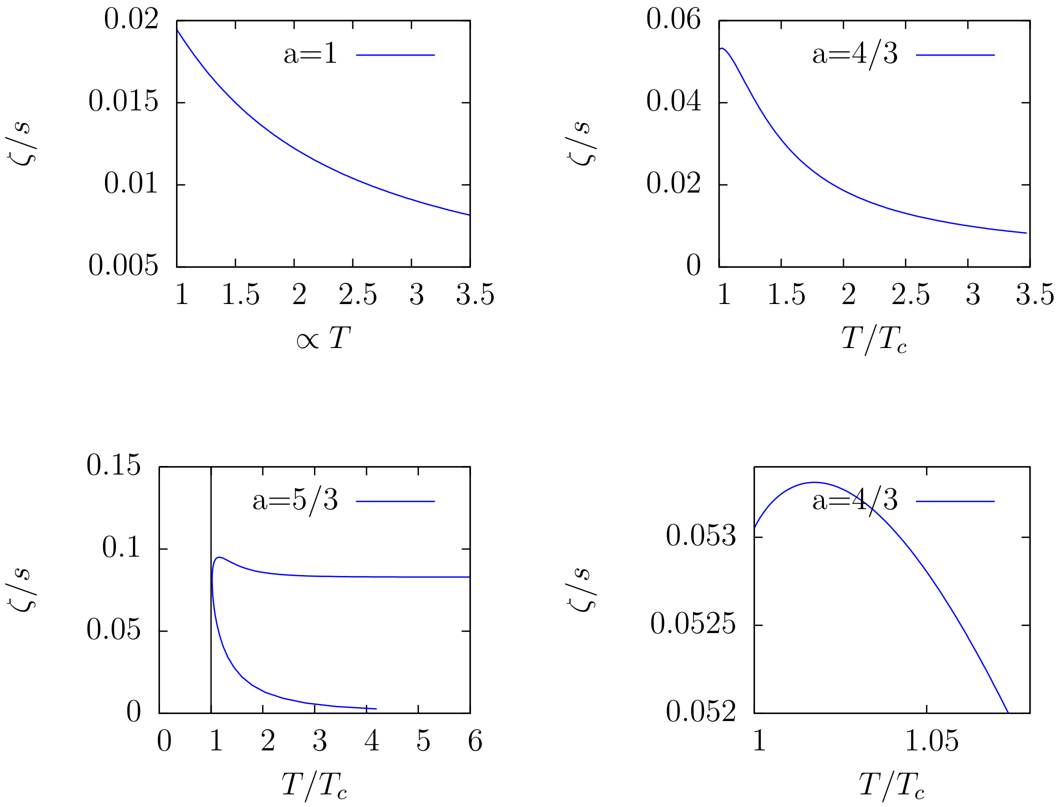


Figure 4.1: *Behaviour of the bulk viscosity / entropy density ratio with respect to temperature for three cases of a in potentials given like in equation (3.1.4). Temperatures are scaled by the critical temperature of each theory, if there exists one.*

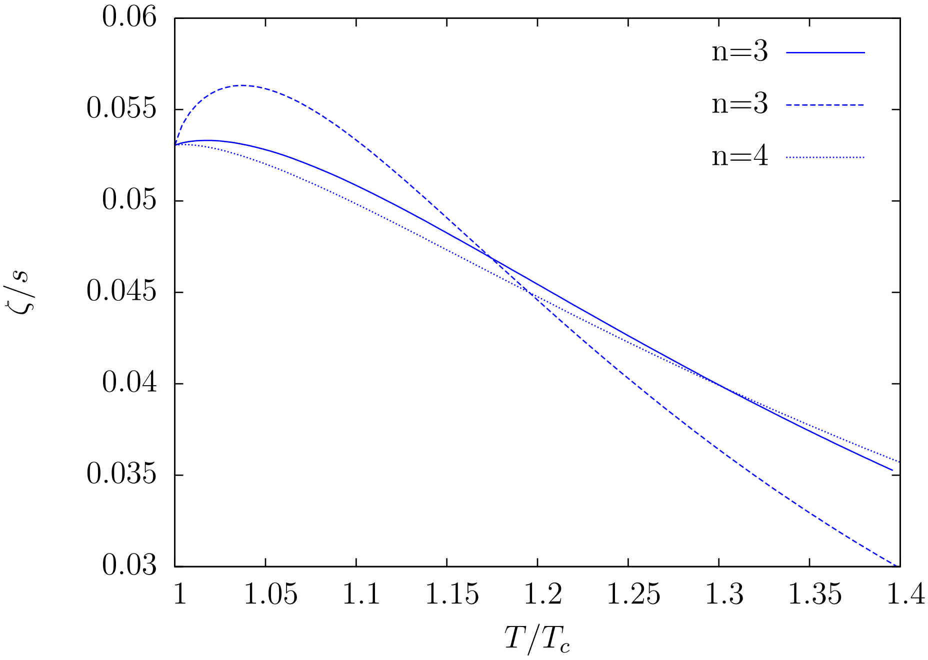


Figure 4.2: *Bulk viscosity / entropy density ratio for marginal backgrounds with continuous phase transitions. The solid, dashed, dotted lines represent $n = 2$, $n = 3$, $n = 4$ respectively, where n is the order of the phase transition according to the discussion at the end of section 3*

Finally, let us concentrate on the marginal case with continuous phase transitions. Before giving results it is interesting to review an analysis made in [18] which concludes that the value of bulk viscosity at the phase transition is universal for the marginal subclass of potentials, i.e. potentials with $\propto e^{\frac{4}{3}}$ IR asymptotics.

To see this we note that near the horizon the adiabatic approximation of [14] becomes valid as the potential asymptotes to an exponential. This implies that the coefficient c_b in (4.0.3) becomes 1. Then the bulk - viscosity / entropy density ratio (4.0.2) simplifies to:

$$\lim_{\Phi_h \to \infty} \frac{\zeta}{s} = \lim_{\Phi_h \to \infty} \frac{3}{32\pi} \left(\frac{V'(\Phi_h)}{V(\Phi_h)} \right)^2 = \frac{3a^2}{32\pi} \tag{4.2.1}$$

independently from the constant prefactor of the potential, the subleading terms and therefore the order of the phase transition.

Chapter 5

Conclusion

We calculated bulk viscosity in theories with gravitational duals as described in section 2 with asymptotically exponential dilaton potentials in the IR. We observed that in backgrounds with first order phase transitions as well as continuous phase transitions with vanishing entropy at the critical point, the bulk viscosity / entropy ratio $\frac{\zeta}{s}$ has a maximum at a temperature just above the critical one. Moreover we have seen that $\frac{\zeta}{s}$ is finite in all of the backgrounds we examined. This justifies the discussion in the appendix B of [14] which applies to our cases as well.

There are several models proposed for the behaviour of the bulk viscosity near criticality, depending on the other critical exponents in the theory [12, 26, 27]. In [17] their compatibility with various theories including $\mathcal{N} = 2^*$ gauge theory plasma [28], the cascading gauge theory plasma [29] and the plasma dual to "exotic black holes" [30], was examined by calculating the critical exponents and comparing them with the behaviour of the bulk viscosity. It would be interesting to do such an analysis for the theories of the type studied in this paper.

Appendix A

Bulk viscosity for exponential dilaton potential

We will compute the coefficient c_b by expanding the solution near the horizon and for low frequencies and comparing them with the known zero frequency solutions to match the boundary conditions.

The zero frequency solution for the exponential potential is found relatively easy since $d(\Phi)$ in equation (4.0.5a) becomes zero.

$$h_{11}'' = \frac{(64 - 9a^2)}{18a}\left(1 + \frac{1}{e^{\frac{9a^2-64}{18a}(\Phi_h - \Phi)} - 1}\right) h_{11}' = 0 \tag{A.0.1}$$

which has the following analytical, complete solution:

$$h_{11}^0 = C_2 + C_1\left(\Phi - \Phi_h\right) - C_1 \frac{18a}{9a^2 - 64} \log\left(e^{\frac{\left(9a^2-64\right)\left(\Phi-\Phi_h\right)}{18a}} - 1\right). \tag{A.0.2}$$

For $a < \frac{8}{3}$ the boundary condition at the boundary fixes $C_2 = 1$.

$$h_{11}^0 = 1 + C_1\left(\Phi - \Phi_h\right) - C_1 \frac{18a}{9a^2 - 64} \log\left(e^{\frac{\left(9a^2-64\right)\left(\Phi-\Phi_h\right)}{18a}} - 1\right). \tag{A.0.3}$$

Near the horizon this solution is:

$$h_{11}^0 = 1 + \frac{18a}{64-9a^2} C_1 \log\left[\left(\frac{64-9a^2}{18a}\right)(\Phi_h - \Phi)\right] + \mathcal{O}\left(\Phi_h - \Phi\right). \quad \text{(A.0.4)}$$

and we see that $C_1 = 0$ if we demand regularity at the horizon for strict zero frequency solutions. Therefore the full zero frequency solution is a constant

$$h_{11}^0 = 1 \quad \text{(A.0.5)}$$

Now we will match the asymptotic expansion of h_{11} with the zero frequency solution. We factor out the asymptotic behaviour (4.0.3) at the horizon and write a series expansion for the corrections:

$$h_{11} = c_b\left(\tilde{\omega}\right)\left(\Phi_h - \Phi\right)^{-i\tilde{\omega}}\left[1 + d(\tilde{\omega})\left(\Phi_h - \Phi\right) + \mathcal{O}\left(\Phi_h - \Phi\right)^2\right] \quad \text{(A.0.6)}$$

Now we expand the coefficients in frequency around zero:

$$h_{11} = \left(\Phi_h - \Phi\right)^{-i\tilde{\omega}}\Big[c_b(0) + c_b(0)d(0)\left(\Phi_h - \Phi\right) + \tilde{\omega}\left[c_b'(0) + c_b(0)d'(0)\left(\Phi_h - \Phi\right)\right] + \mathcal{O}\left(\tilde{\omega}^2\right) + \mathcal{O}\left(\Phi_h - \Phi\right)^2\Big] \quad \text{(A.0.7)}$$

Therefore at zero frequency this solution is:

$$h_{11}(\omega = 0) = c_b(0) + c_b(d)d(0)\left(\Phi_h - \Phi\right) \quad \text{(A.0.8)}$$

We know that in the strict zero frequency limit, $h_{11}\left(\omega = 0\right) = h_{11}^0 = 1$. Therefore we find that

$$c_b(0) = 1$$

and

$$d(0) = 0.$$

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