UNIVERSITY OF CRETE

Spontaneous Symmetry Breaking and Higgs Mechanism in Particle Physics

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OVERVIEW

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1. Historical Introduction

The concept of symmetry inspired many philosophers , artists and mathematicians from the ancient times. In each field , there are different kinds of definitions of symmetry. Although, balance and harmony are synonymous to the word symmetry , in mathematics there is another definition; object is invariant to any varius transformations .The significance of symmetry in physics can be seen in all of the fundamental theories. A great example is the spacetime symmetries of Einstein , according to which the laws of nature are described by equations that are invariant under Lorentz transformations or internal symmetries such as the approximate $SU(2)$ isospin symmetry . Later on, symmetry became a part of the Standard Model of particle physics; a theory that describes the three of the four fundamental forces and their interactions with elementary particles. This theory is governed by some specific kinds of symmetries called gauge symmetries. Electromagnetism and the Strong force that described in the standard model (Q.E.D , Q.C.D) were consistent to the experiments with great accuracy. However, the symmetry of the weak force could not allow non-zero mass for the $W^{\pm}$ and Z bosons, the quanta of weak interactions . Experiments showed that there must be massive particles responsible for the weak interactions with significantly large masses. The physics of superconductors played a crucial role to solve the mystery of massive gauge bosons. Within a superconductor the electromagnetic field becomes a massive field. In other words , the photon acquires non-zero mass. This is caused by the so-called spontaneously symmetry breaking ; the gauge symmetry that the equations of electrodynamics have is not valid by the ground state of the superconductor. Three notable research groups worked to find out the exact mechanism that under symmetry breaking could give mass terms for the $W^{\pm}$, Z bosons. Brout and Englert in Belgium , Higgs in Scotland and Kibble with Guraluik and Hagen in London. In 1964, they found independently the symmetry breaking mechanism, widely known as the *Brout-Englert-Higgs Mechanicm*. According to the mechanism, the initially massless gauge boson of a theory with gauge symmetry, can acquire mass if this symmetry is spontaneously broken by the ground state of the system. Going through the symmetry breaking path, Weinberg in 1967 and Salam in 1968 independently formulated the Electroweak theory. This theory was able to make accurate predictions for the $W^{\pm}$, Z bosons, in agreement with experiments. It turned out that the Universe is like a giant superconductor ; within it the weak interactions bosons cannot be massless. Empty space is filled with the Higgs field which upon quantization gives rise to a new particle, the Higgs Boson. The experimental evidence for the existence of the Higgs boson was crucial for the acceptance of the theory. There were also some others models for the explanation of gauge bosons masses. Multi-Higgs models,extra dimension and brane world models had their own description instead of the Brout-Englert-Higgs mechanism. Finally, in 2012 a particle very much like the Higgs Boson was discovered in the Large Hadron Collider (L.H.C). As Frank Wilczek posted in 2013 " Theories with complex dynamics for the explanation of W, Z masses seem les credible, as simplicity and minimalism carry the day." .This "minimalism triumphant " will go further. Perhaps, Higgs boson research will be able to contribute to cosmological and particle physics problems, yet unsolved.

2. Symmetries

To understand the concept of symmetry breaking of Brout-Englert-Higgs mechanism, some background is needed. Some group theory basics, gauge symmetries and their corresponding bosons will be discussed.

2.1 Group Theory basics

A group G is a set of elements $a, b, c, ...$ with a law of multiplication $(\cdot)$, according to which $a \cdot b$ is another element of G .The multiplication satisfies the following conditions:

1) Associative law: for all $a, b, c, ...$ that are elements of G:

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

2) Unit element : G contains an element called *identity element* I, such that for every element a of the group;

$$a \cdot I = I \cdot a = a$$

3) Existence of inverse : for every element a of G there is an element a^{-1} such that:

$$a \cdot a^{-1} = a^{-1} \cdot a = I$$

A n-dimensional representation of a group G is a mapping M of G into a set of operators acting on a n-dimensional linear vector space V which preserves the group multiplication law; for every element a, b of G the mapping M satisfies: $M(a)M(b) = M(a \cdot b)$.

A simple example of a group is the Cyclic group Cn ; the symmetry group of rotations of a regular polygon with n directed sides. The group

elements are rotations through an angle $\frac{2\pi r}{n}$ $(r = 0, 1, .., n-1)$ about an axis through the center. This is a finite group; has a finite number of elements. However, the case of rotations through an arbitrary angle is more interesting. The continuous groups, the so-called Lie groups, have the appropriate structure to make infinitesimal transformations using infinitesimal generators of the group. The generators form a structure known as Lie algebra. For example, the angular momentum operators J_i satisfying the commutation relations $[J_i, J_j] = i\epsilon_{ijk}J_k$ .The angular momentum operators are the generators of the rotation group $SO(3)$ and the commutation relations form the Lie algebra.

The generators of a group can be defined by infinitesimal transformations. Let $U(\theta)$ be the group element corresponding to the parameter θ and I the identity element corresponding to $\theta = 0$. Then $U(\delta\theta)$ can be expressed as:

$$U(\delta\theta) = \mathrm{I} + i\delta\theta_i T_i$$

The matrices

$$T_i \equiv -i \frac{(}{\partial U} \partial\theta_i)_{\theta=0} \quad , \quad i = 1, ..., N$$

are the generators of the group in the given matrix representation. The group elements can be written in terms of the generators T_i in the exponential form

$$U(\theta) = e^{i\theta_i T_i}$$

which comes from the multiplication of infinite number of infinitesimal transformations.

2.2 Symmetry Transformations

A transformation that leaves our physical system unchanged is called symmetry transformation. In the context of field theories, the lagrangian of the system is invariant under symmetry transformations.

2.2.1 Global Symmetry

Suppose a transformation

$$\psi \longrightarrow e^{i\alpha T}\psi \qquad (2.1)$$

with α being spacetime independent. If the lagrangian of a system is invariant under (2.1) then it has a global symmetry; every point in space transforms according to the same parameter α. For example, the Dirac lagrangian has the form:

$$\mathcal{L}_D = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

It is trivial that it is invariant under transformations such us (2.1).

2.2.2 Gauge Symmetry

Now we suppose the transformation (2.1) but now with spacetime dependence on α ; $\alpha = \alpha(x)$. Now, for every point x there is a different parameter $\alpha(x)$ on the transformation. A theory that is invariant under that transformation has the so-called gauge symmetry. In the case of global symmetry it is easy to see if a theory is invariant. However, due to the spacetime dependence of $\alpha(x)$ the ordinary derivatives of a lagrangian cannot respect the gauge symmetry and a new type of derivative is needed ; the *covariant derivative.*

For simplicity we consider the $U(1)$ gauge transformation which has the form:

$$\psi \longrightarrow e^{i\alpha(x)}\psi \qquad (2.2)$$

The reason that the ordinary derivative is problematic with the gauge invariance is that $\psi(x)$ and $\psi(x+\epsilon)$ transform differently under (2.2) so the derivative of $\psi(x)$ in the direction of the vector n^μ which is defined as

$$n^\mu \partial_\mu \psi = \lim_{\epsilon \longrightarrow 0} \tfrac{1}{\epsilon}[\psi(x+\epsilon\nu) - \psi(x)] \quad (2.3)$$

has complicated transformation law and it must be modified in order to make sure that the lagrangian can become gauge invariant. The new derivative will rise by introducing a scalar quantity $U(y,x)$ that depends on two points while $U(y,y) = 1$ for *zero seperation.* This quantity transforms as:

$$U(y,x) \longrightarrow e^{i\alpha(y)} U(y,x) e^{-ia(x)} \quad (2.4)$$

We can require that $U(y,x)$ to be a pure phase $U(y,x) = e^{i\phi(y,x)}$. Also, from (2.4) it follows that $\psi(y)$ and $U(y,x)\psi(x)$ have the same transformation law:

$$U(y,x)\psi(x) \longrightarrow e^{i\alpha(y)} U(y,x)\psi(x) \quad (2.5)$$

Thus, we can define the so-called *covariant derivative* as:

$$n^\mu D_\mu \psi = \lim_{\epsilon \longrightarrow 0} \tfrac{1}{\epsilon}[\psi(x+\epsilon n) - U(x+\epsilon n)\psi(x)] \quad (2.6)$$

From (2.5) it is clear that $\psi(x+\epsilon n)$ and $U(x+\epsilon n, x)\psi(x)$ transform with the same way, so it is meaningful to subtract them.

For small ϵ we can expand $U(x+\epsilon n, x)$ around the separation of the two points which corresponds to $U = 1$ so we have:

$$U(x+\epsilon n, x) = 1 - ie\epsilon\nu^\mu A_\mu(x) + O(\epsilon^2) \quad (2.7)$$

The coefficient of the displacement ϵn^μ is a new vector field $A_\mu(x)$ while e is an arbitrary constant. From the definition (2.6) and the expansion (2.7) , the *covariant derivative* takes the form:

$$D_\mu \psi = \partial_\mu \psi(x) + ieA_\mu \psi(x) \quad (2.8)$$

In order to find the transformation law of $D_\mu \psi(x)$ we need the transformation law for the field A_μ. Using (2.7) and (2.4) we find that the field transforms as:

$$A_\mu \longrightarrow A_\mu - \frac{1}{e}\partial_\mu \alpha(x) \quad (2.9)$$

From the relations (2.2) and (2.9) it follows that the covariant derivative transforms as:

$$D_\mu \psi(x) \longrightarrow e^{i\alpha(x)} D_\mu \psi(x) \quad (2.10)$$

The covariant derivative transforms with same way as the $\psi(x)$ under the gauge transformation.

An important thing to notice is that the existence of the field A_μ is inevitable for the construction of gauge invariant derivative terms in a lagrangian. Thus, the field A_μ comes from a local phase rotation symmetry i.e by requiring gauge $U(1)$ symmetry. For example, the electromagnetic theory is invariant under gauge $U(1)$ and A_μ corresponds to the photon. Also, a kinetic energy term of the field A_μ that is gauge invariant can be found and is the well-known $(F_{\mu\nu})^2$. Lastly, the gauge invariance idea that discussed for the gauge $U(1)$ is the same for any other group. Again, some vector fields must be occur, in order to fix a suitable covariant derivative.

3. Spontaneous Symmetry Breaking

In this section the Goldstone theorem will be discussed. According to the theorem, if an initial global continuous symmetry is violated by the ground state (vacuum) of the system, a massless scalar field will occur; there is a massless Goldstone boson for every independent broken symmetry. Before that statement, the linear sigma model is a great example to start with.

3.1 The linear sigma model

The lagrangian of this model involves a set of N real scalar fields $\phi^i(x) \quad i = 1, ..., N$ and has the form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}\mu^2(\phi^i)^2 - \frac{\lambda}{4}[(\phi^i)^2] \quad (3.1)$$

where the V is the *mexian hat potential* defined as:

$$V(\phi) = -\frac{1}{2}\mu^2(\phi^i)^2 + \frac{\lambda}{4}[(\phi^i)^2] \quad (3.2)$$

The lagrangian is invariant under the transformation

$$\phi^(i) \longrightarrow R^{ij}\phi^j \qquad (3.3)$$

for any orthogonal matrix R. The R matrices are the group elements of the $O(N)$ group ; the rotation group in N dimensions. The potential V is minimized for any ϕ_0^i that satisfies

$$\phi_0^i = \frac{\mu^2}{\lambda} \qquad (3.4)$$

as it follows from the action of $\frac{\partial}{\partial\phi^i}$ on $V(\phi^i)$. The quantity

$$(\phi_0^i)^2 = (\phi_0^1)^2 + ... + (\phi_0^N)^2 = \frac{\mu^2}{\lambda} \qquad (3.5)$$

is the length of the vector ϕ_0.Its direction is arbitrary: There is an (N- 1) dimensional "sphere" (or "circle") that contains all the possible equivalent vacuum states, so the ground state is degenerate. Thus, we can choose coordinates so that ϕ_0 points in the N^{th} direction

$$\phi_0 = (0, ..., v) \quad , \quad v = \frac{\mu}{\sqrt{\lambda}} \quad . \ (3.6)$$

We take small fluctuations around the vacuum i.e define a set of shifted fields;

$$\phi^i(x) = (\pi^k(x), v + \sigma(x)) \quad for \quad k = 1, ..., N-1 \qquad (3.7)$$

By substituting the shifted fields into, it follows the lagrangian in terms of $\pi(x)$ and $\sigma(x)$ fields;

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\pi^k)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}(2\mu^2)\sigma^2 + \ \ higher \ \ orders \qquad (3.8)$$

This lagrangian describes a massive field σ and a set of $(N-1)$ massless π fields. A way to visualize that is to imagine a $(N1)$ dimensional sphere embodied in N dimensional space. Rotations on the surface of the sphere correspond to the $(N1)$ massless π fields, while radial fluctuations correspond to the massive σ field.The $(N1)$ directions on the sphere are equivalent. Thus, the original $O(N)$ symmetry is hidden , leaving the subgroup $O(N-1)$ as an unbroken symmetry.

3.2 The Goldstone Theorem

Consider a theory with scalar fields $\phi^{\alpha(x)}$.The lagrangian of this theory will be of the form:

$$\mathcal{L} = (derivative\ \ terms) - V(\phi) \quad (3.9)$$

Suppose that ϕ^α is a constant field that minimizes the potential V i.e
$(\frac{\partial V}{\partial \phi^\alpha})_{\phi_0^\alpha} = 0 \quad (3.10)$
The mass matrix comes by the expansion of V around this minimum:

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi_0)^a(\phi - \phi_0)^b(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b})_{\phi_0} + ... \quad (3.11)$$

The coefficient of the quadratic term is a symmetric matrix with eigenvalues equal to the masses of the fields:

$$m^2_{ab} = (\frac{\partial^2 V}{\partial \phi^a \partial \phi^b})_{\phi_0} \quad (3.12)$$

Since ϕ_0 corresponds to minimum, the eigenvalues cannot be negative. According to the Goldstone Theorem, every continuous symmetry of the lagrangian that is not a symmetry of ϕ_0 , leads to a zero eigenvalue of the mass matrix.
In order to prove the theorem, we consider a general continuous symmetry transformation, which has the form

$$\phi^\alpha \longrightarrow \phi^\alpha + g\Delta^\alpha(\phi) \quad (3.13)$$

where g is an infinitesimal parameter and $\Delta^\alpha(\phi)$ a function of ϕ's. We specialize to constant fields so the derivative terms can be ignored. Then, the potential V alone must be invariant under the transformation (3.13).This condition can be written as:

$$V(\phi^\alpha) = V(\phi^\alpha + \alpha\Delta^\alpha(\phi)) \quad or \quad \frac{\partial V(\phi)}{\partial \phi^\alpha}\Delta^\alpha(\phi) = 0 \quad (3.14)$$

By differentiating (3.14) with respect to ϕ_β and setting $\phi = \phi_0$ we have:

$$(\frac{\partial \Delta^\alpha}{\partial \phi^\beta})_{\phi_0}(\frac{\partial V}{\partial \phi^\alpha})_{\phi_0} + \Delta^\alpha(\phi_0)(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b})_{\phi_0} = 0 \quad (3.15)$$

Since ϕ_0 corresponds to minimum , the first term in (3.15) vanishes so the second term must be zero. If the transformation (3.13) is a symmetry of the ground state, then $\Delta^\alpha(\phi_0) = 0$ and the relation is trivial.However,if the symmetry is spontaneously broken by the vacuum, then $\Delta^\alpha(\phi_0) \neq 0$.In this case, $\Delta^\alpha(\phi_0)$ is the vector with zero mass eigenvalue i.e it corresponds to massless field , so the Goldstone theorem has proved.

One thing to notice is the relation between Δ^α and the generator of the transformation (3.13).The group element is written as:

$$R = e^{ig_i T^i} \qquad (3.16)$$

Then, the element for infinitesimal transformation is

$$R = I + ig_i T^i \qquad (3.17)$$

where T^i is the generator that corepónds to the parameter g_i. The transformation goes like this:

$$\phi' \longrightarrow U\phi = (1 + \mathbf{gT})\phi \qquad (3.18)$$

which leads to:

$$\delta\phi = igT \qquad (3.19)$$

It follows that $\Delta^\alpha(\phi_0) \sim T\phi_0$. Then we have:

$$m(T\phi_0) = 0 \qquad (3.20)$$

Thus , the generator acts on ϕ_0 and gives the direction of the massless field.

4. The Brout-Englert-Higgs Mechanism

The whole idea of the Goldstone theorem can be applied to theories with gauge symmetry. In this case, the gauge bosons can become massive particles by "eating" a Goldstone boson. The mechanism, by which spontaneous symmetry breaking generates a mass for a gauge boson is known as the Higgs mechanism. The basic application is the description of weak interactions. Experimentally, the W and Z bosons were found to have large masses, in contrast with the theory. The problem solved by the Glashow-Weinberg-Salam theory of weak interactions, which is an application of the Higgs mechanism to the gauge $SU(2)\times U(1)$ and finding results in agreement with the experimental data.

4.1 The Higgs Mechanism

First, two basic examples of the mechanism must be disqussed.

4.1.1 Abelian Example

Consider a complex scalar field ϕ coupled to the electromagnetic field and to itself. The lagrangian of the system is gauge invariant:

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 \left\|D_\mu\phi\right\|^2 - V(\phi) \ (4.1)$$

The field ϕ and A_μ transform as

$$\phi(x) \longrightarrow e^{i\alpha(x)}\phi(x) \quad (4.2)$$

$$A_\mu \longrightarrow A_\mu - \tfrac{1}{e}\partial_\mu \alpha(x) \quad (4.3)$$

while the covariant derivative D_μ is:

$$D_\mu = \partial_\mu + ieA_\mu \quad (4.4)$$

The potential is the well-known *mexican hat* potential:

$$V(\phi) = -\mu^2 \phi^* \phi + \tfrac{\lambda}{2}(\phi^*\phi)^2 \quad (4.5)$$

we take $\frac{\partial V}{\partial \phi^*}$ we find the minimum of the potential as:

$$\|\phi_0\|^2 = \tfrac{\mu^2}{\lambda} \ (4.6)$$

From all the possible ground states of (6) we choose the vacuum expectation value to be:

$$\phi_0 = \tfrac{\mu}{\sqrt{\lambda}} \quad (4.7)$$

By taking small fluctuations arround the vacuum (7) we have the shifted fields:

$$\phi(x) = \phi_0 + \tfrac{1}{\sqrt{2}}(\varphi_1(x) + i\varphi_2(x)) \quad (4.8)$$

By plugging (4.8) into the potential V we have the potential in terms of the shifted fields:

$$V(\phi) = -\tfrac{1}{2}\mu^4 + \mu^2\phi_1^2 + non \ \ quadratic \ \ terms \quad (4.9)$$

In (4.9) the field ϕ_2 has no mass term while ϕ_1 aquiring mass:

$$m_{\phi_1}^2 = 2\mu^2 \quad (4.10)$$

The kinetic energy term of the lagrangian, after using the field (4.8), becomes:

$$\|D_\mu \phi\|^2 = \tfrac{1}{2}\partial_\mu \phi_1^2 + \tfrac{1}{2}\partial_\mu \phi_2^2 + e^2\phi_0^2 A_\mu A^\mu + \sqrt{2} e \phi_0 A_\mu \partial^\mu \phi_2 + ... \quad (4.11)$$

The third term is the mass term of A_μ field, i.e the photon mass term:

$$m_{A_\mu}^2 = 2e^2\phi_0^2 \quad (4.12)$$

From (4.12) it follows that the gauge boson has non-zero mass when the vacuum expectation value of ϕ_0 is nonvanishing.

So far, we have mass terms for ϕ_1 and A_μ while ϕ_2 is massless. The *Goldstone* boson here corresponds to ϕ_2. However, we can use a gauge fixing called *Unitary gauge* which can eliminate the goldstone boson when it does not appear as an independent physical particle. Using the gauge symmetry according to (4.2) and (4.3) we can choose $\alpha(x)$ in a way that $\phi(x)$ becomes a real field, so ϕ_2 is removed.

Finally, we have a real scalar field ϕ so the lagrangian becomes:

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 + (D_\mu\phi)^2 - V(\phi) \qquad (4.13)$$

By expanding the covariant derivative we end up with the lagrangian

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 + (\partial_\mu\phi)^2 + e^2\phi^2 A_\mu A^\mu - V(\phi) \qquad (4.14)$$

that leads to massive photon for non-zero vacuum expectation value.

4.1.2 Non-Abelian Example

The example is about the $SU(2)$ gauge symmetry. We suppose a field ϕ that transforms as a spinor under $SU(2)$. The generators are the half the *Pauli* matrices so the covariant derivative takes the form:

$$D_\mu\phi = (\partial_\mu - igA^a_\mu T^a)\phi \qquad (4.15) \quad \textit{with} \quad T^a = \tfrac{\sigma^a}{2}$$

Here we introduced three boson fields, one for each generator of $SU(2)$. The lagrangian is as usual

$$\mathcal{L} = \|D_\mu\phi\|^2 - V(\phi) + D.T \qquad (4.16)$$

where $D.T$ is a dynamical term for the boson fields that respects the gauge $SU(2)$ symmetry. The potential V is:

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \qquad (4.17)$$

The potential has the degenerate minima:

$$\phi^\dagger\phi = \frac{\mu^2}{2\lambda} = \frac{v^2}{2} \quad (4.18) \qquad , v = \frac{\mu}{\sqrt{\lambda}}$$

From all of the equivalnet ground states of (4.18) we can choose one of them:

$$\phi_0 = \frac{1}{\sqrt{2}} \; (4.19)$$

The gauge boson masses comes from the kinetic term while using the vacuum expectation value (4.19). We want the quadratic terms of the boson fields A_μ^a . In the vacuum, we have :

$$D_\mu\phi_0 = -igA_\mu^a T^a \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad (4.20)$$

so it follows that:

$$(D_\mu\phi_0)^\dagger = igA_\mu^a \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \end{pmatrix} T^a \quad (4.21)$$

Then, we find the kinetic term:

$$\|D_\mu\phi\|^2 = \frac{1}{2}g^2 A_\mu^a A^{b\mu} \begin{pmatrix} 0 & v \end{pmatrix} T^a T^b \begin{pmatrix} 0 \\ v \end{pmatrix} + \quad ... \quad (4.22)$$

We have for for the anticommutator of the generators the well-known relation of the *Pauli* matrices that leads to:

$$T^a T^b + T^b T^a = \frac{1}{2}\delta^{ab} \quad (4.23)$$

By using (4.23) we have:

$$\|D_\mu\phi\|^2 = \frac{1}{4}g^2 A_\mu^a A^{a\mu} v^2 - \frac{1}{2}g^2 A_\mu^a A^{b\mu} \begin{pmatrix} 0 & v \end{pmatrix} T^b T^a \begin{pmatrix} 0 \\ v \end{pmatrix}$$

and by using the relation $(T^a)^2 = \frac{1}{4}$ we end up with the mass term "$\frac{1}{8}g^2 v^2 A_\mu^a A^{a\mu}$". Thus, the three gauge bosons will have masses:

$$m_{A_\mu} = \frac{gv}{2} \quad (4.24)$$

The three bosons are massive fields. This means that the three generators of $SU(2)$ are broken under the effect of the vacuum.

4.2 The G-W-S Theory of weak interactions

4.2.1 The gauge bosons masses

The theory gives a unified description of *weak* and *electromagnetic* intercations, in which the massless photon corresponds to a particular combination of symmetry generators that remain unbroken.

We begin with a theory with $SU(2)$ gauge symmetry.In order to break the symmetry spontaneously, we introduce a scalar field in the spinor representation of $SU(2)$. Like the non-abelian example we have:

$$D_\mu \phi = (\partial_\mu - igA_\mu^\alpha T^\alpha)\phi \qquad (4.15)$$

The theory leads to a system with no massless gauge bosons. For that reason, we introduce an additional $U(1)$ gauge symmetry. Now the gauge transformation is:

$$\phi \longrightarrow e^{ia^\alpha T^\alpha} e^{i\frac{\beta}{2}}\phi \qquad , \quad T^\alpha = \frac{\sigma^\alpha}{2} \qquad (4.25)$$

The vacuum expectation value has the form (like the non-abelian example):

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad (4.26)$$

The transformation (4.25) for $\alpha^1 = \alpha^2 = 0, \quad \alpha^3 = \beta$ gives:

$$\phi_0 \longrightarrow e^{\frac{i\beta}{2}} \begin{pmatrix} e^{\frac{i\beta}{2}} & 0 \\ 0 & e^{\frac{-i\beta}{2}} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \phi_0 \qquad (4.27)$$

We conclude that vacuum is invariant and for that reason we expect massless gauge boson that corresponds to this combination of generators. There are three more bosons, which will acquire mass with the Higgs mechanism. As usual, the gauge boson masses will rise from the kinetic term $\|D_\mu \phi\|^2$ evaluated at the vasuum expectation value of the scalar field. The $SU(2) \times U(1)_Y$ has the generators, according to the transformation (4.25):

$$T^\alpha = \frac{\sigma^\alpha}{2} \quad \text{and} \quad Y = \frac{I}{2} \quad (4.28)$$

Thus the covariant derivative becomes:

$$D_\mu \phi = (\partial_\mu - igA_\mu^\alpha T^\alpha - i\tfrac{1}{2}g' B_\mu)\phi \quad (4.29)$$

where A_μ^{alpha} and B_μ are the gauge bosons correspond to $SU(2)$ and $U(1)$ respectively. We compute $\|D_\mu \phi\|^2$ for the vacuum expectation value. The term of our interest, that will lead to the masses of gauge bosons, is:

$$\mathcal{L}' = \frac{1}{2}\frac{v^2}{4}[g^2(A_\mu^1)^2 + g^2(A_\mu^2)^2 + (-gA_\mu^3 + g' B_\mu)^2] \quad (4.30)$$

It was well-known from the experiments that the real particles are the W^+, W^-, Z^0 and the photon A_μ.Thus, from the fields A_μ^1 and A_μ^2 we introduce two new fields:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2) \quad (4.31)$$

We do the same for the remaining fields A_{μ^3} and B_μ

$$Z_\mu^0 = \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^3 - g' B_\mu) \quad (4.32)$$

and since the photon is massless:

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g' A_\mu^3 + gB_\mu) \quad (4.33)$$

Using the above definitions of the new fields we end up with the mass terms of the bosons:

$$m_W = \frac{gv}{2} \quad , \quad m_Z = \sqrt{g^2 + g'^2}\frac{v}{2} \quad , m_A = 0 \quad (4.34)$$

Now we rewrite the covariant derivative in terms of the new fields, including explicitly the generator of the $U(1)_Y$. In order to achive this, we seperate T^3 from T^1, T^2 and define the operators $T^\pm$:

$$T^\pm = T^1 \pm iT^2 \quad or \quad T^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2) = \sigma^\pm \quad (4.35)$$

At the end we find the covariant derivative as:

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{1}{\sqrt{g^2+g'^2}} Z_\mu (g^2 T^3 - g'^2 Y) - i\frac{gg'}{\sqrt{g^2+g'^2}} A_\mu (T^3 + Y) \quad (4.36)$$

In the last term of (4.36) the field A_μ couples to $(T^3 + Y)$. If we look again the transformation (4.25) in the case of $\alpha^1 = \alpha^2 = 0, \quad \alpha^3 = \beta$ and take the infinitesimal transformation we have:

$$U = (1 + \frac{i\beta\sigma^3}{2})(1 + \frac{i\beta I}{2}) = 1 + i\beta(T^3 + Y) \quad (4.37)$$

From (4.37) immediately follows that $(T^3 + Y)$ generates the symmetry operation that corresponds to the vacuum state ϕ_0. Thus there is a remaining $U(1)_{EM}$ symmetry. The remaining generator corresponds to the electric charge quantum number.

$$Q = T^3 + Y \quad (4.38)$$

We identify the coefficient of the electromagnetic interaction as the charge e:

$$e = \frac{gg'}{sqrtg^2+g'^2} \quad (4.38)$$

The *weak mixing angle* or *Weinberg angle* θ_W is defined by the transformation that takes us from the basis (A^3, B) to (Z^0, A). This transformation is just a rotatoin of angle θ_W:

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} cos\theta_W & -sin\theta_w \\ sin\theta_w & cos\theta_W \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix} \quad (4.39)$$

Thus, θ_W can be expressed in terms of the couplings g (weak isospin) and g'(weak hypercharge) as:

$$cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \quad , \quad sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad , \quad g = \frac{e}{sin\theta_W} \quad (4.40)$$

From the relations (4.40) and (4.34) we have the relation between the masses:

$$m_W = m_Z cos\theta_W \quad (4.41)$$

Lastly, the final form of the covariant derivative using (4.40) and (4.38) becomes:

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{cos\theta_W} Z_\mu (T^3 - sin^2\theta_W Q) - ieA_\mu Q \quad (4.42)$$

In brief, the initial $SU(2) \times U(1)$ gauge symmetry is broken by the vacuum ϕ_0. That leads to the mass terms of three gauge bosons. The fourth boson remained massless due to the $U(1)_{EM}$ unbroken generator Q which corresponds to the electric charge quantum number. The connection with reality comes with the definition of the new fields in a way that agrees with the experiments i.e the massive $W^{\pm}$, Z^0 and the massless photon.

4.2.2 The one-familly fermion masses

The lagrangian that describes fermions is the *Dirac* lagrangian with ψ being the so-called *Dirac* field:

$$\mathcal{L}_D = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \quad (4.43)$$

The Dirac field can be expressed in terms of a left and a right part ψ_L, ψ_R. The left-handed fermions are assigned to doublets of $SU(2)$, while the right-handed are singlets under $SU(2)$. The two parts couple differently to a gauge field since they belong to different representations of the gauge group. Since the right-handed fields are singlets they have:

$$T^3 = 0 \quad , \quad Q = Y \quad \text{for right-handed} \quad (4.44)$$

The one-familly contains the eletron and its neutrino e^-, ν_e and the top and down quarks u, d. Each one of these have a left-handed part and a right-handed part. We assume that neutrino is massless so it has only a left-handed part to form an $SU(2)$ doublet togrther with the electron.

The left-handed parts are:

$$E_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \qquad Q_L = \begin{pmatrix} u \\ d \end{pmatrix} \quad (4.45)$$

After the discussion above, a mass term like the following

$$\Delta\mathcal{L} = -m_e(\bar{e}_L e_R + \bar{e}_R e_L) \quad (4.46)$$

does not respect the $SU(2) \times U(1)$ gauge symmetry which GWS theory requires. The e_L and e_R are "living" in a different representation of $SU(2)$ and they have different $U(1)_Y$ charges.

The solution to this problem is again a mechanism of spontaneous symmetry breaking. A scalar field ϕ has a vacuum expectation value like in the case of GWS model:

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (4.26)$$

The field ϕ is a spinor under $SU(2)$ with $U(!)_Y$ charge $Y = \frac{1}{2}$ since it is neutral electrically charged. Now we can write an acceptable mass term for the fermions using the scalar field ϕ.

For the electron we have the gauge invariant term

$$\mathcal{L}_e = -\lambda_e \bar{E}_L \phi e_R + h.c \quad (4.47)$$

where λ_e is a new dimensionless coupling constant. The term is gauge $SU(2)$ invariant since $\bar{E}_L$ and ϕ are doublets, while e_R is singlet under $SU(2)$. It is also $U(1)$ gauge invariant, thus the term respects the $SU(2)\times U(1)$ gauge symmetry. By plugging in the vacuum expectation value (4.26) we end up with the electron mass term:

$$\mathcal{L}_e = -\frac{1}{\sqrt{2}}\lambda_e v \bar{e}_L e_R + hc + ... \quad (4.48)$$

It follows that the electron mass is given by:

$$m_e = \frac{1}{\sqrt{2}}\lambda_e v \quad (4.49)$$

The mass terms for the u, d quarks also come form invariant terms, a little more complicated than the electon's term since in the previous case we assume that neutrino ν_e is massless. However, the final mass terms, after evaluating in vasuum expectation value, are simmilar to the electron's:

$$\mathcal{L}_q = -\frac{1}{\sqrt{2}}\lambda_d v \bar{d}_L d_R - -\frac{1}{\sqrt{2}}\lambda_u v \bar{u}_L u_R \quad (4.50)$$

Putting all together we end up with the masses of the one-familly:

$$m_e = \frac{1}{\sqrt{2}}\lambda_e v \quad m_u = \frac{1}{\sqrt{2}}\lambda_u v \quad m_d = \frac{1}{\sqrt{2}}\lambda_d v \qquad (4.51)$$

One thing to notice is that the masses are proportional to v, but we know eperimentally that the electron is much lighter than the vector bosons of weak interactions. The up and down quarks are much lighter too. The λ coupling constants can be determined in a way that there is agreement with experiments. The GWS model cannot explain why the electron is so much lighter than W and Z.

4.2.3 The Higgs Field

The Higgs mechanism is essential in the GWS theory. For small fluctuations arround the vacuum of the scalar field ϕ, which introduced in the theory, a new scalar field rises; the Higgs field. The mass term of such a field will come from the potential $V(\phi)$

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \qquad (4.52)$$

evaluated in the vacuum expectation value ϕ_0:

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad , \quad v = (\frac{\mu^2}{\lambda})^{\frac{1}{2}} \qquad (4.53)$$

Using the unitary gauge discussed in the abelian example the field ϕ is a spinor with real componets. The most general complex-valued two component spinor can be written us

$$\phi = U(x)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \qquad (4.54)$$

where $U(x)$ is a general gauge $SU(2)$ transformation and $h(x)$ is the real valued fluctuating field which coresponds to the Higgs field. We can make a gauge transformation to eliminate $U(x)$ from the lagrangian. A lagrangian consistent with the vacuum expectation value of ϕ is:

$$\mathcal{L} = \|D_\mu\phi\|^2 + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \qquad (4.55)$$

In the unitary gauge, the potential evalueted in the vacuum gives:

$$\mathcal{L}' = -\mu^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 \quad (4.56)$$

We identify the mass term of the Higgs field:

$$m_h = \sqrt{2}\mu = \sqrt{2\lambda}v \quad (4.57)$$

The Higgs field upon quantization gives rise to the so-called Higgs boson; a massive, spin-zero and neutrually charged paticle that discovered in 2012.

5. Experimental Efforts and Discovery

The two large experiments that contributed the most to the Higgs boson detection was CMS and ATLAS at LHC ,the Large Hadron Collider at CERN.During 2011 , hints of the Higgs boson excistance appeared.Both ATLAS and CMS confirmed events of two photons with total energy of 125 GeV occurred as the product of proton collisions.The assumption is that there must be a new particle with mass of 125 GeV which decays into two photos.In addition to that, each experiment found events in which four charged leptons (electrons or muons) carried off simmilar total energies.There was a clear sign that something real is beneath all of these events.These two decay *modes* had the less background "noise" that could allow a Higgss signal detection. However, the signals detected in 2011 were not enough to conclude that a new particle was discovered.

In 2012 , the physicists of ATLAS and CMS managed computer systems to indetify the collisions relevant to the Higgs particle.When they studied the data of the rare photon photon decay and the four leptons decay , they found that a particle that is concistent with a 125 GeV Higgs boson as it described by the standard model.Both ATLAS and CMS independently conclude that the chances of a random fluctuation are causing the Higgs evidence were less than 1 in $3 \cdot 10^6$. Fianally, the discovery of a particle very much like the Higgs boson was real.

In 2013 , Francois Englert and Peter Higgs received the Nobel Price "for the theoritical discovery of a mechanism that contributes to our understanding of the origing of mass of subatomic particles and which recently was confirmed through the discovery of the predicted fundamental particle , by the ATLAS and CMS experiments at CERN's Large Hadron Collider".

Appendix

Gauge Symmetry

- Starting from the relations (2.6) and (2.7) :

$$n^\mu D_\mu \psi = \lim_{\epsilon \longrightarrow 0} \tfrac{1}{\epsilon}[\psi(x+\epsilon n) - U(x+\epsilon n, x)\psi(x)] \quad (2.6)$$

$$U(x+\epsilon n, x) = 1 - ie \cdot \epsilon n^\mu A_\mu(x) + \mathcal{O}(\epsilon^2) \quad (2.7)$$

It follows (2.8) :

$$n^\mu D_\mu \psi = \lim_{\epsilon \longrightarrow 0} \tfrac{1}{\epsilon}[\psi(x+\epsilon n) - \psi(x) + ie\epsilon n^\mu A_\mu(x)\psi(x)]$$

$$n^\mu D_\mu \psi = n^\mu \partial_\mu \psi + i n^\mu A_\mu \psi \Rightarrow n^\mu D_\mu \psi = n^\mu(\partial_\mu \psi + ieA_\mu \psi)$$

Thus:

$$D_\mu \psi(x) = \partial_\mu \psi(x) + ieA_\mu \psi(x) \qquad (2.8)$$

- From (2.4) and (2.7) :

$$U(y,x) \longrightarrow e^{i\alpha(y)} U(y,x) e^{-i\alpha(x)} \quad (2.4)$$

$$U(x+\epsilon n, x) = 1 - ie \cdot \epsilon n^\mu A_\mu(x) + \mathcal{O}(\epsilon^2) \quad (2.7)$$

It follows (2.9):

$$U(x+\epsilon n, x) \longrightarrow e^{i\alpha(x+\epsilon n)} U(x+\epsilon n, x) e^{-\alpha(x)}$$

$$1 - ie\epsilon n^{\mu} A_{\mu} \longrightarrow e^{i[\alpha(x+\epsilon n)-\alpha(x)]}(1 - ie\epsilon n^{\mu} A_{\mu})$$

For ϵ close to zero we can expand :

$$\alpha(x+\epsilon n) = \alpha(x) + n^{\mu}\epsilon\partial_{\mu}\alpha(x)$$

Then we have :

$$1 - ie\epsilon n^{\mu} A_{\mu} \longrightarrow e^{i\epsilon n^{\mu}\partial_{\mu}\alpha(x)}(1 - ie\epsilon n^{\mu} A_{\mu})$$

We now expand the exponential since ϵ is small and we get :

$$1 - ie\epsilon n^{\mu} A_{\mu} \longrightarrow (1 + i\epsilon n^{\mu}\partial_{\mu}\alpha(x))(1 - ie\epsilon n^{\mu} A_{\mu})$$

That leads to the final result (2.9) :

$$A_{\mu}(x) \longrightarrow A_{\mu}(x) - \tfrac{1}{e}\partial_{\mu}\alpha(x) \quad (2.9)$$

• From (2.2) , (2.8) and (2.9) :

$$\psi \longrightarrow e^{i\alpha(x)}\psi \qquad (2.2)$$

$$D_{\mu}\psi(x) = \partial_{\mu}\psi(x) + ieA_{\mu}\psi(x) \quad (2.8)$$

$$A_{\mu}(x) \longrightarrow A_{\mu}(x) - \tfrac{1}{e}\partial_{\mu}\alpha(x) \quad (2.9)$$

It follows the transformation rule of the *covariant derivative* :

$$\partial_{\mu}\psi(x) \longrightarrow e^{i\alpha(x)}[\partial_{\mu}\psi(x) + i\psi(x)\partial_{\mu}\alpha(x)]$$

$$ieA_{\mu}\psi(x) \longrightarrow e^{ia(x)}\psi(x)[ieA_{\mu} - i\partial_{\mu}\alpha(x)]$$

Then we end up with (2.10) :

$$D_{\mu}\psi(x) \longrightarrow e^{i\alpha(x)} D_{\mu}\psi(x) \quad (2.10)$$

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