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Cosmological Perturbations and Screening in ΛDGP Model

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Chapter 1

Introduction

Over the years people have changed their beliefs regarding our universe. In the early 1900s the common opinion was that the universe was flat, infinite and static with only visible mass and radiation. Einstein's equations showed that a universe like that couldn't be static due to gravitational pull. However, the prestige of the physicists who supported the first model was so great that Einstein modified his equations by adding a constant term in order to explain this model. Later on, with the discovery of the expansion of the universe, the big bang theory came up and the Einstein's constant was disregarded. In 1998 the accelerating expansion of the universe was discovered. In order to explain this expansion scientists had to reinsert the constant term back in Einstein's equations. This constant term is called the cosmological constant and represents the dark energy in our universe, a substance whose nature remains a mystery to this day. Some years earlier it was also discovered that there is more matter than we actually observe. As a consequence, the idea of the cold dark matter emerged. At the end of the 20^{th} century scientists had already accepted the ΛCDM model, as the one that best describes the behavior of our universe. According to the ΛCDM model our universe is $4(3+1)$ dimensional, has no curvature which means it is infinite and consists of about 70% dark energy, 30% matter and a neglect portion of radiation. It perfectly predicts the expansion of our universe, the cosmic microwave background radiation and the large scales distribution of galaxies.

However, there are some drawbacks though in the ΛCDM model. Firstly, this model originates from the Einstein's theory of General Relativity which actually can not explain why gravity is much weaker than the other forces. Additionally, General Relativity seems to be incompatible with the Standard Model. While the first one is the theory which describes the fast-moving or massive objects, the Standard Model describes with indubitable accuracy the world at very small scales. Yet when physicists tried to combine them in order to describe very small and massive objects (such as black holes) the results were gibberish. This leads us to conclude that both General Relativity and the Standard Model need modifications. None of them seems to be the ultimate theory, the "Theory of Everything", which completely describes our universe.

There are models that explain why gravity is so weak. A class of these models called "Brane models" suggest that the universe has additional and gravity seems to be weak because it propagates though all dimensions. Some of them assumes that the universe is 5D and we experience it as 4(3+1)-dimensional because the extra dimension is very small. Experiments show that if there exist an extra small dimension it has to be smaller than a few μm[1].

Recently it was shown that the extra dimension has to be not small nor compact. This model is called DGP[1] model[2]. The idea is that all standard model fields are confined to 4(3+1)- dimensions (the brane), while gravity can move freely to the brane and the extra infinite and flat dimension (the bulk). This model give rise to a correct Newtonian potential $1/r$ at small distances, while it changes at very large ones. An extension to this model was proposed by S. Bag, A. Viznyuk, Y. Shtanov and V. Sahni[3] which involved the cosmological constant in the bulk. For this reason, this model is called *phantom brane world* or *ΛDGP* model. Brane world models are also fundamental concepts in string theory. The existence of additional dimensions could be a supporting evidence towards the validity of string theory.

There has been some activity on the phantom brane world model concerning first order perturbations. The problem is though that the results are valid only in sub-Hubble or in super-Hubble regions but never at all cosmological scales. This was also an issue for the ΛCDM model. However, in a recent work []scientist managed to obtain closed forms of first order scalar and vector perturbations in ΛCDM generated by point-like particles. Up until now there hasn't been something similar for the ΛDGP model.

The thesis is organized as follows: In chapter 2 we briefly mention the main points of the ΛCDM model and reproduce the results from first order perturbation generated by point-like particles valid at all cosmological scales. In chapter 3 we discuss the DGP model and the cosmology in it. In chapter 4 we calculate the first

[1]From Dvali Gia, Gabadadze Gregory and Porrati Massimo

order cosmological perturbations valid at all cosmological scales in the aforementioned model. Also, we examine the gravity generated by one stationary point-like particle in his comoving coordinate system. In chapter 5 use experimental data to test the validity of the ΛDGP model. Finally, in chapter 6 we conclude with a discussion.

The Standard Model of Cosmology

Chapter 2

2.1 The ΛCDM model

The dominate model that describes best the behavior of our universe is the ΛCDM. This model assumes that our universe is 4 $(3+1)$-dimensional, homogeneous and isotropic with no curvature (flat). It is described by the Einstein-Hilbert action

$$S = \int \mathrm{d}^4 x \, \sqrt{-g} \left[m^2 (R - 2\Lambda) + \mathcal{L}_m(g_{\mu\nu}, \phi) \right] \tag{2.1}$$

where R is the Ricci curvature scalar of the 4D universe, m is the Planck mass, Λ is the cosmological constant, g is the determinant of the metric tensor $g_{\mu\nu}$[1] and $\mathcal{L}_m$ is the Lagrangian of the Standard Model fields, ϕ. The minus convention will be used and set $m = 1$ and $c = 1$ throughout.

Variation of the action (2.1) leads to the well-known Einstein's equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{2.2}$$

where $T_{\mu\nu}$ is the energy-momentum tensor originating from the variation of $\mathcal{L}_m(g_{\mu\nu}, \phi)$. It contains the information for the distribution of the substances that exist in the universe.

A homogeneous and isotropic universe is described by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric in conformal coordinates

$$ds^2 = a^2(\eta) \left[d\eta^2 - \delta_{ij} dx^i dx^j \right] = w_{\mu\nu} dx^\mu dx^\nu \tag{2.3}$$

where $a \equiv a(\eta)$ is the scale factor and $w_{\mu\nu}$ is the Minkowski metric tensor. The scale factor depends only on the conformal time η, which is a consequence of the homogeneity and the isotropy of the universe. We choose to work with conformal coordinates, because some calculations become easier. Using the FLRW metric and assume that there exist a cosmological constant Λ and nonrelativistic uniform cold dark matter (CDM) density $\bar{\rho}$, Einstein's equations yell

$$\frac{3\mathcal{H}^2}{a^2} = \bar{\rho} c^2 + \Lambda \tag{2.4}$$

and

$$\frac{2\mathcal{H}' + \mathcal{H}^2}{a^2} = \Lambda \tag{2.5}$$

where $\mathcal{H} \equiv \frac{1}{a}\frac{da}{d\eta} = \frac{a'}{a}$ is the Hubble parameter in conformal coordinates. In our calculations we did not consider the existence of radiation. The reason is that measurements have shown that the radiation density is 10^4 times smaller than matter density[5] . For that reason we assume that the effect of the radiation density in the behavior of the Universe is negligible. These two equations above are called Friedman's equations and describes how our universe behaves in the presence of matter and cosmological constant. It is the background stage where every play is acting on.

2.2 First order perturbations and the point-like approximation

Although our universe seems to be homogeneous and isotropic at scales larger than 100 Mpc we can clearly see inhomogeneities in our observations like planets, solar systems, galaxies, clusters etc. at smaller scales. To study

[1] Throughout this paper we will use Greek indexes (like μ, ν...) to represent spacetime coordinates taking values $0, 1, 2, 3$ and Latin indexes (like i, j...) to represent only the special coordinates taking values $1, 2, 3$

these objects, we assume the metric

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 + 2B_i dx^i d\eta - (1-2\Phi)\delta_{ij}dx^i dx^j\right] = g_{\mu\nu}dx^\mu dx^\nu \tag{2.6}$$

where the functions $\Phi \equiv \Phi(\eta, \boldsymbol{x})$ and $\boldsymbol{B} \equiv \boldsymbol{B}(\eta, \boldsymbol{x})$ describes the scalar and vector perturbations of the smooth background metric these inhomogeneities create. The bold font is used to indicate a vector, which determines the position in space where the potentials are evaluated at.

Up until now physicists used perfect fluid perturbations for the energy momentum tensor. In perfect fluid approximation the wavelength of the matter particles is assumed to be very large compared to the intermediate distance and as a consequence they behave like a fluid. In a recent work[4] it was used the point-like approximation, where the wavelength of each particle is assumed to be smaller than their intermediate distances. This approximation give rise to a very interesting result. We will recreate the main points of these calculations.

The energy momentum tensor for a collection of finite point-like particles can be calculated from the well-known formula [6]

$$T^{\mu\nu} = \sum_n \frac{m_n}{\sqrt{-g}}\frac{dx_n^\mu}{d\eta}\frac{dx_n^\nu}{d\eta}\frac{d\eta}{ds_n}\delta^3(\boldsymbol{x}-\boldsymbol{x}_n) \tag{2.7}$$

where $\boldsymbol{x}_n$ is the value of the x coordinate (as defined in the metric (2.6)) where the n^{th} particle is located at and m_n their masses. This energy momentum tensor will be considered as an *extra* contribution to the one creating the background discussed in previous section.

Using the metric from (2.6) and the energy momentum tensor from (2.7) Einstein's equations at first order in perturbation become

$$\Delta\Phi - \frac{9\mathcal{H}^2\Omega_m}{2}\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Phi = \frac{\delta\rho}{2a} \tag{2.8}$$

and

$$\frac{1}{4}\Delta B_i - \frac{3\mathcal{H}^2\Omega_m}{2}B_i + \partial_i\left(\Phi' + \mathcal{H}\Phi\right) = -\frac{1}{2a}\sum_n \rho_n v_n^i \tag{2.9}$$

where $\delta\rho$ is the sum of densities for the different point-like particles and the density ρ_n of a point-like particle is a delta function

$$\delta\rho \equiv \sum_n \rho_n, \qquad \rho_n \equiv m_n\delta^3(\boldsymbol{x}-\boldsymbol{x}_n) \tag{2.10}$$

and $\boldsymbol{v}_n \equiv d\boldsymbol{x}_n/d\eta$ are the peculiar velocities of the point-like particles, which is the velocity if from the total velocity of a particle subtract the Hubble flow at its position. Also we replace $\bar{\rho}$ in favor of Ω_m from the relation

$$\Omega_m = \frac{\bar{\rho}}{3\mathcal{H}a}$$

Observations have shown that peculiar velocities are small compared to the speed of light (smaller than $10^6 m/s$) [7]. For this reason, we treat them as being first order of smallness in perturbation. On the other hand, we do not treat $\delta\rho$ perturbatively. This is because near a galaxy the mass density is orders of magnitudes larger than the background density ($\sim 10^{26}$ times larger). This means that in (2.8) should be present a term $\delta\rho\Phi$. Although this term is first order, it is much smaller than $\delta\rho$ at all scales because $|\Phi| \ll 1$[8].

Taking the divergence of (2.9) if we choose to work in the Poisson gauge ($\partial_i B^i = 0$) we get

$$\Delta\Xi = \partial_i\left(\sum_n \rho_n {v_n}^i\right) \tag{2.11}$$

where $\Xi := -2a(\Phi' + \mathcal{H}\Phi)$. The solution of (2.11) is easy to found because it is the Poisson equation

$$\Xi = \frac{1}{4\pi}\sum_n m_n \frac{(\boldsymbol{x}-\boldsymbol{x}_n)\cdot\boldsymbol{v}_n}{|\boldsymbol{x}-\boldsymbol{x}_n|^3} \tag{2.12}$$

Replacing Ξ in (2.8) and (2.9) we can solve these two equations relatively easy going into Fourier space. The solutions are

$$\Phi = -\sum_n \frac{m_n}{|\boldsymbol{x}-\boldsymbol{x}_n|}\exp(-q_n) + 3\mathcal{H}\sum_n \frac{m_n(\boldsymbol{x}-\boldsymbol{x}_n)\cdot\boldsymbol{v}_n}{|\boldsymbol{x}-\boldsymbol{x}_n|}\frac{1-(1+q_n)\exp(-q_n)}{q_n^2} \tag{2.13}$$

and

$$\begin{aligned}\boldsymbol{B} = \sum_n \Bigg[& \frac{m_n\boldsymbol{v}_n}{|\boldsymbol{x}-\boldsymbol{x}_n|}\frac{(3+2\sqrt{3}q_n+4q_n^2)\exp(-2q_n/\sqrt{3})-3}{q_n^2} \\ & + \frac{m_n(\boldsymbol{x}-\boldsymbol{x}_n)\cdot\boldsymbol{v}_n}{|\boldsymbol{x}-\boldsymbol{x}_n|^3}(\boldsymbol{x}-\boldsymbol{x}_n)\frac{9-(9+6\sqrt{3}q_n+4q_n^2)\exp(-2q_n/\sqrt{3})}{q_n^2}\Bigg]\end{aligned} \tag{2.14}$$

where

$$q_n = |\boldsymbol{x} - \boldsymbol{x}_n|/\lambda, \qquad \lambda = \sqrt{\frac{2a^2}{9\Omega_m \mathcal{H}^2}} \tag{2.15}$$

The gravitational potential of a single, non-moving point-like particle with mass M_0 is

$$\Phi = -\frac{1}{8\pi m^2 a}\frac{M_0}{|\boldsymbol{x}|}\exp\left(-|\boldsymbol{x}|/\lambda\right) \tag{2.16}$$

the current value of the screening depth λ is $\lambda_0 \approx 3700\ Mpc$. We see that at small scales Newton's law of gravity emerge, but at very large distances gravity weakens by the expansion of the universe and give rise to a Yukawa-like potential. This Yukawa behavior comes from the second term in (2.8), which comes from the point like approximation. It is interesting that the currently largest known structures have approximately the same size as the screening depth.

Chapter 3

The ΛDGP model

3.1 Derivation of the effective Einstein's equations

We do not want to start with the DGP action, but rather understand its origin.

The DGP is a 5(4+1)-dimensional model thus we start with the action

$$S_0 = M^3 \int_{\text{bulk}} \Big(\mathcal{R}_{(5)} - \delta(y) L_m(g_{\mu\nu}, \phi)\Big) \tag{3.1}$$

where M is the Planck mass of the five dimensions, $\mathcal{R}_{(5)}$ is the Ricci scalar corresponding to the five dimensions, $L_m(g_{\mu\nu}, \phi)$ is the Lagrangian of the standard model fields ϕ that are confined on the brane with induced metric $g_{\mu\nu}$ and y is the coordinate characterizing the extra dimension.

Variation of this action lead to a $1/r^2$ law of gravity. In order to get the correct $1/r$ law of gravity we have to take a look at the full quantum theory. Calculating the one loop correction of graviton's propagator in this theory we see that at small energy scales we have to add, in the action, the term

$$S_1 = \int_{\text{brane}} R_{(4)} \tag{3.2}$$

where $R_{(4)}$ is the Ricci scalar corresponding to the four dimensions. This is a counterterm that cancel the infinities created by the localized massive scalar [9] and fermion[10, 11] fields running through the 1-loop correction of graviton's propagator[1].

In the action (3.1) we must also add the GHY[2] boundary term

$$S_2 = -2M^3 \int_{\text{brane}} K \tag{3.3}$$

where K is the extrinsic curvature of the brane. This term must be added in order to set the correct boundary conditions on the brane[13].

Finally we would like to add cosmological constant both into the brane (Λ) and to the bulk (Λ_{5D}). Thus the ΛDGP model has the action

$$S = M^3 \left[\int_{\text{bulk}} \Big(\mathcal{R}_{(5)} - 2\Lambda_{5D}\Big) - 2\int_{\text{brane}} K\right] + \int_{\text{brane}} \Big(R_{(4)} - 2\Lambda\Big) - \int_{\text{brane}} L_m(g_{\mu\nu}, \phi) \tag{3.4}$$

Taking the variation of (3.4) and using Gauss-Codacci equations, which project a tensor from a hypersurface to a submanifold, we get the effective 4D Einstein's equations[14]

$$G_{\mu\nu} - \left(\frac{\Lambda_{RS}}{b+1}\right) g_{\mu\nu} = \left(\frac{b}{b+1}\right) T_{\mu\nu} - \left(\frac{1}{b+1}\right)\left[\frac{1}{M^6} Q_{\mu\nu} - \mathcal{C}_{\mu\nu}\right] \tag{3.5}$$

where

$$b = \frac{1}{6}\Lambda l^2\,, \quad l = \frac{2}{M^3}\,, \quad \Lambda_{RS} = \frac{\Lambda_{5D}}{2} + \frac{1}{12}\Lambda^2 l^2 \tag{3.6}$$

are convenient parameters, and

$$Q_{\mu\nu} = \frac{1}{3} E E_{\mu\nu} - E_{\mu\lambda} E^{\lambda}{}_{\nu} + \frac{1}{2}\Big(E_{\rho\lambda} E^{\rho\lambda} - \frac{1}{3} E^2\Big) g_{\mu\nu}, \qquad E_{\mu\nu} \equiv G_{\mu\nu} - T_{\mu\nu}, \qquad E = E^{\mu}{}_{\mu} \tag{3.7}$$

[1]For the massless fields we must add nonlinear terms of Ricci tensor and scalar[12].

[2]Gibbons G. W., Hawking S. W. and York J. W.

The tensor $\mathcal{C}_{\mu\nu}$ comes from the Weyl tensor which is the traceless part of the five dimentional Ricci scalar $\mathcal{R}_{(5)}$. Taking the divergence of (3.5) we get

$$\nabla^{\mu}\Big(\frac{b+1}{m^2}E_{\mu\nu} + T_{\mu\nu} + \frac{1}{M^6}Q_{\mu\nu} - \mathcal{C}_{\mu\nu}\Big) = 0 \tag{3.8}$$

As you can see the elements of $\mathcal{C}_{\mu\nu}$ are not freely specified, but they are constrained through (3.8).

3.2 Cosmology in ΛDGP model

Using the (FLRW) metric (2.3) in the effective Einstein's equations (3.5) we get[3]

$$\frac{\mathcal{H}^2}{a^2} = \frac{\bar{\rho}}{3a^3} + \frac{\Lambda}{3} + \frac{2}{l^2}\left[1 - \sqrt{1 + l^2\left(\frac{\bar{\rho}}{3a^3} + \frac{\Lambda}{3} - \frac{\Lambda_{5D}}{6} - \frac{C}{a^4}\right)}\right] \tag{3.9}$$

The constant C comes from the existence of the Weyl tensor in the bulk. It has a radiation like behavior and for that reason it is called "dark radiation" or "Weyl radiation". We will ignore it since there is practically no radiation in our universe at the current epoch. This is the effective Friedman's equation and describes the behavior of the background. We will also ignore the backreaction of Λ_{5D} to make our calculations easier and to get analytic solutions. After these simplifications it is easy to show that (3.9) becomes

$$\frac{\mathcal{H}}{a} = \frac{1}{l}\left[\sqrt{1 + l^2\left(\frac{\bar{\rho}}{3a^3} + \frac{\Lambda}{3}\right)} - 1\right] \tag{3.10}$$

We will also need the derivative of this equation with respect to conformal time η

$$\frac{d\mathcal{H}}{d\eta} = \mathcal{H}^2\left(1 - \frac{3\Omega_m}{2(1+\Omega_M)}\right) \tag{3.11}$$

where[4]

$$\Omega_m = \frac{\bar{\rho}}{3\mathcal{H}^2 a}, \qquad \Omega_M = \frac{aM^3}{2\mathcal{H}}, \qquad \Omega_\Lambda = \frac{\Lambda}{3\mathcal{H}^2} \tag{3.12}$$

Consequently, (3.8) takes the simple form

$$\Omega_m + \Omega_\Lambda - 2\Omega_M = 1 \tag{3.13}$$

in everything that follows we have replaced $\bar{\rho}$ in favor of $3\mathcal{H}^2\Omega_m a$, M^3 in favor of $2\Omega_M\mathcal{H}/a$ and Λ in favor of $3\mathcal{H}^2\Omega_\Lambda$. It is very convenient since everything we derive may be expressed as functions of Ω_m and Ω_M, only (Ω_Λ is solved for from (3.11)). The advantage of this procedure is twofold, firstly these parameters are dimensionless and we claim rather intuitive to handle, secondly these will make comparison to ΛCDM trivial by simply taking Ω_M to zero.

[3] There is one more solusion with a plus sign in front of the square root, but it recently shown that is has ghost instabilities [15, 16]

[4] We would like to mention that the quantity Ω_M is often defined as $\sqrt{\Omega_l}$ in the related literature

Chapter 4

Perturbation theory in ΛDGP model

4.1 First order scalar and vector perturbation equations

In this section we will extend the linear perturbation scheme developed for the ΛCDM model described in the preceding chapters. We start with the ansatz for the first order McVittie metric on the brane in Cartesian coordinates,

$$ds^2 = a^2(\eta)\left[(1+2\Phi(\eta,\boldsymbol{x}))d\eta^2 + 2B_i(\eta,\boldsymbol{x})d\eta dx^i - (1-2\Psi(\eta,\boldsymbol{x}))\delta_{ij}dx^i dx^j\right] \tag{4.1}$$

where Φ, Ψ and B_i's are respectively the scalar and vector perturbations. Note that unlike the ΛCDM, $\Phi \neq \Psi$ here, owing to the anisotropic stresses originating from the bulk, e.g. [3].

Once again we will consider a collection of finite moving point-like particles. The proper interval for the n-th mass is defined as

$$ds_n = a(\eta)\left[(1+2\Phi) + 2B_i v_n^i - (1-2\Psi)\delta_{ij}v_n^i v_n^j\right]^{1/2} d\eta \tag{4.2}$$

The energy momentum tensor can be calculated using (2.7) and the metric (4.1). Up to first order in perturbation theory the energy momentum tensor is

$$T^{\mu\nu} = \frac{1}{a^5}\begin{pmatrix} (1-2\Phi+3\Psi)\bar{\rho} + \delta\rho(\eta,\boldsymbol{x}) & \sum\limits_n \rho_n v_n \\ \sum\limits_n \rho_n v_n & 0 \end{pmatrix} \tag{4.3}$$

Remember here that $\delta\rho$ is not treated as a perturbation, due to the fact that it is dominant at small scales (see [8]).

The geodesic equation for the n$^{\text{th}}$ particle in (4.2) also reads,

$$(a\boldsymbol{B}|_{\boldsymbol{x}=\boldsymbol{x}_n} - a\boldsymbol{v}_n)' = a\nabla\Phi|_{\boldsymbol{x}=\boldsymbol{x}_n} \tag{4.4}$$

Since we wish to build a perturbation scheme valid all the way to superhorizon scales, we can not assume that the perturbations' spatial variations dominate over the temporal ones, unlike the case of the study of cosmic structures, e.g. [17].

Finally, we come to the perturbation of the Weyl tensor's projection onto the brane, $\delta\mathcal{C}_{\mu\nu}$. Its most generic form is given by, e.g. [3],

$$\delta\mathcal{C}_{\mu\nu} = \frac{1}{a^2}\begin{pmatrix} \delta\rho_{\mathcal{C}} & \partial_i v_{\mathcal{C}} \\ \partial_i v_{\mathcal{C}} & \dfrac{\delta\rho_{\mathcal{C}}}{3}\delta_{ij} - \delta\pi_{ij} \end{pmatrix} \tag{4.5}$$

where $\delta\pi_{ij} = (\nabla_i\nabla_j - g_{ij}\Delta/3)\delta\pi_{\mathcal{C}}$ (Δ stands for the Euclidean 3-Laplacian) is trace free and $\delta\rho_{\mathcal{C}}$, $v_{\mathcal{C}}$ and $\delta\pi_{\mathcal{C}}$ are scalars. In particular, $v_{\mathcal{C}}$ can be regarded as a momentum potential, whose backreaction effects will also be ignored, while considering its time evolution also negligible.

The effective Einstein equations on the brane (3.5), at first order read, after using (4.3), (4.5),

$$\Delta\Psi - \frac{9\mathcal{H}^2\Omega_m}{2m_{\text{eff}}^2}\Psi - 3\mathcal{H}\Psi' - 3\mathcal{H}^2\Phi = \frac{\delta\rho}{2m_{\text{eff}}^2 a} + \frac{\Omega_M}{2m_{\text{eff}}^2 a^2}\delta\rho_{\mathcal{C}} \tag{4.6}$$

which is the 00 component, and for $i \neq j$

$$\frac{\Omega_M}{m_{\text{eff}}^2 a^2}\delta\pi_{ij} - \left(1 - \frac{3\Omega_m}{4m_{\text{eff}}^4}\right)\left(\partial_i\partial_j\left(\Phi - \Psi\right) - \frac{1}{2}\partial_{(i}\left(B'_{j)} + 2\mathcal{H}B_{j)}\right)\right) = 0 \tag{4.7}$$

We also have for the vector perturbation,

$$\frac{1}{4}\Delta B_i - \frac{3\mathcal{H}^2\Omega_m}{2m_{\text{eff}}^2}B_i + \partial_i\left(\Psi' + \mathcal{H}\Phi\right) = -\frac{1}{2m_{\text{eff}}^2 a}\sum_n \rho_n v_n^i \tag{4.8}$$

where Δ as earlier is the Laplacian on the Euclidean 3-space and also the function $m_{\rm eff} \equiv m_{\rm eff}(\eta)$ has been introduced

$$m_{\rm eff}^2 \equiv 1 + \Omega_M \tag{4.9}$$

The ΛCDM limit in the above equations is obtained by letting $\Omega_M \to 0$ in which case we recover the results of Section 2.2 ([4]).

The divergence of (4.8), in the Poisson gauge $\partial_i B^i = 0$, gives again the Poisson equation (2.11), where now $\Xi := -2m_{\rm eff}^2 a(\Psi' + \mathcal{H}\Phi)$. The solution for Ξ is still given from (2.12).

We are interested in distinguishing the ΛDGP model from ΛCDM with respect to the cosmological screening, which as we saw in previous chapters it becomes relevant at very large distances. At that scales, the backreaction from the peculiar velocities are negligible. For this reason, from now on we will ignore them. This means that the vector perturbation (which cause the frame dragging effect) will be zero.

4.2 Solutions ignoring peculiar velocities

From the definition of Ξ and (2.12) in the limit of vanishing peculiar velocities we have (since $\Xi = 0$)

$$\Psi' = -\mathcal{H}\Phi \tag{4.10}$$

With this equation in hand we can simplify (4.6)

$$\Delta\Psi - \frac{9\mathcal{H}^2\Omega_m}{2m_{\rm eff}^2}\Psi = \frac{\delta\rho}{2m_{\rm eff}^2 a} + \frac{\Omega_M}{2m_{\rm eff}^2 a^2}\delta\rho_{\mathcal{C}} \tag{4.11}$$

On the other hand, since the vector perturbation is zero, we can write (4.8) as

$$\frac{\Omega_M}{m_{\rm eff}^2 a^2}\delta\pi_{\mathcal{C}} = \left(1 - \frac{3\Omega_m}{4m_{\rm eff}^4}\right)(\Phi - \Psi) + constant \tag{4.12}$$

and recall that in a marginally closed universe with a *vanishing* bulk cosmological constant, one has [18],

$$\Delta\delta\pi_{\mathcal{C}} = \frac{\delta\rho_{\mathcal{C}}}{2} \tag{4.13}$$

Combining (4.12) and (4.13) to eliminate $\delta\pi_{\mathcal{C}}$ we get

$$\frac{\Omega_M}{2m_{\rm eff}^2 a^2}\delta\rho_{\mathcal{C}} = \left(1 - \frac{3\Omega_m}{4m_{\rm eff}^4}\right)\Delta(\Phi - \Psi) \tag{4.14}$$

In order to solve for Φ and Ψ we want one more equation. This comes from the spatial component of (3.8), after using (4.4) and (4.11), we obtain

$$\frac{\Omega_M}{2m_{\rm eff}^2 a^2}\left(1 - \frac{3\Omega_m}{2m_{\rm eff}^2}\right)\delta\rho_{\mathcal{C}} = \Delta\Phi - \left(1 + \frac{3\Omega_m}{2m_{\rm eff}^4}\right)\Delta\Psi + constant \tag{4.15}$$

We can substitute (4.14) into (4.11) and (4.15) to obtain a system of two equations with only two unknowns, the perturbations Φ and Ψ. The constant in (4.12) and (4.15) has to be zero in order for the potential to be vanishing at infinity. The decomposition of this system is straightforward

$$\Delta\Phi = I\,\Delta\Psi \tag{4.16}$$

and

$$\Delta\Psi - \frac{9\mathcal{H}^2\Omega_{m,\rm eff}}{2}\Psi = \frac{\delta\rho}{2m_{\rm eff,\Psi}^2 a} \tag{4.17}$$

where

$$I \equiv 1 + \frac{4\Omega_M}{1 + 2m_{\rm eff}^2 - \frac{3\Omega_m}{2m_{\rm eff}^4}}, \qquad \Omega_{m,\rm eff} \equiv \frac{\Omega_m}{m_{\rm eff,\Psi}^2}, \quad m_{\rm eff,\Psi}^2 \equiv m_{\rm eff}^2 + (1 - I)\left(m_{\rm eff}^2 - \frac{3\Omega_m}{4m_{\rm eff}^2}\right) \tag{4.18}$$

(4.17) is identical to the one obtained for ΛCDM derived in Section 2.2 ([4]). It is trivial to solve our equation by comparison and using $_{\rm eff}$ subscripts wherever appropriate. This is because the quantities in (4.18) depend only on time. The solution is

$$\Psi\big|_{\rm many\ particle} = -\frac{1}{8\pi m_{\rm eff,\Psi}^2 a}\sum_n \frac{m_n}{|\boldsymbol{x} - \boldsymbol{x}_n|}\, e^{-\frac{|\boldsymbol{x} - \boldsymbol{x}_n|}{\lambda}} \tag{4.19}$$

where

$$\lambda = \sqrt{\frac{2}{9\mathcal{H}^2\Omega_{m,\rm eff}}} \tag{4.20}$$

This is the Ψ potential created by a collection of finite point-like particles valid at all length scales. For a single particle – a single central over-density – the solution for the potential Ψ, valid for all length scales is

$$\Psi\Big|_{\text{one particle}} = -\frac{1}{8\pi m^2_{\text{eff},\Psi} a}\frac{m_0}{r}\ e^{-r/\lambda} \tag{4.21}$$

where m_0 is the mass of the central overdensity and r is the distance from its center. Just like the ΛCDM we also have a screening effect. Although this time it will be different, depending on the value Ω_M characterizing ΛDGP model.

In every case we can find Φ solving (4.16)

$$\Phi = I\ \Psi \tag{4.22}$$

Figures (4.1) $-$ (4.3) elucidate various properties of the gravitational potentials and the screening length. We use the values for cosmological parameters $\Omega_m = 0.3089$ and $H_0 = 67.74\ km\,s^{-1}Mpc^{-1}$ as specified in [5] and we examine the gravitational potentials for one particle with mass $M_\odot = 1.989 \cdot 10^{30}\ kg$. Figure (4.1) depicts the behavior of the effective mass density parameter and the screening length versus Ω_M. The ΛCDM limit is obtained by letting $\Omega_M \to 0$.

We also note that since the screening length is typically of the order of $\mathcal{O}(10^3)$ Mpc (see Figure (4.1)), at length scales comparable of the size of a typical cosmic structure i.e. $\mathcal{O}(100)$ Mpc, (4.21) recovers the $1/r$ fall-off of the gravitational potentials. However, the $1/m^2_{\text{eff},\Phi} \equiv I/m^2_{\text{eff},\Psi}$ and $1/m^2_{\text{eff},\Psi}$ terms present modify Newton's 'constant' in Φ and Ψ respectively and make it time dependent, as discussed in [19].

Figure (4.2) depicts the behavior of the effective Newton's constants for Ψ and Φ. In the $\Omega_M \to 0$ limit both of them aproach unity recovering the ΛCDM limit. Note also that in this limit setting further $\bar{\rho} \to 0$ ($\Omega_m \to 0$) removes the exponential fall off since then $\lambda \to \infty$ (cf., (4.18),(4.20)), yielding Newton's potential for a point mass located in a de Sitter universe. It is easy to verify that, as expected, this is the linearized approximation of the Schwarzschild-de Sitter metric in the McVittie coordinate frame. Similar conclusions hold for the potential $\Phi\big|_{\text{one particle}}$. Finally, we depict the potentials in figure (4.3).

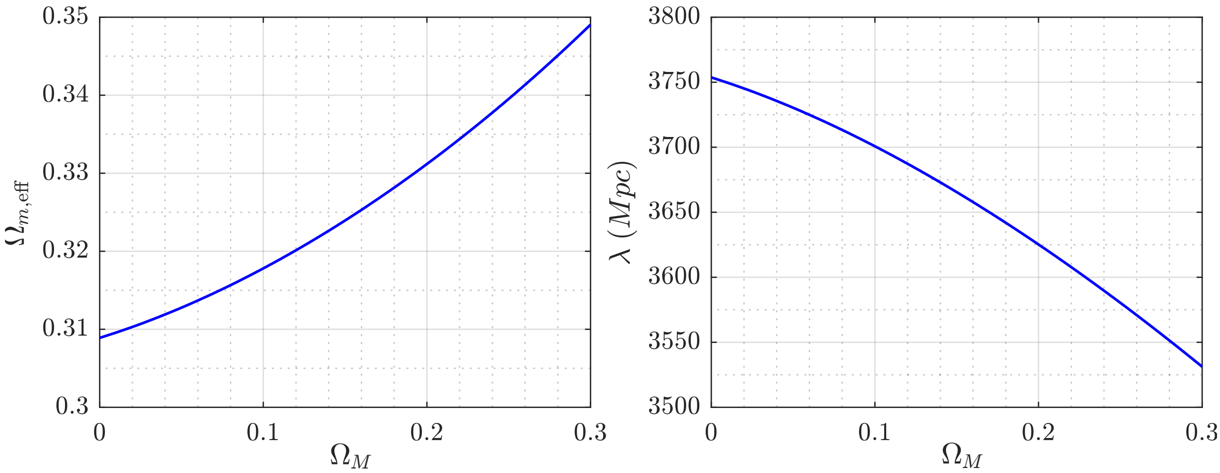

Figure 4.1: Plots of the effective mass density and the screening length versus Ω_M.

Chapter 4

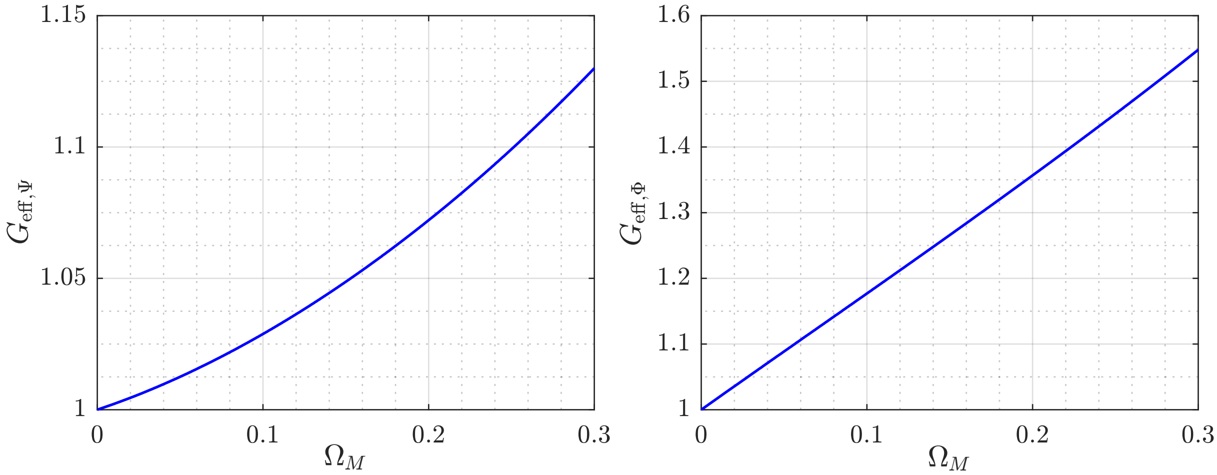

Figure 4.2: Plots of $1/8\pi m^2_{\text{eff},\Psi}$ and $1/8\pi m^2_{\text{eff},\Phi}$ respectively versus Ω_M. These are proportional to the Newton's constant for each potential respectively. Note that the one for Φ decreases faster than Ψ.

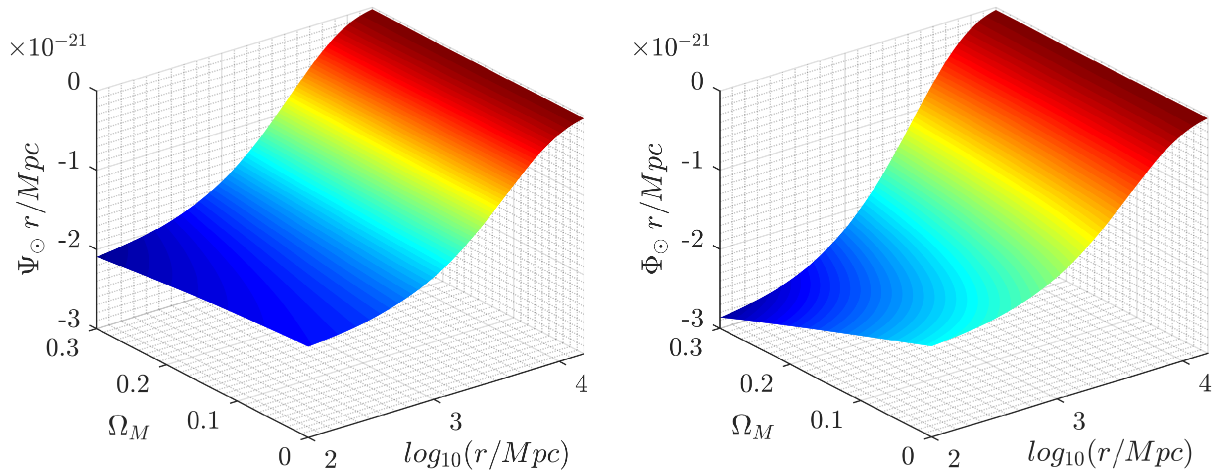

Figure 4.3: Plot of $\Psi_\odot(r)\, r/Mpc$ and $\Phi_\odot(r)\, r/Mpc$ versus Ω_M and $log(r/Mpc)$ for one particle with mass equal to one solar mass.

Cassini's measurement of PPN γ parameter

5

Chapter

In the previous chapter we saw that the Newton's constants change as a function of Ω_M. We can not simply measure one of the constants and define the value of Ω_M, because we could redefine the constant to match our results. We have to measure their relevant difference. As you can see from Figure (4.2) $G_{\text{eff},\Psi}$ and $G_{\text{eff},\Phi}$ have different values in different Ω_M. The question now is how do we make such a measurement?

Let's take a look at the metric in (4.1) (in vanishing velocities) and write it in a more convenient way following [20]

$$ds^2 = a^2(\eta)\left[(1+2\Phi)d\eta^2 - (1-2\gamma\Phi)\delta_{ij}dx^i dx^j\right] \tag{5.1}$$

where $\gamma \equiv \Psi/\Phi$ is widely known as one of the post-Newtonian parameters. The value of γ parameter for General Relativity is $\gamma = 1$. However, in our case we see that it is true only when $\Omega_M \to 0$, which is the ΛCDM limit.

To measure γ parameter we have to observe in accuracy two effects coming from General Relativity: *Time dilation* and the *deflection of light* in a gravitational field. Using (5.1) (for the present time $a(\eta) = 1$) the equation for time dilation is

$$dt_\infty = (1+\Phi)d\eta \tag{5.2}$$

where dt_∞ is the time interval at infinity with zero gravitational potential and $d\eta$ is the time measured by a clock placed at gravitational potential Φ.

The angle of deflection of a light ray is given by the equation

$$\delta\phi = 2\Phi(1+\gamma) \tag{5.3}$$

where in the ΛCDM limit $\gamma = 1$ and (5.3) is our well-known formula for light ray deflection.

In 2003 data for time dilation and light deflection were collected from Cassini mission[21]. The result was

$$\gamma - 1 = (2.1 \pm 2.3) \cdot 10^{-5} \tag{5.4}$$

In ΛDGP model the γ parameter is given by the equation (see Eq. (4.22))

$$\gamma = \frac{1}{I} \tag{5.5}$$

Using (4.18) we can calculate the value of Ω_M

$$\Omega_M = (-1.33 \pm 1.46) \cdot 10^{-5} \tag{5.6}$$

Even though it seems peculiar that Ω_M turns out to be negative, it remains compatible with the value zero within 1σ. We also see that even at 5σ, Ω_M can not be larger than $\sim 10^{-4}$. This extreme value case leads to a Planck mass for the 5 dimension $M \sim 2MeV$ and a characteristic length where gravity change its behavior $l \sim 10^{30}$m or $3 \cdot 10^6 Gpc$.

Future space missions[22] promisses to increase the accuracy of γ parameter up to order of 10^{-9}. These missions will test General Relativity to a unbelievable accuracy and may provide enlightening data for new physics.

Chapter 6

Discussion

We found that in ΛDGP model we still have the screening behavior of gravity just like ΛCDM. We study the behavior of the screening (and the potentials) as a function of Ω_M which characterize the extra dimension of a single stationary point like particle. We saw that Ω_M (which is positive) has to be smaller than $\sim 10^{-4}$ in order to explain the value of post-Newtonian parameter γ which was recently measured by Cassini mission. The small value of Ω_M indicate that ΛCDM model is most probably correct. Future space missions will test General Relativity to a never before accuracy and might give us a more clear view for our universe.

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