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On Jamming Attack in Wireless Networks: A Game Theoretical Analysis

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Abstract

This work is about Jamming, a major threat that affects the security and the quality of communication in wireless networks. This process can be modeled as a two-person zero-sum game between the jammer and the legitimate entity that wants to communicate. We study the Jammer in a communication with thermal-energy constraints where the players can transmit in three different levels of energy, trying to outdo one the other by transmitting at higher power level. In this senario, we investigate and proove the existence of Nash Equilibria for a range of values of two parameters that are related to the matrix of the game. Afterwards, we introduce the Jammer in a cooperative communication environment, in order to formulate analytically how he affects the achieved utility, the choice of the relay by the source and how he chooses the power allocation of his attack. The performance of Decode-and-Forward and Amplify-and-Forward techniques are investigated in this context. Our simulation results show that the Decode-and-Forward technique always gives a higher utility, when the relay succesfully decodes the signal.

Abstract

Dans ce rapport nous nous intèressons au phénoméne de *Jamming*, c'est á dire une menace majeure qui affecte la sécurité et la qualité de la communication dans les resaux sans fil. Ce processus peut être modélisé comme un jeu de deux personnes; connu sous le nom de 'zero-sum' entre le Jammer et l'entité souhaitant communiquer. Nous étudions le Jammer dans une communication avec des contraintes d'énergie thermique où les joueurs peuvent transmettre selon trois niveaux différents d'énergie, en essayant de surpasser l'un l'autre en transmettant avec une puissance plus élevée. Dans ce senario, nous étudions et prouvons l'existence d'équilibres de Nash pour une gamme de valeurs de deux paramètres qui sont liés á la matrice du jeu. Ensuite, nous introduisons le Jammer dans un environnement de communication coopérative, afin de formuler analytiquement comment il affecte l'utilité atteinte, le choix du relais par la source et la façon dont il choisit l'allocation de puissance pour son attaque. Les performances des techniques *Decode-and-Forward* (Décoder-et-Retransmettre) et Amplify-and-Forward (Amplifier-et-Rétransmettre) sont étudiées dans ce contexte. Notre simulation montre que la technique de *Decode-and-Forward* donne toujours une plus grande 'utilité', lorsque le relai décode le signal avec succè s.

Abstract

Η εργασία αυτή ασχολείται με το Jamming, μια από τις κυριότερες απειλές για την ασφάλεια και την ποιότητα της επικοινωνίας στα ασύρματα δίκτυα. Η διαδικασία αυτή μπορεί να μοντελοποιηθεί ως ένα παίγνιο μηδενικού αθροίσματος, μεταξύ δύο ατόμων, τον Jammer και την οντότητα που επιθυμεί να επικοινωνήσει. Μελέταμε τον Jammer σε μια επικοινωνία με ενεργειακούς περιορισμούς λόγω θερμοτήτας, όπου οι παίχτες μπορούν να μεταδώσουν σε τρία επίπεδα ενέργειας. Σε αυτό το σενάριο, διερευνούμε και αποδεικνύουμε την ύπαρξη ισορροπίας Nash για ένα εύρος δύο παραμέτρων που σχετίζονται με τον πίνακα του παιγνίου. Στη συνέχεια, εισάγουμε τον Jammer σε ένα περιβάλλον συνεργατικής επικοινωνίας, προκειμένου να διατυπώσουμε αναλυτικά πώς αυτός επηρεάζει την επιτεύξιμη ωφελιμότητα, την επιλογή του κόμβου-σύνδεσμου από την πηγή και πώς αυτός επιλέγει την κατανομή της ισχύος του στην επίθεσή του. Η επίδοση των τεχνικών Αποκωδικοποίησης-Προώθησης Decode-and-Forward και Ενίσχυσης-Προώθησης Amplify-and-Forward μελετάται σε αυτό το πλαίσιο. Η προσομείωσή μας καταλήγει στο ότι η μέθοδος της Αποκωδικοποίησης-Προώθησης, δίνει πάντα μεγαλύτερη ωφελιμότητα αν ο κόμβος-σύνδεσμος αποκωδικοποιήσει με επιτυχία το σήμα της πηγής.

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Contents

Chapter 1

Introduction

This work is about the modeling of the jamming security issue with Game Theory techniques. In this chapter, firstly we will present an introduction to security issues in wireless networks, by focusing on the jamming problem. We will also introduce our main mathematical tool, that of Game Theory. After giving some background, we will see how it is used in networks and more specifically in jamming situations. Afterwards, we will dispose an introduction in Cooperative Communication, a new approach for routing of packets in wireless networks. The jammer will be studied in this different environment too. Finally, the outline of this work and its contributions will be explained.

1.1 Security Issues and Jamming Attacks

In this section, after some basics about the security issue at wireless networks, we present the threat that is mainly studied in this work: the jammer.

1.1.1 Security in wireless networks

Security is a major concern in wireless networks in order to provide protected communication in hostile environments. The shared wireless medium, the energy constraints and the dynamic topology that characterize them arise more challenges on the security issue. The security threats could be separated into four major categories: passive attacks, active attacks, man-in-the-middle attacks and jamming attacks.

We present briefly those four categories: A passive attack occurs when a malicious user listens or eavesdrops the network traffic. For the active attacks we have numerous exemples like unauthorized access, spoofing (a situation in which one person or program successfully mas-

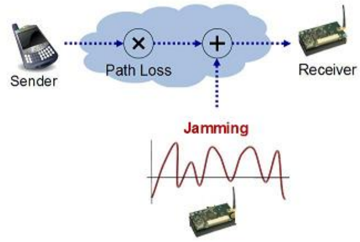

Figure 1.1: A Jamming Situation

querades as another by falsifying data and thereby gaining an illegitimate advantage), flooding attacks, denial-of-service (DoS) attacks etc. The man-in-the-middle attack is a form of active eavesdropping in which the attacker makes independent connections with the victims and relays messages between them. Finally Jamming is a special kind of DoS attack, specific to wireless networks, where an adversary emits radio frequency signals that do not follow an underlying MAC protocol.

1.1.2 Jamming attack

Security in networks has been studied from many perspectives at the different levels of network architecture. Many security threats can be addressed through appropriately designed network security architectures. Our interest focuses on a threat that is not adequately addressed via those methods, Jamming, a situation where a hostile user tries purposefully to interfere with the physical transmission and reception of wireless communications by introducing noise in order to decrease the signal-noise ratio [1].

The shared nature of the wireless medium allows adversaries to observe communications between wireless devices and easily launch DoS attacks that block the wireless medium and prevent wireless devices from communicating. Radio interference attacks (jamming) are not addressable through conventional security mechanisms. A hostile user can transmit continually in a channel, bypassing the medium access protocol and by that he prevents users from being able to communicate legitimately or he introduces packet collisions.

The problem of jamming plays a very important role in ensuring the quality and security of wireless communications. In every jamming situation there exists an entity that wishes to transmit successfully in order to communicate with another and an entity that tries to make this communication difficult by introducing noise, the Jammer. The interest of these two are opposite as the one is trying to outdo the other. In [1] different attack models and philosophies are presented: The constant jammer who continually emits radio signals, the deceptive jammer who constantly injects regular packets to the channel without any gap between, the random jammer who alternates between sleeping and jamming and the reactive jammer who stays quiet when the channel is idle.

1.2 Brief Introduction on Game Theory and its Application in Wireless Networks

Our basic tool for the modeling of jamming situations in wireless networks will be game theory. Here, firstly we will present some basic concepts of game theory that will be useful for our study. Afterwards we will see the use of this mathematical tool in networks and finally we will make an introduction to our problem.

1.2.1 The essentials of game theory

Game Theory is the mathematical study of interaction among independent, self-interested agents/ players. In order to model the player's interest we use the "utility theory" that quantifies his degree of preference across a set of available alternatives. The goal for every player is to maximize his utility function. In the case of more than one agents, the optimal choice for a given player depends on the choices of others. In order to deal with this problem certain subsets of outcomes are identified and they are called solution concepts. In other words, we could say that a solution concept is a formal rule for predicting how the game will be played. These predictions are called "solutions", and describe which strategies will be adopted by players, therefore predicting the result of the game. One of the most fundamental solution concepts is the Nash Equilibrium [2].

Trying to give an intuitive definition of a Nash Equilibrium we could say that a set of strategies is a Nash equilibrium if no player can increase his expected payoff by unilaterally changing his or her strategy. A Nash Equilibrium is a stable strategy profile, as no agent would want to change his strategy if he knew what strategies the other agents were following. A formal definition of Nash Equilibrium is the following:

Let (S, f) be a game with n players, where S_i is the strategy set for player i, $S = S_1 x S_s ... x S_n$ is the set of strategy profiles (i.e. a set of plans of actions for all the situations that may arise in the game) and $f = (f_1(x), ..., f_n(x))$ is the payoff function. Let x_i be a strategy profile of player i and x_{-i} be a strategy profile of all players except for player i. When each player $i \in 1, ..., n$ chooses strategy x_i resulting in strategy profile $x = (x_1, ..., x_n)$ then player i obtains payoff $f_i(x)$. A strategy profile $x^* \in S$ is a Nash Equilibrium (NE) if no unilateral deviation in strategy by any single player is profitable for that player, i.e. :

$$\forall i, x_i \in S_i, x_i \neq x_i^* : f_i(x_i^*, x_{-i}^*) > f_i(x_i, x_{-i}^*)$$

.

The above definition holds when every player's strategy constitues a unique best response to the other agents' strategies. This is the case of a Strict Nash. If not, we have: $\forall i, x_i \in S_i, x_i \neq x_i^* : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*)$, that forms a Weak Nash.

A game can have either a pure-strategy or a mixed Nash Equilibrium. In the case of mixed-strategy a pure strategy, is chosen stochastically. Nash proved that if we allow mixed strategies, then every game with a finite number of players in which each player can choose from finitely many pure strategies has at least one Nash equilibrium. Mixed-strategy NE are necessarily always weak, while pure-strategy NE can be either strict or weak [2].

Another notion from game theory that will be useful in our study is that of the zero-sum game. A game is called zero-sum if for each strategy profile the sum of the utilities of the players for this profile equals to zero. These games represent situations of pure competition as one player's gain come at the expense of the other player. Nash equilibria in zero-sum games can be viewed graphically as a saddle point, where any deviation of the player lowers his utility and increases the utility of the other player [2],[3].

1.2.2 Game theory application in wireless networks

Game theory has been primarily used in Economics, in order to describe the relations between financial entities, companies, consumers etc. Not surprisingly, game theory has also been used in networks, initially to describe routing and resource allocation problems in competitive environment. The evolution of wireless communication gave rise to problems that could be nicely presented through game theory.

The limited transmission resources impose a conflict of interests and every user (player) is called to decide in a distributed way for the strategy that will optimize his payoff. The users of a wireless network are considered rational, which means that they will always try to maximize their utility. From another point of view, in modern wireless networks the idea of incentives for sharing resources arises. These relations of competition and cooperation can be easily described through game theory. Furthermore this mathematical tool offers the concepts and the methods to describe and determine analytically the impact of a specific choice of a user, of different protocols and policies.

In [4] we can see some typical and indicative problems of wireless networks modeled through Game Theory. The following concepts arise from these examples: In some cases the players can mutually increase their payoffs by cooperating (symmetric non-zero sum game). The conflict of interest is that each of them has to provide a service to the other. In other cases players have to share a common resource (usually the wireless channel). At this scenario we can have a zero-sum game, where the gain of a player represents the loss of another, but we can also have a non-zero sum game where the users successfully share. At these examples we can also see that a game can be formulated as a static one, where all the users act simultaneously or as a dynamic one, where players have a sequential reaction.

1.2.3 Modeling jamming with game theory

As mentioned previously, the goal of a Jammer is to cancel the communication of the legitimate user. If the Jammer succeeds, the legitimate user will not be able to transmit and his payoff will be zero. If the Jammer fails, the legitimate user will transmit successfully and the payoff of the jammer will be zero. In other words the utility of the Jammer is exactly the opposite of the utility of the transmitter. So the game between them can be described as a zero-sum game. This kind of formulation exists already in [5] and 6] where a power budget constraint is also taken into account for both players. In [7] although the game is similar, it is formulated as a non-zero sum game with a power budget, because of the use of a cost for the usage of a resource.

Another important issue at the formulation of a game is the utility function. The utility function should include the results of both the Jammer's and the legitimate user's actions. As the study of the Jammer is at the Physical Layer, the most appropriate objective function to express the utility for the players is that of the SNR.

In [8] the jamming game is again formulated as a zero-sum game with an additional constraint about the thermal energy that should not exceed a certain limit. The players decide whether they will transmit or not, according to the thermal energy they have accumulated until now. Pure and mixed strategies are studied in relation to the parameters of the game.

1.3 Cooperative Communication

Cooperative diversity is a form of spatial diversity to combat channel fading through cooperative relaying. In the traditional layered design approach of wireless networks, the route that connects the source with the destination is selected by a protocol of the network layer, and each node

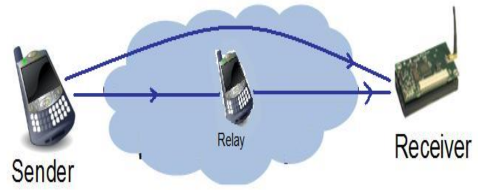

Figure 1.2: Cooperative Communication

along this route is responsible of transmitting and, if necessary, retransmitting the packets to the next hop. Thus, there is no way for the packets to be delivered from an alternative route if a particular link is degraded. This lack of flexibility becomes more critical in order to achieve high quality of service, high data rates and efficient utilization of resources in order to meet the goals of wireless networks [9].

The basic idea of Cooperative Communication is that when a receiver cannot decode a frame, the retransmission is handled not by its original source but rather by a neighbor that overheard the transmission successfully, and may have a better channel to the destination.The cooperative diversity takes advantage of broadcast transmission to send information through multiple relays concurrently. The destination can then choose the best of many relayed signals, or combine information from many signals. By effectively transmitting or processing (semi)independently fading copies of the signal, diversity is a method for directly combating the effects of fading [10].

In order to have a historical background for the cooperative communication, we should get back to the work of Cover and Gamal, on the information theoretic properties of the relay channel [11]. There, we can find the analysis for the capacity of a three-node network consisting of a source, a destination and a relay. Although at this fundamental work the relay's only purpose is to help the main channel, in more recent work users are both information sources and relays [12].

There are two categories of cooperative communication, namely, amplify-and-forward (AF) and decode-and-forward (DF). Under AF, the cooperative relay node performs a linear operation on the signal received from the information source before forwarding it to the destination node. Under DF, the cooperative relay node decodes the received signal, and re-encodes it before forwarding it to the destination node [13].

A capacity, outage and coverage analysis of the model of relay channels and cooperative communication is carried out at [10] and [11]. [13] is focusing on the optimal choice of a relay

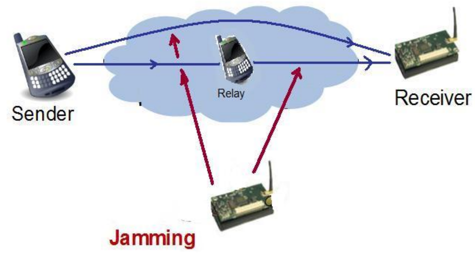

Figure 1.3: A Jamming Situation in Cooperative Communication

between a set of relays in order to maximize the achieved SNR at the receiver, for the Decode and Forward technique while [14] is presenting the Amplify and Forward scenario. Both the A-F and D-F techniques are studied at [15].

Although there seems to be much effort on the selection of an optimal relay ([13], [14], [16],[9]) by studying the SNR at the receiver, there is not much work that incorporates the jamming phenomenon at this problem. Furthermore game theory becomes a popular method to describe security phenomena in cooperative scenario, like in [17],[18], [19]. In our work a scenario of a jammer that attacks a network of cooperative communication with a set of possible relay nodes is studied with the use of game theory.

1.4 Report Organization and Major Contributions

The following work consists of two major parts. The first one is the modeling of the process of communication jamming under temporal energy constraints. The second part introduces the jamming attack in a cooperative communication environment.

1.4.1 Jamming as a dynamic game under energy constraints

In [8], jamming is presented as a two-person zero-sum noncooperative game, where two opponents, a communicator (a transmitter-receiver pair) and a jammer try to outdo one the other by transmitting a signal with a power level greater than that of the adversary. Both players are subject to temporal energy constraints, which account for protection of the communicating and jamming transmitters from overheating. In each slot the players choose randomly their transmission power between two power levels: zero and a positive value. The general behavior of the players' strategies and payoff increment is found to depend on a parameter related to the payoff matrix, called payoff parameter. In order to solve the game the authors of [8] present a backward induction methodology for a grid solution that is based on the 2x2 matrix of the game.

The size of the matrix results from the available choices for each player (zero or positive-value).

In our work, we extend the previous analysis by defining three levels of power transmission: zero, low and high, where a higher level masks a lower and denotes a successful transmission. Energy constraints are considered too, for both players.

The existence of pure and mixed equilibria in this game is investigated, in relation to the payoff parameters that describe the payoff of the players when they both transmit at the high level or both at the low level.

For this game, the matrix is a 3x3 one, so a new methodology has to be formulated in order to obtain a solution for matrices bigger than 2x2. Our analysis show that the game that is described by the bigger matrix can be separated to subgames where the users have two strategies. So each subgame -that is described by a 2x2 matrix is solved separately, under the conditions that must be imposed in order this subgame to exist.

So finally, we have the conditions for the Nash Equilibrium in our game and a methodology in order to search for equilibria in a 3x3 matrix.

1.4.2 Cooperative relaying under the presence of a jammer

In this section we study the jamming problem in an environment of cooperative relaying. Both the jamming security issue and the cooperative communication have been studied extensively, but it seems that there is no sufficient work concerning the behavior of a jammer, and accordingly of the source, in a network of relayed communication. We assume a source, a destination and a pool of available relay nodes. The utility function is the SNR formula and as far as energy constraints are concerned, we take into account the power budget of the source and the jammer. The formulas for the SNR have been studied in the concept of cooperative communication at [14],[15],[20], without the presence of a jammer.

We take into consideration the case of one relay. The source broadcasts a message that will be received from the relay and the destination. The Jammer attacks both the initial broadcast of the source, as well as the channel between the relay node and the destination. We present the analytical analysis for the power allocation behavior of the jammer and the relay selection from the source. An algorithm for both problems is proposed. Both the amplify-and-forward and the decode-and-forward scenario are studied. Finally, a simulation is carried out, in order to compare the performance of the two techniques.

Chapter 2

Jamming as a Dynamic Game under Energy Constraints

2.1 Introduction

Communication Jamming is a power game between two opponents: the Jammer and the Communicator (transmitter-receiver pair). Each of them tries to outdo the other by transmitting a higher power level. Such a situation has been modeled as a two-person zero-sum non-cooperative game in [8], where two strategies (that correspond to two power levels) are available for the players: zero and a positive value. However, the equipment of the transmitter has a limitation on its power heating capability, which leads to an energy constraint for both users, in order to avoid a thermal breakdown.

Our study extends this model, for more than two available strategies for each player. We provide the analytical formulation for a game with three available power levels: zero, low, high. We embed the thermal limitations and we examine the existence of nash equilibria in relation to two payoff parameters that exist when the players transmit on the same level. In order to solve the game a methodology for bigger matrices than the one of [8] is presented.

In this chapter, firstly we will present the formulation of the problem as a game and the payoff as a function of the strategies. Then, we will examine the existence of saddle points and nash equilibria. We will proceed with the solution of the game in a grid form and study the mixed equilibria in relation to the payoff parameters. In the end we will have the contribution of this analysis.

2.2 Formulate the Problem

We study the case in which selected communication and jamming strategies are exercised in sychronism over T time-slots, indexed by 1,2,...,T. In the t_fth slot (denoting the forward time index) the communicator transmits an information signal with power level X_{t_f} and the Jammer transmits a jamming signal with a power level Y_{t_f}. The Communicator can transmit on a high level power (p_h^s), a low level power (p_l^s) or zero. The Jammer can transmit on a high level power (p_h^j), a low level power (p_l^j) or zero, too. The high level power masks the lower.

We infer that at the beginning of any time slot, there is accumulation of thermal energy in the communicating and jamming transmitters owing to past transmissions. Over the current slot duration, a fraction of this energy is dissipated, while the remainder adds on to energy generated by the current slot's transmission. To avoid transmitter failure due to thermal breakdown, the accumulated thermal energy at the end of any slot should not exceed a threshold (temporal energy constraint).

We define the folloiwng:

Z_{t_f} represents the accumulated thermal energy in the communicating transmitter at the end of time slot t_f. W_{t_f} represents the accumulated thermal energy in the jammer transmitter at the end of time slot t_f. δ_C is the fraction of the energy that has not be dissipated by the end of the following time slot, for the Communicator. δ_J is the fraction of the energy that has not be dissipated by the end of the following time slot, for the Jammer.

Assuming that there is no initial accumulated thermal energy, the evolution of the accumulated thermal energy process can be modeles as follows:

For the Communicatior,

$$Z_0 = 0$$

$$Z_{t_f} = \delta_C Z_{t_f-1} + X_{t_f} = \sum_{n=0}^{t_f-1} \delta_C^n X_{t_f-n}$$

$$\text{under the constraint that } Z_{t_f-n} \leq C_m ax$$

For the Jammer.

$$W_0 = 0$$

$$W_{t_f} = \delta_J W_{t_f-1} + Y_{t_f} = \sum_{n=0}^{t_f-1} \delta_J^n Y_{t_f-n}$$

under the constraint that $W_{t_f-n} \leq J_max$

for all $n = 0,..t_f - 1$ and $t_f = 1, ..., T$. And while X_{t_f} takes its values from the set $0, p_h^s, p_l^s$ and Y_{t_f} from the set $0, p_h^j, p_l^j$.

The payoff $G(X_{t_f}, Y_{t_f})$ to the communicator can be described by the following matrix:

$$G = \begin{bmatrix} G(0,0) & G(0,p_l^j) & G(0,p_h^j) \\ G(p_l^s,0) & G(p_l^s,p_l^j) & G(p_l^s,p_h^j) \\ G(p_h^s,0) & G(p_h^s,p_l^j) & G(p_h^s,p_h^j) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & a_l & 0 \\ 1 & 1 & a_h \end{bmatrix}$$

Where $0 < a_l, a_h < 1$, are the payoff parameters.

The overall payoff is the expected value of the average payoff per slot for activities over a sequence of T time slots:

$$G = \frac{1}{T} \sum_{t_f=1}^{T} E[G(X_{t_f}, Y_{t_f})]$$

Let $t = T - t_f$ denote the reverse-time index. For the Communicator, Z_{T-t} admits only those energies that belong to

$$\Phi_t = z : z = p_h^s \sum_{n=0}^{T-t-1} \beta_n \delta_C^n,\ \beta_0, ..., \beta_{T-t-1} \in 0, 1 \cup z : z = p_l^s \sum_{n=0}^{T-t-1} \beta_n \delta_C^n,\ \beta_0, ..., \beta_{T-t-1} \in 0, 1 \cap [0, C_{max}]$$

where $t = 1, ...T$

For the Jammer, W_{T-t} admits only those energies that belong to

$$\Psi_t = w : w = p_h^j \sum_{n=0}^{T-t-1} \beta_n \delta_J^n,\ \beta_0, ..., \beta_{T-t-1} \in 0, 1 \cup w : w = p_l^j \sum_{n=0}^{T-t-1} \beta_n \delta_J^n,\ \beta_0, ..., \beta_{T-t-1} \in 0, 1 \cup [0, J_{max}]$$

where $t = 1, ...T$

We define the following selection probabilities or strategies:

$$ph_t(z, w) = Pr(X_{T-t} = p_h^s | Z_{T-t-1} = z, W_{T-t-1} = w)$$

which denote the probability that the communicator selects power p_h^s at reverse-time t, given that the communicator and jammer have retained z and w units of energy, respectively, from past transmission. In the same way:

$$pl_t(z, w) = Pr(X_{T-t} = p_l^s | Z_{T-t-1} = z, W_{T-t-1} = w)$$

$$qh_t(z, w) = Pr(Y_{T-t} = p_h^j | Z_{T-t-1} = z, W_{T-t-1} = w)$$

$$ql_t(z,w) = Pr(Y_{T-t} = p_h^j | Z_{T-t-1} = z, W_{T-t-1} = w)$$

So the payoff can be expressed:

$$S_0 = E[G(X_T, Y_T)|Z_{T-1}, W_{T-1}]$$

$$S_t = E[G(X_{T-t}, Y_{T-t}) + S_{t-1}|Z_{T-t-1}, W_{T-t-1}]$$

Where, $T = 1, ...T-1$

We can be also write:

$$S_0 = \begin{bmatrix} 1 - ph_0(z,w) - pl_0(z,w) & pl(z,w) & ph(z,w) \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 1 & a_l & 0 \\ 1 & 1 & a_h \end{bmatrix} \times \begin{bmatrix} 1 - qh_0(z,w) - ql_0(z,w) \\ ql_0(z,w) \\ qh_0(z,w) \end{bmatrix}$$

$$S_{t+1} = \begin{bmatrix} 1 - ph_{t+1}(z,w) - pl_0(z,w) & pl_{t+1}(z,w) & ph_{t+1}(z,w) \end{bmatrix}$$
$$\times \begin{bmatrix} S_t(\delta_C z, \delta_J w) & S_t(\delta_C z, p_l^j + \delta_J w) & S_t(\delta_C z, p_h^j + \delta_J w) \\ 1 + S_t(p_l^s + \delta_C z, \delta_J w) & a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) & S_t(p_l^s + \delta_C z, p_h^j + \delta_J w) \\ 1 + S_t(p_h^s + \delta_C z, \delta_J w) & 1 + S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) & a_h + S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) \end{bmatrix}$$
$$\times \begin{bmatrix} 1 - qh_{t+1}(z,w) - ql_{t+1}(z,w) \\ ql_{t+1}(z,w) \\ qh_{t+1}(z,w) \end{bmatrix}$$

The constraints that are imposed by the energy accumulation, force the following conditions:

I)When $z \in \Phi_{t+1} \cap (C_{mid}, C_{max}]$,

$$ph_t(z,w) = 0 \quad pl_t(z,w) = 0$$

II) When $z \in \Phi_{t+1} \cap (\frac{C_{mid}-C}{\delta_C}, C_{mid}]$,

$$ph_t(z,w) = 0 \quad pl_t(z,w) = 1$$

III) When $z \in \Phi_{t+1} \cap [0, \frac{C_{mid}-C}{\delta_C}]$,

$$ph_t(z,w) = 1 \quad pl_t(z,w) = 0$$

IV) When $w \in \Psi_{t+1} \cap (J_{mid}, J_{max}]$,

$$qh_t(z,w) = 0 \quad ql_t(z,w) = 1$$

V)When $w \in \Psi_{t+1} \cap (\frac{J_{mid}-J}{\delta_J}, J_{mid}]$,

$$qh_t(z,w) = 0 \quad ql_t(z,w) = 1$$

VI) When $w \in \Psi_{t+1} \cap [0, \frac{J_{mid}-J}{\delta_J}]$,

$$qh_t(z,w) = 1 \quad ql_t(z,w) = 0$$

The strategy sets P_t for the communicator and Q_t for the Jammer at reverse time $t = 0,..,T-1$ will be:

$$P_t = \left\{ph_t(z,w)\bigcup(\frac{C_{mid}-C}{\delta_C} - z) : z \in \Phi_{t+1}; w \in \Psi_{t+1}\right\} \cup \left\{pl_t(z,w)\bigcup(C_{mid}-z) : z \in \Phi_{t+1}; w \in \Psi_{t+1}\right\}$$

$$Q_t = \left\{qh_t(z,w)\bigcup(\frac{J_{mid}-J}{\delta_j} - w) : z \in \Phi_{t+1}; w \in \Psi_{t+1}\right\} \cup \left\{ql_t(z,w)\bigcup(J_{mid}-w) : z \in \Phi_{t+1}; w \in \Psi_{t+1}\right\}$$

where $\bigcup(.)$ denotes the unit step function.

2.3 Finite Horizon Game: Existence of a Saddle Point

The matrix of the game -denoting the payoff for both players- is as follows:

$C-J$	0	p_l^j	p_h^j
0	$(0,0)$	$(0,0)$	$(0,0)$
p_l^s	$(1,-1)$	$(\alpha_l,-\alpha_l)$	$(-1,1)$
p_h^s	$(1,-1)$	$(1,-1)$	$(\alpha_h,-\alpha_h)$

We can see that there are no pure strategy Nash Equilibria for the game. According to Nash, the game -as a game with a finite number of players and action profiles- it will have at least one mixed-strategy equilibrium. The mixed equilibria will be studied in a following section.

For the finite horizon game, the T is considered finite. Each non trivial element of the strategy sets $P_0, ..., P_{T-1}, Q_0, ...Q_T - 1$ is a probability by definition. Therefore it belongs to the compact convex set $[0,1]$ on the real line. The payoff can be expressed as $G(P_0, ..., P_{T-1}, Q_0, ..., Q_{T-1})$ and is also affine in each of the nontrivial elements of the strategy sets. Therefore it is a continuous functional of them. Hence, the following exist:

$$max_{\{P_0,...,P_{T-1}\}} G(P_0, ..., P_{T-1}; Q_0, ..., Q_{T-1})$$
$$min_{\{Q_0,...,Q_{T-1}\}} G(P_0, ..., P_{T-1}; Q_0, ..., Q_{T-1})$$

While playing the game, the communicator assumes the worst case in which the jammer minimizes the payoff over all possible strategy set sequences $Q_0, ..., Q_{T-1}$, against any sequence that it uses, and chooses a sequence $P_0^{'}, ..., P_{T-1}^{'}$ such that the maximin payoff V_L is achieved:

$$V_L = max_{\{P_0,...,P_{T-1}\}} min_{\{Q_0,...,Q_{T-1}\}} G(P_0, ..., P_{T-1}; Q_0, ..., Q_{T-1}) =$$

$$= min_{\{Q_0,...,Q_{T-1}\}} G(P_0^{'}, ..., P_{T-1}^{'}; Q_0, ..., Q_{T-1})$$

The jammer chooses a sequence $Q_0^{'}, ..., Q_{T-1}^{'}$ in order to achieve the minimax payoff V_U

$$V_U = min_{\{Q_0,...,Q_{T-1}\}} max_{\{P_0,...,P_{T-1}\}} G(P_0, ..., P_{T-1}; Q_0, ..., Q_{T-1}) =$$
$$= max_{\{P_0,...,P_{T-1}\}} G(P_0, ..., P_{T-1}; Q_0^{'}, ..., Q_{T-1}^{'})$$

From the minimax theorem, we het that $V_L \leq V_U$.

A strategy set sequence $\{P_0^{'}, ..., P_{T-1}^{'}\}$ that satisfies $V_L = min_{\{Q_0,...,Q_{T-1}\}} G(P_0^{'}, ..., P_{T-1}^{'}; Q_0, ..., Q_{T-1})$ is the optimal strategy set sequence for the communicator. Accordingly, a strategy set sequence $\{P_0^{'}, ..., P_{T-1}^{'}\}$ satisfying $V_U = max_{\{P_0,...,P_{T-1}\}} G(P_0, ..., P_{T-1}; Q_0^{'}, ..., Q_{T-1}^{'})$ is an optimal strategy set sequence for the Jammer. As mentioned before, the elements of strategy sets $P_0, ..., P_{T-1}, Q_0, ...Q_T - 1$ belong to the compact convex set $[0, 1]$ and as the payoff is a continuous function fo these non-trivial elements, there exists a sequence $P_0^*, ..., P_{T-1}^*$, and a $Q_0^*, ..., Q_{T-1}^*$ such that:

$$V_L \geq min_{\{Q_0,...,Q_{T-1}\}} G(P_0^{'}, ..., P_{T-1}^{'}; Q_0, ..., Q_{T-1}) \geq G(P_0^{'}, ..., P_{T-1}^{'}; Q_0, ..., Q_{T-1})$$

and

$$V_U \leq max_{\{P_0,...,P_{T-1}\}} G(P_0, ..., P_{T-1}; Q_0^{'}, ..., Q_{T-1}^{'}) \leq G(P_0, ..., P_{T-1}; Q_0^{'}, ..., Q_{T-1}^{'})$$

Therefore, the finite horizon game admits a saddle-point, given by the strategy $P_0^*, ..., P_{T-1}^*, Q_0^*, ..., Q_{T-1}^*$. [3]

2.4 The Grid Solution

In this section the finite horizon game will be solved with the technique of backward induction. Firstly we will define the cases that should be studied and the optimal strategies for each of them and afterwards we will proceed to the formulation of grid solutions.

2.4.1 Optimal strategies

In order to obtain a set of optimal strategies, the following cases should be studied:

If we put the power constraints on an axis we get the following:

	$HIGH$		LOW		$ZERO$		$ZERO$	
0		$\frac{C_{mid}-C}{\delta_C}$		C_{mid}		$\frac{C_{max}-C}{\delta_C}$		C_{max}

So we can write:

HIGH		*LOW*		*ZERO*	
0	$\frac{C_{mid}-C}{\delta_C}$		C_{mid}		C_{max}

And for the Jammer:

HIGH		*LOW*		*ZERO*	
0	$\frac{J_{mid}-J}{\delta_J}$		J_{mid}		J_{max}

Where C and J can take the values p_h^s, p_l^s and p_h^j, p_l^j.

The above lead us to the following cases:

1. $C_{mid} \leq z \leq C_{max}; J_{mid} \leq w \leq J_{max}$

   Noone of the players transmits

2. $C_{mid} \leq z \leq C_{max}; 0 \leq w \leq \frac{J_{mid}-J}{\delta_J}$

   Communicator does not transmit. Jammer transmits at high level

3. $C_{mid} \leq z \leq C_{max}; \frac{J_{mid}-J}{\delta_J} \leq w \leq J_{mid}$

   Communicator does not transmit. Jammer transmits at low level

4. $0 \leq z \leq \frac{C_{mid}-C}{\delta_C}; J_{mid} \leq w \leq J_{max}$

   Communicator transmits at high level. Jammer does not transmit.

5. $0 \leq z \leq \frac{C_{mid}-C}{\delta_C}; 0 \leq w \leq \frac{J_{mid}-J}{\delta_J}$

   Communicator transmits at high level. Jammer transmits at high level

6. $0 \leq z \leq \frac{C_{mid}-C}{\delta_C}; \frac{J_{mid}-J}{\delta_J} \leq w \leq J_{mid}$

   Communicator transmits at high level. Jammer transmits at low level

7. $\frac{C_{mid}-C}{\delta_C} \leq z \leq C_{mid}; J_{mid} \leq w \leq J_{max}$

   Communicator transmits at low level. Jammer does not transmit.

8. $\frac{C_{mid}-C}{\delta_C} \leq z \leq C_{mid}; 0 \leq w \leq \frac{J_{mid}-J}{\delta_J}$

   Communicator transmits at low level. Jammer transmits at high level

9. $\frac{C_{mid}-C}{\delta_C} \leq z \leq C_{mid}; \frac{J_{mid}-J}{\delta_J} \leq w \leq J_{mid}$

   Communicator transmits at low level. Jammer transmits at low level

In order to calculate S_0^*:

For the cases 1-3, since the Communicator does not transmit, its payoff will be 0.

For cases 6 and 7, since the Communicator transmits at a higher than the Jammer level, its payoff will be 1.

For cases 4, 8 since the Jammer transmits at a higher than the Communicator level, the payoff for the Communicator will be 0.

For the cases 5 and 9, the players transmit on the same level and the payoff will be given by the game that is described by the matrix: $\begin{bmatrix} 0 & 0 & 0 \\ 1 & a_l & 0 \\ 1 & 1 & a_h \end{bmatrix}$

$$S_0^*(z,w) = \begin{cases} value(\begin{bmatrix} 0 & 0 & 0 \\ 1 & a_l & 0 \\ 1 & 1 & a_h \end{bmatrix}), \text{ if } 0 \le z \le \dfrac{C_{mid} - C}{\delta_C}; 0 \le w \le \dfrac{J_{mid} - J}{\delta_J} \\ 1, \text{ if } 0 \le z \le \dfrac{C_{mid} - C}{\delta_C}; \dfrac{J_{mid} - J}{\delta_J} \le w \le J_{mid} \\ 1, \text{ if } 0 \le z \le \dfrac{C_{mid} - C}{\delta_C}; J_{mid} \le w \le J_{max} \\ 0, \text{ if } \dfrac{C_{mid} - C}{\delta_C} \le z \le C_{mid}; 0 \le w \le \dfrac{J_{mid} - J}{\delta_J} \\ value(\begin{bmatrix} 0 & 0 & 0 \\ 1 & a_l & 0 \\ 1 & 1 & a_h \end{bmatrix}), \text{ if } \dfrac{C_{mid} - C}{\delta_C} \le z \le C_{mid}; \dfrac{J_{mid} - J}{\delta_J} \le w \le J_{mid} \\ 1, \text{ if } \dfrac{C_{mid} - C}{\delta_C} \le z \le C_{mid}; J_{mid} \le w \le J_{max} \\ 0, \text{ if } C_{mid} \le z \le C_{max}; 0 \le w \le \dfrac{J_{mid} - J}{\delta_J} \\ 0, \text{ if } C_{mid} \le z \le C_{max}; \dfrac{J_{mid} - J}{\delta_J} \le w \le J_{mid} \\ 0, \text{ if } C_{mid} \le z \le C_{max}; J_{mid} \le w \le J_{max} \end{cases}$$

In order to calculate S_{t+1}^*

For the case 1,2,3 where Communicator does not transmit, the payoff will be:

$$S_{t+1}(z,w) = S_t(\delta_C z, \delta_J w)$$

For the cases 4 and 7 where the Jammer does not transmit because of the energy constraints,

$$S_{t+1}(z,w) = max(S_t(\delta_C z, \delta_J w), 1 + S_t(p_h^s + \delta_C z, \delta_J w), 1 + S_t(p_l^s + \delta_C z, \delta_J w))$$

For the case 6, the energy constraints affect the Jammer and force him to transmit at the low level. The payoff will be:

$$S_{t+1}(z,w) = max(S_t(\delta_C z, p_l^j + \delta_J w), 1 + S_t(p_h^s + \delta_C z, p_l^j + \delta_J w), a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w))$$

For the case 8, the energy constraints affect the Communicator and force him to transmit at the low level.

$$S_{t+1}(z,w) = min(1 + S_t(p_l^s + \delta_C z, \delta_J w), S_t(p_l^s + \delta_C z, p_h^j + \delta_J w), a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w))$$

For the cases 5 and 9 where both players transmit at the same level, the payoff will be given by the outcome of the game that is described by the matrix that follows:

$$S_{t+1}(z,w) = \begin{bmatrix} S_t(\delta_C z, \delta_J w) & S_t(\delta_C z, p_l^j + \delta_J w) & S_t(\delta_C z, p_h^j + \delta_J w) \\ 1 + S_t(p_h^s + \delta_C z, \delta_J w) & a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) & S_t(p_l^s + \delta_C z, p_h^j + \delta_J w) \\ 1 + S_t(p_l^s + \delta_C z, \delta_J w) & 1 + S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) & a_h + S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) \end{bmatrix}$$

So we can write,

$$S^*_{t+1}(z,w) = \begin{cases} value(\begin{bmatrix} S_t(\delta_C z, \delta_J w) & S_t(\delta_C z, p_l^j + \delta_J w) & S_t(\delta_C z, p_h^j + \delta_J w) \\ 1 + S_t(p_l^s + \delta_C z, \delta_J w) & a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) & S_t(p_l^s + \delta_C z, p_h^j + \delta_J w) \\ 1 + S_t(p_h^s + \delta_C z, \delta_J w) & 1 + S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) & a_h + S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) \end{bmatrix}), \\ \qquad \text{if } 0 \le z \le \dfrac{C_{mid} - C}{\delta_C}; 0 \le w \le \dfrac{J_{mid} - J}{\delta_J} \\ max(S_t(\delta_C z, p_l^j + \delta_J w), 1 + S_t(p_h^s + \delta_C z, p_l^j + \delta_J w), a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w)), \\ \qquad \text{if } 0 \le z \le \dfrac{C_{mid} - C}{\delta_C}; \dfrac{J_{mid} - J}{\delta_J} \le w \le J_{mid} \\ max(S_t(\delta_C z, \delta_J w), 1 + S_t(p_h^s + \delta_C z, \delta_J w), 1 + S_t(p_l^s + \delta_C z, \delta_J w)), \\ \qquad \text{if } 0 \le z \le \dfrac{C_{mid} - C}{\delta_C}; J_{mid} \le w \le J_{max} \\ min(1 + S_t(p_l^s + \delta_C z, \delta_J w), S_t(p_l^s + \delta_C z, p_h^j + \delta_J w), a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w)), \\ \qquad \text{if } \dfrac{C_{mid} - C}{\delta_C} \le z \le C_{mid}; 0 \le w \le \dfrac{J_{mid} - J}{\delta_J} \\ value(\begin{bmatrix} S_t(\delta_C z, \delta_J w) & S_t(\delta_C z, p_l^j + \delta_J w) & S_t(\delta_C z, p_h^j + \delta_J w) \\ 1 + S_t(p_l^s + \delta_C z, \delta_J w) & a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) & S_t(p_l^s + \delta_C z, p_h^j + \delta_J w) \\ 1 + S_t(p_h^s + \delta_C z, \delta_J w) & 1 + S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) & a_h + S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) \end{bmatrix}), \\ \qquad \text{if } \dfrac{C_{mid} - C}{\delta_C} \le z \le C_{mid}; \dfrac{J_{mid} - J}{\delta_J} \le w \le J_{mid} \\ max(S_t(\delta_C z, \delta_J w), 1 + S_t(p_h^s + \delta_C z, \delta_J w), 1 + S_t(p_l^s + \delta_C z, \delta_J w)), \\ \qquad \text{if } \dfrac{C_{mid} - C}{\delta_C} \le z \le C_{mid}; J_{mid} \le w \le J_{max} \\ S_t(\delta_C z, \delta_J w), \text{ if } C_{mid} \le z \le C_{max}; 0 \le w \le \dfrac{J_{mid} - J}{\delta_J} \\ S_t(\delta_C z, \delta_J w), \text{ if } C_{mid} \le z \le C_{max}; \dfrac{J_{mid} - J}{\delta_J} \le w \le J_{mid} \\ S_t(\delta_C z, \delta_J w), \text{ if } C_{mid} \le z \le C_{max}; J_{mid} \le w \le J_{max} \end{cases}$$

In order to obtain the optimal strategies we will start again by their initial values at $t = 0$. So for $ph_0^*, pl_0^*, qh_0^*, ql_0^*$, we have:

$$\text{a) } ph_0^*(z,w) = 1 \quad \text{b) } pl_0^*(z,w) = 0 \quad \text{c) } qh_0^*(z,w) = 1 \quad \text{d) } ql_0^*(z,w) = 0,$$
$$0 \le z \le \frac{C_{mid} - C}{\delta_C}, 0 \le w \le \frac{J_{mid} - J}{\delta_J}$$

$$\text{a) } ph_0^*(z,w) = 1 \quad \text{b) } pl_0^*(z,w) = 0 \quad \text{c) } qh_0^*(z,w) = 0 \quad \text{d) } ql_0^*(z,w) = 1,$$
$$0 \le z \le \frac{C_{mid} - C}{\delta_C}, \frac{J_{mid} - J}{\delta_J} \le w \le J_{mid}$$

$$\text{a) } ph_0^*(z,w) = 1 \quad \text{b) } pl_0^*(z,w) = 0 \quad \text{c) } qh_0^*(z,w) = 0 \quad \text{d) } ql_0^*(z,w) = 0,$$
$$0 \le z \le \frac{C_{mid} - C}{\delta_C}, J_{mid} \le w \le J_{max}$$

$$\text{a) } ph_0^*(z,w) = 0 \quad \text{b) } pl_0^*(z,w) = 1 \quad \text{c) } qh_0^*(z,w) = 1 \quad \text{d) } ql_0^*(z,w) = 0,$$
$$\frac{C_{mid} - C}{\delta_C} \le z \le C_{mid}, 0 \le w \le \frac{J_{mid} - J}{\delta_J}$$

$$\text{a) } ph_0^*(z,w) = 0 \quad \text{b) } pl_0^*(z,w) = 1 \quad \text{c) } qh_0^*(z,w) = 0 \quad \text{d) } ql_0^*(z,w) = 1,$$
$$\frac{C_{mid} - C}{\delta_C} \le z \le C_{mid}, \frac{J_{mid} - J}{\delta_J} \le w \le J_{mid}$$

$$\text{a) } ph_0^*(z,w) = 0 \quad \text{b) } pl_0^*(z,w) = 1 \quad \text{c) } qh_0^*(z,w) = 0 \quad \text{d) } ql_0^*(z,w) = 0,$$
$$\frac{C_{mid} - C}{\delta_C} \le z \le C_{mid}, J_{mid} \le w \le J_{max}$$

$$\text{a) } ph_0^*(z,w) = 0 \quad \text{b) } pl_0^*(z,w) = 0 \quad \text{c) } qh_0^*(z,w) = 0 \quad \text{d) } ql_0^*(z,w) = 0,$$
$$C_{mid} \le z \le C_{max}, 0 \le w \le \frac{J_{mid} - J}{\delta_J}$$

$$\text{a) } ph_0^*(z,w) = 0 \quad \text{b) } pl_0^*(z,w) = 0 \quad \text{c) } qh_0^*(z,w) = 0 \quad \text{d) } ql_0^*(z,w) = 0,$$
$$C_{mid} \le z \le C_{max}, \frac{J_{mid} - J}{\delta_J} \le w \le J_{mid}$$

$$\text{a) } ph_0^*(z,w) = 0 \quad \text{b) } pl_0^*(z,w) = 0 \quad \text{c) } qh_0^*(z,w) = 0 \quad \text{d) } ql_0^*(z,w) = 0,$$
$$C_{mid} \le z \le C_{max}, J_{mid} \le w \le J_{max}$$

Note than when the Communicator does not transmit, the Jammer even if he has the option to transmit, chooses to remain idle as he has no signal to jam. As we demonstrated previously, as z increases, $S_t^*(z, w)$ decreases and as w increases, $S_t^*(z, w)$ decreases.
So we can procede to the formulas of time-slot t and write:

$$\text{a) } ph_t^*(z,w) = 1 \quad \text{b) } pl_t^*(z,w) = 0 \quad \text{c) } qh_t^*(z,w) = 0 \quad \text{d) } ql_t^*(z,w) = 1,$$
$$0 \le z \le \frac{C_{mid} - C}{\delta_C}, \frac{J_{mid} - J}{\delta_J} \le w \le J_{mid}$$

$$\text{a) } ph_t^*(z,w) = 1 \quad \text{b) } pl_t^*(z,w) = 0 \quad \text{c) } qh_t^*(z,w) = 0 \quad \text{d) } ql_t^*(z,w) = 0,$$
$$0 \le z \le \frac{C_{mid} - C}{\delta_C}, J_{mid} \le w \le J_{max}$$

$$\text{a) } ph_t^*(z,w) = 0 \quad \text{b) } pl_t^*(z,w) = 1 \quad \text{c) } qh_t^*(z,w) = 1 \quad \text{d) } ql_t^*(z,w) = 0,$$
$$\frac{C_{mid} - C}{\delta_C} \le z \le C_{mid}, 0 \le w \le \frac{J_{mid} - J}{\delta_J}$$

$$\text{a) } ph_t^*(z,w) = 0 \quad \text{b) } pl_t^*(z,w) = 1 \quad \text{c) } qh_t^*(z,w) = 0 \quad \text{d) } ql_t^*(z,w) = 0,$$
$$\frac{C_{mid} - C}{\delta_C} \le z \le C_{mid}, J_{mid} \le w \le J_{max}$$

$$\text{a) } ph_t^*(z,w) = 0 \quad \text{b) } pl_t^*(z,w) = 0 \quad \text{c) } qh_t^*(z,w) = 0 \quad \text{d) } ql_t^*(z,w) = 0,$$
$$C_{mid} \le z \le C_{max}, 0 \le w \le \frac{J_{mid} - J}{\delta_J}$$

$$\text{a) } ph_t^*(z,w) = 0 \quad \text{b) } pl_t^*(z,w) = 0 \quad \text{c) } qh_t^*(z,w) = 0 \quad \text{d) } ql_t^*(z,w) = 0,$$
$$C_{mid} \le z \le C_{max}, \frac{J_{mid} - J}{\delta_J} \le w \le J_{mid}$$

$$\text{a) } ph_t^*(z,w) = 0 \quad \text{b) } pl_t^*(z,w) = 0 \quad \text{c) } qh_t^*(z,w) = 0 \quad \text{d) } ql_t^*(z,w) = 0,$$
$$C_{mid} \le z \le C_{max}, J_{mid} \le w \le J_{max}$$

And the following two refer to the cases where they both transmit on the same level [8]:

$$\text{a) } ph_t^*(z,w) = 1 \quad \text{b) } pl_t^*(z,w) = 0 \quad \text{c) } qh_t^*(z,w) = 1 \quad \text{d) } ql_t^*(z,w) = 0,$$

$$0 \leq z \leq \frac{C_{mid} - C}{\delta_C}, 0 \leq w \leq \frac{J_{mid} - J}{\delta_J}$$

$$\text{a) } ph_t^*(z,w) = 0 \quad \text{b) } pl_t^*(z,w) = 1 \quad \text{c) } qh_t^*(z,w) = 0 \quad \text{d) } ql_t^*(z,w) = 1,$$
$$\frac{C_{mid} - C}{\delta_C} \leq z \leq C_{mid}, \frac{J_{mid} - J}{\delta_J} \leq w \leq J_{mid}$$

2.4.2 The grid solution of the game

After defining the optimal strategies, we will proceed to the grid solution of the game. The energy constraints define 9 different regions of operation, so the grid solution has the following form:

ZERO	S_{13}^t	S_{23}^t	S_{33}^t
LOW	S_{12}^t	S_{22}^t	S_{32}^t
HIGH	S_{11}^t	S_{21}^t	S_{31}^t
J/C	HIGH	LOW	ZERO

Accordingly, $S_t^*(z,w)$ has a 3x3 structure for all t and can be defined:

$$S_t^*(z,w) = \begin{cases} S_{11}^t, \text{ if } 0 \leq z \leq \frac{C_{mid} - C}{\delta_C}; 0 \leq w \leq \frac{J_{mid} - J}{\delta_J} \\ S_{12}^t, \text{ if } 0 \leq z \leq \frac{C_{mid} - C}{\delta_C}; \frac{J_{mid} - J}{\delta_J} \leq w \leq J_{mid} \\ S_{13}^t, \text{ if } 0 \leq z \leq \frac{C_{mid} - C}{\delta_C}; J_{mid} \leq w \leq J_{max} \\ S_{21}^t, \text{ if } \frac{C_{mid} - C}{\delta_C} \leq z \leq C_{mid}; 0 \leq w \leq \frac{J_{mid} - J}{\delta_J} \\ S_{22}^t, \text{ if } \frac{C_{mid} - C}{\delta_C} \leq z \leq C_{mid}; \frac{J_{mid} - J}{\delta_J} \leq w \leq J_{mid} \\ S_{23}^t, \text{ if } \frac{C_{mid} - C}{\delta_C} \leq z \leq C_{mid}; J_{mid} \leq w \leq J_{max} \\ S_{31}^t, \text{ if } C_{mid} \leq z \leq C_{max}; 0 \leq w \leq \frac{J_{mid} - J}{\delta_J} \\ S_{32}^t, \text{ if } C_{mid} \leq z \leq C_{max}; \frac{J_{mid} - J}{\delta_J} \leq w \leq J_{mid} \\ S_{33}^t, \text{ if } C_{mid} \leq z \leq C_{max}; J_{mid} \leq w \leq J_{max} \end{cases}$$

Taking into consideration the optimal strategies that we have already defined, we can obtain the following results at $t = 0$:

$$\text{a)} S_{11}^0 = a_h \quad \text{b)} S_{12}^0 = 1 \quad \text{c)} S_{13}^0 = 1$$
$$\text{d)} S_{21}^0 = 0 \quad \text{e)} S_{22}^0 = a_h \quad \text{f)} S_{23}^0 = 1$$
$$\text{g)} S_{31}^0 = 0 \quad \text{h)} S_{32}^0 = 0 \quad \text{i)} S_{33}^0 = 0$$

By using the formulas of $S_t^*(z,w)$ at a grid form and at its initial, and by substitution, we obtain the formulas for S^{t+1}.

We should take into account that $S_t^*(z,w)$ decreases with z and increases with w and $a_l, a_h \leq 1$.

So, taking into account the following:

ZERO	S_{13}^t	S_{23}^t	S_{33}^t
LOW	S_{12}^t	S_{22}^t	S_{32}^t
HIGH	S_{11}^t	S_{21}^t	S_{31}^t
J/C	HIGH	LOW	ZERO

we have that:

$$S_{11}^{t+1} =$$

$$value(\begin{bmatrix} S_{11}(\delta_C z, \delta_J w) & S_{11}(\delta_C z, p_l^j + \delta_J w) & S_{11}(\delta_C z, p_h^j + \delta_J w) \\ 1 + S_{11}(p_l^s + \delta_C z, \delta_J w) & a_l + S_{11}(p_l^s + \delta_C z, p_l^j + \delta_J w) & S_{11}(p_l^s + \delta_C z, p_h^j + \delta_J w) \\ 1 + S_{11}(p_h^s + \delta_C z, \delta_J w) & 1 + S_{11}(p_h^s + \delta_C z, p_l^j + \delta_J w) & a_h + S_{11}(p_h^s + \delta_C z, p_h^j + \delta_J w) \end{bmatrix})$$

$$= value(\begin{bmatrix} S_{11}^t & S_{12}^t & S_{13}^t \\ 1 + S_{21}^t & a_l + S_{22}^t & S_{23}^t \\ 1 + S_{31}^t & 1 + S_{32}^t & a_h + S_{33}^t \end{bmatrix})$$

$$S_{12}^{t+1} = max(S_{12}^t, a_l + S_{22}^t, 1 + S_{32}^t)$$

$$S_{13}^{t+1} = S_{23}^{t+1} = max(S_{11}^t, 1 + S_{21}^t, 1 + S_{31}^t)$$

$$S_{21}^{t+1} = min(1 + S_{21}^t, a_l + S_{22}^t, S_{23}^t)$$

$$S_{22}^{t+1} = value(\begin{bmatrix} S_{11}^t & S_{12}^t & S_{13}^t \\ 1 + S_{21}^t & a_l + S_{22}^t & S_{23}^t \\ 1 + S_{31}^t & 1 + S_{32}^t & a_h + S_{33}^t \end{bmatrix})$$

$$S_{31}^{t+1} = S_{32}^{t+1} = S_{31}^{t+1} = S_{11}^t$$

Since the initial conditions S_{11}, S_{12}, S_{13}, S_{21}, S_{22}, S_{23}, S_{31}, S_{32}, S_{33} have been determined, we can solve for $T = 1, ...(T-1)$ -for a finite T, with backward induction.

2.5 The Steady State Solution - Mixed Nash Equilibria

In this section we will examine the mixed equilibria of the game. The matrix of the game has a 3x3 form and the methodology of [8] cannot be used in order to define the values of the payoff parameters for a mixed Nas Equilibrium.

What we propose is the seperation of the matrix to subgames. In order to do that, we will consider the cases that the Communicator mixes between 2 strategies and not 3. Every subgame will be solved seperately by imposing appropriate constraints that will justify the choice of the communicator to play the particular subgame instead of the 3x3 one.

As it occurs from the evolution equation,

$$S^*_{t+1}(z,w) = max_{ph_{t+1},pl_{t+1}} min_{qh_{t+1},ql_{t+1}}$$

$$\sum_{x\in X}\sum_{y\in Y} ph_{t+1}(x/z,w)pl_{t+1}(x/z,w)qh_{t+1}(y/z,w)ql_{t+1}(y/z,w)[f(x,y) + S^*_t(x+\delta_C z, y+\delta_J w)]$$

The optimum payoff appears to increase as t increases and the increment when going from t to $t+1$ is bounded in $[0,1]$, since all the elements of the payoff matrix lie in $[0,1]$.

In order to solve the equation for $S^*_t(z,w)$ when $t \to \infty$, we define the payoff increment

$$\lambda^t_{ij} = S^{t+1}_{ij} - S^t_{ij}, \quad i = 1.2, j = 1,2, t = 0,1,2,...$$

from where we get $S^{t+1}_{ij} = \lambda^t_{ij} + S^t_{ij}$

At a steady state situation as $t \to \infty$, $lim_{t\to\infty} ph^t_{11} = ph^t_{11}$, $lim_{t\to\infty} qh^t_{11} = qh^t_{11}$, $lim_{t\to\infty} ph^t_{22} = ph^t_{22}$, $lim_{t\to\infty} qh^t_{22} = qh^t_{22}$.

In the case the Communicator does not mix all his three strategies, we have to study the following cases, that formulate 4 subgames.:

Case 1 The Communicator mixes between the strategies low and high. The Jammer mixes between the strategies low and high, too.

In order this scenario to occur, the following constraints should be satisfied:

(We consider U() as the utility)

$$U_s(ZERO) < U_s(LOW) \Rightarrow$$

a) $U_s(p^s_0, p^j_l) < U_s(p^s_l, p^j_l)$

b) $U_s(p^s_0, p^j_l) < U_s(p^s_l, p^j_h)$

c) $U_s(p^s_0, p^j_h) < U_s(p^s_l, p^j_h)$

d) $U_s(p^s_0, p^j_h) < U_s(p^s_l, p^j_l)$

and

$$U_s(ZERO) < U_s(HIGH) \Rightarrow$$

a) $U_s(p^s_0, p^j_l) < U_s(p^s_h, p^j_l)$

b) $U_s(p^s_0, p^j_l) < U_s(p^s_h, p^j_h)$

c) $U_s(p^s_0, p^j_h) < U_s(p^s_h, p^j_h)$

d) $U_s(p^s_0, p^j_h) < U_s(p^s_h, p^j_l)$

The above lead to the following:

$$\begin{aligned}
\text{a)}\quad & S_t(\delta_C z, p_l^j + \delta_J w) < a_l + S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) \Rightarrow \\
& S_t(\delta_C z, p_l^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) < a_l
\end{aligned}$$

And in the same way:

$$\begin{aligned}
\text{b)}\quad & S_t(\delta_C z, p_l^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_h^j + \delta_J w) < 1 \\
\text{c)}\quad & S_t(\delta_C z, p_h^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_h^j + \delta_J w) < 1 \\
\text{d)}\quad & S_t(\delta_C z, p_h^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) < a_l \\
\text{e)}\quad & S_t(\delta_C z, p_l^j + \delta_J w) - S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) < 1 \\
\text{f)}\quad & S_t(\delta_C z, p_l^j + \delta_J w) - S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) < a_h \\
\text{g)}\quad & S_t(\delta_C z, p_h^j + \delta_J w) - S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) < a_h \\
\text{h)}\quad & S_t(\delta_C z, p_h^j + \delta_J w) - S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) < 1
\end{aligned}$$

We should formulate the evolution equation for the subgame

S_{11}^t	S_{12}^t
$1 + S_{21}^t$	$a_l + S_{22}^t$

:

$$\begin{aligned}
\text{a)}\quad & S_{22}^{t+1} = value(\begin{bmatrix} S_{11}^t & S_{12}^t \\ 1 + S_{21}^t & a_l + S_{22}^t \end{bmatrix}) \\
\text{b)}\quad & S_{12}^{t+1} = max(S_{12}^t, a_l + S_{22}^t) = a_l + S_{22}^t \\
\text{c)}\quad & S_{21}^{t+1} = min(1 + S_{21}^t, a_l + S_{22}^t) = a_l + S_{22}^t \\
\text{d)}\quad & S_{11}^{t+1} = value(\begin{bmatrix} S_{11}^t & S_{12}^t \\ 1 + S_{21}^t & a_l + S_{22}^t \end{bmatrix})
\end{aligned}$$

Studying at the steady state:

$$\lambda_{11}^t + S_{11}^t = value(\begin{bmatrix} S_{11}^t & S_{11}^t - \lambda_{12}^t \\ 1 + S_{21}^{t+1} - \lambda_{21}^t & a_l + S_{11}^t \end{bmatrix})$$

$$\lambda_{11}^t = value(\begin{bmatrix} 0 & -\lambda_{12}^t \\ 1 - \lambda_{21}^t & a_l \end{bmatrix})$$

And as $\lambda_{11}^t = \lambda_{12}^t = \lambda_{21}^t = \lambda_{22}^t = \lambda_1$, we finaly have:

$$\lambda_1 = value(\begin{bmatrix} 0 & -\lambda_1 \\ 1 - \lambda_1 & a_l \end{bmatrix})$$

Which gives

$$\lambda_1 = \frac{2 - a_l}{3}$$

So, from the constraints -holding those that can be valid- we get:

$$a_l > \frac{1}{2}, \;\; a_l > -1, \;\; 3a_h - a_l > -2$$

Which leads to the following:

$$a_l > \frac{1}{2}$$
$$3a_h - a_l > -2$$

Case 2 The Communicator mixes between the strategies low and high. The Jammer mixes between the strategies low and zero.

The constraints will be:

$$U_s(ZERO) < U_s(LOW) \Rightarrow$$

$$\text{a)} \qquad U_s(p_0^s, p_0^j) < U_s(p_l^s, p_0^j)$$
$$\text{b)} \qquad U_s(p_0^s, p_0^j) < U_s(p_l^s, p_l^j)$$
$$\text{c)} \qquad U_s(p_0^s, p_l^j) < U_s(p_l^s, p_l^j)$$
$$\text{d)} \qquad U_s(p_0^s, p_l^j) < U_s(p_l^s, p_0^j)$$

and

$$U_s(ZERO) < U_s(HIGH) \Rightarrow$$

$$\text{a)} \qquad U_s(p_0^s, p_0^j) < U_s(p_h^s, p_0^j)$$
$$\text{b)} \qquad U_s(p_0^s, p_0^j) < U_s(p_h^s, p_l^j)$$
$$\text{c)} \qquad U_s(p_0^s, p_l^j) < U_s(p_h^s, p_l^j)$$
$$\text{d)} \qquad U_s(p_0^s, p_l^j) < U_s(p_h^s, p_0^j)$$

The above lead to the following constraints:

$$\text{a)} \qquad S_t(\delta_C z, \delta_J w) - S_t(p_l^s + \delta_C z, \delta_J w) < 1$$
$$\text{b)} \qquad S_t(\delta_C z, \delta_J w) - S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) < a_l$$
$$\text{c)} \qquad S_t(\delta_C z, p_l^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) < a_l$$
$$\text{d)} \qquad S_t(\delta_C z, p_l^j + \delta_J w) - S_t(p_l^s + \delta_C z, \delta_J w) < 1$$
$$\text{e)} \qquad S_t(\delta_C z, \delta_J w) - S_t(p_h^s + \delta_C z, \delta_J w) < 1$$
$$\text{f)} \qquad S_t(\delta_C z, \delta_J w) - S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) < 1$$
$$\text{g)} \qquad S_t(\delta_C z, p_l^j + \delta_J w) - S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) < 1$$
$$\text{h)} \qquad S_t(\delta_C z, p_l^j + \delta_J w) - S_t(p_h^s + \delta_C z, \delta_J w) < 1$$

We should formulate the evolution equation for the subgame $\begin{array}{|c|c|} \hline S_{12}^t & S_{13}^t \\ \hline a_l + S_{22}^t & S_{23}^t \\ \hline \end{array}$:

$$\text{a)} \qquad S_{22}^{t+1} = value(\begin{bmatrix} S_{11}^t & S_{12}^t \\ 1 + S_{21}^t & a_l + S_{22}^t \end{bmatrix})$$

$$\text{b)} \qquad S_{12}^{t+1} = max(S_{12}^t, a_l + S_{22}^t) = a_l + S_{22}^t$$

$$\text{c)} \qquad S_{13}^{t+1} = max(S_{12}^t, a_l + S_{22}^t) = a_l + S_{22}^t, \text{for } a_l << 1$$

$$\text{d)} \qquad S_{23}^{t+1} = S_{13}^{t+1}$$

Studying at the steady state:

$$\lambda_{22}^t + S_{22}^t = value(\begin{bmatrix} S_{12}^{t+1} - \lambda_{12}^t & S_{13}^t - \lambda_{13}^t \\ a_l + S_{22}^{t+1} & S_{23}^{t+1} - \lambda_{23}^t \end{bmatrix})$$

$$\lambda_{22}^t = value(\begin{bmatrix} a_l - \lambda_{12}^t & a_l - \lambda_{13}^t \\ a_l & a_l - \lambda_{23}^t \end{bmatrix})$$

And as $\lambda_{12}^t = \lambda_{13}^t = \lambda_{22}^t = \lambda_{23}^t = \lambda_2$, we finaly have:

$$\lambda_2 = value(\begin{bmatrix} a_l - \lambda_2 & a_l - \lambda_2 \\ a_l & a_l - \lambda_2 \end{bmatrix})$$

Which gives

$$\lambda_2 = \frac{a_l}{2}$$

So from the constraints:

$$a_l > 1, \ \frac{a_l}{2} > a_l, \ -\frac{a_l}{2} > a_l, \ a_l > -1$$

Case 3 The Communicator mixes between the strategies zero and low. The Jammer mixes between the strategies low and high.

The constraints will be:

$$U_s(HIGH) < U_s(LOW) \Rightarrow$$

$$\text{a)} \qquad U_s(p_h^s, p_l^j) < U_s(p_l^s, p_l^j)$$

$$\text{b)} \qquad U_s(p_h^s, p_l^j) < U_s(p_l^s, p_h^j)$$

$$\text{c)} \qquad U_s(p_h^s, p_h^j) < U_s(p_l^s, p_h^j)$$

$$\text{d)} \qquad U_s(p_h^s, p_h^j) < U_s(p_l^s, p_l^j)$$

and

$$U_s(HIGH) < U_s(ZERO) \Rightarrow$$

$$\begin{array}{ll} \text{a)} & U_s(p_h^s, p_l^j) < U_s(p_0^s, p_l^j) \\ \text{b)} & U_s(p_h^s, p_l^j) < U_s(p_0^s, p_h^j) \\ \text{c)} & U_s(p_h^s, p_h^j) < U_s(p_0^s, p_h^j) \\ \text{d)} & U_s(p_h^s, p_h^j) < U_s(p_0^s, p_l^j) \end{array}$$

The above lead to the following constraints:

$$\begin{array}{ll} \text{a)} & S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) < a_l - 1 \\ \text{b)} & S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_h^j + \delta_J w) < -1 \\ \text{c)} & S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_h^j + \delta_J w) < -a_h \\ \text{d)} & S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) < a_l - a_h \\ \text{e)} & S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) - S_t(\delta_C z, p_l^j + \delta_J w) < -1 \\ \text{f)} & S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) - S_t(\delta_C z, p_h^j + \delta_J w) < -1 \\ \text{g)} & S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) - S_t(\delta_C z, p_h^j + \delta_J w) < -a_h \\ \text{h)} & S_t(p_h^s + \delta_C z, p_h^j + \delta_J w) - S_t(\delta_C z, p_l^j + \delta_J w) < -a_h \end{array}$$

We should formulate the evolution equation for the subgame

$1 + S_{21}^t$	$a_l + S_{22}^t$
$1 + S_{31}^t$	$1 + S_{32}^t$

:

$$\text{a)} \quad S_{22}^{t+1} = value(\begin{bmatrix} S_{11}^t & S_{12}^t \\ 1 + S_{21}^t & a_l + S_{22}^t \end{bmatrix})$$

$$\text{b)} \quad S_{21}^{t+1} = min(1 + S_{21}^t, a_l + S_{22}^t) = a_l + S_{22}^t, \text{for } a_l << 1$$

$$\text{c)} \quad S_{31}^{t+1} = s_{22}^t$$

$$\text{d)} \quad S_{32}^{t+1} = S_{22}^t$$

Studying at the steady state:

$$\lambda_{22}^t + S_{22}^t = value(\begin{bmatrix} 1 + S_{21}^{t+1} - \lambda_{21}^t & S_{22}^t + a_l \\ 1 + S_{31}^{t+1} - \lambda_{31}^t & 1 + S_{32}^{t+1} - \lambda_{32}^t \end{bmatrix})$$

$$\lambda_{22}^t = value(\begin{bmatrix} 1 + a_l - \lambda_{21}^t & a_l \\ 1 - \lambda 31^t & 1 - \lambda_{32}^{t+1} \end{bmatrix})$$

And as $\lambda_{21}^t = \lambda_{22}^t = \lambda_{31}^t = \lambda_{32}^t = \lambda_3$, we finaly have:

$$\lambda_3 = value(\begin{bmatrix} 1 + a_l - \lambda_3 & a_l \\ 1 - \lambda_3 & 1 - \lambda_3 \end{bmatrix})$$

Which gives

$$\lambda_3 = \frac{1}{2}$$

So from the constraints, by holding those that can be valid:

$$a_l > \frac{3}{2}, \quad a_l > \frac{5}{2}, \quad a_l - a_h > \frac{1}{2}, \quad a_l - a_h > 0, \quad a_h > \frac{1}{2}$$

Which leads to:

$$a_l > \frac{3}{2} a_l - a_h > \frac{1}{2} a_h > \frac{1}{2}$$

Case 4 The Communicator mixes between the strategies zero and low. The Jammer mixes between the strategies zero and low, too.

The constraints will be:

$$U_s(HIGH) < U_s(LOW) \Rightarrow$$

$$\begin{array}{ll}
\text{a)} & U_s(p_h^s, p_l^j) < U_s(p_l^s, p_l^j) \\
\text{b)} & U_s(p_h^s, p_l^j) < U_s(p_l^s, p_0^j) \\
\text{c)} & U_s(p_h^s, p_0^j) < U_s(p_l^s, p_0^j) \\
\text{d)} & U_s(p_h^s, p_0^j) < U_s(p_l^s, p_l^j)
\end{array}$$

and

$$U_s(HIGH) < U_s(ZERO) \Rightarrow$$

$$\begin{array}{ll}
\text{a)} & U_s(p_h^s, p_0^j) < U_s(p_0^s, p_0^j) \\
\text{b)} & U_s(p_h^s, p_0^j) < U_s(p_0^s, p_l^j) \\
\text{c)} & U_s(p_h^s, p_l^j) < U_s(p_0^s, p_l^j) \\
\text{d)} & U_s(p_h^s, p_l^j) < U_s(p_0^s, p_0^j)
\end{array}$$

$$\begin{array}{ll}
\text{a)} & S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) - S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) < a_l - 1 \\
\text{b)} & S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) - S_t(p_l^s + \delta_C z, \delta_J w) < 1 - 1 = 0 \\
\text{c)} & S_t(p_h^s + \delta_C z, \delta_J w) - S_t(p_l^s + \delta_C z, \delta_J w) < 1 - 1 = 0 \\
\text{d)} & S_t(p_h^s + \delta_C z, \delta_J w) - S_t(p_l^s + \delta_C z, p_l^j + \delta_J w) < a_l - 1 \\
\text{e)} & S_t(p_h^s + \delta_C z, \delta_J w) - S_t(\delta_C z, \delta_J w) < -1 \\
\text{f)} & S_t(p_h^s + \delta_C z, \delta_J w) - S_t(\delta_C z, p_l^j + \delta_J w) < -1 \\
\text{g)} & S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) - S_t(\delta_C z, p_l^j + \delta_J w) < -1 \\
\text{h)} & S_t(p_h^s + \delta_C z, p_l^j + \delta_J w) - S_t(\delta_C z, \delta_J w) < -1
\end{array}$$

We should formulate the evolution equation for the subgame

$a_l + S^t_{22}$	S^t_{23}
$1 + S^t_{32}$	$a_h + S^t_{33}$

:

a) $$S^{t+1}_{22} = value(\begin{bmatrix} a_l + S^t_{22} & S^t_{23} \\ 1 + S^t_{32} & a_h + S^t_{33} \end{bmatrix})$$

b) $$S^{t+1}_{23} = max(1 + S^t_{32}, a_l + S^t_{22}) = 1 + S^t_{32}, \text{for } a_l << 1$$

c) $$S^{t+1}_{32} = S^t_{22}$$

d) $$S^{t+1}_{33} = S^t_{33} = S^t_{22}$$

Studying at the steady state:

$$\lambda^t_{22} + S^t_{22} = value(\begin{bmatrix} a_l + S^{t+1}_{21} & S^{t+1}_{23} - \lambda^t_{23} \\ 1 + S^{t+1}_{32} - \lambda^t_{33} & a_h + S^{t+1}_{33} - \lambda^t_{33} \end{bmatrix})$$

$$\lambda^t_{22} = value(\begin{bmatrix} a_l & 1 - \lambda^t_{32} - \lambda^t_{23} \\ 1 - \lambda 32^t & a_h - \lambda^t_{33} \end{bmatrix})$$

And as $\lambda^t_{22} = \lambda^t_{21} = \lambda^t_{32} = \lambda^t_{33} = \lambda_4$, we finaly have:

$$\lambda_4 = value(\begin{bmatrix} a_l & 1 - 2\lambda_4 \\ 1 - \lambda_4 & a_h - \lambda_4 \end{bmatrix})$$

Which does not give a real value for λ_4.

Combining the results from all the above constraints we can conclude that a mixed nash equilibrium exists for the game when:

a)	$a_l - 3a_h < 2$
b)	$a_l - a_h < \frac{1}{2}$
c)	$a_h > \frac{1}{2}$

2.6 Conclusion

In this chapter we have extended the work of [8] for more than 2 strategies. We have formulated the grid solution for the finite horizon game, with the backward induction technique. The main finding is that under certain operating conditions, the jamming game has a mixed nash equilibrium. The value of the conditions have been obtained in relation to the payoff parameters a_h and a_l.

Additionally, we have proposed a methodology in order to end a solution for matrices bigger than 2x2. Our analysis shows that we can seperate the game into 2x2 subgames under conditions. Each game can be solved seperately. In the end, we combine all the constraints of the subgames, so as to find the solution of the initial game.

Like in [8], we assume that the players have knowledge of the payoff parameters, as well as of the parameters that describe the transmitters. Another assumption is that a player can obtain a correct feedback about the past actions of the other.

Chapter 3

Cooperative Relaying under the Presence of a Jammer

3.1 Introduction

In this section we will study the Jammin problem in a cooperative communication environment. The model of a source, a relay node and a destination is used. Firstly we will introduce our problem and the formulations for the source-relay channel. Afterwards we will examine the communication in the relay-destination for the amplify-and-forward scenarion and for the decode=and-forward too. The results of a simulation that compares the two techniques will be demonstrated along with the conclusion that we can draw. Finally, the contribution of this study will be presented.

3.2 Introduction to the Problem

The fundamental idea of the cooperative communication scenario that we are studying is that when a source broadcasts her message, targeting a specific destination, nodes of the network that may be closer to the source than the desirable receiver may also receive the message. As the channel between the source and the destination may have bad characteristics, a relay node will retransmit this message in order to help the communication, so in the end the destination will have a contribution of SNR from both the source and the relay node.

The relay node may transmit the message by simply amplifying it (Amplify-and-Forward case), or can decode the signal and transmit after re-encoding it (Decode-and-Forward case).

If we consider that the communication takes place in two time-slots (in the first we have the source broadcast and in the second the transmission of the relay), the Jammer has two slots to attack. By attacking the first time slot, he affects the channel between the source and the relay as well as the direct channel between the source and the destination. In the second time slots, he affects the channel between the relay and the destination.

We assume a pole of R nodes that could serve as a relay. We study the case where the source has to choose optimally which one will be the node that will finally relay.On the other hand, the Jammer has to decide optimally for the power allocation on the two time-slots, given his power budjet.

For our analysis, we consider that the characteristics of the channels and the choices are known to both players, as well as the choice of one another.

3.3 The Source's Broadcast

In the first time-slot of the communication the source broadcasts her message. This message will be received from the destination -probably through a bad channel- and some of the intermediate nodes that are candidates to act as relays.

Source broadcasts with P_s and we assume a set of $\mathcal{R} = \{1, 2, \cdots, R\}$ R relay nodes.
The jammer attacks the broadcast transmission of the source with power J_0. At the receiver, the signal received from the direct channel between source and destination will be:

$$Y_{s,d} = \sqrt{P_s g_{s,d}} X_s + n + \sqrt{J_0 h_{j,d}} X_0,$$

where $g_{s,d}$: the source-destination channel gain, $h_{j,d}$: the jammer-destination channel gain
And the SNR is given by:

$$SNR_{s,d} \triangleq \gamma_0 = \frac{P_s g_{s,d}}{h_{j,d} J_0 + \sigma^2}$$

3.4 Amplify and Forward Case

In this section -in the context of Amplify-and-Forward scenario- firstly we will formulate the SNR function for the communication between the relay and the destination. The utility function for the Jammer -and accordingly for the source- will be defined and an algorithm that solve's the power allocation problem of the Jammer will be proposed. The utility function will be studied in some scenarios of power allocation, in order to investigate the conditions that should be satisfied for the Jammer. Finally the strategy of the source will be studied and we will proceed to an algorithm for the optimal choice of relay by her.

3.4.1 The relay-destination communication

A relay i receives from the source the following signal:

$$S - R_i : Y_{s,i} = \sqrt{P_s g_{s,i}} X_s + n + \sqrt{J_0 h_{j,i}} X_0$$

According to the AF technique, the relay amplifies $Y_{s,i}$, with power P_i and forwards.

At the destination D the contribution of relay i will be,

$$Y_{i,d} = \sqrt{P_i g_{i,d}} X_{i,d} + n + \sqrt{J_i h_{j,d}} X_i$$

where, $X_{i,d} = \frac{Y_{s,i}}{|Y_{s,i}|}$: the unit energy - transmitted signal that R_i receives from S

So,

$$Y_{i,d} = \frac{\sqrt{P_i g_{i,d}} \left(\sqrt{P_S g_{s,i}} X_S + n + \sqrt{J_0 h_{j,i}} X_0 \right)}{\sqrt{P_s g_{s,i} + \sigma^2 + J_0 h_{j,i}}} + n + \sqrt{J_i h_{j,d}} X_i.$$

The SNR for relay R_i at the destination has the following expression:

$$\begin{aligned} SNR_i &\triangleq \gamma_i = \frac{P_i P_S g_{i,d} g_{s,i}}{P_i g_{i,d}(J_0 h_{j,i} + \sigma^2) + (J_i h_{j,d} + \sigma^2)(P_s g_{s,i} + J_0 h_{j,i} + \sigma^2)} \\ &= \left(\frac{1}{\gamma_{s,i}} + \frac{1}{\gamma_{i,d}} \right)^{-1} \text{ if SNR large,} \end{aligned}$$

where $\gamma_{s,i} = \frac{P_s g_{s,i}}{\sigma^2 + J_0 h_{j,i}}$ and $\gamma_{i,d} = \frac{P_i g_{i,d}}{\sigma^2 + J_i h_{j,d}}$.

The source could choose one or more intermediate nodes to act as relays. The number of relays defines the number of time-slots of the communication. More preciesly, if R relays are used, the number of the time-slots will be $R + 1$. The extra one time-slot is for the first time-slot when the source broadcast the message. Each relay will have a SNR_i contribution at the destination.

3.4.2 Jammer's Utility

For the Jammer, as mentinoed before, the problem that has to be solved is the power allocation at each time slot. Here, we formulate the utility of the Jammer for the case that many relays are used and we propose an algorithm for the power allocation problem. The formula for one relay is directly derived for R=1. Jammer and Source want to minimize and maximize respectively the same formula that expresses the Utility of the game. The utility (for multiple relays) is given by:

$$U_{AFm} = \frac{1}{R+1} \log \left(1 + \sum_{i=0}^{R} \gamma_i \right)$$

And the optimization problem for the Jammer is:

$$\text{Maximize } U_J = -U_{AFm}$$
$$\text{Subject to: } \sum_{i=0}^{R} J_i \le P,$$
$$J_i \ge 0 \quad \forall i \in \mathcal{R}$$

Algorithm Solving the Jammer's Problem:

Algorithm 1 Solving the jammer's problem

1: **Initialization**: set $\mathcal{R}^* = \emptyset$, $J_i = 0, \forall i \in \mathcal{R}\bigcup\{0\}$, set ΔJ to some small value
2: **while** $\sum_{i\in\mathcal{R}\bigcup\{0\}} J_i < 1$ **do**
3: Calculate $\frac{\partial U_J}{J_i}, \forall i \in \mathcal{R}\bigcup\{0\}$
4: Choose $i^* = \text{argmax}_{i\in\mathcal{R}\bigcup\{0\}} \frac{\partial U_J}{\partial J_i}$
5: $J_{i^*} = J_{i^*} + \Delta J$
6: **end while**

3.4.3 Studying the one-relay case

At the case of one relay, i.e. $R = 1$, the SNR for relay R at the destination:

$$\begin{aligned} SNR_R \;&\triangleq\; \gamma_R = \frac{P_R P_s g_{R,d} g_{s,R}}{P_R g_{R,d}(J_0 h_{j,R} + \sigma^2) + (J_R h_{j,d} + \sigma^2)(P_s g_{s,R} + J_0 h_{j,R} + \sigma^2)} \\ &= \left(\frac{1}{\gamma_{s,R}} + \frac{1}{\gamma_{R,d}}\right)^{-1} \text{if SNR large,} \end{aligned}$$

where $\gamma_{s,R} = \frac{P_s g_{s,R}}{\sigma^2 + J_0 h_{j,R}}$ and $\gamma_{R,d} = \frac{P_R g_{R,d}}{\sigma^2 + J_R h_{j,d}}$.

$$U_{AF} = \frac{1}{2}\log\left(1 + \gamma_0 + \gamma_R\right)$$

And the optimization problem for the power allocation in the two time-slots:

$$\text{Minimize} U_{AF}$$
$$\text{Subject to: } J_0 + J_R \le P$$
$$\text{and } J_0, J_R \ge 0$$

In more details we can write:

$$U_{AF} = \frac{1}{2}log(1 + \frac{P_s g_s, d}{h_{j,d} J_0 + \sigma^2} + \frac{P_R P_s g_{R,d} g_{s,R}}{P_r g_{R,d}(J_0 h_{j,R} + \sigma^2) + (J_R h_{J,d} + \sigma^2)(P_s g_{s,R} + J_0 h_{j,R} + \sigma^2)})$$

Our goal is to define some conditions that will indicate when the Jammer should follow a strategy.

After substituing $J_R = P - J_0$, we obtain an one-variable equation

$$U_{AF} = \frac{P_s g_{s,d}}{h_{j,d} J_0 + \sigma^2} + \frac{P_R P_s g_{R,d} g_{s,R}}{P_r g_{R,d}(J_0 h_{j,R} + \sigma^2) + (P g_{S,R} - J_0 h_{J,d} + \sigma^2)(P_s g_{s,R} + J_0 h_{j,R} + \sigma^2)}$$

We can compute the derivative of the above equation:

$$\frac{dU_{AF}}{dJ_0} =$$

$$-\frac{P_S g_{S,d} h_{J,d}}{(h_{J,d} J_0 + \sigma^2)^2} - \frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} h_{J,R} + P g_{S,R} h_{J,R} - h_{J,d} P_S g_{S,R} - 2J_0 h_{J,d} h_{J,R} - h_{J,d}\sigma^2 + \sigma^2 h_{J,R})}{(P_R g_{R,d}(h_{J,R} J_0 + \sigma^2) + (P g_{S,R} - J_0 h_{J,d} + \sigma^2)(P_S g_{S,R} + J_0 h_{J,R}\sigma^2))^2} \tag{3.1}$$

For $J_0 = 0$

$$\left.\frac{dU_{AF}}{dJ_0}\right|_{J_0=0} = -\frac{P_S g_{S,d} h_{J,d}}{\sigma^4} - \frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} h_{J,R} + P g_{S,R} h_{J,R} - h_{J,d} P_S g_{S,R} - h_{J,d}\sigma^2 + \sigma^2 h_{J,R})}{((P_R g_{R,d}\sigma^2) + (P g_{S,R} + \sigma^2)(P_S g_{S,R} + \sigma^2))^2}$$

For $J_0 = P$

$$\left.\frac{dU_{AF}}{dJ_0}\right|_{J_0=P} =$$

$$-\frac{P_S g_{S,d} h_{J,d}}{(h_{J,d} P + \sigma^2)^2} - \frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} h_{J,R} + P g_{S,R} h_{J,R} - h_{J,d} P_S g_{S,R} - 2P h_{J,d} h_{J,R} - h_{J,d}\sigma^2 + \sigma^2 h_{J,R})}{(P_R g_{R,d}(h_{J,R} P + \sigma^2) + (P g_{S,R} - P h_{J,d} + \sigma^2)(P_S g_{S,R} + P h_{J,R}\sigma^2))^2}$$

So we can study the following cases,

(I) , $J_0 = 0$

$$\left.\frac{dU_{AF}}{dJ_0}\right|_{J_0=0} \leq 0 \Rightarrow$$

$$-\frac{P_S g_{S,d} h_{J,d}}{\sigma^4}$$

$$-\frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} h_{J,R} + P g_{S,R} h_{J,R} - h_{J,d} P_S g_{S,R} - h_{J,d}\sigma^2 + \sigma^2 h_{J,R})}{((P_R g_{R,d}\sigma^2) + (P h_{J,d} + \sigma^2)(P_S g_{S,R} + \sigma^2))^2} \leq 0$$

When this inequality is satisfied, the optinal strategy for the Jammer is to invest zero power at the first time-slot, which means that the Jammer does not attack the broadcast transmission of the source.

(II) , $J_0 = P$

$$\left.\frac{dU_{AF}}{dJ_0}\right|_{J_0=P} \geq 0 \Rightarrow$$

$$-\frac{P_S g_{S,d} h_{J,d}}{(h_{J,d}P+\sigma^2)^2}$$

$$-\frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} h_{J,R} + P g_{S,R} h_{J,R} - h_{J,d} P_S g_{S,R} - 2P h_{J,d} h_{J,R} - h_{J,d}\sigma^2 + \sigma^2 h_{J,R})}{(P_R g_{R,d}(h_{J,R}P+\sigma^2) + (P g_{S,R} - P h_{J,d} + \sigma^2)(P_S g_{S,R} + P h_{J,R} + \sigma^2))^2} \geq 0$$

When this inequality is satisfied, the Jammer should invest all his power on the first time-slot. In other words, he should not attack the channel between the relay and the destination.

(III)

$$-\frac{P_S g_{S,d} h_{J,d}}{\sigma^4} - \frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} h_{J,R} + P g_{S,R} h_{J,R} - h_{J,d} P_S g_{S,R} - h_{J,d}\sigma^2 + \sigma^2 h_{J,R})}{((P_R g_{R,d}\sigma^2) + (P h_{J,d} + \sigma^2)(P_S g_{S,R} + \sigma^2))^2} > 0$$

and

$$-\frac{P_S g_{S,d} h_{J,d}}{(h_{J,d}P+\sigma^2)^2} - \frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} h_{J,R} + P g_{S,R} h_{J,R} - h_{J,d} P_S g_{S,R} - 2P h_{J,d} h_{J,R} - h_{J,d}\sigma^2 + \sigma^2 h_{J,R})}{(P_R g_{R,d}(h_{J,R}P+\sigma^2) + (P g_{S,R} - P h_{J,d} + \sigma^2)(P_S g_{S,R} + P h_{J,R} + \sigma^2))^2} < 0$$

These inequalities cover the cases where none of the extreme scenarios of (I) and (II) occurs. The value of J_0 (and $J_R = P - J_0$) can be determined numerically by the equation $\frac{dU}{dJ_0} = 0$.

In order to investigate the behavior of the Jammer, we can impose some additional constraints on the environment of communication, so as to see how the inequalities of the constraints will change.

Firstly, we can compute the inequalities when $h_{J,R} = h_{J,d}$. This constraint is related to the topology of the Jammer and refers to the cases when the gain of the channel between the Jammer and the relay node and the Jammer and the destination are the same.

For the first case. where $J_0 = 0$

$$-\frac{g_{S,d}}{\sigma^4} - \frac{P_R g_{R,d} g_{S,R}(P_R g_{R,d} + P g_{S,R} - P_S g_{S,R})}{(P_R g_{R,d}\sigma^2 + (P g_{S,R} + \sigma^2)(P_S g_{S,R} + \sigma^2))^2} \leq 0$$

The above inequality always holds when $P_R g_{R,d} + P g_{S,R} - P_S g_{S,R} > 0$. The last inequality holds when $P > P_S$ and $g_{S,R}$ is sufficiently big.

For the second case, where $J_0 = P$

$$-\frac{g_{S,d}}{(h_{J,d}P+\sigma^2)^2} - \frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} + P g_{S,R} - P_S g_{S,R} - 2P h_{J,R})}{(P_R g_{R,d}(h_{J,R}P+\sigma^2) + (P g_{S,R} - P h_{J,R} + \sigma^2)(P_S g_{S,R} + P h_{J,R} + \sigma^2))^2} \geq 0$$

This inequality holds when $P_R g_{R,d} + P g_{S,R} - P_S g_{S,R} - 2P h_{J,R} < 0$

Furthermore, if we consider that σ^2 has a very low value -i.e. the noise is very low, then
For the first case

$$-\frac{g_{S,d}}{\sigma^4} - \frac{P_R g_{R,d} g_{S,R}(P_R g_{R,d} + P g_{S,R} - P_S g_{S,R})}{(P_R g_{R,d}\sigma^2 + P g_{S,R} P_S g_{S,R})^2} \leq 0$$

For the second case

$$-\frac{g_{S,d}}{(h_{J,d}P)^2} - \frac{P_R P_S g_{R,d} g_{S,R}(P_R g_{R,d} + P g_{S,R} - P_S g_{S,R} - 2P h_{J,R})}{(P_R g_{R,d} h_{J,R} P + (P g_{S,R} - P h_{J,R})(P_S g_{S,R} + P h_{J,R}))^2} \geq 0$$

3.4.4 Study of the strategy of the source

As we mentioned, we assume a pool of relay nodes. In the case of one-relay that we are examining the source has to choose optimally the node that will be used for the relay communication. The utility function that characterizes the cooperative communication over each relay node, has the following form:

$$U_{AF-Si} = \frac{1}{2} log(1 + \frac{P_s g_{s,d}}{h_{j,d} J_0 + \sigma^2} + \frac{P_{Ri} P_s g_{Ri,d} g_{s,Ri}}{P_r g_{Ri,d}(J_0 h_{j,Ri} + \sigma^2) + (J_{Ri} h_{J,d} + \sigma^2)(P_s g_{s,Ri} + J_0 h_{j,Ri} + \sigma^2)})$$

Where P_{Ri} is the power with which relay i will amplify the received signal, $g_{Ri,d}$ is the gain between relay node i and destination, $g_{s,Ri}$ is the gain between source and relay i, $h_{j,Ri}$ is the gain between relay i and the jammer and J_{Ri} is the power that the Jammer invests for this relay.
So, the problem that the source tries to solve can be formulated as follows:

$$argmax_{i \in R} U_{AF-Si}(i),$$

where R is the set of the relay nodes.

An algorithm that solves the above problem is as follows:

Algorithm 2 Solving the source's problem at the AF case

1: **Initialization**: set $i^* = 0$
2: Calculate $U_{AF-Si}(i).\forall i \in \mathcal{R}$
3: Choose $i^* = \text{argmax}_{i \in \mathcal{R} \bigcup \{0\}} U_{AF-Si}$

3.5 Decode and Forward case

In this section we will study the Jammer and the Source in the case of the Decode-and-Forward case technique. According to that, the relay node decodes and re-encodes the information that he receives from the broadcast of the source and then transmits it. In the sections that follow, we will present the formulations of the SNR for the multiple relays and the one relay case and we will propose an algorithm that solves the Jammer and the Source problem in this context.

3.5.1 Singal-to-Noise ratio achieved at the Destination without the presnce of a Jammer

In the Decode-and-Forward technique the role of the relay node is more active and determining. If the relay-node does not succed to decode the broadcast message of the source then he will not be able to relay any information to the destination.

For the one relay scenario, the achieved SNR is [13]:

$$I2 = \frac{1}{2}\, min\ \{log_2(1 + SNR2_{S-R}), log_2(1 + SNR2_{S-D} + SNR2_{R-D})\}$$

where $SNR2_{S-R}$, $SNR2_{S-D}$, $SNR2_{R-D}$ are the SNR at the source-relay, source-destination, relay-destination channels respectively.

The first term represents the maximum rate at which the relay can reliably decode the source message, while the second term represents the maximum rate at which the destination can reliably decode the source message given repeated transmissions from the source and destination. Requiring both relay and destination to decode the entire codeword without error results in the minimum of the two. [15]

Like in the Amplify-and-Forward technique, the Jammer has to allocate his power on the two time-slots of the communication. In the first time-slot he affects the direct transmission between the source and the destination and the channel between the source and the relay. If his activity in the first time-slot is effective enough, the relay may not be able to decode so the cooperative communication is EKFULIZETAI into a typical communication between a source and a destination.

For the multiple relays scenario, the SNR at the destination is affected from the signal that it receives from all the relay nodes that cooperate.

$$I_{DF} = \begin{cases} log(1 + SNR_{S-D}), SNR_{S-Ri} < \gamma_{Rth} \forall i \in R \\ \frac{1}{k+1} log(i + SNR_{S-D} + \sum SNR_{Ri-D}), SNR_{S-Ri} \geq \gamma_{Rth} \end{cases}$$

Where k is the number of the relays that succesfully decoded the source signal and γ_{Rth} is a threshold for succesful decoding at the relay node . The first case corresponds to the scenario at which no relay can decode the signal of the source. At the second case both the source and the k relays contribute at the achieved SNR at the destination.

The jammer can attack the broadcast of the first time-slot in order to minimize the number of relay nodes that will succesfully decode the signal transmitted from the source as well the direct transmission between the source and the destination. Also it can attack the second time slot (relay-destination channel) in order to affect the SNR at the destination.

3.5.2 Jammer's behavior in the one relay case

After presenting the formulas for the SNR in cooperative communication with decode-and-forward, we introduce the Jammer in this study.

The source broadcasts with P_S. The jammer will attack the broadcast transmission (first time-slot)with power J_0, affecting both the source-destination and the source-relay communication.

So, for the source-destination channel we can write:

$$Y2_{S,d} = \sqrt{P_S g_{s,d}} X_S + n + \sqrt{J_0 h_{j,d}} X_0$$

And the SNR will be:

$$SNR2_{S-D} = \frac{P_S g_{s,d}}{h_{j,d} J_0 + \sigma^2}$$

For the source-relay channel:

$$Y2_{S,R} = \sqrt{P_S g_{s,R}} X_S + n + \sqrt{J_0 h_{j,R}} X_0$$

and the SNR will be

$$SNR2_{S-R} = \frac{P_S g_{s,R}}{\sigma^2 + J_R h_{j,R}}$$

The relay will decode and forward. At the destination:

$$Y2_{R,D} = \sqrt{P_R g_{R,d}} X_R + n + \sqrt{J_R h_{j,d}} X_R$$

where X_R is the signal that the relay node has obtained after the decoding procedure.

If the relay node has succesfully decoded the signal, its contribution to SNR at the destiantion will be:

$$SNR2_{R-D} = \frac{P_R g_{R,d}}{\sigma^2 + J_R h_{j,d}}$$

The Jammer faces the following problem

$$\text{Minimize } U2$$
$$\text{Subject to: } \; J_0 + J_R \leq P$$
$$\text{and } J_0, J_R \geq 0$$

Where,

$$U2 = \frac{1}{2} log(1 + I2)$$

and

$I2 = \frac{1}{2}\ min\ \{log_2(1 + SNR2_{S-R}), log_2(1 + SNR2_{S-D} + SNR2_{R-D})\}$ as mentioned before.

So finally we have the two following optimization problems for the Jammer with the corresponding constraints that describe whether the relay was able to decode the message from the source:

$$\text{Minimize } U_{DF1} = \tfrac{1}{2} log_2(1 + SNR2_{S-R})$$
$$\text{Subject to: } \; J_0 + J_R \leq P$$
$$\text{and } J_0, J_R \geq 0$$
$$\text{and } log_2(1 + SNR2_{S-R}) < log_2(1 + SNR2_{S-D} + SNR2_{R-D})$$
$$\text{or } SNR2_{S-R} < SNR2_{S-D} + SNR2_{R-D}$$

or

$$\text{Minimize } U_{DF2} = \tfrac{1}{2} log_2(1 + SNR2_{S-D} + SNR2_{R-D})$$
$$\text{Subject to: } \; J_0 + J_R \leq P$$
$$\text{and } J_0, J_R \geq 0$$
$$\text{and } log_2(1 + SNR2_{S-D} + SNR2_{R-D}) < log_2(1 + SNR2_{S-R})$$
$$\text{or } SNR2_{S-D} + SNR2_{R-D} < SNR2_{S-R}$$

The final minimum will be $min\,\{minU_{DF1}, minU_{DF2}\}$, with the corresponding strategy. Where,

$$U_{DF1} = \frac{P_S g_{s,R}}{\sigma^2 + J_R h_{j,R}}$$

and

$$U_{DF2} = \frac{P_S g_{S,d}}{h_{j,d} J_0 + \sigma^2} + \frac{P_R g_{R,d}}{\sigma^2 + J_R h_{j,d}}$$

We compute the derivative for the U_{DF1} :

$$\frac{dU_{DF1}}{dJ_R} = -\frac{P_S g_{s,R} h_{j,R}}{(\sigma^2 + J_R h_{j,R})^2}$$

and the partial derivatives for the U_{DF2}:

$$\frac{\partial U_{DF2}}{\partial J_0} = -\frac{P_S g_{S,d} h_{j,d}}{(h_{j,d} J_0 + \sigma^2)^2}$$
$$\frac{\partial U_{DF2}}{\partial J_R} = -\frac{P_R g_{R,d} h_{j,d}}{(h_{j,d} J_R + \sigma^2)^2}$$

In order to find the optimal solution for the U_{DF1},

$$\frac{dU_{DF1}}{dJ_R} = 0 \Rightarrow$$
$$\frac{P_S g_{s,R} h_{j,R}}{(\sigma^2 + J_R h_{j,R})^2} = 0 \Rightarrow$$
$$P_S g_{s,R} h_{j,R} = 0$$

We notice that:

$\frac{dU_{DF1}}{dJ_R} > 0 \Rightarrow P_S g_{s,R} h_{j,R} < 0$ is impossible, while

$\frac{dU_{DF1}}{dJ_R} < 0 \Rightarrow P_S g_{s,R} h_{j,R} > 0$ always stands for $P_S \neq 0$.

As far as the U_{DF2} is concerned, following the same method that was used at the A&F case, we substitue $J_R = P - J_0$,

$$U_{DF2} = \frac{P_S g_{S,d}}{h_{j,d} J_0 + \sigma^2} + \frac{P_R g_{R,d}}{\sigma^2 + P h_{j,d} - J_0 h_{j,d}}$$

and we can obtain the derivative:

$$\frac{dU_{DF2}}{dJ_0} = -\frac{P_S g_{S,d} h_{j,d}}{(h_{j,d} J_0 + \sigma^2)^2} + \frac{P_R g_{R,d} h_{j,d}}{(\sigma^2 + P h_{j,d} - J_0 h_{j,d})^2}$$

For $J_0 = 0$

$$\left.\frac{dU_{DF2}}{dJ_0}\right|_{J_0=0} = -\frac{P_S g_{S,d} h_{j,d}}{\sigma^4} + \frac{P_R g_{R,d} h_{j,d}}{(\sigma^2 + P h_{j,d})^2}$$

For $J_0 = P$

$$\left.\frac{dU_{DF2}}{dJ_0}\right|_{J_0=P} = -\frac{P_S g_{S,d} h_{j,d}}{(h_{j,d} P + \sigma^2)^2} + \frac{P_R g_{R,d} h_{j,d}}{\sigma^4}$$

As expected, when the Jammer invests no power at the direct transmission, only the ratio that describes the relay-destination communication is affected. Similarly when all the power budget of the Jammer is invested at the first time-slot, the relay-destination communication is not affected and only the source-destination communication is deteriorated. The effect at the source-relay channel does not appear at these formulas as they refer to the case of succesful decoding by the relay.

So, we can study the following cases:

(I) , $J_0 = 0$

$$\left.\frac{dU_{DF2}}{dJ_0}\right|_{J_0=0} \leq 0 \Rightarrow -\frac{P_S g_{S,d} h_{j,d}}{\sigma^4} + \frac{P_R g_{R,d} h_{j,d}}{(\sigma^2 + P h_{j,d})^2} \leq 0$$

$$\frac{P_R g_{R,d}}{(\sigma^2 + P h_{j,d})^2} \leq \frac{P_S g_{S,d}}{\sigma^4}$$

(II) , $J_0 = P$

$$\left.\frac{dU_{DF2}}{dJ_0}\right|_{J_0=P} \geq 0 \Rightarrow -\frac{P_S g_{S,d} h_{j,d}}{(h_{j,d} P + \sigma^2)^2} + \frac{P_R g_{R,d} h_{j,d}}{\sigma^4} \geq 0$$

$$\frac{P_S g_{S,d}}{\sigma^4} \geq \frac{P_R g_{R,d}}{(h_{j,d} P + \sigma^2)^2}$$

(III)

$$-\frac{P_S g_{S,d}}{\sigma^4} + \frac{P_R g_{R,d}}{(\sigma^2 + P h_{j,d})^2} > 0$$

$$\text{and} -\frac{P_S g_{S,d}}{(h_{j,d} P + \sigma^2)^2} + \frac{P_R g_{R,d}}{\sigma^4} < 0$$

where J_0 can be defined numerically.

In order to investigate further the cases (I) and (II), we will introduce some extra constraints.

For (I), $J_0 = 0$, we assume $g_{S,d} < g_{R,d}$. Then

$$g_{S,d} < g_{R,d} \Rightarrow$$

$$\frac{P_S g_{S,d}}{\sigma^4} < \frac{P_S g_{R,d}}{\sigma^4} \Rightarrow$$

$$\frac{P_R g_{R,d}}{(h_{j,d} P + \sigma^2)^2} \leq \frac{P_S g_{S,d}}{\sigma^4} < \frac{P_S g_{R,d}}{\sigma^4} \Rightarrow$$

$$\frac{P_R}{(h_{j,d} P + \sigma^2)^2} < \frac{P_S}{\sigma^4} \Rightarrow$$

$$\frac{P_R}{P_S} < \frac{(h_{j,d} P + \sigma^2)^2}{\sigma^4}$$

$$\text{and since} (h_{j,d} P + \sigma^2)^2 > \sigma^4$$

$$P_R > P_S$$

So after the assumption $g_{S,d} < g_{R,d}$ we can see that the inequality $\left.\frac{dU_{DF2}}{dJ_0}\right|_{J_0=0} \leq 0$ holds when

$P_R > P_S$.

For (II), $J_0 = P$, we assume $g_{S,d} > g_{R,d}$. Then

$$\begin{aligned} g_{S,d} > g_{R,d} \Rightarrow \\ \frac{P_R, g_{S,d}}{\sigma^4} > \frac{P_R g_{R,d}}{\sigma^4} \geq \frac{P_S g_{S,d}}{(h_{j,d}P + \sigma^2)^2} \Rightarrow \\ \frac{P_R}{P_S} > \frac{\sigma^4}{(h_{j,d}P + \sigma^2)^2} \Rightarrow P_R < P_S \end{aligned}$$

So after the assumption $g_{S,d} > g_{R,d}$ we can assume that the inequality $\frac{dU_{DF2}}{dJ_0}\Big|_{J_0=P} \geq 0$ holds when $P_R < P_S$.

3.5.3 The strategy of the source

As in the Amplify and Forward case, we assume a pool of relay nodes. The source has to choose optimally the node that will be used for the relayed communication. We consider that the source has full knowledge of the characteristics of the channels and of the jammer's strategy. The utility function that characterizes the cooperative communication for every different available relay node has the following form:

$$U2i = I2i$$

where,

$$I2i = \frac{1}{2}\, min\ \{log_2(1 + SNR2_{S-Ri}), log_2(1 + SNR2_{S-D} + SNR2_{Ri-D})\}$$

The objective of the source is to maximize the utility as follows:

$$argmax_{i \in R} U2i(i)$$

So,

$$\begin{aligned} argmax_{i \in R} Ui_{DF1} = \frac{1}{2} log_2(1 + SNR2_{S-Ri}) \\ argmax_{i \in R} Ui_{DF1} = \frac{1}{2} log_2(1 + \frac{P_S g_{S-Ri}}{\sigma^2 + J_{Ri} h_{j,Ri}}) \end{aligned}$$

Subject to:

$$log_2(1 + SNR2_{S-Ri}) < log_2(1 + SNR2_{S-D} + SNR2_{Ri-D})$$

$$\begin{aligned} SNR2_{S-Ri} < SNR2_{S-D} + SNR2_{Ri-D} \\ \frac{P_S g_{s,Ri}}{\sigma^2 + J_{Ri} h_{j,Ri}} < \frac{P_S g_{S,d}}{\sigma^2 + h_{j,d} J_0} + \frac{P_{Ri} g_{Ri,D}}{\sigma^2 + J_{Ri} h_{j,d}} \end{aligned}$$

or

$$argmax_{i\in R}Ui_{DF2} = \frac{1}{2}log_2(1 + SNR2_{S-D} + SNR2_{Ri-D})$$
$$argmax_{i\in R}Ui_{DF2} = \frac{1}{2}log_2(1 + \frac{P_S g_{s,d}}{\sigma^2 + h_{j,d}J_0} + \frac{P_{Ri} g_{Ri,D}}{\sigma^2 + J_{Ri}h_{j,d}})$$

Subject to:

$$log_2(1 + SNR2_{S-D} + SNR2_{Ri-D}) < log_2(1 + SNR2_{S-Ri})$$
$$SNR2_{S-D} + SNR2_{Ri-D} < SNR2_{S-Ri}$$
$$\frac{P_S g_{S,d}}{\sigma^2 + h_{j,d}J_0} + \frac{P_R g_{Ri,d}}{\sigma^2 + J_{Ri}h_{j,d}} < \frac{P_S g_{S-Ri}}{\sigma^2 + J_{Ri}h_{j,Ri}}$$

So finally, the source's strategy will be
$argmax_{i\in R}min\ \{log_2(1 + SNR2_{S-Ri}), log_2(1 + SNR2_{S-D} + SNR2_{Ri-D})\}$

If we assume that the noise is low enough in order to eliminate σ^2 from our formulas, we can formulate differently the SNR criterion about the succesful decoding:

$$SNR2_{S-Ri} < SNR2_{S-D} + SNR2_{Ri-D}$$
$$\frac{P_S g_{s,Ri}}{J_{Ri}h_{j,Ri}} < \frac{P_S g_{S,d}}{h_{j,d}J_0} + \frac{P_{Ri} g_{Ri,D}}{J_{Ri}h_{j,d}}$$
$$\frac{P_S g_{s,Ri}h_{j,d} - P_{Ri}g_{Ri,D}h_{j,Ri}}{J_{Ri}h_{j,Ri}h_{j,d}} < \frac{P_S g_{S,d}}{h_{j,d}J_0}$$
$$\frac{P_S g_{s,Ri}h_{j,d} - P_{Ri}g_{Ri,D}h_{j,Ri}}{J_{Ri}h_{j,Ri}} < \frac{P_S g_{S,d}}{J_0}$$
$$\frac{J_0}{J_{Ri}} < \frac{P_S g_{S,d}h_{j,Ri}}{P_S g_{S-Ri}h_{j,d} - P_{Ri}g_{Ri,D}h_{j,Ri}}$$

Similarly:

$$SNR2_{S-Ri} > SNR2_{S-D} + SNR2_{Ri-D}$$
$$\frac{J_0}{J_{Ri}} > fracP_S g_{S,d}h_{j,Ri}P_S g_{S-Ri}h_{j,d} - P_{Ri}g_{Ri,D}h_{j,Ri}$$

It holds that $0 \leq \frac{J_0}{J_{Ri}} \leq P - 1$

3.6 Comparing the Strategy of the Source at the DF and AF scenarios

We will proceed to a comparison through simulation for the two techniques that are used in Cooperative Communication, in the case of one relay node.We assuming that the two techniques are applied at exactly the same environment. Meaning, the Jammer's strategy is the same, the source's strategy (same relay node) is the same and the channles have the same characteristics.

With the Amplify and Forward technique the achieved SNR for the source will be:

$$U_{AF} = \frac{1}{2}log(1 + \frac{P_s g_{s,d}}{h_{j,d}J_0 + \sigma^2} + \frac{P_R P_s g_{R,d} g_{s,R}}{P_r g_{R,d}(J_0 h_{j,R} + \sigma^2) + (J_R h_{J,d} + \sigma^2)(P_s g_{s,R} + J_0 h_{j,R} + \sigma^2)})$$

With the Decode and Forward technique:

$$U_{DF1} = \frac{1}{2}log_2(1 + \frac{P_s g_{s,R}}{\sigma^2 + J_R h_{j,R}})$$

Subject to:

$$\frac{P_S g_{s,R}}{\sigma^2 + J_R h_{j,Ri}} < \frac{P_S g_{S,d}}{\sigma^2 + h_{J,d}J_0} + \frac{P_R g_{R,d}}{\sigma^2 + J_R h_{J,d}}$$

or

$$U_{DF2} = \frac{1}{2}log_2(1 + \frac{P_S g_{s,d}}{\sigma^2 + h_{j,d}J_0} + \frac{P_R g_{R,d}}{\sigma^2 + J_R h_{J,d}})$$

Subject to:

$$\frac{P_S g_{S,d}}{\sigma^2 + h_{J,d}J_0} + \frac{P_R g_{R,d}}{\sigma^2 + J_R h_{j,d}} < \frac{P_S g_{S-R}}{\sigma^2 + J_R h_{j,R}}$$

If the criterion $\frac{P_S g_{s,R}}{\sigma^2+J_R h_{j,Ri}} > \frac{P_S g_{S,d}}{\sigma^2+h_{J,d}J_0} + \frac{P_R g_{R,d}}{\sigma^2+J_R h_{J,d}}$ holds, we can see from the simulations that the DF technique achives a better utility for the source than the AF technique.

In our simulation, we assume a pole of 1000 nodes that could act as a relay. The source chooses the one that will offer her the biggest utility, according to the algorithms that were proposed earlier in this chapter.

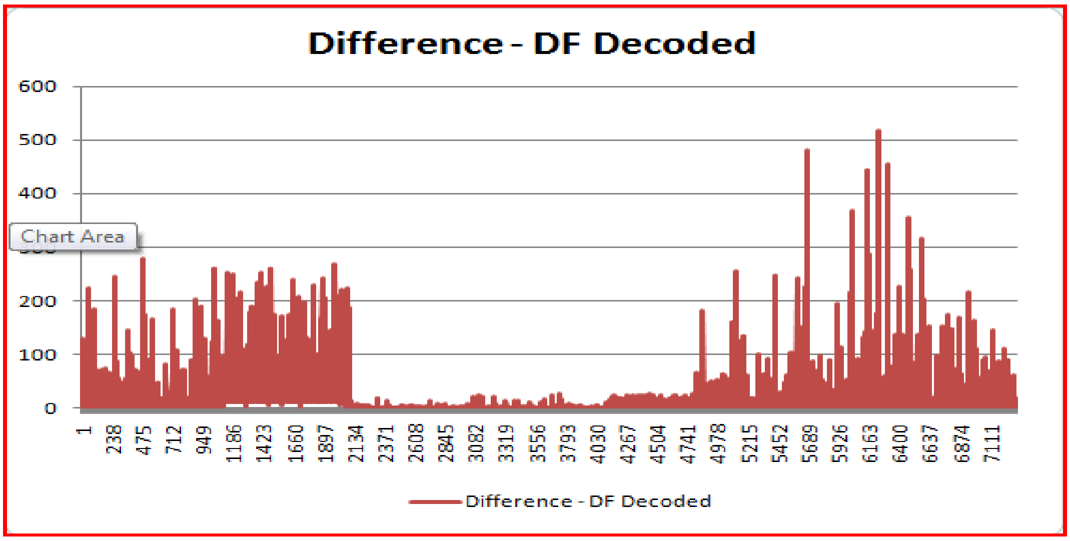

Figure 3.1: The Difference between the DF and the AF technique when the relay decodes succesfully in the first one

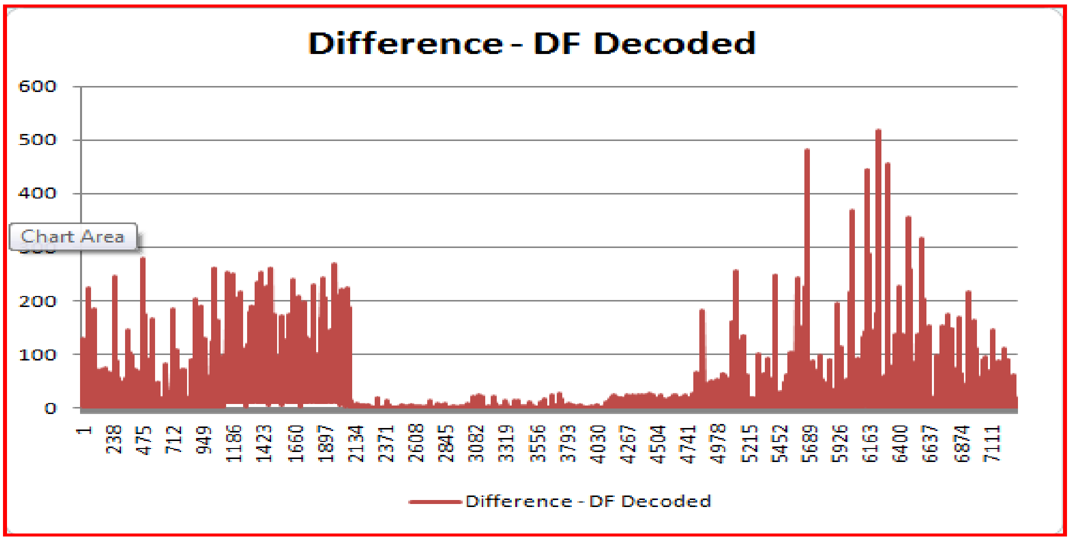

Figure 3.2: The Difference between the DF and the AF technique when the relay does not decode succesfully in the first one

In the simulation we have included various topologies, where the Jammer can be closer to the source or to the destination, or in a random point between them. Furthermore, different values for the power budget of the source and the jammer were tested, as well as for the noise level.

Our goal is to compare the utility for the two techniques. From the results we can say that if -in the Decode-and-Forward technique- the relay decodes succesfully the broadcast message of the source, the utility is bigger from that with the Amplify-and-Forward.

In Figures (3.1) and (3.2) we can see the difference between the utility at the Decode-and-Forward technique minus the utility in the Amplify-and-Forward one. In the case that the decoding was succesful, we can see that the difference was always positive. On the other hand, the difference takes positive and negative values in the case that the decoding was not succesful.

3.6.1 Conclusion

In this chapter we formulated the problem of a Jammer in a Cooperative Communication environment with a source, a relay and a destination. Algorithms that solve the power allocation problem of the Jammer and the relay selection problem of the source were proposed. A simulation for the utility obtained from the source in the D-F and A-F technique, demonstrates that when the relay decodes succesfully the message of the source, the utility achived is always bigger from the one that the source would achieve with the A-F technique.

The case of the use of multiple relays should be investigated in the furure in order to study if the contribution of more nodes has an important affect for the destination and how the performance of the system is deteriorated by the presence of the Jammer.

Chapter 4

Conclusion

We have studied in this work the Jamming attack phenomenon under a game theoretical analysis, in two different situations: Firstly under thermal energy constraints that affect its choices -as well as the choices of the source- for power allocation. Secondly, in a cooperative communication environment with the constraint of a power budget for each player.

In the case of a dynamic game under energy constraints, after formulating the problem and defining the payoff for the players at the end of the game with the backward induction technique, we investigate the existence of nash equilibria. In order to solve the 3x3 matrix of the game, we propose a method in which the game is seperated in 4 subgames with equivalent constraints that justify the choice of the source to mix between two and not three strategies. For the global solution of the initial game we combine all the constraints and we derive the set of inequalities for the payoff parameters a_h and a_l. These inequalities define the conditions under which a mixed Nash equilibria is achieved for our game.

In a future work the assumption of full knowledge of the game parameters by the palyers could be removed. Furthermore, the assumption of correct feedback cannot be always satisfied.

In the second part of our study, we focus on a more specific case of jamming. We introduce the Jammer in the cooperative communication. By using the classic model of a source, a destination and a relay node in cooperative communication, we obtain the analytical formulas that describe the SNR at the destination under the presence of a Jammer. The communication takes place in two time-slots. The source broadcasts in the first one and the relay and the destination receive the message. In the second time-slot the relay wil transmit the message of the source either by just amplifying it (Amplify-and-Forward) or by decoding and re-encoding it (Decode-and-Forward). The Jammer has to decide for his power-allocation on these two time-slots and the source has to deicde for the node that it will use as a relay. We have proposed an algorithm for these problems for both A-F and D-F techniques. Finally we compare the two techniques through a simulation that shows that if the relay node decodes succesfully the message of the source, the utility of the D-F technique is higher than this of the A-F.

The case of cooperative communication in multiple time-slots with the use of multiple relays remains an open issue for mathematical analysis and simulation. Furthermore, the analytical proof for the dominance of D-F technique when the relay decodes succesfully is planed for future work. Games where the players do not have knowledge of all the parameters should also be investigated.

Bibliography

[1] W. Xu , W. Trappe , Y. Zhang , T. Wood, "The feasibility of launching and detecting jamming attacks in wireless networks," In Proc. of the 6th ACM international symposium on Mobile ad hoc networking and computing, May 2005.

[2] K. Leyton-Brown, Y. Shoham, "Essentials of Game Theory. A Concise, Multidisciplinary Intorduction," Morgan and Claypool Publishers, 2008.

[3] J. Von Neuman, O. Morgenstern, "Theory of games and economic behavior," Princeton University Press, Princeton 1953.

[4] M. Felegyhazi, J.P. Hubaux, "Game Theory in wireless networks: A tutorial," EPFL Technical report: LCA-REPORT-2006-002.

[5] E. Altman, K. Avrachenkov, R. Marquez, G. Miller, "Zero-sum constrained stochastic games with independent state processes," Math. Meth. Oper. Res., v.62, 2005, pp.375-386.

[6] E. Altman, K. Avrachenkov, A. Garnaev, "Jamming in wireless networks under uncertainty", In Proc. of GameNets '09, 2009, pp. 585-592.

[7] E. Altman, K. Avrachenkov, A. Garnaev, "A jamming game in wireless networks with transmission cost," In Proc. of the 1st EuroFGI international conference on Network control and optimization, p.1-12, June 05-07, 2007, Avignon, France

[8] R. Mallik, R. Scholtz, G. Papavassilopoulos, "Analysis of an On-Off Jamming Situation as a Dynamic Game," IEEE Transactions on Communications, vol. 48, no. 8, August 2000.

[9] J.N. Laneman, G. Wornell, "Exploiting Distributed Spatial Diversity in Wireless Networks," In Proc. Allerton Conf. Commun., Contr., Computing, Oct. 2000.

[10] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity Part I and Part II,"' IEEE Trans. Commun., vol.51, no. 11, Nov. 2003, pp. 1927-48.

[11] T. Cover, A. El Gamal, "Capacity thoerems for the relay channel," IEEE Transactions Information Theory, vol. IT-25, Sept. 1979, 99. 572-584.

[12] A. Nosratinia, T.E. Hunter, A. Hedayat, "Cooperative communication in wireless networks," IEEE Communications Magazine, , vol. 42, no. 10, Oct. 2004, pp.74-80.

[13] Y. Shi, S. Sharma, Y. Thomas Hou, S. Kompella, "Optimal Relay Assignmant for Cooperative Communications," Proceedings of the 9th ACM international symposium on Mobile ad hoc networking and computing, 2008.

[14] Y. Zhao, R. Adve, T.J. Lim, "Improving Amplify-and-Forward Relay Networks: Optimal Power Allocation versus Selection," IEEE Transactions on Wireless Communications, vol. 6, no. 8, Aug 2007, pp. 3114-3123.

[15] J.N. Laneman, D. Tse, G. Wornell, "Cooperative Diversity in Wireless Networks Efficient Protocols and Outage Behavior," IEEE Transactions in Information Theory, vol. 50, no.12, Dec 2004, pp. 3062-3080.

[16] A. Bletsas, A. Khisti, D. Reed, A. Lippman, "A simple cooperative diversity method based on network path selection," IEEE Journal on Selected Areas in Communications, vol. 24, no. 3, March 2006, pp. 659-672.

[17] S. Dehnie, N. Memon, "Modeling Misbehavior in Cooperative Diversity: A Dynamic Game Approach," EURASIP Journal on Advances in Signal Processing, vol. 2009, Jan 2009.

[18] S. Dehnie, N. Memon, "Detection of misbehavior in cooperative diversity," IEEE Military Communications Conference (MILCOM '08), Nov. 2008, pp. 1-5.

[19] S. Dehnie, N. Memon, "A Stochastic Model for Misbehaving Relays in Cooperative Diversity," IEEE Wireless Communications and Networking Conference (WCNC '08), March 2008, pp.482 - 487.

[20] J. Huang, Z. Han, M. Chiang, H. V. Poor, "Auction-Based Resource allocation for Cooperative Communications", IEEE Journal on Selected Areas in Communications, vol. 26, no. 7, Sept. 2008, pp. 1226-1237.