A multi-layer game-theoretical framework for modeling wireless access and spectrum markets

Georgios Fortetsanakis

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University of Crete
Department of Computer Science

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Doctoral Thesis Committee: Maria Papadopouli, Associate Professor, University of Crete (Advisor)
Apostolos Traganitis, Professor, University of Crete
Costas Courcoubetis, Professor, Singapore University of Technology and Design
Anthony Ephremides, Professor, University of Maryland
Panagiotis Tsakalides, Professor, University of Crete
George Stamoulis, Professor, Athens University of Economics and Business
Xenofontas Dimitropoulos, Assistant Professor, University of Crete

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UNIVERSITY OF CRETE
DEPARTMENT OF COMPUTER SCIENCE

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Doctor of Philosophy degree in Computer Science

Author: ______________________________
Georgios Fortetsanakis

Examination Committee: ______________________________
Maria Papadopouli, Associate Professor, University of Crete

____
Apostolos Traganitis, Emeritus Professor, University of Crete

____
Costas Courcoubetis, Professor, Singapore University of Technology and Design

____
Anthony Ephremides, Professor, University of Maryland

____
Panagiotis Tsakalides, Professor, University of Crete

____
George D. Stamoulis, Professor, Athens University of Economics and Business

____
Xenofontas Dimitropoulos, Assistant Professor, University of Crete

Departmental approval: ______________________________
Angelos Bilas, Professor, Head of the Department of Computer Science

Heraklion, September 2016

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Abstract

Advances in networking and regulatory changes on access and competition rules enable new network architectures, service paradigms, and partnerships. Unlike traditional spectrum and wireless access markets, new types of markets are formed that have larger sizes, are more diverse, and can offer an improved set of services. The analysis of such markets is challenging due to a plethora of phenomena that manifest in different spatio-temporal scales. The main contribution of this thesis is the development of a modular multi-layer modeling framework for analysing wireless access and spectrum markets. This framework employs game theory, queueing-theoretical models, network economics, and clustering algorithms to instantiate a market at multiple levels of detail. It allows providers to distinguish user sub-populations with different profiles that depend on various parameters, such as, the willingness-to-pay, quality of service requirements, and traffic demand and model their decision making separately. As the number of user sub-populations that providers consider increases, the level of detail in the analysis also increases but so does the computational complexity. The goal of a provider is to select the appropriate level of detail that results in high revenue benefits and requires a low computational cost. The framework considers several providers, each potentially offering multiple dataplans. It also models the user decision making in a realistic manner assuming that they do not always make the optimal decisions in terms of the offered prices and quality of service. To reduce the computational complexity even further, it also develops a network decomposition methodology and algorithm based on the theorem of Norton. This algorithm computes equivalent queueing network models for a specific region of interest omitting the details of the entire networks of providers. Based on the modeling framework, various market cases with strong commercial interest have been analysed, e.g., WiFi offloading, secondary spectrum markets for capacity enhancement, pricing via market segmentation, and the flex service, a novel paradigm that allows users to dynamically select their provider. The analysis indicates that when there is a strong correlation between the user

willingness-to-pay and quality of service requirements, providers achieve revenue benefits when they model users with a larger number of sub-populations. When those parameters are independent, in some cases, providers may lose revenue when they model users in the same level of detail. This framework can be the basis of a software tool that enables providers to perform a detailed cost-benefit analysis of different market cases.

Extended abstract in Greek

Η πρόοδος στα ασύρματα δίκτυα σε συνδυασμό με αλλαγές στους ρυθμιστικούς κανόνες για την πρόσβαση στο φάσμα και τον ανταγωνισμό παρέχουν νέες δυνατότητες για δικτυακές αρχιτεκτονικές, υπηρεσίες και συνεργασίες μεταξύ των παρόχων. Σε αντίθεση με τις παραδοσιακές αγορές ασύρματης πρόσβασης και φάσματος, εμφανίζονται νέες αγορές που είναι μεγαλύτερες, πιο ετερογενείς, και παρέχουν νέες και βελτιωμένες υπηρεσίες. Η ανάλυση αυτών των αγορών είναι δύσκολη λόγω μίας πληθώρας φαινομένων που διαδραματίζονται σε διαφορετικές χωρικές και χρονικές κλίμακες. Η κύρια συνεισφορά αυτής της διδακτορικής διατριβής είναι η ανάπτυξη ενός αρθρωτού πλαισίου μοντελοποίησης για την ανάλυση των αγορών ασύρματης πρόσβασης και φάσματος. Το πλαίσιο μοντελοποίησης χρησιμοποιεί εργαλεία από τη θεωρία παιγνίων, τη θεωρία ουρών, τα οικονομικά δικτύων, καθώς και αλγορίθμους ομαδοποίησης για να μοντελοποιήσει αγορές σε διάφορα επίπεδα λεπτομέρειας. Συγκεκριμένα, επιτρέπει στους παρόχους να διακρίνουν υποπληθυσμούς χρηστών με διαφορετικό προφίλ το οποίο εξαρτάται από διάφορες παραμέτρους, όπως την προθυμία πληρωμής, τις απαιτήσεις σε ποιότητα υπηρεσίας και τη ζήτηση και να μοντελοποιούν τις αποφάσεις τους ξεχωριστά. Καθώς ο αριθμός των υποπληθυσμών αυξάνει, το επίπεδο λεπτομέρειας της ανάλυσης αυξάνεται όπως επίσης και η υπολογιστική πολυπλοκότητα. Ο στόχος ενός παρόχου είναι να επιλέξει το κατάλληλο επίπεδο λεπτομέρειας που οδηγεί σε σημαντικά οφέλη στο κέρδος απαιτώντας μικρό υπολογιστικό κόστος. Το πλαίσιο μοντελοποίησης θεωρεί πολλαπλούς παρόχους, καθένας από τους οποίους μπορεί να παρέχει πολλαπλά πακέτα χρέωσης. Επίσης, μοντελοποιεί τη συμπεριφορά των χρηστών με ένα ρεαλιστικό τρόπο θεωρώντας ότι δεν κάνουν πάντα τη βέλτιστη επιλογή με κριτήριο τις προσφερόμενες τιμές και την ποιότητα υπηρεσίας. Για να περιορίσει ακόμα περισσότερο την υπολογιστική πολυπλοκότητα, αναπτύσσει μία μεθοδολογία και ένα αλγόριθμο αποσύνθεσης δικτύων με βάση το θεώρημα του Norton. Αυτός ο αλγόριθμος υπολογίζει ισοδύναμα μοντέλα ουρών για τα δίκτυα μίας περιοχής ενδιαφέροντος, εξαλείφοντας τη λεπτομέρεια ολόκληρων των δικτύων των παρόχων. Με βάση το πλαίσιο μοντελοποίησης, μελετήθηκαν διάφορες τάσεις με ισχυρό εμπορικό ενδιαφέρον στις τηλεπικοινωνιακές αγορές όπως η αποφόρτιση (offloading) των κυψελωτών δικτύων μέσω WiFi, η συμμετοχή των παρόχων σε δευτερεύουσες αγορές φάσματος για την αύξηση της χωρητικότητας των δικτύων τους, η τιμολόγηση με βάση την τμηματοποίηση της αγοράς και το flex service, μία νέα υπηρεσία που παρέχει τη δυνατότητα στους χρήστες να επιλέγουν τον πάροχο τους δυναμικά. Η ανάλυση αναδεικνύει ότι στις περιπτώσεις όπου υπάρχει ισχυρή συσχέτιση μεταξύ της προθυμίας πληρωμής των χρηστών και των απαιτήσεων τους σε ποιότητα υπηρεσίας, οι πάροχοι επιτυγχάνουν σημαντικά οφέλη όταν μοντελοποιούν τους χρήστες με μεγάλο αριθμό υποπληθυσμών. Όταν όμως αυτές οι παράμετροι είναι ανεξάρτητες, σε κάποιες περιπτώσεις, οι πάροχοι μπορεί να καταγράψουν απώλειες κέρδους όταν μοντελοποιούν τους χρήστες στο ίδιο επίπεδο λεπτομέρειας. Το πλαίσιο μοντελοποίησης μπορεί να αποτελέσει τη βάση για την ανάπτυξη ενός λογισμικού που θα πραγματοποιεί μία λεπτομερή ανάλυση κόστους-οφέλους για την υιοθέτηση νέων τεχνολογικών τάσεων στις τηλεπικοινωνιακές αγορές.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 The landscape of wireless access and spectrum markets

The rapid growth of wireless technologies and mobile devices leads to an increased demand for wireless access. According to forecasts, by 2019, the mobile data traffic will exceed the 24.3 exabytes per month worldwide [1]. This explosion of the wireless traffic demand has triggered a plethora of research activities for expanding the capacity of wireless networks and improving the quality of service (QoS). To achieve this goal, an efficient allocation of the available electromagnetic spectrum should be performed. Besides performance reasons, the efficient spectrum utilization is important from an economic point of view: spectrum is a scarce resource of high economic value (its worldwide value is approximately 1 trillion USD) for both the society and the wireless industry with a wide variety of active business stakeholders.

Spectrum markets define the process through which providers acquire licenses to operate at certain portions of the spectrum. In traditional spectrum markets, the state assigns nation-wide licenses that are valid for a long time period to various interested parties through appropriate auction mechanisms [2, 3]. Due to these *static* allocation mechanisms, a large percentage of the available spectrum remains underutilized. To improve the efficient use of spectrum, more dynamic forms of spectrum access and trading have emerged.

In *secondary spectrum markets*, licence holders resell their spectrum access rights in fine spatial and temporal granularities [4, 5, 6, 7, 8, 9, 10]. A large research effort has been performed in the design of truthful, collusion-resistant, and effective auction mechanisms for secondary spectrum markets [4, 5, 6, 7]. Given the rapid changes in the user traffic demand, such markets can play an important role in improving the QoS of wireless networks.

To utilize the available spectrum more efficiently, opportunistic spectrum access paradigms can also be applied [11, 12]. In these paradigms, some intelligent devices called *cognitive radios* can function at licensed portions of the spectrum given that they do not affect the QoS of the licensed systems. Specifically, those devices can sense the licensed spectrum, find currently unused bands (called spectrum holes) and use them to communicate. On the other hand, the Body of European Regulators for Electronic Communications (BEREC) envisages measures for roaming users and consumer empowerment to boost consumer choices. Although traditionally users subscribe to or prepay for specific cellular operators/providers for network access, new access paradigms can be designed in which users will be able to select their provider dynamically each time they perform a new session.

To increase the capacity of wireless networks, new wireless access paradigms have been proposed including, *dynamic network selection*, *femtocells*, and *cooperative spectrum access*. In dynamic network selection, users can choose among a set of different available networks (e.g., 3G, LTE, and WiFi networks) to connect to the Internet [13, 14]. Users can dynamically select the best available network at small time scales (e.g., at the beginning of each session). That way a load balancing can be performed among the different networks improving the overall QoS.

The cooperation among users can also be a critical parameter in improving the performance of wireless networks. In the femtocell paradigm, users can access the Internet by connecting to the home wireless routers of other users instead of connecting to the cellular network of their provider [15, 16, 17]. This can significantly alleviate the congestion in cellular networks and improve the utilization of the existing wired network infrastructure. Additionally, in cooperative spectrum access, users can forward the traffic of other users forming a multi-hop network that expands the coverage of existing wireless networks [18, 19, 20, 21].

Another popular method for expanding the capacity of cellular networks is *mobile data offloading* [22, 23, 24, 25, 26, 27, 28, 29, 30]. To cope with the increase of the traffic demand, providers may serve a part of the data traffic that is originally targeted to their cellular networks by a complementary network infrastructure that has already been deployed (e.g., based on a WiFi network, femtocells). This is a cost effective alternative to the leasing of additional spectrum or the extension of the network infrastructure. Two types of WiFi offloading schemes have been proposed, namely, the *delayed offloading* and *on the spot offloading*. In delayed offloading, users wait until they are in the coverage of an access point (AP) until sending their delay-tolerant traffic [22, 23, 24, 25]. In on-the-spot offloading, users opportunistically transfer data via WiFi whenever they are in the coverage of an AP [26, 27]. It is also assumed that the offloading process can be initiated either by the service providers [27] or by the wireless users [30].

The cooperation among providers can also play an important role in the design of more efficient wireless networks and markets. One form of cooperation could be the leasing of network resources to other providers (e.g., access to a network infrastructure). Due to such types of agreements, a new type of service providers has appeared, the mobile virtual network operators (MVNOs) [31, 32]. These providers do not own a network infrastructure but sublease network resources from other providers. Another form of cooperation among providers is the infrastructure sharing [33]. Providers can share a part of their network infrastructure to reduce their capital and operational expenditures and improve their network coverage and QoS.

In general, the landscape of wireless markets is under drastic changes. New markets that are of larger size (in the number of users and providers), more heterogeneous (in the user population and services), and more dynamic are formed. The modeling of such markets is a challenging problem due to a plethora of technological and economic aspects that affect their evolution.

1.2 State of the art in modeling of wireless markets

There is a plethora of models for wireless markets in the literature [34, 35, 28, 23, 36, 13, 14, 18, 19, 20, 21, 4, 37, 38, 9, 39, 40, 41]. These models can be classified into two general categories, namely, the microscopic and macroscopic ones. Microscopic models usually focus on a short spatial and temporal scale and evaluate the impact of various technical aspects on the performance of a wireless market (i.e., on the revenue of providers and user satisfaction). Examples of such technical aspects include mobile data offloading [28, 23], femtocells [36], network selection mechanisms for users [13, 14], cooperative spectrum access schemes for primary and secondary users in cognitive radio networks [18, 19, 20], and multihop access paradigms [21]. Such models consider each provider and user as a distinct entity with its own characteristics and utility function. They perform a detailed analysis on a local scale and are accurate but usually not scalable.

On the other hand, macroscopic approaches focus on large-scale wireless markets and make various simplifications in the modeling of the user population to decrease the computational complexity [4, 37, 38, 9, 39]. For example, they either consider a homogeneous user population or provide models for the aggregate user traffic demand and behaviour. Other studies define a probability distribution expressing the different preferences of users with respect to the QoS [40, 41].

Unlike the traditional wireless markets, emerging ones are larger (in the number of users, providers, and devices), more heterogeneous (in terms of user profiles and services), and more complex and dynamic (e.g., in the interactions of providers and clients and their decision making). Modeling such markets is challenging due to various business, network, and service related phenomena that manifest at different spatio-temporal scales. Furthermore, the computational and scalability issues when analyzing the evolution of such markets at large scales (e.g., at a nation-wide level) are prominent. Modeling such markets microscopically or macroscopically may not be effective. Microscopic models are accurate but not scalable and can not be applied in large-scale or nation-wide markets. On the other hand, macroscopic models may result in inaccuracies due to the simplifications in the assumptions they make.

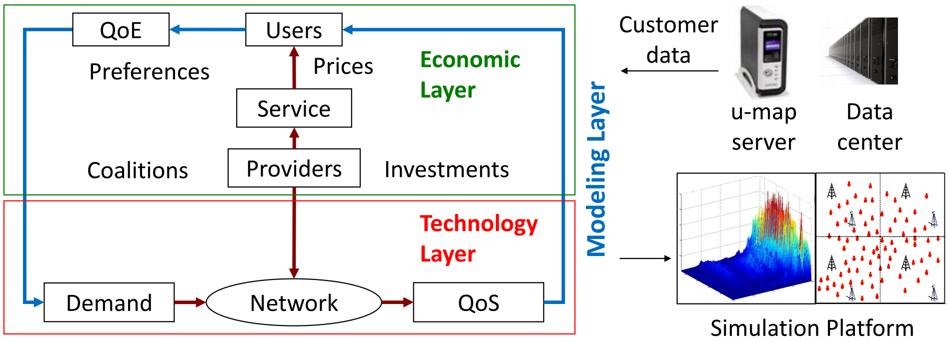


Figure 1.1: Modeling and analysis of wireless markets.

1.3 Main objectives of this Ph.D. thesis

In response to the above mentioned challenges, this Ph.D. thesis develops a multi-layer modeling framework for analysing the emerging wireless access and spectrum markets. The main objective is to design the framework to be general enough in order to study different trends in wireless markets with a strong commercial interest, such as, market segmentation, mobile data offloading, secondary spectrum markets, infrastructure sharing, and network virtualization. Using this modeling framework providers will be able to comparatively analyse different market cases and evaluate their benefits from adopting new wireless technologies and services.

Another important objective is to design the modeling framework to be multi-layer to address the tradeoff between scalability and accuracy. In contrast to the previous approaches that are either purely microscopic or macroscopic, this framework allows the instantiation of a market at multiple levels of detail. At the microscopic level, the various entities are modeled in fine temporal and spatial detail, while at the macroscopic level, entities are described as homogeneous populations. Between these levels, various *mesoscopic levels* are defined in which entities are modeled at different degrees of detail. By selecting the appropriate mesoscopic level, one can achieve the desired tradeoff between accuracy and computational complexity.

In summary, the objective of this thesis is to provide a methodology, algorithms, set of tools, and software that will allow providers to analyse different market cases in an accurate and computationally efficient manner. That way, they will be able to better design their business plan and strategies in the emerging wireless access and spectrum markets.

1.4 Methodology

The proposed framework models wireless markets at two distinct layers, the technological and the economic ones. The technological layer models the networks of providers as

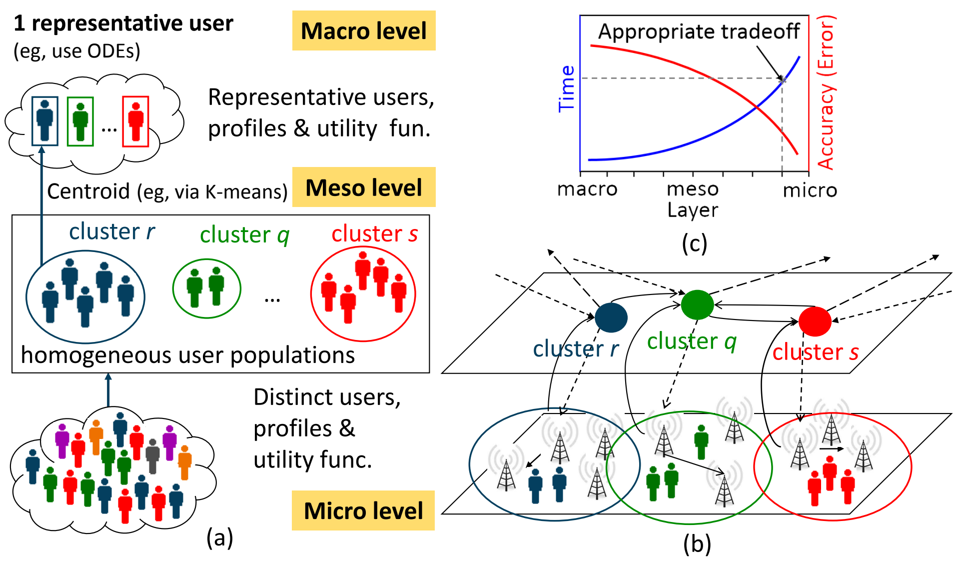


Figure 1.2: Multi-layer modeling framework for the analysis of wireless markets. (a) The user clusters can be derived based on hierarchical clustering algorithms. (b) A clustering approach on the spatial domain. (c) An example of the accuracy and scalability tradeoff (it does not correspond to real data but just illustrates the tradeoff).

queueing networks and the user traffic demand with appropriate stochastic processes. It also estimates the QoS that is offered by the providers. The economic layer models the interaction of users and providers using game theory. Providers design their dataplans and services and select their prices aiming to maximize their revenue, while users select the most appropriate provider that better satisfies their requirements with respect to the offered prices and QoS. The main components of the modeling framework are presented in Fig. 1.1. This figure has been inspired by the one that Jean Walrand provided in his tutorial talk in ACM Sigmetrics 2008.

To define the mesoscopic levels of detail, machine learning and data mining algorithms have been employed (Fig. 1.2). In a "coarse-graining" procedure that results to a loss of information in a controlled and hierarchical fashion, the individual entities of the microscopic level (e.g., users) are replaced by clusters with certain attributes (Fig. 1.2a). Instead of modeling the decision making of each distinct user, the mesoscopic levels consider a number of user clusters reducing significantly the computational complexity. Then, based on the requirements of a specific study, the appropriate mesoscopic level can be selected that achieves the desired tradeoff between accuracy and complexity (Fig. 1.2c).

To model the cellular networks of providers, and user traffic demand, the modeling framework also employs Markovian processes and queueing networks. In order to attain a succinct level of realism, often a large number of states needs to be taken into account. This results in computationally expensive operations. Under certain conditions (e.g., local bal-

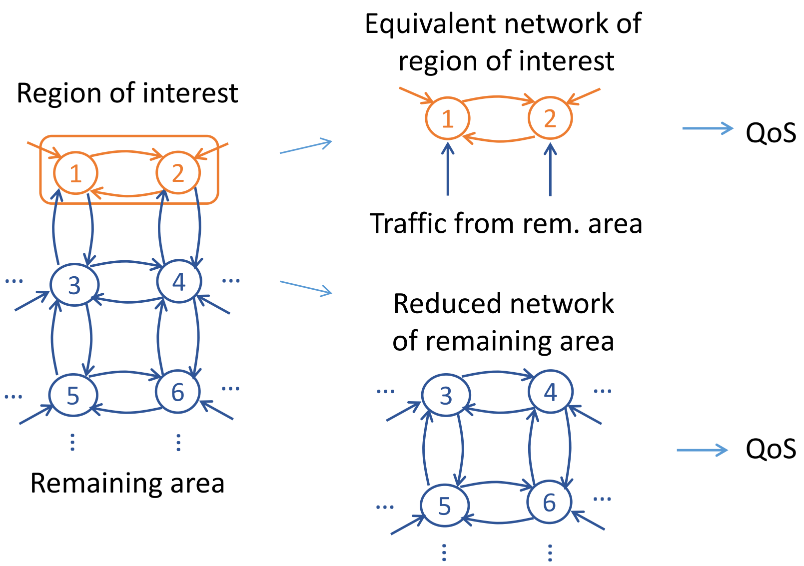


Figure 1.3: The network decomposition technique based on the theorem of Norton.

ance, quasi reversibility) equilibrium solutions possess a product form but break down if the state space becomes too large. This can be especially frustrating, as one is often interested in the characteristics that are related to the behaviour of parts of the networks of providers that depend only indirectly on the other states of these networks. Decomposition methods can be applied to restrict the analysis to the part of interest, provided that the accuracy of this isolated analysis remains tolerable. This thesis develops a network decomposition technique based on the theorem of Norton [42]. This technique estimates equivalent queueing network models for a specific region of interest and estimates the QoS of this region independently of the remaining area of the network (Fig. 1.3).

1.5 Technical challenges

As mentioned earlier, the analysis of wireless markets involves various technological and economic parameters (Fig. 1.1). Specifically, it incorporates models for the networks of providers, user traffic demand, and mobility pattern, as well as, the economic interactions of users and providers. The development of all those models requires knowledge from multiple scientific fields, such as, queueing theory, game theory, network economics, dynamical systems, and data mining. The design and implementation of the modeling framework involves various technical challenges related to the mathematical analysis as well as numerical analysis and programming-related issues.

The computation of the Nash equilibriums (NEs) of users and providers is in general a difficult problem. For example, continuous games can be analysed efficiently only under certain assumptions: The strategy space should be rectangular and the utility functions of players should be twice-continuously differentiable [43, 44, 45]. In some cases, those assumptions may be restrictive. For example, in markets in which users are completely rational in their service selection, the utility functions of providers have discontinues derivatives. To resolve this issue, we have provided a novel algorithm for the computation of the Nash equilibrium (NE) of providers presented in Chapter 7.

Another important mathematical issue is the existence of closed-form solutions for the equilibriums of users and providers. Although in many cases the conditions of equilibrium can be formulated as simple non-linear systems of equations, often such systems can not be solved analytically. In such cases, the equilibriums can only be computed numerically. This creates a computational complexity issue, as the numerical computation of the equilibriums of users and providers should be performed for each possible market configuration.

During the implementation of the framework, various other numerical-analysis related issues have emerged. For example, the computation of the NE of providers requires the numerical evaluation of the derivatives of the provider utility functions. Traditional methods for evaluating the derivatives, such as [46, 47], are computationally expensive. To resolve this issue, we have proposed a problem-specific treatment to efficiently compute those derivatives (described in the Appendix 7.D of Chapter 7).

The computation of the market equilibriums often requires the solution of non-linear systems of equations and optimization problems. Providing a good initial guess for the solutions of those problems is not a trivial task. A good initial guess can significantly reduce the execution time of computing a solution and in some cases, it can affect the ability of the algorithm to converge. To provide a good initial guess, often it is required to solve an additional optimization problem to find a point that is close enough to the solution, such as, the required execution time for the algorithm to converge is reduced.

Finally, during the implementation of the framework in matlab, some programming-related issues arose. To write computationally efficient programs in matlab, one should use matrices and vectors instead of the more conventional type of programming, like in C or Java, using "for loops". This can sometimes complicate the programming process. In general, it requires an additional effort to implement the modeling framework in a computationally efficient manner.

1.6 Contributions

This thesis makes several contributions:

1. It develops a multi-layer game-theoretical framework for analysing wireless markets. The framework is general and can be used to study various market cases with a strong commercial interest. It also allows providers to model users at different levels of detail

and controls the tradeoff between the accuracy and computational complexity. Except from modeling and analysing wireless markets, the tools and algorithms provided in this framework can be used as a basis to study market cases from other research areas (e.g., from the areas of public health and cloud computing).

2. It applies a network decomposition technique based on the theorem of Norton. This technique provides equivalent queueing network models for a specific region of interest omitting the details of the entire networks of providers. Often in the analysis of a market, providers are interested in the performance of a specific part of their network. In such cases, they can apply the theorem of Norton and restrict the analysis in their region of interest achieving significant computational gains.

3. It uses the multi-layer modeling framework to analyse the performance of three market cases, namely, pricing via market segmentation, WiFi offloading, and capacity planning. It performs an extensive set of experiments considering different scenarios with respect to the user profiles and utility functions, mobility pattern, traffic demand, and pricing strategies of providers. It also provides some conclusions and provider guidelines for those market cases.

4. It models the user decision making in a realistic manner. In contrast to most game-theoretical approaches that model users as rational entities, this thesis assumes that users are not completely rational. Except from the price and QoS, other psychological and social aspects affect the user decisions. The effect of all those aspects is captured by a noise parameter in the user decision making process. This thesis compares the performance of markets with completely rational entities with markets in which users are characterised by a certain degree of irrationality.

5. It develops an event-based simulator that can be used as an alternative method for analysing wireless markets. The simulator models in detail all the events that happen in a market including session generations and terminations, price adaptation of providers, and user service selections. It also models some phenomena at multiple levels of detail from the microscopic to the macroscopic one.

1.7 Other related research activities outside the context of this thesis

The research of this Ph.D. thesis is related with various other activities of the Mobile Computing group of ICS-FORTH, under the supervision of Prof. Maria Papadopouli. One relevant activity was the development of the u-map [48, 49], a crowdsourcing monitoring system that collects network measurements and correlates them with user quality of experience (QoE) feedback. In a real wireless market, the measurements collected by u-map can be used to train the models of the framework presented in this thesis. In other words, a system like u-map can be a useful tool for collecting the necessary data about the users to better design the models of their utility functions and behaviour and select their parameters (Fig. 1.1).

Another research activity was performed in collaboration with Neurocom S.A. The main goal of this activity was to investigate the parameters that affect the adoption of a new service in wireless markets. This research focused on studying the parameters that affect the user service selection, including the price and QoS as well as other psychological and social aspects, such as, brand name and brand equity, reputation of a provider, market share of a provider, length of a customer association with a provider, and force of habit.

1.8 Roadmap

The structure of this Ph.D. thesis is as follows: Chapter 2 presents the publications that are related to this Ph.D. thesis. Chapter 3 presents the multi-layer game-theoretical framework for analysing wireless markets. The framework considers detailed models for the networks of providers, user traffic, and mobility pattern. It also models the economic interactions of users and providers as a two-stage game. The first stage models the competition of providers through the price setting, while the second stage models the user service and provider selection. The framework allows providers to model users at multiple levels of detail by distinguishing different user sub-populations and modeling their decision making separately. It also models the user behaviour in a realistic manner assuming that they do not always make the optimal decisions in terms of price and QoS. Finally, the framework providers a network aggregation technique based on the theorem of Norton that computes equivalent queueing network models only for a specific region of interest omitting the details of the entire networks of providers. This results in significant computational gains.

The modeling framework is general and can be used to study various market cases with a strong commercial interest. Chapter 4 presents the analysis of a market in which providers apply a market segmentation approach for designing their dataplans and selecting their prices. Providers, distinguish various user sub-populations with different profiles and requirements and consider the decision making of those sub-populations when estimating their utility functions. That way, they target the most suitable market segments improving their revenue. The analysis indicates the benefits of providers from modeling users at different levels of detail and from offering a different number of dataplans.

Chapter 5 analyses the benefits of providers from applying the WiFi offloading, an alternative method for expanding the capacity of the cellular networks and improving the offered QoS. In WiFi offloading, a provider deploys a complementary network infrastructure that mainly consists of WiFi APs and femtocells and offloads a part of its data traffic to this infrastructure. This can significantly alleviate the congestion at the cellular network of the provider improving the QoS. The analysis evaluates the benefits of the offloading in a wireless oligopoly as well as its impact on the offered QoS and overall market equilibriums.

Chapter 6 focuses on the problem of capacity planning. Due to the spatial heterogeneity of the traffic demand, some regions of the networks of providers may become congested. To improve their QoS in these regions, providers may participate in a secondary spectrum market to purchase additional spectrum. In such markets, the spectrum allocation is performed according to an appropriate auction mechanism. This chapter analyses a VCG spectrum auction and evaluates the benefits of providers from participating in this auction at different market scenarios with respect to the user utility function and the available spectrum for sale.

To model wireless markets in a realistic manner, the framework assumes that users are not completely rational in their decision making. Specifically, it assumes that except form the price and QoS, various other psychological and social aspects also affect the user decisions. This assumption is meaningful and realistic in markets in which the decisions are made by real users. However, they may be some market cases in which the decision making is performed by software agents. Those agents make their decisions in a completely rational manner. Chapter 7 presents a modeling framework for analysing wireless markets with completely rational entities at the macroscopic level. It also uses this modeling framework to analyse a wireless oligopoly of a small city and compares its performance with other markets in which users are not completely rational.

Instead of performing a mathematical analysis of wireless markets, an alternative method for evaluating their performance can be applied using an event-based simulator. Chapter 8 presents such a simulator and describes in detail its main components. It also presents appropriate tools and algorithms to analyse wireless markets at multiple levels of detail from the microscopic to the macroscopic one. Based on this simulator, the performance of a wireless oligopoly of a small city was analysed. This market offers the flex service, a novel paradigm that allows users to select their provider dynamically each time they perform a session. The analysis indicates the benefits of the flex service from the perspective of users and providers. Finally Chapter 9 presents the concluding remarks of this Ph.D. thesis.

Chapter 2

Related publications

The main contributions of this Ph.D. thesis have been included in the following publications:

Paper 1:
Georgios Fortetsanakis and Maria Papadopouli, "Multi-Layer Game-Theoretical Analysis of Wireless Markets via Market Segmentation", submitted to the IEEE/ACM Transactions on Networking.

Abstract:
Market segmentation is important in the design of the marketing strategies of providers in the telecom industry: To improve their performance, providers often target specific user sub-populations to better satisfy their requirements and achieve higher revenues. However, existing models of wireless markets usually do not consider the effect of market segmentation on provider decisions. They are either microscopic, focusing on a specific technical aspect (e.g., protocol, network topology, technology) at a fine scale or macroscopic modeling wireless markets at a large-scale, e.g., considering homogeneous user populations. In contrast to these approaches, this work introduces a multi-layer game-theoretical framework. It allows providers to model users at different levels of detail by considering a different number of user sub-populations. It also models the behaviour of users in a realistic manner assuming that they do not always make the optimal decisions in terms price and quality of service (QoS). The analysis indicates that in general, the larger the number of the customer sub-populations that providers consider, the higher their revenue gains and reduction of disconnected users. However, in some cases, when providers consider the same number of user sub-populations, their competition becomes more prominent, resulting in revenue loss.

My contribution:
The idea came up from discussions with my advisor. I derived all the mathematical models under the supervision of my advisor, implemented them in matlab, and performed all the experiments presented in the performance evaluation. I also wrote most of the paper. My advisor provided many comments for the improvement of the writing and presentation as well as feedback on the performed experiments and results.

Paper 2:
Georgios Fortetsanakis, Ioannis Dimitriou, and Maria Papadopouli, "A Game-Theoretical Analysis of Wireless Markets using Network Aggregation", **IEEE Transactions on Mobile Computing** (to appear).

Abstract:
Modeling wireless access and spectrum markets is challenging due to a plethora of technological and economic aspects that affect their performance. This work develops a modeling framework for analysing such markets using network economics, game theory, and queueing networks. The framework models the service selection of users as well as the competition and coalition among providers. It also develops tools and algorithms to analytically compute the Nash equilibriums (NEs) under the presence of discontinuities in the derivatives of the utility functions of providers. The analysis of different market scenarios reveals various interesting trends in the offered prices, market share, and revenue of providers depending on the user utility function, traffic demand, and mobility pattern. It also demonstrates the role of the quality of service (QoS) in the user utility function in reducing the intensity of competition and allowing for higher prices and revenue. However, the analysis of large-scale markets exhibits a high computational complexity. To improve the computational efficiency, we developed a network aggregation methodology based on the theorem of Norton. This aggregation allows the construction of equivalent networks for a specific region of interest, omitting the details of the entire networks of providers. We demonstrate the aggregation algorithm in the context of capacity planning.

My contribution:
The idea came up from discussions with my advisor. I derived the mathematical models for the economic interaction of users and providers under the supervision of my advisor, while our collaborator Dr. Ioannis Dimitriou derived the mathematical models for the queueing networks of providers and the network aggregation technique based on the theorem of Norton presented in Sections 3.1 and 5.1, respectively. I implemented all the mathematical models in matlab and performed all the experiments. I also wrote most of the paper. My advisor provided much help in the editing process as well as comments on this work.

Paper 3:
Georgios Fortetsanakis and Maria Papadopouli, "On Multi-layer Modeling and Analysis of Wireless Access Markets", **IEEE Transactions on Mobile Computing**, Jan. 2015, vol.14, no. 1, pp. 113-125.

Abstract:
Advances in networking and regulatory changes on access and competition rules enable new network architectures, service paradigms, and partnerships, opening new opportunities for business cases. Unlike traditional cellular-based markets, new spectrum and wireless access markets are formed that have larger sizes, are more diverse, and can offer an improved set of services. The analysis of such markets is challenging due to a plethora of phenomena that manifest in different spatio-temporal scales. The main objective of this work is the development of a modular multi-layer modeling framework and simulation platform for analyzing wireless access markets. This framework employs game theory and queueing-theoretical models to instantiate a market at multiple spatio-temporal scales. At a microscopic layer, it models each entity of the market in a fine level of detail. By applying various aggregations, it also models the average behavior of certain clusters of entities. In that way, it can analyze a certain phenomenon at the appropriate level of detail, addressing the tradeoff between the loss of accuracy and computational complexity. The analysis then focuses on the flex service, a novel paradigm which allows users to select their provider dynamically. The proposed framework is used to model and analyze the performance of markets that offer the flex service. It employs various metrics, such as blocking probability, percentage of disconnected users, social welfare, and profit, to assess whether this service is beneficial to users, regulators, and providers, respectively. Furthermore, it highlights various challenges in modeling such markets and demonstrates the advantages in using the proposed multi-layer framework.

My contribution:
A long-term goal of the research of my advisor was the development of multi-layer models for telecom markets. This has triggered discussions from which the idea of this paper came out. I derived the mathematical models under the supervision of my advisor and implemented them in matlab. I also performed all the experiments. Our collaborator Dr. Ioannis Dimitriou provided feedback on numerical methods for estimating the stationary distribution of a finite-state two-dimensional Markov chain. The paper was written jointly with my advisor that also provided valuable feedback on the performed experiments.

Paper 4:
Georgios Fortetsanakis and Maria Papadopouli, "How Beneficial is the WiFi Offloading? A Detailed Game-Theoretical Analysis in Wireless Oligopolies", **IEEE WoWMoM**, Coimbra, Portugal, Jun. 2016.

Abstract:
The rapid advances in networking, mobile computing, and virtualization, lead to a dramatic increase in the traffic demand. A cost-effective solution for serving it, while maintaining a good quality of service (QoS), would be to offload a part of the traffic originally targeted for cellular base stations (BSs) to a WiFi infrastructure. Related work on the WiFi offloading often considers markets with a single provider and omits parameters, such as the effect of the offloading on the perceived QoS by users, the capacity of the WiFi infrastructure, and competition of providers. In contrast to these approaches, this paper develops a detailed modeling framework for analysing the WiFi offloading using network economics, game theory, and queueing networks. It also proposes a novel network aggregation technique to reduce the computational complexity of the analysis. Using this framework, the performance of WiFi offloading was evaluated under various scenarios with respect to the bandwidth of BSs and APs, coverage of WiFi, and user preferences. Our results highlight that it is not always profitable for providers to invest in a large WiFi infrastructure. The limited capacity of the WiFi APs restricts the benefits of the offloading.

My contribution:
I came up with the idea behind this paper with help from my advisor. I derived all the mathematical models under the supervision of my advisor and implemented them in matlab. I also performed all the experiments and wrote most of the paper. My advisor provided comments for the improvement of the writing and presentation. Additionally, I presented the paper at IEEE WoWMoM 2016.

Paper 5:
Georgios Fortetsanakis, Maria Papadopouli, Gunnar Karlsson, Manos Dramitinos, and Emre A. Yavuz, “To Subscribe, or not to Subscribe: Modeling and Analysis of Service Paradigms in Cellular Markets”, **IEEE DySPAN**, Bellevue, Washington, Oct. 2012.

Abstract:
Traditionally customers subscribe to specific providers and are served by accessing base stations (BSs) of the network of their provider. Inevitably subscribers with relatively “high” usage pattern and data-rate requirements are subsidized by the ones with lower usage and data-rates. As the wireless technology advances, a diverse set of services will be available. This paper introduces the “flex service” paradigm that allows a customer to dynamically access BSs of different providers based on various criteria, such as profile, network conditions, and offered prices. “Flex users” can select the appropriate provider and BS on a per-session basis. This work considers a diverse customer population with respect to their demand, their preference on data-rate over price, their tolerance on the blocking probabilities of their sessions, and their willingness-to-pay for certain services. Users can dynamically decide to buy a long-term subscription or become flex users. In this paper, we develop a rich framework for modeling and analysis of such markets in

different spatio-temporal scales. We analyse the evolution of markets with the flex service paradigm, focusing on whether it can improve the quality-of-service (QoS), social welfare, flexibility and further enhance the competition among providers. The main contribution of this paper is detailed modeling and in-depth performance analysis of such complex markets, in different spatial and temporal scales. It considers the perspective of clients, providers, and regulators. It demonstrates the benefits of markets with the flex service paradigm and compares them with the ones that only offer subscription contracts.

My contribution:
The idea came up from discussions with my advisor, Prof. Gunnar Karlsson, and Dr. Emre A. Yavuz during my visit at KTH Royal Institute of Technology during the summer of 2011. I derived all the mathematical models under the supervision of my advisor and implemented them in matlab. I also performed all the experiments. Dr. Manos Dramitinos and Dr. Emre A. Yavuz provided comments and feedback on this work. The paper was written jointly with my advisor. I also presented the paper at IEEE DySPAN 2012.

Paper 6:
Georgios Fortetsanakis, Markos Katsoulakis, and Maria Papadopouli, "A Novel Multi-Layer Framework for Modeling the Evolution of Spectrum Markets and Cognitive-Radio Devices", **IEEE DySPAN**, Aachen, Germany, May 2011.

Abstract:
This work presents a novel multi-layer modelling framework for the evolution of spectrum markets of multiple spectrum/network operators that provide wireless access to users. It integrates models of the channel, mobility, user preference, network operators (providers), infrastructure deployment, user distribution, and price-adaptation mechanisms. Providers aim to maximize their own profit, while clients decide based on criteria, such as the financial cost of the access, transmission rate, and required transmission power. This paper gives a brief description of the modelling framework and a novel price-adaptation algorithm for providers. It presents how this framework can be used to instantiate a cellular-based market in a small city. Finally, it analyzes the evolution of this market under different topologies and user profiles, summarizing the main performance results.

My contribution:
The idea for this paper came up from discussions with my advisor and Prof. Markos Katsoulakis. I derived the mathematical model under the supervision of my advisor and implemented it in matlab. I also performed all the experiments. The paper was written jointly with my advisor. I also presented the paper at IEEE DySPAN 2011.

Paper 7:
Michalis Katsarakis, Georgios Fortetsanakis, Paulos Charonyktakis, Alexandros Kostopoulos, and Maria Papadopouli, "On User-Centric Tools for QoE-Based Recommendation and Real-Time Analysis of Large-Scale Markets", **IEEE Communications Magazine**, Sep. 2014.

Abstract:
This article focuses on mechanisms that empower users with Quality of Experience (QoE) recommendations and smart real-time analytics. It presents a user-centric recommendation system (called u-map) that enables users to collect network measurements and subjective opinion scores about the performance of various services. It also cross-correlates measurements obtained by u-map to provide geo-statistics, user profiles, and QoE prediction models for different services. The article also presents CoRLAB, a modular multi-layer framework for modeling and assessing various markets, services, and their evolution under a diverse set of customer populations and conditions. U-map feeds CoRLAB with user measurements and feedback in (semi) real-time. The article discusses how u-map and CoRLAB have been used to analyze telecommunication markets and services. It highlights the main research results, challenges, and potential research directions.

My contribution:
The idea behind the paper came up from my advisor. I wrote the Section 3 which is related to the contributions of my Ph.D. thesis and helped in the writing of the remaining Sections by providing comments.

During my M.Sc. and Ph.D. studies at the department of Computer Science of the University of Crete and at the Institute of Computer Science of the Foundation for Research and Technology-Hellas, I also participated in some other research activities in the mobile computing group that resulted in several publications:

Paper 8:
Alexandros Kostopoulos, Ioanna Papafili, Georgios Fortetsanakis, and Maria Papadopouli, "Pricing Wireless Access Services: The Effect of Offloading and Users' Bounded Rationality", **IEEE/IFIP WONS**, Cortina d' Ampezzo, Italy, January 2016.

Abstract:
Offloading through WiFi access networks has been recently proposed as a cost-effective solution for coming up against the unprecedented increase in the mobile data traffic volume. However, apart from reducing the operational costs of a network operator, WiFi access can be also promoted as an alternative low-cost service for users with low willingness-to-pay.

In this paper, we consider a monopolistic scenario of a Mobile Virtual Network Operator (MVNO) offering LTE and WiFi access services and make the optimal pricing decisions. We further show that the presence of reluctant users to switch to the WiFi service could increase the profits of the MVNO.

My contribution:
Our collaborator Dr. Alexandros Kostopoulos jointly with my advisor came up with the idea behind this paper. I provided comments on the mathematical model and on the writing of the paper.

Paper 9:
Georgios Fortetsanakis, Michalis Katsarakis, Maria Plakia, Nikos Syntychakis, and Maria Papadopouli, "Supporting Wireless Access Markets with a User-Centric QoE-Based Geo-Database", **ACM MobiArch**, Istanbul, Turkey, Aug. 2012.

Abstract:
This paper presents the u-map, a novel user-centric geo-database for wireless access markets that enables clients to upload information about their profile, their quality-of-experience (QoE) feedback for a service, traffic demand, network/spectrum conditions (e.g., interference, coverage), providers, and their position (e.g., GPS-based measurements) in a spatio-temporal geo-database. To evaluate the impact of the u-map on wireless access markets and study the evolution of such markets, we have developed an economic-driven modeling framework. The framework integrates models of the channel, clients and network operators, wireless infrastructures, types of interaction, and price adaptation in a modular manner. We have implemented a simulation platform based on this framework and instantiated a cellular-based duopoly. Via simulations, we analyzed the impact of the u-map, user profiles, and network coverage of providers on the evolution of the market. The analysis demonstrates that the u-map can be beneficial to users in their network operator selection process. We also developed a proof-of-concept implementation of the u-map and performed a preliminary analysis. Finally, the paper highlights the research directions that need to be explored for developing a robust and effective mechanism.

My contribution:
The idea behind this paper came up from my advisor. I wrote the Section 4 of the paper which is related to my research, implemented the respective models and performed the experiments discussed in Section 4.2. I also contributed in the writing of the other Sections of the paper and provided comments. Additionally, I presented the paper at ACM MobiArch 2012.

Paper 10:
Ilias Tsompanidis, Georgios Fortetsanakis, Toni Hirvonen, and Maria Papadopouli, "A Comparative Analysis of the Perceived Quality of VoIP under various Wireless Network Conditions", **International Conference on Wired/Wireless Internet Communications (WWIC)**, Lulea, Sweden, June 2010.

Abstract:
This paper performs a comparative analysis of the perceived quality of (unidirectional, non-interactive) VoIP calls under various wireless network conditions (e.g., handover, high traffic demand). It employs the PESQ tool, E-model and auditory tests to evaluate the impact of these network conditions on the perceived quality of VoIP calls. It also reveals the inability of PESQ and E-model to capture the quality of user experience. Furthermore, it shows that the network condition and the evaluation method exhibit statistically significant differences in terms of their reported opinion score values. Finally, the paper highlights the benefits of the packet loss concealment of the AMR 12.2 kb/s and the QoS mechanisms under these network conditions.

My contribution:
The idea for this paper came up from my advisor and our collaborator Ilias Tsompanidis. I contributed in the writing of this paper and derived a mathematical model for the probability of packet loss.

Paper 11:
Ilias Tsompanidis, Georgios Fortetsanakis, Toni Hirvonen, and Maria Papadopouli, "Analyzing the Impact of various Wireless Network Conditions on the Perceived Quality of VoIP", **IEEE LANMAN**, Long Branch, New Jersey, USA, May 2010.

Abstract:
This paper focuses on a comparative statistical analysis of the performance of VoIP calls under various situations, namely, during a handover and under different background traffic conditions at a wireless access point (AP). Using empirical-based measurements, it demonstrates that these network conditions exhibit distinct statistical behaviour, in terms of SNR, packet losses and end-to-end delays, and thus, impact the VoIP user quality in a different manner. The analysis shows that both network conditions and codec type, as well as their interaction, have a significant effect on the PESQ MOS values. Moreover, it indicates statistically highly significant differences between the estimations of the PESQ and E-model. Finally, it highlights the benefits of the packet loss concealment of the AMR 12.2 kb/s under these network conditions.

My contribution:
The idea for this paper came up from my advisor and our collaborator Ilias Tsompanidis. I contributed in the writing of this paper and derived a mathematical model for the probability of packet loss.

Chapter 3

Modeling framework

3.1 Modeling requirements and objectives

In the design of our modeling framework, we have set several objectives. First of all, the framework should be general and expressive enough to model a variety of market cases with different wireless technologies, access paradigms, and services. This will allow providers to comparatively analyse different market scenarios and evaluate the profitability of various strategies, such as, adopting a new wireless technology, offering a new service, or investing in additional network infrastructure and spectrum.

Another important aspect in the design of the framework is the degree of realism. The models of the networks of providers, user traffic demand and mobility, as well as the economic interactions of users and providers should be realistic and capture the phenomena that manifest in real wireless markets. This will result in accurate and meaningful predictions that will be useful to the providers.

The scalability is another desired property of the framework. Providers should be able to analyse different market scenarios at multiple spatial and temporal scales, from the market of a small neighbourhood up to a nation-wide market. This raises accuracy and computational complexity issues. For example, a microscopic model is accurate but requires a large computational complexity especially in the case of large-scale markets. On the other hand, a macroscopic model can be applied on a large-scale but due to its simplifications, it may result in inaccuracies. Therefore, it is important to be able to select the appropriate level of detail when modeling a market of interest that will result in a good tradeoff between accuracy and computational complexity.

We have developed a game-theoretical modeling framework for wireless markets that satisfies the above objectives. The framework defines multiple levels of detail that are called *mesoscopic*. At each mesoscopic level, a provider applies a clustering algorithm on the profiles of users and determines a set of clusters. Each of those clusters is modeled as a

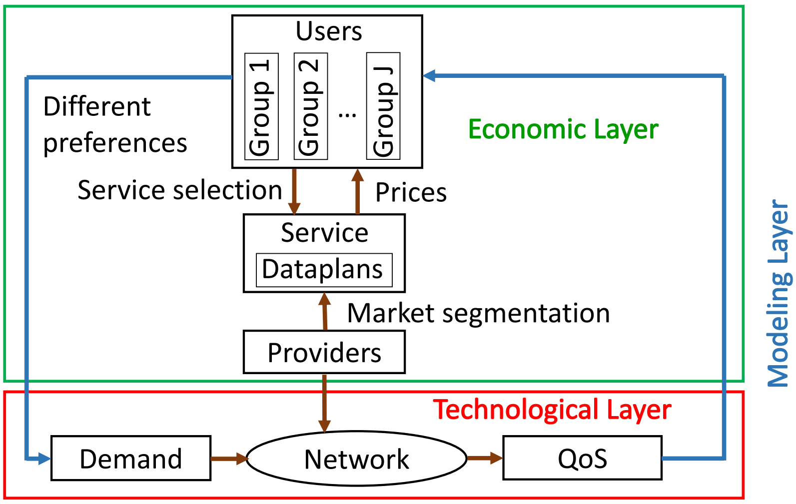


Figure 3.1: The main modules of wireless markets.

homogeneous sub-population with a *representative profile* that is the centroid of the profiles of all users belonging in that cluster. Then, the provider can model the decision making of these clusters of users by a system of ordinary differential equations. Using our modeling framework, providers can select the appropriate level of detail that improves their revenue while requiring a low computational complexity.

Except from the user heterogeneity, another aspect that can result in an increased computational complexity when analysing wireless markets is the large size of the networks of providers. However, often, in a study, the objective is the analysis of the performance of a specific region of interest. In such cases, we can apply network aggregation techniques that compute equivalent network models only for the region of interest omitting the details of the entire networks of providers. In this chapter, we develop such an aggregation technique based on the theorem of Norton [42] for queueing networks. This technique can result in significant computational gains.

The structure of this chapter is as follows: Section 3.2 presents our multi-layer game-theoretical framework for modeling wireless markets, while Section 3.3 presents the network decomposition technique based on the theorem of Norton. Finally, Section 3.4 presents the concluding remarks.

3.2 Multi-layer modeling of wireless markets

The evolution of wireless access and spectrum markets is affected by a variety of technological and economic aspects. Our framework provides elaborate models of those aspects distinguishing two different layers, the technological and the economic one (Fig. 3.1). The technological layer models the cellular networks of providers as queueing networks and the user traffic demand with appropriate stochastic processes. It also estimates the QoS of providers based on the average and variance of data rate. The economic layer models the interaction among a set of providers and a heterogeneous user population. Each provider can model users at different levels of detail by considering a different number of user sub-populations with distinct characteristics and preferences. It also offers different dataplans to users that produce different traffic demand and selects their prices aiming to maximize its revenue, while each user selects a provider considering the offered prices and quality of service (QoS) guarantees.

In our framework, the user population is divided into segments, each having different characteristics and preferences. For example, the *business user* segment is characterized by a high willingness-to-pay and QoS requirements, while the *low-profile user* segment has a low willingness-to-pay and a high tolerance on a degraded QoS. The *value-for-money user* segment is characterized by a low willingness-to-pay and high QoS requirements, while the *lenient user* segment has a high willingness-to-pay and high tolerance on a degraded QoS. Throughout this chapter, we use the terms users and customers interchangeably (similarly for the terms group and segment).

A *two-stage game* defines the interaction of users and providers. The first stage instantiates the *competition of providers* and the second one the *user decision making*. A *population game* models the user decisions: members of each user group could either select to become subscribers of a certain provider or remain disconnected. The decisions of these users are modeled by the Logit dynamics, a system of ordinary differential equations. They are based on a utility function that depends on the price and QoS and a noise parameter that defines how much trust users place on this utility function, capturing the user "irrationality" and "stickiness" to a provider. On the other hand, the competition of providers is modeled as a *normal-form game* in which providers strategically select their prices to optimize their revenue. The utility functions of providers depend on the offered prices and the equilibrium of users (Fig. 3.2). Our framework models a wireless access market of I providers and a user population of J groups. Each provider has deployed a network of wireless BSs and offers long-term subscriptions, which are *best-effort data services*. The following subsections describe the components of our modeling framework in more detail.

3.2.1 The queueing networks of providers

Each provider (e.g., provider i) has deployed a number of BSs (K_i) covering a geographical region (e.g., a city). We assume that in all BSs, the available bandwidth is shared equally among connected users (i.e., processor-sharing discipline). For LTE cellular BSs, this bandwidth allocation models a scheduler that divides the OFDMA resources among users in a fair manner. Such fair allocation of bandwidth among connected users has been also assumed in other studies of wireless markets (e.g., [50]). Users generate requests to connect to a base station (BS) to start a session. During a session, a user transmits and receives data via that BS. The session generation of a group of users j follows a Poisson process with a

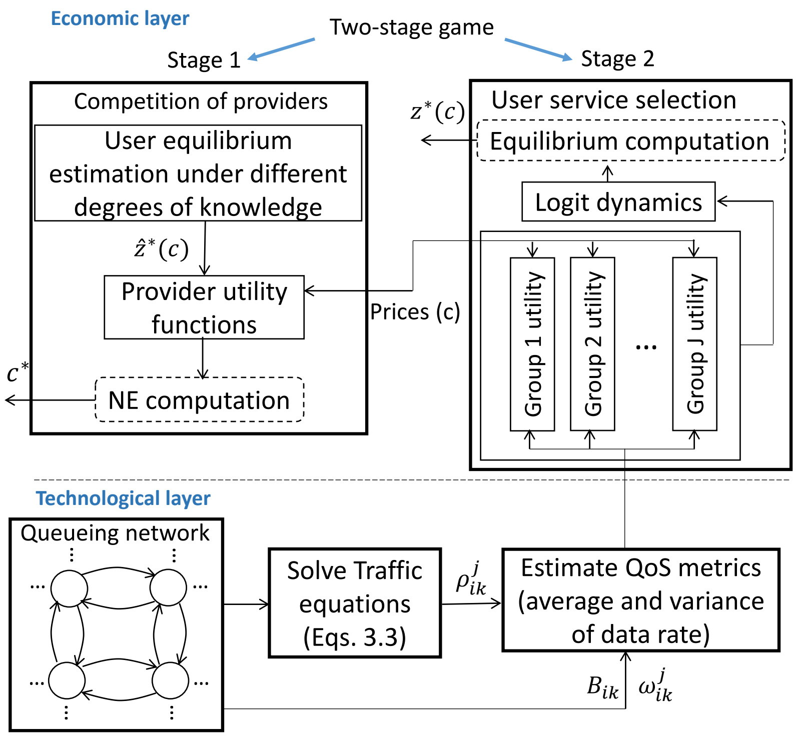


Figure 3.2: Main components of the modeling framework.

total rate of λ_j. This rate is allocated across providers according to the probability vector $z_j = (z_{j0}, z_{j1}, ..., z_{jI})$. The ratio of members of the group j that select the provider i is indicated by z_{ji}, while z_{j0} indicates the ratio of members of that group that select the disconnection. The vector $z = (z_1, ..., z_J)$ corresponds to the probability vector of each user group and shows how the entire user population is divided among the available providers and disconnection state.

Each provider (e.g., provider i) has deployed a number of BSs (K_i) covering a geographical region (e.g., a city). We assume that in all BSs, the available bandwidth is shared equally among connected users (i.e., processor-sharing discipline). For LTE cellular BSs, this bandwidth allocation models a scheduler that divides the OFDMA resources among users in a fair manner. Such fair allocation of bandwidth among connected users has been also assumed in other studies of wireless markets (e.g., [50]). Users generate requests to connect to a base station (BS) to start a session. During a session, a user transmits and receives data via that BS. The session generation of a group of users j follows a Poisson process with a total rate of λ_j. This rate is allocated across providers according to the probability vector $z_j = (z_{j0}, z_{j1}, ..., z_{jI})$. The ratio of members of the group j that select the provider i is indicated by z_{ji}, while z_{j0} indicates the ratio of members of that group that select the disconnection. The vector $z = (z_1, ..., z_J)$ corresponds to the probability vector of each user group and shows how the entire user population is divided among the available providers and disconnection state.

The mobility of members of the group j in the network of a provider is modeled with a Markov-chain in which a state corresponds to the coverage area of a BS. The total session generation rate of the members of the group j that select the provider i is further divided among its BSs ($k = 1, ..., K_i$) according to the probabilities ω^j_{ik}. These probabilities correspond to the stationary distribution of the Markov chain modeling the user mobility. Note that the handovers at a BS k of the provider i are modeled with a Poisson process of total rate v_{ik}. This rate is estimated according to the fluid flow mobility model [51]. We also assume that handovers are performed in a seamless manner. Table 3.1 defines the queueing-theoretical parameters for the members of the group j when connected at the network of the provider i. Let us now focus on a simple case in which all users select the provider i (i.e., $z_{ji} = 1$ for all $j = 1, ..., J$). The total session arrival rate at a BS k from members of the group j (γ^j_{ik}) consists of the new sessions ($a^j_{ik} = \omega^j_{ik}\lambda_j$) and handover sessions from neighbouring BSs (Fig. 3.3):

Table 3.1: **Queueing-theoretical parameters for users of group j when connected at the network of provider i**

Parameter	Description
K_i	Number of BSs of the provider i
λ_j	Total session generation rate
$z_{ji}(z_{j0})$	Ratio of subscribers (disconnected users)
ω^j_{ik}	Steady-state probability for a user of group j to be located within the coverage of BS k
v_{ik}	Departure rate from BS k due to handover
μ_{ik}	Session service rate at BS k
d_{ik}	Total departure rate from BS k ($d_{ik} = v_{ik} + \mu_{ik}$)
$p^{(j)*}_{i,m,k}$	Conditional prob. of handover from BS m to BS k given that a handover occurs
$p^{(j)}_{i,m,k}$	Unconditional prob. of handover from BS m to BS k $\left(p^{(j)}_{i,m,k} = v_{im}p^{(j)*}_{i,m,k}/d_{im}\right)$
γ^j_{ik}	Total session arrival rate at BS k
a^j_{ik}	Arrival rate of new sessions at BS k
ρ^j_{ik}	Traffic intensity at BS k
n_i	Vector indicating the number of users at each BS
$Q_i(n_i)$	Stationary distribution of number of users at BSs
B_{ik}	Bandwidth at BS k
$R_{ji}(z)$	Average data rate
$V_{ji}(z)$	Variance of data rate

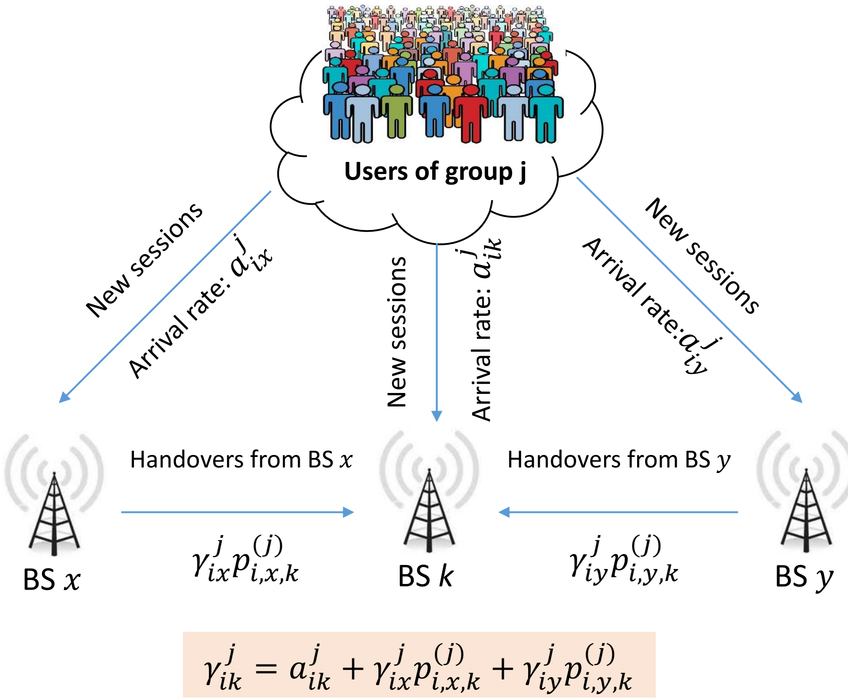


Figure 3.3: Session arrivals at a BS of the provider i from members of the group j.

$$\gamma^j_{ik} = a^j_{ik} + \sum_{m=1}^{K_i} \gamma^j_{im} p^{(j)}_{i,m,k} \tag{3.1}$$

The traffic intensity generated by the users of the group j at the BS k of the provider i (ρ^j_{ik}) is equal to the ratio of the total session arrival rate at the BS k from members of the group j (γ^j_{ik}) over the total session departure rate at that BS (d_{ik}). However, to characterize the performance of the network of the provider i, the total traffic intensity introduced by all user groups at each BS needs to be estimated. By summing the Eq. 3.1 over all user groups, we derive the traffic equations for the network of the provider i:

$$\sum_{j=1}^{J} \gamma^j_{ik} = \sum_{j=1}^{J} a^j_{ik} + \sum_{m=1}^{K_i} \sum_{j=1}^{J} \gamma^j_{im} p^{(j)}_{i,m,k} \tag{3.2}$$

We now define the total session arrival rate at the BS k of the provider i from all user groups $\gamma_{ik} = \sum_{j=1}^{J} \gamma^j_{ik}$, the total arrival rate of new sessions at the BS k of the provider i $a_{ik} = \sum_{j=1}^{J} a^j_{ik}$, and the average unconditional probability of a handover from the BS m to the BS k over all user groups $p_{i,m,k} = \sum_{j=1}^{J} \gamma^j_{im} p^{(j)}_{i,m,k} / \gamma_{im}$. The corresponding average conditional probability of a handover from the BS m to the BS k given that a handover occurs is defined as $p^*_{i,m,k} = p_{i,m,k} d_{im} / v_{im}$ Then, the traffic equations can be rewritten as in Eq. 3.3:

$$\gamma_{ik} = a_{ik} + \sum_{m=1}^{K_i} \gamma_{im} p_{i,m,k} \tag{3.3}$$

The queueing network of the provider i is modeled as a Markov chain. Each state corresponds to a vector $n_i = (n_{i1}, ..., n_{iK_i})$ indicating the number of connected users at all BSs. State transitions correspond to various types of events including session arrivals, terminations, and handovers. The stationary distribution of the Markov chain is estimated by solving the global-balance equations. Such equations set the arrival rate at each state of the Markov chain equal to the departure rate from that state. Due to the Markovian property and the processor-sharing discipline of our system, the global-balance equations can be simplified into a set of local-balance equations [52]. Unlike global-balance equations, local-balance equations focus on the session arrivals and departures at specific BSs. According to these equations (Eqs. 3.4), the rate leaving a state n_i due to the departure of a user at a specific BS k is equal to the rate entering that state due to the arrival of a user at the BS k either due to a new session or a handover (Eq. 3.4a). Furthermore, the rate leaving the state n_i due to the arrival of a new session at a BS is equal to the rate entering that state due to the termination of a session at a BS (Eq. 3.4b).

$$d_{ik} Q_i(n_i) = a_{ik} Q_i(n_i - e_{ik}) + \sum_{m=1}^{K_i} v_{im} p^*_{i,m,k} Q_i(n_i - e_{ik} + e_{im}) \tag{3.4a}$$

$$\sum_{k=1}^{K_i} a_{ik} Q_i(n_i) = \sum_{k=1}^{K_i} \mu_{ik} Q_i(n_i + e_{ik}) \tag{3.4b}$$

In Eqs. 3.4, e_{ik} is a vector with all entries equal to 0 except the k-th entry which is equal to 1. Fig. 3.4 illustrates the local-balance equations for a network of two BSs. Given that $\rho_{ik} = \sum_{j=1}^{J} \rho_{ik}^{j} < 1$ for each BS of the provider i, the stationary distribution of the number of connected users at all BSs can be derived as follows:

$$Q_i(n_i) = \prod_{k=1}^{K_i} (1 - \rho_{ik}) (\rho_{ik})^{n_{ik}} \tag{3.5}$$

By substituting Eq. 3.5 in the local-balance equations (Eqs. 3.4) and using simple algebra, we derive the traffic equations (Eq. 3.3). This proves the validity of Eq. 3.5. Given that the stationary distribution is in product form, each BS can be viewed as an independent M/M/1 queue with the processor-sharing discipline.

In the general case in which not all users select the provider i (i.e., $z_{ji} < 1$), we can replace γ_{ik}^{j}, a_{ik}^{j}, and ρ_{ik}^{j} with $z_{ji}\gamma_{ik}^{j}$, $z_{ji}a_{ik}^{j}$, and $z_{ji}\rho_{ik}^{j}$, respectively and Eqs. 3.1-3.5 still hold. In this case, the average number of connected users at the BS k of the provider i

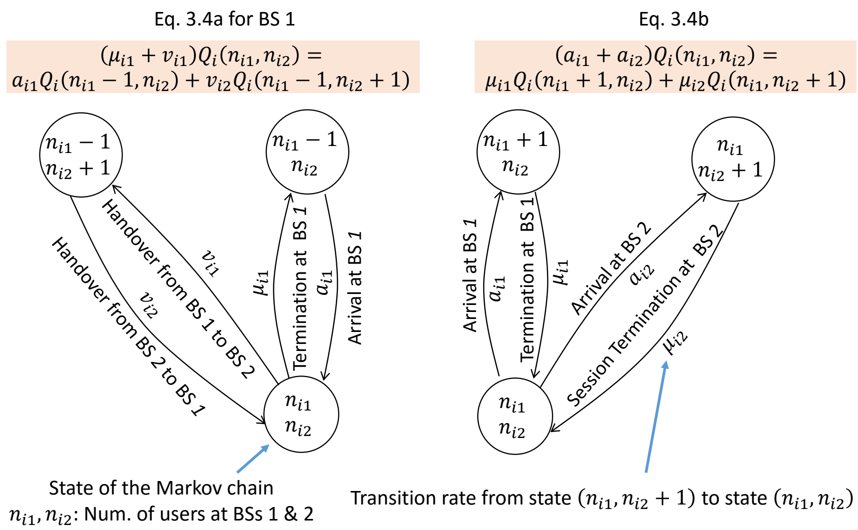


Figure 3.4: Local-balance equations for a network of the provider i consisting of two BSs.

is $E[N_{ik}] = \frac{\rho_{ik}}{1-\rho_{ik}} = \frac{\sum_{j=1}^{J} z_{ji}\rho_{ik}^{j}}{1-\sum_{j=1}^{J} z_{ji}\rho_{ik}^{j}}$ [53], where ρ_{ik} is the traffic intensity introduced by all user groups ($\rho_{ik} = \sum_{j=1}^{J} z_{ji}\rho_{ik}^{j}$). When a new user arrives at the BS k, it shares the available bandwidth along with all other currently connected users at that BS. Therefore, the amount of bandwidth that a new user gets when it connects to the BS k is $\frac{B_{ik}}{E[N_{ik}]+1} = B_{ik}(1-\sum_{j=1}^{J} z_{ji}\rho_{ik}^{j})$, where B_{ik} is the total bandwidth of that BS. The average data rate as perceived by a user of the group j at the network of the provider i can be computed as the weighted average of the data rate achieved at each BS (Eq. 3.6):

$$R_{ji}(z) = \sum_{k=1}^{K_i} \omega_{ik}^{j} B_{ik}(1 - \sum_{l=1}^{J} z_{li}\rho_{ik}^{l}) \tag{3.6}$$

The spatial variability of data rate affects the QoS. Thus, the utility function of the members of the group j (Eq. 3.8) incorporates the average data rate (Eq. 3.6) and variance of data rate which is defined as a polynomial of second degree with respect to z (Eq. 3.7):

$$V_{ji}(z) = \sum_{k=1}^{K_i} \omega_{ik}^{j} \left(B_{ik}(1 - \sum_{l=1}^{J} z_{li}\rho_{ik}^{l}) - R_{ji}(z) \right)^2 \tag{3.7}$$

The user service selection process employs the average and variance of data rate. The sub-games modeling the user service selection and competition of providers are described in Subsections 3.2.2 and 3.2.3, respectively.

Table 3.2: **Main parameters of a wireless market**

Parameter	**Description**
I	Number of providers
J	Number of user groups
N_j	Number of users in group j
N	Total number of users
c	Vector with the prices of all providers
H	User strategies
$f_j\left(R_{ji}(z)\right)$	Impact of average data rate on utility function of group j
$w_V^j\ (w_P^j)$	Weight of variance of data rate (price) for users of group j
$u_{ji}\,(z;c)$	Utility function of group j
d_j	Average traffic demand in MB that a member of group j produces in a period of 1 month
$z(t)$	User strategy profile at time t
$z^*(c)$	Equilibrium of users when price vector is c
P	Providers
C	Provider strategy profiles
$\sigma_i(c)$	Utility function of provider i

3.2.2 User service selection

The user service selection process is modeled by a population game. Each member of a user group can choose among $I+1$ available strategies $H = \{0, 1, ..., I\}$. Strategies $1, 2, ..., I$ correspond to subscriptions with the providers $1, 2, ..., I$, respectively, while strategy 0 denotes the disconnection state. We assume that each group corresponds to a homogeneous sub-population, and as such, the utility attained when selecting a specific strategy is the same for all users in that group. Therefore, it suffices to describe the service selection of the members of the group j with a probability distribution over the set of strategies (H). This distribution $z_j = (z_{j0}, z_{j1}, ..., z_{jI})$ is the *strategy profile* of the group j indicating how members of this group are divided among the different strategies (subscriptions and disconnection). The strategy profile of the entire user population consists of the strategy profiles of all groups ($z = (z_1, ..., z_J)$). Additionally, the *market share* that corresponds to a strategy i is the average percentage of customers over all user groups that select this strategy, i.e., $z_i = \sum_{j=1}^{J} z_{ji} N_j / N$. All parameters of a wireless market are defined in Table 3.2.

3.2.2.1 Utility function of the group j

A user from the group j selects a strategy (i.e., a subscription or disconnection) based on the average and spatial variance of the achievable data rate at the networks of providers and the offered prices:

$$u_{ji}(z;c) = \begin{cases} f_j\left(R_{ji}(z)\right) - w_V^j V_{ji}(z) - w_P^j c_i(d_j) & \text{if } i = 1, ..., I \\ 0 & \text{if } i = 0 \end{cases} \tag{3.8}$$

The function f_j is concave, strictly increasing, and non-negative and defines the impact of the average data rate ($R_{ji}(z)$) on the utility of the members of the group j. In the analysis, we assume that f_j is exponential $f_j(x) = w_R^j(\tau_j - \exp(-h_j x))$. The parameter w_R^j expresses the willingness-to-pay of the members of the group j. The larger the w_R^j, the larger the maximum price that users from the group j can pay. The parameter h_j defines the sensitivity of the members of the group j to low data rate. The larger the h_j, the larger the tolerance of users to a low data rate. The impact of the variance of data rate ($V_{ji}(z)$) and price of the subscription of the provider i ($c_i(d_j)$) is assumed to be linear and their significance is indicated by the positive weights w_V^j and w_P^j, respectively. The price that the members of the group j pay when selecting the subscription with the provider i depends on the average traffic demand they produce in a period of a month (d_j) according to Eq. 3.9:

$$c_i(d_j) = \begin{cases} c_{i1} & \text{if } 0 < d_j \leq D_1 \\ c_{i2} & \text{if } D_1 < d_j \leq D_2 \\ ... & \\ c_{iS_i} & \text{if } D_{S_i-1} < d_j \leq D_{S_i} \end{cases} \tag{3.9}$$

3.2.2.2 Dataplans

The provider i offers S_i distinct dataplans each for different traffic demand levels. Depending on which interval the traffic demand of a user lies, that user pays a different price. In other words, the provider i charges each user with a flat rate that depends on its level of traffic demand according to Eq. 3.9. We assume that each user is aware of its own average traffic demand per month when selecting a service [1]. Furthermore, when a user selects the disconnection (i.e., $i = 0$), it attains utility equal to 0. The parameters of the utility function u_{ji} (i.e., w_R^j, τ_j, h_j, w_V^j, w_P^j, and d_j) constitute the profile of the members of the group j.

In our analysis, we distinguish four main categories of user groups, namely, the *business users*, *low profile users*, *value-for-money users*, and *lenient users*. Business users have

[1] In this chapter, the term service refers to a subscription with a provider or the disconnection, while the term dataplan refers to the offered price of a provider for a specific interval of traffic demand. A user selects a service and pays the price corresponding to its level of traffic demand.

Table 3.3: **The four main user categories**

Category	Willingness to pay (w_R^j)	tolerance on low data rate (h_j)
Business users	$>50\%$ percentile	$<50\%$ percentile
Low-profile users	$<50\%$ percentile	$>50\%$ percentile
Value-for-money users	$<50\%$ percentile	$<50\%$ percentile
Lenient users	$>50\%$ percentile	$>50\%$ percentile

a high willingness-to-pay (i.e., high value of w_R^j) but are highly sensitive on low data rate (i.e., low value of h_j). Low-profile users are the opposite. They can not pay a high price but are more tolerant on low data rates. Value-for-money users are the most demanding ones in the market. They have a low willingness-to-pay and can not tolerate low values of data rate. Finally, lenient users are characterized by a high willingness-to-pay and do not consider the data rate important. These users make their decisions considering mainly the price, i.e., they search for the cheapest service. The characteristics of these four categories of users are given in Table 3.3[2].

3.2.2.3 Evolution of user decision making

Based on the utility functions of all user groups, the evolution of the strategy profile of users ($z(t)$) is described by the Logit dynamics, a system of ordinary differential equations (Eq. 3.10):

$$\frac{dz_{ji}(t)}{dt} = \frac{r}{1 + \sum_{k \neq i} \exp\left(\frac{u_{jk}(z(t);c) - u_{ji}(z(t);c)}{\epsilon}\right)} - r z_{ji}(t) \tag{3.10}$$

Note that the evolution of the user decisions manifests at a much slower time scale compared to the time scale at which sessions arrive and depart at the BSs of a provider. Specifically, the session arrivals and departures are performed at a time scale of minutes, while the user decisions manifest at a time scale of days or even months. Therefore, when the user strategy profile changes from z to z' over the course of several days, there is enough time for the queueing network of a provider to reach the equilibrium, and therefore, we can safely assume that the average data rate changes from $R_{ji}(z)$ to $R_{ji}(z')$ and the spatial variance of data rate changes from $V_{ji}(z)$ to $V_{ji}(z')$. The parameter r controls the speed of the dynamics, while ϵ is the noise and takes values in the interval $[0, \infty)$. When $\epsilon = 0$, users are completely "rational", i.e., always select the strategy that maximizes their utility function. Specifically, if the strategy k has slightly larger utility than strategy i that has been selected by a rational user, this rational user will then switch to strategy k. However, this behaviour is not realistic.

[2] We have distinguished only 4 main user categories to make the discussion of the performance analysis easier. However, one can define a larger number of user categories with different thresholds on the willingness-to-pay and tolerance on low data rate.

Users are reluctant to switch to another provider when the additional benefits are small. According to a survey study, users should be offered additional benefits of around 40% before they are highly likely to change their provider [54]. Several aspects, such as, the brand name and brand equity [55], reputation of a provider [56, 57], market share of a provider [58], length of customer association with a provider [59], and force of habit [60], affect the user decision making. In this work, we model the aggregate effect of those aspects by the noise of the Logit dynamics. As the noise increases from 0 towards infinity, users become more "sticky" with their selected service and change provider only when the benefits in terms of price and QoS are large enough. In the extreme case in which the noise parameter tends to infinity, users become completely irrational: they select their services randomly and are not affected by their utility function at all.

To compute the equilibrium of users for a given set of prices offered by the providers, we solve the system of the Logit dynamics (Eq. 3.10) starting from an initial point at which all user groups are uniformly distributed across the available strategies (i.e., subscriptions with providers and disconnection). The point at which the Logit dynamics converge is the user equilibrium.

3.2.3 Competition of providers

The competition of providers is modelled as a normal-form game $(P, C, \{\sigma_i\}_{i \in P})$. In this game, each provider (say provider i) offers S_i distinct dataplans to users. The strategy of that provider $c_i = (c_{i1}, ..., c_{iS_i})$ is a vector containing the prices of all those dataplans. Each price is selected from a closed interval $[0,\ C_i^{max}]$. The strategy space of providers is the set of all possible combinations of prices that can be offered in the market and is a rectangle of the form $C = [0,\ C_1^{max}]^{S_1} \times [0,\ C_2^{max}]^{S_2} \times ... \times [0,\ C_I^{max}]^{S_I}$. Each point of the strategy space $c = (c_1, ..., c_I)$ is a vector containing the offered prices of all providers.

We assume that each provider can model the user population at different levels of detail. Specifically, it applies a clustering algorithm on the profiles of all user groups[3] and determines a number of clusters. It models the decision making of those clusters of users based on the system of Logit dynamics (Eq. 3.10). For each set of offered prices c, a provider solves the system of Logit dynamics modeling the decision making of these clusters of users starting from uniform initial conditions and computes the corresponding equilibrium point $(\hat{z}^*(c))$. Based on this equilibrium, the utility function of the provider i is defined according to Eq. 3.11:

$$\sigma_i(c) = \sum_{s=1}^{S_i} \sum_{j: D_{s-1} < d_j \leq D_s} N_j \hat{z}_{ji}^*(c) c_{is} \tag{3.11}$$

This function computes the sum of the revenue collected by each of the offered dataplans

[3] The profile of a user group (say group j) is a vector containing the parameters of the utility function of that group, i.e., $(w_R^j, \tau_j, h_j, w_V^j, w_P^j, d_j)$.

of the provider i. For a dataplan s, only the payments of the clusters corresponding to this dataplan are considered (i.e., the user clusters that produce an average traffic demand lying in the interval $(D_{s-1}, D_s]$).

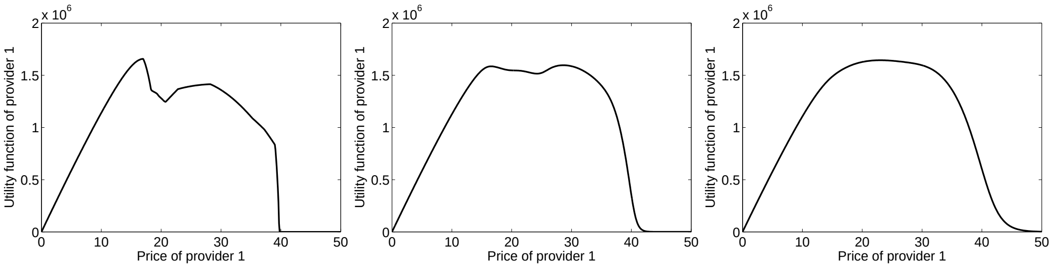
When a user is rational (i.e., $\epsilon = 0$), it selects the provider that maximizes its utility function. Let us assume that such a user has selected the provider i. If at a certain point the subscription of another provider k becomes slightly more profitable compared to the subscription of the provider i, the user will switch providers. This extreme behaviour of rational users introduces technical difficulties in the analysis: Various discontinuities appear in the derivatives of the utility functions of providers. We developed a methodology to analytically compute the NEs of providers under such discontinuities. This methodology divides the strategy space of providers into various subsets called "regions" in which the provider utility functions are continuously differentiable and analyses the game of providers at each one of those regions separately. At the final step, it combines the results of the analysis from all the regions to compute the global NEs of providers. A more detailed description of this methodology can be found in Chapter 7.

This modeling approach can effectively compute the NEs of providers, when users are completely rational and providers model users at the macroscopic level (i.e., considering only 1 cluster) and offer only 1 dataplan[4]. However, what happens when providers have a more rigorous knowledge about the market segments? In such a case, the computation of the global NE becomes challenging: As the number of user clusters increases, the number of (hyper-)surfaces at which the utility functions of providers have discontinuous derivatives in the strategy space of providers also increases.

This issue is mainly due to the assumption of the user rationality. Under this assumption, users can drastically change their behaviour even under small deviations in their utility function. To more realistically model the user behaviour, we use the Logit dynamics (Eq. 3.10). Among other standard population dynamics, such as, the replicator, best response, BNN, and Smith dynamics, these are the only ones that can model the user irrationality [61]. As explained earlier, their noise parameter (ϵ) indicates how much the user decisions deviate from the optimal ones based on their utility function capturing the effect of other factors that influence users (e.g., psychological/social factors).

The noise of the Logit dynamics does not only make the model of users more realistic but also simplifies the analysis by "smoothing out" the discontinuities in the derivatives of the utility functions of providers. An example of the effect of noise is shown in Fig. 3.5. This figure presents the utility function of a provider (say provider 1) offering only 1 dataplan in an oligopoly with 5 distinct user groups, when the prices of its competitors remain fixed. When the noise is equal to zero (i.e., $\epsilon = 0$), the rational behaviour of users results in various discontinuities in the derivative of the utility function of the provider 1 (Fig. 3.5a). When the value of noise slightly increases (i.e., $\epsilon = 0.5$), the discontinuities are smoothed

[4]In the worst case, the algorithm needs to solve two non-linear systems of I equations and a system of $2I$ non-linear inequalities.



(a) $\epsilon = 0$ (b) $\epsilon = 0.5$ (c) $\epsilon = 1.5$

Figure 3.5: Effect of noise on the utility function of a provider in a market with 5 user groups.

out (Fig. 3.5b). If the noise increases a little further (i.e., $\epsilon = 1.5$), the utility function of the provider becomes concave. This function (Fig. 3.5c) does not deviate significantly from the corresponding one when users are completely rational (Fig. 3.5a).

In each market case, by selecting a sufficient amount of noise, the utility functions of providers become concave. This simplifies the estimation of a global NE significantly. One should simply set the derivatives of the utility functions of providers with respect to their prices equal to 0:

$$\frac{\partial \sigma_i(c)}{\partial c_{is}} = 0, \text{ for all } i = 1, \ldots, I \text{ and } s = 1, \ldots, S_i \tag{3.12}$$

When the utility functions of providers are concave, a solution of the system 3.12 (c^*) is guaranteed to be a global NE of the game of providers. At this point, the utility functions of providers are maximized given the prices of their competitors [62] and therefore, no provider has the incentive to change its strategy. Our algorithm for the estimation of a NE proceeds as follows: First the system of Eqs. 3.12 is solved. If a solution is reported, it will be then verified whether or not it corresponds to a global NE (i.e., at which the utility function of each provider is maximized given the prices of the other providers). This final verification step is necessary given that the concavity of the utility functions of providers is not always guaranteed. For example, if the selected value of noise is not sufficiently large, the utility function of one or more providers may not be concave, and thus, the solution of Eqs. 3.12 does not necessarily correspond to a global NE. This possibility is ruled out by the final verification step.

3.3 Network decomposition technique

Often the analysis of wireless markets focuses on the performance a specific region of interest. For example, when a specific part of the network of a provider becomes congested, that provider may need to lease additional spectrum or extend its network infrastructure to improve the offered QoS in this region. To reduce the computational complexity of the analysis in such cases, a queueing network aggregation methodology based on the theorem of Norton [42] can be applied. This methodology proceeds as follows: First, the network of a provider is divided into two parts, the *region of interest* (e.g., congested region) and the *remaining area*. Then, appropriate Poisson sources are estimated modeling the input traffic from the BSs of the remaining area to the BSs of the region of interest. The equivalent network of the region of interest is formed by the subgraph of the original network that contains only the BSs of the region of interest adding the Poisson sources modeling the input traffic from the remaining area. Bellow the procedure is described in more detail.

3.3.1 Constructing the equivalent queueing network of the region of interest

Consider the network of a provider and let $s = \{M_1, ..., M_n\}$ be the subset that contains the BSs of the region of interest and $\hat{s} = \{1, ..., K\}$ be the subset that contains all BSs in the remaining area. The first step of the algorithm constructs a *reduced queueing network of the remaining area* in which all BSs of the region of interest are removed one by one by a "shortening" process. Each time a BS is removed, the service rate of its corresponding queue is set equal to infinity (i.e., all input traffic is immediately forwarded to the output of the queue). This is performed in n phases. At the l-th phase, the BS M_l is removed and only the BSs $\hat{s} \cup \{M_{l+1}, ..., M_n\}$ remain present. The transition probability matrix $P^{(l)}$ and new session arrival rates at the l-th phase are estimated based on the following recursive formulas:

$$p_{k,v}^{(l)} = p_{k,v}^{(l-1)} + \frac{p_{k,M_l}^{(l-1)} p_{M_l,v}^{(l-1)}}{1 - p_{M_l,M_l}^{(l-1)}} \tag{3.13}$$

$$a_{ik}^{(l)} = a_{ik}^{(l-1)} + a_{iM_l}^{(l-1)} \frac{p_{M_l,k}^{(l-1)}}{1 - p_{M_l,M_l}^{(l-1)}} \tag{3.14}$$

At the end of the n-th phase, the reduced queueing network of the remaining area has been constructed and the corresponding traffic equations for this network (Eq. 3.3) are solved to estimate the total input traffic of each BS $k \in \hat{s}$ of the remaining area (i.e., $\hat{\gamma}_{ik}$) and the corresponding traffic intensity (i.e., $\hat{\rho}_{ik}$). Then, for each BS of the region of interest $M_t \in s$, the Poisson source that models the input traffic from the remaining area has a rate of $\sum_{k \in \hat{s}} \hat{\gamma}_{ik} p_{k,M_t}$. An example of the construction of the reduced network is shown in Fig. 3.6. At the first phase, the BS 1 is removed (in the middle). The new transition and session arrival rates (e.g., $p_{2,2}^{(1)}, a_{i2}^{(1)}$) are computed based on Eqs. 3.13 and 3.14. At the second phase, the BS 2 is removed and the reduced network is formed (at the right).

The construction of the equivalent queueing network of the region of interest is performed as follows: A subgraph of the original queueing network of the provider is selected

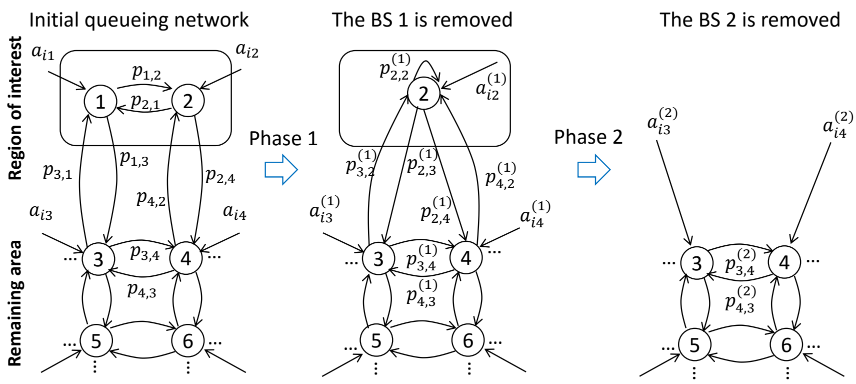


Figure 3.6: Example of the construction of the reduced network.

that contains only the BSs of the region of interest. Then, at each BS of this region, the corresponding Poisson source that models the input traffic from the remaining area (computed earlier) is added to the total input traffic of that BS. By solving the traffic equations of this network, we can compute the traffic intensity that corresponds to each station $M_t \in s$ of the region of interest (i.e., ρ_{iM_t}). Based on the traffic intensities of the BSs in the region of interest and the traffic intensities of the BSs in the remaining area, we can compute models for the average and spatial variance of the achievable data rate.

When a change is performed in the network of the region of interest, we modify the equivalent queueing network of this region accordingly. To estimate the average and spatial variance of data rate, we only need to solve the traffic equations of the modified queueing network of the region of interest and keep the traffic intensities corresponding to the BSs of the remaining area unchanged.

The computational complexity of solving the traffic equations for the entire networks of providers is of $O(K_i^3)$, where i is the provider with the largest number of BSs. On the other hand, if the largest subnetwork of providers in the region of interest has k_j BSs, then by applying the theorem of Norton, the computational complexity of estimating performance metrics for this region becomes $O(k_j^3)$. Usually, K_i is much larger than k_j indicating that the application of the theorem of Norton results in significant computational gains.

3.4 Concluding remarks

This chapter develops a methodology and set of tools that allow providers to model wireless markets at multiple levels of detail. First, it defines a method based on clustering algorithms that identifies user clusters with different profiles and requirements and models their decision making separately. By increasing the number of clusters, the accuracy in the modeling of the user population increases but so does the computational complexity. Using the proposed framework, a provider can select the appropriate number of clusters that models accurately the heterogeneity of the user population and their decision making while requiring a low computational complexity.

Except from the user population, the framework can also apply aggregations on the spatial domain. Often, providers are interested in the performance of a specific region. For example, if a specific part of their network becomes congested, they may need to perform a better capacity planning in this region by purchasing additional spectrum or extending the network infrastructure. To decrease the computational complexity of the analysis in such cases, aggregation methods can be applied. This chapter has developed such an aggregation method based on the theorem of Norton for queueing networks. This method estimates equivalent queueing network models for a specific region of interest omitting the details of the entire network of a provider. This can significantly reduce the computational complexity of the analysis.

We have used our multi-layer modeling framework and the network decomposition technique based on the theorem of Norton to analyse three market cases with a strong commercial interest, namely, pricing via market segmentation, WiFi offloading, and capacity planning. The detailed analysis of these cases is presented in Chapters 4, 5, and 6, respectively.

Chapter 4

Market segmentation-driven analysis of wireless markets

4.1 The concept of market segmentation and its impact on wireless markets

Market segmentation is an important aspect in almost all types of markets in the modern economy [63]. It is the process of dividing a large diverse customer population into homogeneous sub-populations that can be interested in specific services and products [64]. A seller makes offers to specific sub-populations to better satisfy their requirements and preferences. That way, the overall customer satisfaction is improved compared to applying homogeneous marketing strategies allowing for sellers to achieve higher revenues. Although the concept of market segmentation was introduced by Smith in the 1950s [65], it is still an important focal point in the academic research and marketing practices [66, 67, 68].

In the telecom markets, the identification of the different user sub-populations has received considerable attention [69]. Various clustering and data mining algorithms have been proposed to identify the different market segments [70, 71, 72, 73]. Software tools visualize the user sub-populations enabling the more efficient design of marketing strategies [67, 74]. However, despite the extensive knowledge of the user characteristics and preferences in wireless markets, few modeling and analysis methods apply a market segmentation approach for the design of the services of providers and optimal pricing.

Existing models of wireless markets that evaluate the profitability of different strategies for providers (e.g., in the price determination) are based on game theory and are either microscopic or macroscopic. Microscopic approaches usually study specific technical aspects (e.g., protocol, network topology, technology) on a short spatial and temporal scale, often considering networks with a small number of base stations (BSs) and markets with a single provider [13, 18, 20, 75, 76, 28, 23]. On the other hand, macroscopic approaches mainly focus on large-scale markets considering homogeneous populations to make the analysis tractable [4, 37, 38, 9, 39].

To the best of our knowledge, there are no game-theoretical studies focusing on large-scale wireless markets that consider the effect of market segmentation. This is due to the technical difficulties that the modeling of different user sub-populations introduces in the analysis. These challenges are enhanced by the assumption of the user *rationality* usually considered in game theory. Specifically, it is assumed that each user selects the service that maximizes a utility function that depends on the price and quality of service (QoS). However, in practice, various psychological and social aspects affect the user decision making [54, 77, 56].

In this chapter, we model a market using the multi-layer framework of Chapter 3 and assume that providers can offer multiple dataplans and apply a market segmentation approach when setting their prices. Specifically, they identify different user groups and offer services targeted to the specific needs of those groups. The framework models a market at an elaborate degree of detail, considering various technological and economic aspects. It models the cellular networks of providers as *queueing networks* and the user traffic demand with appropriate stochastic processes. It also estimates the perceived QoS of each user group based on the average and variance of data rate at the network of a provider. Furthermore, it models the user service selection in a realistic manner considering apart from the price and QoS, aspects such as the "stickiness"/loyalty to a provider.

Our analysis indicates that by modeling users in a high degree of detail, providers make better pricing decision achieving significant revenue gains and reducing the percentage of disconnected users compared to modeling users macroscopically. However, there are cases in which providers may lose revenue if they consider the same number of sub-populations when modeling users. The structure of this chapter is as follows: Section 4.2 defines an example of a wireless oligopoly of a small city. Section 4.3 presents the performance of this market when providers identify a different number of user sub-populations when setting their prices, while Section 4.4 presents the benefits of providers when they offer a different number of dataplans in the market. Finally, Section 4.5 presents the concluding remarks and some guidelines for providers.

4.2 An example of a wireless oligopoly

We have used the modeling framework of Chapter 3 and instantiated a wireless access market of a small city, represented by a rectangle of 14.4 km x 12.5 km. In this market there are 4 providers and a population of $300,000$ users. Each provider has deployed a cellular network covering the entire city. The BSs at each network are placed on the sites of a triangular grid, with a distance between two neighbouring sites of 1.6 km. The maximum data rate with which a BS can serve sessions is 25, 22, 19, and 16 Mbps for the providers 1, 2, 3, and 4, respectively. The average size of a session is 10 MB. Furthermore, the session service rate of a BS is $\mu_1 = 18.75$, $\mu_2 = 16.50$, $\mu_3 = 14.25$, and $\mu_4 = 12.00$ sessions/min for the providers 1, 2, 3, and 4, respectively. We also consider different cases for the user traffic demand. Specifically, for different average session generation rates over all users, from 0 up to 1.5 sessions/hour, we estimate the impact on the provider and user equilibriums.

4.3 Modeling users at different levels of detail

We consider a heterogeneous user population consisting of 100 distinct groups (which corresponds to the most detailed picture of the user population). Each group of users is characterised by a different utility function defined according to Eq. 3.8. The maximum willingness-to-pay (w_R^j) and tolerance on low data rate (h_j) of these groups follow normal distributions of mean 30 and 0.6 and standard deviation of 7.6 and 0.3, respectively[1]. In real markets those two parameters may also be correlated. For example, users with a large willingness-to-pay (high w_R^j) usually demand a high QoS and are less tolerant on low data rate (low h_j). To evaluate the impact of the correlation among the user willingness-to-pay and tolerance on low data rate, we defined two scenarios in which those two parameters are correlated and independent, respectively. Specifically, in the first scenario, the cross correlation of w_R^j and h_j is equal to -0.85 (Fig. 4.1a). This means that users with a large willingness-to-pay (high w_R^j) usually are less tolerant on low data rate (low h_j). In the second scenario, the cross correlation of w_R^j and h_j is equal to 0 and the maximum willingness-to-pay and data rate requirements of groups are completely independent (Fig. 4.1b). At both scenarios, it is assumed that the traffic demand (d_j) is the same for all user groups and providers offer only one dataplan. Additionally, the weight of data rate variabil-

[1]Those distributions were selected to model a diverse user population. However, the modeling framework is modular and allows the use of other distributions possibly resulting from empirical studies of real wireless markets.

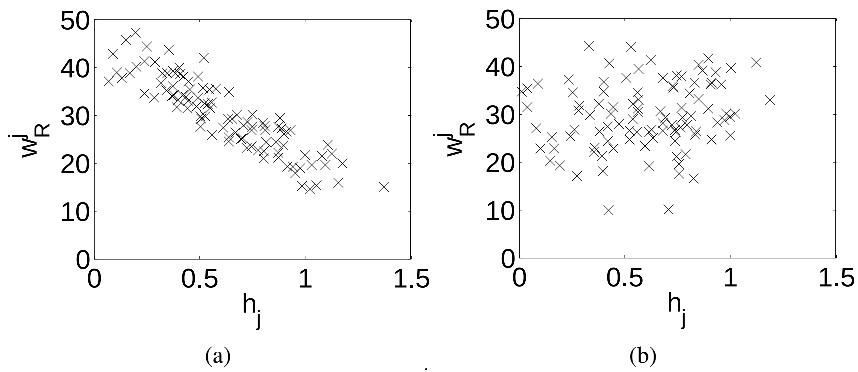


Figure 4.1: User group distribution when w_R^j and h_j are correlated (a) and when they are independent (b), respectively.

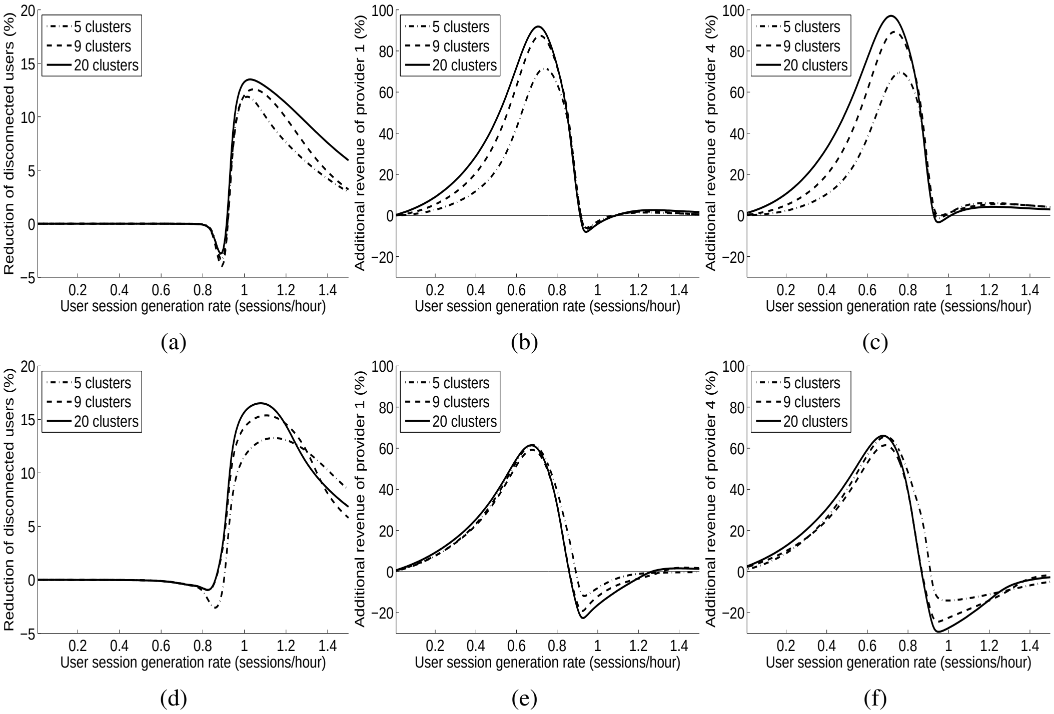


Figure 4.2: Performance gains when providers model users at different levels of detail compared to macroscopic modeling when w_R^j and h_j are correlated (top) and when they are independent (bottom), respectively.

ity (w_V^j) is set equal to 0 for all user groups, while the noise value (ϵ) that characterises the user decision making (Eq. 3.10) is set equal to 1.5.[2]

For each scenario, we have defined different market cases in which providers model the user population at different degrees of detail when estimating their utility functions. Each provider applies a clustering algorithm (e.g., K-means) on the profiles of the 100 user groups and determines a set of 5, 9, or 20 *representative user clusters*. Providers consider these sets of clusters when estimating their utility functions. As the number of clusters increases, the modeling of the user population becomes more accurate but the computational complexity of estimating the NE of providers increases. Our objective is to select the most appropriate level of detail that results in large performance gains for providers at a low computational complexity. Fig. 4.2 shows the *additional* benefits of users and providers obtained when providers model the user population at different levels of detail compared to modeling users macroscopically (i.e., with only 1 cluster) when w_R^j and h_j are correlated (Figs. 4.2a - 4.2c)

[2]In our performance evaluation, we have selected the minimum value of noise that results in concave provider utility functions. However, in real markets, the noise parameter should reflect the user behaviour as captured in real-world customer data.

and when they are independent (Figs. 4.2d - 4.2f), respectively.

Case a: When the maximum willingness-to-pay (w_R^j) and tolerance on low data rate (h_j) are correlated, modeling the user population at a higher level of detail (with a larger number of clusters) pays off for providers and users. Providers increase their revenue in almost all cases (Figs. 4.2b and 4.2c) and the percentage of disconnected users is significantly reduced (Fig. 4.2a). When providers model users at the macroscopic level, they apply a *homogeneous marketing* strategy by selecting their prices considering only aggregate profiling information for the entire population, without detailed information for the different sub-populations that reflect the diversity of the market. The provider 1 offers a price that is close to the prices of the other providers and attracts both business and low-profile users. Additionally, the providers 3 and 4 offer relatively high prices losing customers and revenue form the low-profile user groups. In such a market, except from the revenue of providers, the performance of users is also sub-optimal. A large percentage of users ends up selecting the provider 1 resulting in a degradation of the offered quality of service. Furthermore, due to the relatively high prices of the providers 3 and 4, more low-profile users become disconnected. In other words, under a macroscopic view of the market, providers make suboptimal decisions, which have a negative impact on their revenue and performance of users.

When providers consider a larger number of clusters for the estimation of their utility functions (e.g., 5, 9 or 20 clusters), they obtain a more detailed picture of the market, and as a result, the performance of the market improves. The provider 1 offers a higher price compared to the other providers and attracts mostly business users, while its share of low-profile users drops. On the contrary, the providers 3 and 4 offer low prices and attract the largest percentage of low-profile users, while their share of business users becomes low. The strong providers (i.e., the ones with the largest cellular capacity) focus on users with high willingness-to-pay and QoS requirements, while the weak providers focus on users with low willingness-to-pay and QoS requirements. This improves the overall performance of users (Fig. 4.2a) and reduces the intensity of competition allowing for a higher revenue for providers (Figs. 4.2b and 4.2c). In general, as the degree of detail in modeling the user population (i.e., number of clusters) increases, the performance of the market improves. However, the computational complexity of computing the equilibriums of users and providers increases. When the number of user clusters that providers consider increases, the execution time also increases in a non-linear way (Fig. 4.3). It is in the interest of a provider to select the appropriate number of clusters that results in high revenue benefits and at the same time requires a low execution time (e.g., 9 clusters).

Under high traffic demand, a large decrease of the benefits of providers is observed from a certain point onwards (Figs. 4.2b and 4.2c). At this point, the capacity of the networks of providers is reached and disconnected users start appearing. The first users that become disconnected are the ones with a low willingness-to-pay. Providers enter a competition and restrain their prices aiming to prevent those users from becoming disconnected. Even the provider 1 enters the competition regardless of its focus on business users. As the traffic demand keeps increasing, it becomes less beneficial for providers to keep those users

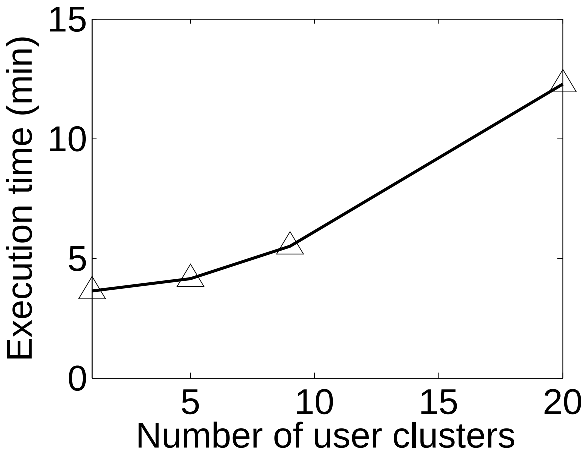


Figure 4.3: Execution time needed to perform the market analysis at different levels of detail.

in the market. This weakens the competition allowing for a small recovery of the revenue gains of providers.

Case b: If the maximum willingness-to-pay (w_R^j) and tolerance on low data rate (h_j) are independent, the modeling of users at a finer level of detail is beneficial for users: The percentage of disconnected users is significantly reduced (Fig. 4.2d). Interestingly, the performance of providers is not always improved. Under a low traffic demand, providers achieve significant revenue gains, while under large traffic demand, they lose revenue compared to a macroscopic modeling of users (Figs. 4.2e and 4.2f). Furthermore, the larger the number of clusters, the more prominent the losses. This is a counter-intuitive result: One would expect that the larger the degree of knowledge about the user population, the more significant the benefits of the providers. To explain this phenomenon, we should focus on the distribution of the user groups in this scenario (shown in Fig. 4.4).

In general, we can distinguish four different types of groups: the business users, low-profile users, lenient users, and value-for-money users. The revenue losses are mainly due to the pressure that value-for-money users and lenient users introduce in the market. Value-for-money users have a low willingness-to-pay (i.e., low w_R^j) and small tolerance on low data rate (i.e., low h_j). Satisfying the requirements of those users can be difficult. Providers should offer services of high data rate on low prices. Under a large traffic demand, when the capacity of the networks of providers is reached, value-for-money users start becoming disconnected. When modeling users at high levels of detail, providers are aware of the presence of value-for-money users and restrict their prices in an effort to attract these users given that they correspond to a significant percentage of the market (around 25%). This results in a steep decrease of the provider revenues (Figs. 4.2e and 4.2f). As the number of clusters increases, providers become aware of more "extreme" cases of value-for-money users with stricter price and data rate requirements. Therefore, they become more aggressive

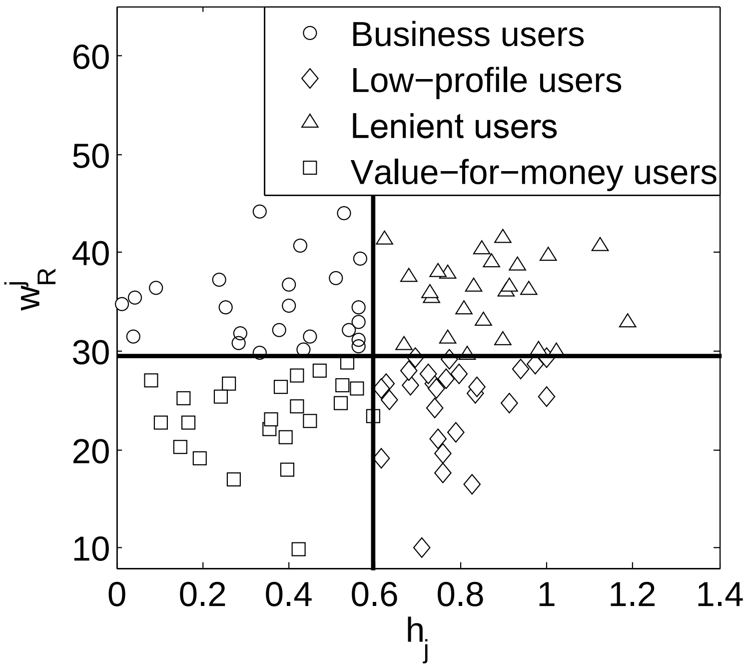


Figure 4.4: Distribution of user groups for independent w_R^j and h_j.

in the decrease of their prices, and as a result, they lose more revenue.

In the case of high traffic demand, value-for-money users eventually become disconnected and their influence weakens allowing for a slow recovery of the provider revenues. However, in the case of the provider 4, its revenue gain always remains negative under large traffic demand (Fig. 4.2f). This is due to the influence of lenient users. These users have a high willingness-to-pay (high w_R^j) and high tolerance on low data rate (high h_j). Price is the parameter that mainly drives their decisions. Under a large traffic demand, disconnected users appear in the value-for-money, low-profile and business users. However, almost all lenient users remain connected due to their low data rate requirements and high willingness-to-pay. Therefore, as the traffic demand increases, the influence of lenient users in the market intensifies. This strengthens the competition of providers keeping their revenues low.

4.3.1 Impact of different degrees of knowledge

We repeated the analysis, considering now a market in which only the provider 1 models users with 9 clusters, while all other providers model users macroscopically. Figs. 4.5a, 4.5b, and 4.5c show the differences in the prices, market share, and revenue of providers, respectively, compared to a market in which all providers model users macroscopically when w_R^j and h_j are correlated. Figs. 4.5d, 4.5e, and 4.5f present the same differences when w_R^j and h_j are independent.

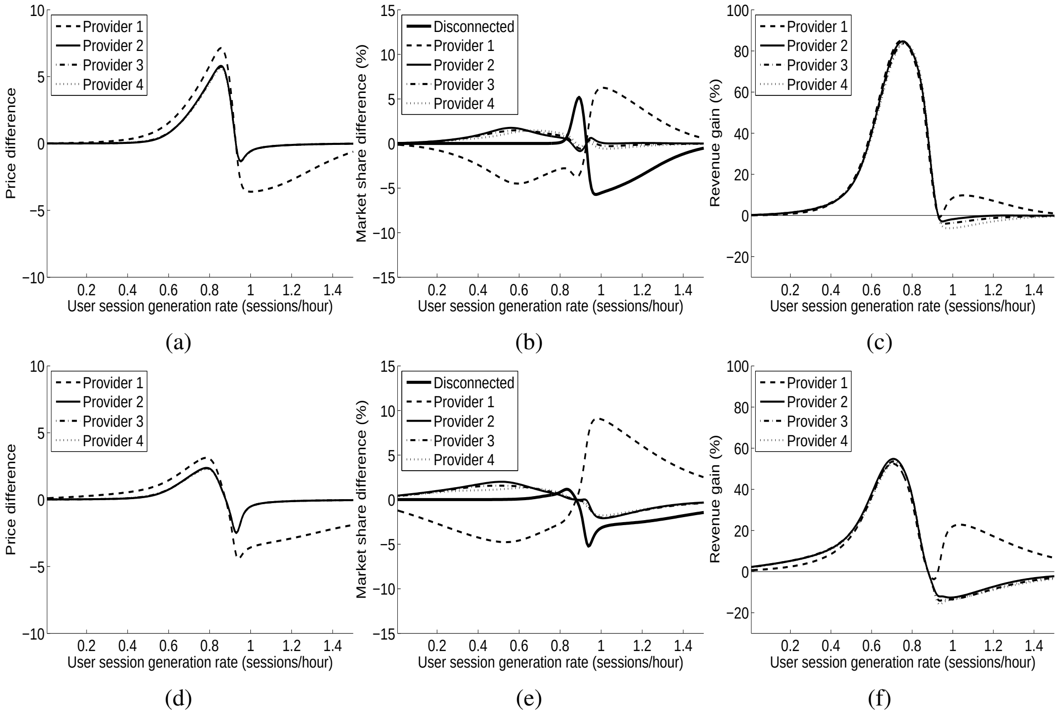


Figure 4.5: Performance gains of a market in which only the provider 1 models users with 9 clusters, while all other providers model users macroscopically compared to a market in which all providers model users macroscopically. The top (bottom) figures correspond to a scenario in which w_R^j and h_j are correlated (independent), respectively.

The provider 1 always achieves revenue gains. This is observed both when w_R^j and h_j are correlated and when they are independent (Figs. 4.5c and 4.5f, respectively). With its higher degree of knowledge, the provider 1 can influence the market to its benefit outsmarting the other providers. This is an important result which shows that the benefits of a provider from modeling the user population in a high level of detail strongly depend on the level of knowledge of the other providers about users.

Another interesting trend is that the effect of the knowledge of the provider 1 on the revenues of the other providers is twofold. Under small traffic demand, the provider 1 raises its price compared to the one offered at the macroscopic level (Figs. 4.5a and 4.5d). Given that it is the most influential provider in the market, its price increase provides also an opportunity to the other providers to raise their prices. This results in significant revenue gains for all provider (Figs. 4.5c and 4.5f). However, when the traffic demand becomes large, suddenly, the provider 1 “turns against” the other providers. When the capacity of the networks of providers is reached and disconnected users appear, it reduces its price bellow the prices of the other providers to attract those users (Figs. 4.5a and 4.5d). Given their macroscopic view of the market, the other providers do not react. This results in an increase of the revenue gain of the provider 1 at the expense of the revenues of the other providers (Figs. 4.5c and 4.5f).

Let us now explain the behaviour of the provider 1 in more detail. When the user traffic demand is low, the provider 1 realizes that it is more beneficial to focus on business users increasing its price. This results in a decrease of its market share compared to the one obtained at the macroscopic level (Figs. 4.5b and 4.5e). However, this decrease of market share is useful because it improves the quality of service of the provider 1 making its subscription more appealing to business users. Given their improved satisfaction, those users can pay a higher price to the provider 1 increasing its revenue. This trend persists as the traffic demand increases until the capacity of the networks of providers is reached. From this point onwards, the quality of service drops substantially and some of the business users become dissatisfied and decide to disconnect. Then, the provider 1 suddenly changes its strategy. Instead of focusing on the business users, it realizes that it is more beneficial to restrict its price in order to prevent users with low willingness-to-pay from becoming disconnected and attract them to its network. Other providers do not realize this early enough due to their macroscopic view of the market and they do not react. That way, the provider 1 achieves significant revenue gain at the expense of the revenues of the other providers.

We repeated the previous analysis considering now that only the provider 4 models users with 9 clusters, while all other providers model users macroscopically (Fig. 4.6). The provider 4 achieves revenue gains both when w_R^j and h_j are correlated (Fig. 4.6c) and when they are not (Fig. 4.6f). Additionally, when the traffic demand is low, all providers achieve revenue gains, while under large traffic only the provider 4 gains additional revenue at the expense of the revenues of the other providers (Figs. 4.6c and 4.6f). However, given that the provider 4 is the weakest one in the market, its influence is low, and therefore, the observed tends of Fig. 4.6 are subtler compared to Fig. 4.5.

4.3.2 Sensitivity analysis

To strengthen our results, we performed a sensitivity analysis. We studied a market in which the 100 user groups differ in the weight of data rate variability (w_V^j) except from the willingness-to-pay (w_R^j) and tolerance on low data rate (h_j). Again, we defined two scenarios in which these parameters are correlated and independent, respectively. We observed similar trends as in Fig. 4.2. At high levels of detail, when w_R^j, h_j, and w_V^j are correlated, providers almost always achieve revenue gains, while there is a significant reduction of disconnected users compared to the macroscopic case. However, when the parameters w_R^j, h_j, and w_V^j are independent, again there are some revenue losses for providers under a large traffic demand. We also performed an experiment in which the capacities of the 9 central BSs of each provider are increased by 50 %. We observed the same trends as in Fig. 4.2 but with the corresponding curves shifted to the right: Given the increase of the capacity of the central BSs of providers, disconnected users appear later.

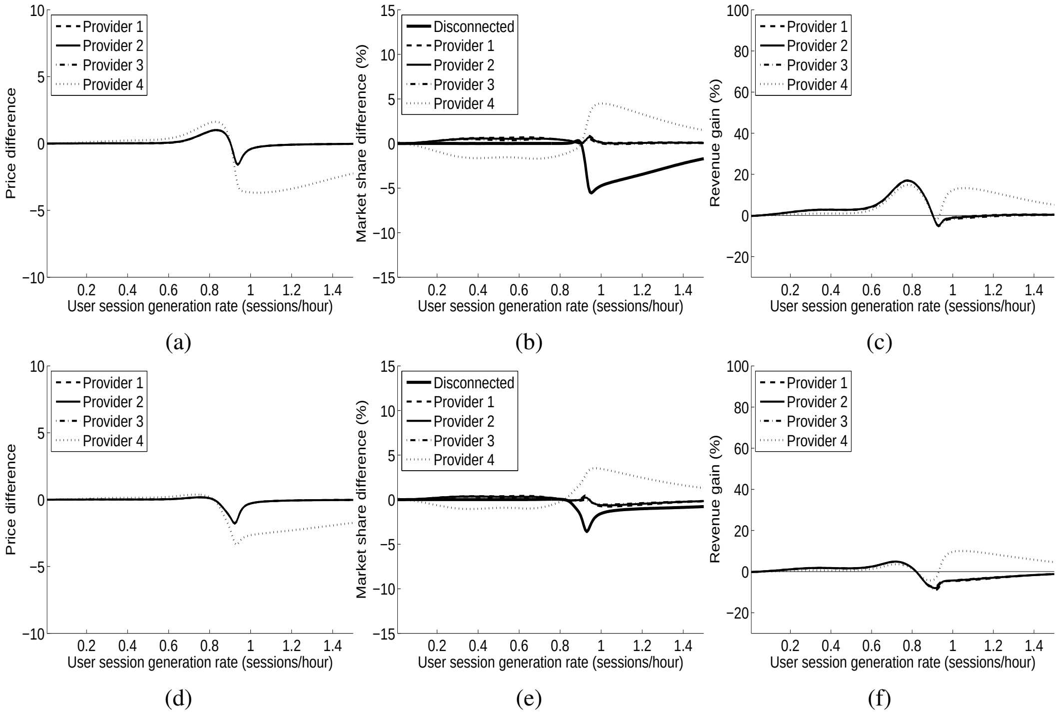


Figure 4.6: Performance gains of a market in which only the provider 4 models users with 9 clusters, while all other providers model users macroscopically compared to a market in which all providers model users macroscopically. The top (bottom) figures correspond to a scenario in which w_R^j and h_j are correlated (independent), respectively.

4.4 Offering of multiple dataplans

We have defined a market in which users of different groups deviate not only in their willingness-to-pay (w_R^j) and tolerance on low data rate (h_j) but also in their traffic demand (d_j). As in Section 4.3, w_R^j and h_j follow normal distributions, while the average session generation rate over all user groups ($\overline{\lambda} = \sum_{j=1}^{J} \lambda_j / \sum_{j=1}^{J} N_j$) varies from 0 up to 1.5 sessions/hour. The normalized session generation rate $n_j = \frac{\lambda_j / N_j}{\overline{\lambda}}$ of user groups follows a normal distribution of mean 1 and standard deviation 0.2.[3] Some user groups correspond to "heavy" users producing a large amount of traffic (users groups with a large n_j), while other groups correspond to "light" users (groups with a low n_j). Again we distinguish two market scenarios. In the first scenario, the cross correlation of w_R^j and h_j is equal to -0.85, while the cross correlation of w_R^j and n_j is 0.85. This means that users with a large willingness-

[3]Note that λ_j is the total session generation rate of the members of the group j, N_j is the number of users in group j, and N is the total number of users in the entire population (see Tables 3.1 and 3.2).

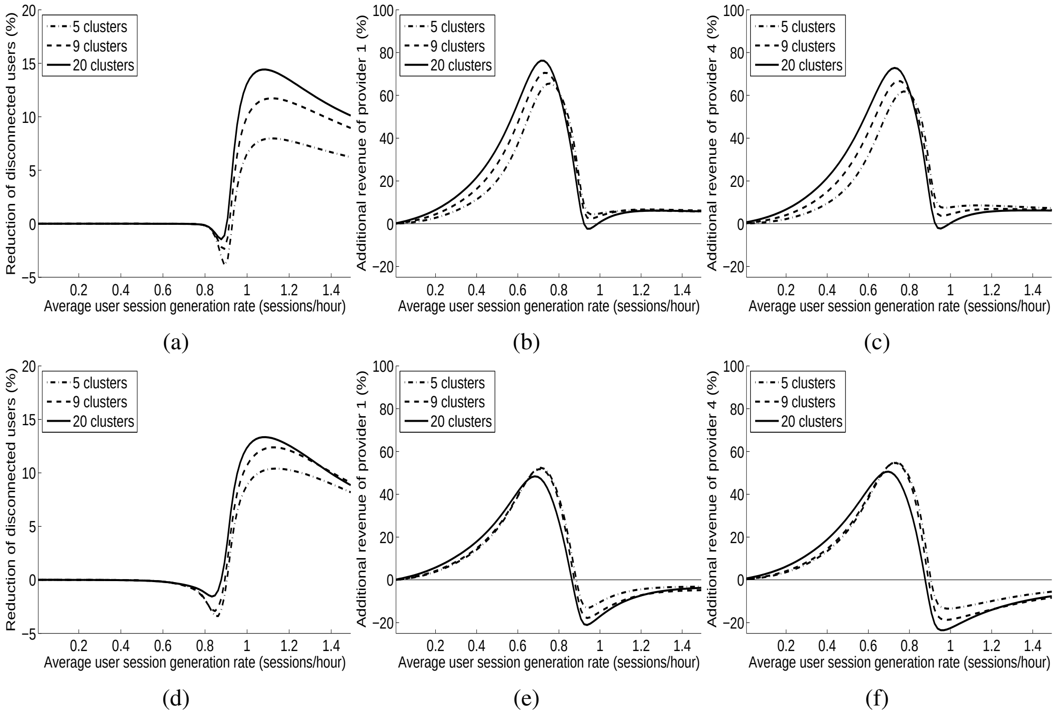


Figure 4.7: Performance gains when providers model users at different detail and offer 3 dataplans compared to modeling users macroscopically and offering 1 dataplan when w_R^j, h_j, and n_j are correlated (top) and independent (bottom), respectively.

to-pay (high value of w_R^j) usually are heavy users (high value of n_j) and less tolerant on low data rate (low value of h_j). In the second scenario, the cross correlation among w_R^j, h_j, and n_j is equal to 0 and the maximum willingness-to-pay, data rate requirements and traffic demand of groups are completely independent. In this set of experiments, we set the noise parameter of the Logit dynamics modeling the user decision making equal to 2.

We assume that a provider i offers a total number of S_i dataplans each corresponding to a different interval of traffic demand and flat-rate price. The first dataplan applies to light users with a normalized session generation rate up to $100/S_i\%$ percentile, while the last dataplan corresponds to heavy users with normalized session generation rate lying in the interval from $(S_i - 1) * 100/S_i\%$ up to 100% percentile. In other words, the range of the values of the normalized user session generation rate is divided into S_i segments. Users groups belonging to different segments are charged with a different price. Providers can also model users at different levels of detail by estimating a different number of user clusters. We have analysed a market in which providers offer 3 dataplans to users. The performance gains obtained when providers model users at different levels of detail compared to when they model users macroscopically and offer only 1 dataplan are presented in Fig. 4.7. The top (bottom) figures correspond to the case that w_R^j, h_j, and n_j are correlated (independent), respectively.

Similar trends are observed as in Fig. 4.2. When w_R^j, h_j, and n_j are correlated, in most cases, providers achieve revenue benefits when they model users at a higher level of detail (Figs. 4.7b and 4.7c), while the reduction of the percentage of disconnected users becomes more prominent (Fig. 4.7a). However, in a small interval around 0.95 session/hour, providers lose a small amount of revenue when they model users in higher detail. When the number of user clusters increases, an increase of the number of dataplans is required to perform a better pricing. With a larger number of dataplans, providers can charge the different user clusters more efficiently achieving higher revenue. In the cases of 9 clusters and 20 clusters, when the number of dataplans is increased above 3, the revenue losses around 0.95 sessions/hour are reduced.

When w_R^j, h_j, and n_j are independent, the observed trends are exactly the same as in Fig. 4.2. An increase of the number of user clusters results in a more prominent reduction of the percentage of disconnected users (Fig. 4.7d). Additionally, under a low traffic demand, an increase of the number of clusters always results in revenue benefits for providers. However, under a large traffic demand, modeling users with a large number of clusters results in revenue losses compared to the macroscopic case (Figs. 4.7e and 4.7f). Again those revenue losses are due to the existence of value-for-money users and lenient users. Those users intensify the competition of providers under a large traffic demand resulting in lower offered prices and revenue.

We have also studied a market in which providers offer a different amount of dataplans (i.e., 1, 3, or 5 dataplans). In this market, each provider models the users with 9 clusters. The performance gains compared to a market in which providers model users macroscopically and offer only 1 dataplan are depicted in Fig. 4.8. In the top (bottom) figures the parameters w_R^j, h_j, and n_j are correlated (independent), respectively. When w_R^j, h_j, and n_j are correlated, if providers offer only 1 dataplan, at an interval of the traffic demand around 0.9 sessions/hour, they achieve revenue losses compared to the macroscopic case (Figs. 4.8b and 4.8c). Those losses are mainly due to the limited amount of degrees of freedom when setting the prices of providers. Specifically, providers offer the same price both to heavy users with a large willingness-to-pay and to light users with a low willingness-to-pay. To prevent light users from becoming disconnected, providers restrict their prices regardless of the high willingness-to-pay of heavy users losing revenue. However, when providers offer 3 or 5 dataplans, the heavy and light users are charged with a different price. Therefore, providers are able to offer a high price to heavy users and a low price to light users always achieving revenue benefits (Figs. 4.8b and 4.8c).

When w_R^j, h_j, and n_j are independent, a counter intuitive result is observed. An increase in the number of offered dataplans does not result in revenue benefits for providers. On the contrary, under a large traffic demand, the offering of a larger number of dataplans results in revenue losses (Figs. 4.8e and 4.8f). In this case, the willingness-to-pay and traffic

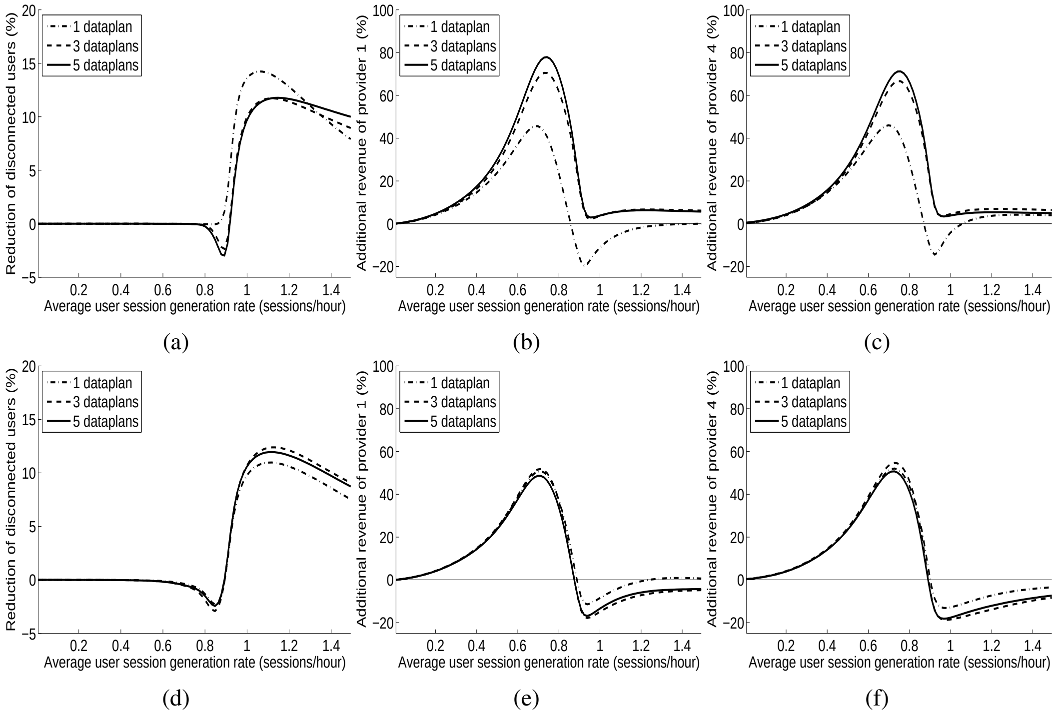


Figure 4.8: Performance gains when providers model users with 9 clusters and offer a different number of dataplans compared to macroscopic modeling when w_R^j, h_j, and n_j are correlated (top) and when they are independent (bottom), respectively.

demand of user clusters are independent. This means that the average willingness-to-pay of heavy and light users is almost the same. When providers offer different dataplans to these users, their competition is intensified. Given that the light users produce a low amount of traffic, their admission at the network of a provider does not significantly affect its QoS. Therefore, providers have the incentive to reduce their offered prices to light users in order to attract them to their networks entering a price war. Additionally, providers can not charge the heavy users with a high price due to their relatively low willingness-to-pay (almost the same as the one of light users). Therefore, providers would lose revenue by offering a larger number of dataplans in this market.

4.5 Lessons learned

This chapter analyses the impact of market segmentation on the performance of a wireless access market of a small city. Providers model users at multiple levels of detail by considering a different number of user clusters with distinct characteristics, utility functions, and requirements. They also offer multiple dataplans depending on the level of the user traffic demand. By performing a market segmentation, providers can identify the most suitable user clusters to target to improve their revenue. In market scenarios in which there is a strong correlation between the user willingness-to-pay and QoS requirements (i.e., markets with mostly business and low-profile users), market segmentation is beneficial. The strong providers (i.e., the ones with the largest cellular capacity) focus on user clusters with high willingness-to-pay and QoS requirements, while the weak providers focus on clusters with low willingness-to-pay and QoS requirements. This market segmentation improves the user satisfaction and reduces the intensity of competition improving the revenue of providers.

In general, one would expect that when providers model users in a higher level of detail (considering a larger number of clusters) or when they offer a larger number of dataplans, they make better decisions improving their revenue. Our analysis indicates that this is always true in market scenarios in which there is a strong correlation between the user willingness-to-pay and QoS requirements. In other market cases in which those two parameters are completely independent, a different trend is observed. Providers improve their revenue when they model users in a higher level of detail only under a low user traffic demand, while they achieve revenue losses in the case of a large traffic demand. Additionally, the offering of a large number of dataplans is not beneficial and it may also result in revenue losses. In such market cases, when all providers model users in a high level of detail, they end up enhancing their competition and lose revenue compared to applying a homogeneous marketing strategy.

Another important result that our analysis reveals is that when a provider models users at a high level of detail, its revenue benefits strongly depend on the knowledge of the other providers about users. As mentioned earlier, in markets in which the user willingness-to-pay and QoS requirements are completely independent, if all providers model users at a high level of detail, they may enhance their competition losing revenue. However, when only one provider models users at a high level of detail and all other providers model users macroscopically, the first provider always achieves significant revenue benefits.

Our modeling framework can be a useful tool that can help providers select the optimal level of detail for modeling users and the number of offered dataplans that will improve their revenue. This selection depends on a variety of factors including, the correlation among the willingness-to-pay and QoS requirements in the user population, the level of the user traffic demand, and the level of knowledge of the other providers about users. Specifically, a provider should first estimate the profiles of its customers in terms of willingness-to-pay, QoS requirements, and traffic demand. Then, it can use our modeling framework to select the optimal number of clusters and dataplans that will maximize its revenue for a specific market setting.

Chapter 5

On the impact of WiFi offloading in wireless markets

5.1 Use of WiFi offloading for increasing the capacity of wireless networks

According to forecasts, the wireless traffic will exceed the 24.3 exabytes per month world-wide by 2019 [1]. To cope with this explosion of traffic demand, providers aim to increase the capacity of their cellular networks. Traditional solutions for expanding the capacity involve the leasing of additional spectrum or the extension of the network infrastructure. However, such solutions are costly and time consuming. A cost-effective alternative for increasing the capacity that has received considerable attention is the data offloading: A provider leases access to some third-party WiFi access points (APs) and femtocells and offloads a part of its traffic to these APs. Alternatively, the provider can deploy its own WiFi infrastructure. This can alleviate the congestion of its cellular network.

Most approaches that study the WiFi offloading claim that it is beneficial for providers to offload as much traffic as possible to WiFi APs (e.g., [27, 28, 26, 23]). Their main argument is that a large volume of offloaded traffic alleviates the congestion in cellular networks and reduces the operational costs. Such studies usually omit the effect of the limited capacity of WiFi APs on the quality of service (QoS) and its long-term impact on the revenue of providers. Only a few studies consider the effect of the offloading on the QoS [50, 78]. Typically, they only focus on the physical layer (e.g., on the achievable data rate and SINR) and omit economic aspects, such as, the effect of the offloading on the competition of multiple providers and user decisions. In this chapter, we evaluate the impact of the offloading using the multi-layer game-theoretical framework of Chapter 3. The framework models markets with *multiple* competing providers that can perform WiFi offloading and a population of users that select their provider considering the offered prices and QoS. Several questions drive this research: What is the optimal percentage of traffic that should be offloaded to a WiFi network? How does the coverage of WiFi, user traffic demand, and user preference affect it? What is its impact on users? Under which cases is it beneficial for providers to invest in WiFi infrastructure? To answer these questions, we have evaluated the performance of WiFi offloading under various scenarios based on the bandwidth of cellular BSs, the coverage of the WiFi network, and the user utility.

Our framework employs detailed queueing-theoretical models of the networks of providers, base stations (BSs), access points (APs), user arrivals, departures, and handovers and provides a methodology to analyse large-scale wireless markets. It is also modular, in that, it can incorporate different user utility functions and traffic demand, mobility patterns, and network topology models. The analysis highlights that it is not always profitable for a provider to invest in a large number of WiFi APs. The benefits of the offloading increase with the WiFi coverage but with a diminishing rate. Furthermore, it is not beneficial to offload the entire traffic that can be served by APs. From a certain point onwards the APs will become congested. This will result in a QoS degradation reducing the revenue of providers.

The structure of this chapter is as follows: Section 5.2 presents the related work in the modeling of WiFi offloading. Section 5.3 presents our experiment setting for evaluating the impact of WiFi offloading on a wireless oligopoly of a small city. Sections 5.3.1 and 5.3.2 present the benefits of WiFi offloading when providers have deployed an LTE or 3G network infrastructure, respectively. Finally, Section 5.4 presents the concluding remarks.

5.2 State of the art in modeling of WiFi offloading

The problem of WiFi offloading has received considerable attention. The proposed approaches can be classified into two general categories, namely the *delayed offloading* and *on-the-spot offloading*. In delayed offloading, users wait until they are in the coverage of an AP before sending their delay-tolerant traffic [23, 24, 25]. In on-the-spot offloading, users opportunistically transfer data via WiFi whenever there are in the coverage of an AP [26, 27]. When a user moves out of the coverage of the AP with which it has been associated, a vertical handover is performed back to the cellular network of its provider. The delayed offloading cannot be always an appropriate option, especially for real-time applications.

Prior studies of offloading have made various simplifications. Most of them analyse the optimal decisions of a single provider and do not study the impact of offloading in markets with multiple competing providers [26, 28, 23, 24]. Only a few papers focus on scenarios with multiple providers that can perform WiFi offloading [27]. Typically, they assume that the larger the volume of the offloaded traffic, the larger the benefits for providers [28, 27, 24]. Such approaches usually consider only the operational cost omitting other aspects. There are also several studies that investigate the impact of the offloading on the QoS [50, 78]. However, they focus on the physical layer omitting economic aspects, such as, the competition of providers and user decisions. In addition to the theoretical approaches, there are experimental evaluations of the benefits of WiFi offloading [79, 80]. Incentive mechanisms to enable third-party resource owners to share their infrastructure have also been proposed [81]. In contrast to the above approaches, this chapter focuses on the on-the-spot offloading and evaluates the impact of the offloading on the competition of multiple providers, the offered QoS and the equilibriums of users and providers.

5.3 Experiment setting

Using the modeling framework of Chapter 3, we instantiated a wireless oligopoly of a small city of 180 km^2 with 4 providers and a population of $300,000$ users.

Network infrastructure. Each provider has deployed a cellular network covering the entire city. The BSs at each network are placed on the sites of a triangular grid, with a distance between two neighbouring sites of 1.6 km. The city is divided into 9 equally sized rectangular areas. We assume that a WiFi AP infrastructure has been deployed at different areas in the city. A provider with access to the APs of a specific area may offload a part of its mobile data traffic to these APs. A number of the new sessions that are generated within the coverage of an AP (say AP q) may be served by that AP. Furthermore, when a user moves into the coverage of that AP during a session, a vertical handover can be performed to that AP. From all the above sessions, a provider offloads to the AP q a certain percentage denoted as *offloading percentage*. Note that the offloading percentage is the same for all APs of a provider. When a user who is being served by an AP moves out of the coverage of that AP, a vertical handover is performed back to the cellular network of its provider. The goal of a provider is to select the *optimal offloading percentage* that will maximize its revenue in the market. Apart from the traffic of cellular BSs, the WiFi APs may also serve their own WiFi customers. This reduces the effective capacity of APs which is available for offloading. We have performed a set of experiments in which the optimal offloading percentage of a provider was estimated under different cases with respect to: (a) the bandwidth of cellular BSs, (b) the coverage of the WiFi network, and (c) the user utility function.

Bandwidth of wireless stations. We distinguish two scenarios with respect to the capabilities of the BSs and APs: the 3G scenario with BSs of lower bandwidth than that of APs (e.g., [26]) as well as the LTE scenario with BSs that have bandwidth larger than that of WiFi APs. In this analysis, the bandwidth of an AP is 6 Mbps. In 3G, the maximum data rate with which a BS can serve sessions is 5, 4.5, 4, and 3.5 Mbps for the providers 1, 2, 3, and 4, respectively, while in LTE, the maximum data rates of providers are 25, 22, 19, and 16 Mbps, respectively.

WiFi infrastructure. We assumed that in the city of interest, there exists a WiFi infrastructure. In our experiments, either the strongest provider (provider 1) or the weakest one (provider 4) may perform offloading. With respect to the WiFi coverage, we have considered cases in which APs are located in one, two, or three different areas close to the city center. We have also defined different cases in which the number of APs that correspond to a single BS varies from 1 up to 15. It is assumed that the APs associated with a BS are located within its cell and their coverage areas do not overlap. 16 APs are required to cover a cell, and therefore, the larger the number of APs per BS, the larger the WiFi coverage. Furthermore, an AP serves sessions from its own WiFi customers with a total arrival rate of 2 sessions per minute. Due to the traffic demand of WiFi customers, the effective bandwidth of APs that can be used for offloading is reduced from 6 to around 3.3 Mbps.

User utility function. In this set of experiments, we model users as a homogeneous population (i.e., as 1 group). The user utility function is defined as follows:

$$u_i(z_i; c) = \begin{cases} w_R\left(\tau - \exp(-hR_i(z))\right) - w_V V_i(z) - w_P c_i & \text{if } i = 1, ..., I \\ 0 & \text{if } i = 0 \end{cases} \tag{5.1}$$

Note that in the definition of the user utility, we do not use the index j since there is only 1 user group. We assume that the dependence of the user utility on the average data rate ($R_i(z)$) is exponential and the values of w_R, τ, and h are equal to 30, 1, and 0.6, respectively. The parameters w_R defines the user willingness-to-pay, while the parameter h defines the tolerance of users on low values of data rate. The user utility function has a diminishing derivative with respect to the data rate. For each case, we computed the market equilibriums given that no offloading is performed, and then, we computed the *optimal offloading percentage* for a provider, its *additional revenue* and the *decrease in disconnected users*.

5.3.1 The LTE scenario

We evaluated the performance of the WiFi offloading in the case of LTE. In this scenario, the average traffic demand of a user varied from 0.9 up to 1.5 sessions/hour. Fig. 5.1 presents the benefits of WiFi offloading. In general, as the number of APs associated with a BS increases, the additional revenue from the offloading also increases but in a diminishing manner (Figs. 5.1a and 5.1d). By increasing the number of APs per BS up to 6, the optimal offloading percentage is equal to 100% (Figs. 5.1b and 5.1e) which results in an increased revenue. However, an increase of the number of APs above this threshold has a diminishing "return" due to the decrease of the optimal offloading percentage (Figs. 5.1b and 5.1e). In other words, it is profitable for a provider to invest in WiFi infrastructure up to a certain threshold (e.g., around 6 APs per BS in this scenario). Above this threshold, the investment would not be beneficial.

Under large WiFi coverage, the low optimal offloading percentage (e.g., Figs. 5.1b and 5.1e) is due to two opposing trends: The increased offloading percentage alleviates the congestion at BSs allowing for higher data rates for users. On the other hand, it also results in a larger number of users being served by APs, which have lower bandwidth compared to that of BSs. Therefore, the data rate of these users decreases. In general, it is not always profitable for a provider to offload all traffic that can be served by APs. The optimal offloading percentage achieves the best load balancing among APs and BSs. It is selected in

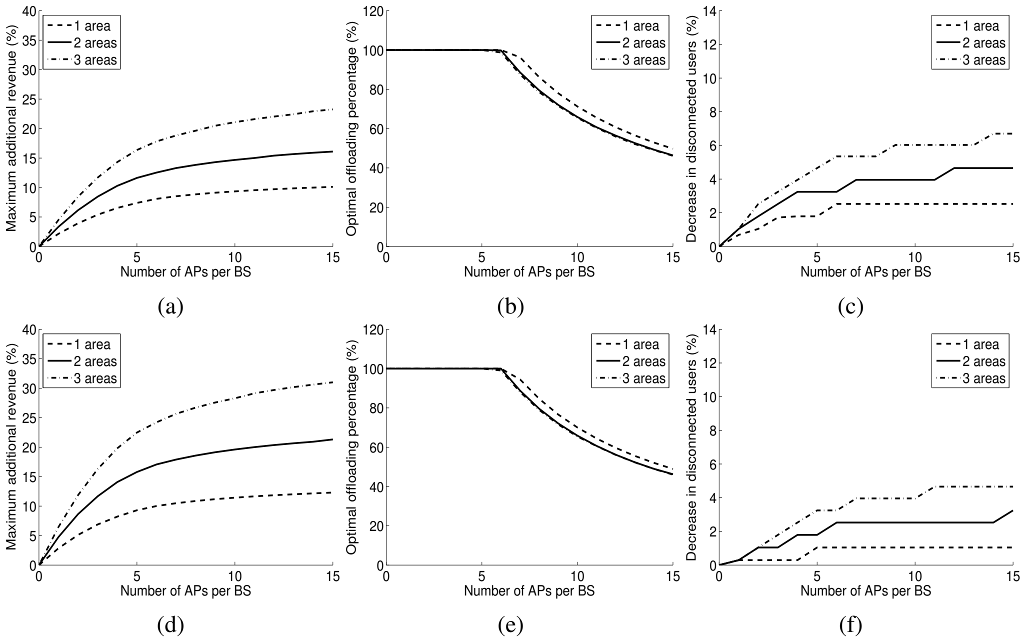


Figure 5.1: Benefits of offloading in the LTE scenario. Only the provider 1 (4) performs offloading in top (bottom) figures, respectively.

such a way to improve the average data rate, attract more users, and maximize the revenue of the provider. As expected, an increase in the number of APs per BS results in a larger decrease in the percentage of disconnected users (Figs. 5.1c and 5.1f). Furthermore, when the number of areas with a WiFi infrastructure expands, the revenue of a provider grows and the percentage of disconnected users drops. Surprisingly, it also results in a slight decrease of the optimal offloading percentage (Figs. 5.1b and 5.1e). This is due to the phenomenon discussed earlier. In general, an increase in the WiFi coverage results in a decrease of the optimal offloading percentage. Let us now discuss the case in which the provider 4 performs the offloading. Although the percentage increase of its revenue is higher compared to that of the provider 1 (Figs. 5.1a and 5.1d), the corresponding absolute increase of its revenue and the reduction of disconnected users are lower compared to the provider 1 (Figs. 5.1c and 5.1f). Therefore, from the perspective of the overall market performance, it is more beneficial for the provider with the largest capacity to perform the offloading.

5.3.2 The 3G scenario

We repeated the analysis of Section 5.3.1 for the 3G scenario. In this scenario, the traffic demand of a user varied from 0.15 up to 0.25 sessions/hour. Fig. 5.2 presents the results. The percentage increase of the revenue of providers is higher in the 3G scenario compared

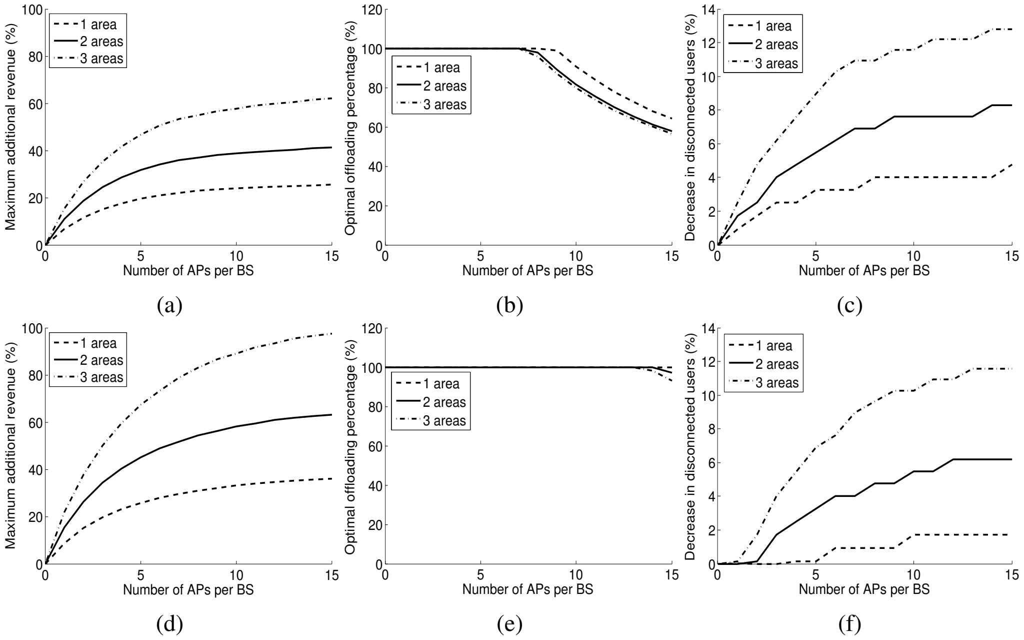


Figure 5.2: Benefits of offloading in the 3G scenario. Only the provider 1 (4) performs offloading in top (bottom) figures, respectively.

to the LTE one (Fig. 5.1). There is also a more prominent decrease in the percentage of disconnected users. The bandwidth of BSs in the 3G scenario is closer to the one of APs compared to the LTE scenario increasing the benefits of the offloading.

The larger effective bandwidth of the BSs of the provider 1 (i.e., 5 Mbps) compared to that of APs (i.e., 3.3 Mbps) results in a reduction of the optimal offloading percentage of the provider 1 above a certain threshold (Fig. 5.2b). On the contrary, the bandwidth of the BSs of the provider 4 is very close to that of APs (i.e, 3.5 Mbps) resulting in an optimal offloading percentage equal to 100% for almost all cases (Fig. 5.2e). In general, the closer the effective bandwidth of APs to that of BSs, the more traffic the BSs can offload to these APs without negatively affecting the QoS.

5.4 Lessons learned

We performed an extensive evaluation of the WiFi offloading under various scenarios. Our results highlight the benefits of the WiFi offloading to users and providers. It is not always beneficial for providers to invest in a large number of APs. The additional revenue of a provider from the offloading increases with the WiFi coverage but with a diminishing rate. Our framework can be used to enable providers to design their business plan for offloading.

Specifically, a provider has mainly two options for performing WiFi offloading. Either, it can lease access to third-party WiFi APs and femtocells, or it can deploy its own WiFi infrastructure. In both cases, a detailed cost-benefit analysis is required to indicate whether or not the offloading is a viable option for the provider.

In the first case, the cost that the provider pays to the third-party AP owners can be determined by appropriate double or reverse auction mechanisms (e.g., [82, 83]). In such cases, our framework can be used by providers to estimate their expected benefits for different outcomes of the auction and determine their optimal bids. When a provider deploys its own WiFi infrastructure, the capital and operational expenditures (CAPEX/OPEX) should be estimated. The CAPEX involves investment costs for the WiFi and backhaul equipment and installation, as well as, the WiFi core network equipment (servers, gateways, and portals) [84]. On the other hand, the OPEX involves per month operational costs for WiFi and backhaul site rental and maintenance, as well as, the traffic backhaul cost [84]. Specifically, the provider can perform a detailed cost analysis for deploying and maintaining the WiFi infrastructure for an extended time period of several years. Based on the expected traffic demand, the provider can use our framework to estimate its additional revenue due to the offloading for different cases of WiFi coverage and AP density. Given the additional revenue and required cost, the provider can then select the optimal infrastructure deployment that maximizes its total benefit. The outcome of such an analysis estimates the expected return of investment (ROI) and indicates whether the WiFi offloading is a viable business plan for the provider.

Chapter 6

On capacity planning

6.1 Participating in secondary spectrum markets for leasing spectrum

Due to the spatial heterogeneity of the user traffic demand, certain regions of the networks of providers may become congested. To improve their quality of service (QoS) in the these regions, providers can purchase additional spectrum by participating in a secondary market [85]. In such a market, spectrum licenses are traded in a short spatial and temporal scale. This means that providers can lease additional spectrum only at their specific regions of interest improving their overall QoS.

In secondary spectrum markets, the spectrum is allocated via auction mechanisms. Determining the optimal bidding strategy in such auctions can be challenging. Providers should estimate their benefits from all possible allocations of spectrum considering the amount of spectrum they can get as well as the amount of spectrum that their competitors acquire. In general, a provider does not only aim to purchase additional spectrum to improve its QoS but also to minimize the amount of spectrum that its competitors can get. If a large amount of spectrum is allocated to the competitors of a provider, these providers will get an advantage possibly attracting more customers. Through its bidding, a provider declares its willingness-to-pay for all possible allocations of spectrum. Usually, a provider favours more the outcomes in which it acquires the required amount of spectrum, while its competitors do not get a large portion of the spectrum.

To estimate its optimal bidding strategy in such auctions, a provider can use our multi-layer game-theoretical framework. That way, it can compute the provider and user equilibriums for each possible allocation of spectrum and estimate its revenue benefits. Based on these benefits, the provider can then determine its bidding strategy. For example, in a second-price auction (e.g., *a Vickrey-Clarke-Groves (VCG)* auction), a provider has the incentive to submit its true willingness-to-pay. In this case, it can submit its estimated revenue benefits for each possible allocation of spectrum. We have performed an experiment in which the networks of providers become congested at a specific region of interest. Those providers participate in a secondary spectrum market to lease additional spectrum and improve their QoS. In this market, the spectrum is allocated according to a VCG auction.

The structure of this chapter is as follows: Section 6.2 defines a VCG spectrum auction. Section 6.3 analyses the problem of capacity planning in a wireless oligopoly. Finally, Section 6.4 presents the concluding remarks and some provider guidelines.

6.2 A VCG spectrum auction

Let us consider a scenario in which the networks of providers become congested at a certain region. To improve their QoS, providers may engage in a secondary spectrum market. We assume that the spectrum allocation in such a market is performed according to a VCG auction [86, 87]. The VCG auction has been used for spectrum allocation in wireless markets [4, 88]. In this auction, license holders offer a certain amount of spectrum for sale which is divided into Q equally sized chunks. The outcome of the auction is an allocation vector $q = (q_1, ..., q_I)$ in which q_i is the total number of spectrum chunks that have been allocated to the provider i and $\sum_{j=1}^{I} q_j \leq Q$. Note that some spectrum chunks may not be allocated to any provider. Each provider submits a bid for all possible outcomes of the auction (i.e., for each possible value of the allocation vector q). The bid of a provider i for a specific allocation vector q is denoted as $b_i(q)$ and is the total amount that the provider i is willing to pay to the spectrum seller if the outcome of the auction is the vector q. The optimal allocation is the one that maximizes the sum of the bids of all providers. The cost that a provider pays is equal to the *externality* it causes to other providers. This externality is the total utility reduction that is caused by that provider (e.g., provider i) and is computed as follows: Let q^* be the optimal allocation of spectrum and $\overline{q}$ the optimal allocation without considering the participation of the provider i in the auction. The cost that the provider i pays is equal to $\sum_{j \neq i} b_j(\overline{q}) - \sum_{j \neq i} b_j(q^*)$. The total amount of spectrum that is allocated to a provider in the auction is divided equally among its BSs in the region of interest.

It has been proven that the VCG auctions are truthful [89], i.e., the best strategy for a provider is to submit its true willingness-to-pay. However, one of the drawbacks of the VCG mechanism is its computational complexity. Specifically, each provider should submit a bid for each possible allocation of spectrum. The number of allocations increases exponentially with the number of spectrum chunks. In practice, a small number of chunks is selected (e.g., around 8) to reduce the computational requirements. Determining the optimal bidding strategy of a provider in VCG auctions requires the estimation of the market equilibriums for all possible allocations of spectrum. The estimation of these equilibriums imposes a significant computational burden due to the large size of the networks of providers. Even if the additional spectrum is purchased at a specific region, in order to obtain the necessary information to determine the market equilibriums, we have to take into account the entire network of BSs. To overcome such problems, we can apply the network aggregation methodology based on the theorem of Norton described in Chapter 3.

6.3 Performance analysis setting

We used the multi-layer framework of Chapter 3 and instantiated a wireless access market of a small city, represented by a rectangle of 14.4 km x 12.5 km. In this market there are 4 providers and a population of $300,000$ users.

Network infrastructure. Each provider has deployed a cellular network covering the entire city. The BSs at each network are placed on the sites of a triangular grid, with a distance between two neighbouring sites of 1.6 km. The maximum data rate with which a BS can serve sessions is 25, 22, 19, and 16 Mbps for the providers 1, 2, 3, and 4, respectively. The average size of a session is 10 Mbytes. Furthermore, the session service rate of a BS is $\mu_1 = 18.75$, $\mu_2 = 16.50$, $\mu_3 = 14.25$, and $\mu_4 = 12.00$ sessions/min for the providers 1, 2, 3, and 4, respectively.

The user mobility among the BSs of a provider is modeled by a Markov chain. The transition probabilities from a BS to its neighbouring BSs are determined according to a Zipf distribution ($f(k; s, N) = \frac{1/k^s}{\sum_{n=1}^{N} 1/n^s}$, where N is the number of neighbouring BSs, k is the order of a specific neighbouring BS with respect to its distance from the center of the topology, and s is the exponent characterizing the distribution). In general, the closer a neighbouring BS is to the center of the network topology, the larger is the transition probability to this BS. Furthermore, the larger the exponent of the Zipf distribution, the larger the concentration of user traffic demand towards the center of the topology. In this series of experiments, we set s equal to 0.075. This results in the area around the center of the city to become congested. We varied the user session generation rate from 0 to 1.5 sessions/hour and performed a VCG spectrum auction in the central region.

User utility function. In this set of experiments, we again model users as a homogeneous population. The user utility function is defined as follows:

$$u_i(z_i; c) = \begin{cases} w_R\,(\tau - \exp(-hR_i(z))) - w_V V_i(z) - w_P c_i & \text{if } i = 1, ..., I \\ 0 & \text{if } i = 0 \end{cases} \tag{6.1}$$

We assume that the dependence of the user utility on the average data rate ($R_i(z)$) is exponential and the values of w_R, τ, and h are equal to 30, 1, and 0.6, respectively. The parameters w_R defines the user willingness-to-pay, while the parameter h defines the tolerance of users on low values of data rate.

We defined several market scenarios with respect to the amount of available spectrum for sale (BW) and weight of data rate variability in the user utility function (w_V). The term BW defines the additional bandwidth per BS that corresponds to the available spectrum for

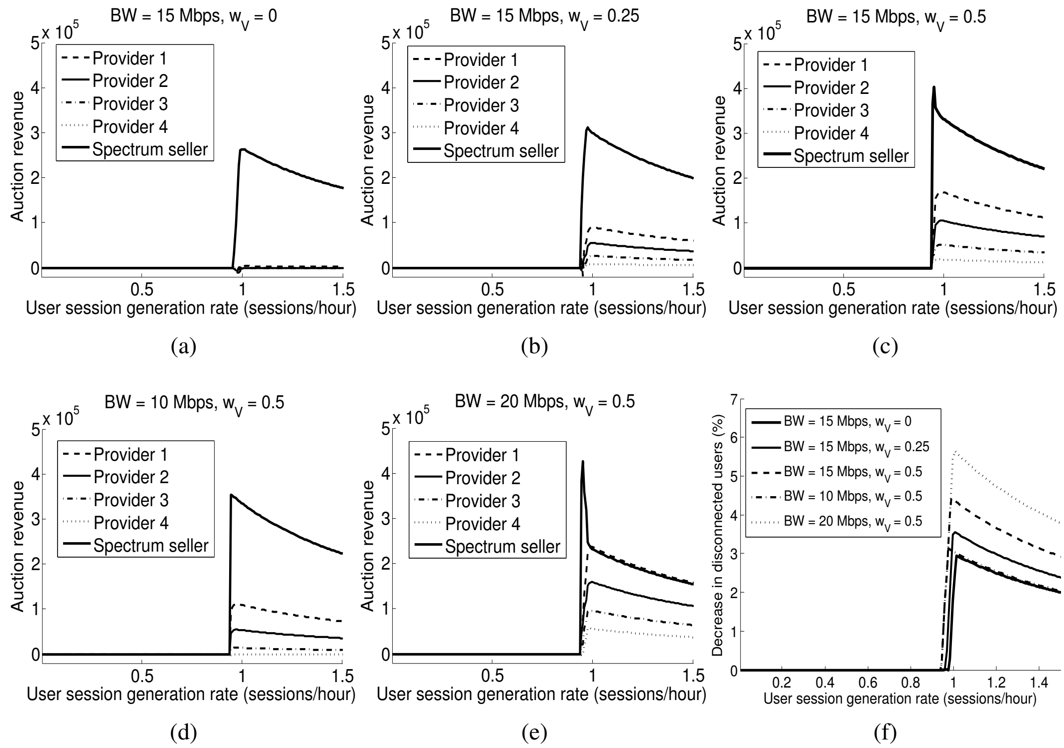


Figure 6.1: Performance of a VCG spectrum auction. (a)-(e) Revenue of providers and spectrum seller for different cases of auctioned spectrum (BW) and weight of data rate variability (w_V). (f) Decrease in the percentage of disconnected users.

sale. Fig. 6.1 presents the revenue of providers in the auction (additional revenue due to the purchased spectrum minus the paid cost) and the revenue of the spectrum seller.

In general, the spectrum auction is beneficial only when the user traffic demand is large (i.e., when there are disconnected users in the market). In the case of low user traffic demand (i.e., there are no disconnected users), the additional amount of spectrum intensifies the competition of providers, reducing prices and revenue. In such cases, it is not profitable for providers to participate in the auction and the additional revenues of all providers and the spectrum seller is equal to 0. In cases with disconnected users, the spectrum auction is always profitable.

Interestingly, when the weight of data rate variability (w_V) is equal to 0, each provider gets almost no additional revenue from the auction and all the profit is collected by the spectrum seller (Fig. 6.1a). In this case, the increase of the average data rates of providers due to the extra spectrum is similar resulting in similar bids. Therefore, based on the charging scheme of the VCG auction (Section 6.2), the winner achieves a very small revenue. When the w_V is increased, the profits are divided among the spectrum seller and providers more fairly (Figs. 6.1b and 6.1c). A provider is not interested in acquiring all the available spectrum but only such an amount that improves its average and spatial variance of data rate and maximizes its revenue. The purchase of all the available spectrum may enhance the variance of data rate reducing the user utility and the revenue of the provider. This lessens the competition resulting in multiple winners in the auction each of which achieves additional revenue.

An increase in the amount of spectrum to be auctioned results in an increase of the total revenues of the spectrum seller and providers (Figs. 6.1c - 6.1e). Interestingly, when the amount of additional spectrum is above a certain threshold, the revenue of the spectrum seller decreases (Fig. 6.1e), while the revenues of providers increase. As mentioned earlier, providers are not interested in acquiring all the available spectrum. The more is the available spectrum, the weaker the competition. This results in an advantage for providers over the spectrum seller. In general, it is in the interest of the spectrum seller to offer such an amount of spectrum to achieve a high revenue and keep the competition of providers intense. Finally, Fig. 6.1f presents the perspective of users. As expected, when the amount of spectrum to be auctioned increases, the decrease in the percentage of disconnected users is more prominent. Furthermore, an increase in the weight of the data rate variability also results in a larger reduction of disconnected users. The additional bandwidth at the central region improves the variance of data rate increasing the user utility. This increase in the user utility becomes larger when the weight of data rate variability increases leading to a larger reduction of the percentage of disconnected users.

To reduce the computational complexity of the bid estimation in the VCG auction, we employed the network aggregation method based on the theorem of Norton presented in Chapter 3. Our analysis indicates that this method reduces the execution time of the estimation of the QoS by 83%. The error of this estimation is on the order of 0.019 Mbps and 0.01 Mbps^2 for the average and variance of data rate, respectively. These results indicate that the application of the network aggregation method based on the theorem of Norton significantly reduces the computational complexity, while it achieves a high level of accuracy in the estimation of the QoS.

6.4 Lessons learned

The model of the user utility function that providers consider when determining their bidding strategy in a VCG spectrum auction is crucial. If providers assume that the user utility depends only on the average data rate, they aim to purchase all the available spectrum to maximize the achievable data rate in their networks making similar bids. In this case, the winner of the auction pays a price close to its bid and achieves a very low additional revenue. However, when providers assume that the user utility is affected by the spatial variance of data rate, their competition weakens and there are multiple winners in the auction that achieve significant revenue benefits. Therefore, it is beneficial for providers to make their bids aiming to offer a similar data rate in all geographical regions of their network and not just increase the average value of data rate across their network.

Our modeling framework can be used by providers to design their optimal bidding strategy in secondary spectrum markets. Specifically, providers can predict their revenue benefits when a specific spectrum allocation is performed. Then, based on these revenue benefits, they can determine their optimal bidding strategy in the spectrum auction.

Our analysis also indicates that when the amount of the available spectrum for sale is above a certain threshold, the competition of providers weakens resulting in revenue losses for the spectrum seller. In general, the spectrum seller should offer such an amount of spectrum that will keep the competition of providers intense in order to achieve a high revenue.

Chapter 7

Analysis of markets with rational entities

7.1 Degree of rationality in the decisions of real users and software agents

Our game-theoretical modeling framework is a powerful tool that can be used to analyse a variety of different market scenarios. It allows providers to model users at different levels of detail (by considering a different number of user clusters when they estimate their utility function) and controls the tradeoff between the accuracy and computational complexity. Additionally, it models the behaviour of users in a realistic manner assuming that they do not always make the optimal decisions in terms of price and quality of service (QoS) but are also affected by a variety of other psychological and social aspects. Those aspects result in a user behaviour that is not completely rational.

In real markets, when users select their provider and service, a degree of irrationality always affects their decision making. However, in some market cases, the decisions could be made by software agents. Such agents can be programmed to make optimal decisions in terms of price and QoS, and therefore, they are completely rational. This rational behaviour of software agents introduces challenges in the analysis. Discontinuities appear in the derivatives of the utility functions of providers making the computation of their Nash equilibrium (NE) a difficult task.

In this chapter, we propose a set of tools and algorithms to analyse wireless markets when all the involved entities (i.e., providers and users) are completely rational. To compute the NE of providers in such markets, we apply a novel methodology. This methodology analyses the game of providers at various subsets of the strategy space at which the utility functions of providers have a continuous derivative and combines the results from these subsets to compute the global Nash equilibriums (NEs).

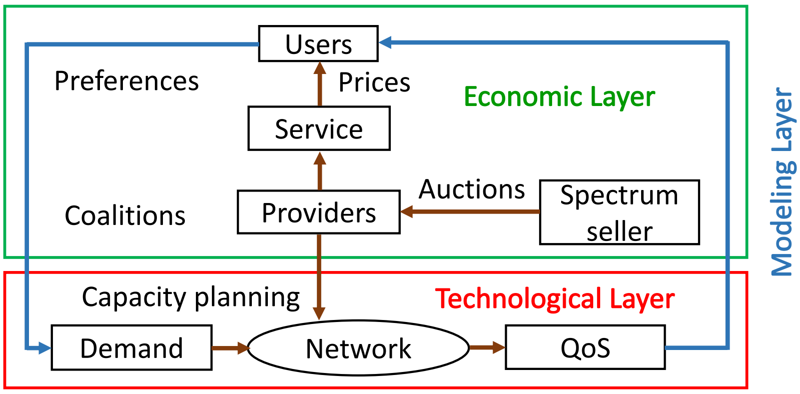


Figure 7.1: The main modules of wireless markets.

The structure of this chapter is as follows: Section 7.2 presents the modeling framework for analysing wireless markets at a macroscopic level of detail when all the involved entities are completely rational. Section 7.3 analyses the performance of a wireless oligopoly of a small city, while Section 7.4 compares this wireless oligopoly with markets in which there is a degree of irrationality in the user decision making. Finally, section 7.5 presents the concluding remarks.

7.2 Modeling framework for a macroscopic analysis with rational entities

We model wireless markets with two main types of entities, namely, the providers and users. Providers offer various types of services to users aiming to maximize their revenue, while users select the service that best satisfies their requirements with respect to price and QoS. In this sense, users are completely rational in their decisions. Providers try to balance price and QoS to keep their customers satisfied and increase their revenue.

We have developed a modeling framework that consists of two layers, the technological layer and the economic one (Fig. 7.1). The technological layer models the cellular networks of providers as *queueing networks* and the user traffic demand with appropriate stochastic processes. It also estimates the QoS of providers based on the average and variance of data rate. The economic layer models the market as *a two-stage game*. The first stage instantiates the *competition of providers* and the second one the *user decision making*. A *population game* models the user decisions: each user could either select to become a subscriber of a certain provider or remain disconnected based on a utility function that depends on the price and QoS. On the other hand, the competition of providers is modeled as a *normal-form game* in which providers strategically select their prices to optimize their revenue. The utility functions of providers depend on the offered prices and the NE of users (Fig. 7.2). Our framework models a wireless access market of I providers and N users.

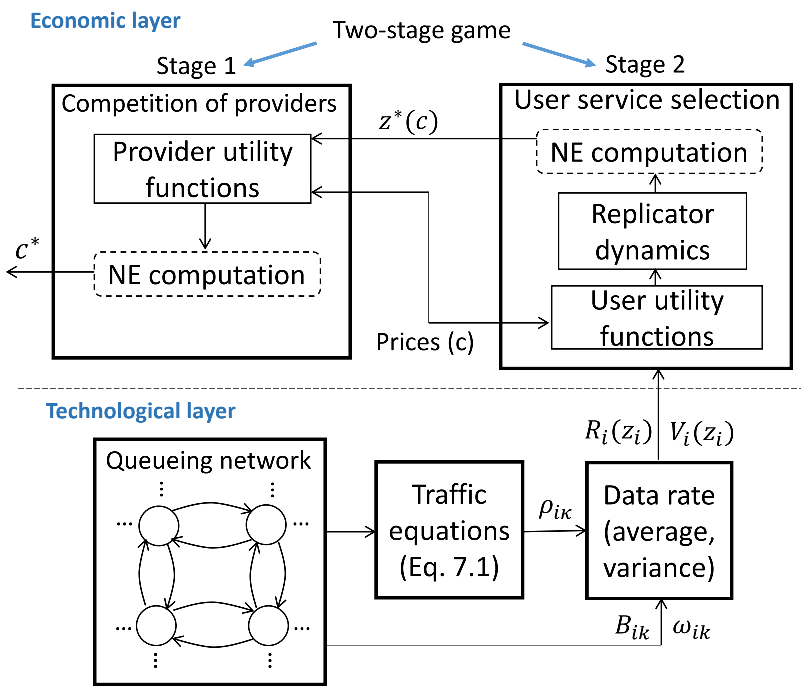


Figure 7.2: Main components of the modeling framework.

Each provider has deployed a network of wireless BSs and offers long-term subscriptions, which are *best-effort data services*. The following subsections describe the components of the modeling framework in more detail. The modeling framework of this chapter is a preliminary version of the framework presented in Chapter 3. In this framework, users are completely rational in their decision making and are modeled as a homogeneous population with only 1 group, while providers can offer only 1 dataplan.

7.2.1 The queueing networks of providers

As in Chapter 3, each provider (e.g., provider i) has deployed a number of BSs (K_i) covering a geographical region (e.g., a city). We also assume that in all BSs the available bandwidth is shared equally among connected users (processor-sharing discipline). For LTE cellular BSs, this bandwidth allocation models a scheduler that divides the OFDMA resources among users in a fair manner. Users generate requests to connect to a BS to start a session. During a session, a user transmits and receives data via that BS. The user session generation follows a Poisson process with a total rate of λ. This rate is allocated across providers according to the current *market share* $z = (z_0, z_1, ..., z_I)$. The ratio of subscribers of the provider i is indicated by z_i, while z_0 indicates the ratio of disconnected users. The user mobility in the network of a provider is modeled with a Markov-chain in which a state corresponds to the coverage area of a BS. The total session generation rate of subscribers of the provider i

is further divided among its BSs ($k = 1, ..., K_i$) according to the probabilities ω_{ik}. These probabilities correspond to the stationary distribution of the user mobility in the network of the provider i. Note that the handovers at a BS k of the provider i are modeled with a Poisson process of rate v_{ik}. This rate is estimated based on the fluid flow mobility model [51]. Table 7.1 defines the parameters of the queueing network of the provider i. Let us now focus on a simple case in which all users select the provider i (i.e., $z_i = 1$). The total session arrival rate at a BS k (γ_{ik}) consists of the new sessions ($a_{ik} = \omega_{ik}\lambda$) and handover sessions from all neighbouring BSs (Fig. 7.3):

$$\gamma_{ik} = a_{ik} + \sum_{m=1}^{K_i} \gamma_{im} p_{m,k}^{(i)} \tag{7.1}$$

The traffic intensity at the BS k of the provider i (ρ_{ik}) is equal to the ratio of the total session arrival rate at the BS k (γ_{ik}) over the total session departure rate at that BS (d_{ik}).

The queueing network of the provider i is modeled as a Markov chain. Each state

Table 7.1: **Parameters of queueing network of provider i**

Parameter	**Description**
K_i	Number of BSs
λ	Total session generation rate of users
$z_i(z_0)$	Ratio of subscribers (disconnected users)
ω_{ik}	Steady-state probability for a user to be located within the coverage of BS k
v_{ik}	Departure rate from BS k due to handover
μ_{ik}	Session service rate at BS k
d_{ik}	Total departure rate from BS k ($d_{ik} = v_{ik} + \mu_{ik}$)
$p_{m,k}^{(i)*}$	Conditional prob. of handover from BS m to BS k given that a handover occurs
$p_{m,k}^{(i)}$	Unconditional prob. of handover from BS m to BS k $\left(p_{m,k}^{(i)} = v_{im} p_{m,k}^{(i)*} / d_{im}\right)$
γ_{ik}	Total session arrival rate at BS k
a_{ik}	Arrival rate of new sessions at BS k
ρ_{ik}	Traffic intensity at BS k
n_i	Vector indicating the number of users at each BS
$Q_i(n_i)$	Stationary distribution of number of users at BSs
B_{ik}	Bandwidth at BS k
$R_i(z_i)$	Average data rate
$V_i(z_i)$	Variance of data rate

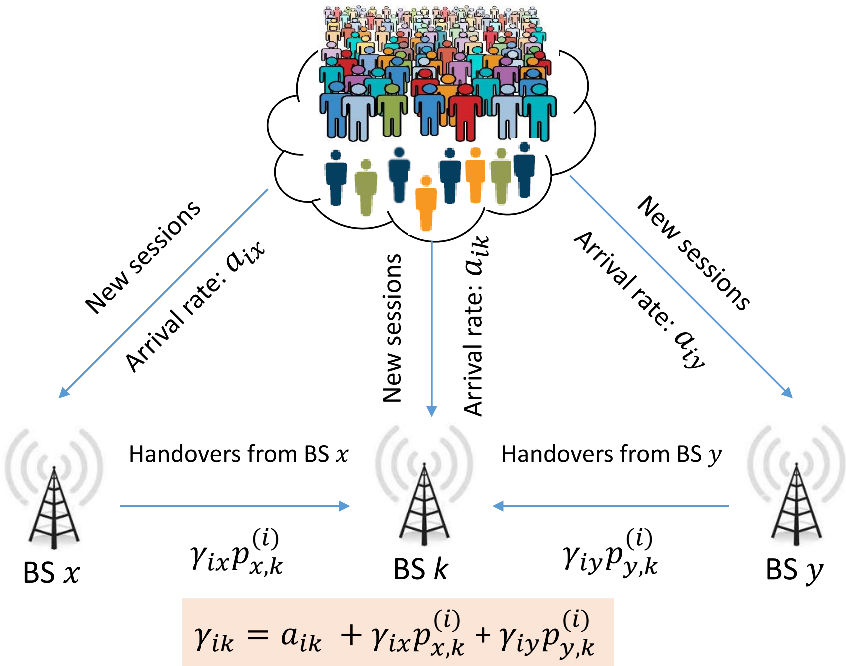


Figure 7.3: Session arrivals at a BS of the provider i.

corresponds to a vector $n_i = (n_{i1}, ..., n_{iK_i})$ indicating the number of connected users at all BSs. State transitions correspond to various types of events including session arrivals, terminations, and handovers. The stationary distribution of the Markov chain is computed by solving the global-balance equations. Such equations set the arrival rate at each state of the Markov chain equal to the departure rate from that state. Due to the Markovian property of our system and the processor-sharing discipline, the global-balance equations can be simplified into a set of local-balance equations [52]. Unlike global-balance equations, local-balance equations focus on the session arrivals and departures at specific BSs. According to these equations (Eqs. 7.2), the rate leaving a state n_i due to the departure of a user at a specific BS k is equal to the rate entering that state due to the arrival of a user at the BS k either due to a new session or a handover (Eq. 7.2a). Furthermore, the rate leaving the state n_i due to the arrival of a new session at a BS is equal to the rate entering that state due to the termination of a session at a BS (Eq. 7.2b).

$$d_{ik} Q_i(n_i) = a_{ik} Q_i(n_i - e_{ik}) + \sum_{m=1}^{K_i} v_{im} p_{m,k}^{(i)*} Q_i(n_i - e_{ik} + e_{im}) \tag{7.2a}$$

$$\sum_{k=1}^{K_i} a_{ik} Q_i(n_i) = \sum_{k=1}^{K_i} \mu_{ik} Q_i(n_i + e_{ik}) \tag{7.2b}$$

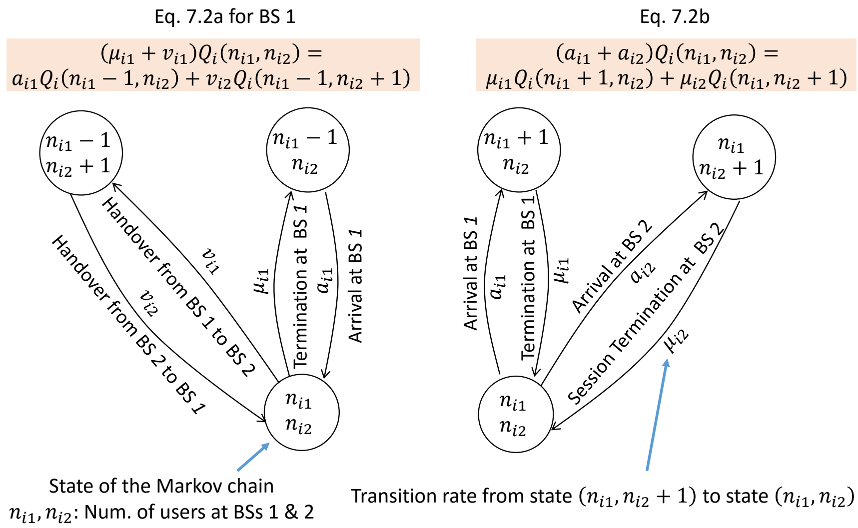


Figure 7.4: Local-balance equations for a network of the provider i consisting of two BSs.

In Eqs. 7.2, e_{ik} is a vector with all entries equal to 0 except the k-th entry which is equal to 1. Fig. 7.4 illustrates the local-balance equations for a network of two BSs. Given that $\rho_{ik} < 1$ for each BS of the provider i, the stationary distribution of the number of connected users at all BSs can be derived as follows:

$$Q_i(n_i) = \prod_{k=1}^{K_i} (1-\rho_{ik})\,(\rho_{ik})^{n_{ik}} \tag{7.3}$$

By substituting Eq. 7.3 in the local-balance equations (Eqs. 7.2) and using simple algebra, we derive the traffic equations (Eq. 7.1). This proves the validity of Eq. 7.3. Given that the stationary distribution is in product form, each BS can be viewed as an independent M/M/1 queue with the processor-sharing discipline.

In the general case in which not all users select the provider i (i.e., $z_i < 1$), we can replace γ_{ik}, a_{ik}, and ρ_{ik} with $z_i\gamma_{ik}$, $z_i a_{ik}$, and $z_i\rho_{ik}$, respectively and Eqs. 7.1-7.3 still hold. In this case, the average number of connected users at the BS k of the provider i is $E[N_{ik}] = \frac{z_i\rho_{ik}}{1-z_i\rho_{ik}}$ [53]. When a new user arrives at the BS k, it shares the available bandwidth along with all other currently connected users at that BS. Therefore, the amount of bandwidth that a new user gets when it connects to the BS k is $\frac{B_{ik}}{E[N_{ik}]+1} = B_{ik}(1-z_i\rho_{ik})$, where B_{ik} is the total bandwidth of that BS. The average data rate of a user session at the network of the provider i is the weighted average of the data rate achieved at each BS (Eq. 7.4):

$$R_i(z_i) = \sum_{k=1}^{K_i} \omega_{ik} B_{ik}(1-z_i\rho_{ik}) \tag{7.4}$$

The spatial variability of data rate affects the QoS. Thus, the user utility function (Eq. 7.6) incorporates the average data rate (Eq. 7.4) and variance of data rate which is defined as a polynomial of second degree with respect to z_i (Eq. 7.5).

$$V_i(z_i) = \sum_{k=1}^{K_i} \omega_{ik} \left(B_{ik}(1 - z_i\rho_{ik}) - R_i(z_i)\right)^2 \tag{7.5}$$

The user service selection employs the average and variance of data rate in the decision making process (Fig. 7.2). The sub-games modeling the user service selection and competition of providers are described in Subsections 7.2.2 and 7.2.3, respectively.

7.2.2 User service selection

The user service selection process is modeled by a population game. Each user chooses among $I + 1$ available strategies $H = \{0, 1, ..., I\}$. Strategies $1, 2, ..., I$ correspond to subscriptions with the providers $1, 2, ..., I$, respectively, while strategy 0 denotes the disconnection state. We assume that the population of users is homogeneous, and as such, the utility attained when selecting a specific strategy is the same for all users. Therefore, it suf-

Table 7.2: **Parameters of a wireless market**

Parameter	**Description**
I	Number of providers
N	Number of users
c	Vector with the prices of all providers
H	User strategies
$f\left(R_i(z_i)\right)$	Impact of average data rate on user utility
w_V (w_P)	Weight of variance of data rate (price)
$u_i\left(z_i; c\right)$	User utility function
$z(t)$	Market share of users at time t
$z^*(c)$	User NE
P	Providers
C	Provider strategy profiles
$\sigma_i(c)$	Utility function of provider i
$\sigma_i^r(c)$	Utility function of provider i restricted in the region r of the strategy space of providers
$g_j^r(c) \leq 0$	j-th constraint used to define the region r of the strategy space of providers
A_r	Set of price vectors corresponding to the region r of the strategy space of providers

Table 7.3: **Dependence of user utility on average data rate**

Name	**Formula**	**Parameters**
Linear	$w_R R_i(z_i)$	w_R
Exponential	$w_R\left(\tau - e^{-hR_i(z_i)}\right)$	w_R, τ, h
Logarithmic	$w_R \ln\left(h\left(R_i(z_i) - q\right)\right)$	w_R, h, q
Isoelastic	$w_R\left(h\left(R_i(z_i) + q\right)^{\kappa} - \tau\right)$	w_R, h, q, κ, τ

fices to describe the service selection of users with a probability distribution over the set of strategies (H). This distribution $z = (z_0, z_1, ..., z_I)$ is the *user strategy profile* also denoted as *market share*. All parameters of a wireless market are defined in Table 7.2.

7.2.2.1 User utility function

A user selects a strategy (i.e., a subscription or disconnection) based on the QoS and price:

$$u_i(z_i; c) = \begin{cases} f\left(R_i(z_i)\right) - w_V V_i(z_i) - w_P c_i & \text{if } i = 1, ..., I \\ 0 & \text{if } i = 0 \end{cases} \tag{7.6}$$

The function f is concave, strictly increasing, and non-negative and defines the impact of the average data rate ($R_i(z_i)$) on the user utility. The analysis considers four different functions f, namely, a *linear*, *logarithmic*, *exponential*, and *isoelastic* one (Table 7.3). The impact of the variance of data rate ($V_i(z_i)$) and price of the subscription of the provider i (c_i) is assumed to be linear and their significance is indicated by the positive weights w_V and w_P, respectively. Furthermore, when the user selects the disconnection state (i.e., $i = 0$), it attains utility equal to 0.

7.2.2.2 User population dynamics

Based on the user utility function, the evolution of the market share of users ($z(t)$) is described by the replicator dynamics, a system of ordinary differential equations (Eq. 7.7):

$$\frac{dz_i(t)}{dt} = z_i(t)\left(u_i\left(z_i(t); c\right) - \sum_{j \in H} z_j(t) u_j\left(z_j(t); c\right)\right) \tag{7.7}$$

Depending on the initial conditions, the replicator dynamics may converge to different equilibrium points. However, not all these equilibriums are NEs (e.g., the equilibrium where all users are disconnected), since at a NE, no user has the incentive to change its strategy. Every population game admits at least one NE (as can be proven by applying Kakutani's fixed point theorem [90]).

7.2.2.3 Computation of the user NE

At a user NE, we can divide the set of strategies H into two disjoint subsets X and Y, such that: (i) X is non-empty, (ii) all strategies in X correspond to the same utility, (iii) all strategies in Y correspond to a market share of 0. The ability to construct the sets X and Y is proven in the Appendix 7.A. To compute a NE, we distinguish the following cases with respect to the sets X and Y. Depending on the case, a different system of equations needs to be solved.

Case (a). The subscriptions of all providers correspond to the same utility and there are no disconnected users: $X = \{1, ..., I\}$ and $Y = \{0\}$.

$$u_i\left(z_i; c\right) = u_1\left(z_1; c\right)\ \forall i \in \{2, ..., I\},\ \sum_{j=1}^{I} z_j = 1 \tag{7.8}$$

For the solution of Eqs. 7.8 $z^1(c) = (z_0^1(c), z_1^1(c), ..., z_I^1(c))$ to be a NE, additional conditions should be satisfied (inequalities 7.9). First, $z^1(c)$ should be a valid probability distribution and c should lie in the strategy space of providers (inequalities 7.9b and 7.9c, respectively). Furthermore, the utility of the disconnection should be less than or equal to the utilities of the subscriptions (inequality 7.9a). Otherwise, there is a contradiction to the definition of a NE.

$$u_1\left(z_1^1(c); c\right) \geq 0 \tag{7.9a}$$
$$z_i^1(c) \geq 0,\ \forall i \in \{1, ..., I\} \tag{7.9b}$$
$$c_i \geq 0,\ \forall i \in \{1, ..., I\} \tag{7.9c}$$

When the conditions 7.9 are true, the solution of Eqs. 7.8 ($z^1(c)$) is a user NE for the price vector c. No user has the incentive to change its strategy at that equilibrium. In general, two types of transitions may happen, namely, (i) a subscriber may change provider, and (ii) a subscriber may become disconnected. However, in this case, none of these can occur: a transition of type (i) is not profitable, since all subscriptions have equal utility at the equilibrium (Eqs. 7.8), and a transition of type (ii) reduces the user utility since all subscriptions have higher utility than the disconnection (Eqs. 7.8 and 7.9a).

Case (b). All strategies, including the disconnection, correspond to the same utility: $X = H$ and $Y = \emptyset$.

$$u_i(z_i; c) = 0\ \forall i \in \{1, ..., I\},\ \sum_{j=0}^{I} z_j = 1 \tag{7.10}$$

For the solution of Eqs. 7.10 $z^2(c) = (z_0^2(c), z_1^2(c), ..., z_I^2(c))$ to be a NE, additional conditions should be satisfied (inequalities 7.11). The vector $z^2(c)$ should be a valid probability distribution (inequalities 7.11a and 7.11b) and c should lie in the strategy space of providers (inequality 7.11c).

$$\begin{aligned} z_0^2(c) &\geq 0 && (7.11a)\\ z_i^2(c) &\geq 0,\ \forall i \in \{1, ..., I\} && (7.11b)\\ c_i &\geq 0,\ \forall i \in \{1, ..., I\} && (7.11c) \end{aligned}$$

When the conditions 7.11 are true, the solution of Eqs. 7.10 ($z^2(c)$) is a user NE for the price vector c. At that equilibrium, no user has the incentive to change its strategy. Except from (a) and (b), other cases can be defined in which the subscriptions of one or more providers belong in the set Y (i.e., obtain a market share of 0). However, as it will be shown in Section 7.2.3, the computation of the user NE in these cases is not necessary because providers set their prices aiming to achieve a strictly positive market share and revenue. Thus, we can ignore the other cases in which the subscription of one or more providers corresponds to a market share of 0[1].

7.2.3 Competition of providers

The competition of providers is modeled as a continuous normal-form game $(P, C, \{\sigma_i\}_{i \in P})$. In this game, each provider (say provider i) selects a price for its subscription (c_i) belonging in a closed interval $[0, C_i^{max}]$. The strategy space of providers is the set of all possible combinations of prices that can be offered in the market and is a rectangle of the form $C = [0,\ C_1^{max}] \times ... \times [0,\ C_I^{max}]$. Each point of the strategy space $c = (c_1, ..., c_I)$ is a vector containing a specific price for each provider and corresponds to a user NE $z^*(c) = (z_0^*(c), z_1^*(c), ..., z_I^*(c))$. Based on this equilibrium, the utility function of a provider i is defined as $\sigma_i(c) = N z_i^*(c) c_i$ and indicates the total revenue of the provider i in the market.

In general, continuous games can be analysed efficiently provided that they have a rectangular strategy space and twice continuously differentiable utility functions [91, 43]. However, in our case, there exists a finite set of surfaces in the strategy space, at which, the derivatives of the utility functions of providers are discontinuous. Those surfaces divide the strategy space into a finite number of regions. At the interior of each region, the set of user strategies that obtain a strictly positive market share at the user NE is fixed. Fig. 7.5 depicts two examples of the strategy space of providers in a simple case of a duopoly under large and small user traffic demand (Figs. 7.5a and 7.5b, respectively). These figures have been constructed by computing the user NE over a set of prices. At the interior of each region, all strategies that correspond to a strictly positive market share at the user NE are listed. The

[1]Note that the user NE is unique, if the utility functions $u_i(z_i; c)$ are strictly decreasing in z_i (proven in the Appendix 7.B). However, when $w_V > 0$, there are cases in which the above condition does not hold, and therefore, there may exist multiple user NEs (i.e., there may exist multiple solutions for Eqs. 7.8 and 7.10). In such cases, we select the solution that the non-linear equation solver computes starting from an initial point that corresponds to an equal market share for all strategies.

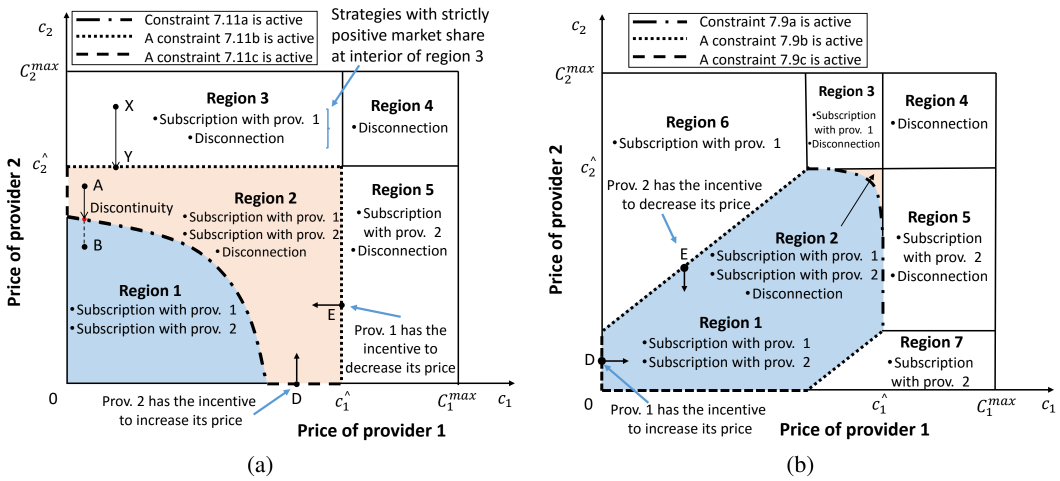


Figure 7.5: Examples of the provider strategy space in a duopoly under large user demand (a) and small demand (b).

region 1 (region 2) is composed by the price vectors that satisfy the conditions of case (a) (case (b)) of Section 7.2.2.3, respectively. The region 2 is larger in Fig. 7.5a compared to Fig. 7.5b, since under large traffic demand, disconnected users appear more frequently. In the case of low traffic demand (Fig. 7.5b), there are two additional regions (the regions 6 and 7) where all users become subscribers of one provider.

The segmentation of the strategy space appears in markets with multiple providers, each offering a unique dataplan for a price and with users that make rational decisions. In such markets, a NE can be narrowed down in the regions 1 and 2. This can be easily proven by contradiction. If a NE existed outside those regions, then at least one provider would obtain a market share of 0. However, this provider would have the incentive to reduce its price and obtain a strictly positive market share and revenue. This contradicts the definition of a NE. For example, if a NE existed in the region 3 of Fig. 7.5a (e.g., at the point X), then the provider 2 would obtain a market share of 0. However, this provider would have the incentive to reduce its price to obtain a strictly positive market share and revenue.

At the boundaries of regions, the derivatives of the provider utility functions are discontinuous. Let us explain that with an example. Assume that the currently offered prices correspond to the point A of Fig. 7.5a. At this point, the user NE contains subscribers of both providers and disconnected users. If the provider 2 starts reducing its price, it will attract disconnected users to its network leaving the provider 1 unaffected. If we assume that some subscribers of the provider 1 switch to the provider 2, then the QoS of the provider 1 will be slightly increased, making the utility of its subscription larger than 0. This will result in a "flow" of disconnected users to the provider 1 until its utility becomes again equal to 0. Therefore, the market share of the provider 1 remains unaffected by the reduction of the price of the provider 2. When the offered prices reach the boundary of the region 1, no disconnected users will remain. If the provider 2 keeps reducing its price, it will attract users from the provider 1. In other words, the utility function of the provider 1 remains fixed as long as the prices remain in the region 2, while it decreases when the prices enter the region 1. Furthermore, when disconnected users join the provider 2 in the region 2, the utility of disconnection does not change, while when subscribers of the provider 1 switch to the provider 2 in the region 1, the utility of the subscription of the provider 1 is improved. This means that for the same deviation of the price of the provider 2 around the point of the boundary, a smaller number of subscribers of the provider 1 should switch to the provider 2 in the region 1 compared to the number of disconnected users that should join the provider 2 in the region 2 until the user equilibrium is reached. This results in a discontinuity of $\partial z_2^*(c)/\partial c_2$. Therefore, the derivatives of the provider utility functions are discontinuous on the boundary. Similar arguments can be made for the boundaries of other regions.

7.2.3.1 Computation of the NEs of the game of providers

To compute the NEs of providers, we propose a novel algorithm (illustrated in Fig. 7.6): First, the strategy space is split into the different regions. Two separate games are defined for the regions 1 and 2. The problem of computing the NEs of a game with its strategy space restricted in a single region is a generalized Nash equilibrium problem (GNEP) [44, 45]. The final step checks whether the NEs corresponding to the regions 1 and 2 are also global NEs of the game of providers. Let us now describe the algorithm in more detail.

Computation of NEs in the region 1. The region 1 is the set of price vectors that satisfy the constraints 7.9 (case (a) of Section 7.2.2.3). At a NE, the price of a provider is a

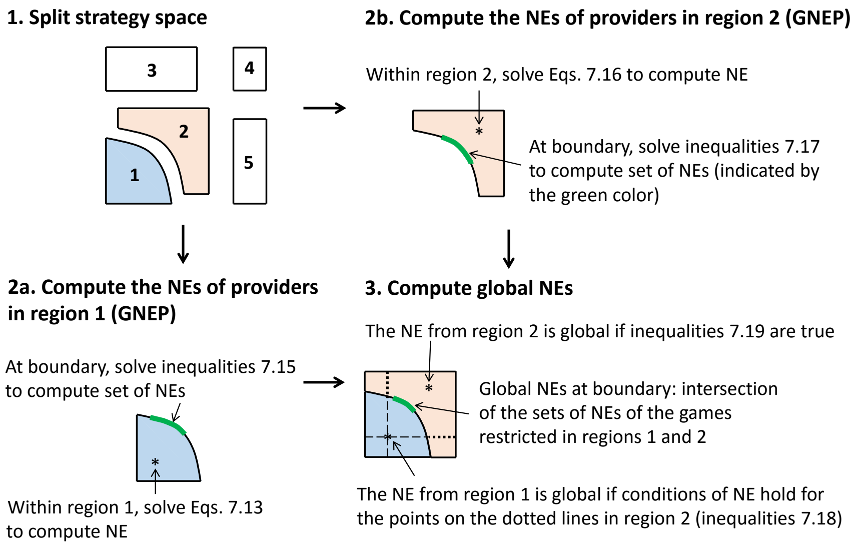


Figure 7.6: The main idea for computing the NEs of providers.

best response to the prices of its competitors. To compute its best response, a provider solves an optimization problem to select from the set of available prices the one that maximizes its revenue. As it will be shown bellow, all NEs lie either in the interior of the region 1 or at the set of points at which the constraint 7.9a is active[2] (Fig. 7.5b). The local maxima of the utility functions of providers at these sets of points satisfy the linear independence constraint qualification (LICQ)[3], and thus, at a NE, the Karush-Kuhn-Tucker (KKT) conditions of the optimization problems of individual providers should be satisfied [92]. The system 7.12 combines the KKT conditions of these problems. The derivation of this system is described in the Appendix 7.C.

$$\frac{\partial \sigma_i^1(c)}{\partial c_i} = \sum_{j=1}^{2I+1} \lambda_{ij} \frac{\partial g_j^1(c)}{\partial c_i},\ i = 1, \ldots, I \tag{7.12a}$$

$$g_j^1(c) \leq\ 0,\ j = 1, \ldots, 2I+1 \tag{7.12b}$$

$$\lambda_{ij} \geq\ 0,\ i = 1, \ldots, I \text{ and } j = 1, \ldots, 2I+1 \tag{7.12c}$$

$$\lambda_{ij} g_j^1(c) =\ 0,\ i = 1, \ldots, I \text{ and } j = 1, \ldots, 2I+1 \tag{7.12d}$$

To compute a NE, we distinguish various cases with respect to the location of that equilibrium. A point at which at least one of the constraints 7.9c is active can not be a NE: At such a point, there is always a provider with price equal to 0 that has the incentive to increase its price and attain a strictly positive utility (e.g., point D in Fig. 7.5b). Similarly, a point at which a constraint 7.9b is active can not be a NE. At such a point, there is always a provider with 0 market share that has the incentive to reduce its price and attain a strictly positive utility (e.g., point E in Fig. 7.5b). Therefore, all NEs lie either in the interior of the region 1 or at the set of points at which only the constraint 7.9a is active. In the interior of the region 1, all inequalities 7.12b are strict and therefore, based on the complementary slackness KKT conditions (Eq. 7.12d), all Lagrange multipliers are equal to 0. This reduces the system 7.12 into the following system:

$$\frac{\partial \sigma_i^1(c)}{\partial c_i} = 0,\ \text{for all } i = 1, \ldots, I \tag{7.13}$$

Standard numerical analysis methods are used to solve this system[4]. If the solution satisfies the constraints 7.12b (i.e., the inequalities 7.9) and corresponds to a global maximum of the utility functions of providers, it is a NE. Let us now focus on the set of points at which only

[2] A constraint $g(c) \leq 0$ is active when the equality holds.

[3] Consider a local maximum x^* of an optimization problem with continuously differentiable objective and constraint functions. If the gradients of the active inequality constraints and the gradients of the equality constraints are linearly independent at x^*, the KKT conditions should be satisfied at x^*.

[4] The derivatives of the provider utility functions are computed numerically according to the method described in the Appendix 7.D.

the constraint 7.9a is active. Based on the complementary slackness KKT conditions (Eq. 7.12d) and the fact that all other constraints except 7.9a are not active, we derive that all Lagrange multipliers are equal to 0 except those corresponding to the constraint 7.9a. This reduces the system 7.12 into the following system:

$$\frac{\partial \sigma_i^1(c)}{\partial c_i} - \lambda_{i1} \frac{\partial g_1^1(c)}{\partial c_i} = 0, \ i = 1, \ldots, I \tag{7.14a}$$

$$\lambda_{i1} \geq 0, \ i = 1, \ldots, I \tag{7.14b}$$

The derivative $\frac{\partial g_1^1(c)}{\partial c_i}$ is always positive, for all different user utility functions considered in this work. Therefore, the system of Eqs. 7.14 is reduced to the following inequalities:

$$\frac{\partial \sigma_i^1(c)}{\partial c_i} \geq 0, \ i = 1, \ldots, I \tag{7.15}$$

This system of inequalities restricted at the points in which the constraint 7.9a is active corresponds to a feasibility problem that can be solved efficiently. Such a problem may have uncountable solutions and therefore, the set of NEs on the surface separating the regions 1 and 2 may be infinite.

Computation of NEs in the region 2. To compute the NEs of the game of providers in the region 2, we follow a similar procedure. The region 2 is the set of price vectors that satisfy the constraints 7.11. As in the case of the region 1, a point at which at least one of the constraints 7.11c or 7.11b is active can not be a NE (e.g., points D and E in Fig. 7.5a). Therefore, the NEs could either lie in the interior of the region 2 or at the set points at which the constraint 7.11a is active. To search for a NE in the interior of the region 2, the following system of equations should be solved:

$$\frac{\partial \sigma_i^2(c)}{\partial c_i} = 0, \ \text{for all } i = 1, \ldots, I \tag{7.16}$$

The term $\sigma_i^2(c)$ corresponds to the utility function of the provider i restricted in the region 2. If the solution of Eq. 7.16 satisfies the constraints 7.11, it is a NE of the game of providers restricted in the region 2. Furthermore, a point c at which the constraint 7.11a is active is a NE, if the following conditions hold:

$$\frac{\partial \sigma_i^2(c)}{\partial c_i} \leq 0, \ i = 1, \ldots, I \tag{7.17}$$

Again, solving the inequalities 7.17 restricted at the points at which the constraint 7.11a is active corresponds to a feasibility problem with potentially uncountable solutions.

Computation of global NEs. Let us denote the sets of price vectors corresponding to the regions 1 and 2 as A_1 and A_2, respectively. The games restricted in these regions can be then defined as $\Gamma_1 = \big(P, A_1, \{\sigma_i^1\}_{i \in P}\big)$ and $\Gamma_2 = \big(P, A_2, \{\sigma_i^2\}_{i \in P}\big)$, respectively. A more

general game $\Gamma = (P, A_1 \cup A_2, \{\sigma_i\}_{i \in P})$ that is restricted on the union of the regions 1 and 2 can be now formed. The following set of theorems (proven in the Appendix 7.F) relate the NEs of the game Γ with the NEs of the games Γ_1 and Γ_2.
Theorem 1: A point $c^* \in A_1 \cap A_2$ is a NE of the game Γ, if and only if, it is a NE of the games Γ_1 and Γ_2.

Theorem 2: A point $c^* \in A_1 \backslash A_2$ is a NE of the game Γ, if and only if, it is a NE of the game Γ_1 and the following conditions are true:

$$\sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^2(c_i, c_{-i}^*),\ \forall c_i : (c_i, c_{-i}^*) \in A_2, \forall i \in P \tag{7.18}$$

Theorem 3: A point $c^* \in A_2 \backslash A_1$ is a NE of the game Γ, if and only if, it is a NE of the game Γ_2 and the following conditions are true:

$$\sigma_i^2(c_i^*, c_{-i}^*) \geq \sigma_i^1(c_i, c_{-i}^*),\ \forall c_i : (c_i, c_{-i}^*) \in A_1, \forall i \in P \tag{7.19}$$

In the inequalities 7.18 and 7.19, the point c^* is also denoted as (c_i^*, c_{-i}^*), where c_i^* is the price of the provider i and c_{-i}^* is a vector containing the prices of all other providers except i, at c^*. Theorem 2 implies that if there exists a NE in the interior of the region 1 (i.e., solution of Eqs. 7.13) and if the conditions of NE hold for points lying in the region 2 (inequalities 7.18), then it is also a global NE. For example, in Fig. 7.6, a NE in the interior of the region 1 is global if the inequalities 7.18 hold. The dotted lines in the region 2 correspond to the points $(c_i, c_{-i}^*) \in A_2$ considered in the inequalities 7.18. Similarly, Theorem 3 implies that if there exists a NE in the interior of the region 2 (i.e., provided by Eqs. 7.16), then it is also a global NE, if the conditions of inequalities 7.19 are satisfied. Furthermore, according to the Theorem 1, the set of NEs at the surface separating the regions 1 and 2 is the intersection of the sets of NEs of the games restricted in the regions 1 and 2, respectively (Fig. 7.6).

An algorithm for the computation of a NE. The KKT system 7.12 is a set of necessary conditions for a point to be a NE of the game of providers restricted in the region 1. These conditions are also sufficient only if the utility functions of providers are concave in their prices [62]. Given that the utility functions of providers are concave, the methodology described in Section 7.2.3.1 is guaranteed to compute a global NE if one exists. However, there are scenarios in which the utility functions of providers are not concave in the region 1. In such a case, while the KKT conditions 7.12 are still necessary for a set of prices c to be a NE, they are not sufficient. Therefore, when computing a solution of these conditions, we should verify if it is a NE or not. Specifically, we should verify whether or not this solution corresponds to a global maximum of the utility functions of providers with respect to their prices.

Our algorithm for the computation of a NE proceeds as follows: First, it attempts to compute a NE at the interior of the region 2 by solving Eqs. 7.16 and checking the conditions 7.19. If a global NE is computed, it is returned, otherwise, the algorithm attempts to compute a NE at the interior of the region 1 by solving Eqs. 7.13. If a solution is computed, the algorithm verifies whether or not it corresponds to a global maximum of the utility functions of providers. If it corresponds to a global maximum of the utility functions of providers, it is a global NE and is returned. If it corresponds to a local maximum of the utility functions of providers, the algorithm reports it as a "local NE". Finally, if the solution corresponds to a local minimum for at least one of the providers or if no solution was computed for Eqs. 7.13, the algorithm searches for a NE at the surface separating the regions 1 and 2 by solving the inequalities 7.15 and 7.17. Note that if the utility functions of providers are not concave at the interior of the region 1, there may be scenarios in which there is no pure-strategy NE of the game of providers. In such cases, our algorithm will not report a NE.

7.2.3.2 Cooperation of providers: computation of the Pareto optimal solution

Until now, we had assumed that providers are fully competitive. Let us now define a market case in which providers cooperate aiming to optimize a common objective function. An example of such a function could be the sum of the utility functions of individual providers (i.e., $S(c) = \sum_{i=1}^{I} \sigma_i(c)$). The price vector c that maximizes the function $S(c)$ is a *Pareto optimal solution*. The closer a competitive NE to a Pareto optimal solution, the more efficient [71]. A Pareto optimal solution can be computed by solving an optimization problem in which the function to be minimized is $-S(c)$. The constraints of the optimization problem define the strategy space of providers. As mentioned earlier, the utility functions of providers are not continuously differentiable at the entire strategy space (C). However, we could again divide the strategy space C into a number of regions (as shown in Fig. 7.5a) in which the utility functions of providers have a continuous derivative.

To compute the Pareto optimal solution, we only need to search the regions 1 and 2. At all points outside these regions, the market share of one or more providers is equal to 0. We can reduce the prices of these providers by such an amount that we reach the boundary of the region 1 or region 2. The value of the objective function at the point of the boundary is the same as the value at the initial point because only providers with a market share of 0 changed their prices. For example, if the currently offered prices correspond to the point X of Fig. 7.5a, the price of the provider 2 can be reduced until it reaches the point Y on the boundary of the region 2. The value of the objective function ($S(c)$) at points X and Y is the same because only the price of the provider 2 which has a market share of 0 at both X and Y has changed. Therefore, we can restrict our attention in the regions 1 and 2. When searching for the Pareto optimal solution at the regions 1 and 2, we solve two optimization problems in which the function to be minimized is $-S(c)$, under the corresponding constraints defined by the inequalities 7.9 and 7.11, respectively. The Pareto optimal solution is the one that corresponds to the highest value for the function $S(c)$.

7.3 Analysis of a wireless oligopoly

We implemented the modeling framework in Matlab and instantiated a wireless access market of a small city, represented by a rectangle of 14.4 km x 12.5 km. In this market there are 4 providers and a population of $300,000$ users. Each provider has deployed a cellular network covering the entire city. The BSs at each network are placed on the sites of a triangular grid, with a distance between two neighbouring sites of 1.6 km. The maximum data rate with which a BS can serve sessions is 25, 22, 19, and 16 Mbps for the providers 1, 2, 3, and 4, respectively. The average size of a session is 10 Mbytes. Furthermore, the session service rate of a BS is $\mu_1 = 18.75$, $\mu_2 = 16.50$, $\mu_3 = 14.25$, and $\mu_4 = 12.00$ sessions/min for the providers 1, 2, 3, and 4, respectively.

7.3.1 Impact of traffic demand

We analysed the performance of our modeling framework and estimated the impact of the user traffic demand on the market equilibriums. Specifically, we varied the session generation rate of each user from 0 up to 1.5 sessions/hour. Furthermore, we assumed that the dependence of the user utility function (Eq. 7.6) on the average data rate is exponential ($f(x) = w_R \left(\tau - e^{-hx}\right)$), where w_R, τ and h are equal to 30, 1, and 0.6, respectively, while the weight of data rate variability (w_V) was set equal to 0. Fig. 7.7 shows the prices and revenue of providers, and market share of users. Figs. 7.7a, 7.7b, and 7.7c correspond to the NE, while Figs. 7.7d, 7.7e, and 7.7f correspond to the Pareto optimal solution.

The type of the NE varies with the user traffic demand. When the traffic demand is close to 0, the competition is not sustainable and the providers 3 and 4 obtain a market share of 0. As the traffic demand increases, the provider 3 and subsequently the provider 4 enter the market (Fig. 7.7b). Furthermore, low traffic demand results in low prices (Fig. 7.7a). This is due to the high intensity of competition. The price is the most significant parameter that drives the user decisions leading to a price war. As the traffic demand increases, the competition weakens. The effect of market share on the achievable data rate becomes more prominent and the intensity of competition is reduced leading to higher prices. In this case, the NE of providers lies in the interior of the region 1 (Fig. 7.5a), as explained in Section 7.2.3.

The prices of providers increase with the user traffic demand until a certain threshold. At this threshold, the NE moves from the interior of the region 1 to the surface separating the regions 1 and 2. In this equilibrium, the prices of providers have been slightly decreased. We observe this decrease of prices for a relatively small range of traffic demand around 0.95 sessions/hour (Fig. 7.7a). Then, the NE moves in the interior of the region 2: the prices become fixed and independent of the traffic demand and disconnected users appear (Figs. 7.7a and 7.7b, respectively).

As mentioned in Section 7.2.3.2, in the Pareto optimal solution, providers cooperate

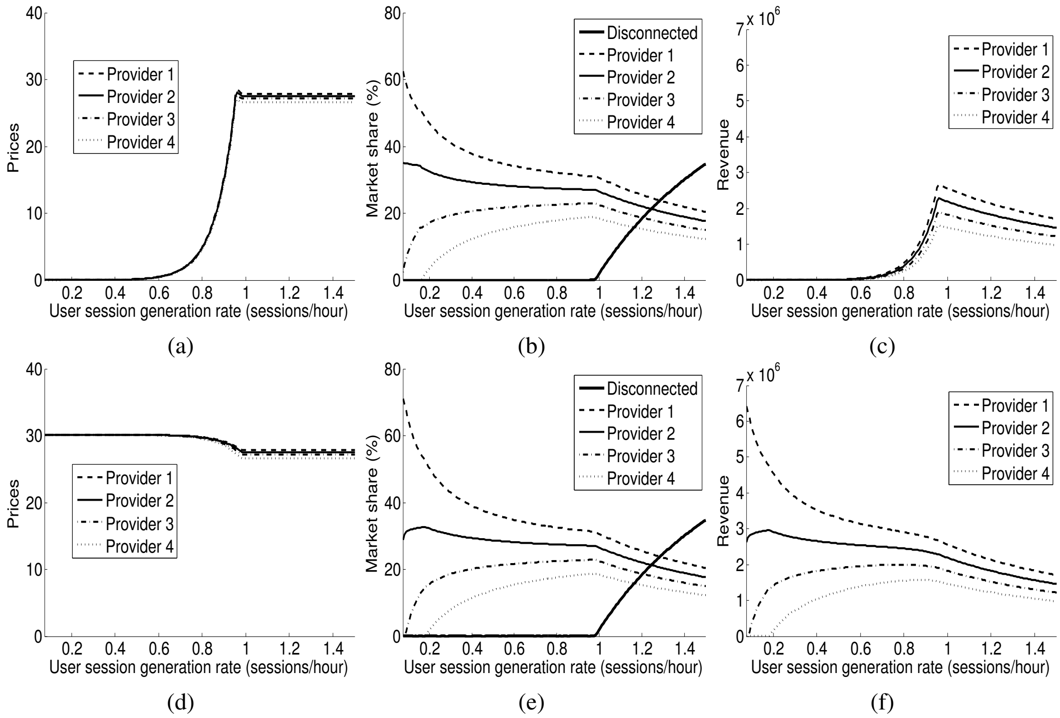


Figure 7.7: Competitive market equilibrium (top) and Pareto optimal solution (bottom) as a function of the user traffic demand with $w_V = 0$.

aiming to optimize a common objective function. When the user traffic demand is low, the Pareto optimal solution follows the reverse trend compared to the NE. Specifically, the prices of providers start at a high level and are reduced as the traffic demand increases (Fig. 7.7d). At the NE, due to the effect of competition, the prices start at a low level and increase as the traffic demand increases (Fig. 7.7a). As explained earlier, in the NE, the low traffic demand intensifies the competition, which has as a result reduced prices. On the other hand, in the Pareto optimal solution, providers are cooperative and offer prices at the level of the user willingness-to-pay. The larger the user traffic demand, the lower the achievable data rate in the networks of providers, and therefore, the lower the user willingness-to-pay, resulting in lower prices. The gap between the NE and the Pareto optimal solution decreases as the traffic demand increases until the two solutions become identical. This happens, when the generated traffic becomes too large to be served by the networks of providers. In this case, the competition between the providers is nullified and the prices become fixed and independent of the traffic demand. The above analysis demonstrates that a QoS-based user utility function can act as a catalyst in the competition and allow for higher prices. The absence of the QoS may trigger a price war. This also indicates that monitoring the QoS in real markets could be proven beneficial to providers.

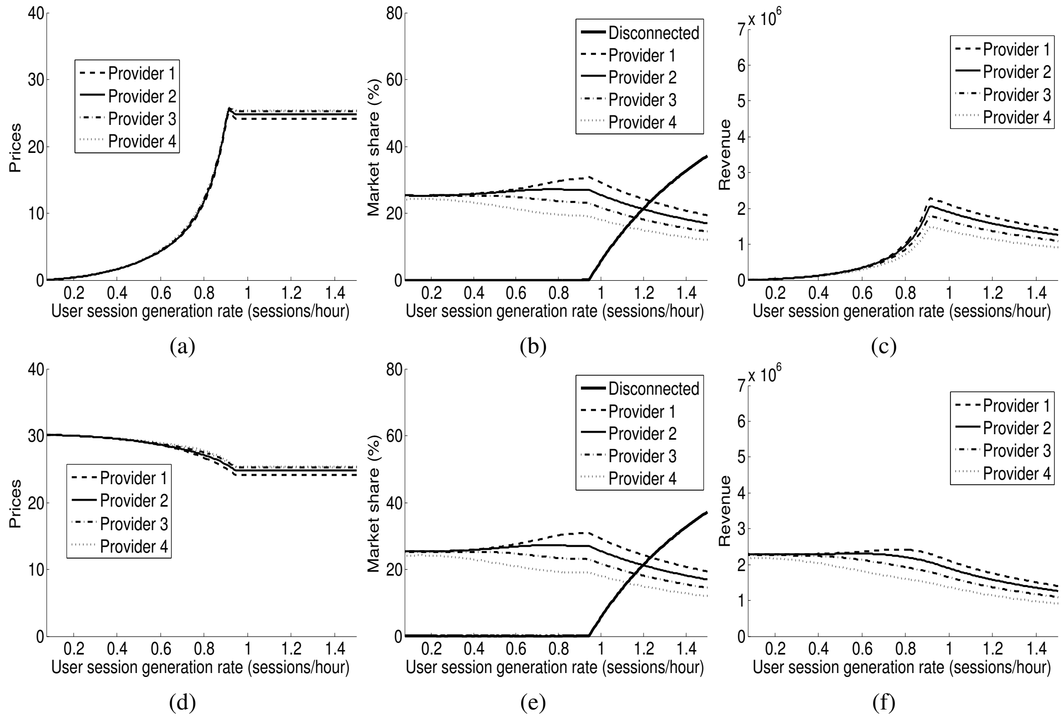


Figure 7.8: Competitive market equilibrium (top) and Pareto optimal solution (bottom) as a function of the user traffic demand with $w_V = 0.5$.

7.3.2 Impact of data rate variability

. This analysis was repeated by increasing the weight of data rate variability ($w_V = 0.5$). Fig. 7.8 shows the market equilibriums. Similar trends with Fig. 7.7 are observed: under low traffic demand, the NE lies in the interior of the region 1 and the prices are increasing with traffic demand (Fig. 7.8a). Above a certain threshold, the NE moves at the surface separating the regions 1 and 2 in which it remains for a small interval. Finally, the NE moves in the interior of the region 2 in which the prices remain fixed and disconnected users appear (Fig. 7.8b).

Despite the similarities, there are also important differences. Under low traffic demand, a large weight of data rate variability in the user utility function prevents the movement of large crowds towards a specific provider weakening the competition and increasing the prices. On the contrary, in case of large traffic demand, the variance of data rate reduces the user utility resulting in lower prices and revenue (e.g., Fig. 7.8a compared to Fig. 7.7a). Furthermore, under low traffic demand, a large weight of data rate variability results in similar market share of providers (Fig. 7.8b) compared to the case of low weight, in which the providers with the largest amount of bandwidth have a clear advantage (Fig.

7.7b). An increase in the market share of the large providers results in an increase of their data rate variance. This interrupts the flow of users to these providers. Another interesting trend is that the ordering of prices in Fig. 7.8a is reversed compared to Fig. 7.7a. This is due to the larger data rate variability in the networks of large providers compared to small ones resulting in slightly larger prices for small providers. Despite this, the ordering in the market share and revenue (Figs. 7.8b and 7.8c) remains the same as in Figs. 7.7b and 7.7c, respectively.

7.3.3 Impact of average data rate

The impact of the average data rate on the user willingness-to-pay is expressed through the function f (Eq. 7.6). The analysis has considered linear, exponential, logarithmic, and isoelastic functions (Table 7.3). However, in the case of linear functions, the assumed linear dependence of the user willingness-to-pay on the average data rate causes a pathology. Specifically, providers set their prices in such a way that there are at most two connected users at each of their BSs, on average. This is a not realistic outcome especially when there is a large amount of bandwidth available at BSs. The user willingness-to-pay is not linearly dependent on the average data rate. For example, if two users have data connections of 1 Mbps and 10 Mbps, respectively, an increase of the data rate by 1 Mbps will have a different impact on each of these users. This trend can be expressed by functions with a diminishing derivative with respect to the data rate. Examples of such functions that are commonly used in the economic literature are the exponential, logarithmic, and isoelastic ones [93].

7.3.4 Impact of network heterogeneity

The user mobility among the BSs of a provider is modeled by a Markov chain. The transition probabilities from a BS to its neighbouring BSs are determined according to a Zipf distribution ($f(k; s, N) = \frac{1/k^s}{\sum_{n=1}^{N} 1/n^s}$, where N is the number of neighbouring BSs, k is the order of a specific neighbouring BS with respect to its distance from the center of the topology, and s is the exponent characterizing the distribution). In general, the closer a neighbouring BS is to the center of the network topology, the larger is the transition probability to this BS. Furthermore, the larger the exponent of the Zipf distribution, the larger the concentration of user traffic demand towards the center of the topology. We have defined several scenarios in which, we varied the exponent of the Zipf distribution from 0 to 0.14 keeping all other parameters fixed. In general, as the exponent increases, the user traffic demand becomes more concentrated towards the center of the topology. This results in a decreased average value and increased spatial variance of data rate. Therefore, in the corresponding market equilibriums, the prices and revenue of providers decrease, while the percentage of disconnected users increases.

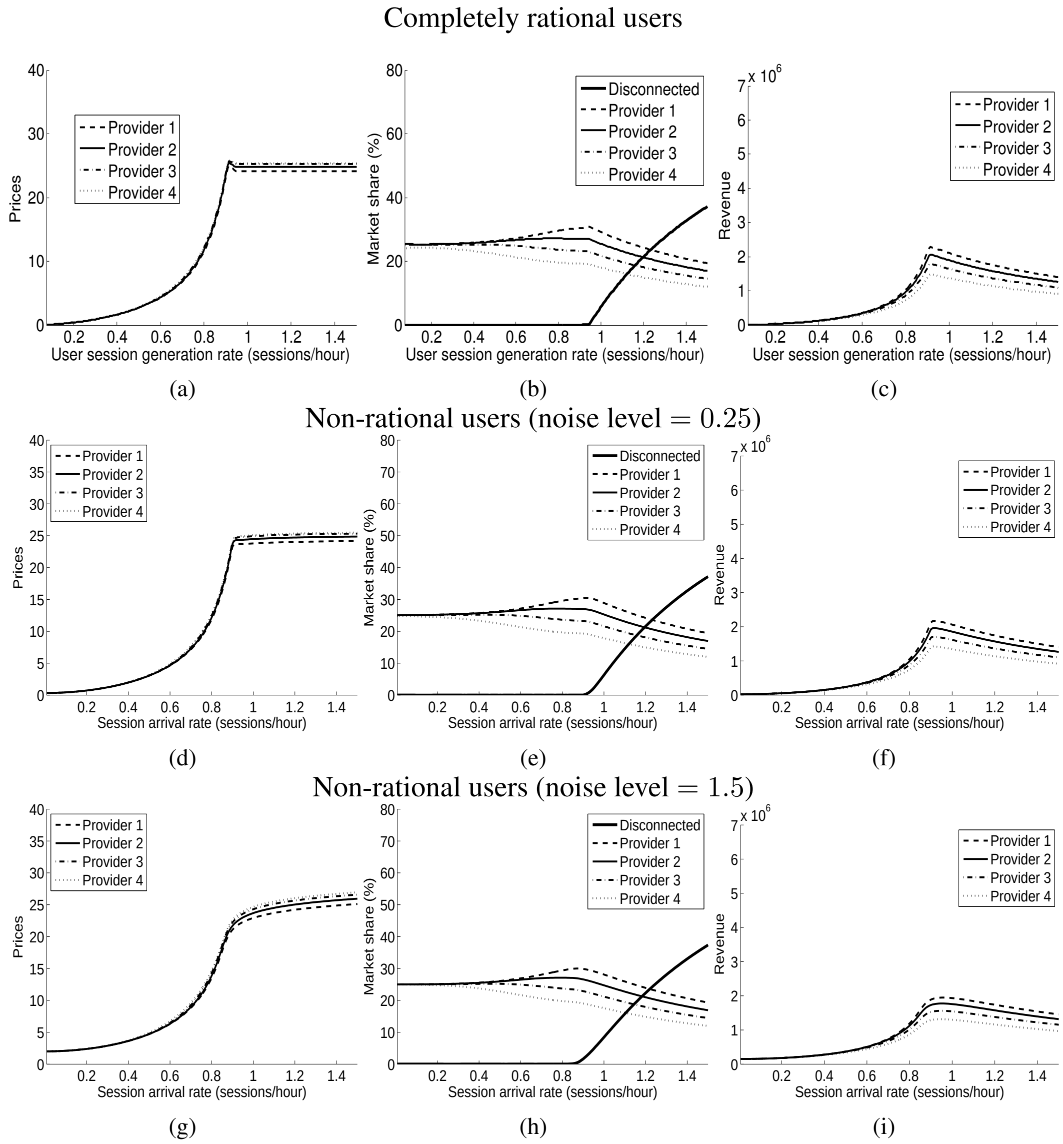


Figure 7.9: Performance of markets with different degrees of user rationality.

7.4 Comparison with markets in which users are not completely rational

We performed an additional experiment to compare the performance of a market with completely rational users with markets in which users are characterized by a certain degree of irrationality. Those markets were analysed at the macroscopic level according to the framework presented in Chapter 3. Figs. 7.9a - 7.9c present the offered prices, market share, and revenue of providers, respectively, in a market with completely rational users. The analysis of this market was performed according to the methodology presented in this chapter. On the other hand, Figs. 7.9d - 7.9f present the same performance metrics for a market in which users are not completely rational. The decision making of those users was modeled by the Logit dynamics (Eq. 3.10) and was affected by a noise level of 0.25. Finally, Figs. 7.9g - 7.9i correspond to a market in which users are affected by a noise level of 1.5 when they make their decisions. Except from the user irrationality, all other parameters of the market are kept the same as in Section 7.3.

In general, as the value of noise decreases, the performance metrics become closer to the ones of Figs. 7.9a - 7.9c. In other words, when the noise decreases towards 0, the results of the analysis performed using the multi-layer framework of Chapter 3 converge to the results obtained when applying the modeling framework of this chapter, which assumes that users are completely rational.

If we now study the Figs. 7.9a - 7.9i in more detail, we can observe some additional interesting trends. When users are completely rational, the prices of providers are increasing with the user traffic until a certain threshold. Then, the prices are slightly decreased, and finally, they become fixed and independent of the traffic demand (Fig. 7.9a) and disconnected users appear (Fig. 7.9b).

When the value of noise increases, this rapid change in the offered prices of providers is smoothed out. When the noise is equal to 0.25 the transition from increasing prices to fixed prices (Fig. 7.9d) is more smooth compared to the case of completely rational users (Fig. 7.9a). When the noise becomes equal to 1.5, this transitions has been completely smoothed out and the offered prices of providers resemble a sigmoid-like function of the user traffic demand (Fig. 7.9g). In this case, when the capacity of the networks of providers is reached and disconnected users appear, the prices continue to increase with the user traffic demand instead of becoming fixed but with a much smaller rate (Fig. 7.9g).

7.5 Concluding remarks

This chapter contributes with a novel methodology for the analytical estimation of the NEs of users and provider in markets in which users make their decisions in a completely rational manner. This methodology can be applied at the macroscopic level in which users are modeled as a homogeneous population. Based on this methodology, we have analysed a wireless oligopoly of a small city. We have distinguished scenarios in which providers are fully competitive with ones in which providers cooperate to optimize a common objective.

The analysis has demonstrated that the QoS is an important parameter of the user utility function which can reduce the intensity of competition and allow for higher prices and revenue. It has also illustrated that the model of the user utility function is important. For example, when it considers the effect of the spatial variance of data rate except from its average value, different trends are observed in the offered prices, market share, and revenue of providers. The analysis has also shown that at the macroscopic level, there is a convergence of the performance metrics produced by the framework of Chapter 3 with the ones produced by the framework presented in this chapter when the value of noise tends to 0.

7.A Estimation of the user NE

For each user NE $z^* = (z_0^*, z_1^*, ..., z_I^*)$, we can prove the following lemmas:

Lemma 1: All strategies with a strictly positive market share at a user NE z^* should correspond to the same utility.
Proof: If the Lemma 1 does not hold, there will exist at least two strategies i, j with a strictly positive market share such that $u_i(z_i^*; c) > u_j(z_j^*; c)$. Users that have selected the strategy j would have the incentive to select the strategy i as it offers a higher utility. This contradicts the definition of a NE.

Lemma 2: All strategies with a market share of 0 at a user NE z^* should correspond to a utility that is lower than or equal to the utility of the strategies with a strictly positive market share.
Proof: If the Lemma 2 does not hold, there will be at least two strategies i, j such that the strategy i has a market share of 0 (i.e., $z_i^* = 0$) and the strategy j has a strictly positive market share (i.e., $z_j^* > 0$) and $u_i(z_i^*; c) > u_j(z_j^*; c)$. Users that have selected the strategy j would have the incentive to select the strategy i as it offers a higher utility. This contradicts the definition of a NE.

Lemma 3: At a user NE z^*, we can divide the set of strategies H into two disjoint subsets X and Y, such that: (i) X is non-empty, (ii) all strategies in X correspond to the same utility, (iii) all strategies in Y correspond to a market share of 0. The pair X and Y is not necessarily unique.
Proof: For a user NE z^*, we will construct two sets X and Y that satisfy the conditions of the Lemma 3. The set of all strategies with a strictly positive market share at z^* is non-empty (given that $\sum_{j \in H} z_j^* = 1$) and is denoted as X_1. Based on the Lemma 1, all strategies in X_1 correspond to the same utility (U). All other strategies in $H - X_1$ correspond to a market share of 0. Therefore, according to the Lemma 2, all strategies in $H - X_1$ correspond to a utility that is lower than or equal to U. We divide the set $H - X_1$ into two subsets, one containing the strategies with utility equal to U and one containing the strategies with utility strictly lower than U (denoted as X_2 and X_3, respectively). For each subset $S \subseteq X_2$, we can define the sets $X = X_1 \cup S$ and $Y = (X_2 - S) \cup X_3$. The set X is non-empty (given that X_1 is non-empty) and all strategies in X correspond to a utility equal to U. All strategies in Y correspond to a market share of 0. Therefore, the sets X and Y satisfy the conditions of the Lemma 3. If $X_2 = \emptyset$, then based on the above construction process, the sets X and Y are unique. Otherwise, they are not unique.

7.B Uniqueness of user NE

As mentioned in Section 7.2.2, every population game admits at least one NE (as can be proven by applying Kakutani's fixed point theorem). Now, we will prove the uniqueness of the user NE for each price vector c of providers in the case that the functions $u_i(z_i;c)$ are strictly decreasing in z_i.

Theorem 1: If the utility functions $u_i(z_i;c)$ are strictly decreasing in z_i, for each price vector c of providers, there exists a unique user NE.
Proof: First, we will prove that at a user NE z^*, all strategies with a strictly positive market share should correspond to the same utility. If this is not true, then there will be two strategies k and j with strictly positive market share, such that $u_k(z_k^*;c) < u_j(z_j^*;c)$. This contradicts the definition of NE at z^* because users that have selected the strategy k would have the incentive to select the strategy j as it corresponds to a higher utility. Let us now assume that for a specific price vector c, there exist multiple user NEs. We select an arbitrary pair of such equilibriums $z^+ = \left(z_0^+, z_1^+, ..., z_I^+\right)$ and $z^- = \left(z_0^-, z_1^-, .., z_I^-\right)$. When a user selects a strategy that has a strictly positive market share at the equilibrium z^+, it will obtain a utility denoted as U^+. Similarly, when a user selects a strategy with a strictly positive market share at the equilibrium z^-, it will obtain a utility denoted as U^-. If we assume that $z^+ \neq z^-$, then there will be two different strategies $k, j \in H$, such that $z_k^+ > z_k^-$ and $z_j^+ < z_j^-$. First, given that $z^+ \neq z^-$, there will be at least a strategy $k \in H$, such that $z_k^+ \neq z_k^-$. Without loss of generality, we assume that $z_k^+ > z_k^-$, then:

$$z_k^+ > z_k^- \Rightarrow 1 - z_k^+ < 1 - z_k^- \Rightarrow \sum_{i \neq k} z_i^+ < \sum_{i \neq k} z_i^- \tag{7.20}$$

We still need to prove that there exists a strategy $j \neq k$, such that $z_j^+ < z_j^-$. We will prove that by contradiction. Specifically, we assume that $z_i^+ \geq z_i^-$ for all $i \neq k$. Then, $\sum_{i \neq k} z_i^+ \geq \sum_{i \neq k} z_i^-$. This contradicts the inequality 7.20. In summary, we have proven that if $z^+ \neq z^-$, then there will be two different strategies $k, j \in H$, such that $z_k^+ > z_k^-$ and $z_j^+ < z_j^-$. We will now distinguish two cases:

Case (a): None of the strategies k, j is the disconnection. As mentioned earlier, each strategy, except the disconnection, has a strictly decreasing utility function. Therefore, the functions $u_k(z_k;c)$ and $u_j(z_j;c)$ are strictly decreasing in z_k and z_j, respectively, and it follows that:

$$z_k^+ > z_k^- \geq 0 \Rightarrow U^+ = u_k(z_k^+;c) < u_k(z_k^-;c) \leq U^- \tag{7.21a}$$
$$z_j^- > z_j^+ \geq 0 \Rightarrow U^- = u_j(z_j^-;c) < u_j(z_j^+;c) \leq U^+ \tag{7.21b}$$

The final inequality in 7.21a is true because if $z_k^- > 0$, then $u_k(z_k^-; c) = U^-$ (as shown earlier, all strategies with a strictly positive market share should correspond to the same utility, i.e., U^-), while if $z_k^- = 0$, then $u_k(z_k^-; c) \leq U^-$ (otherwise there is a contradiction in the definition of NE at z^-). A similar argument can be made for the final inequality of 7.21b. The inequalities 7.21a and 7.21b contradict each other.

Case (b): One of the strategies k, j is the disconnection. Without loss of generality, we assume that $k = 0$ and $j \neq 0$. Given that the utility function $u_j(z_j; c)$ is strictly decreasing in z_j, it follows that:

$$z_0^+ > z_0^- \geq 0 \Rightarrow U^+ = u_0(z_0^+; c) = u_0(z_0^-; c) \leq U^- \tag{7.22a}$$
$$z_j^- > z_j^+ \geq 0 \Rightarrow U^- = u_j(z_j^-; c) < u_j(z_j^+; c) \leq U^+ \tag{7.22b}$$

The inequalities 7.22a and 7.22b contradict each other. Therefore, z^+ and z^- should coincide. We have proven that for each arbitrary pair of user NEs z^+ and z^- at c, those equilibriums should coincide. Therefore, each vector of offered prices c corresponds to a unique user NE.

7.C The GNEP problem

Consider the game of providers with its set of strategies restricted in the region 1. This region corresponds to an orthogonally convex set[5] $A_1 \subset C$ that can be defined by a number of inequalities. The orthogonal convexity of the regions 1 and 2 is proven in the Appendix 7.E.

$$A_1 = \left\{c \in C : g_j^1(c) \leq 0, \forall j \in \{1, ..., 2I + 1\}\right\} \tag{7.23}$$

Each provider $i \in P$ is characterized by its utility function $\sigma_i^1 : A_1 \to R$ which is restricted on the set A_1. A strategy profile $c \in A_1$ is a vector containing the prices offered by all providers and can be annotated as (c_i, c_{-i}), where c_i is the price of the provider i and c_{-i} is a vector containing the prices of all other providers except i. To proceed with the analysis, we make the following assumption: $\sigma_i^1(\cdot, c_{-i})$ and $g_j^1(\cdot, c_{-i})$ are continuously differentiable $\forall c_{-i}, i = 1, ..., I, j = 1, ..., 2I + 1$.

A game satisfying the above condition is annotated as $\Gamma_1 = (P, A_1, \{\sigma_i^1\}_{i \in P})$. One necessary and sufficient condition for a point $c^* \in A_1$ to be a Nash equilibrium of the game Γ_1 is that for each provider i, its chosen strategy (c_i^*) is a best response to the strategies of all other providers.

[5] A set S in the Euclidean space is called *orthogonally convex*, if any segment parallel to any of the coordinate axes connecting two points of S lies totally within S.

$$\sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^1(c_i, c_{-i}^*),\ \forall c_i \in X_i^1(c_{-i}^*) \tag{7.24}$$

The set $X_i^1(c_{-i}^*)$ contains all possible prices that can be offered by the provider i when the prices of all other providers are c_{-i}^* ($X_i^1(c_{-i}^*) = \{c_i : (c_i, c_{-i}^*) \in A_1\}$). In general, the price of a provider $i \in P$ at the Nash equilibrium c^* can be determined by solving the following optimization problem.

$$\begin{aligned} &\underset{c_i \in X_i^1(c_{-i}^*)}{\text{minimize}} && -\sigma_i^1(c_i, c_{-i}^*) \\ &\text{subject to} && g_j^1(c_i, c_{-i}^*) \leq 0,\ j = 1, \ldots, 2I+1. \end{aligned} \tag{7.25}$$

The Lagrange function of the problem is defined in Eq. 7.26.

$$L_i^1(c_i, c_{-i}^*, \lambda_i) = -\sigma_i^1(c_i, c_{-i}^*) + \sum_{j=1}^{2I+1} \lambda_{ij} g_j^1(c_i, c_{-i}^*) \tag{7.26}$$

The vector $\lambda_i = (\lambda_{i1}, \lambda_{i2} \ldots, \lambda_{i2I+1})$ contains the Lagrange multipliers that correspond to all inequality constraints. The KKT conditions for the problem (7.25) are defined in detail in the system 7.27.

$$\begin{aligned} \frac{\partial L_i^1(c_i, c_{-i}^*, \lambda_i)}{\partial c_i} &= \ 0 \\ g_j^1(c_i, c_{-i}^*) &\leq \ 0,\ j = 1, \ldots, 2I+1 \\ \lambda_{ij} &\geq \ 0,\ j = 1, \ldots, 2I+1 \\ \lambda_{ij} g_j^1(c_i, c_{-i}^*) &= \ 0,\ j = 1, \ldots, 2I+1 \end{aligned} \tag{7.27}$$

We can now combine the KKT conditions of the optimization problems of individual providers to the general system 7.28.

$$\frac{\partial L_i^1(c, \lambda_i)}{\partial c_i} = \ 0,\ i = 1, \ldots, I \tag{7.28a}$$

$$g_j^1(c) \leq \ 0,\ j = 1, \ldots, 2I+1 \tag{7.28b}$$

$$\lambda_{ij} \geq \ 0,\ i = 1, \ldots, I \text{ and } j = 1, \ldots, 2I+1 \tag{7.28c}$$

$$\lambda_{ij} g_j^1(c) = \ 0,\ i = 1, \ldots, I \text{ and } j = 1, \ldots, 2I+1 \tag{7.28d}$$

The NEs of the game of providers restricted in the region 1 can either lie in the interior of the region 1 or at the surface separating the regions 1 and 2. All local maxima of the utility function of each individual provider (solutions of the problem 7.25) at these sets of points satisfy the LICQ constraint qualification. Therefore, the conditions of the system 7.28 are satisfied at each pure-strategy NE of the game of providers restricted in the region 1 [92]. A similar argument can be made for the game of providers restricted in the region 2.

7.D Estimation of the derivatives of the utility functions of providers

In the region 1, the user NE $z^*(c)$ corresponds to the solution of Eqs. 7.8, i.e. to $z^1(c)$. Therefore, the derivatives of the provider utility functions in the region 1 are derived as follows:

$$\frac{\partial \sigma_i^1(c)}{\partial c_i} = N \frac{\partial z_i^1(c)}{\partial c_i} c_i + N z_i^1(c) \tag{7.29}$$

Given that the user NE $z^1(c)$ is estimated numerically, no closed-form solution can be determined for the function $\frac{\partial z_i^1(c)}{\partial c_i}$ and therefore, for the function $\frac{\partial \sigma_i^1(c)}{\partial c_i}$. However, the user NE at the region 1 satisfies the Eqs. 7.8:

$$\begin{aligned} u_i(z_i^1(c); c) = u_1(z_1^1(c); c)\ \forall i \in \{2, ..., I\} \Rightarrow \\ g_i(z_i^1) - w_P c_i = g_1(z_1^1) - w_P c_1 \end{aligned} \tag{7.30}$$

In Eq. 7.30, $g_i(z_i^1) = f\left(R_i(z_i^1)\right) - w_V V_i(z_i^1)$. Now, we will differentiate Eq. 7.30 with respect to c_k:

$$\frac{\partial g_i(z_i^1)}{\partial z_i^1} \frac{\partial z_i^1(c)}{\partial c_k} - w_P \frac{\partial c_i}{\partial c_k} = \frac{\partial g_1(z_1^1)}{\partial z_1^1} \frac{\partial z_1^1(c)}{\partial c_k} - w_P \frac{\partial c_1}{\partial c_k}$$

Based on Eqs. 7.8, $z_1^1 = 1 - \sum_{j=2}^{I} z_j^1$. We substitute this expression in the above equation.

$$\frac{\partial g_i(z_i^1)}{\partial z_i^1} \frac{\partial z_i^1(c)}{\partial c_k} - w_P \frac{\partial c_i}{\partial c_k} = -\frac{\partial g_1(z_1^1)}{\partial z_1^1} \sum_{j=2}^{I} \frac{\partial z_j^1(c)}{\partial c_k} - w_P \frac{\partial c_1}{\partial c_k}\ \forall i \in \{2, ..., I\} \tag{7.31}$$

Note that in Eq. 7.31, all terms are known except from the terms $\frac{\partial z_j^1(c)}{\partial c_k}$ for $j \in \{2, ..., I\}$. Therefore, it is a linear system of $I-1$ equations with $I-1$ unknowns. Similar systems can be defined for all $k \in \{1, ..., I\}$. By solving these systems of equations, we estimate the values of $\frac{\partial z_i^1(c)}{\partial c_i}$ needed for the estimation of the derivatives of the utility functions of providers (Eq. 7.29). The derivatives of the provider utility functions in the region 2 are estimated similarly.

7.E Orthogonal convexity of regions 1 and 2

As mentioned in the Appendix 7.C, for the formulation of the GNEP problems that correspond to the regions 1 and 2, it is required that these regions are orthogonally convex. The orthogonal convexity of these regions is proven by the following theorems.

Theorem 1: The region 2 is orthogonally convex.
Proof: Consider two price vectors $c^a = (c_1, ..., c_i^a, ..., c_I)$ and $c^b = (c_1, ..., c_i^b, ..., c_I)$ in the region 2 (i.e., c^a, $c^b \in A_2$) with $c_i^b > c_i^a$ (Fig. 7.10). The vector c^a differs from c^b only in the price of the provider i. Take an arbitrary point $c = (c_1, ..., c_i, ..., c_I)$ on the line segment connecting c^a and c^b. We will prove that this point belongs in the region 2.

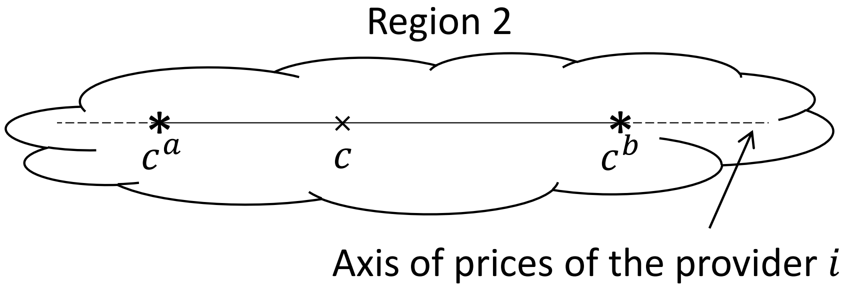


Figure 7.10: The vectors c^a, c^b, and c.

Given that $c^a, c^b \in A_2$, there exist two market share vectors that are NEs of the game of users at c^a and c^b, respectively. At these NEs, the utilities of all strategies are equal to 0 (case b of Section 7.2.2.3). We denote these vectors as $z^+ = (z_0^+, z_1^+, ..., z_i^+, ..., z_I^+)$ and $z^- = (z_0^-, z_1^-, ..., z_i^-, ..., z_I^-)$. The vector z^+ is the one in which the market share of the provider i is the largest (i.e., $z_i^+ > z_i^-$) [6] and corresponds to one of the price vectors c^a or c^b which we denote as c^+, while the other is denoted as c^-. The price vectors c^+ and c^- (i.e., c^a and c^b) belong in the region 2 and therefore, the utilities of all strategies at the equilibriums z^+ and z^- are equal to zero (see case b in Section 7.2.2.3). Based on the definition of the user utility function (Eq. 7.6), the following equations are derived:

$$f\left(R_i(z_i^+)\right) - w_V V_i(z_i^+) = w_P c_i^+ \tag{7.32a}$$
$$f\left(R_i(z_i^-)\right) - w_V V_i(z_i^-) = w_P c_i^- \tag{7.32b}$$

[6] Note that z_i^+ and z_i^- cannot be equal because in such a case, the utility that corresponds to the strategy i could not be equal to zero at both z^+ and z^-.

We define a new function $g_i(z_i) = f(R_i(z_i)) - w_V V_i(z_i)$. This function is continuous. Specifically, $f(R_i(z_i))$ is continuous as a composite of the continuous functions $f(x)$ and $R_i(z_i)$. Furthermore, $g_i(z_i)$ is continuous as the sum of the continuous functions $f(R_i(z_i))$ and $-w_V V_i(z_i)$. According to the intermediate value theorem, there exists a $z_i \in [z_i^-, z_i^+]$ such that $g_i(z_i) = w_P c_i$. We can now construct a market share vector z^* which is a NE of the game of users at c as follows:

$$z^* = (z_0^*, z_1^*, ..., z_i^*, ..., z_I^*) = \left(1 - \sum_{j \neq 0,i} z_j^+ - z_i, z_1^+, ..., z_i, ..., z_I^+\right) \tag{7.33}$$

It is evident that $\sum_{j=0}^{I} z_j^* = 1$. Furthermore, $z_j^* \geq 0$ for $j \neq 0, i$ as part of the user NE at c^+, $z_i^* = z_i \geq z_i^- \geq 0$, and $z_0^* = 1 - \sum_{j \neq 0,i} z_j^+ - z_i \geq 1 - \sum_{j \neq 0} z_j^+ = z_0^+ \geq 0$. In summary, z^* is a valid probability distribution and when the market share of users becomes equal to z^* at c, the utilities of all strategies become equal to 0 (i.e., it satisfies the conditions of Eqs. 7.10 and 7.11). Therefore, c belongs in the region 2. A similar argument can be made for all other points on the line segment connecting c^a and c^b. Therefore, the entire line segment is included in the region 2. This proves that the region 2 is orthogonally convex.

Theorem 2: If the utility functions $u_i(z_i; c)$ are strictly decreasing in z_i, the region 1 is orthogonally convex.
Proof: Consider two price vectors $c^+ = (c_1, ..., c_i^+, ..., c_I)$ and $c^- = (c_1, ..., c_i^-, ..., c_I)$ belonging in the region 1 (c^+, $c^- \in A_1$). The vector c^+ differs from c^- only in the price of the provider i and we assume that $c_i^+ > c_i^-$ (Fig. 7.11). Take an arbitrary point $c = (c_1, ..., c_i, ..., c_I)$ on the line segment connecting c^+ and c^-. We will prove that this point also belongs in the region 1.

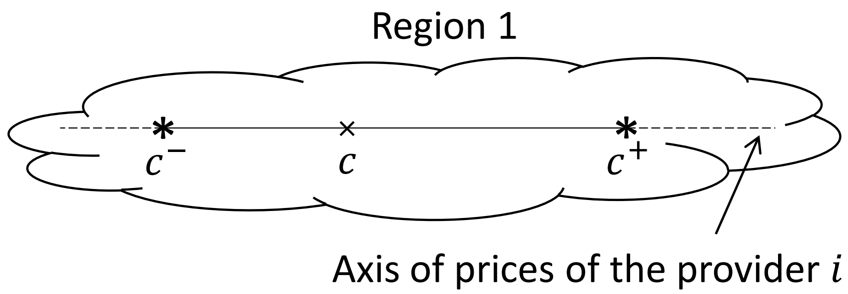


Figure 7.11: The vectors c^+, c^-, and c.

Given that $c^+, c^- \in A_1$, there exist two market share vectors $z^+ = (z_0^+, z_1^+, ..., z_i^+, ..., z_I^+)$ and $z^- = (z_0^-, z_1^-, ..., z_i^-, ..., z_I^-)$ that are NEs of the game of users at c^+ and c^-, respectively. Additionally, the market share of the disconnection at these equilibriums is equal to zero (i.e., $z_0^+ = 0$, $z_0^- = 0$) (see case a in Section 7.2.2.3). At the point

c, there exists a unique user NE z^* (proven in the Appendix 7.B). We will distinguish cases for this NE. First, we will assume that this equilibrium contains positive market share for the disconnection (i.e., $z_0^* > 0$). Given that $z_0^* > 0$, there exists a strategy $j \in \{1, ..., I\}$ such that $z_j^+ > z_j^*$. This can be easily proven by contradiction. If $j = i$, then:

$$\begin{aligned} z_i^+ > z_i^*, c_i^+ > c_i \Rightarrow \\ u_i(z_i^*; c_i) > u_i(z_i^+; c_i) > u_i(z_i^+; c_i^+) \end{aligned} \tag{7.34}$$

Given that the disconnection has a strictly positive market share at z^*, then according to the Lemmas 1 and 2 of the Appendix 7.A, it follows that $u_i(z_i^*; c_i) \leq 0$ and based on the inequalities 7.34 $u_i(z_i^+; c_i^+) < 0$. This contradicts the definition of NE at c^+ because users that have selected the strategy i would have the incentive to select the disconnection as it offers higher utility compared to the strategy i [7]. If $j \neq i$, then:

$$z_j^+ > z_j^* \Rightarrow u_j(z_j^*; c_j) > u_j(z_j^+; c_j) \tag{7.35}$$

Given that the disconnection has a strictly positive market share at z^*, it follows that $u_j(z_j^*; c_j) \leq 0$. Therefore, based on the inequality 7.35 $u_j(z_j^+; c_j) < 0$. This also contradicts the definition of NE at c^+. In summary, at c, it is not possible for a NE to have a positive market share for the disconnection (i.e., $z_0^* = 0$).

We will now prove that at z^*, the subscriptions of all providers correspond to the same utility. According to the Lemmas of the Appendix 7.A, all strategies with a strictly positive market share at z^* correspond to the same utility (U). We divide the set of strategies $H - \{0\}$ into two subsets X and Y, such that, X contains all strategies with utility equal to U and Y contains all strategies with utility strictly lower than U, at z^*. Note that, at z^*, there are no strategies with utility larger than U. Additionally, all strategies in Y correspond to a market share of 0. Otherwise, there would be a contradiction in the definition of NE. Let us now assume that not all subscriptions of providers correspond to the same utility, at z^*. Then, the set Y will be non-empty. If $i \in Y$, then:

$$\begin{aligned} z_i^+ \geq z_i^* = 0, c_i^+ > c_i \Rightarrow \\ U > u_i(z_i^*; c_i) \geq u_i(z_i^+; c_i) > u_i(z_i^+; c_i^+) \end{aligned} \tag{7.36}$$

Given that c^+ belongs in the region 1, at z^+, the subscriptions of all providers correspond to the same utility (see case a in Section 7.2.2.3). Therefore, $u_i(z_i^+; c_i^+) = u_k(z_k^+; c_k)$ for all $k \in X$. Furthermore, at z^*, all strategies in X correspond to utility equal to U (i.e. $u_k(z_k^*; c_k) = U$ for all $k \in X$). By substituting the above expressions in Eq. 7.36, it follows

[7] Note that in Eq. 7.6, the utility that corresponds to a provider $i \in \{1, ..., I\}$ depends only on the market share (z_i) and price (c_i) of that provider. To simplify the analysis in this proof, we will denote the utility that corresponds to a provider i as $u_i(z_i; c_i)$ instead of $u_i(z_i; c)$.

that:

$$\begin{aligned} &U = u_k(z_k^*; c_k) > u_k(z_k^+; c_k)\ \forall k \in X \Rightarrow \\ &z_k^* < z_k^+\ \forall k \in X \Rightarrow \sum_{k\in X} z_k^* < \sum_{k \in X} z_k^+ \end{aligned} \tag{7.37}$$

However, at z^*, all strategies with a strictly positive market share belong in X and therefore $\sum_{k\in X} z_k^* = 1$. Based on Eq. 7.37, it follows that $\sum_{k\in X} z_k^+ > 1$ which is a contradiction. If $i \in X$, then we distinguish two additional cases:

Case (a): There exists a strategy $k \in Y$ such that, $z_k^- > 0$. In this case, we can easily prove by contradiction that there exists another strategy $j \in X$ such that, $z_j^* > z_j^-$.

$$\text{If } j = i\text{: } z_i^* > z_i^-, c_i > c_i^- \Rightarrow u_i(z_i^*; c_i) < u_i(z_i^-; c_i^-) \tag{7.38a}$$

$$\text{If } j \neq i\text{: } z_j^* > z_j^- \Rightarrow u_j(z_j^*; c_j) < u_j(z_j^-; c_j) \tag{7.38b}$$

Given that c^- belongs in the region 1, at z^-, the subscriptions of all providers correspond to the same utility (see case a in Section 7.2.2.3). Therefore, $u_j(z_j^-; c_j) = u_i(z_i^-; c_i^-) = u_k(z_k^-; c_k)$. Based on Eqs. 7.38, it follows that $u_j(z_j^*; c_j) < u_k(z_k^-; c_k)$. However, the strategy k belongs in Y and its market share is equal to 0 at z^* (i.e., $z_k^* = 0$). Therefore, $z_k^- > 0 = z_k^* \Rightarrow u_k(z_k^-; c_k) < u_k(z_k^*; c_k)$. By combining the above expressions, it follows that $u_j(z_j^*; c_j) < u_k(z_k^*; c_k)$. This contradicts the definition of NE at z^* because users that have selected the strategy j would have the incentive to select the strategy k as it offers higher utility.

Case (b): $z_k^- = 0$ for all strategies $k \in Y$. In this case, all strategies $k \in Y$ correspond to a market share of 0 at both z^* and z^-. Therefore, $u_k(z_k^*; c_k) = u_k(z_k^-; c_k)$ for all $k \in Y$. Furthermore, given that c^- belongs in the region 1, at z^-, the subscriptions of all providers correspond to the same utility, i.e., $u_k(z_k^-; c_k) = u_i(z_i^-; c_i^-) = u_j(z_j^-; c_j)$ for all $j \in X - \{i\}$. By combining the above equations, it follows that $u_k(z_k^*; c_k) = u_i(z_i^-; c_i^-) = u_j(z_j^-; c_j)$ for all $j \in X - \{i\}$. Furthermore, at z^*, the utilities of all strategies in Y are strictly lower than the utility of strategies in X, i.e., $u_k(z_k^*; c_k) < u_j(z_j^*; c_j)$ for all $j \in X$. Based on the above expressions, it follows that $u_j(z_j^*; c_j) > u_j(z_j^-; c_j)$ for all $j \in X - \{i\}$. Additionally, given that $c_i^- < c_i$, it follows that $u_i(z_i^*; c_i) > u_i(z_i^-; c_i^-) > u_i(z_i^-; c_i)$. Therefore, $u_j(z_j^*; c_j) > u_j(z_j^-; c_j)$, for all $j \in X$. However, the functions $u_j(z_j; c_j)$ are strictly decreasing with respect to z_j and therefore:

$$z_j^* < z_j^-\ \forall j \in X \Rightarrow \sum_{j \in X} z_j^* < \sum_{j\in X} z_j^- \tag{7.39}$$

Given that the set X contains all strategies with strictly positive market share at z^*, it follows that $\sum_{j\in X} z_j^* = 1$. If we substitute this expression in Eq. 7.39, $\sum_{j\in X} z_j^- > 1$. This is a contradiction. In summary, at z^*, the utilities of all subscriptions are the same and the market share of the disconnection is equal to 0. Therefore, c belongs in the region 1. A similar argument can be made for all other points on the line segment connecting c^+ and c^-. Therefore, the entire line segment is included in the region 1. This proves that the region 1 is orthogonally convex.

7.F Estimation of global NEs

Let us denote the sets of price vectors that correspond to the regions 1 and 2 as A_1 and A_2, respectively. The games of providers restricted in these regions can be then defined as $\Gamma_1 = \left(P, A_1, \{\sigma_i^1\}_{i\in P}\right)$ and $\Gamma_2 = \left(P, A_2, \{\sigma_i^2\}_{i\in P}\right)$, respectively. A more general game $\Gamma = (P, A_1 \cup A_2, \{\sigma_i\}_{i\in P})$ that is restricted on the union of the regions 1 and 2 can now be formed. The following set of theorems relate the NEs of the game Γ with the NEs of the games Γ_1 and Γ_2.

Theorem 1: A point $c^* \in A_1 \cap A_2$ is a NE of the game Γ, if and only if, it is a NE of the games Γ_1 and Γ_2.
Proof: Assume that a point $c^* \in A_1 \cap A_2$ is a NE of the game Γ. This implies the following inequalities.

$$\sigma_i(c_i^*, c_{-i}^*) \geq \sigma_i(c_i, c_{-i}^*),\ \forall c_i \in X_i(c_{-i}^*) \tag{7.40}$$

The set $X_i(c_{-i}^*)$ contains all possible prices that can be offered by the provider $i \in P$ when the prices of all other providers are c_{-i}^* ($X_i(c_{-i}^*) = \{c_i : (c_i, c_{-i}^*) \in A_1 \cup A_2\}$). If we define $X_i^1(c_{-i}^*) = \{c_i : (c_i, c_{-i}^*) \in A_1\}$ and $X_i^2(c_{-i}^*) = \{c_i : (c_i, c_{-i}^*) \in A_2\}$ as the sets of prices that can be offered by the provider i at the games Γ_1 and Γ_2, respectively, when the prices of all other providers are c_{-i}^*, we deduce the following inequalities.

$$\begin{aligned} \sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^1(c_i, c_{-i}^*),\ \forall c_i \in X_i^1(c_{-i}^*) \\ \sigma_i^2(c_i^*, c_{-i}^*) \geq \sigma_i^2(c_i, c_{-i}^*),\ \forall c_i \in X_i^2(c_{-i}^*) \end{aligned} \tag{7.41}$$

Therefore, c^* is a NE of the games Γ_1 and Γ_2. Let us now consider the reverse statement and assume that a point $c^* \in A_1 \cap A_2$ is a NE of the games Γ_1 and Γ_2. This implies the following inequalities.

$$\begin{aligned} \sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^1(c_i, c_{-i}^*),\ \forall c_i \in X_i^1(c_{-i}^*) \\ \sigma_i^2(c_i^*, c_{-i}^*) \geq \sigma_i^2(c_i, c_{-i}^*),\ \forall c_i \in X_i^2(c_{-i}^*) \end{aligned} \Rightarrow \tag{7.42}$$

$$\sigma_i(c_i^*, c_{-i}^*) \geq \sigma_i(c_i, c_{-i}^*),\ \forall c_i \in X_i^1(c_{-i}^*) \cup X_i^2(c_{-i}^*) \Rightarrow \tag{7.43}$$

$$\sigma_i(c_i^*, c_{-i}^*) \geq \sigma_i(c_i, c_{-i}^*),\ \forall c_i \in X_i(c_{-i}^*) \tag{7.44}$$

Therefore, the point c^* is a NE of the game Γ.

Theorem 2: A point $c^* \in A_1 \backslash A_2$ is a NE of the game Γ, if and only if, it is a NE of the game Γ_1 and the following conditions are true.

$$\sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^2(c_i, c_{-i}^*),\ \forall c_i \in X_i^2(c_{-i}^*), \forall i \in P. \tag{7.45}$$

Proof: Assume that a point $c^* \in A_1 \backslash A_2$ is a NE of the game Γ. This implies the following inequalities.

$$\sigma_i(c_i^*, c_{-i}^*) \geq \sigma_i(c_i, c_{-i}^*),\ \forall c_i \in X_i(c_{-i}^*) \Rightarrow \tag{7.46}$$

$$\begin{aligned} \sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^1(c_i, c_{-i}^*),\ \forall c_i \in X_i^1(c_{-i}^*) \\ \sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^2(c_i, c_{-i}^*),\ \forall c_i \in X_i^2(c_{-i}^*) \end{aligned} \tag{7.47}$$

Therefore, the point c^* is a NE of the game Γ_1 and $\sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^2(c_i, c_{-i}^*)$, $\forall c_i \in X_i^2(c_{-i}^*)$. Let us now consider the reverse statement and assume that a point $c^* \in A_1 \backslash A_2$ is a NE of the game Γ_1 and $\sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^2(c_i, c_{-i}^*)$, $\forall c_i \in X_i^2(c_{-i}^*)$. Then, we can deduce the following inequalities.

$$\left.\begin{aligned} \sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^1(c_i, c_{-i}^*),\ \forall c_i \in X_i^1(c_{-i}^*) \\ \sigma_i^1(c_i^*, c_{-i}^*) \geq \sigma_i^2(c_i, c_{-i}^*),\ \forall c_i \in X_i^2(c_{-i}^*) \end{aligned}\right. \Rightarrow \tag{7.48}$$

$$\sigma_i(c_i^*, c_{-i}^*) \geq \sigma_i(c_i, c_{-i}^*),\ \forall c_i \in X_i^1(c_{-i}^*) \cup X_i^2(c_{-i}^*) \Rightarrow \tag{7.49}$$

$$\sigma_i(c_i^*, c_{-i}^*) \geq \sigma_i(c_i, c_{-i}^*),\ \forall c_i \in X_i(c_{-i}^*) \tag{7.50}$$

Therefore, the point c^* is a NE of the game Γ.

Theorem 3: A point $c^* \in A_2 \backslash A_1$ is a NE of the game Γ, if and only if, it is a NE of the game Γ_2 and the following conditions are true.

$$\sigma_i^2(c_i^*, c_{-i}^*) \geq \sigma_i^1(c_i, c_{-i}^*),\ \forall c_i \in X_i^1(c_{-i}^*), \forall i \in P. \tag{7.51}$$

Proof: Similar as Theorem 2.

Chapter 8

A preliminary analysis of wireless markets using an event-based simulator

8.1 Event-based simulator: an alternative approach for analysing wireless markets

So far, this thesis has developed mathematical tools and algorithms to analyse wireless markets at multiple levels of detail and compute their corresponding equilibriums. An alternative methodology for the analysis of such markets could be also applied using an event-based simulator. Such a simulator replays in detail all the events that happen in a specific market, such as, session arrivals and terminations, user service and provider selection, user movement, and price adaptations of providers. Then, it estimates the evolution of the market and computes various performance metrics.

We have developed such an event-based simulator that can analyse the performance of a wireless access market of a small city. To reduce the computational complexity, we have provided a methodology that models various phenomena in such a market at a mesoscopic or macroscopic level of detail. This chapter defines in detail the components of the event-based simulator. It also analyses the performance of the wireless access market of a small city at multiple levels of detail from the microscopic to the macroscopic one and evaluates the tradeoff between the accuracy and computational complexity. As a driving force, it also analyses the impact of the *flex service*, a novel paradigm that allows users to select their provider dynamically each time they perform a new session.

The event-based simulator presented in this chapter corresponds to a preliminary framework for analysing wireless markets that was developed during the early stages of this Ph.D. The mathematical frameworks presented in Chapters 7 and 3 were developed later.

The structure of this chapter is as follows: Section 8.2 presents the components of the event-based simulator, while Section 8.4 presents the performance of the wireless access market of a small city when simulated at multiple levels of detail from the microscopic to the macroscopic. Finally Section 8.5 presents the concluding remarks.

8.2 Components of the event-based simulator

Wireless access markets are complex systems that involve various economic and technological parameters. In such markets, there are mainly two types of entities, the providers and users. A provider has deployed a cellular infrastructure and offers subscription and flex services to users. Two important decision making processes take place: (A) Periodically, providers set the prices of their services aiming to maximize their revenue. (B) Each user periodically selects the most appropriate provider and service. While a user has an active subscription or flex service, it initiates *sessions*. During a session, a user transmits and receives data via a BS. The price for a service as well as the service selection of a user last for the time period of an *epoch*. Note that in our framework, the epoch has a fixed duration and lasts for several days (or months). A session lasts for several minutes (Fig. 8.3).

For an educated selection of the service and provider, users should consider their traffic demand, willingness-to-pay, and requirements, as well as the cost of the service. However, a typical user is not willing to process such complex information. A software agent running on the user mobile device could in a (semi-)automatic manner select the best service on behalf of the user. In this work, we assume the presence of such a software agent, called *client* throughout this work. We also assume the existence of a server attached to a data repository that collects measurements about the sessions, traffic demand, and user profiles through a crowdsourcing monitoring system called *u-map*. The client uploads measurements about its sessions on the u-map server. Statistics on u-map data are available to clients and providers. We have developed and evaluated the u-map prototype [48]. This work uses u-map to represent the collected knowledge about the users and performance of services. The main components of the market are presented in Fig. 8.1 and are described in more detail in the following subsections.

Infrastructure: Each provider has deployed a cellular topology that offers wireless access via its BSs to clients in a small city. Providers divide their channels into time slots according to TDMA. The interference power at a BS during a time slot is computed considering the contribution of all interfering devices at co-channel BSs.

Clients: To start a session, a client generates a request to connect to a BS. The duration of a session and the *off duration* (i.e., the time interval between the end of a session and the start of the immediately next one of the same client) are given by theoretical distributions.

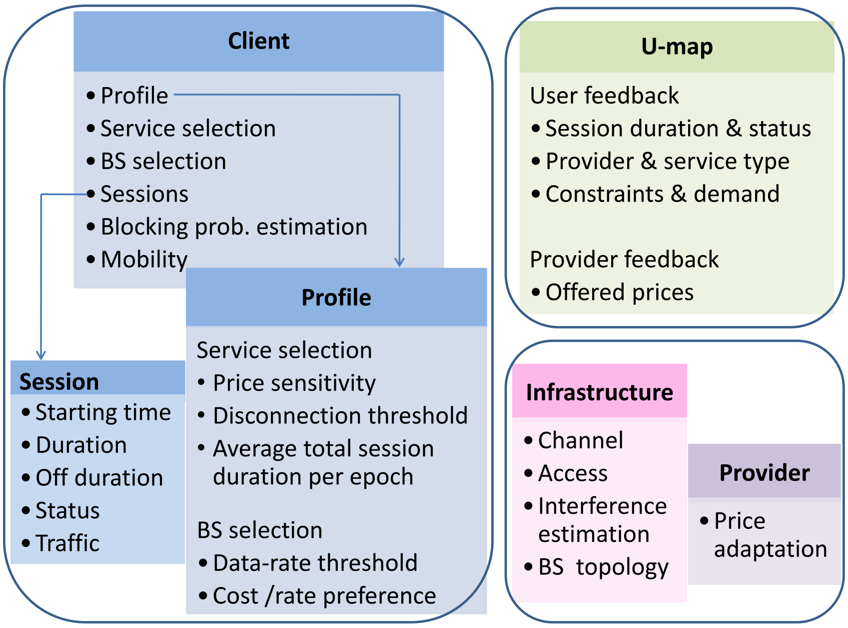


Figure 8.1: Main entities and modules.

To initiate a session, the client needs to *first* select a BS. The BS selection is based on either the *data-rate* or *price*. Specifically, a *price-conscious* client selects the BS that minimizes its cost spending, while a *QoS-conscious* client selects the BS that maximizes its achievable data rate. Furthermore, each client is characterized by a *data-rate threshold* (R_u) which is the minimum required data rate. When a client can not satisfy its data-rate requirement with any BS in its neighborhood, its session becomes *blocked*.

Service types: Two service types are considered, the *subscription* and *flex service*. Periodically, a client is required to select a *service type* or remain *disconnected*. During a disconnection period, the client *does not have any active service*, and thus, *cannot* initiate sessions. A subscriber or flex user needs to first select a BS in order to start a session. A flex user may select a BS of *any* provider, while a *subscriber* of a certain provider may select *only* BSs of *that provider*. We assume that a flex user has a “basic” subscription with the network infrastructure of each provider in order to receive notifications, incoming calls or other data sessions. However, in order to initiate and complete a session, a flex user is required to select a provider dynamically.

Charging: The subscription and flex services are charged according to different schemes: The pricing for subscriptions is a *flat rate* scheme that charges the user with a specific fixed cost (per epoch). On the other hand, the flex users are charged for each session in a cost-per-minute manner. The service (i.e., flex service, or subscription) or the disconnection lasts for an epoch. The selection of the service type is performed once during each epoch. The BS selection takes place before the start of each session. Fig. 8.2 illustrates

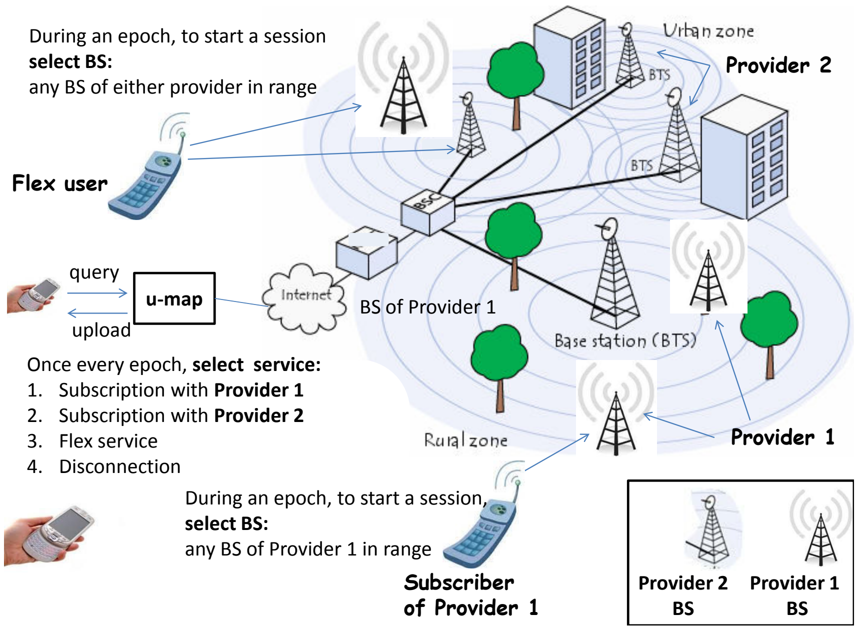


Figure 8.2: An example of a wireless access duopoly.

the decision-making mechanism of a typical subscriber and a typical flex user in a wireless access duopoly.

u-map: u-map is modeled as a data structure which stores information about users and providers. Periodically, each client reports the status (*successful or blocked*) and duration of its sessions along with its service type, client id, provider id, and constraints. Clients obtain information about the *average blocking probability per service*, which is computed as the percentage of the blocked sessions that were made during the last w epochs, as reported by clients of that service in u-map.

8.3 Game-theoretical modeling

The evolution and performance of wireless access markets involve networking and business aspects that manifest various spatio-temporal dependencies and localities. For example, user profiles, network topology, and channel conditions may vary across regions. Also, local decisions of users and providers may be required. This motivated us to develop a *game-theoretical* modeling framework and simulation platform that encompass multiple levels of detail, from a microscopic level to a macroscopic one.

The framework considers a wireless access market that consists of two providers $P = \{1, 2\}$ and a population of n users $U = \{1, ..., n\}$. This market is modeled using *two distinct games*, one for the *competition of providers* and one for the *user decision making*. The game of providers takes place at *epochs* (i.e., time periods of fixed duration). At the beginning of each epoch, providers adapt the prices for their services aiming to maximize their revenue. Each user selects a type of service depending on its price and performance (according to a utility function).

Our modeling framework is configurable and parameterized based on the channel, infrastructure and network topology, type of users (e.g., service, demand, mobility, constraints, preferences), providers (e.g., price adaptation, services), and available information. The simulation environment that implements this framework is also modular, in that, it can instantiate and implement different models for the aforementioned parameters. The games of users and providers are described in detail in the following subsections.

8.3.1 Game of users

The service selection of users is modeled as a *population game*. Each member of the population can choose among four available strategies $H = \{1, 2, 3, 4\}$. Strategies 1 and 2 correspond to subscriptions with Providers 1 and 2, respectively, strategy 3 indicates the flex service, and strategy 4 denotes the disconnection state. All users have the same options with respect to service selection, and as such, the set of all possible combinations of user choices (i.e., set of user *strategy profiles*) is defined as $S = H^n$. The selection of a strategy (i.e., service) is based on a *utility function* that expresses the tradeoff between quality of service and price. In this work, the utility function is a linear combination of blocking probability and price.

$$\pi_{ui}(s) = \begin{cases} -B_i(s_i^u) - b_u c_i^s & \text{if } i = 1, 2 \\ -B_i(s_i^u) - b_u c_u^f(s_i^u) & \text{if } i = 3 \\ -k_u & \text{if } i = 4 \end{cases}, \tag{8.1}$$

Utility function: Eq. 8.1 defines the utility of a user $u \in U$, when the strategy profile of all users is $s \in S$ and the user u switches to strategy $i \in H$ from its current service. The strategy profile resulting from this transition is defined as s_i^u. $B_i(s_i^u)$ is the blocking probability that is associated with the service i, and c_i^s is the subscription rate of Provider i. The client estimates the price that the user u pays per epoch when it selects the flex service $c_u^f(s_i^u) = (a_1^u c_1^f + a_2^u c_2^f) d_u \, (1 - B_3(s_i^u))$, where d_u is the average total duration of all sessions of user u per epoch. The probabilities a_1^u and a_2^u indicate the frequencies with which user u has selected Providers 1 and 2, respectively, during the last w epochs (as reported by u-map). The duration d_u is divided between providers according to the probabilities a_1^u and a_2^u. The total cost is estimated based on the time spent at the network of each provider and their flex rates (c_1^f and c_2^f, respectively). Only non-blocked sessions contribute to the total payment (indicated by the term $(1 - B_3(s_i^u))$). All the parameters of Eq. 8.1 are estimated with the data collected in u-map.

The user *profile* is expressed by the utility function, which is parameterized based on the price sensitivity (b_u), disconnection threshold ($-k_u$), and average total session duration per epoch (d_u) of a user (u), respectively. The price sensitivity is a positive value that indicates the importance of price compared to blocking probability in the user utility function. The higher the price sensitivity, the larger the significance of price on the user decision making. The disconnection threshold is the minimum utility acceptable by a user. If this utility can not be achieved with any of the offered services, the client remains *disconnected for an epoch*. Finally, the average total session duration per epoch is the average demand in minutes that a user generates per epoch. The profile of a user $u \in U$ with respect to service selection is annotated as $(b_u, -k_u, d_u)$. In summary, the game of users can now be defined by the triplet $g = (U, S, \{\pi_{ui}\}_{u \in U, i \in H})$.

The analytical study of the user population game is challenging. For the estimation of the Nash equilibriums (NEs), a closed form of the blocking probabilities is required. However the analytical estimation of the blocking probabilities in the presence of the flex service is difficult even when all users are identical with respect to their profile (see Section 8.3.3). For that reason, we proceed to study the game of users via simulations. Specifically, for a given set of offered prices, we simulate the user movement, session generation, and service selection. During service selection, a client first estimates the total cost for each service as well as the corresponding blocking probabilities. For that, we assume that the client can approximately estimate the average total duration of its sessions per epoch based on the history of performed sessions via u-map. Then, it checks whether the utilities of the various services are higher than its disconnection threshold ($-k_u$). The *blocking probability of subscribers of a certain provider* is estimated as the percentage of blocked sessions performed by all subscribers of that provider during the last w epochs at u-map. The *blocking probability for the flex service* is estimated as the percentage of blocked sessions performed by all flex users during the last w epochs at u-map. The client selects the service with the highest utility value above the disconnection threshold. If there is no such service, the user becomes disconnected for an epoch.

After the service-type selection, a user who is not disconnected initiates sessions. For each session, a client first selects a BS based on either data rate or price. As mentioned earlier, price-conscious clients select the BS that minimizes their cost spending, while data rate conscious clients select the BS that maximizes their achievable data rate. The decision making of users is shown in Fig. 8.3.

8.3.2 Game of providers

The competition of providers is modeled as a *normal-form game* $(P, C, \{\sigma_p\}_{p \in P})$, where P is the set of providers and C is the set of strategy profiles. A strategy profile $c \in C$ contains the offered subscription and flex rates of both providers $c = (c_1^s, c_1^f, c_2^s, c_2^f)$, where c_p^s and c_p^f are the subscription and flex rates of provider p, respectively. Each provider can

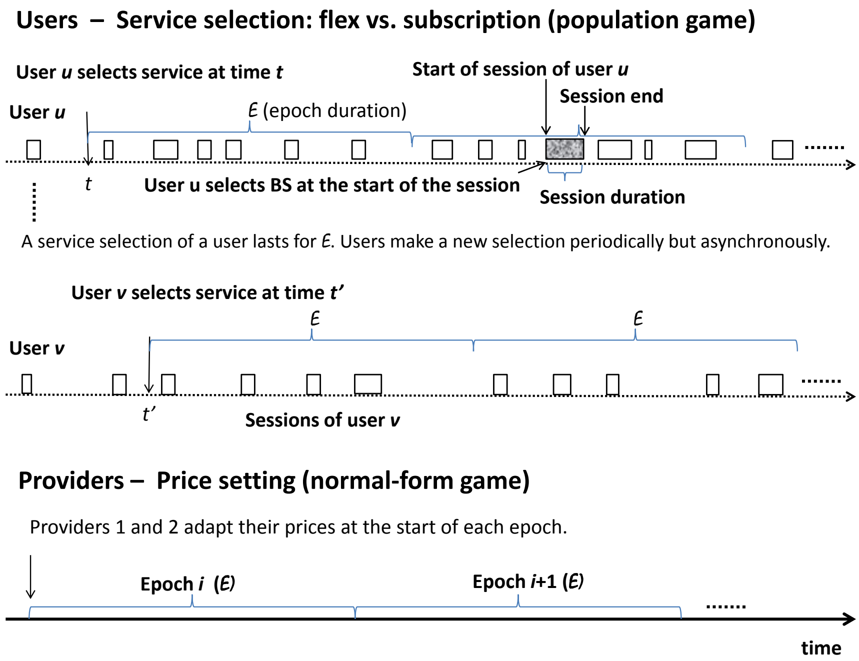


Figure 8.3: Decisions of providers and users during the evolution of a market.

choose its subscription and flex rates from two finite and discrete sets of prices C^s and C^f, respectively and as such the game is finite. The utility function of each provider $p \in P$ is defined as $\sigma_p(c) = U_p^s(c) + U_p^f(c)$, where $U_p^s(c)$ is the total revenue from subscribers and $U_p^f(c)$ is the total revenue from flex users.

Empirical game: The lack of a closed form solution for the equilibrium of the user population game prevents us from estimating closed form solutions for the functions $U_p^s(c)$ and $U_p^f(c)$. For that reason, we define the game of providers as an *empirical game* [94]. An empirical game is exactly the same as a normal-form game with the difference that there is no analytical expression for the utility functions of players. Instead, it provides a *game simulator* Θ that for any strategy profile $c \in C$, it reports the corresponding revenues ($\{\sigma_p(c)\}_{p \in P}$). A formal definition of the empirical game is $G = (P, C, \Theta)$. Various algorithms have been proposed in the literature to solve empirical games (estimate pure strategy NEs). Examples of such algorithms are (tabu) best response [95], and minimum regret first search [96]. In this work, we use the best response due to its simplicity, although other algorithms can be incorporated easily. Specifically, at the beginning of each epoch providers adapt their rates in two phases: their subscription rate at the first phase and their flex rate at the second phase. At the first phase, all providers (in parallel) simulate the market for all possible subscription rates they can offer keeping the subscription rate of the other provider and flex rates unchanged. Each provider selects the subscription rate that maximizes its immediate revenue. In the second phase, a similar process is followed: all providers (in parallel) simulate the market, computing their profit for each flex rate they can offer, keeping fixed the flex rate of the other provider and subscription rates. Then, each provider selects the flex rate that maximizes its immediate revenue.

In contrast to classical normal-form games, empirical games are analysed via simulations. Therefore, the computational complexity of the simulator Θ is an important parameter. In our case, at the microscopic level, the game simulator generates a large number of events which increases the execution time for the best response algorithm in the order of several days! This motivated us to explore the multi-layer aspect: model the game of users (service selection) at a mesoscopic or macroscopic level to reduce the computational complexity.

8.3.3 Multi-layer modeling

As mentioned earlier, the microscopic modeling framework describes each user as a distinct entity. However, the diversity and large size of the user population makes the analysis challenging due to the estimation of many different utility functions. To reduce the computational complexity of the game simulator, various aggregations are performed. Specifically, we divide the user population into a set of *clusters* $U_J = \{1, ..., J\}$, each corresponding to a "homogeneous" population, i.e., its members are characterized by the same *representative profile*. The representative profile of a cluster $j \in U_J$, denoted as $(b_j, -k_j, d_j)$, corresponds

Table 8.1: **Parameters of the game of users at the micro (meso) level**

Parameter	Description
$U(U_J)$	Set of users (clusters) at micro (meso) level
n	Number of users at micro level
J	Number of clusters at meso level
H	Set of user strategies
$S(S_J)$	Set of user strategy profiles at micro level (meso level with J clusters)
s	User strategy profile at micro level
z	User strategy profile at meso level
$\pi_{ui}(\pi_{ji})$	Utility function of user u (cluster j) when selecting strategy i
B_i	Blocking probability of service i
$b_u(b_j)$	Price sensitivity of user u (cluster j)
$-k_u(-k_j)$	Disconnection threshold of user u (cluster j)
$d_u(d_j)$	Average total session duration per epoch of user u (cluster j)
$a_i^u(a_i^j)$	Probability for flex user u (a flex user in cluster j) to select Provider i

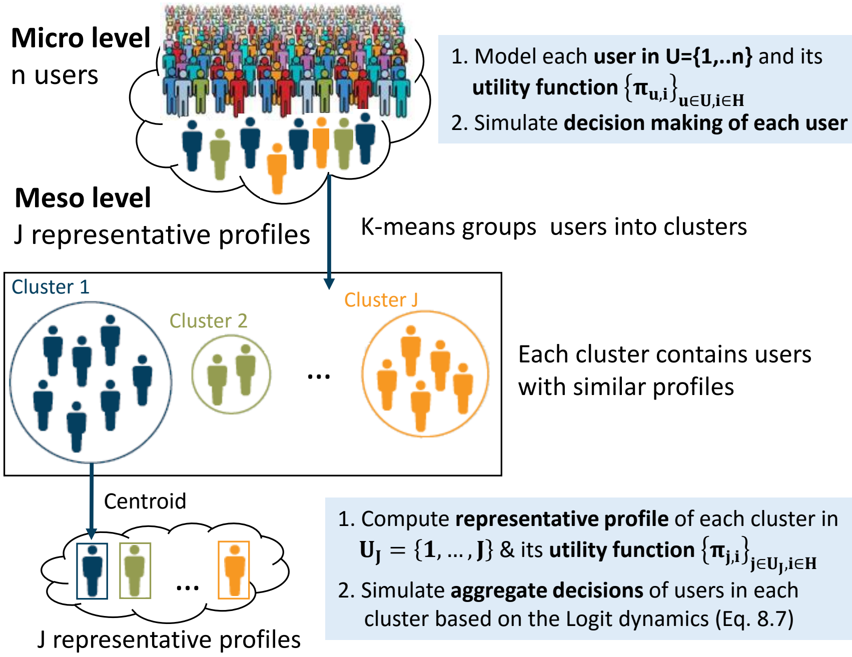


Figure 8.4: Game of users at multiple levels of detail.

to the centroid of that cluster, as reported by a clustering algorithm on the profiles of all users of the microscopic modeling. As mentioned, the profile of each user u is modelled as a vector $(b_u, -k_u, d_u)$ containing the price sensitivity, disconnection threshold, and average total session duration per epoch of that user. This work employs the K-means algorithm [97]. The aim of this algorithm is to partition the user profiles into (J) clusters such that each user profile belongs to the cluster with the nearest centroid. The number of user clusters (J) determines the mesoscopic level. Fig. 8.4 illustrates the transformation of the user population from the microscopic level to the mesoscopic level via clustering. Tables 8.1 and 8.2 summarize the main parameters of the modeling framework at these levels.

Table 8.2: **Parameters of the game of providers**

Parameter	Description
P	Set of providers
c_i^s	Subscription rate of Provider i
c_i^f	Flex rate of Provider i
C	Set of provider strategy profiles
σ_p	Utility function of Provider p
U_p^s	Utility of Provider p from subscribers
U_p^f	Utility of Provider p from flex users

Due to the homogeneity of a cluster of users, all its members are "assigned" the same utility, when selecting the same strategy. Unlike the microscopic level that simulates the decisions of each user with respect to service, the mesoscopic levels estimate the *percentage of users* of each cluster that choose each service. That is, for each cluster, the strategies of its members can be represented by a probability distribution over the set of strategies (H). The set of all probability distributions over H is defined as $\Delta(H)$, and the set of strategy profiles of all clusters is $S_J = \Delta(H)^J$. The utility of the users of cluster j when they select the strategy $i \in H$ is defined as follows:

$$\pi_{ji}(z) = \begin{cases} -B_i(z) - b_j c_i^s & \text{if } i = 1, 2 \\ -B_i(z) - b_j c_j^f(z) & \text{if } i = 3 \\ -k_j & \text{if } i = 4 \end{cases}, \tag{8.2}$$

In Eq. 8.2, $z \in S_J$ is the user strategy profile. It is composed by J probability distributions ($z = (z_1, ..., z_J)$), one for each cluster. The probability distribution $z_j = (z_{j1}, ..., z_{j4})$ shows the distribution of users of cluster j across the available strategies $i \in H$. Indices 1 and 2 correspond to subscriptions with Providers 1 and 2, respectively, index 3 indicates the flex service, and index 4 denotes the disconnection state. $B_i(z)$ is the blocking probability associated with the service i and $c_j^f(z)$ is the price that a flex user in cluster j pays per epoch $c_j^f(z) = (a_1^j c_1^f + a_2^j c_2^f) d_j \, (1 - B_3(z))$. The total session duration of any flex user of cluster j per epoch (d_j) corresponds to the total session duration spent at the network of each provider. The probabilities a_1^j and a_2^j indicate the frequencies with which the user chooses Providers 1 and 2, respectively. Thus, the total cost of the flex service is estimated according to the time spent at the network of each provider and the corresponding flex rates (c_1^f and c_2^f, respectively). Again blocked sessions are ignored for the estimation of the cost. Formally, at the mesoscopic level, the game of users is defined as $g_J = (U_J, S_J, \{\pi_{ji}\}_{j \in U_J, i \in H})$.

The macroscopic level corresponds to a mesoscopic level with *only* one user cluster. That is, the user population is assumed homogeneous and the user choices are described by one probability vector over the set of strategies. Formally, at the macroscopic level, the definition of the user population game becomes $g_1 = (U_1, S_1, \{\pi_i\}_{i \in H})$. The set U_1 contains one user cluster with the representative profile $(b, -k, d)$ which is computed as the centroid of the profiles of users at the microscopic level, and $S_1 = \Delta(H)$.

Blocking probability model: To estimate the blocking probabilities at the mesoscopic and macroscopic levels, we employ a simple Markovian queueing-theoretical model. Specifically, we focus on a small region containing one BS per provider. The session and off durations of each user follow exponential distributions. The parameters of the distributions are selected so that the average session generation and service rates over all users coincide with the ones at the microscopic level. The BS of Provider 1 (2) is modeled as a *finite queue* with m_1 (m_2) servers, respectively. A server of a BS corresponds to a time slot (in a TDMA scheme). A session is blocked when there are no available slots to serve the user request.

The total session generation rate of all users is λ and is composed by the session generation rates of all user clusters. The session generation rate for a cluster $j \in U_J$ is denoted as λ_j^c. Eq. 8.3 estimates the session generation rates for subscribers of Providers 1 and 2, λ_1^s and λ_2^s, respectively, and flex users λ^f by summing the contributions of each user cluster.

$$\lambda_1^s = \sum_{j=1}^{J} z_{j1}\lambda_j^c,\ \lambda_2^s = \sum_{j=1}^{J} z_{j2}\lambda_j^c,\ \lambda^f = \sum_{j=1}^{J} z_{j3}\lambda_j^c \tag{8.3}$$

A new session of a subscriber can only be served by the BS of its own provider, while a new session of a flex user can be served by either provider. In general, a flex user selects a BS according to a probability distribution $(\delta, 1-\delta)$. With respect to the BS selection, we consider two types of user preference, the price preference, and rate preference one. In price preference, a flex user selects the BS of the cheapest provider (i.e., $\delta = 1$ if Provider 1 is the cheapest, otherwise $\delta = 0$), while in rate preference, it selects the BS that maximizes its achievable data rate (δ is the probability that the achievable data rate in the network of Provider 1 is larger compared to the one of Provider 2).

When the BS of one provider is fully utilized, a flex user can only connect to the BS of the other provider assuming that the criteria for the BS selection are satisfied. In general, when there are available slots at both BSs, the session arrival rate at the BS of Provider 1 is $\lambda_1 = \lambda_1^s + \delta\lambda^f$, while when the BS of Provider 2 is fully utilized, the arrival rate for Provider 1 becomes $\lambda_1' = \lambda_1^s + \lambda^f$. Similarly, the arrival rates λ_2 and λ_2' of Provider 2 are defined.

A large user population (compared to the number of time slots) is considered. Thus, the total session generation rate is not affected by the number of users being served [53]. We then model the market using a Markov chain with a two dimensional state space as

Table 8.3: **Parameters of the Markov-chain model**

Parameter	**Description**
λ	Total session generation rate
μ	Session service rate
λ_j^c	Session generation rate for users of cluster j
λ_i^s	Session generation rate for subscribers of Provider i
λ^f	Session generation rate for flex users
m_i	Time slots available at the BS of Provider i
δ	Probability for flex user to select the BS of Provider 1 when both BSs are not fully utilized

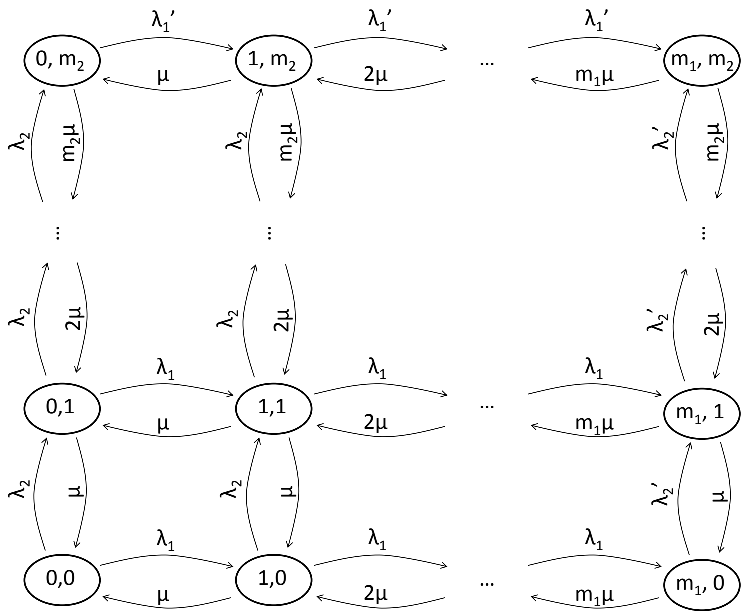


Figure 8.5: The Markov-chain model of the cellular network. The state (x, y) corresponds to the total number of time slots x and y that serve sessions at the BSs of Provider 1 and Provider 2, respectively.

shown in Fig. 8.5. To estimate the blocking probabilities for the three types of services, the steady-state distribution of the Markov chain is computed:

$$\Phi = (\varphi_{0,0}, \varphi_{1,0}, ..., \phi_{m_1,0}, ..., \varphi_{0,m_2}, \varphi_{1,m_2}, ..., \varphi_{m_1,m_2}) \tag{8.4}$$

where $\varphi_{x,y}$ is the steady-state probability for the state (x, y). The probabilities that the BSs of Providers 1 and 2 are fully utilized (Φ_1 and Φ_2, respectively) and the probability that both BSs are fully utilized (Φ_3) are defined as follows:

$$\Phi_1 = \sum_{y=0}^{m_2} \varphi_{m_1,y}, \ \Phi_2 = \sum_{x=0}^{m_1} \varphi_{x,m_2}, \ \Phi_3 = \varphi_{m_1,m_2} \tag{8.5}$$

The Markov chain of Fig. 8.5 corresponds to a "*finite-loss* system of two queues with overflow" and there is no simple analytical solution for the forward Kolmogorov equations of such a system [98, 99]. On the other hand, numerical methods can be employed to solve the system of Kolmogorov equations. By taking advantage of the structure of the transition rate matrix of the Markov chain, we determine the steady-state probabilities in a low computational cost [100, 101]. In this work, we applied the method proposed in [100].

As in the microscopic level modeling, a user aims to achieve a target data rate at the start of a session. If this data rate can not be achieved, the session is blocked. The probability that an arbitrary user can achieve its target data rate in the network of Provider i is denoted as q_i. The blocking probabilities (B_i) for all services ($i = 1, 2, 3$) can now be defined as follows:

$$\begin{aligned} B_1 &= (1 - q_1) + q_1\Phi_1,\, B_2 = (1 - q_2) + q_2\Phi_2 \\ B_3 &= (1 - q_1)(1 - q_2) + q_1(1 - q_2)\Phi_1 + q_2(1 - q_1)\Phi_2 + q_1 q_2\Phi_3 \end{aligned} \tag{8.6}$$

To estimate the blocking probability of sessions of subscribers of Provider i, we distinguish two cases: (1) a subscriber of Provider i cannot achieve its target data rate at the start of a session, which happens with probability $1 - q_i$. In this case, that session becomes blocked, and (2) with probability q_i, a subscriber of Provider i can achieve its target data rate. In that case, the session gets blocked only when the BS of Provider i is fully utilized. This happens with probability Φ_i. Therefore, the overall blocking probability for a subscriber of Provider i is $(1 - q_i) + q_i\Phi_i$.

To estimate the blocking probability of a session of a flex user (given by Eq. 8.6), we can distinguish the following cases: With probability $(1 - q_1)(1 - q_2)$, the flex user cannot achieve the target rate in the network of either provider and its session is blocked. The user achieves its target rate only in the network of Provider 1 (Provider 2) with probability $q_1(1 - q_2)$ (with probability $q_2(1 - q_1)$), and its session becomes blocked with probability Φ_1 (Φ_2), respectively. Finally, with probability $q_1 q_2$, the user can achieve its target rate in either of the two networks of providers. In that case, its session gets blocked when the BSs of both providers are fully utilized, which occurs with probability Φ_3.

User population dynamics: Based on the user utility function (Eq. 8.2) and blocking probability model (Eqs. 8.4, 8.5, 8.6), the user population game can be simulated. Specifically, we simulate the user evolution by applying the smoothed best response dynamics [102], also known as Logit dynamics, a system of ordinary differential equations (8.7).

$$\frac{dz_{ji}(t)}{dt} = r * \left(\frac{1}{1 + \sum_{\kappa \neq i} G_{j\kappa i}(z(t))} - z_{ji}(t) \right) \tag{8.7}$$

The parameter r controls the *speed of the dynamics*, while $G_{j\kappa i}(z(t))$ are functions of the difference in utility that a user in cluster j achieves when it selects the strategy $\kappa \in H$ compared to when it selects the strategy $i \in H$ (Eq. 8.8).

$$G_{j\kappa i}(z(t)) = \exp\left(\frac{\pi_{j\kappa}(z(t)) - \pi_{ji}(z(t))}{\epsilon} \right) \tag{8.8}$$

The term ϵ is a parameter that indicates the user irrationality: when ϵ tends to zero, users are completely rational, while when ϵ increases, a degree of irrationality is introduced in the user decision making.

8.3.3.1 Convergence and complexity issues

The existence of a NE in the game of providers is guaranteed, since according to the theorem of Nash, each finite game has at least one NE [91]. For the estimation of a NE, a closed form of the utility functions of providers is required. However, the utilities of providers (i.e., profit) depend on the equilibrium of the user population game. This equilibrium indicates the portion of users that choose each service, which is a metric necessary for the estimation of the utilities of providers. To derive the equilibrium points of the user population game, a closed form of the blocking probability for all services is necessary. The estimation of the blocking probabilities of the various services is based on the queuing-theoretical model presented above. As mentioned, there is no simple analytical solution for the forward Kolmogorov equations of such a model. The lack of a closed form solution for the blocking probabilities prevents the provision of analytical expressions for the equilibrium points of the user population game and for the NEs of the game of providers.

Our scenarios involve complex games for which it is very difficult to derive the expected utilities of providers, even if complete policies for all providers are given. Empirical game theory is a relatively recent research direction in game theory for analysing such complex games [94]. Various algorithms have been proposed in the literature to solve empirical games and estimate evolutionary stable NEs [95, 96]. These algorithms converge to a pure strategy NE if one exists. In most of the simulation scenarios presented in this work, the best response algorithm converges to a pure strategy NE. In cases in which the game does not have pure strategy NEs (all NEs are mixed strategy ones), the best response algorithm converges to a periodic solution (market oscillations).

At the beginning of each epoch, providers adapt their prices based on the best response algorithm. Specifically, they *simulate the market at the macroscopic level for all possible subscription and flex rates* and select the rates that maximize their revenue. The computational complexity of the process is $O(L \times M)$, where L is the number of subscription and flex rate levels and M is the complexity of simulating the game of users at the macroscopic level. After the price determination, during the epoch, users are simulated at a mesoscopic level with J clusters. The ODE solver performs a number of steps (N) to determine the evolution of the user strategy profile (z) depending on the *stiffness* of the problem.

The field of numerical analysis focuses on the convergence of the various ODE solvers, without providing an algorithmic complexity of the methods to achieve a certain level of accuracy in the solution. At each step, the blocking probabilities are estimated by computing the steady state of the Markov chain defined in Fig. 8.5 ($O(m_1 \times {m_2}^3)$) and the right side of Eqs. 8.7 ($O(J)$). The overall complexity for the simulation of the game of users at the mesoscopic level with J clusters (M_J) is of $O(N \times (m_1 \times {m_2}^3 + J))$. In general, the larger the number of user clusters, the higher the accuracy, but also the larger the computational

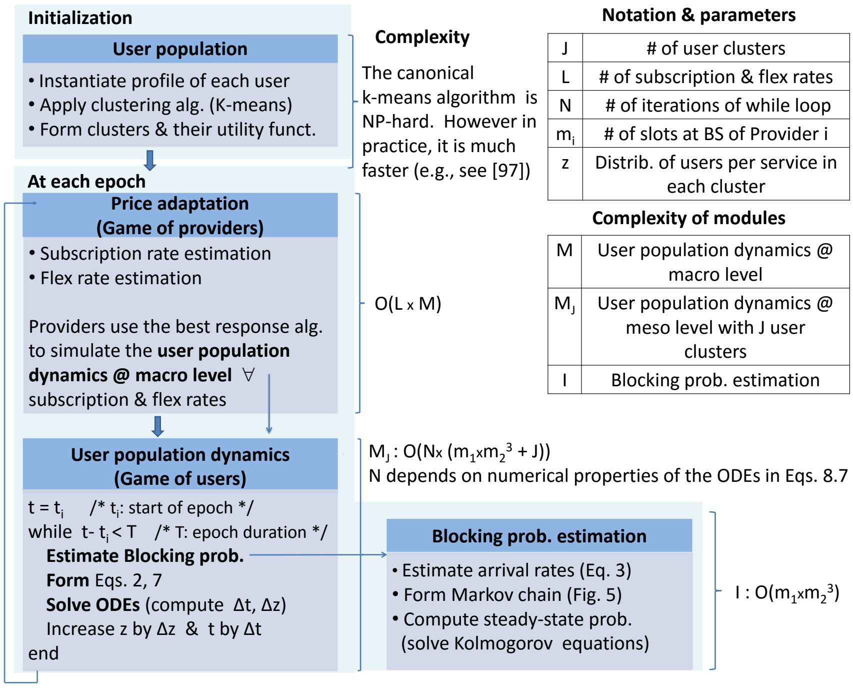


Figure 8.6: Algorithms at the mesoscopic level and their complexity.

complexity (see also Sec. 8.4.4). Fig. 8.6 outlines the computational complexity of the analysis of a market.

Although this work focuses on a duopoly, the framework can be easily extended to incorporate a larger number of providers. However, the state space of the corresponding Markov chain increases resulting to a significant computational complexity for the blocking probability estimation.

8.4 Performance evaluation

8.4.1 Simulation setup

We implemented the modeling framework in Matlab and instantiated various wireless access markets for its performance evaluation. For the analysis of the flex service, we instantiated a wireless access market of a small city, represented by a rectangle of 11 km x 9 km and performed simulations for different scenarios. The parameters of the user profiles remained *fixed throughout a simulation scenario*. Each experiment (i.e., simulation run) represents the evolution of the market during a period of 50 epochs, each lasting 5 days.

Wireless network infrastructure: Each provider has a cellular network that consists of 49 BSs placed on a triangular grid, with a distance between two neighboring sites of 1.6 km.

Moreover, each provider owns bandwidth of 5.6 MHz, that is divided into 28 channels of 0.2 MHz width. These channels are allocated to BSs according to a frequency reuse scheme with spatial reuse factors of 4 and 7, for Provider 1 and Provider 2, respectively. Each channel is further divided into three time slots in a TDMA scheme, resulting in 21 time slots per BS of Provider 1 and 12 slots per BS of Provider 2. A single time slot of a given BS can be offered to *only one client*. A client can use *only one* time slot of a given BS, and can be associated with *only one* BS during a given session. Although, this work assumes a TDMA protocol for wireless access, the framework can employ other medium access protocols, such as CSMA or CDMA, by modifying the user utility function appropriately (e.g., using the achievable data rate as an indicator of the QoS). The channel quality is described based on the *Okumura Hata* path-loss model for small cities considering the contribution of shadowing to the channel gain [103, 104]. The maximum transmission power that a client can use is 2 Watts. The chosen parameters correspond to a *typical microcellular network* of a small city [105] and the channel availability of BSs was determined to match the demand of the user population.

User population: There are 28000 users in total, distributed according to a uniform distribution in the simulated region of this small city. The parameters of the user profiles follow Gaussian distributions. The mean values of the disconnection threshold ($-k_u$) and data-rate threshold (R_u) are -0.2 and 0.1 Mbps, respectively, while their standard deviation is 0.01. On the other hand, the price sensitivity (b_u) is the same for all users (equal to 0.025).[1] In each scenario, a user selects the BS with the best data rate.

Client demand: A client generates a sequence of session requests. The session duration follows a Pareto distribution of mean 5 min (the scale and shape parameters are equal to 3.890 and 4.500, respectively). The off period follows a log-normal distribution with different location parameter for each user, selected with a uniform distribution from $[4.068, 6.215]$, and a scale parameter equal to 0.368. This corresponds to a client demand (d_u) that varies from 33 to 267 minutes (in total) per epoch. The Pareto and log-normal distributions have been used in studies of wireless traffic to describe the session on and off durations [106].

Estimation of blocking probability: For the service selection of users, the blocking probability is estimated based on the status of recent sessions reported at u-map during the last 2 epochs.

User mobility: During off periods, clients move with a pedestrian speed with a maximum value of 1 m/s, while they remain stationary during sessions. In this work, we do not consider handovers: a client remains connected at the *same BS for the entire duration of the session*. In general, a session could be blocked not only at its initiation (as in this work) but also during a handover between BSs. The estimation of the blocking probability of handover sessions in an on-going work.

[1] This value was chosen to transform the prices of the subscription and flex services in the same scale as the values of the blocking probability.

To highlight the impact of the flex service, two market types were simulated: a *subscriber-only market* (baseline), in which each user can either become a subscriber of a provider or remain disconnected, and a *mixed market*, in which users have the additional option of becoming flex users. The analysis evaluates the impact of the flex service on the evolution of the market, using metrics that can provide insights to regulators, users, and providers. The performance of a provider is characterized by its revenue, while the performance of a client is indicated by the blocking probability of its sessions. Furthermore, the session blocking probability, social welfare, market share, and percentage of disconnected users are computed. The *session blocking probability of a client* is the ratio of its blocked sessions over the total number of session requests. The *social welfare* is defined as the sum of the *net benefit* of all users and providers. The net benefit of a provider is its revenue, while the net benefit of a user is the difference of the *user valuation* for wireless connectivity and what the user paid for his/her wireless access. The user valuation for wireless connectivity is the price that the user is willing to pay when the blocking probability is zero. Finally, the market share is a vector that shows the percentage of users that choose each service. Our reported results are average statistics over all epochs.

8.4.2 Analysis of flex service at the micro level

To evaluate the performance of the flex service, we performed a series of experiments. The game of users was simulated at the microscopic level, while the game of providers was executed at the macroscopic level to reduce the computational complexity of the price setting algorithm. The flex service improves the performance of users. It significantly reduces the percentage of disconnected users (Fig. 8.7b), increases the social welfare (Fig. 8.7c), and improves the blocking probability in most scenarios (Fig. 8.7a). Provider 1 has an advantage in the revenue over Provider 2 due to the larger channel availability of its BSs. In some cases, the market manifests strong oscillations. The oscillations are caused by the relatively "stale" data in the estimation of the blocking probability that is employed for the service selection of users. The blocking probability is estimated by u-map, based on historical feedback that users upload in u-map about the status of their sessions. Specifically, in certain epochs, the percentage of subscribers of Provider 2 falls close to zero. After some epochs, u-map reports a low blocking probability for the subscribers of Provider 2. This low blocking probability in conjunction with a low subscription rate motivates many users to become subscribers of Provider 2. The massive "flow" of users to Provider 2 results in an increased blocking probability. Users will realize a performance degradation after some time (due to the uploading delay), and then, they will once again abandon Provider 2. This phenomenon will be repeated (Fig. 8.7f). The intensity of the oscillations depends on the flex service, market share, uploading frequency, and number of epochs based on which the blocking probabilities of the various services are estimated. The higher the intensity of the oscillations, the higher the average blocking probability. As a result, in certain scenarios, the blocking probability in the mixed market is slightly larger compared to the subscriber-only

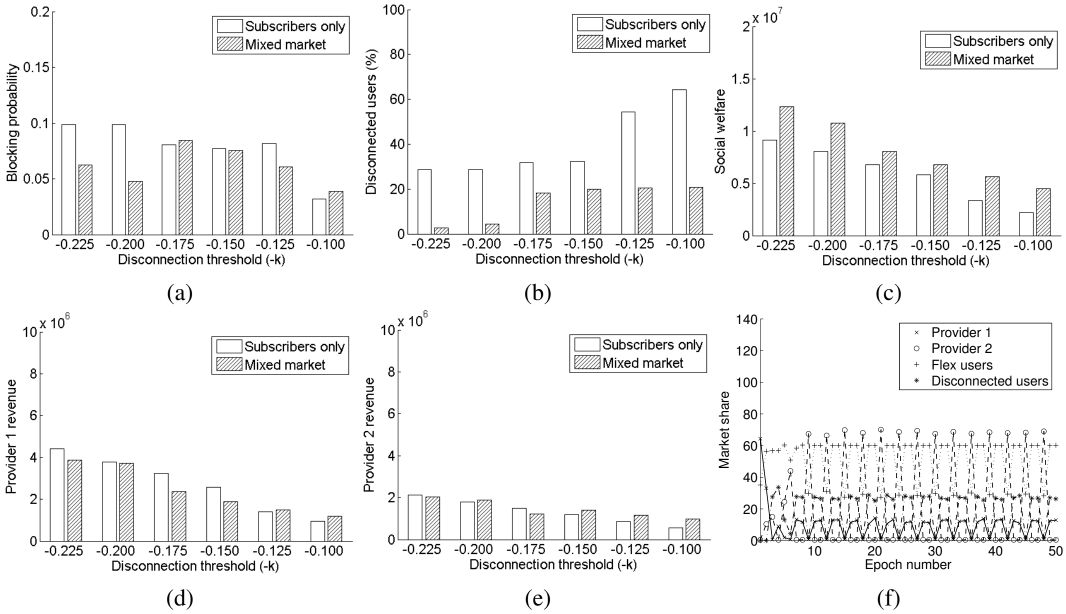


Figure 8.7: Performance at the microscopic level: (a) Blocking prob. (b) Disconnected users (percent) (c) Social welfare (d), (e) Revenue of providers (f) Market share in the mixed market with a disconnection threshold of -0.175.

one (Fig. 8.7a). The use of "stale" data causes these oscillations, and the suboptimal performance. Similar oscillations have been also observed in agent economies and biological systems. An extensive discussion can be found in [107]. To improve the estimation of the blocking probability, the Markov-chain model described in Section 8.3.3 can be employed.

In our earlier study [107], the user utility function was not a weighted sum of the blocking probability and cost. Instead, there were two types of scenarios in which all users were selecting the service that minimizes either the cost or the blocking probability. Part of the user profiles were a willingness-to-pay and a blocking probability tolerance threshold. The benefits of the flex service were also prominent under such user profiling, namely an improvement in the number of disconnected users, blocking probability, and social welfare as well as some trends (e.g., oscillations in market share). The flex service was a preferable choice for users with a low blocking probability tolerance and users with low traffic demand. Regarding providers, that study indicated some interesting phenomena: When clients select BS based on the data rate, the revenue of providers increases in the mixed market compared to the subscriber-only one. The reverse trend holds when clients select BS based on the price. Furthermore, under large user population (e.g., 28000 users), the provider with most resources outperforms in terms of revenue (this was also observed here), while under small population size (e.g., 5000 users), the provider with the best channel quality has the advantage.

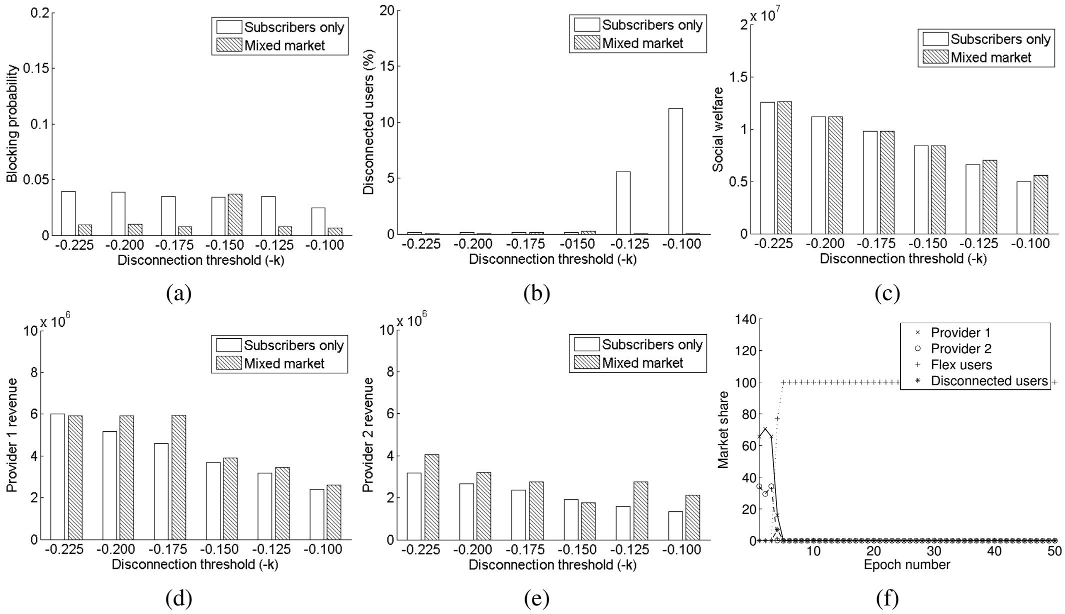


Figure 8.8: Performance at the macroscopic level: (a) Blocking prob. (b) Disconnected users (percent) (c) Social welfare (d), (e) Revenue of providers (f) Market share in the mixed market with a disconnection threshold of -0.175.

8.4.3 Comparative analysis at micro & macro levels

We performed a series of simulations of the game of users and the game of providers at the macroscopic level. As it happens also at the microscopic level, the flex service improves the performance of users. Specifically, it reduces the percentage of disconnected users (Fig. 8.8b) and blocking probability (Fig. 8.8a) in most cases. Furthermore, in the mixed market, the social welfare is similar or slightly larger compared to the subscriber-only one (Fig. 8.8c). Another common trend of the microscopic and macroscopic levels is that Provider 1 has the advantage in terms of revenue (Figs. 8.8d and 8.8e). The impact of the mean value of the user disconnection threshold ($-k$) was also analysed. At both the microscopic and macroscopic level, the revenue of the providers decreases, when the disconnection threshold of users increases. The higher the disconnection threshold, the more likely a user will choose to be disconnected. This has as a result low prices and revenue.

Significant differences of the performance of the two levels also exist. The macroscopic-level models underestimate the blocking probability compared to the microscopic-level ones (Figs. 8.7a, 8.8a). This is due to the difference in the market share at the two levels. In the mixed market, at the macroscopic level, the user population usually converges to a stable equilibrium point in which there are only flex users (e.g., Fig. 8.8f). In general, the flex service is characterized by a much lower blocking probability compared to subscriptions. Therefore, at the macroscopic level, the tendency of users to select the flex service results in a reduced average blocking probability. On the other hand, at the microscopic level, the market share exhibits intense oscillations, as explained earlier. These oscillations cause an increased number of users to select a certain service, during some epochs, raising the average blocking probability. Furthermore, the percentage of disconnected users is larger at the microscopic level compared to the macroscopic one (Figs. 8.7b, 8.8b). The market oscillations at the microscopic level result in very high blocking probabilities at some epochs. Therefore, in the subsequent epochs, many users choose to become disconnected. Moreover, at the microscopic level, the user traffic demand varies. This means that at the microscopic level, there are users with higher demand than at the macroscopic level. When these users choose the flex service, they may need to pay a high price which sometimes surpasses their tolerance, leading to an increased percentage of disconnected users.

In some cases, providers exhibit different performance depending on the level. Specifically, at the macroscopic level, in most scenarios, the revenue in the mixed market is larger compared to the subscriber-only one (Figs. 8.8d, 8.8e). On the contrary, at the microscopic level, the revenue in the mixed market is larger compared to the subscriber-only one when the disconnection threshold is relatively high (e.g., -0.125, -0.100). In the remaining scenarios, the revenue in the mixed market is similar or even smaller compared to the subscriber-only one (Figs. 8.7d, 8.7e). This is due to the level that the price setting algorithm simulates the game of users. The estimation of the revenue at the macroscopic level (for the price setting) is not as accurate as the estimation at the microscopic level, leading to suboptimal performance.

As mentioned earlier, at the macroscopic level, in the mixed market, the user population usually converges to an equilibrium, in which there are only flex users. Therefore, the offered subscription rates do not significantly affect the performance of the market. For example, in some scenarios, Provider 2 can not attract subscribers by offering a very low subscription rate. In such scenarios, the market usually converges to an equilibrium in which Provider 2 offers the lowest possible subscription rate. However, users still choose the flex service and the performance of the market is not affected by this strategy of Provider 2. On the contrary, at the microscopic level, conditions are different. The population of users is heterogeneous, possibly with users that prefer subscriptions over the flex service. For example, a user with high traffic demand may prefer the flat rate of subscriptions. Therefore, by offering a very low subscription rate, Provider 2 may lose profit from these users. Furthermore, the very low subscription rate of Provider 2 intensifies the oscillations. This may result to an increased blocking probability that may surpass even the one of the subscriber-only market.

The above example shows that pricing decisions performed at the macroscopic-level may lead to suboptimal performance when used at the microscopic level (even if they are optimal at the macroscopic level), both from the perspective of users and providers. This further motivates the design of the mesoscopic levels. The price setting algorithm could be formulated at a mesoscopic level to achieve a good tradeoff between accuracy and computa-

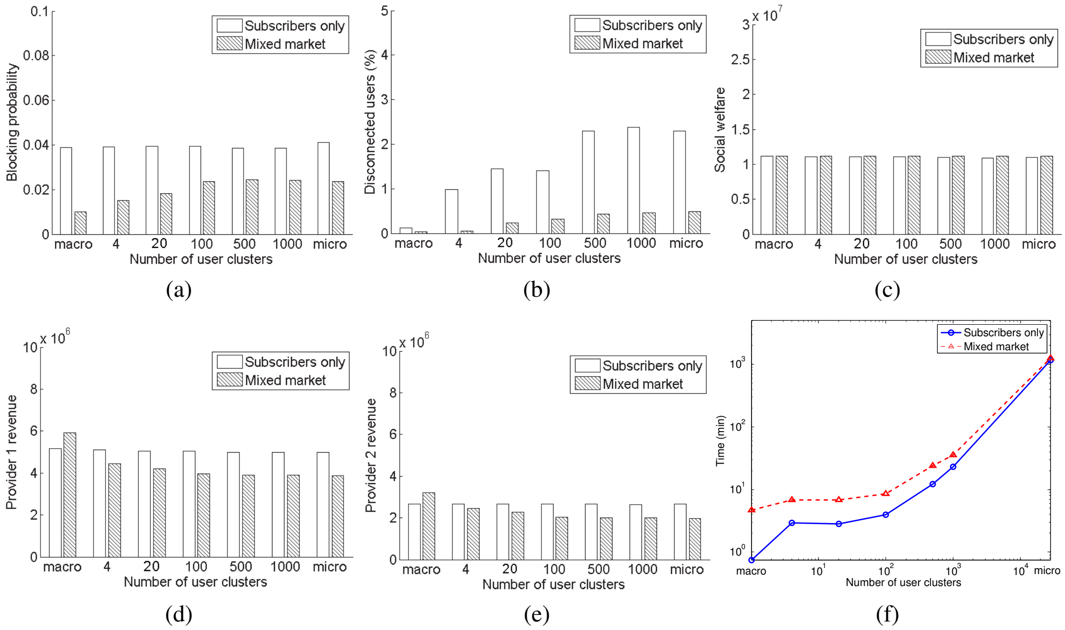


Figure 8.9: Performance at multiple levels of detail: (a), (b) Blocking prob. and disconnected users (percent) (c) Social welfare (d), (e) Revenue of providers (f) Execution time.

tional complexity. This can prevent providers from taking wrong decisions, and at the same time, keeps the computational complexity of the price setting algorithm relatively low.

8.4.4 Performance at mesoscopic levels

To quantify the tradeoff between accuracy and computational complexity, we simulated the game of users at several levels of detail. Specifically, we defined games with $1, 4, 20, 100, 500, 1000$, and 28000 user clusters according to the mesoscopic modeling. The games with 1 and 28000 clusters correspond to the macroscopic and microscopic levels, respectively. In each experiment, the game of providers was simulated at the macroscopic level.

In general, the larger the number of user clusters, the closer the performance of the mesoscopic level to the microscopic level (Figs. 8.9a - 8.9e). The microscopic level "corresponds" to the ground truth, since it models each user as a distinct entity, with its complete profile (as opposed to other levels that apply aggregations). The larger the number of user clusters, the more accurate the results. Furthermore, as mentioned in Section 8.3.3.1, the larger the number of user clusters, the larger the computational complexity. We quantified this trend and measured that the difference in the execution time of the microscopic and macroscopic levels is up to three orders of magnitude! (Fig. 8.9f).

The macroscopic level tends to overestimate the revenue of providers, especially in the mixed market (Figs. 8.9d and 8.9e). Specifically, at the macroscopic level, the estimated revenue in the mixed market is higher compared to the subscriber-only one. However, the reverse trend appears in all other levels. As mentioned, to reduce the computational requirements, the price setting is performed at the macroscopic level. An inaccurate prediction of revenue may mislead a provider in taking decisions that result in suboptimal performance. Instead of simulating the game of providers at the macroscopic level, a mesoscopic level that achieves a good tradeoff between accuracy and computational complexity can be selected (e.g., the mesoscopic level with 100 user clusters). Actually, in experiments in which the simulator of the game of providers was based on this mesoscopic level, we did observe an increased revenue of providers compared to the corresponding case at the macroscopic level.

8.5 Concluding remarks

This chapter presents a methodology for analysing wireless markets that uses an event-based simulator. It also shows the importance and practical merit of this methodology by investigating the roll out of a new service, the flex service. The analysis demonstrates the following trends: the flex service dramatically reduces the percentage of disconnected users, decreases the blocking probabilities, and improves the social welfare.

Using the proposed methodology, we have analysed a wireless oligopoly of a small city at the macroscopic and microscopic levels. The two levels manifest similarities: e.g., the flex service improves the performance of the market with respect to the blocking probability and percentage of disconnected users. The analysis also reveals important differences. Interestingly, while at the macroscopic level, the revenue of both providers is larger in the mixed market compared to the subscriber-only one, at the microscopic level, this does not always hold. Furthermore, the market exhibits intense oscillations at the microscopic level, while at the macroscopic level, it usually converges to a stable equilibrium. The competition among providers as well as the delay in the dissemination of the user feedback via u-map cause strong oscillations at the microscopic level. In addition, the difference in the execution time of the microscopic and macroscopic levels is prominent.

The multi-layer modeling pays off. The simulation of a market at multiple levels of detail provides a quantitative measure of the tradeoff between accuracy and complexity and can help us select the appropriate level.

Chapter 9

Conclusions

This thesis presents a modular multi-layer modeling framework for analysing wireless markets. The framework employs queueing theory to model the networks of providers and user traffic demand as well as game theory to model the user service selection and the competition of providers. It allows providers to model users at different levels of detail by distinguishing different user sub-populations and modeling their decision making separately. It also models the user decision making in a realistic manner assuming that they do not always make the optimal decisions in terms of the offered prices and quality of service. Except from those parameters, a variety of psychological and social aspects affect the user decisions that are captured by a noise parameter in the user service selection process.

To decrease the computational complexity of the analysis even further, the framework provides a network aggregation technique based on the theorem of Norton. Many times, providers are interested in the performance of a specific region of interest within their network. For example, they may need to participate in a secondary spectrum market to purchase additional spectrum and improve their QoS at a congested part of their network. In such cases, the theorem of Norton can compute equivalent queueing network models only for the region of interest omitting the details of the entire networks of providers. This results in significant computational gains.

The proposed modeling framework is a powerful tool that can be used to study various market cases with a strong commercial interest. In this thesis, we have analysed three such cases, namely, pricing via market segmentation, WiFi offloading, and capacity planning. In pricing via market segmentation, providers distinguish various user sub-populations with different profiles and requirements and design their dataplans aiming to improve the satisfaction of those sub-populations and maximize their revenue. Using our modeling framework providers can select the optimal level of detail for modeling users such that they improve their revenue. Our analysis indicates that in market scenarios in which there is a strong correlation between the user willingness-to-pay and QoS requirements, when providers model users with a larger number of sub-populations or offer a larger number of dataplans they achieve revenue benefits. In other market cases in which those two parameters are completely independent, a different trend is observed. In such markets, providers improve their revenue when they model users in a higher level of detail only under a low traffic demand, while they achieve revenue losses in the case of a large traffic demand. Additionally, the offering of a large number of dataplans in such a market is not beneficial and in some cases it may also result in revenue losses.

In WiFi offloading, a provider can serve a part of its data traffic by a complementary network infrastructure that mainly consists of WiFi APs and femtocells. Our analysis indicates that it is not always profitable for providers to invest in a large WiFi infrastructure. The benefits of a provider from the offloading increase with the coverage of the WiFi infrastructure but with a diminishing rate. Our framework can be extended to enable providers to design their business plan for offloading. Specifically, it can incorporate the capital (e.g., investment for WiFi, backhaul equipments, installation) and operational (e.g., WiFi and backhaul site rentals, maintenance) expenditures for supporting the offloading. Based on that, it can perform a cost-benefit analysis to estimate whether offloading is profitable.

In capacity planning, providers can purchase additional spectrum at a congested part of their network to improve their offered QoS. In this thesis, we have analysed such a market in which the available spectrum is allocated to providers according to a VCG auction. The analysis indicates that the model of the user utility function that providers consider is crucial. If providers assume that users are only affected by the average achievable data rate, they end up making similar bids and therefore, the winner of the auction does not achieve significant benefits. However, when providers assume that the user satisfaction is also affected by the spatial variance of the achievable data rate, then competition of providers weakens in the auction allowing for multiple winners each of which achieves significant additional revenue benefits.

The proposed framework could also be used to study problems from other research areas except from wireless markets. For example, it could model the economic interactions of autonomous systems/networks on the Internet. Some of these networks may forward traffic of other networks for a price. These networks corresponds to the providers of our framework, while their customers correspond to the users. Each customer network has its own utility function that depends on the offered price and bandwidth. The framework could also be used to model routing decisions of nodes in a network, where each link may be associated with a different cost and bandwidth. Another example is from the area of public health in which several primary and secondary health institutions may form agreements for serving patients. In general, this framework can model various types of markets in which a set of service providers with limited resources serve a set of customers with different characteristics and demand. The use of this framework may provide interesting insight for the performance of such markets.

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