University of Crete

Department of Economics

M.Sc. in Economic Theory and Policy

[Value at Risk and combined forecasts: a forecasting evaluation performance]

Katerina Kastrinaki, RN: 231

Supervisor: Tsiotas G.

Member of the board: Koukouritakis M.
Member of the board: Tzinious M.

Abstract

Given the growing need for managing market risk, risk prediction plays an increasing role in finance. Value at risk (VaR), is a measure of market risk used by financial institutions. Interpreting the VaR as the quantile of future portfolio values conditional on current information, the conditional autoregressive value at risk (CAViaR) model specifies the evolution of the quantile using an autoregressive process and estimates the parameters with regression quantiles. However, uncertainty with regard to model selection in CAViaR model estimators, raises the issue of indentifying the best quantile predictor. In this study, we propose an AIC and an Equal Weighted method that generates combinations of conditional VaR estimators based on single CAViaR models. The aim of the research, is to compare single CAViaR models against combined ones for their ability to forecast VaR. We apply this method to real financial data, providing empirical support for our results.

Introduction

Financial crisis has (once again) called into question risk management practices and whether risk measures can be forecast accurately enough for that purpose. This paper adds to this challenge by proposing semi-parametric conditional autoregressive VaR (CAViaR) models and specifically, evaluating them for forecasting tail risk, after-crisis period, for three financial index returns. The motivation is to generate more accurate and efficient forecasts of VaR for index returns, by using single CAViaR models or combined CAViaR models, to help achieve better risk measurement and risk management practice. We attempt this, by examine which model after all, has the better performance and the most desirable results.

Financial markets and products continue to become increasingly complex, and risk management and regulations need to keep pace with this rapid process. The Basel II Accord is designed to monitor and encourage sensible risk taking, using appropriate models to calculate VaR and daily capital charges. VaR is now a standard tool in risk management and became highly important following the 1995 amendment to the Basel Accord, whereby banks and other Authorized Deposit-taking Institutions (ADIs) were permitted to use internal models to forecast daily VaR. VaR was pioneered by J.P. Morgan Corporation, via their RiskMetrics system, in 1993 and is more formally defined by Jorion (1996).

In this work, we define the VaR as a measure of risk and we mention some methods of evaluating VaR. Also, we propose a new semi-parametric family of quantile risk CAViaR models and try to estimate the VaR for these models, in a time horizon for two given confidence level. Finally, we introduce combined CAViaR forecasts based on single CAViaR models and discuss the selection of optimal CAViaR model.

The research is structured as follows. In Chapter 1, some basic informations for VaR are reviewed and CAViaR specifications, are presented. Also, in this Chapter, the criteria for measuring VaR performance are discussed. Chapter 2 discusses the combined CAViaR forecasts based on single CAViaR models and how other researchers analyze their structure. In addition, in this chapter we discuss our perspective of analysis the combined forecasts using the AIC measure. Empirical analysis is conducted in Chapter 3 on European financial indices, including IBEX index, OSEBENCH index and SMI index to forecast VaR. Some concluding remarks are given in Chapter 4. Finally, some technically details are given in the appendix.

Contents

Chapter 1

Value at Risk (VaR)

1.1 Risk Management

Risk management is the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events or to maximize the realization of opportunities. Risk management's objective is to assure uncertainty does not deflect the endeavor from the business goals.

Risks can come from various sources including uncertainty in financial markets, threats from project failures (at any phase in design, development, production, or sustainment life-cycles), legal liabilities, credit risk, accidents, natural causes and disasters, deliberate attack from an adversary, or events of uncertain. There are two types of events i.e. negative events can be classified as risks while positive events are classified as opportunities. Several risk management standards have been developed including the Project Management Institute, the National Institute of Standards and Technology, actuarial societies, and ISO standards. Methods, definitions and goals vary widely according to whether the risk management method is in the context of project management, security, engineering, industrial processes, financial portfolios, actuarial assessments, or public health and safety.

Risk sources are identified and located in human factor variables, mental states and decision making as well as infrastructural or technological assets and tangible variables. Strategies to manage threats (uncertainties with negative consequences) and risk sources typically include avoiding the threat, reducing the negative effect or probability of the threat, transferring all or part of the threat to another party, and even retaining some or all of the potential or actual consequences of a particular threat, and the opposites for opportunities (uncertain future states with benefits).

Management of risk is a central core in the management sector of each company. More specifically, is the way with which companies approach the risks associated with their actions, intending profits. Center of attention in a right risk management is the acknowledgement and operation of these risks. In this way, the access possibility of the company's goals enhances. The management of risk could be applied in all operations, in order to deal with problems in advance. The management of risk is separated in three basic parts: the acknowledgement of risk, the quantitative definition of risk and the control or mitigation of his consequences. By the analysis of the plans for the time, the cost and the quality is possibly arise outcomes far away of basic aims and interests. Also, the company should know how to act if something goes wrong. Specifically, the company should consider how risky an operation is and which technique should be followed in order to define the risk quantitatively.
Some of these techniques are:

- The expected value
- The sensibility analysis
- The Monte Carlo method

- The condition of failure analysis
- The Program evaluation and review technique (PERT)

As long as, the risk, has been defined, a short of procedure are demanded in order to ensure that the possibility of risk should be diminished or the consequences should be mitigated. Management of risk enables a better depiction of reality and an important benefit as far as the improvement of plants is concerned. From the phase of planning, the sectors that need more attention are already indicated. In that way, risks should be managed quickly and presently. In addition, in that way, risk should be valued and risk's course should be followed in the past. This cognition should be deployed in future projects.

1.2 Market Risk

Market risk is the possibility for an investor to experience losses due to factors that affect the overall performance of the financial markets in which he is involved. Market risk, also called, "systematic risk", cannot be eliminated through diversification, though it can be hedged against. Sources of market risk include recessions, political turmoil, changes in interest rates, natural disasters and terrorist attacks. Companies in the United States are required to detail how their productivity and results may be linked to the performance of the financial markets. This is meant to provide a reflection of how a company is exposed to financial risk. For example, a company providing derivative investments or foreign exchange futures may be more exposed to financial risk than companies who do not provide these types of investments. This information helps investors and traders make decisions based on their own risk management rules.

The two major categories of investment risk are market risk and specific risk. Specific risk, also called "unsystematic risk", is tied directly to the performance of a particular security and can be protected against through investment diversification. One example of unsystematic risk is, a company declaring bankruptcy, making its stock worthless to investors. Market risk exists because of price changes. The standard deviation of changes in prices of stocks, currencies or commodities is referred to as price volatility. Volatility is rated in annualized terms. It may be expressed as an absolute number, such as $10, or a percentage of the initial value, such as 10 %. To measure market risk, investors and analysts use the value at risk method (VaR). VaR method is a well known and established risk management method, but it comes with some assumptions that limit its correctness. We should discuss this subject on a great scale below.

1.3 Definition of VAR

1.3.1 The Origin and Development of VaR

In the late 1970s and 1980s, a number of major financial institutions started work on internal models to measure and aggregate risks across the institution as a whole. They started work on these models in the first instance for their own internal risk management purposes as firms became more complex, it was becoming increasingly difficult, but also increasingly important, to be able to aggregate their risks taking account of how they interact with each other, and firms lacked the methodology to do so [1].

Extreme price movements in the financial markets are rare, but important. The stock market crash on Wall Street in October 1987 and other big financial crises such as the Long Term Capital Management have attracted a great deal of attention among practitioners and researchers, and some people even called for government regulations on the derivative markets. In recent years, the seemingly large daily price movements in high-tech stocks have further generated discussions on market risk and margin setting for financial institutions. As a result, VaR has become a widely used measure of market risk in risk management. The new risk system was highlighted in JP Morgans 1993 research conference and aroused a great deal of interest from potential clients who wished to buy or lease it for their own purposes. The subsequent adoption of VaR systems was very rapid, first among securities houses and investment banks, and then among commercial banks, pension funds and financial or non-financial institutions.

Meanwhile, other financial institutions had been working on their own internal models, and VaR software systems were also being developed by specialist companies that concentrated on software but were not in a position to provide data. The resulting systems differed quite considerably from each other. Even where they were based on broadly similar theoretical ideas, there were still considerable differences in terms of subsidiary assumptions, use of data, procedures to estimate volatility and correlation. Besides, not all VaR systems were based on portfolio theory, some systems were built using historical simulation approaches that estimate VaR from histograms of past profit and loss data, and other systems were developed using Monte Carlo simulation techniques.

Risk management has experienced a revolution in recent years, started by VaR, which was developed in response to the financial / derivative disasters of the late 1980s and early 1990s. VaR was pioneered in 1993, part of the "Weatherstone 4:15 pm" daily risk assessment report, in the RiskMetrics model at JP Morgan (1996). Subsequently, the Group of Thirty (G-30) advised financial institutions to value positions using market prices and to assess financial risks via VaR [2].

Concluding, the need to improve control of financial risks has led to a uniform measure of risk, the VaR measure, which the private sector is increasingly adopting as a first line of defense against financial risks. Regulations and central banks, also, provided the impetus behind VaR. The Basel Committee on Banking Supervision announced in April 1995 that capital adequacy requirements for commercial banks are to be based on VaR. In December 1995, the Securities and Exchange Commission issued a proposal that requires publicly traded U.S. corporations to disclose information about derivatives activity, with a VaR measure as one of three possible methods for making such disclosures. Thus, the unmistakable trend is toward more-transparent financial risk reporting based on VaR measures [3]. Despite some criticism of VaR, for example, it does not measure the magnitude of the loss for violations and it is not “coherent” , it is recommended in Basel II and is widely used in industry. For details see Jorion (2001).

1.3.2 Measuring VaR

To formally define a portfolio's VaR, one first must choose two quantitative factors: the length of holding horizon and the confidence level. The significance of the quantitative factors depends on how they are to be used. If the resulting VaRs are directly used for the choice of a capital cushion, then the choice of the confidence level is crucial. This choice should reflect the company's degree of risk aversion and the cost of a loss exceeding the VaR. Higher risk aversion, or greater costs, implies that a larger amount of capital should be available to cover possible losses, thus leading to a higher confidence level. In contrast, if VaR numbers are used only to provide a companywide yardstick to compare risks among different markets, then the choice of confidence level is not very important.

To compute the VaR of a portfolio, define W_0 as the initial investment and y as its rate of return. The portfolio value at the end of the target horizon is W= $W_0(1 + y)$. Define μ and σ as the annual mean and the standard deviation of y, respectively, and Δ_t as the time interval considered. VaR is defined as the dollar loss relative to what was expected, that is VaR =E(W) - W*= $W_0(\mu - y^*)$, where W* is the lowest portfolio value at given confidence level α. Finding VaR is equivalent to identifying the minimum value, W*, or the cutoff return, y* [3].

In its most general form, VaR can be derived from the probability distribution for the future portfolio value, f(w). At a given confidence level, α, we wish to find the worst possible realization, W*, such that the probability of exceeding this value is α, where $\alpha = \int_{W^*}^{\infty} f(w)dw$ or such that the probability of a value lower than W^* is 1 - α, where 1 - $\alpha = \int_{-\infty}^{W^*} f(w)dw$ [3].

In order to understand in depth the notion of VaR, the reader should recall that the VaR on a portfolio is the maximum loss we might expect over a given holding or horizon period, at a given confidence level α. Mathematically that is,

$$\alpha = \Pr(y_t \leq -\text{VaR}|F_{t-1}),$$

where y_t is the log return series at time t given from $y_t = [log(P_t) - log(P_{t-1})] \cdot 100$, P_t express close price of the index at time t, α is the given confidence level and F_{t-1} denotes the information set at time t-1. Models and methods for VaR forecasting are an ongoing debate for financial practitioners and statisticians [4].

The VaR is thus proportional to a quantile in the conditional one-step-ahead forecast distribution for the observations. Hence, the VaR is defined contingent on two arbitrarily chosen parameters. As we mention above these are the holding or horizon period, which is the period of time over which we measure our portfolio profit or loss, and which might be daily, weekly, monthly, or whatever and a confidence level α, which indicates the likelihood that we will get an outcome no worse than our VaR, and which might be 50%, 90%, 95%, 99% or indeed any fraction between 0 and 1. The most commonly used range is the 95th to 99th percentile range. The choice of these components by risk managers greatly affects the nature of the VaR model. The VaR is illustrated in the figure below, which shows a common probability density function (pdf) of profit/loss (P/L) over a chosen holding period [1].

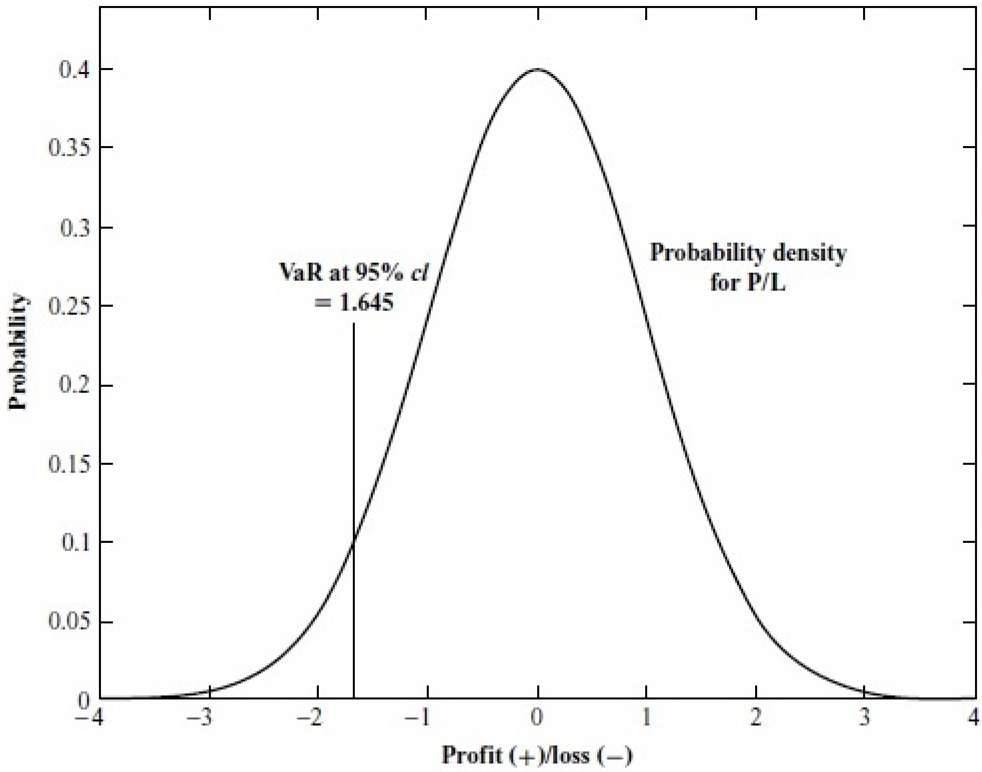

Figure 1.1 Value at Risk (Note: Produced using the "normal var figure" function)

To get the VaR, we must choose a confidence level (α). If this is 95%, say, then the VaR is given by the negative of the point on the x-axis that cuts off the top 95% of observations from the bottom 5% of tail observations. In this case, the relevant x-axis value is -1.645, so the VaR is 1.645. The negative value of observations corresponds to a positive VaR, indicating that the worst outcome at this level of confidence is a loss of 1.645.

In practice, the point on the x-axis corresponding to our VaR will usually be negative and, where it is, will correspond to a (positive) loss and a positive VaR.

However, this x-point can sometimes be positive, in which case it indicates a profit rather than a loss, and in this case the VaR will be negative. This also makes sense: if the worst outcome at this confidence level is a particular profit rather than a loss, then the VaR, the likely loss, must be negative. As mentioned already, the VaR is contingent on the choice of confidence level, and will generally change when the confidence level changes. We should also remember that the VaR is contingent on the choice of holding period as well, and so we should consider how the VaR varies with the holding period.

The VaR figure has two important characteristics. The first is that it provides a common consistent measure of risk across different positions and risk factors. It enables us to measure the risk associated with a fixed-income position, say, in a way that is comparable to and consistent with a measure of the risk associated with equity positions. VaR provides us with a common risk yardstick, and this yardstick makes it possible for institutions to manage their risks in new ways that were not possible before. The other characteristic of VaR is that it takes account of the correlations between different risk factors [1]. If two risks offset each other, the VaR allows for this offset and tells us that the overall risk is fairly low. If the same two risks do not offset each other, the VaR takes this into account as well and gives us a higher risk estimate. Clearly, a risk measure that accounts for correlations is essential if we are to be able to handle portfolio risks in a statistically meaningful way.

VaR information can be used in many ways:

1. Senior management can use it to set their overall risk target, and from that determine risk targets and position limits down the line. If they want the firm to increase its risks, they would increase the overall VaR target, and vice versa.

2. Since VaR tells us the maximum amount we are likely to lose, we can use it

to determine capital allocation. We can use it to determine capital requirements at the level of the firm, but also right down the line, down to the level of the individual investment decision: the riskier the activity, the higher the VaR and the greater the capital requirement.

3. VaR can be very useful for reporting and disclosing purposes, and firms increasingly make a point of reporting VaR information in their annual reports.

4. We can use VaR information to assess the risks of different investment opportunities before decisions are made. VaR-based decision rules can guide investment, hedging and trading decisions, and do so taking account of the implications of alternative choices for the portfolio risk as a whole.

5. VaR information can be used to implement portfolio-wide hedging strategies that are otherwise rarely possible.

6. VaR information can be used to provide new remuneration rules for traders, managers and other employees that take account of the risks they take, and so discourage the excessive risk-taking that occurs when employees are rewarded on the basis of profits alone, without any reference to the risks they took to get those profits.

In short, VaR can help provide for a more consistent and integrated approach to the management of different risks, leading also to greater risk transparency and disclosure, and better strategic management [1].

1.3.3 Criticisms of VaR

Most risk practitioners embraced VaR with varying degrees of enthusiasm, and most of the debate over VaR dealt with the relative merits of different VaR systems, the pros and cons of Risk Metrics, of parametric approaches relative to historical simulation approaches, and so on. However, there were also, those who warned that VaR had deeper problems and could be dangerous.

A key issue was the validity of the statistical and other assumptions underlying VaR, especially the transfer of mathematical and statistical models from the physical sciences where they were well suited to social systems where they were often invalid. Such applications often ignore important features of social systems, the ways in which intelligent agents learn and react to their environment, the non-stationarity and dynamic interdependence of many market processes, and so forth features that undermine the plausibility of many models and leave VaR estimates wide open to major errors [1].

A related argument was that VaR estimates are too imprecise to be of much use, and empirical evidence suggests that different VaR models can give very different VaR estimates. To make matters worse, VaR models were exposed to considerable implementation risk as well, so even theoretically similar models could give quite different VaR estimates because of the differences in the ways in which the models are implemented. It is therefore difficult for VaR advocates to deny that VaR estimates can be very imprecise. In other words, if VaR estimates are too inaccurate and users take them seriously,they could take on much bigger risks and lose much more than they had bargained for [1] .

Also, if VaR measures are used to control or remunerate risk-taking, traders will have an incentive to seek out positions where risk is over or underestimated and trade them. They will therefore take on more risk than suggested by VaR estimates so our VaR estimates will be biased downwards and their empirical evidence suggests that the magnitude of these underestimates can be very substantial. Other suggest that the use of VaR might destabilise the financial system. Thus, VaR players are dynamic hedgers, and need to revise their positions in the face of changes in market prices. If everyone uses VaR, there is a danger that this hedging behaviour will make uncorrelated risks become very correlated and firms will bear much greater risk than their VaR models might suggest [1].

VaR has its drawbacks as a risk measure and some of these are fairly obvious. VaR estimates can be subject to error and VaR systems can be subject to model risk (i.e., the risk of errors arising from inappropriate assumptions on which models are based) or implementation risk (i.e., the risk of errors arising from the way in which systems are implemented). However, such problems are common to all risk measurement systems, and are not unique to VaR. More specifically, the VaR only tells us the most we can lose if a tail event does not occur, it tells us the most we can lose 95 % of the time, or whatever, but tells us nothing about what we can lose on the remaining 5% of occasions. If a tail event (i.e., a loss in excess of VaR) does occur, we can expect to lose more than the VaR, but the VaR figure itself gives us no indication of how much that might be. However, it is not always feasible to use information about VaRs at multiple confidence levels, and where it is not, the failure of VaR to take account of losses in excess of itself can create some perverse outcomes. Also, a VaR measure can discourage diversification of risks because it fails to take into account the magnitude of losses in excess of VaR [1].

But there is also a deeper problem with VaR. Sub-additivity, which means that aggregating individual risks does not increase overall risk, is thus a highly desirable property for any risk measure. Unfortunately, VaR is not generally sub-additive, and can only be made to be sub-additive if we impose the (usually) implausible assumption that returns, are normally (or slightly more generally, elliptically) distributed [1] [5].

1.4 VaR Classification

The existing VaR methods have been classified into three broad categories: parametric, semiparametric and nonparametric. **Parametric** approaches involve a parameterization of the behavior of prices, with conditional quantiles estimated using a conditional volatility forecast and an assumption for the shape of the distribution. An example is a GARCH volatility model with a Student-t distribution or perhaps an asymmetric t distribution [6]. A notable benefit of a parametric method is the complete formation of the conditional returns distribution. A significant pitfall of a parametric approach is that the specification of the variance equation and the choice of distribution may be wrong [7] [8].

The **semiparametric** VaR category includes applications of extreme value analysis and methods based on quantile regression, such as the CAViaR models introduced by Engle and Manganelli (2004). Using an autoregressive framework, CAViaR models aim to derive the evolution of the desired quantile rather than extracting the quantile from an estimate of a complete distribution or from a volatility estimate. The approach has the advantage of allowing the shape of the conditional returns distribution to be time-varying, and for the time-variation to be different for different quantiles of the distribution. The CAViaR models introduced by Engle and Manganelli (2004) are presented below.

The most widely used **non-parametric** method is historical simulation. With this method, the VaR is estimated as the quantile of the empirical distribution of historical returns from a moving window of the most recent periods. The advantage of historical simulation is that it requires no distributional assumption and that it is easy to compute. However, the VaR estimation can be poor and slow to converge to the actual VaR, especially for the extreme quantiles. In other words, the VaR forecast might be inaccurate due to inadequate rolling window of risk factors [9]. Another difficulty, is in the choice of the number of observations to include in the moving window[9]. A moving window that is too small leads to large sampling errors, while too many observations in the moving window results in sluggish adaptation to the dynamic changes in the true distribution. Some researchers attempt to overcome this issue through their exponentially weighted approaches to VaR estimation [7] [8] .

Hendricks(1996) examine three most common categories of VaR models, equally weighted moving average approaches, exponentially weighted moving average approaches, and historical simulation approaches. The first two approaches or “variance-covariance” VaR approaches, assume normality and serial independence and an absence of nonlinear positions such as options. The dual assumption of normality and serial independence creates ease of use for two reasons [10]. First, normality simplifies value-at-risk calculations because all percentiles are assumed to be known multiples of the standard deviation however normality may under-estimate the extreme outcomes[11]. Second, serial independence means that the size of a price move on one day will not affect estimates of price moves on any other day. In the same spectrum and according to a distribution assumption or not, the range of different methods that have been developed for VaR estimation and forecasting in the literature could be also categorized, as indirect and direct methods [2].

1.4.1 Parametric approaches: GARCH and RiskMetrics models

Much of the literature on VaR forecasting focuses on GARCH and RiskMetrics models as benchmarks. More precisely, the GARCH(1,1) model with Gaussian and Student-t errors and the IGARCH(1,1) of RiskMetrics model with Gaussian errors, are considered in the empirical analysis. For a log return series y_t, let $y_t = \mu_t + \alpha_t$. Then α_t follows a GARCH(r,s) model if

$$\alpha_t = \sigma_t\ \varepsilon_t,\ {\sigma_t}^2 = \alpha_0 + \sum_{i=1}^{r} \alpha_i \alpha_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$

and we also assume that

$$\alpha_0 > 0,\ \alpha_i \geq 0\ ,\ \beta_j \geq 0 \text{ and } \sum_{i=1}^{max(r,s)} \alpha_i + \beta_i < 1.$$

where again ε_t is a sequence of iid random variables with mean 0 and variance 1. It is understood that $\alpha_i = 0$ for i > r and $\beta_j = 0$ for j>s. The latter constraint on $\alpha_i + \beta_j$ implies that the unconditional variance of α_t is finite, whereas it's conditional variance ${\sigma_t}^2$ evolves over time. Also, ε_t is often assumed to be a standard normal or standardized Student-t distribution or generalized error distribution.

GARCH model encounters an important weakness. For instance, it responds equally to positive and negative shocks. In addition, recent empirical studies of high-frequency financial time series indicate that the tail behavior of GARCH models remains too short even with standardized Student-t innovations [1]. For parametric methods, also RiskMetrics models are used. The model is specified as follows:

$$y_t = \alpha_t,$$
$$\alpha_t = \sigma_t \varepsilon_t, \text{ where } \varepsilon_t \sim \text{iid N(0,1)},$$
$$\sigma_t^2 = (1- \lambda)\ \alpha_{t-1}^2 + \lambda\ \sigma_{t-1}^2.$$

Under each model, the one-step-ahead VaR at α-quantile level is computed, as $VaR_t = \mu_t + D_\alpha^{-1}\ \sigma_t$, where D_α^{-1} is the inverse cumulative distribution function for the distribution D [4].

1.4.2 Semiparametric approaches: The CAVIAR models

Despite VaR's conceptual simplicity, its measurement is a very challenging statistical problem, and none of the methodolology developed so far gives satisfactory solutions. Because VaR is simply a particular quantile of future portfolio values, conditional on current information, and because the distribution of portfolio returns typically changes over time, the challenge is to find a suitable model for time-varying conditional quantiles. The problem is to forecast a value each period that will be exceeded with probability (1- α) by the current portfolio, where $\alpha \in (0,1)$ represents the confidence level associated with the VaR. Let y_t, t=1 until T denote the time series of returns and T denote the sample size. We want to find VaR such that $\alpha = \Pr(y_t \leq -\text{VaR}|F_{t-1})$. We remind that F_{t-1} denotes the information set at time t-1.

More specifically, Engle and Manganelli (2004), propose a different approach to quantile estimation, thus instead of modeling the whole distribution they model the quantile directly. The distribution of stock market returns is auto-correlated, consequently the VaR, which is tightly linked to the standard deviation of the distribution, must exhibit similar behavior. In order to formalize this characteristic, it is used use some type of autoregressive specification, which is called Conditional Autoregressive Value at Risk (CAViaR) models [13]. CAViaR models, usually, employ GARCH-type specifications, giving rise to the Indirect GARCH (IG), Symmetric absolute value (SAV), and Asymmetric slope (AS) CAViaR models. Also, Jeon J. and Taylor J.(2013) introduce in their article, the indirect AR(1)-GARCH(1,1)(I) CAViaR model. These are:

The **Indirect GARCH(1,1) (IG) CAViaR model**,

$$f_{t,\alpha}(\beta) = (\beta_0 + \beta_1 f^2_{t-1,\alpha}(\beta) + \beta_2 y^2_{t-1})^{1/2}.$$

The **Symmetric absolute value (SAV) CAViaR model**,

$$f_{t,\alpha}(\beta) = \beta_0 + \beta_1 f_{t-1,\alpha}(\beta) + \beta_2 |y_{t-1}|.$$

The **Asymmetric slope (AS) CAViaR model**,

$$f_{t,\alpha}(\beta) = \beta_0 + \beta_1 f_{t-1,\alpha}(\beta) + (\beta_2 I_{\{y_{t-1}>0\}} + \beta_3 I_{\{y_{t-1}<0\}}) \cdot \mid y_{t-1} \mid .$$

The **Indirect AR(1)-GARCH(1,1)(I) CAViaR model**,

$$f_{t,\alpha}(\beta) = \beta_4\, y_{t-1} + (\, \beta_0 + \beta_1\, ((\, f_{t-1,\alpha}(\beta) - \beta_2 y_{t-2})^2) + \beta_3\, ((\, y_{t-1} - \beta_2 y_{t-2})^2))^{1/2}.$$

The **Threshold CAViaR model**,

$$f_{t,\alpha}(\beta) = \begin{cases} \beta_0 + \beta_1 f_{t-1,\alpha}(\beta) + \beta_2 |y_{t-1}|, & y_{t-1} \leq r \\ \beta_3 + \beta_4 f_{t-1,\alpha}(\beta) + \beta_5 |y_{t-1}|, & y_{t-1} > r, \end{cases}$$

where $f_{t,\alpha}(\cdot)$ denotes the α-level conditional quantile.

The first equation is exactly equivalent to the dynamic quantile function for a GARCH(1, 1) model with an identically independently distributed (iid) symmetric error distribution and mean = 0. The model, thus, allows efficient estimation for GARCH(1, 1) quantiles with unspecified error distribution. This is an advantage: GARCH models are typically estimated under a parametric error distribution. However, it is well known that standard GARCH models tend to overreact to large return shocks, essentially they overreact to a variance increase (since they are squared). As such, we prefer the absolute value model types, which depict in second and third equation.

The first two CAViaR models respond symmetrically to positive and negative observations, with linear responses and parameters. To account for financial market asymmetry, via the leverage effect, the SAV-CAViaR model was extended to AS-CAViaR model[13]. Hence, the AS-CAViaR model responds differently to positive and negative observations as the indicator function, I($\cdot$), is designed specifically to capture the asymmetric leverage effect. This effect is the tendency for volatility to be greater following a negative return than a positive return of equal size. Also, I-CAViaR model allows for the conditional mean to be time-varying [8].

In general, CAViaR models are semi-parametric in nature: dynamics are specified but error distributions are not [12]. In the same spirit, recent work has been devoted to threshold nonlinear specifications such as the Threshold and Threshold Indirect GARCH CAViaR models. As Gerlach et al.(2011) and Tsiotas (2014) point out that in CAViaR team models there is a threshold nonlinear extension of the VaR model. In these spesifications all parameters in the volatility (and mean) equations were allowed to change between regimes, based on an observed threshold. As a result, the **Threshold (T) CAViaR** model is given by the last equation. Here, r is the threshold value, typically set as r = 0, or estimated. This specification called the T-CAViaR model, includes both the SAV-CAViaR (r =∞) and the AS-CAViaR (r = 0, $\beta_0 = \beta_3$, $\beta_1 = \beta_4$) models as special cases.

The CAViaR models' estimation is treated using a standard quantile regression approach. Thus, one can estimate the one-step-ahead conditional quantile, $Q_{t+1,\alpha} = f_{t+1,\alpha}(\beta)$, at a nominal level α by minimizing with respect to β, as follows[14]:

$$min_{\beta} L_{\alpha}(\varepsilon_{1:T}^{t+1}, \beta) \equiv min_{\beta} \sum_{t} (\alpha - I_{\varepsilon_{t+1<0}}) \cdot \varepsilon_{t+1},$$

over the sample of $y_1,...,y_T$ observations. Here, $(\alpha - I_{\varepsilon_{t+1<0}}) \cdot \varepsilon_{t+1}$ stands for the quantile loss function with error $\varepsilon_{t+1} = y_{t+1}$ - $f_{t+1,\alpha}(\beta)$.

The general dynamic quantile model may be written

$$y_t = f_{t,\alpha}\,(\beta, y_{t-1}) + \varepsilon_t\ ,$$

where y_t is the observation at time t, y_{t-1} are the explanatory variables, β are unknown parameters and ε_t is an error term. The function $f_{t,\alpha}(\cdot)$ defines the dynamic link between y_t and y_{t-1} and is usually linear in β and y_{t-1}, an aspect that is extended here. The conditional α quantile level is then[12]

$$q_\alpha \left(y_t \mid \beta, y_{t-1} \right) = f_{t,\alpha}(\hat{\beta}, y_{t-1}),$$

where $\hat{\beta}$ is the solution of the above minimization.

1.4.3 Non-Parametric approaches: Historical Simulation Approach

Arguably the simplest way to estimate VaR is to use the sample quantile estimate based on historic return data, which is referred to as historical simulation (HS). There are several varieties of this method, with various advantages and disadvantages (for details see Dowd, 2002). We entertain the most popular way which we call (naive) HS, and the most successful way, which is filtered historical simulation (FHS).

For HS, the VaR estimate for t+1 is given by the empirical α-quantile, $\hat{Q}_\alpha(\cdot)$, of a moving window of w observations up to date t, that is $Va\hat{R}_{t+1} = -\hat{Q}_\alpha(y_t, y_{t-1}, ..., y_{t-w+1})$ [6]. For example, with a moving window of length, say, w=1000 observations, the 5% VaR estimate is simply the negative of the 50^{th} sample order statistic. Notice that, besides ignoring the oftentimes blatant non-iid nature of the data, predictions extending beyond the extreme returns observed during the past w observations are not possible with this method. Also, the resulting VaR estimates can exhibit predictable jumps when large negative returns either enter into or drop out of the window.

For FHS, a location-scale model (such as the equation for a time-varying variance, captured by a GARCH(r,s) process, see above) is used to prefilter the data. VaR forecasts are then generated by computing the VaR from paths simulated using draws from the filtered residuals [6].

1.5 Testing VaR models

Generally speaking, back-testing a VaR model means checking whether the realized daily returns are consistent with the corresponding VaR produced by an interval model of a financial institution, over an extended period of time. In this paragraph we discuss assessing the accuracy of VaR estimates and forecasts. The Basel II Accord requires financial institutions to use back-testing, so that at least one year of actual returns are compared with VaR forecasts. There are some common criteria for comparing the forecasting performance of VaR models and we are going to discuss below.

The Basel Committee (1996) classified the reasons for model back-testing failures into the following categories[4]:

1. Basic integrity of the model: The system is unable to capture the risk of the positions or there is a problem in calculating volatilities and correlations.

2. Model's accuracy could be improved: Risk of some instruments not measured with sufficient precision.

3. Bad luck, or markets moved in a fashion that could not be anticipated by the model. For instance, volatilities or correlations turned out to be significantly different than what was predicted.

4. Intra-day trading: There is a change in positions after the VaR estimates were computed.

1.5.1 Back-testing

Before we can use risk models with confidence, it is necessary to validate them. Back-testing is the critical issue in model validation. More specifically, back-testing is the application of quantitative, typically statistical, methods to determine whether a model's risk estimates are consistent with the assumptions on which the model is based. Back-tests are a critical part of the risk measurement process, as we rely on them to give us an indication of any problems with our risk measurement models (e.g., such as misspecification, underestimation of risks, etc.). In other words, back-testing is a key part of the internal model's approach to market risk management as laid out by the Basel Committee on Banking Supervision (1996).

Having completed our preliminary data analysis, we turn now to formal statistical back-testing. All statistical tests are based on the idea that we first select a significance level, and then estimate the probability associated with the null hypothesis being "true".

Typically, we would accept the null hypothesis if the estimated value of this probability, the estimated prob-value, exceeds the chosen significance level, and reject it otherwise. The higher the significance level, the more likely we are to accept the null hypothesis, and the less likely we are to incorrectly reject a true model (i.e., to make a Type I error, to use the jargon). However, it also means that we are more likely to incorrectly accept a false model (i.e., to make a Type II error). Any test therefore involves a trade-off between these two types of possible error.

In principle, we should select a significance level that takes account of the likelihoods of these errors (and, in theory, their costs as well) and strikes an appropriate balance between them. However, in practice, it is very common to select some arbitrary significance level such as 5% and apply that level in all our tests. A significance level of this magnitude gives the model a certain benefit of the doubt, and implies that we would reject the model only if the evidence against it is reasonably strong: for example, if we are working with a 5% significance level, we would conclude that the model was adequate if we obtained any prob-value estimate greater than 5%.

A test can be said to be reliable if it is likely to avoid both types of error when used with an appropriate significance level. In other words, if we use a 99% confidence interval, we expect to find exceptions in 1% of the instances. By determining a range of the number of exceptions that we would accept, we must strike a balance between rejecting an accurate model (Type I error) and accepting an inaccurate model (Type II error) [11].

1.5.2 Violation Rate

A common criterion to compare VaR models is the violation rate, defined as the proportion of observations for which the actual return is more extreme than the forecasted VaR level, over the forecast period [12] or as the proportion of violations. The violation rate (here after $\hat{\alpha}$) is defined as:

$$\hat{\alpha} = \frac{\sum_{t=1}^{T} I(y_t < -f_{t,\alpha(\beta)})}{T}$$

where T is the sample size. A forecast model's $\hat{\alpha}$ should be close to the nominal level α. We employed the ratio $\hat{\alpha}/\alpha$, to help compare the competing models, where models with $\hat{\alpha}/\alpha \approx 1$ are most desirable. When $\hat{\alpha} < \alpha$ risk and loss estimates are conservative (higher than actual), while alternatively, when $\hat{\alpha} > \alpha$, risk estimates are lower than actual and financial institutions may not allocate sufficient capital to cover likely future losses. Here solvency outweighs profitability and for models where $\hat{\alpha} / \alpha$ are equidistant from 1, lower or conservative rates are preferred, for example, $\hat{\alpha} / \alpha = 0.9$ is preferred to $\hat{\alpha} / \alpha = 1.1$ [12].

1.5.3 Other Back-tests

Back-testing represents a way to test how well VaR estimates would have performed in the past, i.e., how often was the actual 1-day (or 10-day) loss greater than the 95% (or 99%) VaR measure. Before presenting some commonly discussed back-tests, let's initially recall that y_t is the observed returns and $f_{t,\alpha(\beta)}$ that is the respective one-day VaR defined for a quantile level α. Now define a violation sequence by the following indicator function, also called in the literature "hit sequence":

$$H_t = \begin{cases} 1 \quad , \quad \text{if} \quad y_t > f_{t,\alpha(\beta)}, \\ 0 \quad , \quad \text{if} \quad y_t \leq f_{t,\alpha(\beta)} \end{cases}$$

and compute the numbers of violations N = $\sum_{t=1}^{T} H_t$.

Three standard hypothesis-testing methods for evaluating and testing the accuracy of VaR models are analyzed below: the unconditional coverage (UC) test of Kupiec (1995): a likelihood ratio test, the conditional coverage (CC) test of Christoffersen (1998): a joint test, combining a likelihood ratio test for independence of violations and the UC test and the Dynamic Quantile (DQ) test of Engle and Manganelli (2004). From these tests we used in our work the conditional coverage CC test of Christoffersen [16]. Readers should skim through the original papers for specific details. Let us, present a detailed description for each one of them[17]:

(i) Kupiec (1995) : Some of the earliest proposed VaR back-tests is due to Kupiec (1995), which proposes a nonparametric unconditional coverage test based on the proportion of exceptions. Assume a sample size of T observations and a number of violations of N. The objective of the test is to know whether or not $\hat{\alpha} \equiv$ N/T is statistically equal to α. In other words the null hypothes is :

$$H_0 : \hat{\alpha} = \alpha$$

The probability of observing N violations over a sample size of T is driven by a Binomial distribution and null hypothesis H_0: $\hat{\alpha} = \alpha$ can be verified through a LR test of the form:

$$LR_{us} = 2ln\frac{\hat{\alpha}^N(1-\hat{\alpha})^{T-N}}{\alpha^N(1-\alpha)^{T-N}},$$

which follows (under the null hypothesis) the chi-squared distribution with one degree of freedom. Kupiec (1995) finds that the power of his test is generally poor in finite samples, and the test becomes more powerful only when the number of observations is very large.

(ii) Christoffersen (1998) : The unconditional property does not give any information about the temporal dependence of violations, and the Kupiec (1995) test ignores conditioning coverage, since violations could cluster over time, which should also invalidate a VaR model. In this sense, Christoffersen (1998) extends the previous LR statistic to specify that the hit sequence should also be independent over time. We should not be able to predict whether the VaR will be violated, since if we could predict it, then, that information could be used to construct a better risk model [17]. The proposed test statistic is based on the mentioned hit sequence H_t, and on T_{ij}, that is defined as the number of days in which a state j occurred in one day, while it was at state i the previous day. The test statistic, also depends on π_i, which is defined as the probability of observing a violation, conditional on state i the previous day. It is also assumed that the hit sequence follows a first order Markov sequence with transition matrix given by

$$\Pi = \begin{bmatrix} 1-\pi_0 & 1-\pi_1 \\ \pi_0 & \pi_1 \end{bmatrix}$$

Note that under the null hypothesis of independence, we have that $\pi = \pi_0 = \pi_1 = (T_{01} + T_{11}) / T$ and the following LR statistic can thus, be constructed:

$$LR_{ind} = 2ln\frac{(1-\pi_0)^{T_{00}}(\pi_0)^{T_{01}}(1-\pi_1)^{T_{10}}(\pi_1)^{T_{11}}}{(1-\pi)^{T_{00}+T_{10}}(\pi)^{T_{01}+T_{11}}}.$$

The joint test, also known as, conditional coverage test, includes unconditional coverage and independence properties and is simply given by $LR_{cc} = LR_{uc} + LR_{ind}$, where each component follows a chi-squared distribution with one degree of freedom, and the joint statistic LR_{cc} is asymptotically distributed as $x^2_{(2)}$. An interesting feature of this test is that a rejection of the conditional coverage may suggest the need for improvements on the VaR model, in order to eliminate the clustering behavior. On the other hand, the proposed test has a restrictive feature, since it only takes into account the autocorrelation of order 1 in the hit sequence.

(iii) Engle and Manganelli(2004) : They, also, suggested a specification test, also known as dynamic conditional quantile (DQ) test, which involves running the following regression

$$DQ_{oos} = (\text{ H i}t_t' X_t[X_t' X_t]^{-1} X_t' \text{ H i}t_t) / (\alpha(1-\alpha)),$$

where X_t =[c, V_t (α), Z_t], Z_t denotes lagged Hit_t , Hit_t = I_t (α) - α and

$$I_t(\alpha) = \begin{cases} 1 \quad , \quad \text{if} \quad y_t < f_{t,\alpha(\beta)} \\ 0 \quad , \quad \text{if} \quad y_t \geq f_{t,\alpha(\beta)}, \end{cases}$$

The null hypothesis is the independence between Hit_t and X_t. Under the null, the proposed metric to evaluate one-step-ahead forecasts (DQ_{out}) follows a x^2_q in which q= rank(X_t). Note that the DQ test can be used to evaluate the performance of any type of VaR methodology (and not only the CAViaR family) [17]. For specific details, reader should skim to the article of Engle and Manganelli(2004) in which is in.

In conclusion, under the null hypothesis, Kupiec (1995) employs a likelihood ratio to test whether VaR estimates, on average, provide correct coverage of the lower α percent tails of the forecast distributions. Christoffersen (1998) develops an independence test, employing a two-state Markov process, and combines this with the UC test to develop a joint likelihood ratio conditional coverage test, that examines whether VaR estimates display correct conditional coverage at each point in time. The conditional coverage test thus examines simultaneously whether the violations appear independently and the unconditional coverage is α. The DQ test is also a joint test of the independence of violations and correct coverage. It employs a regression-based model of the violation-related variable "hits", defined as $I(y_t < -f_{t,\alpha(\beta})$ - α which will on average be α if unconditional coverage is correct. A regression-type test is then employed to examine whether the "hits" are related to lagged "hits", lagged VaR forecasts, or other relevant regressors, over time, a model producing accurate and independent violations and "hits" will not be. The DQ test is well known to be more powerful than the CC test. The tests and criteria above do not consider whether the magnitude of the VaR forecasts is appropriate, only that the violations occur independently and in the right proportion [4].

Chapter 2

Combined Forecasts

In the VaR literature, the model selection has been based on non-statistical and statistical tests, such as the coverage tests [14]. Generally speaking, statistical analyses require the consideration of more than one model. Thus, one could investigate how these specifications can estimate VaR at a pre-specified nominal level, given the alternative conditional quantile models. The existing model selection strategies have led to considerable instability on CAViaR model selection. Specifically, non-statistical tests instability is considerable among data series and the nominal level used [18].

The same model selection strategies have led researchers to adopt models that produce biased parametric and forecast estimates. An alternative strategy consists of introducing combined (or weighted) estimators, which can potentially improve the estimator stability and model uncertainty with the suitable assigned weights. Combining VaR estimators can potentially capture extreme and rare financial events that are not detected by a single model case. Also, the well-documented bias of VaR estimation calls for the use of combined VaR forecasts to improve VaR validation.

If it is not clear which of two forecasts performs better, a combination can be the best option [19]. Combining methods include information contained in each of individual forecast. The combined forecasts should be applied if several different models can be combined to obtain better forecast, there is no certainty about the future state of the object forecast, and where large forecasting error involves a high cost. By combining, forecasters, should be able to reduce inconsistency in estimates and to cancel out biases to some extent [7]. The work by Bates and Granger (1969) often is considered to be the seminal article on combining forecasts. They combined two separate sets of forecasts of airline passenger data to form a composite set of forecasts. They concluded that the composite set of forecasts can yield lower mean-square error than either of the original forecasts. Past errors of each of the original forecasts are used to determine the weights to attach to these two original forecasts in forming the combined forecasts. They, also, examined different methods of deriving these weights [19].

In this chapter, we briefly introduce the idea of combined VaR forecasts in order to obtain better forecasts. Also, we try to review comparative empirical forecasting studies, that have considered the combining VaR forecasting methods and model averaging techniques. Finally, we try to explain the four combined methods: Simple Average Combining (SimpAvg), Unrestricted Linear Combination (LinearComb), Weighted Averaged Combining (WtdAvg) and Weighted Averaged Combining Optimized using Exponential Weighting (WtdAvgExp) [7].

2.1 Model Averaging

Statistics literature includes attempts to resolve this model selection uncertainty by adopting averaging strategies for predictors. This approach intends to use weighted estimators to improve the forecasting accuracy and/or variability by assigning an appropriate weight. Model averaging (MA) is a technique that assumes uncertainty about the type of model with which data are generated.

Statistical thinking in many aspects of science have been profoundly influenced by the introduction of AIC (Akaikes information criterion) as a tool for model selection and as a basis for model averaging. In their paper, Link and Barker (2006), advocate the Bayesian paradigm as a broader framework for multimodel inference, one in which model averaging and model selection are naturally linked, and in which the performance of AIC-based tools is naturally evaluated. Prior model weights, implicitly associated with the use of AIC are seen to highly favor complex models: in some cases, all but the most highly parameterized models in the model set are virtually ignored a priori. They suggest the usefulness of the weighted BIC (Bayesian information criterion) as a computationally simple alternative to AIC, based on explicit selection of prior model probabilities rather than acceptance of default priors associated with AIC. They note, however, that both procedures are only approximate to the use of exact Bayes factors. Assume a set of models M = (M_1, M_2,..., M_R), Akaikes information criterion is defined by

$$AIC_i = -2log[g(y_t|\theta^{(i)}, M_i)] + 2k_i,$$

where k_i is the number of parameters in the model. Models with smaller values of AIC are favored on the basis of fit and parsimony. AIC weights for a collection of models are proportional to exp(-1/2 AIC) [20].

Link and Barker (2006) argue that the use of model weights in prediction requires their interpretation as posterior model probabilities. This observation raises the question as to which set of prior model weights are implicitly chosen when one uses AIC weights. The answer provides valuable insights into the operating characteristics of AIC in multimodel inference, explaining its well-documented tendency to favor highly parameterized models. They, also, recommend that analysts use weighted BIC (Bayesian information criterion) as a computationally simple alternative to AIC, based on explicit selection of prior model probabilities, rather than the default choice implicit to the use of AIC. The weighted BIC (and AIC, as a special case) use approximate rather than exact Bayes factors, which are the fundamental quantities for updating prior to posterior model probabilities.

Link and Barker(2006), also, assume a set of priors on parameters, one for each model in M, they denote the prior on parameters $\theta^{(i)}$ of model M_i by g($\theta^{(i)}$ | , M_i). They, finally, introduce a collection of prior probabilities (π_1, π_2, ..., π_R) assigned to the collection M, independent of the data; π_i = Pr(M_i) is the prior probability that model M_i is true.

The Bayesian information criterion is defined by

$$BIC_i = -2 \log[g(y_t \mid \theta^{(i)}, M_i)] + k_i \log(T),$$

and $\theta^{(i)}$ is the maximum likelihood estimator of the parameters for model i, k_i is the number of parameters in model i, and T is the sample size. Taking into account the previous equations we can obtain approximate posterior probabilities:

$$w_i \approx Pr(M_i|y_t) \approx \frac{exp(-BIC_i/2)\pi_j}{\sum_j exp(-BIC_j/2)\pi_j}$$

Assigning uniform prior probabilities to the set M, π_i [1/R], yields what are commonly referred to as BIC weights. This last equation can be thought of as a generalized BIC weight.

$$w_i \approx Pr(M_i|y_t) \approx \frac{exp(-BIC_i/2)}{\sum_r^R exp(-BIC_r/2)}$$

We could replace BIC with AIC, resulting in what has been called smoothed AIC (AIC) or weighted AIC (WAIC). The weights are

$$w_i \approx Pr(M_i|y_t) \approx \frac{exp(-AIC_i/2)}{\sum_{r}^{R} exp(-AIC_r/2)}.$$

In our thesis, we specify the weights w_i, in the case of CAViaR models as:

$$AIC_i = -2log[L_\alpha(\varepsilon_{1:T}^{t+1}, \beta)] + 2k_i$$

$$BIC_i = -2log[L_\alpha(\varepsilon_{1:T}^{t+1}, \beta)] + 2k_i log(T)$$

where $L_\alpha(\varepsilon_{1:T}^{t+1}, \beta)$ is the CheckLoss function with error $\varepsilon_{t+1} = y_{t+1}$ - $f_{t+1,\alpha}(\beta)$, k_i is the number of parameters in each CAViaR model i and T is the sample size.

2.2 Combined estimating methods

Granger(1989) and Granger et al., (1989) introduce the idea of using quantile regression to combine quantile forecasts. Using simulated data, Taylor and Bunn (1998) assess the usefulness of different restrictions on the parameters of the quantile regression combination. More recently, Zou and Yang (2004) have introduced a new combined method for time-series forecasting called aggregate forecasting through exponential re-weighting (AFTER). Giacomini and Komunjer (2005) have, also, introduced combining estimation to the conditional quantile literature. They have stated that one can, in principle, compare the out-of-sample average loss implied by alternative quantile forecasts in CAViaR estimations by choosing the appropriate loss function.

Another interesting research (Jeon and Taylor , 2013), proposes VaR estimation methods that are a synthesis of CAViaR time series models and implied volatility. Forecast combining methods, with weights estimated using quantile regression, are considered. Results, for daily index returns indicate that the newly proposed methods are comparable or superior to individual methods, such as the standard CAViaR models and quantiles constructed from implied volatility and the empirical distribution of standardized residuals. In addition, Tsiotas (2014) propose a quasi-Bayesian model averaging method that generates combinations of conditional VaR estimators based on single CAViaR models. Finally, Ratuszny (2015), use four methods of combining forecasts in order to check whether simultaneous use of information both from historical time series and regarding markets' expectation can improve accuracy of forecasts.

2.2.1 Combined CAViaR models

Regarding the implementation of combined estimates two important issues should be managed, according to Tsiotas (2014). The first is the choice of the loss function and the second is the assignment of weights in the combinations. Tsiotas (2014), introduce, a loss quantile function designed for the case where the quantile estimate is expressed as a linear combination of two rival CAViaR specifications. Given that the estimation of a given quantile model is based on the minimization of the CheckLoss function $L_\alpha(\varepsilon_{1:T}^{t+h}, \beta)$, the objective function for the combined CAViaR model becomes

$$L_\alpha(\varepsilon_{1:T}^{t+h}, \hat{\beta}, w, A_{1+2}) \equiv \sum_t (\alpha - I_{\varepsilon_{t+h,1+2<0}}) \cdot \varepsilon_{t+h}(\hat{\beta}, w),$$

where $\varepsilon_{t+h}(\hat{\beta}, w)$ is the α-quantile error given the estimated parameter vector $\hat{\beta} = (\hat{\beta_1}, \hat{\beta_2})$ from the CAViaR modelsA_1 and A_2.

Thus, given the $\hat{\beta}$ estimates, the w =(w_1,w_2) vector can be derived by solving a minimization problem, such as the following:

$$min_w L_\alpha(\varepsilon_{1:T}^{t+h}, \hat{\beta}, w, A_{1+2}) \equiv \sum_t (\alpha - I_{\varepsilon_{t+h,1+2<0}}) \cdot \varepsilon_{t+h}(\hat{\beta}, w).$$

Also, regarding the weights w, we let them lie in some compact subset of R^2 and we assume that $w_1 + w_2$ =1, with $w_1 \geq 0$ and $w_2 \geq 0$.

2.2.2 Simple Average Combining (SimpAvg)

In order to analyze the basic methodology of Jeon and Taylor(2013), let us first introduce the idea of Implied Quantile(IQ). More specifically, Jeon and Taylor(2013) construct an IQ estimator for period t as the product of implied volatility recorded in the previous period, $\sigma_{t-1}^{implied}$, and the quantile, of an empirical distribution, which we construct as the distribution of the in-sample values of y_t , defined earlier, standardized by implied volatility. The IQ estimator can be expressed in the following form:

$$Q_t^{IQ}\,(\,\alpha) = Q^{Emp}\,(\,\alpha)\,\sigma_{t-1}^{implied}$$

In basing quantile estimation on implied volatility, the IQ approach captures the market's expectation of future risk. Another advantage is that the method does not assume a particular distribution for the asset returns, and it involves no parameter estimation. We also considered the use of a Gaussian assumption, but the post-sample forecasting results were comfortably superior for the empirical distribution. It is interesting to note that this simple approach to capturing an "implied quantile" assumes returns standardised with implied volatility are i.i.d. By contrast, the CAViaR models allow for the shape of the conditional distribution to be time varying, as well as the volatility. The IQ method is similar to the filtered historical simulation, but different in that we use implied volatility instead of the variance estimated from a GARCH model [8].

The simplest and most widely used forecast combining method is to take the simple arithmetic mean of the individual forecasts. We consider the simple average of the quantile forecasts from the IQ method and one CAViaR model (Jeon and Taylor, 2013):

$$VaR_t\,(\alpha) = \tfrac{1}{2}\,Q_t^{IQ}(\alpha) + \tfrac{1}{2}\,VaR_t^{CAViaR}\,(\alpha)$$

The method will be here denoted as SimpAvg according to Jeon and Taylor (2013) nomenclature. The aim of this approach is to determine the combination of forecasts with lower error variance than in case of individual forecasts.

2.2.3 Unrestricted Linear Combination (LinearComb)

A traditional approach to combining is to compute linear combinations of forecasts, called also regression method (Jeon and Taylor, 2013). The method will be denoted as LinearComb according to Jeon and Taylor (2013) nomenclature. Forecast is formed on the basis of an IQ forecast and one of CAViaR models (Jeon and Taylor, 2013):

$$VaR_t\ (\alpha) = \gamma_1 + \gamma_2\ Q_t^{IQ}(\alpha) + \gamma_3 VaR_t^{CAViaR}\ (\alpha)$$

The parameters γ_2 and γ_3 inform about the dynamics of forecasted variable. If the sum of the parameters γ_2 and γ_3 is less than unity, the individual predictions are more volatile than the risk measure VaR. If the sum of the parameters is greater than one, then the individual forecasts are of less dynamic than VaR [8].

There are several difficulties with this combination method. The first is related to collinearity of individual forecasts. If the individual predictions are quite good, they would not differ significantly and this entails the phenomenon of collinearity. Consequently, the low-significance and high randomness of estimated weights are obtained. Another issue is the autocorrelation of the random component, caused by autocorrelation of dependent variable. The third issue is related with the inability to impose zero restrictions for correlation between the errors of individual forecasts, when examining the behavior of individual forecasts in the past. In addition, regression method requires large data sets, which in case of time series is fulfilled. The advantage of this method is the lack of restrictions on the parameters and lack of assumptions about unbiasedness of individual forecasts [7].

2.2.4 Weighted Averaged Combining (WtdAvg)

The Weighted Averaged Combining method is based on the relation between forecast error in the past [8]. In this approach the unbiasedness of quantile forecast is assumed. Error variance of combined forecast will be equal or smaller than of the individual forecasts. The method in research will be noted as WtdAvg according to Jeon and Taylor (2013) nomenclature. The resultant quantile forecast is of the form (2627), without constant, where combining weights are constrained to be between zero and one.

$$VaR_t\ (\alpha,\ w) = w\ Q_t^{IQ} + (1\text{-}\ w) VaR_t^{CAViaR}\ (\alpha)$$

Bunn (1989) noted greater robustness of the method compared with regression method. Taylor and Bunn (1998) pointed out that the value of the weight indicates the relative explanatory powers of the two quantile predictors.

2.2.5 Weighted Averaged Combining Optimized using Exponential Weighting (WtdAvgExp)

The method is similar to Weighted Averaged Combining but additionally the Exponential Weighting factor for the optimization of the combining weight is applied. The factor gives greater weight to the more recent observations in the quantile regression optimization. In this way the nonstationarity problem of weights is solved. This is particularly important when the time series exhibits time-varying and cyclical volatility. Boudoukh et al. (1998) insist that such an approach is a reasonable compromise between statistical precision and adaptation to the latest information. Exponentially Weighted Quantile Regression (EWQR) method solves the following minimizing problem (Jeon, Taylor, 2013):

$$min_w\ \Big(\sum_{t|y_t \leq -VaR_t(\alpha,w)} \lambda^{T-t}\ \alpha\ |\ y_t\ \text{-}\ VaR_t(\alpha,\omega)\ |\ +$$

$$+ \sum_{t|y_t > -VaR_t(\alpha,w)} \lambda^{T-t} \text{ (1- } \alpha) \mid y_t \text{ - } VaR_t(\alpha,w) \mid)$$

where VaR_t (α, w) are expressed in the previous paragraph. A lower value of the decay parameter λ implies faster exponential decay, and hence more weight is given to the recent observations and less historical information is captured. This method is noted as WtdAvgExp according to Jeon and Taylor (2013).

2.3 One-step-ahead combined estimates

For the alternative single CAViaR models, we form pairs of weighted combined one-step-ahead CAViaR estimates. These are, the SAV+AS-CAViaR model:

$$f_{t+1,SAV+AS}\ (\alpha,\beta) = wf_{t+1,SAV}\ (\alpha,\beta) + (1\text{-}w)\ f_{t+1,AS}\ (\alpha,\beta)$$

the SAV+I-CAViaR model:

$$f_{t+1,SAV+I}\ (\alpha,\beta) = wf_{t+1,SAV}\ (\alpha,\beta) + (1\text{-}w)\ f_{t+1,I}\ (\alpha,\beta)$$

and the AS+I-CAViaR model:

$$f_{t+1,AS+I}\ (\alpha,\beta) = wf_{t+1,AS}\ (\alpha,\beta) + (1\text{-}w)\ f_{t+1,I}\ (\alpha,\beta).$$

Also, for the alternative single CAViaR models, we form pairs of equal weighted combined one-step-ahead CAViaR estimates. In this specification we put w equal to 1/2. These are, the SAV+AS-CAViaR(EqualW) model:

$$f_{t+1,SAV+AS}\ (\alpha,\beta) = 1/2 f_{t+1,SAV}\ (\alpha,\beta) + (1\text{-}(1/2))\ f_{t+1,AS}\ (\alpha,\beta)$$

the SAV+I-CAViaR(EqualW) model:

$$f_{t+1,SAV+I}\ (\alpha,\beta) = 1/2 f_{t+1,SAV}\ (\alpha,\beta) + (1\text{-}(1/2))\ f_{t+1,I}\ (\alpha,\beta)$$

and the AS+I-CAViaR(EqualE) model:

$$f_{t+1,AS+I}\ (\alpha,\beta) = 1/2 f_{t+1,AS}\ (\alpha,\beta) + (1\text{-}(1/2))\ f_{t+1,I}\ (\alpha,\beta).$$

Chapter 3

Empirical data analysis

3.1 Our data

In order to apply the methodology on real data, a researcher needs to construct the historical series of portfolio returns and to choose specification of the functional form of the quantile. We considered three daily European stock market indices: the **IBEX 352SYTE(Spain)**[1], the **SMI(SSMI) (Switzerland)** [2] and the **OSE BENCH IDX GI(Norway)**[3]. The data were obtained from Yahoo Finance

[1]The IBEX 35 is the benchmark stock market index of the Bolsa de Madrid, Spain's principal stock exchange. Initiated in 1992, the index is administered and calculated by Sociedad de Bolsas and comprising the 35 most liquid Spanish stocks, traded in the Madrid Stock Exchange General Index.

[2]The Swiss Market Index (SMI) is Switzerland's stock market index, which makes it the most important in the country. As a price index, the SMI is not adjusted for dividends, but a performance index that takes account of such distributions is available (the SMIC - SMI Cum Dividend). The SMI was introduced on 30 June 1988 at a baseline value of 1500 points. Its composition is examined once a year. The securities contained in the SMI currently represent more than 90 % of the entire market capitalisation, as well as of 90% trading volume. Because the SMI is considered to be a mirror of the overall Swiss stock market, it is used as the underlying index for numerous derivative financial instruments such as options, futures and index funds.

[3]The OSE BENCH IDX GI(Norway) is an investable index containing a representative selection of all listed shares on Oslo Brs, an online market place where all trading is done through computer networks. Oslo Brs is the only independent stock exchange within the Nordic countries. Trading starts at 09:00am and ends at 04:30pm local time on all days of the week except weekends and holidays declared by Oslo Brs in advance. The stock exchange offers a full product range in-

and covered the period from January 1, 2002 to September 30, 2016, approximately 3700 observations. Figures 3.1,3.2 and 3.3 below, show plots of these series. The log return series were generated by taking logarithmic differences of the daily close price, $y_t = [log(P_t) - log(P_{t-1})] \cdot 100$, where P_t is the close price at time t. It is evident that the return series are clearly characterized by basic features which are typical in financial return series, such as a mean near zero and clusters of high and low volatility. Small differences in end-dates across indices occurred due to different indices-specific non-trading days.

cluding equities, derivatives and fixed income instruments. The OSEBENCH is revised on a half year basis and the changes are implemented on December 1 and June 1.

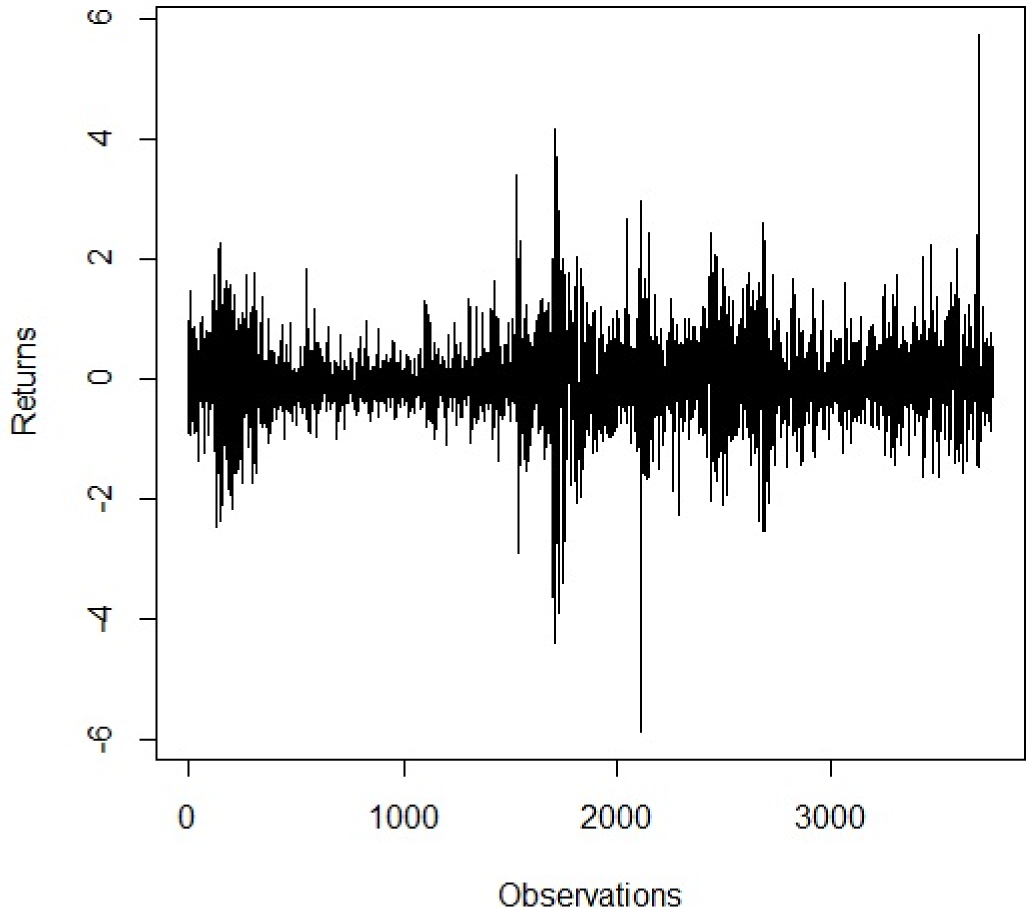

Figure 3.1 Plot of IBEX return data series

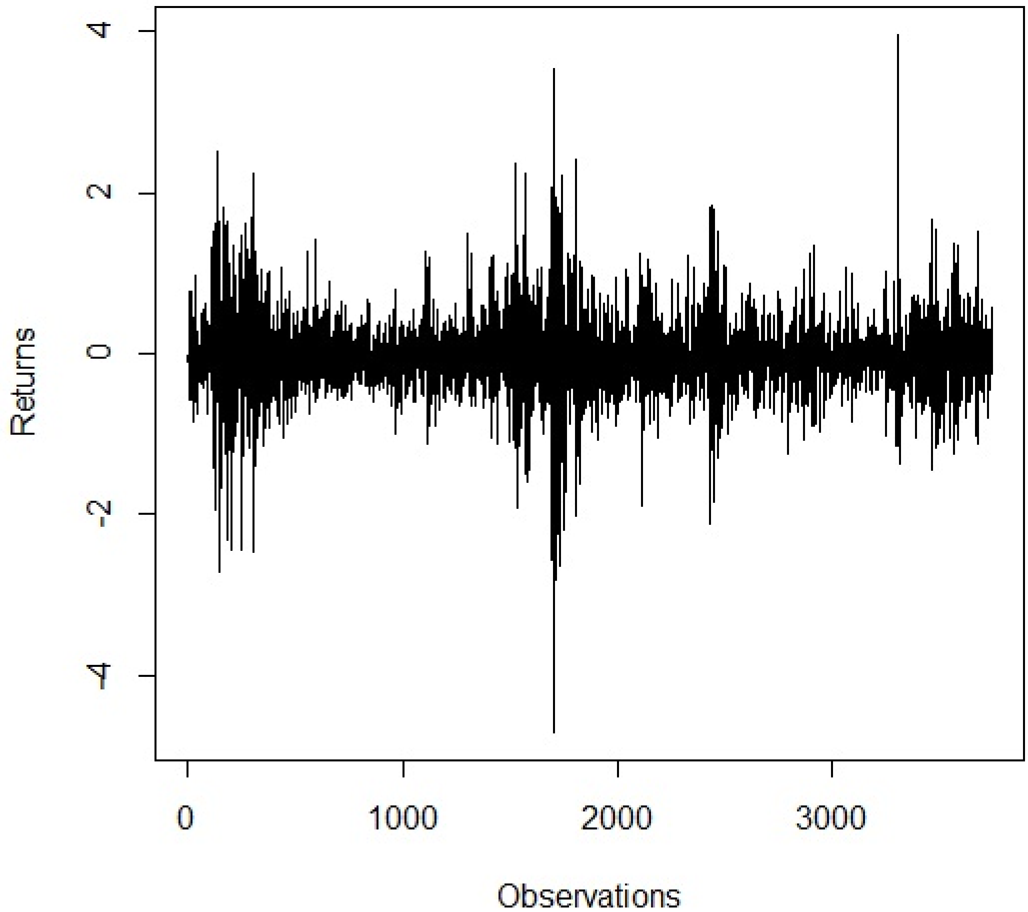

Figure 3.2 Plot of SMI return data series

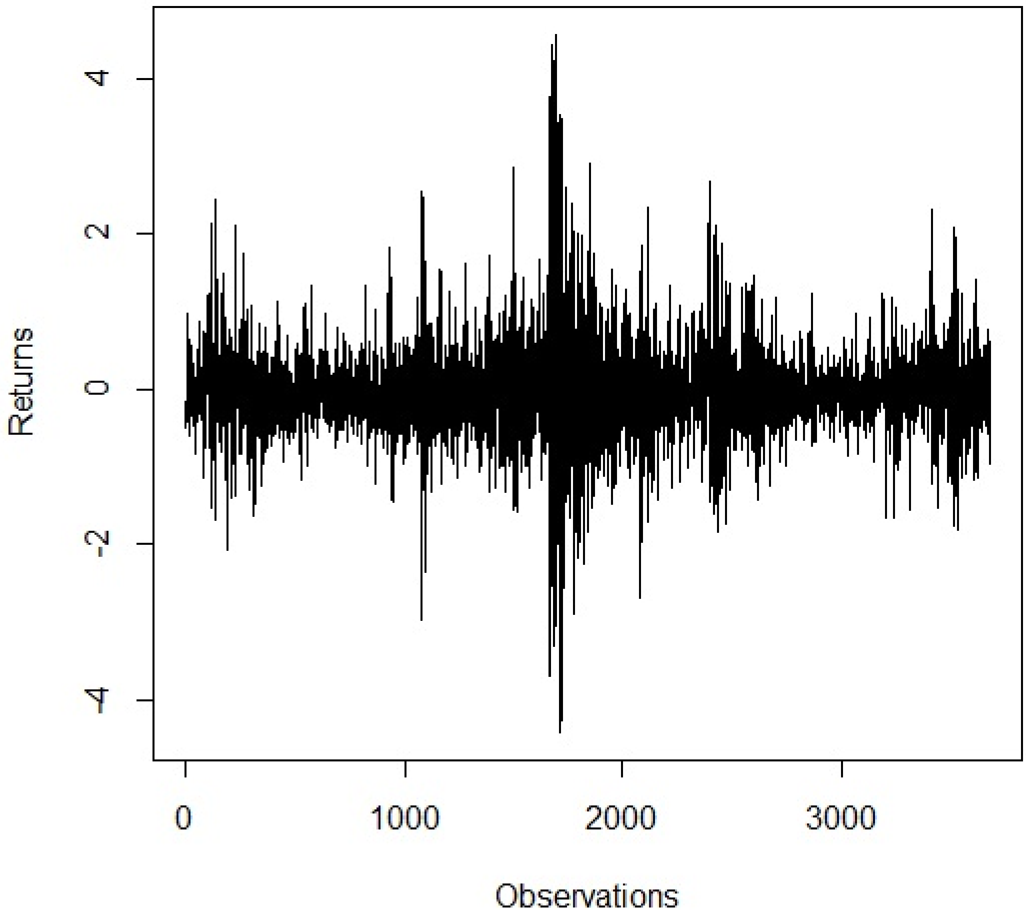

Figure 3.3 Plot of OSEBENCH return data series

3.2 Descriptive Statistics

Let us, first, introduced some basic statistical information about the terms skewness, kurtosis and Jarque-Bera test.

3.2.1 Skewness-Kurtosis

Consider a series y_t and t=1,...,T with mean μ and standard deviation σ. Let μ_r = E $[(x-\mu)^r]$ be the r-th central moment of y_t with $\mu_2 = \sigma^2$. The coefficients of skewness and kurtosis are defined as

$$\tau = \frac{\mu_3}{\sigma^3} = \frac{E[(x-\mu)^3]}{E[(x-\mu)^2]^{3/2}}$$

and

$$\kappa = \frac{\mu_4}{\sigma^4} = \frac{E[(x-\mu)^4]}{E[(x-\mu)^2]^2}.$$

If y_t is symmetrically distributed, then μ_3 and thus τ will be 0. Sample estimates of τ and κ can be obtained on replacing the population moments μ_r by the sample moments $\overline{\mu_r} = T^{-1}\sum_{t=1}^{T}(y_t - \overline{y_t})^r$. If y_t is iid and normally distributed, then $\sqrt{T}\ \overline{\tau} \rightarrow$ N(0,6) and $\sqrt{T}\ \overline{\kappa} \rightarrow$ N(0,24).

When a distribution is positively skewed, we shall in fact have Mean $>$ Median $>$ Mode, and the presence of observations on the right hand side of a distribution makes it positively skewed. On the other hand, the presence of observations to the left hand side of a distribution make it negatively skewed and the relationship between mean, median and mode is: Mean $<$ Median $<$ Mode. In a symmetrical distribution, the Mean, Median and Mode are equal to each other [25].

Skewness measures the lack of symmetry of the frequency curve of a distribution, whereas, kurtosis is a measure of the relative peakedness of its frequency curve. A measure of kurtosis was previously given and was defined as κ. The value of κ= 3 for a mesokurtic curve. When $\kappa > 3$, the curve is more peaked than the mesokurtic curve and is called as leptokurtic. Similarly, when $\kappa < 3$, the curve is less peaked than the mesokurtic curve and is called as platykurtic curve [25] .

3.2.2 Jarque-Bera Test

In statistics, the Jarque-Bera test is a test of whether sample data have the skewness and kurtosis matching a normal distribution. The test statistic JB is defined as

$$JB = \frac{T-c+1}{6}(\tau^2 + \frac{1}{4}(\kappa-3)^2)$$

where T is the number of observations (or degrees of freedom in general), τ is the skewness, κ is the kurtosis, and c is the number of regressors. The JB statistic asymptotically has a chi-squared distribution with two degrees of freedom, so the statistic can be used to test the hypothesis that the data are from a normal distribution. The null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being zero. Samples from a normal distribution have an expected skewness of 0 and an expected excess kurtosis of 0 (which is the same as a kurtosis of 3). As the definition of JB shows, any deviation from this increases the JB statistic [26].

3.2.3 Summary Statistics

The time series of returns were checked for the presence of the following features: fatter tails than in the normal distribution (identified on the basis of the quantile plots, histograms and the Jarque-Bera test), skewness, kurtosis(see Table 3.1). The returns display a degree of kurtosis approximately 6.5 (leptokurtic kurve) and positive skewness is evident both in three cases, indicating considerable asymmetry. The Jarque Bera-test reject normality at the 5% and 1% level in three cases. As we notice, small amount of p-values have a result the rejection of the null hypothesis of normality. The variance of the time series returns is the highest in the case of OSEBENCH index. As a consequence, “standard” methods, based on the assumption of normality, tend not to suffice, which has led to various alternative strategies for VaR prediction. The most prominent of these are outlined in the following subsections.

Table 3.1: Summary statistics of return index series financial series

	IBEX	OSEBENCH	SMI
mean	-0.00054	-0.01553	-0.00284
variance	0.43276	0.43539	0.27228
skewness	0.08149	0.55494	0.11905
kurtosis	6.4651	6.35902	6.82533
min.	-5.85587	-4.40319	-4.68501
max.	5.72628	4.55046	3.93925
Jarque-Bera test	224.53(< 2.2e-16)	6394.1 (<2.2e-16)	7283.4(< 2.2e-16)

3.3 Parametric Estimation and VaR forecast

Before analyzing the empirical data results, let us first explain the basic methodology of our research. We investigate the sampling performance of a family of conditional VaR models, the CAViaR models. In practice, we limit our attention to a few representative specifications, including the SAV-CAViaR, AS-CAViaR and I-CAViaR models. Our aim is two-fold. Firstly, we estimate the parameters ($\hat{\beta}$) of each model and test the Violation Rate (here after $\hat{\alpha}$) or the ratio $\hat{\alpha}/\alpha$ of the individual CAViaR models for the three financial indices. Secondly, we examine the ability of a single CAViaR model to correctly estimate the nominal level α via $\hat{\alpha}$, by employing weighted CAViaR models. In other words, we determine whether a single CAViaR model can forecast VaR in the presence of combined VaR dynamics.

To this end, we employ the following strategy: first, we use quantile regression to estimate the β for each model in a within sample estimation. In our thesis, the within sample period is from 01/01/2002 until 30/12/2008 and the out-of-sample period is from 01/01/2009 until 30/09/2016, in our first set of results. Also, we estimate the β for each model in a within sample period from 01/01/ 2002 until 30/12/ 2013 and apply back-testing in a out-of-sample period from 01/01/2014 until 30/09/2016. Consequently, we use the estimated $f_{t,\alpha}(\beta)$ in order to estimate the $\hat{\alpha}$ / $\alpha$for each model.

In other words, we used the first observations in the within sample period to estimate the models and the last observations for out-of-sample testing. We estimated 1% and 5% 1-day VARs ($f_{t,\alpha}(\beta)$), using the SAV-CAViaR, AS-CAViaR and I-CAViaR models. Finally, we test our estimates in the out-of-sample period by calculating some test-values. As we mention above, we compare the estimated $\hat{\alpha}$ to the assigned nominal level α. This non-statistical validation test estimates the empirical nominal level (or $\hat{\alpha}$) which takes the following form for a m forecast or test sample size and a n within sample size:

$$\hat{\alpha} = \frac{\sum_{t=n+1}^{m} I(y_t < -f_{t,\alpha(\beta)})}{m}.$$

Our second aim is to generate a combination of conditional VaR estimates based on single CAViaR models by calculating weights, which are derived from AIC measure. At first, we calculate CheckLoss functions of each model by using the estimated betas and use these CheckLoss functions in order to calculate the AIC and estimate the weights. Again, we aim to make a forecast by calculating the estimated $\hat{\alpha}$ in the out-of-sample period and test our estimates by calculating some test-values. In the end, we generate three new combined estimates based on the single CAViaR models, by applying the weighted coefficients and using alternately the three CAViaR models. Basic concept is to investigate the performance of the combined CAViaR estimates compared to that of the individual estimates. Also, working with the same way, we generate VaR estimates based on single CAViaR models of equal weights.

3.3.1 VaR forecast for the 2009-2016 period

In order to provide a complete introduction of results, we divide our results into groups of results. Thus, we work in two forecasting periods. In this paragraph we illustrate the results for the 2009-2016 forecasting period and in the next paragraph we illustrate some results for the 2014-2016 forecasting period. Consequently, the full sample was divided into a learning sample or within sample period: January 1, 2002 to December 30, 2008 and a forecast or testing sample or out-of-sample period: the trading days from January 1,2009 to September 30, 2016, and the results illustrate in this paragraph. Likewise, the full sample was divided into a learning sample : January 1, 2002 to December 30, 2013 and a forecast sample period: the trading days from January 1,2014 to September 30, 2016, and the results illustrate in the next paragraph.

Table 3.2: Estimated parameters of CAViaR models for index SMI

$\hat{\beta}$	SAV-CAViaR	AS-CAViaR	I-CAViaR
α =0.05			
$\hat{\beta}_0$	-0.0078(0.0110)	-0.0204(0.0068)	0.8731 (0.0185)
$\hat{\beta}_1$	0.8545 (0.0169)	0.8988 (0.0081)	-0.1136(0.0545)
$\hat{\beta}_2$	-0.2739(0.0303)	-0.2952(0.0143)	0.2190 (0.0430)
$\hat{\beta}_3$		-0.0143(0.0205)	-0.2470(0.0317)
$\hat{\beta}_4$			0.0160 (0.0076)
α =0.01			
$\hat{\beta}_0$	-0.0074(0.0159)	-0.0216(0.0105)	0.8780 (0.0094)
$\hat{\beta}_1$	0.8418 (0.0145)	0.9215 (0.0088)	0.0033(0.0316)
$\hat{\beta}_2$	-0.4364(0.0186)	-0.3481(0.0150)	0.5019 (0.0293)
$\hat{\beta}_3$		-0.0188(0.0206)	-0.1701(0.0367)
$\hat{\beta}_4$			0.0225(0.0217)

Note: Entries in brackets next to each estimated parameter, represent standard errors, boxed values indicate significant parameter according to t statistics.

Table 3.2 reports the value of the estimated parameters and the corresponding standard errors for the three CAViaR models, SAV-CAViaR, AS-CAViaR and I-CAViaR model applied to the SMI index for nominal level α =0.05 and α =0.01. We should check the hypothesis that β =0. Thus, our initial hypothesis, H_0 is β =0 and our alternative hypothesis, H_1 is $\beta \neq 0$. We could use the type

$$t = \frac{\hat{\beta} - \beta}{s_{\hat{\beta}}}$$

which follows t-student distribution with T-2 degrees of independence. For example, at α=0.05 and for the SAV model $\hat{\beta_2}$ = 0.8545 and $s_{\hat{\beta_2}}$ = 0.0169. Consequently,

$$t = \frac{0.8545}{0.0169} = 50.56$$

From the table 6 page 435 (Phoinos,1999) for T-2= 3742-2=3740 degrees of independence at α=0.05, we found, $|t_{3740,0.05}|$ = 1.96. The degrees of independence in the table approach the ∞. And so, because, $50.56 > 1.96$, H_0 hypothesis is rejected at α=0.05 and $\hat{\beta_2}$ is a statistically significant estimator. In the same way, we calculate the significance of our rest estimators. Among others, we notice the strong significance of $\hat{\beta_2}$ estimators for SAV and AS- CAViaR models at both nominal levels. Likewise, we notice strong significance of $\hat{\beta_1}$ and $\hat{\beta_3}$ estimators for I-CAViaR model at both nominal levels [25].

As we mention before, VaR is forecast one day ahead for each day in the forecast sample of approximately 1900 returns, using a range of competing models. Table 3.3 shows the ratios of the estimated ($\hat{\alpha}$) at the given nominal levels, α=0.05, 0.01, across the three models and three return data series. An estimated $\hat{\alpha}$ equals to the given nominal level is highly desirable. The best model's ratio in each index is that which approaches enough to 1 and is boxed. The results are quite different from α=0.05 to 0.01. However, all the models are much closer in performance and ratios closer to 1 across most models. Also, the majority of the results do not under-estimate or over-estimate the risk, as the most of the results are not differ enough from 1.

Specifically, as we notice in Table 3.3 at α =5%, AS-CAViaR model does not display a $\hat{\alpha}$ /α close enough to the optimum level, for none of the data sets. On the other hand, SAV-CAViaR show a $\hat{\alpha}/\alpha$ close enough to 1 both in OSEBENCH and SMI return data series. I-CAViaR introduces the desirable effect for the IBEX case. For α=1%, we notice controversial results, as each model introduce a good performance in each case. The SAV-CAViaR model shows the best performance in three out of six cases, at both nominal levels, whereas the AS-CAViaR model appears less successful results in comparison with other models. Finally, I-CAViaR model appears relatively successful in achieving second place, with two desirable results close to 1, in IBEX and SMI data set series and three out of six times achieving the second desirable result, at both nominal levels.

Table 3.3: Ratio for $\hat{\alpha}/\ \alpha$=0.05, 0.01 for each model across the three indices

Models	Ibex	Osebench	Smi
α = 0.05			
SAV-CAViaR	0.96823	1.169945	1.069087
AS-CAViaR	1.190116	1.25063	1.190116
I-CAViaR	1.028744	1.190116	1.089259
α =0.01			
SAV-CAViaR	1.159859	1.260716	1.563288
AS-CAViaR	1.563288	1.008573	1.210287
I-CAViaR	1.512859	1.664145	1.059002

Note: Entries in each table represent the $\hat{\alpha}/\alpha$. Boxed numbers indicate the favored model.

Table 3.4: CheckLoss functions

Models	Ibex	Osebench	Smi
α =0.05			
S.CheckLoss	99.19611	93.65973	90.86233
AS.CheckLoss	93.85468	92.46308	85.38442
I.CheckLoss	97.17359	95.00548	85.78754
α =0.01			
S.CheckLoss	26.45912	25.84891	23.5111
AS.CheckLoss	24.92244	25.39028	21.93466
I.CheckLoss	25.14795	25.5322	21.99209

Table 3.4 includes the CheckLoss functions. The results are based on all IBEX,SMI and OSEBENCH return data series and cover nominal levels α of 1% and 5%. As CheckLoss functions evaluate the loss we should choose the model, which display the least value of a CheckLoss function. AS-CAViaR is the model which appears the best performance, as six out to six cases displays the least value of a CheckLoss function. I-CAViaR model is in the second place, as appears the relatively highly values in five to six cases. Finally, S-CAViaR shows the worst results.

Table 3.5: Estimated weights from AIC function

Models	Ibex	Osebench	Smi
α =0.05			
weights.SandAS	0.1024 0.8975	0.4754 0.5245	0.0963 0.9036
weights.SandI	0.4971 0.5028	0.8419 0.1580	0.1769 0.8230
weights.ASandI	0.8965 0.1034	0.8546 0.1453	0.6685 0.3314
α =0.01			
weights.SandAS	0.4333 0.5666	0.5672 0.4327	0.4284 0.5715
weights.SandI	0.5852 0.4147	0.6988 0.3011	0.5598 0.4401
weights.ASandI	0.6485 0.3514	0.6389 0.3610	0.6291 0.3708

Note: The first number indicates the weight for the first written model and the second number indicates the weight for the first written model.

Based on the aforementioned posterior simulation results, Table 3.5 reports the estimated weights and Table 3.6 reports the one-step-ahead $\hat{\alpha}/\alpha$ ratio and the corresponding Christoffersen's conditional coverage test. The combined CAViaR models SAV+AS-CAViaR, SAV+I-CAViaR and AS+I-CAViaR derive from the AIC Weighted Individual CAViaR forecasts. The combined CAViaR models SAV+AS-CAViaR(EqW), SAV+I-CAViaR(EqW) and AS+I-CAViaR(EqW) derive from Equal Weighted Individual CAViaR forecasts, thus weights are equal to 0.5. The estimated weights add up to 1. The results are based in IBEX,SMI and OSEBENCH return data series and cover nominal levels α of 1% and 5%. The most desirable ratio in each index is boxed, while bolded numbers indicate the model is rejected by CC test (at a 5% level). Naturally, we prefer models with ratios close to 1, while again ratios less than 1 are preferred to those above 1 that are equidistant from 1.

First, at $\alpha = 5\%$, in all return data series, except OSEBENCH, one of the Equal Weights combined models ranks first, with $\hat{\alpha}/\alpha$ closest to 1. Specifically, in IBEX return data series the SAV+AS-CAViaR(EqW) ranks first with 1.0085 and in SMI return data series the SAV+I-CAViaR(EqW) ranks first with 1.0388. Further, AS-CAViaR in three cases, I-CAViaR especially in OSEBENCH return data series, the SAV+AS-CAViaR model in three cases and the AS+I-CAViaR model in three cases, all have ratios mostly above 1. Thus, this group of models consistently under-estimates 5% risk levels in these markets. Also, in IBEX return data series SAV-CAViaR model, the SAV+I-CAViaR model and SAV+I-CAViaR(EqW) model display values below 1, 0.9682, 0.9178 and 0.9178 accordingly. Thus, this group of models, in IBEX return data series, over-estimates 5% risks levels.

As far as $\alpha = 1\%$ is concerned, a different story applies. As a consequence, SAV+AS-CAViaR, SAV+AS-CAViaR(EqW) and SAV+I-CAViaR(EqW) models, in IBEX return data series, display the most desirable result, all with 1.1094. Possibly, this effect is due to the fact that the cumulative AIC weights are close to 0.5. Also, in OSEBENCH return data series, a different story applies. Here the AS-CAViaR model has the best performance with 1.0085. Also, SAV+AS-CAViaR and SAV+AS-CAViaR(EqW) models over-estimates 1% risks levels, with the same value 0.9581. In SMI return data series, at $\alpha = 1\%$, AS+I-CAViaR and AS+I-CAViaR(EqW) models have the best performance with 1.0085. Further, SAV-CAViaR in three cases, AS-CAViaR in IBEX and OSEBENCH return data series, I-CAViaR in three cases, SAV+I-CAViaR in three cases and AS+I-CAViaR in IBEX return data series, all have ratios mostly above 1. Thus, this group of models under-estimates 1% risk levels.

At α =5%, AIC Weighted CAViaR models and single CAViaR models do not display a ratio $\hat{\alpha}/\alpha$ close enough to the optimum level, except in OSEBENCH data set series. Thus, the AIC Weighted CAViaR models places second in one out of three data sets, while the Equal Weighted CAViaR models places first in two out of three data set at α =5%. At α=1%, we notice controversial and confusing results, as AIC Weighted CAViaR models introduce a $\hat{\alpha}/\alpha$ ratio close enough to the optimum level. For example, both in the case of index IBEX for SAV+AS-CAViaR model and in the case of index SMI for AS+I-CAViaR model, we have desirable results. Also, in IBEX data set series SAV+AS-CAViaR(EqW) and SAV+I-CAViaR(EqW) and AS+I-CAViaR(EqW) in SMI data set series rank first. In OSEBENCH data set series, a different story applies as AS-CAViaR has the best performance. Thus, the AIC Weigthed CAViaR models places first in two out of three data sets and Equal Weighted CAViaR models places first in two out of three data sets, too.

Table 3.6: CAViaR model evaluation using the check-loss function applied to financial data series

Models	Ibex	Osebench	Smi
α =0.05			
SAV-CAViaR	0.9682(0.3925)	1.1699(0.2219)	1.0690(**0.00005**)
AS-CAViaR	1.1901(0.1301)	1.2506(0.7864)	1.1901(0.9919)
I-CAViaR	1.0287(0.0907)	1.1901(0.0717)	1.0892(0.6928)
SAV+AS-CAViaR	1.1094(0.1163)	1.2405(0.2918)	1.1497(0.5026)
SAV+I-CAViaR	0.9178(0.1163)	[1.1396](0.1119)	1.0892(0.3857)
AS+I-CAViaR	1.1497(0.0848)	1.2002(0.6571)	1.1598(0.2390)
SAV+AS-CAViaR(EqW)	[1.0085](0.6283)	1.2304(0.3128)	1.0993(**0.0479**)
SAV+I-CAViaR(EqW)	0.9178(0.2175)	1.1497(**0.0291**)	[1.0388](0.2457)
AS+I-CAViaR(EqW)	1.0388(0.5351)	1.1497(0.1072)	1.1598(0.2390)
α =0.01			
SAV-CAViaR	1.1598(0.4623)	1.2607(0.3228)	1.5632(0.3209)
AS-CAViaR	1.5632(0.0887)	[1.0085](0.1851)	1.2102**(0.0328)**
I-CAViaR	1.5128(0.3369)	1.6641(0.2904)	1.0590(0.5024)
SAV+AS-CAViaR	[1.1094](0.4923)	0.9581(0.1641)	1.2102(0.4430)
SAV+I-CAViaR	1.2607(0.4241)	1.2607(0.3228)	1.3615(0.3878)
AS+I-CAViaR	1.6137(0.5262)	1.0590(0.2074)	[1.0085](0.5231)
SAV+AS-CAViaR(EqW)	[1.1094](0.4923)	0.9581(0.1641)	1.2607(0.4241)
SAV+I-CAViaR(EqW)	[1.1094](0.4821)	1.1598(0.2686)	1.3615(0.3878)
AS+I-CAViaR(EqW)	1.6137(0.3131)	1.1094(0.2309)	[1.0085](0.5231)

Note: Entries in each table represent the $\hat{\alpha}/\alpha$. Numbers in brackets next to each ratio represent the p-values of the conditional LR test. Boxed numbers indicate the favored model, bold indicates the model is rejected by CC test (at a 5% level).

Examining both nominal levels, SAV+I-CAViaR(EqW) and SAV+AS-CAViaR(EqW) display the best performance as they rank first in two out of six data set series. The AS-CAViaR model shows the worst performance (value far enough from 1) in three out of six cases, in three data set series and at α= 5%, 1%. The AS+I-CAViaR model has the second place among all the models at α=1% in OSEBENCH data set, the I-CAViaR model has the second place at α=5% in IBEX data set, the SAV+I-CAViaR(EqW) and AS+I-CAViaR(EqW) models have the second place at α=5% in OSEBENCH data set. SAV-CAViaR has the second place at α=5% in SMI data set and at α=1% in IBEX data set. The AS+I-CAViaR model has the second place among all the models at α=1% for OSEBENCH data set and I-CAViaR displays second place in SMI data set series, at α = 1%. Thus, SAV-CAViaR and I-CAViaR models appear good performance, and they take the second place, at both nominal levels.

Among the single models, the SAV-CAViaR model shows the best performance in three out of six cases and I-CAViaR model has the second place as it shows the best performance in two out of six cases. While, among single models, the single AS-CAViaR model shows the worst performance (value far enough from 1) in four out of six cases, at both nominal levels. Among AIC Weighted CAViaR models, the AS+I-CAViaR model shows the worst performance(value far enough from 1) in three out of six cases, and SAV+I-CAViaR the best performance (value close enough to 1) in three out of six cases, at both nominal levels. Among Equal Weighted CAViaR models, the SAV+I-CAViaR(EqW) model shows the worst performance (value far enough from 1) in three out of six cases, however the same model appears relatively successful values of $\hat{\alpha}/\alpha$ ratio at both nominal levels.

Also, SAV+AS-CAViaR(EqW) model performs good enough to take the second place, among Equal Weighted CAViaR models, at both nominal levels. Table 3.6 contains the p-values of the conditional LR test for each model over the IBEX, OSEBENCH and SMI index, at $\alpha = 1\%, 5\%$. First, at $\alpha = 5\%$, none of the models are rejected in IBEX return data series, as the LR-p-values are higher that the 0.05, with H_0: $\hat{\alpha} = \alpha$. In OSEBENCH return data series, the SAV+I-CAViaR(EqW) model with LR-p-value 0.0291, is rejected. Finally, in SMI return data series, the SAV-CAViaR model and the SAV+AS-CAViaR(EqW) model with LR-p-values 0.00005 and 0.0479, respectively, are rejected. At 1% confidence level, none of the models are rejected in IBEX and OSEBENCH return data series, as the LR-p-values are higher that the 0.05. Finally, in SMI return data series AS-CAViaR model with LR-p-value 0.0328, is rejected. However, before reject a model, it is reasonable to check our estimates with other tests.

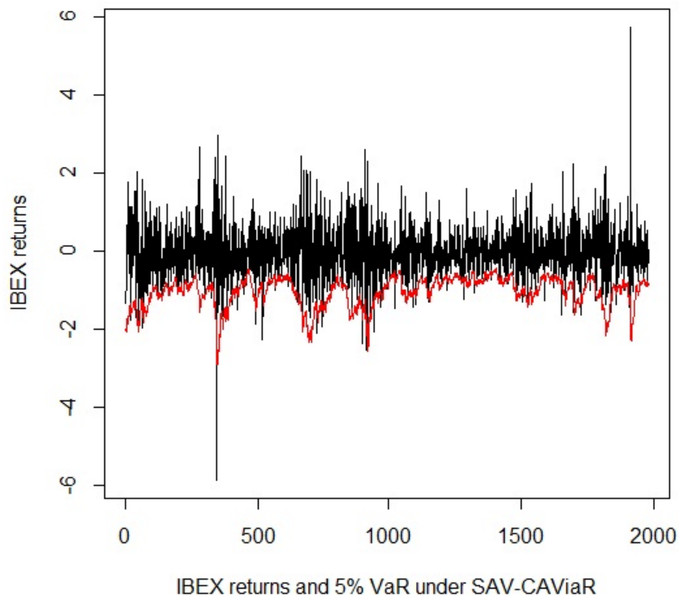

Figure 3.4 Plot of IBEX returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under SAV-CAViaR.

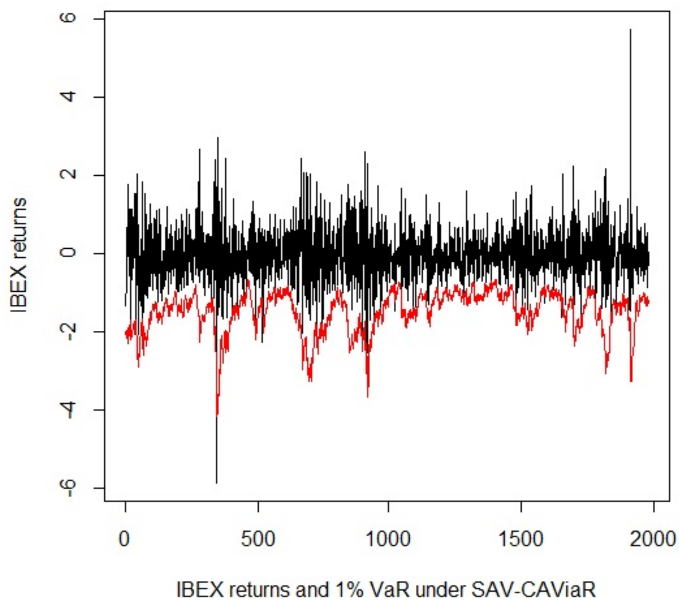

Figure 3.5 Plot of IBEX returns January 2009 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under SAV-CAViaR.

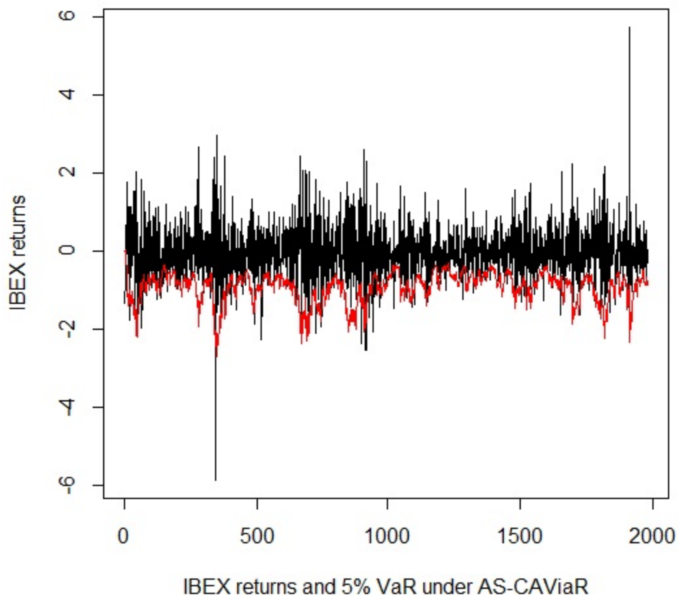

Figure 3.6 Plot of IBEX returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under AS-CAViaR.

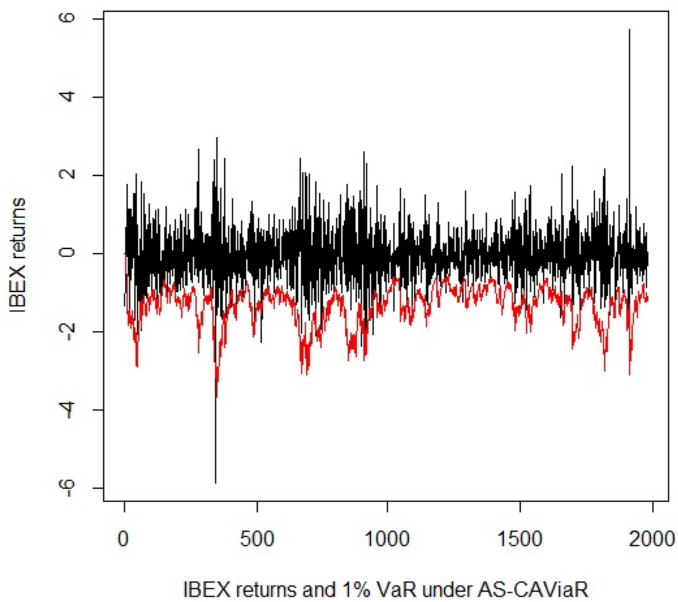

Figure 3.7 Plot of IBEX returns January 2009 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under AS-CAViaR.

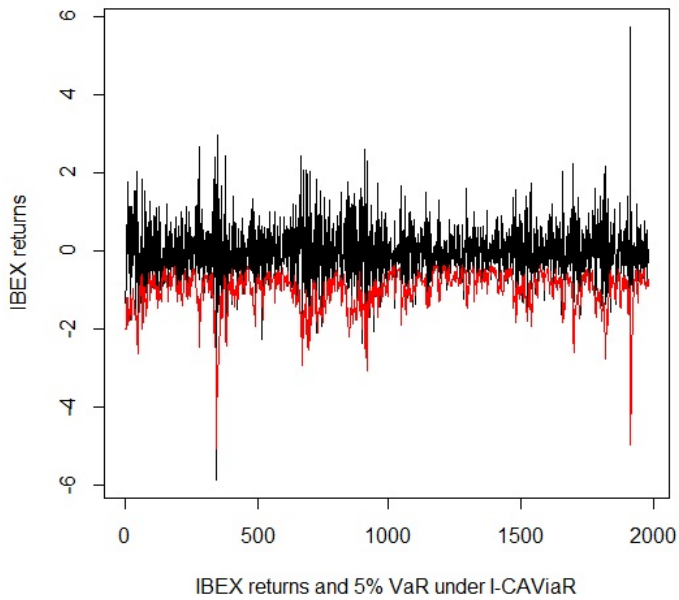

Figure 3.8 Plot of IBEX returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under I-CAViaR.

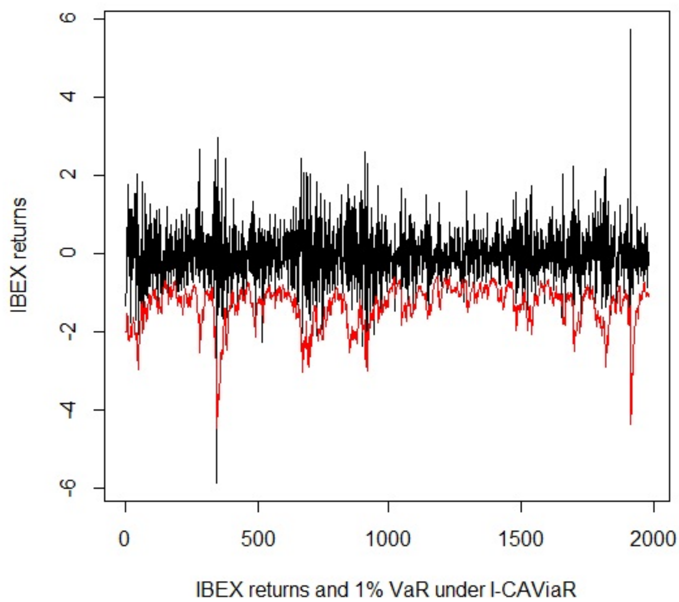

Figure 3.9 Plot of IBEX returns January 2009 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under I-CAViaR.

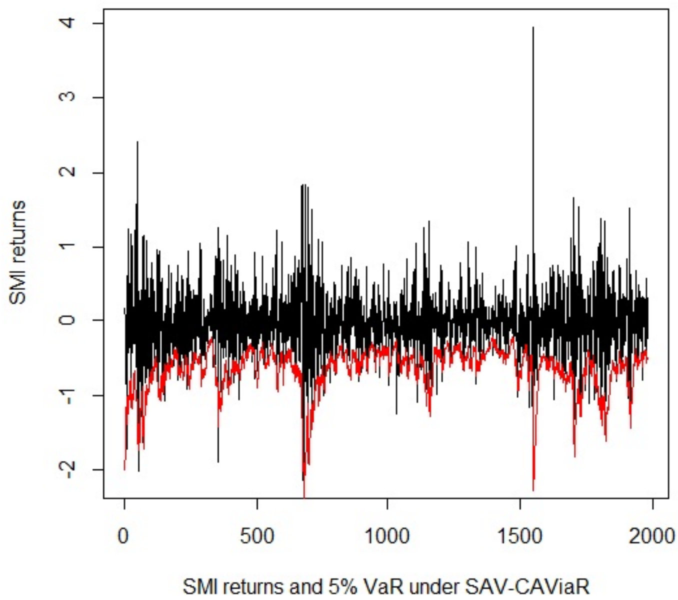

Figure 3.10 Plot of SMI returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under SAV-CAViaR.

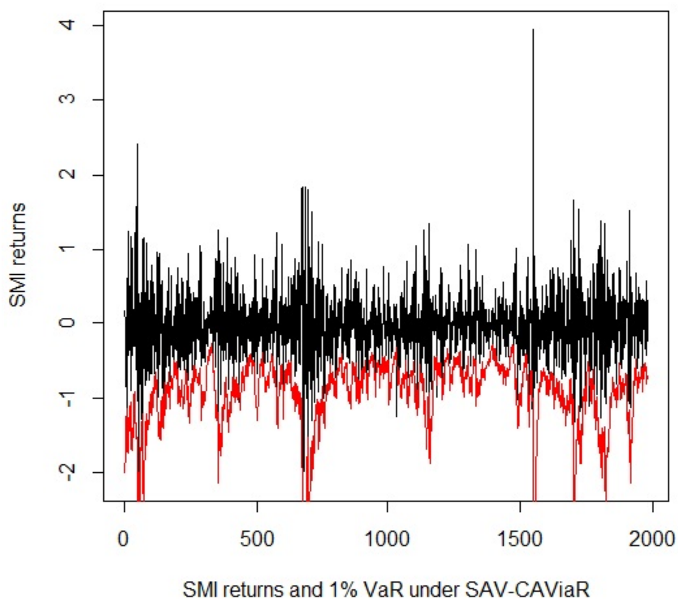

Figure 3.11 Plot of SMI returns January 2009 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under SAV-CAViaR.

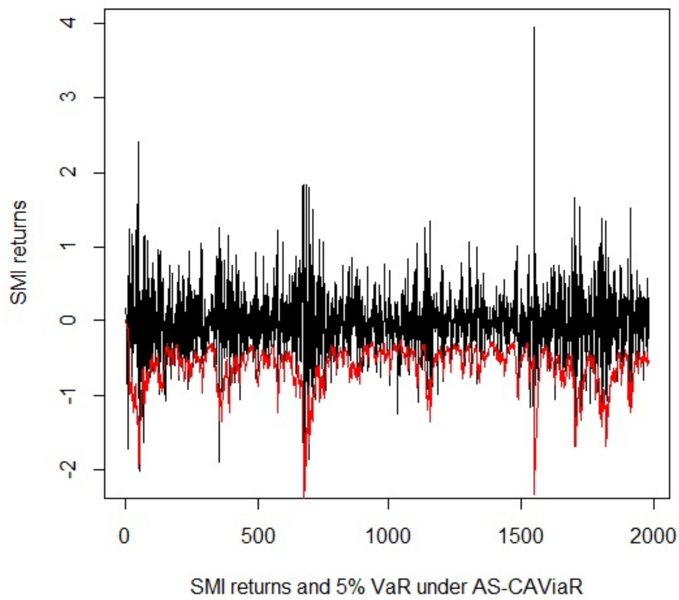

Figure 3.12 Plot of SMI returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under AS-CAViaR.

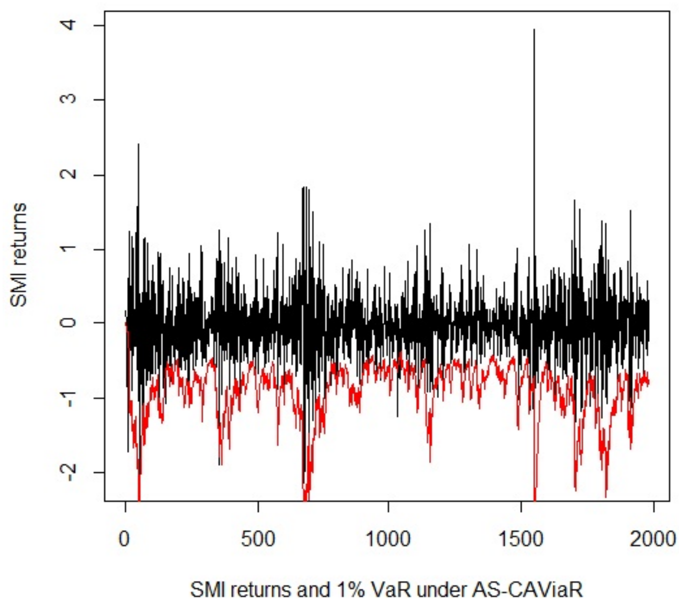

Figure 3.13 Figure 3.8 Plot of SMI returns January 2009 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under AS-CAViaR.

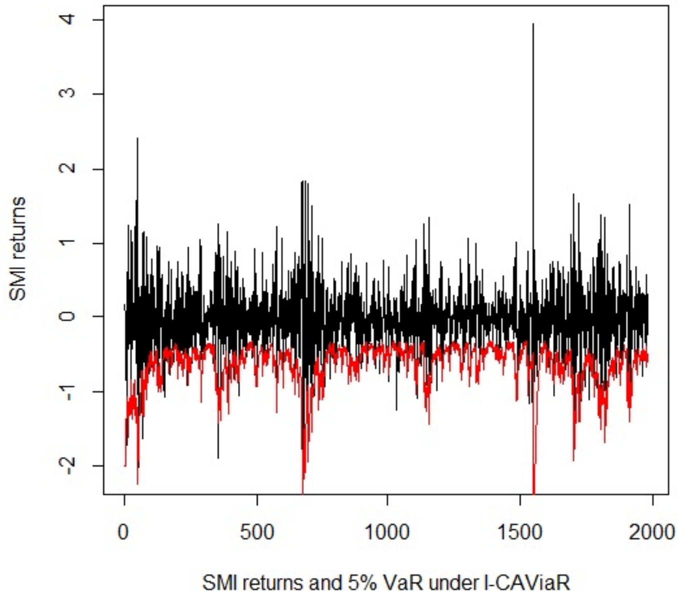

Figure 3.14 Plot of SMI returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under I-CAViaR.

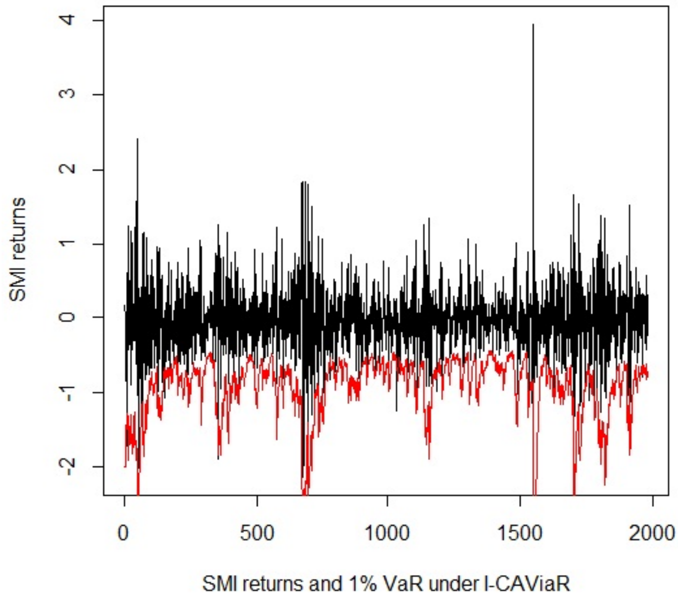

Figure 3.15 Plot of SMI returns January 2009 to Semptember 2016(black lines), together with1% forecasted VaR(red lines) under I-CAViaR.

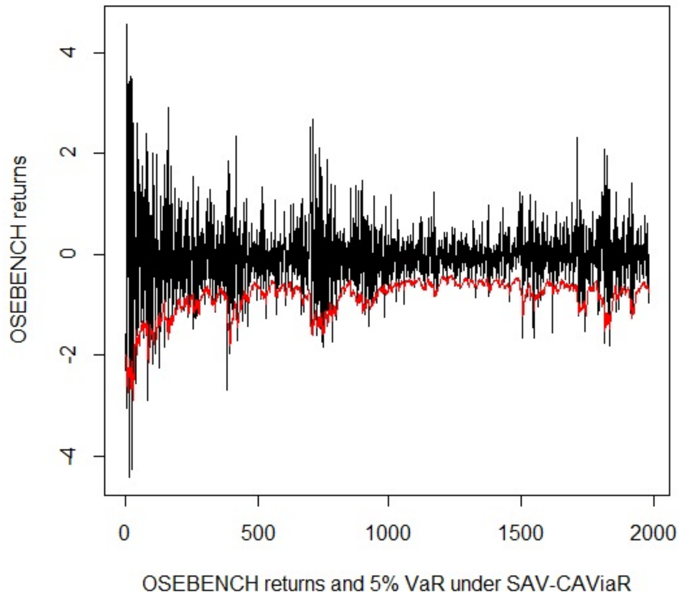

Figure 3.16 Plot of OSEBENCH returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under SAV-CAViaR.

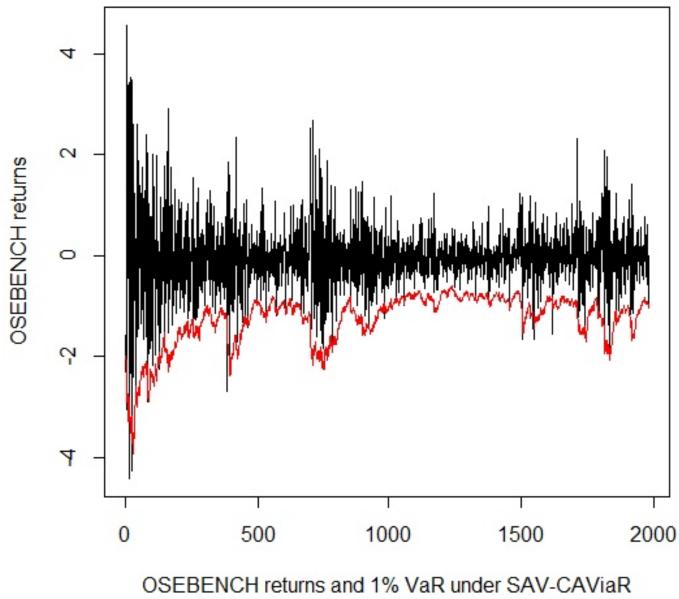

Figure 3.17 Plot of OSEBENCH returns January 2009 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under SAV-CAViaR.

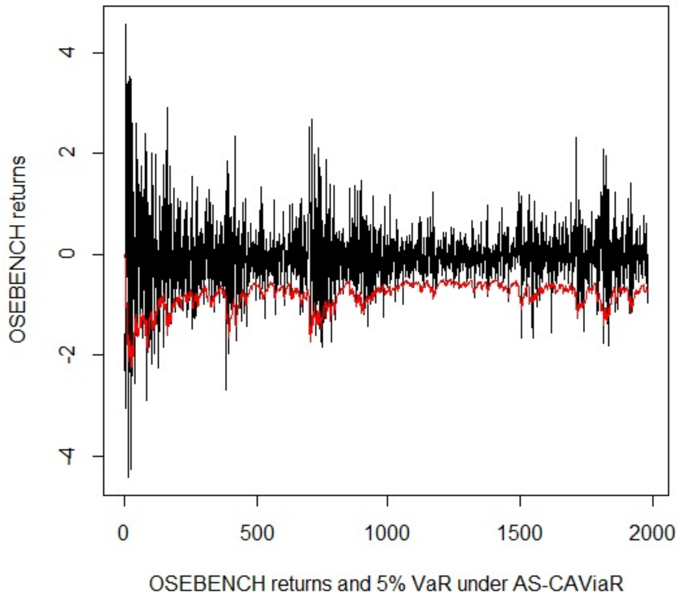

Figure 3.18 Plot of OSEBENCH returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under AS-CAViaR.

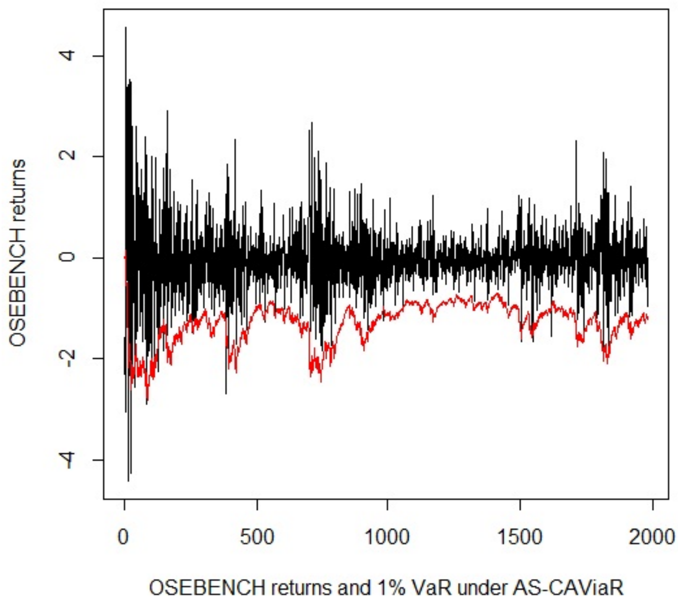

Figure 3.19 Plot of OSEBENCH returns January 2009 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under AS-CAViaR.

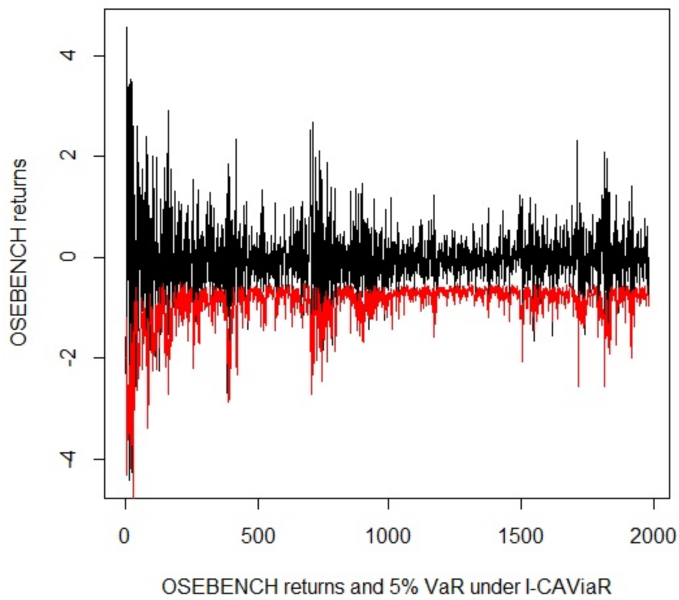

Figure 3.20 Plot of OSEBENCH returns January 2009 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under I-CAViaR.

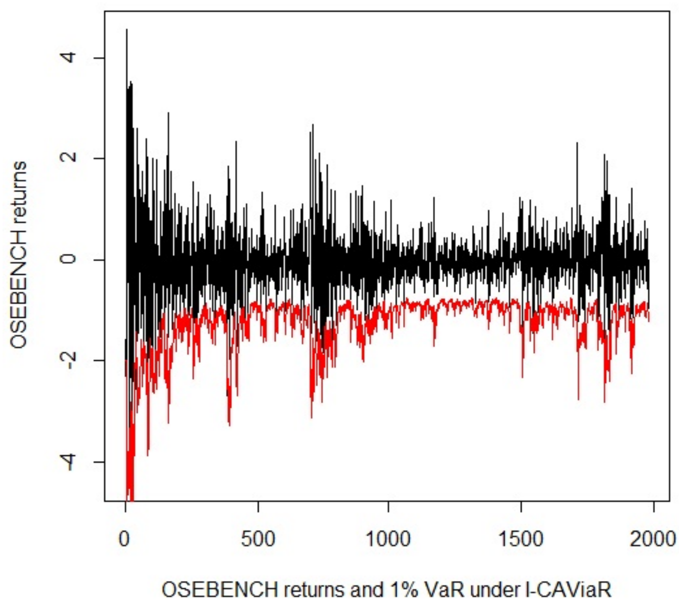

Figure 3.21 Plot of OSEBENCH returns January 2009 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under I-CAViaR.

Figures 3.4-3.21 illustrate the VaR forecasts in IBEX,SMI and OSEBENCH return data series, for the SAV-CAViaR, AS-CAViaR and I-CAViaR models, at α = 5% ,1%. As we refer above, the forecast sample includes the after-crisis period. More specifically, includes the period from January 1,2009 to September 30, 2016. All the series and their VaR's seem to have high-volatility periods towards their start, in each case corresponding to the effects of the 2008 financial crisis .

3.3.2 VaR forecast for the 2014-2016 period

Table 3.7 reports the one-step-ahead $\hat{\alpha}/\ \alpha$ ratio and the corresponding Christoffersens conditional coverage test. The combined CAViaR models SAV+AS-CAViaR, SAV+I-CAViaR and AS+I-CAViaR derive from the AIC Weighted Individual CAViaR forecasts. The combined CAViaR models SAV+AS-CAViaR(EqW), SAV+I-CAViaR(EqW) and AS+I-CAViaR(EqW) derive from Equal Weighted Individual CAViaR forecasts, thus weights are equal to 0.5. The estimated weights add up to 1. The results are based on IBEX,SMI and OSEBENCH return data series and cover nominal levels α of 1% and 5%. The most desirable ratio in each index is boxed, bold indicates the model is rejected by CC test (at 5% level). Naturally, we prefer models with ratios close to 1, while again ratios less than 1 are preferred to those above 1 that are equidistant from 1.

Table 3.7: CAViaR model evaluation using the check-loss function applied to financial data series

Models	Ibex	Osebench	Smi
α =0.05			
SAV-CAViaR	0.8226(0.4800)	0.9705 (0.7506)	0.7194(0.1716)
AS-CAViaR	0.9645 **(0.0231)**	1.1176**(0.0180)**	0.8920 (0.1906)
I-CAViaR	0.7943(0.9092)	0.9117(0.6259)	0.6618(0.2091)
SAV+AS-CAViaR	0.9361(0.0784)	0.9411(0.6875)	0.8920 (0.1906)
SAV+I-CAViaR	0.7659(0.9708)	0.9705 (0.7506)	0.6043(0.2522)
AS+I-CAViaR	0.8226(0.8487)	1.0588**(0.0443)**	0.8920 (0.1906)
SAV+AS-CAViaR(EqW)	0.7943(0.9092)	0.9411(0.6875)	0.6618(0.2091)
SAV+I-CAViaR(EqW)	0.7659(0.9708)	0.9411(0.6875)	0.6618(0.2091)
AS+I-CAViaR(EqW)	0.7943(0.9092)	0.8823 (0.5661)	0.6330(0.2300)
α =0.01			
SAV-CAViaR	0.9929 (0.7076)	1.6176(0.1641)	1.0071 (0.1426)
AS-CAViaR	1.4184(0.1259)	1.4705**(0.0062)**	1.4388**(0.0001)**
I-CAViaR	0.8510(0.7480)	0.4411(0.8703)	0.5755(0.8294)
SAV+AS-CAViaR	1.1347(0.6680)	0.7352(0.7853)	0.8633(0.7463)
SAV+I-CAViaR	0.7092(0.7891)	1.0294 (0.0556)	0.7194(0.7876)
AS+I-CAViaR	1.2765(0.6292)	0.7352**(0.0242)**	0.5755(0.8294)
SAV+AS-CAViaR(EqW)	1.1347(0.6680)	0.7352(0.7853)	0.8633(0.7463)
SAV+I-CAViaR(EqW)	0.7092(0.7891)	0.5882(0.8276)	0.8633(0.7463)
AS+I-CAViaR(EqW)	0.8510(0.1031)	0.7352**(0.0242)**	0.5755(0.8294)

Note: Entries in each table represent the $\hat{\alpha}/\alpha$. Numbers in brackets next to each ratio represent the p-values of the conditional LR test. Boxed numbers indicate the favored model, bold indicates the model is rejected by CC test (at a 5% level).

First, at α = 5%, in all return data series, except IBEX, one of the AIC Weighted CAViaR models ranks first, with $\hat{\alpha}/\ \alpha$ closest to 1. Specifically, in OSEBENCH return data series the SAV+I-CAViaR ranks first with 0.9705 and in SMI return data series the SAV+AS-CAViaR and AS+I-CAViaR models rank first with 0.8920. Also, SAV-CAViaR and AS-CAViaR illustrate a ratio closest to 1, for the same return data series. Further, all the models in IBEX and SMI return data series and the majority of the models in OSEBENCH return data series, have ratios mostly below 1. Thus, this group of models consistently over-estimates 5% risk levels in these markets. As far as α = 1% is concerned, none of the Equal Weighted CAViaR models display a ratio closest to 1. As a cosenquence, SAV-CAViaR, dispalys the best performance in IBEX and SMI return data series with 0.9929 and 1.0071, respectively. SAV+I-CAViaR model, display the most desirable result, in OSEBENCH return data series, with 1.0294. Also, SAV-CAViaR and AS-CAViaR models under-estimates 1% risks levels, with 1.6176 and 1.4705, in OSEBENCH return data series and AS-CAViaR under-estimates 1% risk leves in SMI return data series, with 1.4388.

At α =5%, Equal Weighted CAViaR models does not display a ratio $\hat{\alpha}/\alpha$ close enough to the optimum level. Thus, the AIC Weighted CAViaR models places second in two out of three data sets, while the single CAViaR models places first in three out of three data sets at α=5%. At α=1%, we notice controversial and confusing results, as both AIC Weighted CAViaR models, specifically SAV+I-CAViaR model, and single CAViaR models(SAV-CAViaR) introduce a $\hat{\alpha}/\alpha$ ratio close enough to the optimum level. Thus, the AIC Weighted CAViaR models places second in one out of three data sets. However, single CAViaR models places first in two out of three data sets. Examining both nominal levels, SAV-

CAViaR model display the best performance as it ranks first in three out of six data set series. The AS+I-CAViaR(EqW) model shows the worst performance (value far enough from 1) in three out of six cases, in three data set series and at α= 5%, 1%.

Among the single models, the SAV-CAViaR model shows the best performance in three out of six cases and AS-CAViaR model has the second place as it shows the best performance in two out of six cases. While, among single models, I-CAViaR model shows the worst performance (value far enough from 1) in four out of six cases, at both nominal levels. Among AIC Weighted CAViaR models, the SAV+I-CAViaR model shows the worst performance (value far enough from 1) in three out of six cases, and SAV+AS-CAViaR the best performance (value close enough to 1) in four out of six cases, at both nominal levels, however the same model appears relatively successful values of $\hat{\alpha}$ / α ratio at both nominal levels. Among Equal Weighted CAViaR models, the SAV+I-CAViaR(EqW) and AS+I-CAViaR(EqW) models show the worst performance (value far enough from 1) in three out of six cases.

Also, Table 3.7 contains the p-values of the conditional LR test for each model over the IBEX, OSEBENCH and SMI index, at α = 1%, 5%. First, at α = 5%, none of the models are rejected in SMI return data series, as the LR-p-values are higher that the 0.05 and we accept our H_0 : $\hat{\alpha} = \alpha$. In OSEBENCH return data series, AS-CAViaR and SAV+I-CAViaR models, are rejected. Finally, in IBEX return data series, the AS-CAViaR model with LR-p-values 0.0231 is rejected. At 1% confidence level, none of the models are rejected in IBEX return data series, as the LR-p-values are higher that the 0.05. Finally, in OSEBENCH and SMI return data series, AS-CAViaR, AS+I-CAViaR, AS+I-CAViaR(EqW) and AS-CAViaR, are rejected respectively. However, before reject a model, it is reasonable to check our estimates with other tests.

Let us introduce some plots, which are illustrated the VaR forecasts in IBEX, SMI and OSEBENCH return data series, for the SAV-CAViaR, AS-CAViaR and I-CAViaR models, at $\alpha = 5\%$,1%. As we refer above, the forecast sample includes the period from January 1,2014 to September 30, 2016. We illustrate the plots in Figures 3.22-3.33 below.

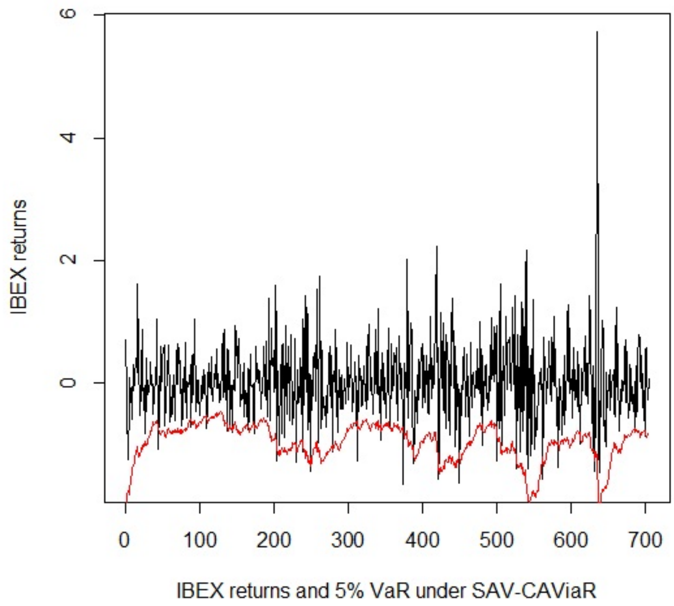

Figure 3.22 Plot of IBEX returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under SAV-CAViaR.

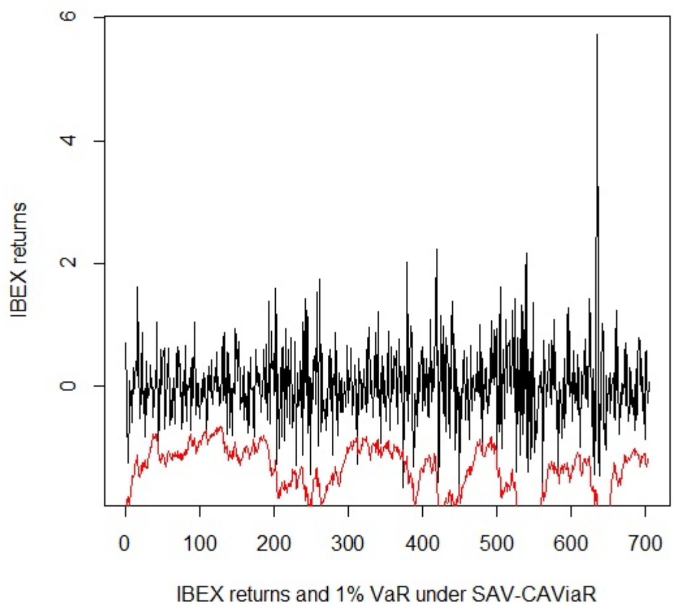

Figure 3.23 Plot of IBEX returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under SAV-CAViaR.

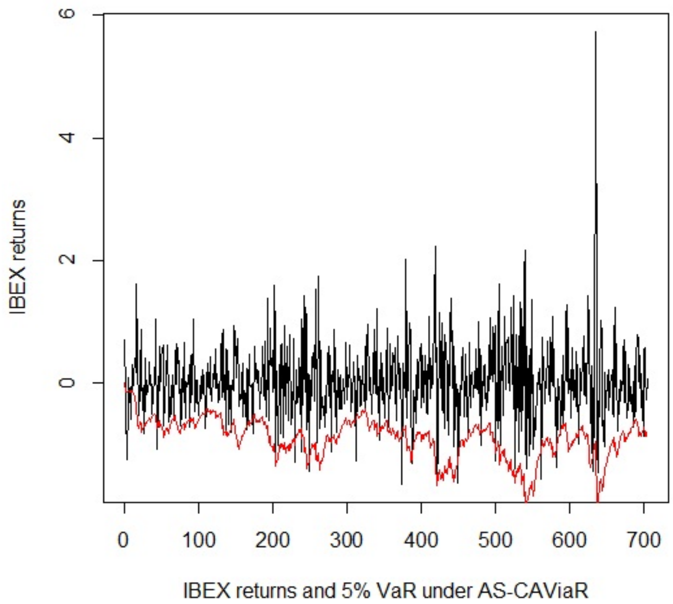

Figure 3.24 Plot of IBEX returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under AS-CAViaR.

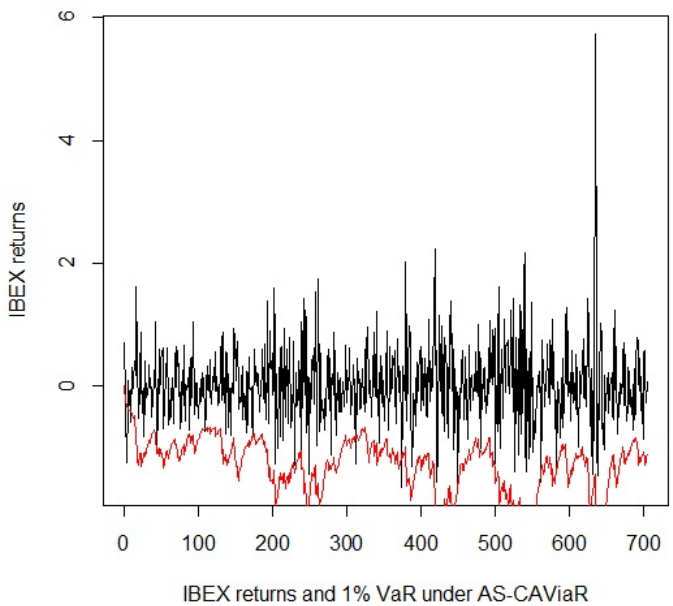

Figure 3.25 Plot of IBEX returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under AS-CAViaR.

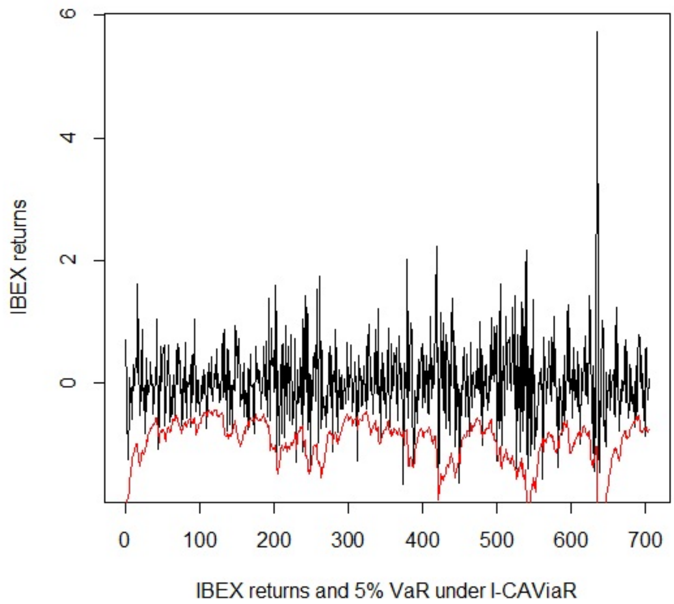

Figure 3.26 Plot of IBEX returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under I-CAViaR.

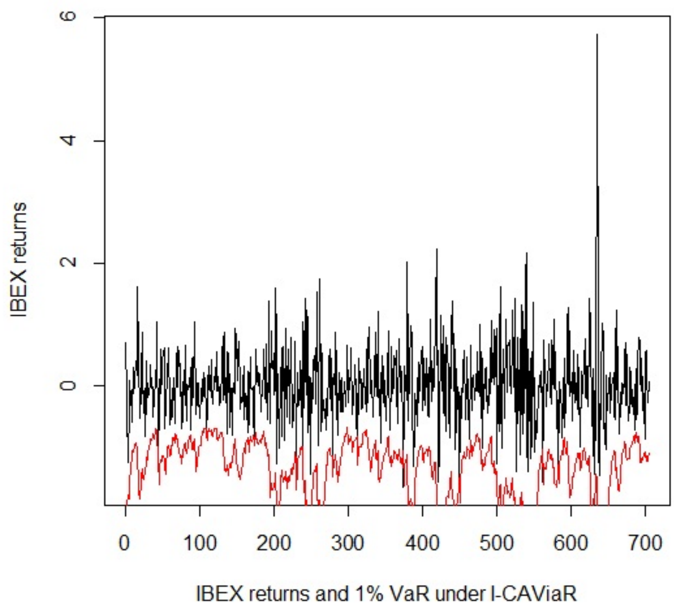

Figure 3.27 Plot of IBEX returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under I-CAViaR.

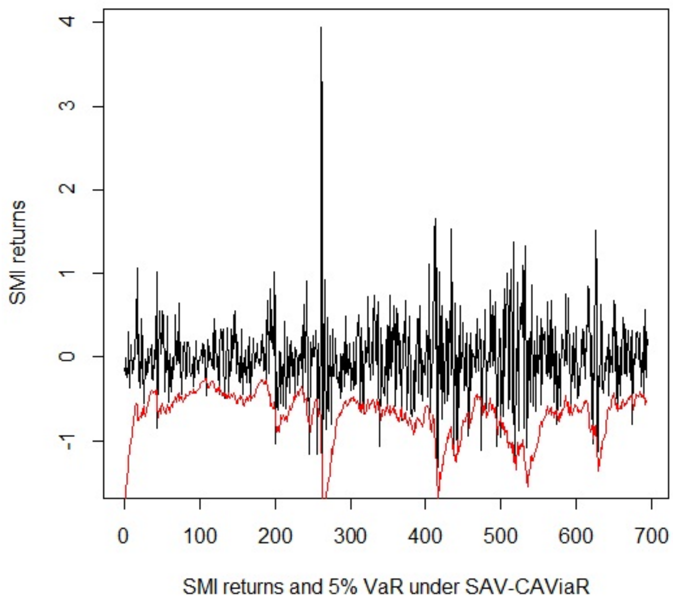

Figure 3.28 Plot of SMI returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under SAV-CAViaR.

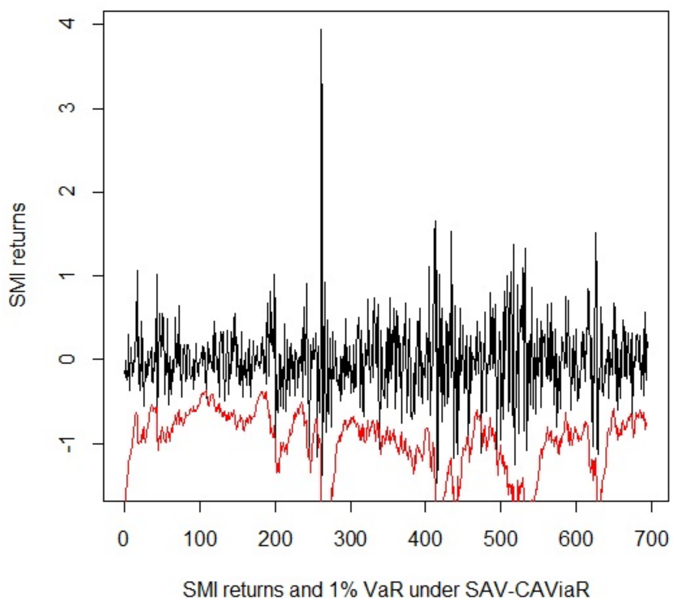

Figure 3.29 Plot of SMI returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under SAV-CAViaR.

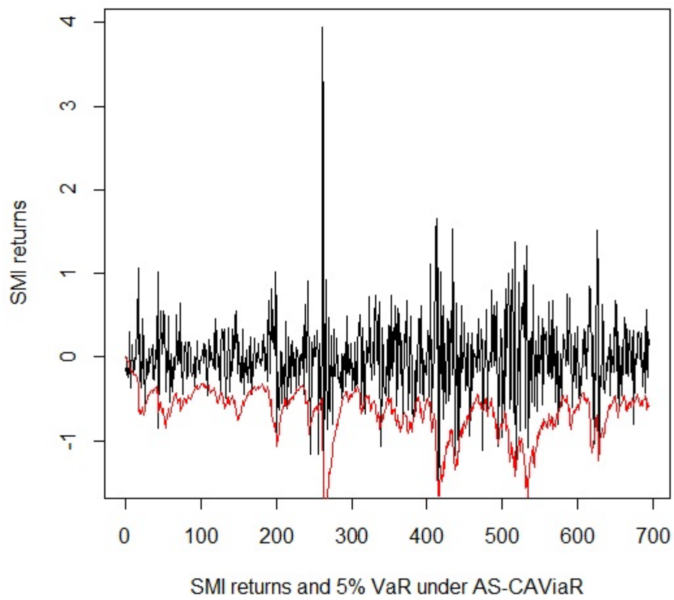

Figure 3.30 Plot of SMI returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under AS-CAViaR.

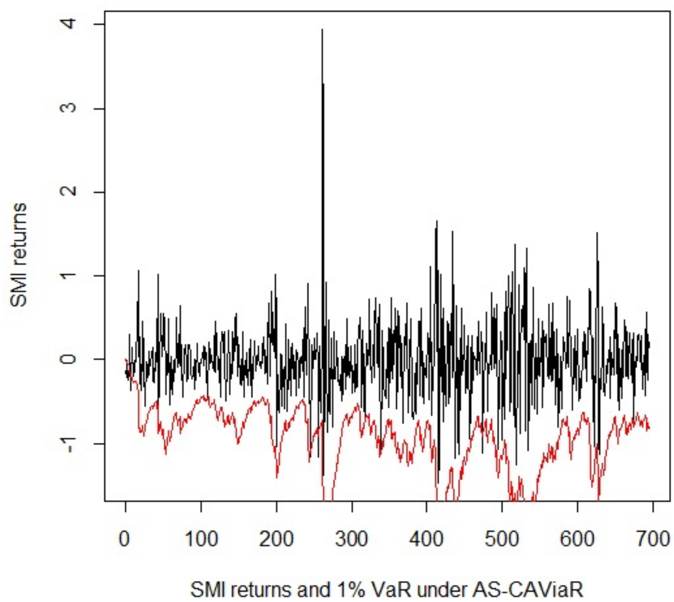

Figure 3.31 Figure 3.32 Plot of SMI returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under AS-CAViaR.

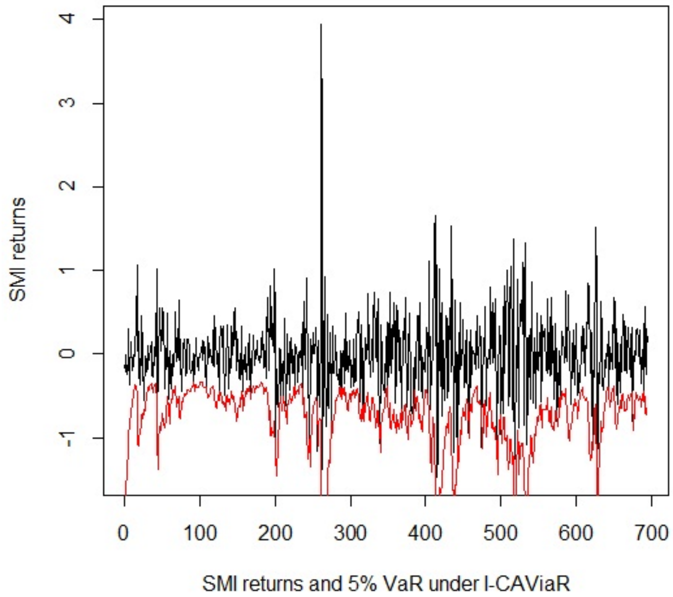

Figure 3.32 Plot of SMI returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under I-CAViaR.

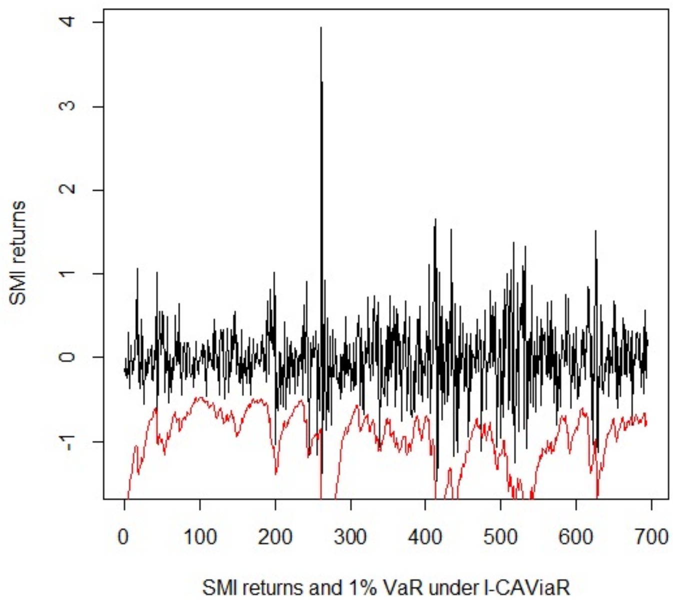

Figure 3.33 Plot of SMI returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under I-CAViaR.

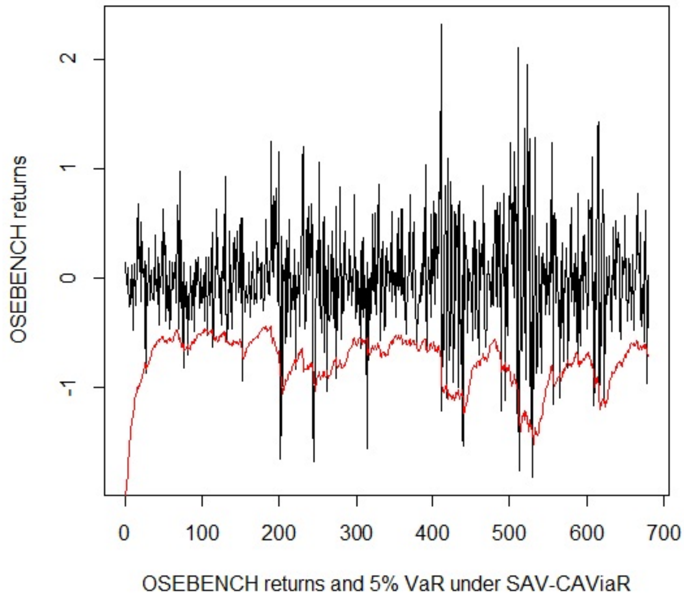

Figure 3.35 Plot of OSEBENCH returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under SAV-CAViaR.

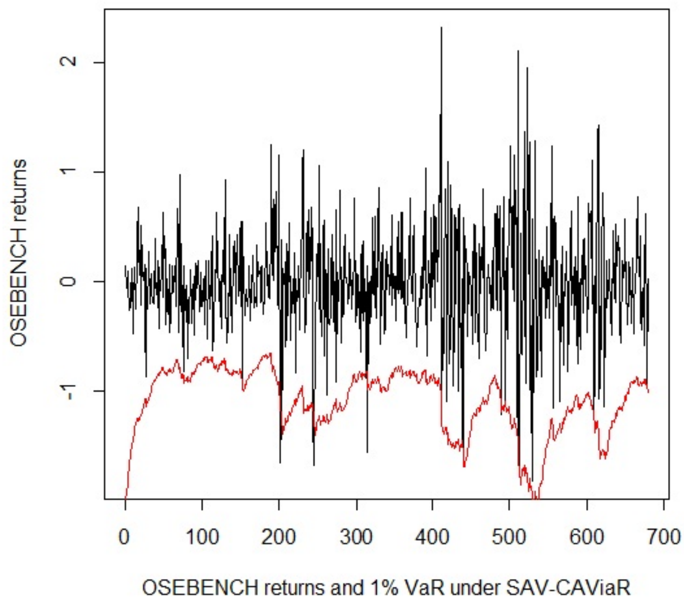

Figure 3.36 Plot of OSEBENCH returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under SAV-CAViaR.

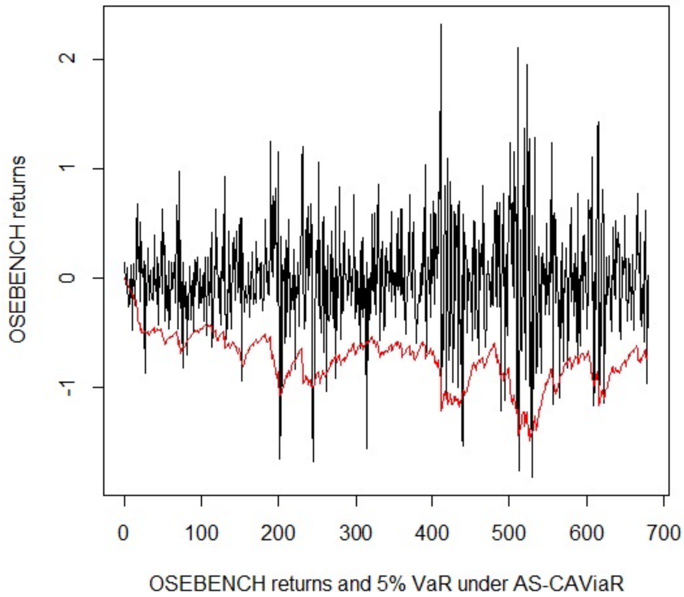

Figure 3.37 Plot of OSEBENCH returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under AS-CAViaR.

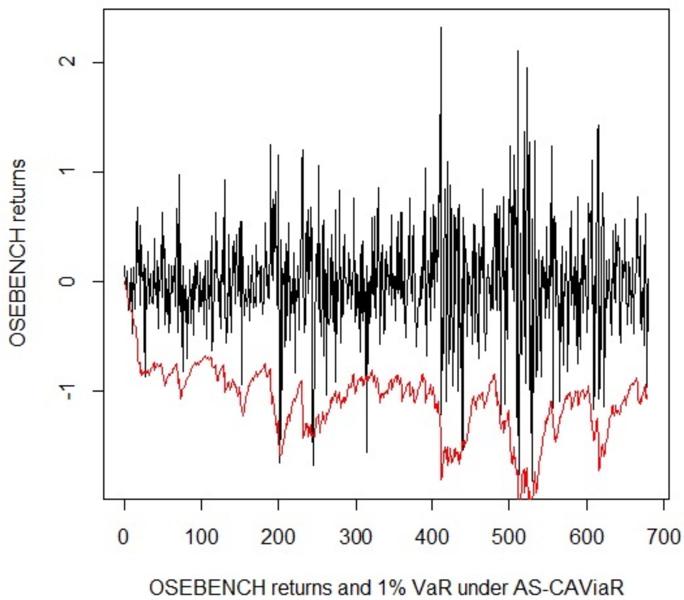

Figure 3.38 Plot of OSEBENCH returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under AS-CAViaR.

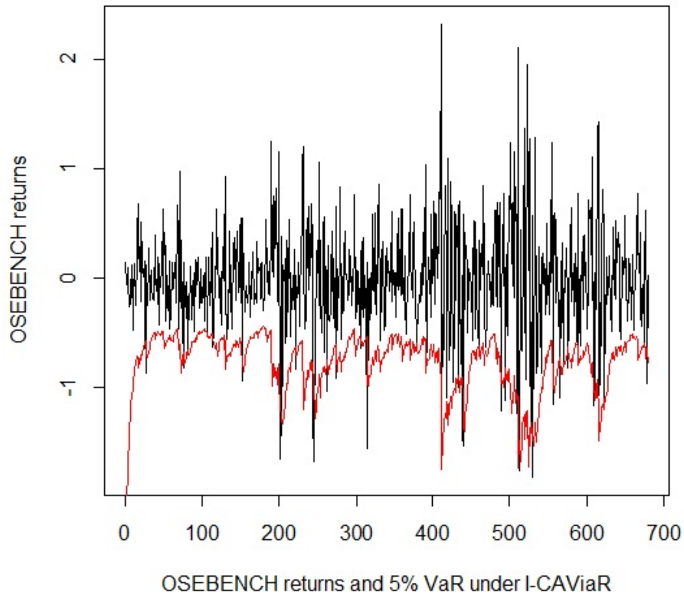

Figure 3.15 Plot of OSEBENCH returns January 2014 to Semptember 2016(black lines), together with 5% forecasted VaR(red lines) under I-CAViaR.

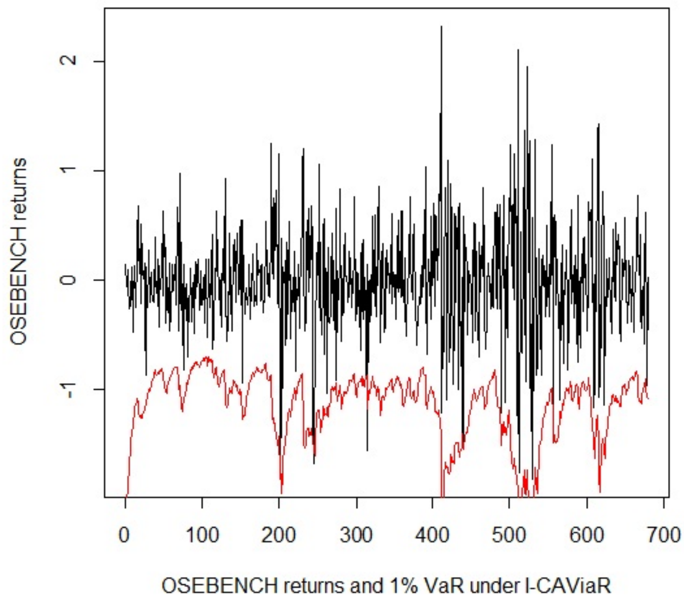

Figure 3.18 Plot of OSEBENCH returns January 2014 to Semptember 2016(black lines), together with 1% forecasted VaR(red lines) under I-CAViaR.

Chapter 4

Concluding Remarks

The importance of VaR became institutional in August 1996, when U.S. bank regulators adopted a "market risk" supplement to the Basel Accord of 1988. VaR has, thus, become a risk measure for setting capital-adequacy standards of U.S. commercial banks [23]. The data used in our empirical application consist of 15 years, approximately,of daily returns on IBEX, SMI and OSEBENCH. The data were obtained from Yahoo Finance and covered the period from January 1, 2002 to September 30, 2016. After the theoretical model specification and estimation based on the Engle-Manganelli methodology[13],we implement an applied exercise.

Our goal is to test and compare alternative models of one-step-ahead forecasts of $f_{t,\alpha}(\hat{\beta})$, the conditional α-level quantile of the distribution of y_t. We perform the evaluation in an out-of-sample period. This involves dividing the sample of size T into an within-sample part of size n and an out-of sample part of size m, so that T = m+ n. The within-sample portion is used to produce the estimates of betas, and the evaluation is performed over the remaining out-of-sample portion. In particular, we estimate three CAViaR models, the SAV-CAViaR model, the AS-CAViaR model and the I-CAViaR model and we consider the 5% and 1% VaR forecasts. Since none of the forecasts of single models is dominant and there is no universally accepted accepted ranking of the various methods, we decided to check if a forecast combination method based on AIC or in equal weights, may display better results compared to individual forecast. In practice, we assess whether a combined conditional CAViaR forecast can produce better predictions than the estimates based on single CAViaR models at nominal levels of 1% and 5%.

First, we use real financial data to validate the $\hat{\alpha}$ /α generated by our single CAViaR models at both nominal levels. The results for the 2009-2016 forecast period, show that the SAV-CAViaR model shows the best performance in three out of six cases, at both nominal levels, whereas the AS-CAViaR model appears less successful results in comparison with other models. Second, we create new estimates, by combining single CAViaR models alternatively. Finally, we display the forecast estimates from both single and combined CAViaR models and we statistically and non-statistically assess whether the single or weighted models display the best performance at nominal levels of 1% and 5%.

The empirical results for the 2009-2016 forecast period, indicate that the combined (both the AIC Weighted and Equal Weighted) CAViaR models are competitive to the single model alternatives. More specifically, although we notice controversial results, examining all the models, SAV+I-CAViaR(EqW) and SAV+AS-CAViaR(EqW) models display the best performance as they rank first in two out of six data set series. The AS-CAViaR model shows the worst performance(value far enough from 1) in three out of six cases, in three data set series and at α= 5%, 1%.

As far as the empirical results for the 2014-2016 forecast period, is concerned, we notice quite different results. In other words, the combined (both the AIC Weighted and Equal Weighted) CAViaR models display a ratio $\hat{\alpha}$ /α closest to 1 in three out of six data set series, while single CAViaR models display a ratio $\hat{\alpha}$ $/\alpha$ closest to 1 in five out of six data set series. Especially, at α=1%, both the AIC and Equal Weighted CAViaR models dispaly a bad performance, as their results, are strongly far enough from 1.

Perhaps, each of the findings should be useful for practitioners and institutions. However, many additional questions emerge for future research. Due to time limitations, we only focus on 1-day forecasting, in three models and in three return data series. Extensions to include more than three return data sets and other estimating VaR models are potential directions for further research.

Bibliography

[1] Dowd K.: Measuring Market Risk.John Wiley & Sons Ltd;2002.

[2] Gerlach RH, Zudi L, Huange H.: Exponentially smoothing the skewed laplace distribution for value-at-risk forecasting. Journal of Forecasting; 2013;32:534550.

[3] Jorion P. Risk: Measuring the risk in Value at Risk.Financial Analysts Journal; 1996; 52: 47-56.

[4] Chen C.,Gerlach R.,Hwang B., McAleer M.: Forecasting Value-at-Risk Using Nonlinear Regression Quantiles and the Intra-day Range;2011.

[5] Yamai Y., Yoshiba T.:On theValidity of Value-at-Risk: Comparative Analyses with Expected Shortfall. Institute for Monetary and Economic Studies;2002;57-86.

[6] Kuester K., Mittnik S., Paollela M.: Value-at-risk prediction: a comparison of Alternative strategies. Journal of Financial Economics;2006;4:5389.

[7] Ratuszny E.: Risk Modeling of Commodities using CAViaR Models, the Encompassing Method and the Combined Forecasts;2015;15:129-156.

[8] Jeon J.,Taylor J.: Using CAViaR Models with Implied Volatility for Value at Risk Estimation. Journal of Forecasting;2013;32:62-74.

[9] Boudoukh J., Richardson M., Whitelaw R.: The best of both worlds. Risk; 1998;11:64-67.

[10] Hendricks D.: Evaluation of Value at Risk Models using Historical Data. FRBNY Economic Policy Review;1996;37-69.

[11] Li H., Fan Y., Li Y., Zhou Y., Jin Z., Liou Z.: Approaches to VaR. Investment Practice Project;2011-2012.

[12] Gerlach R., Chen C., Chan N..: Bayesian time-varying quantile forecasting for value-at-risk in financial markets. Journal of Business Economics Statistics; 2011;29:481492.

[13] Engle RF, Manganelli S. CAViaR: Conditional autoregressive value at risk by regression quantiles. Journal of Business Economics Statistics.2004;22:367381.

[14] Tsiotas G.A quasi-Bayesian model averaging approach for conditional quantile models.Journal of Statistical Computation and Simulation;2015;85:19631986.

[15] Kupiec P. Techniques for verifying the accuracy of risk management model.Journal of Derivatives;1995;3:73-84.

[16] Christofferesen P. Evaluating interval forecasts. International Economic Review; 1998;39:841-862.

[17] Gaglianone W., Lima L.R, Linton O.: Evaluating VaR models via quantile regressions,Working Paper Series;2008;161:1-56.

[18] Chen C., Gerlach R., Lin E., Lee W.: Bayesian forecasting for financial risk management, pre and post the global financial crisis.Journal of Forecasting; 2012;31:661687.

[19] Bates J., Granger C.: The combination of forecasts. Operations Research Quarterly;1969;20:451-468.

[20] Link W. ,Barker R. :Model weights and the foundations of multimodel inference.Ecology; 2006;26262635.

[21] Taylor J.,Bunn D.: Combining forecast quantiles using quantile regression: Investigating the derived weights, estimator bias and imposing constraints. Journal of Applied Statistics;1998;25:193-206.

[22] Zou H., Yang Y.: Combining time series models for forecasting. International Journal of Forecasting;2004;20:69-84.

[23] Giacomini R. ,Komunjer I.: Evaluation and Combination of Conditional Quantile Forecasts.American Statistical Association Journal of Business & Economic Statistics; 2005; 23:416-431.

[24] Bunn D.: Forecasting with more than one model. Journal of Forecasting; 1989;8:161-166.

[25] Phoinos D.: Statistics. Zhth;1999.

[26] Bera A. : A test for normality of observations and regression residuals. International Statistical Review;1987;55: 163172.

Appendix

R is in an open code application of the object-oriented mathematical programming language S. R has developod from statistics all over the world and is a free software, which is covered by the GNU General Public License. This license, also, ensures the dissemination and modification of the software. Syntactically and functionally, R has many similarities with the language S in which is used the known statistical package S^+.

R is a complete software for management of data, estimation and graphical representation. Also, R is a completely organised and composed system and is not a specific and rigid toll, such as other data analysis softwares. R uses new methods of data analysis and has been expanded to a big collection of packages. Although, an important part of programs written in R concern specific parts od data analysis. The last statistical methodologies, are available for use in R.

For our estimations we use the following packages:

- e1071: for the Descriptive Statistics
- ucminf: for the CAViaR estimation models-Backtesting