UNIVERSITY OF CRETE

DEPARTMENT OF PHYSICS

MSc THESIS

Fundamental Physical Constants; Theory and Experiment

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UNIVERSITY of CRETE
DEPARTMENT of PHYSICS

THESIS

MASTER OF SCIENCE
IN
- PHYSICS -

Defended by
CHARIKLEIA TROULLINOU

Fundamental Physical Constants; Theory and Experiment

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Date of the defense:
14/07/2013

Περίληψη

Η εργασία αυτή αποτελεί μια σύντομη αναφορά μερικών θεμελιωδών φυσικών σταθερών. Η ομάδα των σταθερών που περιγράφονται περιλαμβάνει τη σταθερά της λεπτής υφής (α), την ταχυτητα του φωτός (c), τη βαρυτική σταθερά (G), καθώς επίσις και τις μάζες δύο σωματίων φορέων θεμελιωδών αλληλεπιδράσεων, συγκεκριμένα του φωτονίου και του βαρυτονίου. Ιδιαίτερη έμφαση δίνεται στις πιο πρόσφατες και ακριβής τιμές τους, όπως αυτές προκύπτουν μέσα από μια αλληλουχία θεωρητικών εκτιμήσεων και πειραμάτων.

Abstract

A brief review for the values of some fundamental physical constants is presented. The sets of constants discussed here, include the fine structure constant (α), the speed of light (c), the gravitational constant (G), as well as the masses of 2 bosons-carriers of fundamental interactions, namely the photon and the graviton. The determination of their latest and most accurate numerical values is studied through a chain of theoretical estimations and experimental considerations.

Acknowledgements

I would like to thank Prof. Tsamis for his guidance and invaluable help. I am grateful to Prof. Zezas, Prof. Pavlidou, Prof. Tassis, Prof. Charmandaris, Prof. Woodard and all the scientists of the Physics Department who devoted time and helped me understand various physical phenomena. Also, I would like to thank Prof. Tomaras and Prof. Rakitzis for the significant support during my studies at the University of Crete. The list of friends and colleagues-graduate students who have helped at various stages of this project is too long to be presented here in its entirety. However, I would like to deeply thank them all. Finally, I feel obliged to mention G. Panopoulou, K. Roubedakis, E. Xypakis and I. Iatrakis. Without their contribution the completion of this thesis would be even more difficult.

This master thesis has been co-financed by the European Union (European Social Fund, ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) under "Funding through graduate scholarships from the Hellenic State Scholarship Foundation after an individual evaluation. Academic year 2011-2012"

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1 Introduction

To a great extent the description of physical reality is based on the form of a few basic equations and their implications. The electromagnetic phenomena are understood in the framework of Maxwell equations. Einstein's theory of relativity and Newton's theory of gravity suffice to explain the behaviour of celestial motions and astrophysical events. While quantum mechanics and quantum field theory provide the foundations for the comprehension of the atomic structure and the elementary forces in Nature.

Writing the basic equations encountered in these theories, some significant quantities have to be assumed for granted. These emerging parameters are the Fundamental Physical Constants. Serving as conversion factors between physical units or interpreted as coupling constants of the principal interactions, they hold a prominent position in the foundations of Physics. Their numerical values, covering a wide range of magnitudes, are in direct correlation with definitions and determinations of the SI units.

The role of the Fundamental Physical Constants in modern physics is of great importance as well. Their universality indicates that they unify separate domains of physics. In fact, all of them appear in self-consistent formulae that interrelate one another. This observation has challenged theoretical physicists to investigate further ideas concerning the ultimate unification of fundamental forces. Additionally, the precise estimation of their values provide arguments concerning the validity of limitation principles.

The fact that the numerical values of the Fundamental Physical constants are reference points for many sciences, makes their accurate determination extremely important. This process usually relies on the prosperous combination of theoretical calculations with experimental measurement and innovative technological applications. What is more, it is considered an interdisciplinary task, which involves the investigation of seemingly unrelated phenomena from various fields of Physics.

The sets of constants discussed here, include the fine structure constant (α), the speed of light (c), the gravitational constant (G), as well as the masses of 2 bosons-carriers of fundamental interactions, namely the photon and the graviton. The determination of their latest and most accurate numerical values is studied through a chain of theoretical estimations and experimental constraints.

2
The Photon Mass

Light has played an exceptional role in the progress of physics through the ages. Today it is known that the photon is the quantum of light and that its properties are in strong correlation with the well known Maxwell equations. According to them the photon is thought to be massless and moving in a steady speed c. However, physics is an experimental science, which means that in order for the value of the photon mass to be experimentally defined we first have to doubt this statement and to probe a massive electrodynamic theory.

2.1 Massive Electrodynamics

The Lagrangian used to describe a massive vector spin-1 field was first proposed by Proca in 1936 in his attempt to describe the four-states of electron and positrons with a Lorentz four-vector. Proca theory is a generalization of Maxwell electrodynamics and it can be derived if we add a mass term in the classical QED Lagrangian:

$$L_{Proca} = -\frac{1}{4\mu_0} F_{\alpha\beta} F^{\alpha\beta} - j_\alpha A^\alpha + \frac{\mu_\gamma^2}{2\mu_0} A_\alpha A^\alpha \tag{2.1}$$

where $A_\alpha = (\mathbf{A}, \frac{i\phi}{c})$ is the massive photon vector field, $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ the antisymmetric field strength tensor, and $j_\alpha = (\mathbf{J}, ic\rho)$ the four-vector current density with the symbols ρ, $\mathbf{J}$ and μ_0 corresponding to the charge and current densities and the permeability of free space respectively. From the variation of Lagrangian with respect to A_μ follows:

$$\partial_\alpha F^{\alpha\beta} + \mu_\gamma^2 A^\beta = \mu_0 j^\beta \Rightarrow \Box A^\beta - \partial_\alpha \partial^\beta A^\alpha + \mu_\gamma^2 A^\beta = \mu_0 j^\beta \tag{2.2}$$

$$\partial_\beta \Box A^\beta - \partial_\beta \partial_\alpha \partial_\beta A^\alpha + \mu_\gamma^2 \partial_\beta A^\beta = \mu_0 \partial_\beta j^\beta \Rightarrow \partial_\beta A^\beta = \mu_0 \partial_\beta j^\beta \tag{2.3}$$

Combining 2.2 and 2.3, we obtain the equation of motion for the Proca field A_α

$$\boxed{(\Box - \mu_\gamma^2) A^\alpha = \mu_0 j^\alpha} \tag{2.4}$$

Thus, in the region of no charge or current density ($j^{\alpha} = 0$) the equation is simply the Klein-Gordon equation of the photon with mass m related to the μ_γ through the equation: $\mu_\gamma = \frac{mc}{\hbar}$. The photon mass is usually encountered in the definition of the Compton wavelength of photon $\lambda = \mu_\gamma^{-1}$ and it is indicative of the exponential fall of the "Yukawa-like" potential formed by charge q at the origin of the coordinate system:

$$V(r) = \frac{q}{4\pi\epsilon_0}\frac{e^{-\mu_\gamma r}}{r} \tag{2.5}$$

Taking into account the Proca Lagrangian, one has to modify the Maxwell equations respectively so that they include the mass term:

$$\nabla \cdot \mathbf{B} = 0 \tag{2.6}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{2.7}$$

$$\nabla \times \mathbf{B} + \mu_\gamma^2 \mathbf{A} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J} \tag{2.8}$$

$$\nabla \cdot \mathbf{E} + \mu_\gamma^2 \phi = 4\pi\rho \tag{2.9}$$

The correlation relations of the electric and magnetic field $\mathbf{E}$ and $\mathbf{B}$ with the vector and scalar potentials:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla \cdot \phi \quad , \quad \mathbf{B} = \nabla \times \mathbf{A} \tag{2.10}$$

have remained unaffected by the introduction of mass along with the continuity equation regarding ρ and $\mathbf{J}$: $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$. In addition the formula giving the force due to the presence of $\mathbf{E}$ and $\mathbf{B}$ on a point charge q, moving with velocity $\mathbf{u}$, has not been altered:

$$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{u} \times \mathbf{B}}{c}\right) \tag{2.11}$$

Apart from the use of different Electrodynamic equations, the presence of a massive photon has arisen several implications, which effectively contribute to the experimental determination of the value of the photon mass.

- *dispersion relation.* The speed of light is no longer a dispersionless constant. Actually it is easy to see that through the relativistic equation of energy: $E^2 = p^2c^2 + m^2c^4$, the dispersion relation reads as[1]:

  $$k^2c^2 = \omega^2 - \mu_\gamma^2 c^2 \tag{2.12}$$

  A direct consequence of the above dispersion relation is that the photon can be regarded as a wave packet with group velocity given by the formula:

  $$\upsilon_g = \frac{d\omega}{dk} = c\left(1 - \frac{\mu_\gamma^2 c^2}{\omega^2}\right)^{-\frac{1}{2}} \simeq c\left(1 + \frac{\mu_\gamma^2 c^2}{2\omega^2}\right) \tag{2.13}$$

[1] Here the symbol c refers to the ultimate speed of light, or otherwise the speed of light with the largest frequency"velocity of infinitely high-frequency photons".

The fact that the group velocity is a function of the frequency ω has triggered many laboratory experiments designed to determine the photon mass. Comparing the group velocities of two wave packets with different frequencies ω_1 and ω_2 :

$$\frac{\Delta v}{c} = \frac{v_2 - v_1}{c} = \frac{\mu_\gamma^2}{8\pi^2}(\lambda_2^2 - \lambda_1^2) + O[(\mu_\gamma \lambda_1)^4] \tag{2.14}$$

we conclude that the velocity difference is proportional to the square wavelength difference of the wave packets with proportionality constant the square of the photon mass. However, due to uncertainties regarding the intrinsic measurements, the results derived from the terrestrial experiments have reached a limit $m < 8 \cdot 10^{-40} g$. (4). Thus, the hopes lie in the region of extraterrestrial experiments which study the time interval needed for the travel of two waves with different wavelengths λ_1 and λ_2 moving through the same distance L:

$$\Delta t \equiv \frac{L}{v_{g1}} - \frac{L}{v_{g2}} \approx \frac{L}{8\pi^2 c}(\lambda_2^2 - \lambda_1^2)\mu_\gamma^2 \tag{2.15}$$

For astronomical distances of order of $L \sim 10^3 ly$ the value of the photon mass seems to be $m < 10^{-44} g$. Other experimental attempts may not even exceed the distance of the radius of the Earth (Kroll effect) while they study the behaviour of low-frequency waves and thus deducing limits $m < 4 \cdot 10^{-39} g$.

- *longitudinal photon.* A basic property of a massless photon is that it has only two possible polarization states (λ), both of which are transverse to the momentum four-vector k. However, this is not valid in the case of the non-zero photon mass. Namely the expansion of photon field is written:

$$A_\mu(\mathbf{k}, \lambda; x) = N_k e^{i(\omega_k t - \mathbf{k}\cdot\mathbf{x})} \epsilon_\mu(\mathbf{k}, \lambda) \quad , \quad \omega_k = \sqrt{\mathbf{k}^2 + m^2} \tag{2.16}$$

where the four-vectors $\epsilon_\mu(\mathbf{k}, \lambda)$ constitute a four dimensional orthonormal system $\epsilon_\mu(\mathbf{k}, \lambda)\epsilon(\mathbf{k}, \lambda') = g_{\lambda,\lambda'}$ and at the same time satisfy the completeness relation

$$\sum_{\lambda=0}^{3} g_{\lambda\lambda'} \epsilon_\mu(\mathbf{k}, \lambda) \epsilon_\nu(\mathbf{k}, \lambda') = g_{\mu\nu} \tag{2.17}$$

For the massive field, it is possible to construct a well defined four-vector $\epsilon(\mathbf{k}, 3) = \left(\frac{\mathbf{k}}{m}, \frac{\mathbf{k}}{|\mathbf{k}|} \cdot \frac{k_0}{m}\right)$ which is directed along the direction of the four-momentum k. Furthermore, the introduction of the mass term affects the form of the Poynting vector:

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B} + \mu_\gamma^2 \phi \mathbf{A}) \tag{2.18}$$

Since now the electromagnetic vector is decomposed as: $\mathbf{A} = \mathbf{A}^T + \mathbf{A}^L$, where $\nabla \cdot A^T = 0$ and $\mathbf{A}^L = \nabla A^L$, the presence of the μ_γ in 2.18 causes additional longitudinal radiation. Experimentally this effect does not seem to be that useful since the mass of the photon is thought to be too small to be detected using this property.

- *violation of the gauge invariance.* It is rather obvious that the Lagrangian 2.1 is not invariant under a gauge transformation:

$$V \to V' = V - \frac{\partial \Lambda}{\partial t} \quad , \quad \mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla \Lambda \tag{2.19}$$

where Λ is an arbitrary but well behaved scalar function. In fact, the gauge invariance is replaced by the apparent Lorentz invariance of the Proca Lagrangian which means that the Lorentz condition 2.3 is automatically verified. The violation of gauge invariance is actually the main argument stated against the theory of massive electrodynamics. Nevertheless, it can be revised since by introducing a new scalar field Φ carrying electric charge q the gauge invariance is automatically restored and the photon mass can be expressed in terms of q and Φ. Thus, the enhancement of the theory by an extra field can guarantee the existence of a massive photon theory satisfying gauge invariance.

2.2 Experimental approaches

The above implications of the existence of a massive photon played a significant role in the experimental determination for the upper limit of the photon mass. Several studies have been conducted regarding the longitudinal electromagnetic radiation, the deviation from Coulomb's law and the dispersion relation of the photon momentum. Nevertheless, the more effective experiments are those based on extra-terrestrial mechanisms and especially on the magnetohydrodynamic phenomena taking place in the interstellar medium.

The generally accepted and most precise value of the photon mass has been obtained from the work of D. D. Ryutov : $\boxed{m_\gamma < 1.5 \cdot 10^{-53} g \simeq 8.41 \cdot 10^{-21} \frac{eV}{c^2}}$ The theoretical background of Ryutov's consideration is the Proca electrodynamics and the modifications which follow in the magnetohydrodynamic (MHD) equations. Thus according to (5) the following equations, derived from 2.6-2.9, are used to describe astrophysical processes with characteristic phase velocities small compared to that of light (c). [1]

$$\rho\frac{d\mathbf{u}}{dt} = -\nabla p + \frac{1}{c}\mathbf{j}\times\mathbf{B} \tag{2.20}$$

$$\nabla\times\nabla\times\mathbf{B} + \frac{\mathbf{B}}{\lambda^2} = \frac{4\pi}{c}\nabla\times\mathbf{j} \tag{2.21}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \tag{2.22}$$

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{\mathbf{u}\times\mathbf{B}}{c}\right) \tag{2.23}$$

Another factor that plays a vital role in the form of equations (2.20)–(2.23) is the scale L of the processes as compared with the Compton wavelength λ (6). For $L > \lambda$ the Proca equations should definitely be used and in our case the second term in the left hand side of 2.21 should be neglected.

The outcome of Ryutov's theoretical analysis can be applied in the Solar wind problem and the trajectories of charged particles near the orbit of Pluto. In fact, at distances larger than 2 AU the magnetic field lines, responsible for the Solar wind, are affected by the Solar rotation and the line-tying effect. Therefore they form a spiral like shape. For a coordinate system with the Sun being at its origin, we can conclude that the dominant component at the scale of Pluto's orbit is the azimuthal one (B_ϕ). This observation demands, the current density at this region derived from :

$$\nabla\times\nabla\times B_\phi\hat{\phi} + \frac{B_\phi\hat{\phi}}{\lambda^2} = \frac{4\pi}{c}\nabla\times\mathbf{j} \tag{2.24}$$

to be approximately:

$$\mathbf{j} = \frac{cB_\phi r}{4\pi\lambda^2}\hat{\theta} + \frac{cB_\phi r}{4\pi\lambda^2}\hat{r} \tag{2.25}$$

Consequently there should be a force in this region :

$$\mathbf{f} = \mathbf{j}\times\mathbf{B} \Rightarrow |\mathbf{f}| = \frac{B_\phi r}{4\pi\lambda^2} \tag{2.26}$$

which affects the trajectory of the plasma. Specifically, if this force (per unit volume) is greater than the centrifugal one $\left(f_c = \frac{\rho v^2}{r}\right)$, the interplanetary plasma should not move in the radial direction. However such an observation has not been verified which leads to the condition :

$$f < \frac{\rho v^2}{r} \Rightarrow \frac{B_\phi^2 r}{4\pi\lambda^2} < \frac{\rho v^2}{r} \tag{2.27}$$

Inserting the values of r, B,v obtained from astrophysical observation at the scale of the Pluto's orbit one can come up with a wavelength $\lambda > 2\cdot 10^{13}cm$ which corresponds to the aforementioned mass limit.

[1] The following formulas are given in the CGS system which is more convenient for astrophysical measurements.

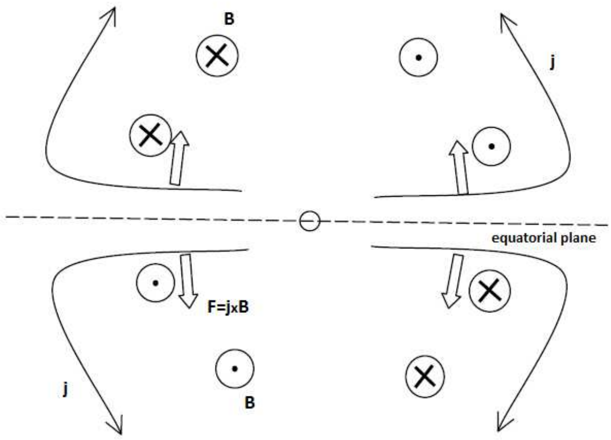

Figure 2.1: Magnetohydrodynamic phenomena at the outer edges of the heliosphere. - The rotation of the sun makes the magnetic field **B** to have an almost torodial direction denoted by $\odot$ and $\otimes$, with respect to the equatorial plane. Since the charged particles in this region follow trajectories displayed by the line arrows (current **j**), the force $\mathbf{f} = \mathbf{j} \times \mathbf{B}$ exerted on them should have the direction pointed by the boxed arrows. (6)

Except from the Ruytov's approach some other attempts have given stricter constraints on the photon mass. The best of them was first proposed by Chibisov (9) and later supported by Adelberger, Dvali and Gruzinov (7) and it gives a photon mass limit of $m < 3 \cdot 10^{-27} eV/c^2$. When Proca electrodynamics is considered, their model deduces that the average magnetic pressure exerted on the interstellar medium of our galaxy takes the form:

$$p_{magnetic} = \frac{\mathbf{B}^2}{24\pi} - \frac{m^2\mathbf{A}^2}{24\pi} \tag{2.28}$$

If one assumes that the interstellar medium is a stable system, this magnetic pressure should be equal to the sum of the plasma pressure and the plasma kinetic energy. In the framework of conventional electrodynamics, where the second term of the above equation is not present, the magnitude of the standard magnetic pressure is comparable to the other two that form the sum. Therefore, the magnitude of the $\frac{m^2\mathbf{A}^2}{24\pi}$ term should not exceed the standard magnetic pressure, which demands that $m^2\mathbf{A}^2 \lesssim B^2$. When this constraint is combined with the approximate form of the vector potential in galactic scales R; $A \sim B \cdot R$, it leads to the aforementioned upper bound. The opponents of the above idea doubt its reliability because it makes use of a kind of virial theorem which is not applicable at the scale of the galactic cluster, since there is no indication that the system is necessarily isolated.

In conclusion, the research for the limits of the photon mass is in continual development. Better understanding of extra galactic astrophysics along with the collection of more precise astronomical data could radically contribute to the further constrain of the photon mass value. It is reasonable though to assume that if the existence of a non zero photon mass is valid, then the ultimate value of the corresponding Compton wavelength should not exceed the length of $c \cdot T$ where T is the age of the universe.

3
The graviton mass

The gravitational theory proposed by Einstein in 1916 is considered one of the most well established theories in physics. Based on a geometric point of view, general relativity (GR) satisfies the Equivalence principle, it contains field equations with linear relations in the 2nd derivatives and in the limit of the weak fields, the Newton's gravitational law is recovered.

From the perspective of quantum field theory, there have been several attempts to quantize gravity. In this section, the field equations of a massive graviton will be presented. After a brief discussion about the problems encountered through the introduction of a graviton mass, its existence will be studied from a phenomenological point of view. Finally experimental considerations, evaluating the value of the graviton mass, will be presented.

3.1 Pauli-Fierz theory and v-DVZ discontinuity

A vital problem of the quantization of gravity is that GR is a perturbatively non renormalizable theory, since the gravitational coupling constant is negative dimensional. However, if one tries to introduce the Feynman rules for this theory, one should begin with the introduction of the symmetric graviton field $h_{\mu\nu}(x)$, which can be considered a small fluctuation of flat spacetime.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3.1}$$

where $\eta_{\mu\nu}$ is the Minkowski metric.

Inserting the above expansion of the metric in the expression for the Christoffel symbols and the Ricci tensor, one obtains:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left\{ \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} + \frac{\partial g_{\rho\mu}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\rho}} \right\} \rightarrow \Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} \eta^{\lambda\rho} \left\{ \frac{\partial h_{\rho\nu}}{\partial x^{\mu}} + \frac{\partial h_{\rho\mu}}{\partial x^{\nu}} - \frac{\partial h_{\mu\nu}}{\partial x^{\rho}} + \mathcal{O}(h^2) \right\} \tag{3.2}$$

$$R_{\mu\nu} \equiv R^{\lambda}_{\mu\lambda\nu} \equiv \frac{\partial \Gamma^{\lambda}_{\mu\lambda}}{\partial x^{\nu}} - \frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^{\lambda}} + \Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\nu\eta} - \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\lambda\nu} \to$$
$$R_{\mu\nu} = \frac{1}{2}\left\{\Box h_{\rho\nu} + \frac{\partial^2}{\partial x^{\nu}\partial x^{\mu}}h^{\lambda}_{\lambda} - \frac{\partial^2}{\partial x^{\lambda}\partial x^{\mu}}h^{\lambda}_{\nu} - \frac{\partial^2}{\partial x^{\lambda}\partial x^{\nu}}h^{\lambda}_{\mu} + \mathcal{O}(h^2)\right\} \tag{3.3}$$

The Einstein field equations up to second order terms in h are:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu} \Rightarrow$$
$$\Box(h_{\mu\nu} - g_{\mu\nu}h) + \partial_{\nu}\partial_{\mu}h - \partial_{\lambda}\partial_{\mu}h^{\lambda}_{\nu} - \partial_{\lambda}\partial_{\nu}h^{\lambda}_{\mu} - \eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}h_{\alpha\beta} = 16\pi G T_{\mu\nu} \tag{3.4}$$

where $G_{\mu\nu}$ is the Einstein tensor.

The first modification of massless gravity was attempted by Pauli and Fierz in 1939. Assuming an approximately flat spacetime, they added a mass term to the Einstein field equation:

$$G_{\mu\nu} - m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = -8\pi G T_{\mu\nu} \tag{3.5}$$

The form of the above equation is in accordance with the “potential energy” density being bounded from below. In fact, any other linear combination of $h_{\mu\nu} - \eta_{\mu\nu}h$ leads to the appearance of an unnatural particle moving faster that light (“tachyon").

Alternatively, the modified Einstein field equations can be derived from the **Pauli-Fierz action**:

$$S = \int d^4x(-\frac{1}{2}\partial_{\lambda}h_{\mu\nu}\partial^{\lambda}h^{\mu\nu} + \partial_{\mu}h_{\nu\lambda}\partial^{\nu}h^{\mu\lambda} - \partial_{\mu}h^{\mu\nu}\partial_{\nu}h + \frac{1}{2}\partial_{\lambda}h\partial^{\lambda}h$$
$$- \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)) \tag{3.6}$$

Both the action and the modified Einstein field equations correspond to the dynamics of a spin-2 massive boson coupling to matter.

The Pauli-Fierz model as presented here seems to be contradictory to the basic idea of general relativity, since it requires the choice of a particular spacetime metric, namely the Minkowski metric, through which the mass effects are defined. Additionally, a careful reader may have noticed that the above theory does not respect the general coordinate covariance, i.e the action does not preserve its form under a general coordinate transformation $x \to x'$. However the coordinate invariance can be restored through the Stuckelberg trick just as the Lorentz invariance is restored in the massive photon theory.

Apart form the Pauli Fierz theory, several ideas have been proposed, claiming the existence of a massive graviton. Most of them, though, have to overcome an obstacle known as **vDVZ-discontinuity**. Van Dan, Veltman and separately Zakharov compared the results of massive gravity to the ones from GR in the limit that m_g tends to zero ($m_g \to 0$) and discovered that the observable quantities do not coincide. More specifically, in their article (16) Van Dam and Veltman estimated that in the small mass limit the bending angle of light by a massive body approaches $\frac{3}{4}$ of the Einstein's result. The physical interpretation of this phenomenon lies in the fact that a massive spin-2 field carries 5 polarizations, while a massless one only 2. Thus in the massless limit, the 5 spin states of massive graviton become 2 helicity states of a massless graviton, 2 helicity states of a massless vector and a single massless scalar. This remnant of the $m_g \to 0$ limit behaves like a longitudinal graviton and it is the one responsible for the observed discontinuity.

The stronger opposition to the vDVZ-discontinuity was proposed some years later by Vainstein, who observed that the above study refers to linear theories. Through generalization to a complete non-linear theory, strong non-linearities appear in the massless limit and they can compensate for the action of the spin-0 field.

3.2 Experimental estimations of m_g

Even though the existence of a graviton with non-zero mass is accompanied by several theoretical problems, a lot of research has been conducted for the examination of this issue phenomenologically. In correspondence with the photon mass theory, we could expect that the implications of a massive graviton will be quite similar. In particular from linearised massive gravity theories, it follows that a massive graviton verifies the Klein-Gordon equation. Indeed, if we consider only first order terms of h from the expression of $G_{\mu\nu}$ as expressed in 3.4, we get :

$$(\nabla^2 - \frac{1}{c^2}\partial_t^2 - m^2)(h_{\mu\nu} - \eta_{\mu\nu}h) = 0 \tag{3.7}$$

Thus, we identify the wave equation and conclude that gravitational waves should no longer be dispersionless. This observation if applied to the physics of binary pulsars can give significant data for the determination of m_g. Binary pulsars are astronomical objects that have been used as nature's laboratories. Taking advantage of the strong gravitational potential in their vicinities, scientists use them to test the theory of gravity. They consist of a pulsar (rotating neutron star with period of 10^{-2} to 10^{-1}) and a non pulsating companion star that may be a white dwarf or a neutron star. Starting from a linearised theory of gravity, Finn and Sutton (17) calculate the rate of the energy loss in a binary pulsar as a function of the mass. The upper bound for m_g derived from the discrepancy between the observed and the predicted orbital decay rates of PSR B1913+16 and PSR B1534+12 was estimated $m_g \leqslant 7,6 \times 10^{-20} \frac{eV}{c^2}$.

Other considerations for the graviton mass bounds have been triggered by the claim that for $m_g \neq 0$, the gravitational force will come from a Yukawa-type potential: $\sim \exp(-m_g r)/r$. In (12) the analysis of the behaviour of gravity at large distances and its deviation from the $V(r) \sim \frac{1}{r^2}$ formula for galaxy clusters ($r \sim 580$ kpc) gave $m_g \sim 10^{-29} \frac{eV}{c^2}$.

Currently the best approved bound of m_g comes from **weak gravitational lensing data** from stellar clusters (14). These effects have been proved to be valuable for the progress of astrophysics and cosmology. Numerous data have provided a great field for testing GR as well as indications for dark matter.

The geometry of a gravitational lens is presented in Figure 3.1 . When light propagates in a vicinity near a massive object, the latter's gravitational potential causes both the deflection of the light beam and the modification of the observed object's size and shape. This deformation of the final image is described by the deflection angle:

$$\vec{\beta} = \vec{\theta} - \frac{D_{ds}}{D_s}\hat{a}(D_d\theta) \Rightarrow \vec{a} \equiv \frac{D_{ds}}{D_s}\hat{a}(D_ds) = \frac{4GM}{c^2}\frac{D_{ds}}{D_sD_d}\frac{\vec{\theta}}{|\vec{\theta}|^2} \tag{3.8}$$

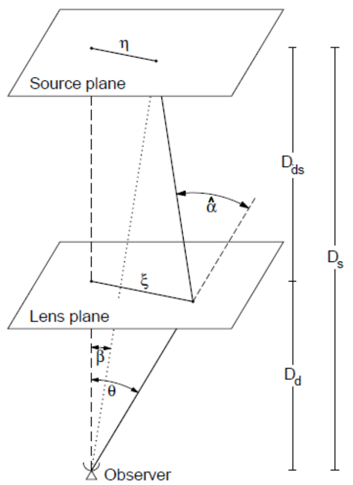

Figure 3.1: The basic geometry of gravitational lenses - When a beam of light passes through the lens plane the beam of light is deflected. Therefore while β should be the angle of sight of a distant object with respect to the perpendicular to the source plane, the observed angle is θ and it is related to the previous one through the deflection angle α. The symbols D_d, D_{ds}, D_s correspond to the angular diameter distances between the objects present in the schematic (15).

Nevertheless, if the lens is considered to be a collection of point masses such as an entire galaxy or clusters, then one should introduce the surface density of the lens $\Sigma(D_d\theta)$ as compared to the critical one, $\Sigma_{cr} = \frac{c^2}{4\pi G}\frac{D_s}{D_dD_{ds}}$. Then, the function of the deflection angle $\vec{a}$ generalizes to :

$$\vec{a}(\vec{\theta}) = \frac{1}{\pi}\int_{\Re^2} d^2\theta' \kappa(\theta') \frac{\vec{\theta} - \vec{\theta'}}{|\vec{\theta} - \vec{\theta'}|^2} \tag{3.9}$$

which can be written as

$$\vec{a} = \vec{\nabla}\Psi \quad , \quad \Psi(\vec{\theta}) = \frac{1}{\pi}\int_{\Re^2} d^2\theta' \kappa(\theta') \log(\vec{\theta} - \vec{\theta'}) \tag{3.10}$$

Actually this deflection potential Ψ is the 2-d analogue of the Newtonian gravitational potential and it satisfies the Poisson equation $\nabla^2\Psi = 2\kappa(\vec{\theta})$.

The above analysis refers to massive objects being in a specified comoving distance w. If we wish to consider a group of points lying in various w , we should use the formula:

$$a(\vec{\theta}, w) = \frac{2w}{c^2}\int_0^1 dy(1-y)\nabla_\perp\Phi(wy\vec{\theta}, wy) \quad , \quad y \equiv \frac{w'}{w} \tag{3.11}$$

where we assumed that the universe is flat, i.e. k=0, the scale factor $f_K(w) = w$. the potential Φ is the 3-dimensional generalization of the deflection potential Ψ.

The basic features used to describe the effects of gravitational lensing are:

- convergence (indicative of the magnification of the source)
- shear (the amount of stretching of the background objects in the tangential direction with respect to the foreground mass).

The convergence κ is related to the deflection angle

$$\kappa(\vec{\theta}) = \frac{1}{2}\nabla_\theta\vec{a}(\vec{\theta}) = \frac{1}{2}\frac{\partial\vec{a}(\vec{\theta})}{\partial\theta_i} \tag{3.12}$$

and consequently it is related to the potential Φ derived from the Poisson equation.

$$\nabla^2\Phi = \frac{3H_0^2}{2a}\Omega_0^2\delta \tag{3.13}$$

where δ stands for the density of the source. [1]

The most common method of constraining the cosmological parameters is based on Fourier space and the important tool is the power spectrum $P_\kappa(l)$:

$$< \hat{\kappa}(l)\hat{\kappa}(l') >= (2\pi)^2\delta(\vec{l} - \vec{l'})P_\kappa(l) \tag{3.14}$$

Being simply the Fourier transform of the correlation function of convergence, the power spectrum $P_\kappa(l)$ depends on the 3-dim matter power spectrum $P_\delta(\frac{l}{w}, w)$:

$$P_\kappa(l) = \frac{9H_0^4\Omega_m^2}{4c^2}\int_0^{w_s} dw\left(1 - \frac{w}{w_s}\right)^2 P_\delta\left(\frac{l}{w}, w\right) \tag{3.15}$$

This equation is derived in the framework of conventional gravity provided that the source is a point mass at $w = w_s$. If instead we assume that the gravitational field is Yakuwa-type i.e. $\sim \frac{e^{-m_g r}}{r}$, then the potential Φ should be modified to :

$$(\nabla^2 - m^2)\Phi = \frac{3H_0\Omega_m^2}{2a}\delta \tag{3.16}$$

[1] This formula is simply an expression from Newtonian gravity $\nabla_r^2\Phi' = 4\pi G\rho$ where we considered a matter dominated space and the density $\rho = (1+\delta)\overline{\rho}$ appears to have small perturbations δ.

This would imply a modification for the power spectrum $P_\kappa^m(l)$ given by:

$$P_\kappa^m(l) = \frac{9H_0^4\Omega^2}{4c^2}\int_0^{w_s} dw \frac{P_\delta(\frac{l}{w}, w)}{a^2(w)} \left[\frac{\frac{l^2}{w^2}}{\frac{l^2}{w^2} + m^2}\right]^2 \tag{3.17}$$

By comparing and contrasting the results for P_κ and P_κ^m to the observational data from a cluster of stars at redshift of $z = 1.2$, Choudhury, Joshi, Mahajan and McKellar estimated that $m_g \leq 100Mpc^{-1}$. More specifically, the group examined the behaviour of the power spectrum of P_κ, which is defined as:

$$\gamma^2(\theta) = \frac{2}{\pi\theta^2}\int_0^\infty dl P_\kappa(l) J_1^2(l\theta) \tag{3.18}$$

J_n stands for the Bessel function of order n. Fixing the including cosmological parameters the aforementioned scientists obtained the plot of γ^2 as a function of θ (Figure 3.2)

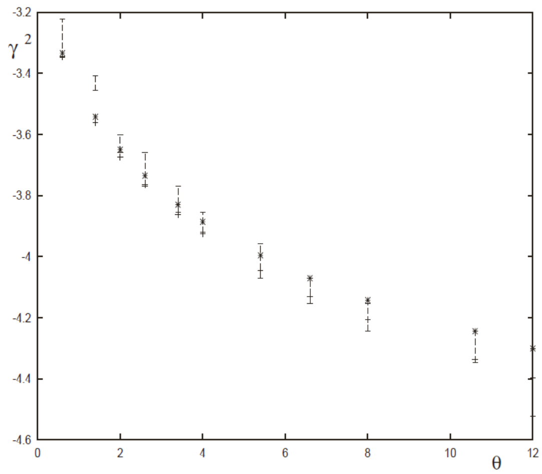

Figure 3.2: Diagrammatic representation of the parameter γ as a function of the angle θ - The data have been obtained in the framework of both the usual Newtonian gravity $(*)$ and the modified one which includes a photon with $m^{-1} = 100Mpc$ $(+)$.

They made the observation that the model of the massive gravity with $m^{-1} = 100$Mpc can give points lying in the region defined by the experimental error bars. Thus the bound $m^{-1} = 100$Mpc corresponding to $\boxed{m \leq 7 \cdot 10^{-32}\frac{eV}{c^2}}$ is the best one for the graviton mass compatible to observational data.

4 The Fine Structure Constant

The fine structure constant is a dimensionless quantity defined as:

$$\alpha = \frac{\mu_0 c e^2}{2h} = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{e^2}{\hbar c} = \frac{e^2}{4\pi} \tag{4.1}$$

where the e is the elementary charge of the electron, $\hbar = \frac{h}{2\pi}$ is the Planck constant, c is the speed of light in vacuum, ϵ_0 the electric and μ_0 the magnetic permittivity of free space. The last two definitions are most commonly encountered in the literature. They are expressed in the electrostatic cgs units ($4\pi\epsilon_0 = 1$) and the natural units ($c = h = 1$) respectively.

Lying principally in the field of atomic physics, the fine structure constant α was named from the analysis of the relativistic splitting of the degenerate levels of Bohr atom. Sommerfield was the first to introduce α in 1916 through the fine structure formula:

$$\frac{W(n,k)}{m_0 c^2} = \left\{ 1 + \frac{\alpha^2 Z^2}{[(n-k) + \sqrt{k^2 - \alpha^2 Z^2}]^2} \right\}^{1/2} - 1 \tag{4.2}$$

in his attempt to express the electron's binding energy $W = E - E_0$ in terms of the principal n and azimuthal k quantum numbers in hydrogen-like atoms.

The fact that it is a dimensionless constant, appearing in almost every electromagnetic phenomenon, interested well-acknowledged scientists in the early 1900s who tried to determine its value and understand it. Through these attempts α obtained various interpretations:

- It can be viewed as the squared ratio of the electron's elementary charge over the Planck charge: $\alpha = \left(\frac{e}{q_p}\right)^2$
- It correlates the classical electron radius r_e to the Bohr radius a_0 and the Compton wavelength of the electron λ_e: $r_e = \frac{\alpha \lambda_e}{2\pi} = \alpha^2 a_0$
- It is classified among the coupling constants of the fundamental interactions. In particular, it is used to determine the strength of the electromagnetic interaction in the low energy limit.

This last property made α extremely interesting from the theoretical point of view and caused experimental physicists to evaluate it in the framework of QED. However, high precision experiments which lie in different areas of physics have been proposed too. Some of them use phenomena like the quantized Hall effect or the Josephson effect and they are known as non QED determination methods.

4.1 Quantum electrodynamics, α and a_e

The history of α is strongly correlated with the transition from quantum mechanics to the quantum theory of fields. Actually the exact measurement of the anomalous magnetic moment of electron a_e was a triumph of physics that also allowed a precise measurement for α. The calculation of a_e from the Feynman diagrams and the correction when additional virtual photons are added is the process that gave the dependence of a_e from α with impressive precision.

The scattering of an incoming electron of momentum p from a fixed electromagnetic potential A_μ is given through the interaction term $\overline{\psi}\gamma^\mu A^\mu \psi$ in the Lagrangian density:

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \overline{\psi}[\gamma_\mu(\partial^\mu + ieA^\mu) + m]\psi \tag{4.3}$$

In the language of Feynman diagrams, the interaction of an electron of momentum p with a heavy target of momentum k is presented in the picture.

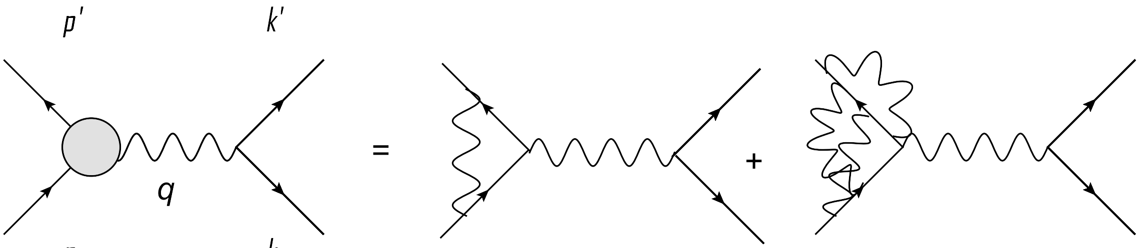

Figure 4.1: Feynman diagrams corresponding to the scattering of an electron of momentum p from a heavy target. - The loop corrections up to 2nd order have been considered.

The use of the gray circle indicates the fact that in the general case every possible combination of additional virtual photons can be considered. In terms of the calculations the sum of all the possible vertices is denoted as $-ie\Gamma^\mu(p, p')$ instead of $-ie\gamma^\mu$ used for the simple diagram (i). Therefore the amplitude for the scattering from a heavy target of initial momentum k is:[1]

$$i\mathfrak{M} = ie^2(\overline{u}(p')\Gamma^\mu(p,p')u(p))\frac{1}{q^2}(\overline{u}(k')\gamma_\mu u(k)) \tag{4.4}$$

The requirement that the fermion should be on shell constrain Γ^μ to be a function of p'^μ, p^μ, γ^μ. Its precise form can be further determined if we take into consideration the Lorentz invariance, the discrete symmetries of QED as well as the Ward identity. Applying the above ideas and using the Gordon identity:

$$\overline{u}(p')\gamma^\mu u(p) = \overline{u}(p')\left[\frac{p'^\mu + p^\mu}{2m} + \frac{i\sigma^{\mu\nu}q^\nu}{2m}\right]u(p) \tag{4.5}$$

we conclude that:

$$\Gamma^\mu(p,p') = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2) \tag{4.6}$$

where $F_1(q^2)$ and $F_2(q^2)$ are the form factors and $q = p' - p$ the momentum transfer or the momentum of the virtual photon.

We could constrain the problem even more and study the motion of a non-relativistic electron in the region of a non-zero electrostatic potential $A^{cl}_\mu(x) = (\Phi(\vec{x}), 0) \to A^{cl}_\mu(q) = (2\pi\delta(q^0)\Phi(\vec{q}), 0)$. Then, by comparison of $i\mathfrak{M}$ to the amplitude derived from the first order Born approximation $f(p', p)$ we identify that to leading order $F_1(0) = 1$.

In the same way if we consider the limiting case of an electron moving under the influence of a static vector potential $A^{cl}_\mu(x) = (0, \vec{A}^{cl}_\mu(\vec{x}))$, we can estimate the electron's magnetic moment. More specifically, $i\mathfrak{M}$ can be written:

$$i\mathfrak{M} = -i2me\xi'^\dagger\left(\frac{-1}{2m}\sigma^k[F_1(0) + F_2(0)]\right)\xi\tilde{B}^k(q) \tag{4.7}$$

where the k component of the magnetic field induced due to A_{cl} is $\tilde{B}^k(q) = -i\epsilon^{ijk}q^i\tilde{A}^j_{cl}$. Comparing again the above result to the corresponding amplitude obtained from the Born approximation of an electron moving around a potential well: $V(x) = -<\vec{\mu}><\vec{B}(\vec{x})>$, we get

$$\vec{\mu} = \frac{e}{m}[F_1(0) + F_2(0)]\xi'^\dagger\frac{\vec{\sigma}}{2}\xi \tag{4.8}$$

Thus we conclude that the electron's magnetic moment is given by the equation:

$$\vec{\mu} = \frac{e}{2m}g\vec{s} \tag{4.9}$$

where g is the Lande factor : $g = 2(F_1(0) + F_2(0)) = 2(1 + F_2(0))$

A more detailed calculation of the $F_2(0)$ to leading order of perturbation theory is presented below in order to derive its dependence on α. We write Γ_μ as $\Gamma^\mu = \gamma^\mu + \delta\Gamma^\mu$. The correction in $\delta\Gamma^\mu$ to first order in α is obtained from the diagram of the Figure 4.2.

[1] The $\overline{u}(p)$ are the well known 4-vectors obeying the Dirac equation: $(\gamma^\mu p_\mu - m)u(p) = 0$ and they correspond to the positive energy solutions. According to the spin, they can be written as $u^s(p) = \binom{\sqrt{p\times\sigma}\xi^s}{\sqrt{p\times\overline{\sigma}}\xi^s}$, $s = 1, 2$. The ξ's are normalized vectors like $\xi^1 = \binom{1}{0}, \xi^2 = \binom{0}{1}$, so that the following normalization relation holds for the u's: $\overline{u}^r(p)u^s(p) = 2m\delta^{rs}$

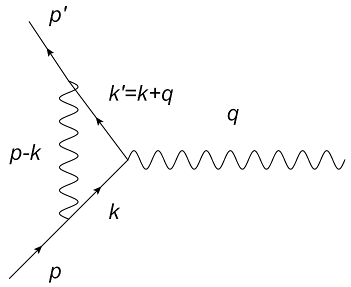

Figure 4.2: The Feynman diagram corresponding to the amplitude $\overline{u}\delta\Gamma^{\mu}u$ - The scattering of a fermion with initial momentum p is presented in the 1-loop diagram

$$
\begin{aligned}
ie\delta\Gamma^{\mu} &= \int \frac{d^Dk}{(2\pi)^D}(-ie\gamma^{\rho})\frac{i(\not{p}' - \not{k} + m)}{(p'-k)^2 - m^2_i\epsilon}(-ie\gamma^{\mu})\frac{i(\not{p} - \not{k} + m)}{(p-k)^2 - m^2 + i\epsilon}(-ie\gamma^{\sigma})\frac{-i\eta_{\rho\sigma}}{(k^2 + i\epsilon)} \\
&= e^3 \int \frac{d^Dk}{(2\pi)^D}\frac{\gamma^{\nu}(\not{p}' - \not{k} + m)\gamma^{\mu}(\not{p} - \not{k} + m)\gamma_{\nu}}{[(p'-k)^2 - m^2 + i\epsilon][(p-k)^2 - m^2 + k][k^2 + i\epsilon]}
\end{aligned}
\tag{4.10}
$$

Applying the Schwinger trick and shifting the variables: $k' = k - xp' - yp$:

$$
\begin{aligned}
\delta\Gamma^{\mu} = -2ie^2 &\int_0^1 dx \int_0^1 dy \int_0^1 dz\delta(1 - x - y - z) \\
&\int \frac{d^Dk}{(2\pi)^D}\frac{\gamma^{\nu}[(1-x)\not{p}' - y\not{p} - \not{k} + m]\gamma^{\mu}[-x\not{p} + (1-y)\not{p} - \not{k} + m]\gamma_{\nu}}{[k^2 + xyq^2 - (1-z)^2m^2 + i\epsilon]^3}
\end{aligned}
\tag{4.11}
$$

Manipulating the γ matrices of the numerator and applying the Dirac equations for the spinors $\overline{u}'\not{p} = m\overline{u}'$, $\not{p}u = mu$, the product $\overline{u}\delta\Gamma^{\mu}u$ splits into two terms:

$$
\begin{aligned}
\overline{u}\delta\Gamma^{\mu}u = -2ie^2 &\int_0^1 dx \int_0^1 dy \int_0^1 dz\delta(1 - x - y - z) \\
\int \frac{d^Dk}{(2\pi)^D}\overline{u}&\left\{\frac{\gamma^{\mu}[k^2\frac{(D-2)^2}{D} + q^2(-2(1-x)(1-y) + \epsilon xy) + m^2[4 - 8(x+y) + 2(x+y)^2 - \epsilon(x+y)^2]]}{[k^2 + xyq^2 - (1-z)^2m^2 + i\epsilon]^3}\right\}u \\
+ \quad \overline{u}&\left\{\frac{\frac{(p'^{\mu} + p^{\mu})}{2m}m^2[4(x+y)(1-x-y) + 2\epsilon(x+y)^2]}{[k^2 + xyq^2 - (1-z)^2m^2 + i\epsilon]^3}\right\}u
\end{aligned}
\tag{4.12}
$$

Using the Gordon identity 4.5 we have:

$$
\overline{u}\left[\frac{p'^{\mu} + p^{\mu}}{2m}\right]u = \overline{u}\left[\gamma^{\mu} - \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}\right]u
\tag{4.13}
$$

and therefore:

$$\overline{u}\delta\Gamma^\mu u = -2ie^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz\delta(1-x-y-z)$$
$$\int \frac{d^Dk}{(2\pi)^D}\overline{u}\left\{\frac{\gamma^\mu[k^2\frac{(D-2)^2}{D} + q^2(-2(1-x)(1-y)+\epsilon xy) + m^2[4-4(x+y)-2(x+y)^2+\epsilon(x+y)^2]]}{[k^2+xyq^2-(1-z)^2m^2+i\epsilon]^3}\right\}u$$
$$+ \quad \overline{u}\left\{\frac{-i\sigma^{\mu\nu}q_\nu}{2m}\frac{m^2[4(x+y)(1-x-y)+2\epsilon(x+y)^2]}{[k^2+xyq^2-(1-z)^2m^2+i\epsilon]^3}\right\}u \tag{4.14}$$

Comparing the above formula to 4.6 we identify the factor multiplying the $\frac{i\sigma^{\mu\nu}q_\nu}{2m}$ as the F_2 form factor, namely [1];

$$\begin{aligned}F_2^{1-loop}(q^2) &= -2ie^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz\delta(1-x-y-z)\\ &\int \frac{d^Dk}{(2\pi)^D}\overline{u}\left\{\frac{m^2[-4(x+y)(1-x-y)-2\epsilon(x+y)^2]}{[k^2+xyq^2-(1-z)^2m^2+i\epsilon]^3}\right\}u\\ &= -2ie^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz\delta(1-x-y-z)\frac{\Gamma(3-\frac{D}{2})}{\Gamma(3)}\frac{(-im^2)}{(4\pi)^{D/2}}\\ &[-4z(1-z)-2\epsilon(1-z)^2][(1-z)^2m^2-xyq^2-i\epsilon]^{\frac{D}{2}-3}\end{aligned} \tag{4.16}$$

If we consider the case of interest where $q^2 = 0$ and apply the additional constraint of 4 dimensions (i.e. $D \to 4 \Rightarrow D = 4 - \varepsilon$ and finally taking the limit where both $\epsilon, \varepsilon \to 0$), we obtain:

$$F_2^{1-loop}(q^2 = 0) = \frac{\alpha}{2\pi} \tag{4.17}$$

The above result was first presented by Schwinger in 1948. The fact that the theoretical estimation was in a good agreement with the experimental determination of the anomalous magnetic moment of electron a_e was a strong evidence in favour of quantum electrodynamics and a memorable achievement for physics in general. The following years this estimation was not only theoretically verified but also extended to higher orders of α, obtained by considering the contribution of diagrams of more than 1-loop. Quite contrary to the 1-loop case, for higher loop diagrams both ultraviolet and infrared divergences emerge. They are eliminated through renormalization techniques, so that the values corresponding to physical processes reveal no infinities.

The most general formula giving the anomalous magnetic moment of the electron a_e contains contributions from other interactions besides the electromagnetic one.

$$a_e = a_e(QED) + a_e(hadronic) + a_e(electronic) \tag{4.18}$$

[1] In this calculation the method of dimensional regularization was used as well as the relation:

$$\int \frac{d^Dk}{(2\pi)^D}\frac{(k^2)^\alpha}{(k^2-a^2+i\epsilon)^\beta} = \frac{i(-1)^{\alpha-\beta}}{(4\pi)^{D/2}}\frac{\Gamma(\frac{D}{2}+\alpha)}{\Gamma(\frac{D}{2})\Gamma(\beta)}(a^2-i\epsilon)^{\frac{D}{2}+\alpha-\beta} \tag{4.15}$$

The QED's contribution is of course the major one and it can be written as a function of terms depending on the masses of electron m_e, muon m_μ and tauon m_τ. Since a_e is dimensionless and it is required to contain no divergences, only the fractions of lepton masses should be inserted.

$$a_e(QED) = A_1 + A_2(\frac{m_e}{m_\mu}) + A_2(\frac{m_e}{m_\tau}) + A_3(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}) + \cdots \tag{4.19}$$

What is more, each A_i term is perturbatively expanded in terms of α

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \cdots \qquad i = 1, 2, 3, \cdots \tag{4.20}$$

At present $A_1^{(2)}, A_1^{(4)} \& A_1^{(6)}$ are known analytically while the functions of higher α are evaluated through the use of specially designed computer programs. In extremely precise experimental measurements of a_e, which will be described in the next section, the contribution from hadronic and electroweak interactions cannot be neglected (37),(31):

$$a_e(hadronic) = 1.689(20) \times 10^{-12} \quad , \quad a_e(electroweak) = 0.030 \times 10^{-12} \tag{4.21}$$

4.2 High precision experiments for α

As stated above, the pursue of determining α numerically is a task of great interest which required the cooperation of scientists from various branches of physics. Atomic physics experimentalists where the first to deal with this issue, mainly by trying to measure more and more accurately the difference in energy for different atomic levels, such as the transition $2P_{3/2} - 2P_{1/2}$ of the Hydrogen atom. Even the most accurate atomic methods, though do not seem to exceed the precision of 1ppm= 10^{-6}. This is due to the high complexity that characterizes the study of $2e^-$ electron atomic energy levels as well as the lack of information as far as the structure of the proton is concerned.

Nowadays, the uncertainty of the most accurate acclaimed methods is better than 10^{-7}. These experiments cover a wide range from high precision particle experiments to applications of solid state physics. According to the way of determining the fine structure constant, the corresponding experimental methods are classified as direct and indirect. This classification will be used for the following brief presentation of the most accurate experiments.

4.2.1 Indirect methods

This category includes measurements of parameters such as $\frac{e}{h}$ or $\frac{m}{h}$ and R_∞. The great majority of these experiments make use of the formula giving the Rydberg constant in terms of α, m_e the electron mass, the speed of light c and Planck constant h:

$$R_\infty = \frac{m_e c^4}{2h}\alpha^2 \tag{4.22}$$

One of the methods is based on the measurement of the fraction $\frac{h}{m_n}$, m_n being the neutron mass. This fraction can be encountered in the de Broglie formula relating the wavelength λ to the velocity u of a particle: $\lambda = \frac{h}{m_n u}$. Measuring the parameters λ and u of a neutron beam, an experimental group in Grenoble managed to determine the quotient $\frac{h}{m_n} = 3.956\ 033\ 320\ (302) \cdot 10^{-7} m^2 s^{-1}$. The measurement was probed through the Bragg scattering of a neutron beam from a silicon crystal. Therefore, replacing the above value in the following formula :

$$\alpha^2 = 2\frac{R_\infty}{c}\frac{m_n}{m_e}\frac{h}{m_n} \tag{4.23}$$

they concluded to the value: $\frac{1}{\alpha} = 137.\ 036\ 010\ 82(524)$ uncertainty ($0.039ppm$). The uncertainty of α^{-1} follows from the accuracy to which the values of R_∞ and the atomic masses of neutron M_n and electron M_e are known.

$$R_\infty \sim 10\ 973\ 731.\ 568\ 34\ (24)\ m^2 s^{-1}$$
$$M_n = 1.\ 008\ 664\ 923\ 43\ (221) u$$
$$M_e = 0.\ 000\ 548\ 579\ 9111\ (12) u$$

An advantage of this estimation lies in the fact that it requires no QED calculation. Quite to the contrary, its validity depends on the correctness of formula 4.23, which is actually a modification of eq. 4.22.

Similar ideas giving even more accurate measurements have been proposed lately. In particular, an innovative experiment was carried out in 2007 by Bouchendira et al.(34). Instead of determining the quotient $\frac{h}{m_n}$, they estimated the fraction $\frac{h}{m_{Rb}}$, through the measurement of the recoil velocity $u_r = \frac{\hbar k}{m_{Rb}}$, that is acquired by a Rb atom when it absorbs a photon of momentum $\hbar k$. More specifically, through these techniques many recoils are transferred to the atoms Rb^{87} at rest and excite them from the $F = 2$ hyperfine sub-level to the $F = 1$ one. While at this state, the atoms are bombarded through recoils from Bloch oscillations which cause the increase of their velocity by $2u_r$ accompanied by no simultaneous excitation. Finally, this velocity is measured through the interaction of the Rb atoms with the 2 pulses, causing their transition to F=2 hyperfine level. Having determined the ratio $\frac{h}{m_{Rb}}$, the group combined it with the values of the other parameters in the relation:

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_{Rb}}{m_e}\frac{h}{m_{Rb}} \tag{4.24}$$

and they concluded to $\boxed{\alpha^{-1}\left(\frac{h}{m_{Rb}}\right) = 137.\ 035\ 99\ 037\ (91)}$ (uncertainty $6.6 \cdot 10^{-10}$)

The phenomena of solid state physics and in particular the quantum Hall and the Josephson effects have triggered some other independent measurements of α. The AC Josephson effect describes the current created in the edges of a Josephson junction due to tunnelling of Cooper pairs in 2 superconductors separated by an insulating material. When a fixed voltage is applied across the junction, an AC current is induced. The frequency of I_C is related to the voltage V_{DC} through:

$$\omega = \frac{1}{h} 2eV_{DC} \tag{4.25}$$

Thus the measurement of ω and V_{DC} allows the determination of $\frac{e}{h}$ and consequently of α since:

$$\alpha^{-2} = \frac{c}{4R_\infty \gamma_p} \frac{\mu_p}{\mu_B} \frac{2e}{h} \tag{4.26}$$

where μ_p is the magnetic moment of the proton in water, μ_B the Bohr magnetic moment and γ_p the gyromagnetic ratio of the proton in the water.

4.2.2 Direct methods

An example of a direct experimental method is the measurements of the Hall effect. The Hall effect measurement belongs to the direct experimental methods. During this type of experiment, the behaviour of a thin layer of (almost 2-dimensional) electron gas is studied while it is subjected to a perpendicular strong magnetic field B in low temperature conditions. The voltage across the sample V_H is related to the current I_H flowing along the sample through the Hall resistance:

$$R_H^{(n)} \equiv \frac{V_H}{I_H} = \frac{h}{e^2 n} \qquad n = 1, 2, \cdots \tag{4.27}$$

which appears to be quantized. The measurement of R_H directly implies the determination of the fine structure constant, since for a certain integer value of n it explicitly includes α

$$\frac{1}{\alpha} = \frac{2nR_H^{(n)}}{\mu_0 c} \tag{4.28}$$

The most competitive value of α was obtained through data from the estimation of the anomalous magnetic moment of the electron a_e, also known as $g-2$ experiments. Actually the measurement of a_e carried out by the University of Washington group in 1987 was for many years considered as the cornerstone of the high energy precision experiments determining α (α^{-1} = 137.035 998 83 (50) uncertainty $3.7 \cdot 10^{-9}$) (22). This accuracy was superseded by the latest determination of α^{-1} accomplished by the Harvard group of Hanneke et al.(35)

Both measurements aim to confine the electron's motion in order to achieve better control over it and longer observation time. This is done with the use of the Penning trap; a device that imposes electric and magnetic field to a charged particle. The composite motion of an electron in a Penning trap can be thought of as the combination of 2 distinguished motions:

- a harmonic oscillation with frequency ω_z induced on the axial direction due to the weak static electric quadrupole potential $V(z,r) \sim z^2 - \frac{\rho^2}{2}$, where z, ρ are used for denoting the cylindrical coordinates.

- an epitrochoid; a radial motion caused by the magnetic and electric fields and characterized by 2 frequencies; namely the magnetron frequency ω_- and the modified cyclotron frequency ω_+. The well known cyclotron frequency, $\omega_c \equiv \frac{q}{m}B = \omega_+ + \omega_-$ is related to ω_z, ω_+ and ω_- through $\omega_c^2 = \omega_+^2 + \omega_-^2 + \omega_z^2$.

The above description refers to an ideal Penning trap. However in the real situation there are various misalignments from the axial symmetry which are responsible for the slight modification of the frequencies of the cyclotron ν_c , of the axial motion ν_z, and of the magnetron motion ν_m to $\overline{\nu}_c$, $\overline{\nu}_z$, and $\overline{\nu}_m$ respectively. Now the invariance theorem of the frequencies is stated as: $\nu_c^2 = \overline{\nu}_c^2 + \overline{\nu}_z^2 + \overline{\nu}_m^2$. The energy of the moving electron can be written in terms of the quantum numbers $n = 0, 1, \cdots$ which form the cyclotron energy levels and $m_s = \pm\frac{1}{2}$ corresponding to the spin energy levels, as:

$$E(n, m_s) = \frac{g}{2}h\nu_c m_s + \left(n + \frac{1}{2}\right)h\overline{\nu}_c - \frac{1}{2}h\delta\left(n + \frac{1}{2} + m_s\right)^2 \tag{4.29}$$

The relativistic effects cannot be considered negligible if such high accuracy is required in the result. In the above formula they have been taken into account through the introduction of the energy shift δ .The calculation of $g/2$ relies on the measurement of 2 frequencies:

$$\frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_s - \nu_c}{\nu_c} = 1 + \frac{\nu_a}{\nu_c} \tag{4.30}$$

where $\nu_s \equiv g\frac{eB}{2\pi m}$ is the spin frequency, $\nu_c \equiv \frac{\omega_c}{2\pi} = \frac{eB}{2\pi m}$ and $\nu_a = \nu_s - \nu_c$ the anomaly frequency. When the relativistic effects and the phenomena of misalignment as well as the interaction with the cavity of the trap $\left(\frac{\Delta g_{cav}}{2}\right)$ are included, the above formula takes the form:

$$\frac{g}{2} = \frac{\overline{\nu}_c + \overline{\nu}_a}{\nu_c} \simeq 1 + \frac{\nu_a - \overline{\nu}_z^2/2f_c}{f_c + 3\delta/2 + \overline{\nu}_c^2/2\overline{f}_c} + \frac{\Delta g_{cav}}{2} \tag{4.31}$$

Therefore the measurement of the Lande factor of the electron is reduced to the determination of the frequencies:

$$\overline{f}_c \equiv \overline{\nu}_c - \frac{3}{2}\delta \qquad \overline{\nu}_a = \frac{g}{2}\nu_c - \overline{\nu}_c \tag{4.32}$$

which is accomplished through techniques of quantum-jump spectroscopy. When the highly accurate measurement $\frac{g}{2} = 1.001\ 159\ 652\ 180\ 73\ 28\ (3)\quad (0.28ppt)$ (35) is combined with the theoretical formula :

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + a_{\mu\tau} + a_{hadr} + a_{weak} \tag{4.33}$$

the following result is obtained:

$$\boxed{\alpha^{-1}(a_e) = 137.035\ 999\ 084\ (51)} \qquad (0.37ppb) \qquad (4.34)$$

The formula 4.33 is simply a rearrangement of 4.19. For the deduction of the above impressively small uncertainty, the contributions of both the uncertainties of the theoretical 4.21 and experimental calculation were considered. Currently the precision of this measurement is expected to be further improved through the development of numerical calculation of Feynman diagrams. This indicates that the value $\alpha^{-1}(a_e)$ is in strong correlation with the validity of the Standard Model and QED calculations in particular. What is more, a stringent value for the fine structure constant will shed some light on issues like that of the hadronic or electroweak nature of a_e and it will answer questions regarding the structure of the electron.

5 The speed of light

Of all the physical constants, the speed of light seems to be the one which has caused the greatest impact to the progress of physics. The measurement of its value has abetted many noticeable physicists. Nowadays its values is fixed and exact $c = 299\ 792\ 458 m/s$. It allows no experimental errors and it is used for the definition of the metre.

5.1 Brief Historical Review

The question of whether light propagates instantaneously or not has raised the attention of the scientific community even before the 17th century. Due to its high velocity, light was believed to travel with infinite speed in free space. The first to question this claim was Galileo who tried to measure the time required for a light beam to travel the distance between two nearby hills. Even though this experiment did not reveal any outstanding results, the endeavours of measuring the speed of light were not abandoned.

The idea that the speed of light is finite was actually verified by Roemer through the astronomical observations of the eclipses of Io, one of Jupiter's satellites. This first evidence was followed by numerous other attempts of estimating the value of c. The most well known of them are briefly presented in the Table 5.1.

From the theoretical point of view, the speed of light has proved to play a fundamental role too. Many conflicts have been reported before the dual nature of light was realized, namely that it acts as a particle and a wave. The Maxwell theory of electromagnetism indicated some years later that visible light is only a short band of the broad radiation spectrum. It also related the wavelength λ with the frequency f of radiation. This theory along with the evidence supporting the lack of luminiferous aether were the motivations of Einstein's special theory of relativity. Form his perspective, the speed of light does not depend on the motion of the source or the observer. It is the upper bound of the velocity at the universe. Therefore any observed contractions of length or time intervals should not be attributed to the existence of aether. They should rather be understood as the outcomes of the Lorentz transformations, required to go from the one spacetime

Table 5.1: Experiments giving the value of c

Date	Scientist	Method of experiment	Value of c in $\frac{km}{s}$
1676	O.Roemer & Huygens	moons of Jupiter	220 000
1729	J. Bradley	Stellar Aberration	30 000
1849	A. Fizeau	Toothed Whell	31 000
1862	L. Foucault	Rotating Mirror	298 000 ± 500
1879	A. Michelson	Rotating Mirror	299 910 ± 50
1926	A. Michelson	Rotating Mirror	299 796 ± 4
1947	Essen & Gorden-Smith	Cavity Resonator	299 792 ± 3
1958	K.D. Froome	Radio Interferometer	299 792. 5 ± 0.1
1973	Evenson et al	Lasers	299 792. 457 ± 0, 001
1983		Adopted Value	299 792. 458 EXACT

frame of reference to the other. The ultimate picture of the speed of light was given after the introduction of General Relativity according to which travelling at the speed of light means following world-lines tangent to the null vectors.

The above theoretical ideas modified the experimental techniques that have been applied for the precise measurement of c. All in all, these techniques have been obtained from nearly any branch of physics and they estimate the value of c:

(i) either by measuring the distance travelled by light in a given amount of time

(ii) or by making use of the appearance of c in basic electromagnetic formulae:

$$c^2 = \frac{1}{\mu_0 \epsilon_0} \quad , \quad c = \lambda f \tag{5.1}$$

and trying to determine with the greatest precision the values of the correlated parameters.

Actually the method (i) triggered the most accurate experiments for the estimation of c carried out just before 1983 when its value was defined as exact. Evenson et. al. (45) through high precision experiments determined the wavelength and the frequency of the radiation from a stabilized methane He-Ne laser at $3.39\mu m$. The choice of this particular source was of decisive importance, since a monochromatic and stable electromagnetic wave with short wavelength would minimize the experimental errors. Various other experimental approaches from the field of atomic physics (Blaney et al using the CO_2 laser, stabilized on $9.32\mu m$ R(12) transition of CO_2 and Baird et al (41) using the frequency of CH_4-stabilized laser) took place in the mean time. Averaging their results, the following value of c was obtained: $\boxed{c = 299\ 792\ 458.\ 05 \pm 0.75m/s}$ The fact that these independent measurements all concluded to the same value led the scientific community to adopt it as a constant. Thus, from 1983 according to the Conference Generale de Poid et Measures, the metre is defined to be the path travelled by light in vacuum during a time interval of 1/299 792 458 m/s. Therefore the arising question now is with what precision we know the value of the meter and the answer comes from astrophysical data. Actually the principal unit of length used now is the Astrophysical unit (1AU) which is known to be the distance between the Earth and the Sun. Its value is fixed through gravitational models and observations of our Solar System. More specifically the AU is defined through the relation:

$$GM_{Sun}[m^3 s^{-2}] = k^2 AU[m]^3/86400[s]^2 \tag{5.2}$$

Thus by determining the values of GM_{Sun} and the Gaussian gravitational constant $k = 0.017\ 202\ 098\ 95$ one can get the currently most accurate value of $1AU = 149\ 597\ 870\ 700\ 0(3)m$ (42)

5.2 Variation of c

Even though the speed of light is considered to be constant as a matter of metrology, there are several theories claiming its variation. Most of these models are derived from cosmological theories and suggest c to be a function of the frequency of radiation ν. This dispersion relation can be justified if one considers it as a consequence of the non zero photon mass 2.12.

The accuracy of the Varying Speed of Light (VSL) theories has been tested through analysis of astrophysical data coming from observations of flare stars or lately from Gamma Ray Bursts (GRBs). The astronomical objects used initially for this purpose were the flare stars; main sequence stars of class M demonstrating occasional rapid fluctuations in brightness. The limit taken from their study is $\frac{\Delta c}{c} < 10^{-6}$ which is much greater than the one obtained from laboratory experiments $\frac{\Delta c}{c} < 10^{-8}$.

A stricter limit on the above fraction was obtained from the analysis of data coming from Gamma Ray Bursts(GRBs). They are explosive events emitting flashes of gamma rays and they are considered to take place in distant galaxies. Their great intensity suggests that most probably the are generated through supernova explosions which do not reveal their associated energy isotropically. Known as the brightest electromagnetic events in the universe, they have duration that may range from $(10^{-2} - 10^{3})s$ and their rise times are of about $10^{-4}s$. Usually the main γ-ray flash is followed by an "afterglow" which includes photons of longer wavelengths lying from the radio to the X-ray spectrum. From the experimental point of view, an indication of the variation of c can be obtained if we consider the difference in time Δt required for 2 photons of different frequency ν_1 and

ν_2 to travel the distance D between the source and the observer:

$$\frac{\Delta c}{c} < \frac{c\Delta t}{D} \tag{5.3}$$

This technique was applied to a group of GRBs whose data were analysed by Schaefer (43). In particular, Schaefer took under consideration a sample of GRBs which contain various types of wavelengths and determined their distance from the observer through arguments about their luminosity (Fenimore's relation) including the values of cosmological parameters such as the Hubble constant $H_0 = 65km/s/Mpc$ and $\Omega = 1$. Therefore by comparing the arrival times for signals of different frequencies ν, he verified a general dispersion relation for the speed of light:

$$V = c\left(1 + \frac{A}{\nu^2}\right) \tag{5.4}$$

through which he could draw some conclusions as far as the photon mass is concerned.

The analysis of (43) obtains an upper limit for the possible Lorentz violation, since it is claimed that the ratio $\frac{\Delta c}{c}$ can take values lying from $2.5 \cdot 10^{-12}$ to $6.3 \cdot 10^{-21}$. Furthermore, it is discussed that the whole argument depends on some main assumptions. Namely, to obtain this limit one assumes that every photon regardless of its wavelength, should start from the exact same point of the supernova responsible for the particular GRB. Additionally, the plausible interaction with the electrons in the meantime is ignored so that any observable delay between those photons should be attributed to the dispersion of c.

All in all, the speed of light is classified among the constants known with an impressive accuracy. Its value is considered fixed by definition and any deviation from it needs to be justified both theoretically and experimentally. Other current studies in the field of varying speed of light theories suggest there is a limit in energy scales for which the Lorentz invariance holds. As one approaches the Planck scales quantum effects seem to play a greater role and can cause the change of c between different observers. This postulate is also probed through the observation of GRBs. (44)

6 The Gravitational Constant

Alternatively known as the "Newton's constant", this fundamental constant holds a prominent position in the most well known law of physics, the universal law of gravitation. The formula correlating the gravitational force F between 2 particles of masses m_1 and m_2 to their intermediate distance r:

$$F = G\frac{m_1 \cdot m_2}{r^2} \tag{6.1}$$

was stated by Newton in 1687. Since then numerous experiments have been performed in order to determine the value of the proportionality constant G. Contrary to the rest of the fundamental constants though, the value of G is today known with the low accuracy of just $12ppm$.

In the following, we will discuss the theoretical significance of G and we will briefly present the latest laboratory experiments that contributed to the CODATA 2010 (47) value: $G = 6.673\ 84\ (80) \cdot 10^{-11}\ m^3kg^{-1}s^{-2}$.

6.1 General relativity in the weak field limit

The role of G in the classical physics manifests itself when viewed in the framework of Newtonian theory/ It is encountered in the Einstein's theory of relativity too, where it can be conceived in 2 ways:

(i) as a proportionality constant in the field equations, which correlates the energy momentum $T_{\mu\nu}$ to the Einstein curvature tensor

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{6.2}$$

(ii) in the weak field limit, where the familiar Newton's gravitational law emerges in the form of the Poisson equation:

$$\nabla^2\Phi = -4\pi G\rho \tag{6.3}$$

In the above formula, Φ is the gravitational potential formed due to the presence of matter with mass density ρ.

Actually, starting from Einstein's equations we can derive the geodesic equations:

$$\frac{d^2x^\lambda}{ds^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0 \tag{6.4}$$

They give the trajectories of a particle moving in a spacetime which is described by the metric $g_{\mu\nu}$ and can be parametrized through the proper distance s: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. In this framework the Christoffel symbols are given by:

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2}g^{\mu\rho}\left[\frac{\partial g_{\nu\rho}}{\partial x^\lambda} + \frac{\partial g_{\rho\mu}}{\partial x^\nu} - \frac{\partial g_{\nu\lambda}}{\partial x^\rho}\right] \tag{6.5}$$

Considering the particle to move slowly with respect to light $\left(\frac{d\mathbf{x}}{dt} << c\right)$ under the influence of a static and weak gravitational field, the equation of motion takes the form:

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{00}\left(\frac{dx^0}{ds}\right)^2 = 0 \tag{6.6}$$

The requirement of a static field $\left(\frac{\partial g_{\mu\nu}}{\partial x^0} = 0\right)$ imposes the condition:

$$\Gamma^\mu_{00} = -\frac{1}{2}g^{\mu\nu}\frac{\partial g_{00}}{\partial x^\nu} \tag{6.7}$$

Additionally, the metric for a weak field can be expressed as the sum of the Minkowskian one $\eta_{\mu\nu}$ and a small perturbation $h_{\mu\nu}$, $|h_{\mu\nu}| << 1$, i.e. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Consequently, to first order in the perturbation: $\Gamma^\alpha_{00} = -\frac{1}{2}\eta^{\alpha\beta}\frac{\partial h_{00}}{\partial x^\beta}$ and we obtain :

$$\frac{d^2x}{ds^2} = \frac{1}{2}\left(\frac{dt}{ds}\right)^2\nabla h_{00} \tag{6.8}$$

which is simply: $\frac{d^2x}{ds^2} = -\nabla\Phi$ with Φ defined through $h_{00} = \frac{2\Phi(x)}{c^2}$. If one takes into consideration the equivalence of gravitational and inertial mass, $m_G = m_I$, the above equation is Newton's law for a massive particle moving in a gravitational field Φ coming from the relation 6.3.

Since the gravitational attraction is the major mechanism present in astronomical scales, it would be expected that astronomical objects provide the best estimations about G values. However, this is not the case. The reason is that the measurements related to the planetary orbits include the product of GM, also known as standard gravitational parameter, where M is the source's mass. An example is the statement of Kepler's law. The techniques of celestial mechanics are not sufficient to determine separately the masses of planets or stars in an independent way or at least are in a preliminary stage.

The fact the the gravitational interaction is by far the weakest introduces another problem which has to overcome. A rough estimation of how greater the electromagnetic force is compared to gravity can be obtained if we consider 2 protons interacting in both ways:

$$F_{em} = \frac{e^2}{4\pi r} \quad , \quad F_{gr} = \frac{Gm^2}{r} \tag{6.9}$$

In analogy to the fine structure constant α one can deduce a coupling constant for gravity:

$$\alpha_g = \frac{Gm^2}{\hbar c} \tag{6.10}$$

For the case of the proton interactions $\frac{\alpha_g(m)}{\alpha} \sim 10^{-40}$ This indicates that in order to carry out as accurate experiments as possible, the conditions of the experimental settings should assure the elimination of any other interaction apart from gravity between the participating objects.

The significance of G is not constrained to the area of classical physics and celestial mechanics. When combined with the other fundamental constants: speed of light (c) ,reduced Planck constant ($\hbar$), it can be used to construct the Planck units:

$$l_P \equiv \left(\frac{\hbar G}{c^3}\right)^{1/2} \quad t_P \equiv \left(\frac{\hbar G}{c^5}\right)^{1/2} \quad M_P \equiv \frac{\hbar c}{G} \tag{6.11}$$

They are considered to determine the boundaries after which quantum fluctuations of a non stationary background cannot be neglected.

6.2 Experiments

Despite the various formulae in which the gravitational constant is encountered, the determination of its value seems to be a quite difficult task. The reasons responsible for this fact, are that gravity is an extremely weak interaction compared to the other fundamental forces and that it cannot be shielded from the rest of the phenomena.

The first successful experimental attempt for the measurement of G is attributed to Cavendish. Over 300 years ago, he considered the interaction of 2 pairs of masses forming a torsion balance. A schematic of the experimental apparatus can be seen in the picture. It consists of a rod (A) in whose edges two spheres of the same mass are attached, and a dumbbell (B) of 2 less massive spheres. The dumbbell is suspended from a thin wire and it is allowed to twist. The rod (A) which has the same swivel axis, is moved to approach the lightest balls. The rod is twisted until the equilibrium is accomplished. The change of the angle θ between the initial and final position of the rod is measured through the deflection of light from a mirror attached on the wire. This primitive attempt of Cavendish gave the value: $G = (6.67 \pm 0.07) \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ when the value of the earth's density was taken into account.

Nowadays, most of the apparatuses used for the determination of G are a modification of the Cavendish torsion balance. The results of 11 such experiments were used by the Committee on Data for Science and Technology (CODATA) (47), in order to obtain a weighted mean for the value of G: $G = 6.673\,84\ (80) \cdot 10^{-11}\ m^3 kg^{-1} s^{-2}$. The latest four of them are mentioned in the Table 6.1 . It is of note that the presented values are not in

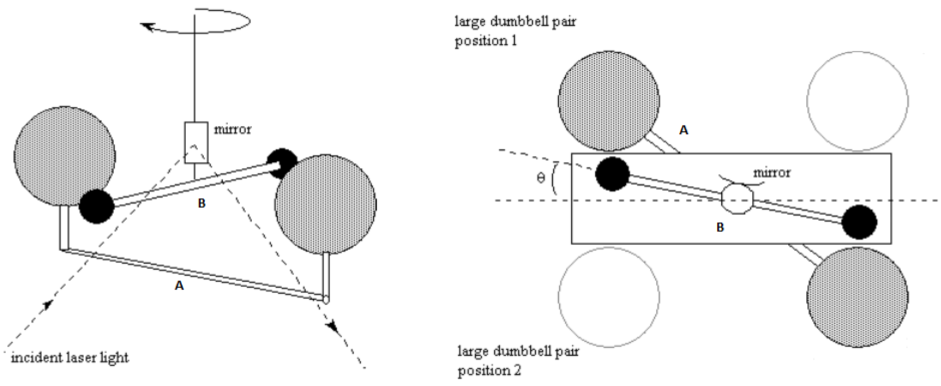

Figure 6.1: An illustration of the Cavendish torsion balance

agreement even when the experimental errors are taken into account. This leads to a big uncertainty for the CODATA average which is attributed to the various systematic errors.

Table 6.1: The latest experiments determining the value of G (47)

Experimental group & Institution	Method of experiment	Value of G in $\frac{10^{11}G}{m^2kg^{-1}s^{-2}}$	Uncertainty
Hu et al (2005) **HUST-05** Huazhong University of Science and Technology	Fiber torsion balance, dynamic mode	6.672 28(87)	$1.3 \cdot 10^{-4}$
Schlamminger et al (2006) **UZur-06** University of Zurich	Stationary body, weight change	6.674 25(12)	$1.9 \cdot 10^{-5}$
Tu et all (2010) **HUST-09** Huazhong University of Science and Technology	Fiber torsion balance, dynamic mode	6.673 49(18)	$2.7 \cdot 10^{-5}$
Parks & Faller (2010) **JILA-10** University of Colorado	Suspended body, displacement	6.672 34(14)	$2.1 \cdot 10^{-5}$

Two of the above measurements rely on the *time-of-swing method*. This technique resembles the Cavendish experiment. The apparatus, that was used, consists of a system of rectangular glass which is suspended from a fiber and interacts gravitationally with a pair of spheres. The frequency ω_0^2 corresponding to the oscillation of this pendulum without the presence of the spheres is related to the inertial mass of the rectangular glass (I) and the torsion constant (K). When the pair of spheres is put near and away from the rectangular mass the frequency of oscillation gets slightly distorted:

$$\omega_n^2 = \frac{K_n + GC_{gn}}{I} \quad , \quad \omega_f^2 = \frac{K_f + GC_{gf}}{I} \tag{6.12}$$

where C_{gn} and C_{gf} are constants referring to the mass distributions of the pendulum and the spheres. This comes naturally, if one considers that the torque exerted on the pendulum's mass is a sum of two components; the torque produced by the twisted fiber and the one from gravitational interaction.

$$\tau_{tot} = -K\theta + \tau_g(\theta) \tag{6.13}$$

Eliminating $K_n = K_{gf} = K$ from the above relations, one can express the gravitational constant as:

$$G = \frac{I(\omega_n^2 - \omega_f^2)}{C_{gn} - C_{gf}} = \frac{I\Delta(\omega^2)}{\Delta C_g} \tag{6.14}$$

Therefore, measuring the change of the angular oscillation frequencies for the 2 possible configurations of the pendulum and the 2 spheres, the group obtained data for the estimation of the value of G.

The above description though is quite simplified. Various parameters have to be taken into consideration for the experiment to be as accurate as possible. First of all, in the formulas 6.14 there is no dependence on the spring constant of the torsion fiber K . This indicates the assumption that the parameter K is not affected by the frequency of the oscillatory motion. However, if such great accuracies are pursued, theories of elasticity doubt this assumption (Kuroda effect (48)). Therefore, taking into account the theorems of elasticity as well as the contributions from the other parts of the pendulum in the motion, we obtain:

$$G = \frac{I\Delta(\omega^2)}{\Delta C_g}\left[1 - \frac{\Delta K}{I\Delta(\omega_n)} + \frac{I_m}{I}\frac{K^2(\omega^2)}{K_m^2}\right] \tag{6.15}$$

The symbols I_m and K_m are related to the magnetic damper which is connected to the torsion fiber and it is used to constrain the simple motions of the pendulum. In particular they correspond to the latter's moment of inertia and the torsion constant of the prehanger fiber from which it is suspended respectively. Some elements that improved the accuracy of HUST-09 compared to the one of HUST-05 experimental estimation of G are the conduction of two independent experiments, the better knowledge of the homogeneous densities and the remote control of the motion of the spherical masses between the two configurations

The other latest experiment – JILA-10 (51) – was performed in 2004 and the data were analysed and published in 2010. The procedure relies on a simple pendulum method. Here, two pendulums are suspended and left to interact gravitationally with 4 cylinders, which are moved periodically in inner and outer positions with respect to the virtual line

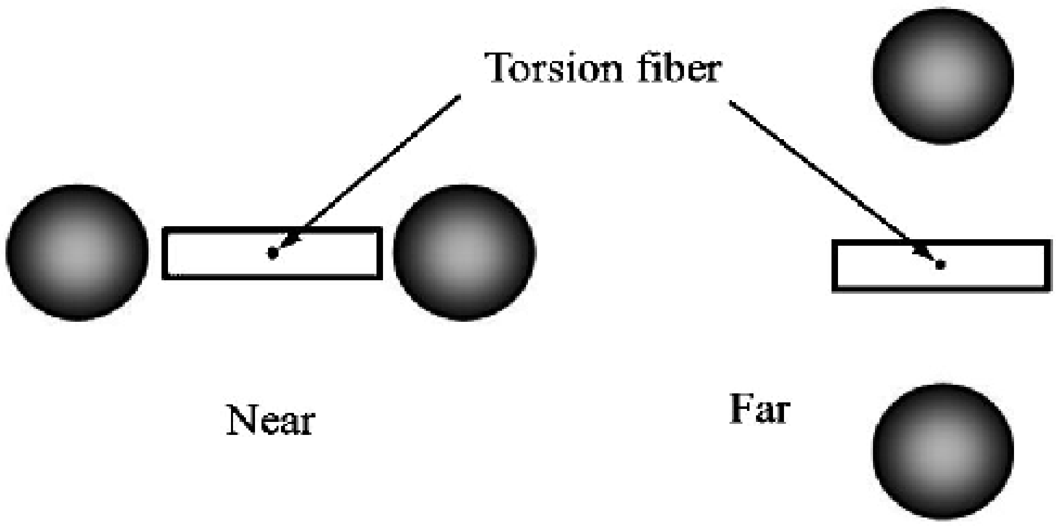

Figure 6.2: The apparatus for the HUST experiment - from (49)
right: Schematic of the cutaway view of the apparatus
left: Top view of the two configurations of the pair of spheres and the rectangular glass

connecting them. This interaction is counterbalanced by the regular pendulum force. The separation between the two pendulums is measured through a Fabry-Perot interferometer. The outcome of this experiment is a value of G smaller compared to the rest of the measurements. At the same time its small uncertainty does not justify such a discrepancy.

In conclusion, the accurate estimation of G continues to be a challenge for modern experimental physics. Up to now the accepted value is the one obtained from the CODATA-10 and it appears to have a big uncertainty, compared to the other physical constants. Better knowledge of the inelastic phenomena and their contribution to the determination of G will help to obtain more accurate knowledge on this constant. Subsequently such an accuracy may be enough to verify theories which assume that the Gravitational constant may be related to other coupling constants of the known interactions in the framework of string theory.

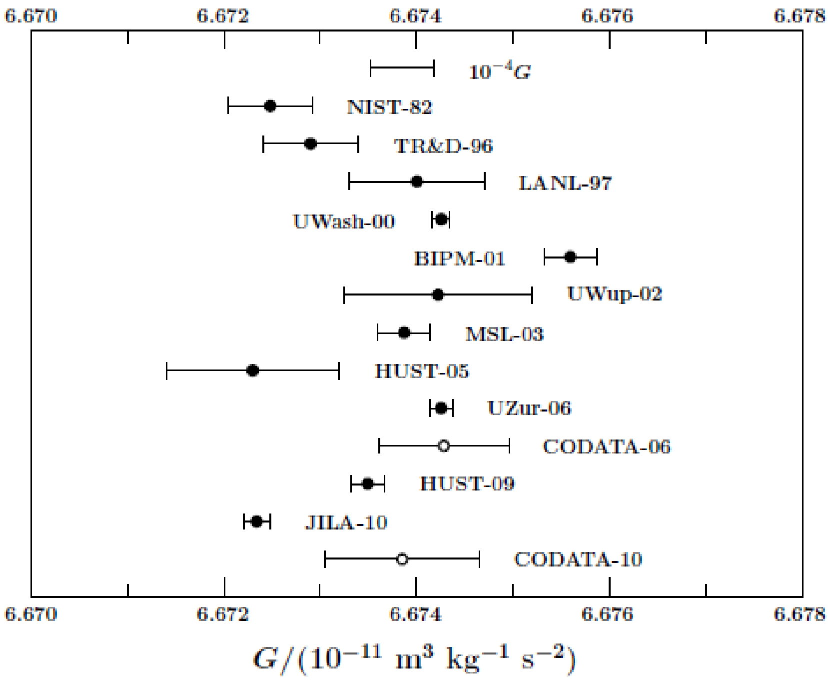

Figure 6.3: The values of the gravitational constant as obtained from the latest 11 experimental estimations. - It is of note that the CODATA-10 value is slightly moved to the left which is principally due the JILA-10 contribution.

7 Conclusion

There is no doubt that the study of fundamental physical constant passes through various branches of physics. Such an interdisciplinary approach has been attempted in the above presentation. The values of the constants have been obtained either through high precision experimental considerations of through processes determining an upper bound. A table recapitulating the main results is presented below.

Table 7.1: The currently approved values of the Fundamental Physical constants studied.

Physical constant	Symbol	Value or Upper bound
Photon mass	m_γ	$< 1.5 \cdot 10^{-53} g \simeq 8.41 \cdot 10^{-21} \frac{eV}{c^2}$
Graviton mass	m_g	$< 7 \cdot 10^{-32} \frac{eV}{c^2}$
Fine Structure Constant	α	137.035 999 084 (51) (0.37 ppb)
Speed of light	c	299 792 458 $\frac{m}{s}$ exact
Gravitational constant	G	6.673 84 (80) $\cdot 10^{-11} \frac{m^3}{kgs^2}$

Provided that the endeavour for the most accurate determination of the physical constants will be continued in the future, many beneficial consequences can occur. Apart from the higher precision in the calculations where these constants are included, the scientific society will have the chance to check the limits of validity for known fundamental theories (such as QED or general relativity). The simultaneous development of applied studies such as frequency metrology and space navigation are among the side effects of this research.

Another intriguing topic concerns the theoretical motivations for the time variability of the constants of nature. If such ideas are verified for constants related to the strength of fundamental interactions (like α and G), this would imply that the correspondence between experimental results and theories depends on the particular time that measurements were performed.

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Photon mass

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