Cosmic-ray air-shower simulations across the ankle: Combining mixed Galactic composition with new physics above 50 TeV

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Abstract

Observations of extensive air showers (EAS) initiated by Ultra High Energy Cosmic Rays (UHECR), when interpreted in the conventional way, indicate a transition to heavy primaries at energies $> 10^{18.3}$ eV. However, at energies $> 10^{18.9}$ eV, the arrival directions of UHECRs display a dipole anisotropy. This, along with new studies showing that the Galactic magnetic field is stronger than previously believed, disfavor heavy composition, as heavy nuclei are strongly deflected by magnetic fields and thus would lose any traces of anisotropy. One solution to this problem could be the appearance of new effects beyond the Standard Model in particle interactions at center-of-mass (CM) energies $\gtrsim 50$ TeV, which would increase the cross-section of the interaction and the multiplicity of the interaction products, and lead to the composition of UHECRs appearing heavy-like.

In this work, we study the effects of this scenario on the composition-sensitive cosmic-ray observables using EAS simulations for primaries in the energy region $10^{17} - 10^{20}$ eV. For that we produce a model for the transition from Galactic to extragalactic primaries, assuming a mixed composition Galactic and a single composition extragalactic component. We perform simulations for showers induced by each different primary species using the CORSIKA software, appropriately modified to include the new effects for first collisions at CM energies $> 50$ TeV, and study the observables produced by them. We find that the new effect reproduces perfectly the data from Pierre Auger Observatory for a proton-Air cross-section of $\sim 740$ mb and an increase in first-collision products of a factor of 3, at 140 TeV CM energy.
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Chapter 1

Overview

The term cosmic rays (CR) usually refers to high-energy particles arriving to Earth from the outer-space, comprising a background of radiation permeating our galaxy and the extra-galactic space. They mostly consist of protons and fully-ionized atomic nuclei (up to iron nuclei) plus a small percentage of solitary electrons, gamma rays, high-energy neutrinos (> TeV) and antimatter particles. Colliding with atoms and molecules of Earth’s atmosphere (mainly oxygen and nitrogen) they produce a cascade of lighter particles (called air shower), such as pions, muons, electrons and neutrinos [69].

Cosmic rays are classified into primary and secondary: primary cosmic rays are those particles accelerated at astrophysical sources while secondaries are those particles produced in interaction of the primaries with interstellar medium or the Earth’s atmosphere. Thus electrons, protons and helium, as well as carbon, oxygen, iron, and other nuclei synthesized in stars, are primaries. Nuclei such as lithium, beryllium, and boron (which are not abundant products of stellar nucleosynthesis) are secondaries. Antiprotons and positrons are also in large part secondary [19].

There are three main observables used to study the cosmic rays: the energy spectrum, the chemical composition of the primary particles, and the distribution of their arrival directions. All of these properties are connected to each other. A complete theory of cosmic rays and their secondaries has to explain all of the features and observables simultaneously. Despite the theoretical challenges of describing cosmic-ray transport and interaction with proper astrophysical input, the observational results can be separated quite clearly [21].

1.1 Cosmic Ray Energy Spectrum

The cosmic ray all particle energy spectrum, usually given in units of an energy-weighted particle flux,

$$J(E) \cdot E^b = \text{(particles)} \cdot \text{(length)}^{-2} \cdot \text{(time)}^{-1} \cdot \text{(solid angle)}^{-1} \cdot \text{(energy)}^{b-1} \quad (1.1)$$

covers more than 13 orders of magnitude in energy and 34 orders of magnitude in flux.
This huge range in energy and flux level of cosmic rays makes it impossible to observe the complete spectrum with one single instrument, and thus their energy spectrum has been measured by many experiments, which exploit different techniques and are located at various atmospheric depths depending on the energy region they wanted to study.

The flux at the lowest energies as it is measured e.g. by the Voyager spacecrafts or the Alpha Magnetic Spectrometer (AMS-02) experiment yields hundreds of particles per second for a square-meter sized detector area which is challenging for the data processing but decreases statistical uncertainties fast. At the other edge of the spectrum observatories like the Telescope Array (TA) and the Pierre Auger Observatory (PAO) cover several hundred square-kilometer of surface area to collect a flux that amounts to only a few ultra-high energy cosmic rays (UHECRs) per square kilometer and year. Also the detection techniques differ very much for low and high energy cosmic rays: Where cosmic rays up to energies of \(E = 10^4 - 10^5\) GeV can be detected directly cosmic rays above this energy can only be made visible via their imprint on Earth via their produced air showers [21].

Fig. 1.1 shows the unweighted all-particle cosmic-ray flux (number of particles per \(m^2 sr s GeV\)) across all their energies. The CR spectrum is to a great extent featureless, which is surprising, considering the vast variety of different astrophysical objects that may be their sources, from protostars to super-massive black holes in active galaxies, with different properties and located in all kinds of environments. For a large part of the energy range (above \(\sim 10\) GeV) it is found to follow a power law of the form,

\[
dN(E)/dE = J(E) \propto E^{-\gamma}
\]

where \(\gamma\) is the spectral index. The overall cosmic-ray flux is well described with a spectral index of \(\gamma \approx 2.6 - 2.7\) for energies between \(\sim 10^9\) eV and \(\sim 3 \times 10^{15}\) eV [18]. This makes it likely that cosmic-ray production and propagation is governed by the same mechanism, at least in this energy range. At higher energies, three small — but nevertheless important — features can be observed in the spectrum:

1. a steepening, around \(3 \times 10^{15}\) eV, referred to as the knee, where \(\gamma\) changes from \(\sim 2.7\) to \(\sim 3.0\) [47].

2. a further steepening, around \(5 \times 10^{17}\) eV, referred to as the second knee, where \(\gamma\) decreases to \(\sim 3.3\) [48].

3. a pronounced flattening, around \(3 \times 10^{18}\) eV, referred to as the ankle, where \(\gamma\) returns to \(\sim 2.7\).

These spectral features suggest significant changes in the properties of CRs, which can be attributed to changes in the chemical composition, the location of the sources or propagation effects.

The features of the CR energy spectrum can be better observed in Fig. 1.2, where the cosmic ray flux as a function of energy is multiplied by \(E^{2.6}\) to emphasize the spectral shape.
1.1.1 The knee

The cosmic-ray knee is spread across the total energy spectrum over several orders of magnitude in energy due to its rigidity dependence.

The most prominent change in slope is around an energy $E \approx (1 - 5) \times 10^{15}$ eV, where the increase of $\gamma$ is of $\sim 15\%$. After the discovery of the first break of the knee, another knee-like feature was detected in the energy spectrum around $E \approx 6 \times 10^{16}$ eV which is exactly at 26 times higher energy than the first knee. This feature is more noticeable when only the heavy component of the total flux is taken into account.

This has motivated the interpretation that both features are formed by the same underlying charge-dependent process giving a light (proton) $E \approx (3 - 5) \times 10^{15}$
Figure 1.2: All particle energy spectrum of CRs from air shower data in the energy range of $10^{13} - 10^{21}$ eV. The flux of CRs is multiplied by energy to 2.6. The experiments that contribute data to this graph are displayed. Taken from [19].

GeV) and a heavy (iron) ($E \approx 80 \times 10^{15}$ GeV) knee, according to the idea of rigidity-dependent\(^1\) cutoffs in the spectra of individual nuclei [18, 21, 13]. The standard interpretation of the knee is that particles of energy below and around it are accelerated at galactic astrophysical objects, mainly at supernova remnants and possibly at powerful binary systems. The knee itself is thus a result of reaching the maximum energy of such accelerators [51].

1.1.2 The second knee

The second knee is more subtle compared to the first knee. The analysis of this structure is difficult, and there is not yet a widely accepted interpretation of it, partially because at these energies there is no element-resolved flux data available. It is still unclear whether the knee and the second knee are different features of one population.

One explanation of the second knee may be a general rigidity-dependent process (as in e.g. [73]). Another one is that cosmic rays are accelerated by two Galactic source classes which have different maximum rigidities leading to two distinct knees in the spectrum, see e.g. [11, 55].

\(^1\)Rigidity is defined as particle momentum divided by charge, $R \equiv p/Z \propto E/Z$
1.1.3 The ankle

Cosmic rays are accelerated in various astrophysical processes and therefore can be classified with respect to their source as galactic CRs (the ones that originate from our Galaxy) and extragalactic CRs (the ones that originate from external galaxies). Somewhere in the energy range between $10^{17}$ and $10^{19}$ eV the transition from Galactic to Extragalactic CRs is expected to take place.

The highest energy cosmic rays are likely to originate in extragalactic sources, given the strength of Galactic magnetic fields and the lack of correlations with the Galactic plane. Low energy cosmic rays are easily created and contained in the Galaxy, so a transition region should occur in some intermediate energy.

The arrival direction of galactic CRs is approximately isotropic, due to diffusive propagation in the galactic magnetic field. On the other hand, cosmic-ray arrival directions at energies $> 10^{18.9}$ eV exhibit a dipole anisotropy. Auger has reported the observation of a dipole of amplitude $6.6^{+1.2}_{-0.8}$% for cosmic rays with energies above $8 \times 10^{18}$ eV. The direction of the dipole indicates that these CRs are extragalactic [65].

As mentioned previously, the ankle is a flattening of the spectrum to a spectral index close to 2.7. This feature has been observed by multiple experiments (see e.g. [68, 11, 12]), and there are two main interpretations of it.

The first one is that the ankle is the region where the transition from Galactic to extragalactic CRs happens, as there both populations contribute equally to the flux. So around the ankle energies Galactic CRs end and above the ankle different sources (probably in the nearby supercluster of galaxies) are active [44]. Ankle transition models initially were considered to oppose models for the origin of Galactic CRs, as in older models acceleration in supernova remnants (SNRs) was predicted to weaken around $10^{15}$ eV [50]. However, modifications of the standard SNR case such as magnetic field amplification in SN shocks [22], or a different progenitors such as Wolf-Rayet star winds [26], and trans-relativistic supernovae [28] may explain the energy gap from $10^{15}$ to $10^{18}$ eV [49].

The second interpretation is that the dip structure in the region of the ankle is the result of electron-positron pair-production losses during the propagation of extragalactic protons (extragalactic protons interacting with the CMB) [23]. The models that are based on this interpretation are called dip models [24]. In those models, the transition between the galactic and extragalactic components takes place at lower energy, in the second knee region.

Analyzing the properties of the ankle in order to favour one of scenarios is complicated and a huge part in that depends on studying the CR composition in this region, as each model predicts a different composition.

1.1.4 Other features

Besides the three features we mentioned above, there are deviations from the power-law behavior in the energy spectrum below $10^9$ eV and above $10^{19.3}$ eV.
Below $10^9$ eV, the departure from the power-law fit is caused by the diffusion of the CRs coming from outside the solar system through the solar wind [18]. Above $10^{19.3}$ eV we observe a strong and fast suppression of the spectrum. This suppression can be attributed to two different scenarios. The first is the appearance of the so-called Greisen-Zatsepin-Kuzmin (GZK) effect [37, 75] in those energies.

The GZK effect is caused by the interaction of CRs with the cosmic microwave background (CMB), due to the larger cross-section of these interactions at EeV energies. Specifically, above the threshold of about $10^{19.6}$ eV CRs interact with the CMB producing pions, and these interactions continue until their energy fall below the pion production threshold. Thus CRs with energies above this threshold do not reach Earth.

The second scenario is that the observed suppression of the spectrum is simply due to the sources reaching their maximum energy of acceleration. Those two scenarios are difficult to disentangle, and the suppression may be a result of a combination of them. Current measurements are compatible with both a GZK cut-off and a maximum energy of the sources [21].

1.2 Acceleration of Cosmic Rays

The origin of cosmic rays is one of the main problems in astroparticle physics and remains still largely unsolved. As can be seen in Figures 1.1 and 2.1 all different species share the power-law behaviour ($J(E) \propto E^{-\gamma}$, with $\gamma \sim 2.7 - 3.3$) as already mentioned. This may suggest that they share a common origin, i.e. the mechanism for them to acquire such energies must be essentially the same.

The acceleration of charged particles is easily achieved in the presence of electric fields. However, large-scale electric fields are destroyed by omnipresent astrophysical plasmas throughout the universe. On the other hand, magnetic fields are ubiquitous in astrophysical objects. Their fluctuations in space and time imply the presence of transient electric fields which can supply a consequent amount of energy to charged particles.[49]

There are various acceleration mechanisms discussed in the literature, however the most the most commonly cited one is the so-called Fermi acceleration.

1.2.1 Fermi acceleration

The current leading theory for acceleration of cosmic rays was proposed by Fermi in 1949 [33]. Fermi’s idea was that a relativistic particle with initial energy $E_0$ could accelerate through elastic "collisions" with a nonuniform, magnetic field\(^2\).

Let us assume that with each "collision" the relativistic test particle gains energy $\Delta E = \xi E$. This way, after $n$ collisions the test particle has:

$$E_n = E_0(1 + \xi)^n$$

\(^2\)By collision here we mean the reflection off the irregularities in the magnetic field itself, not with other particles.
and the number of collisions needed to reach a certain energy $E$ is:

$$n = \frac{\ln \left( \frac{E}{E_0} \right)}{\ln(1 + \xi)} \quad (1.4)$$

If the probability of the particle to to abandon the acceleration region after each scattering is $P_{\text{esc}}$, the probability of the particle remaining in this region after $n$ collision is $P_n = (1 - P_{\text{esc}})^n$, and the integral energy spectrum, i.e. the fraction of particles with energy $E > E_n$ will be given by:

$$N(> E) \propto \sum_{m=n}^{\infty} (1 - P_{\text{esc}})^m = \frac{(1 - P_{\text{esc}})^n}{P_{\text{esc}}} = \frac{1}{P_{\text{esc}}} \left( \frac{E}{E_0} \right)^{-\gamma} \quad (1.5)$$

where in the last equality we used Eq. 1.4. In a differential form this is written as:

$$\frac{dN}{dE} \propto E^{-(1+\gamma)} \quad (1.6)$$

The spectral index $\gamma$ is:

$$\gamma = \frac{\ln \left( \frac{1}{1 - P_{\text{esc}}} \right)}{\ln(1 + \xi)} \sim \frac{P_{\text{esc}}}{\xi} \quad (1.7)$$

in the limit of $P_{\text{esc}}, \xi \ll 1$.

If we assume that there is a characteristic time for a single acceleration cycle, $T_{\text{cycle}}$, then we can write the total time needed to escape from the acceleration region as $T_{\text{esc}} = T_{\text{cycle}}/P_{\text{esc}}$. This way, we can rewrite Eq. (1.7) as:

$$\gamma = \frac{T_{\text{cycle}}}{\xi T_{\text{esc}}} \quad (1.8)$$

Moreover, if the test particle has been accelerating for a time, $t_{\text{accel}}$, the number of its possible collisions is $n \leq t_{\text{accel}}/T_{\text{cycle}}$ and therefore the maximum energy after $t_{\text{accel}}$ (from Eq. (1.3)) is:

$$E = E_0(1 + \xi)^{t_{\text{accel}}/T_{\text{cycle}}} \quad (1.9)$$

From Eq.(1.5) we see that this mechanism leads to a power law spectrum, as desired. Furthermore, Eq.(1.9) shows that higher final energies require more acceleration time. Therefore the maximum energy that a particle can reach is limited by the time frame of an accelerator [35].

In his original paper, Fermi suggested that CRs are accelerated in moving "magnetic clouds", which contain the magnetic field irregularities that produce the scatterings we discussed above. These magnetic clouds are randomly moving clouds of gas with embedded magnetic fields. The processes can be understood qualitatively in the following way. Let us suppose that the first collision occurs when the particle is moving along with the direction of the magnetic field (rear end collision). Then, in this scattering, the particle loses energy and is thrown back against the direction of the field flow. The particle subsequently engages in a frontal collision, which results in an energy gain and sends the particle back into the field’s forward motion. If the field were stationary, there would be on average an equal number
of these two collision types, and so the particle would in total neither gain nor lose energy. However, since in the model the field are moving with random velocities, the probability of energy increases outweighs the probability of losses, leading in an overall gain in energy of order:

$$\xi \propto \beta^2$$

(1.10)

where $\xi$ is again the average fractional energy gain, $\Delta E/E$ per "collision", however here a "collision" a pair of frontal and rear-end collisions, and $\beta = V/c$ where $V$ is the velocity of the plasma flow. This version is known as "Second Order Fermi Acceleration" since it is the second order term in an expansion of $\beta$. This acceleration model explains a power law in the energy spectrum, but the fact that the speed of the clouds is non-relativistic ($\beta \ll 1$) and the small cloud dimensions ($\sim 1$ pc) lead to an inefficient mechanism that would need tens of Gigayears to cause an appreciable acceleration.

Nevertheless, this idea evolved in another iterative process that can accelerate CRs. It is based on the idea of a shock wave moving in a hydromagnetic environment such as the one generated by a SNR, when exploding gas travels faster than the speed of sound of a medium. This model can be qualitatively understood by considering a particle bouncing back and forth across the shock front and elastically colliding with the magnetic turbulence both in front of (upstream) and behind (downstream) the shock. The downstream edge of the front converges on the upstream edge: in either the upstream or downstream rest frame the other side of the shock is approaching, similar to a ping-pong ball bouncing back and forth between two paddles moving toward each other [62]. Therefore, collisions with the particle are frontal in both the upstream and downstream frame, increasing the energy of the particle with every collision and thus resulting in an overall gain in energy of order:

$$\xi \propto \beta$$

(1.11)

This is known as "First Order Fermi Acceleration". This mechanism is very promising, being the most effective and probable one, since shock waves are expected to be present in different astrophysical environments, and it leads to simple powerlaw predictions for the spectrum of the accelerated population (see, e.g., [35]). One other advantage of the diffusive shock acceleration mechanism is that it naturally provides a power law spectrum whose predicted index $\gamma$ is within the range of the experimental measurements. Depending on the exact geometry of the shock and on its relativistic nature, the combination of the energy gain per crossing and of the escape probability leads to a power law index of exactly 2 for the case of a strong nonrelativistic shock in an ideal gas and to indexes between 2.1 and 2.4 for relativistic shocks [51]. The further interaction of the cosmic rays between the source and Earth allow for the remaining difference between the observed spectrum and that produced at the source.

### 1.2.2 The Hillas Criterion

Acceleration mechanism aside, there is a basic argument for each potential accelerator that shows if an object can be considered as a candidate source or not: the Hillas Criterion. As first pointed out by Hillas [44], a particle escapes the accelerator as
soon as the gyroradius $r_g$ of the particle exceeds the size of the object $R$, 

$$r_g = \frac{E}{|q|cB} \leq R \quad (1.12)$$

with $E$ and $Ze$ as the energy and charge of the particle, $B$ as the magnetic field of the accelerator (cgs units). In the relativistic particle limit $v \to c$, the energy is thus constrained:

$$E_{max} \leq Z e c B R \quad (1.13)$$

This is the famous Hillas Formula that gives a concrete prediction for the conditions to be present in an accelerator in order to get particles to a certain maximum energy. In typical units, the maximum energy can be expressed as

$$E_{max} = Z \cdot 10^{18}\text{eV} \cdot \left(\frac{B}{\mu\text{G}}\right) \cdot \left(\frac{R}{\text{kpc}}\right) \quad (1.14)$$

For the acceleration at relativistic shocks, it was pointed out the Hillas formula needs to be modified:

$$E_{max} \leq Z e c B \Gamma_{sh} \beta_{sh} R \quad (1.15)$$

Here, $\Gamma_{sh}$ is the boost factor of the shock front and $\beta_{sh} = v_{sh}/c$ is the shock velocity in units of the speed of light [21].

The Hillas criterion allows for astrophysical objects to be studied regarding their typical magnetic fields $B$ and their extensions $R$ in order to get a first estimate of their potential to accelerate cosmic rays to the knee, to the ankle or to the absolute maximum of observed particle energies in the Universe. Since the gyroradius of UHECRs in Galactic magnetic fields ($r_g \sim 110\text{kpc}Z^{-1}(\mu\text{G}/B)(E/100\text{EeV})$) is much larger than the thickness of the Galactic disk, confinement in the Galaxy is not maintained at the highest energies. Therefore, after some energy CRs observed at Earth transition to extragalactic ones.

Figure 1.3 is an example of a Hillas plot which, for a given maximum energy $E_{max}$ of the accelerated particle, shows the relation between the source’s magnetic field strength $B$ and its size $R$. Sources above the top line are able to accelerate protons up to $10^{21}$ eV, while sources above the bottom line are able to accelerate iron up to $10^{20}$ eV.

## 1.3 Extensive Air Showers

While the CR spectrum extends up to more than $10^{20}$ eV, for increasing values of primary energy the flux drops fast, and above about $10^{15}$ eV the primary CR flux becomes very low ($< 1$ particle/$m^2\cdot\text{year}$). This is why flying balloons or spacecrafts in the upper part of the atmosphere for a direct detection of UHECRs is ineffective – a small detector of a few square meters would require huge amounts of time just to observe a few of them. Space telescopes are also not an alternative, for the same reason.

Therefore, in the highest energies, CRs are detected indirectly, by measuring the secondary particles that they generate due to their interaction with the air molecules.
The particle cascades following the interaction of a cosmic ray with a molecule of the atmosphere are referred to as Extensive air showers (EAS). High energy CRs produce air showers which reach the ground, e.g. a proton of initial energy $10^{19}$ eV coming vertically into the atmosphere produces at sea level about $3 \times 10^{10}$ particles, with an extension at ground over a few $km^2$ [18].

In order to measure those EASs two main different types of detectors are used: surface detectors which measure the secondary particles of the EAS at ground level, and radiation detectors that measure the electromagnetic radiation emitted during the shower development in the atmosphere. Since the flux at higher energies is so low, these detectors have to cover very large areas.

Four main components can be distinguished in the EASs:

1. **Electromagnetic (EM) component**: it is composed by photons, electrons and positrons created mainly through neutral pion decay ($\pi^0$ has a lifetime of $\approx 8.4 \times 10^{-17}$ s and decays into two photons). Their energy is mostly in the range of 1 to 10 MeV. Most particles arriving at the ground level are photons, followed by electrons. In each generation of particles in the cascade about

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**Figure 1.3:** Hillas diagram. Above the blue (red) line protons (iron nuclei) can be confined to a maximum energy of $E_{\text{max}} = 10^{20}$ eV. The most powerful candidate sources are shown with the uncertainties in their parameters. Taken from [49].

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25% of the total energy of the shower is transferred to the EM component, and eventually, around 90% of the primary particle's energy is dissipated by the EM component (the remaining 10% is carried by muons and neutrinos). The EM component also produces the dominant part of the Cherenkov radiation.

2. **Muonic component**: it consists of muons which are the third most abundant particles arriving to the ground. Muons come mainly from $\pi^+, \pi^-, K^+$, and $K^-$ decays and have an average energy of about 1 GeV. Since muons as well decay electromagnetically, the muonic component also contributes to the EM component. The muonic component of an EAS differs from the electromagnetic component for two main reasons. First, muons are generated through the decay of cooled ($E_{\pi^\pm} \leq 1$ TeV) charged pions, and thus the muon content is sensitive to the initial baryonic content of the primary particle. Furthermore, since there is no “muonic cascade”, the number of muons reaching the ground is much smaller than the number of electrons. Second, the muon has a much smaller cross section for radiation and pair production than the electron, and so the muonic component of an EAS develops differently than does the electromagnetic component. The smaller multiple scattering suffered by muons leads to earlier arrival times at the ground for muons than for the electromagnetic component.

3. **Hadronic component**: it is composed by protons, neutrons, pions, and kaons, generated at the top of the atmosphere. The interaction of a baryonic CR with an air molecule high in the atmosphere leads to a cascade of secondary mesons and baryons. The first few generations of charged pions interact again, producing the hadronic component. It then produces all the other components, as it is transformed into them very fast. This component is thus rare at the ground level and it is difficult to detect it due to the small size of its footprint on the ground.

4. **Neutrinos** are not visible in CR detectors. They are produced in the decays of pions, muons, and kaons of the shower. The energy taken away by neutrinos and the energy carried by muons are correlated. [18, 51] These components of the air-shower can be seen schematically in Fig. 1.4.

As the cascade develops in the atmosphere, the number of particles in the shower increases until the energy of the secondary particles is reduced to the level where ionization losses dominate. At this point the density of particles starts to decline. A well-defined peak in the longitudinal development, $X_{\text{max}}$, occurs where the number of $e^\pm$ in the EM shower reaches its maximum (see Fig. 1.5). $X_{\text{max}}$ increases with primary energy, as more cascade generations are required to degrade the secondary particle energies.

The indirect detection has been efficient in reconstructing the energy and the arrival direction of cosmic rays. EASs can also be used in studying the CR composition, since the characteristics of the EASs depend on the primary type. The differences between the EASs started by different primary types are used to determine the nature of the primary. However, identifying the primary mass composition turns out to be a difficult task as we will see in Section 1.4.
A central part of many such composition analyses through air showers is evaluating $X_{\text{max}}$. For showers of total energy $E$, heavier nuclei have smaller $X_{\text{max}}$ because the shower is already subdivided into $A$ nucleons when it enters the atmosphere. The average depth of maximum $\langle X_{\text{max}} \rangle$ scales approximately as $\ln(E/A)$, as we will see in the next section. Therefore, since $\langle X_{\text{max}} \rangle$ can be determined directly from the longitudinal shower profiles measured with a fluorescence detector, the CR composition can be extracted after estimating $E$ from the total fluorescence yield. Indeed, the parameter often measured is $D_{10}$, the rate of change of $\langle X_{\text{max}} \rangle$ per decade of energy.

The basic properties of the development of a shower can be deduced from a simplified model which describes the evolution of a pure electromagnetic cascade and was introduced by Heitler [43] as well as from its extension to hadronic showers by Matthews [52].

### 1.3.1 Heitler’s model of electromagnetic showers

In Heitler’s model, the EM shower evolves via two processes: bremsstrahlung radiation and electron-positron pair production. It assumes that all particles (electrons, positrons and photons) have the same radiation length $\lambda_r$ in the medium (which is the air for EAS), which means that they all go through an interaction at a standard distance $d = \lambda_r \ln 2$. When the medium is air, $\lambda_r = 37 g/cm^2$. After each interaction, two particles of equal energy are produced for each existing particle. Photons...
Figure 1.5: Particles interacting near the top of the atmosphere initiate an EM and hadronic cascade. Its profile is shown on the right. The different detection methods are illustrated. Mirrors collect the Cherenkov and nitrogen fluorescent light, arrays of detectors sample the shower reaching the ground, and underground detectors identify the muon component of the shower. The number of particles as a function of the amount of atmosphere penetrated by the cascade ($X$ in g/cm$^2$) is known as the longitudinal profile. The integrated longitudinal profile provides a calorimetric measurement of the energy of the primary CR, with a relatively small uncertainty due to the correction for energy lost to neutrinos and particles hitting the ground. From [17].

generate an $e^+/e^-$ pair while electrons produce radiation via bremsstrahlung. The cross-sections are assumed to be independent of energy and collision energy losses are ignored.

The evolution of the shower thus happens in fixed steps and can be imagined as a particle tree with branches that bifurcate every $d$ (see Fig. 1.6). After $n$ steps there will be $N_n = 2^n$ particles in the shower, and each one will carry an energy $E = E_0/N_n$, where $E_0$ is the energy of the primary particle. The creation of particles stops when their energy becomes smaller than the one necessary for the bremsstrahlung or pair production. This critical energy $E_{\gamma}^c$ is the energy at which the rate of radiative energy loss (via bremsstrahlung) is equal to the rate of collisional energy loss (by ionization). The critical energy in air is $E_{\gamma}^c = 80$ MeV. In this model, the maximum development of the shower is reached when the energy of each electromagnetic particle is equal to $E_{\gamma}^c$, and from then on particles interact with atmosphere losing energy until they are absorbed.

Despite being highly simplified, this model accurately reproduces 3 properties of EM cascades:

- the number of particles at the cascade maximum is proportional to the energy of the primary particle
  \[ N_{\text{max}} = E_0/E_{\gamma}^c \]  

- the depth of the cascade maximum (along the shower axis) is logarithmic with
energy
\[ X_{\text{max}} = X_1 + X_D = X_1 + \lambda_r \log\left(\frac{E_0}{E_c^\gamma}\right) \] (1.17)

where \(X_1\) is the depth of the first interaction and \(X_D\) the additional column density required for the shower to reach its maximum development.

- the evolution rate of \(X_{\text{max}}\) with the logarithm of the energy, named elongation rate, defined as
\[ D_{10} = \frac{dX_{\text{max}}}{d\log E_0} = 2.3\lambda_r \] (1.18)

is given by \(\lambda_r\) in the medium. In air, the elongation rate is about 85 g/cm².

Extensive simulations of electromagnetic cascades are in agreement with these properties, however the particle number at maximum and the ratio of electrons to photons are overestimated by the model, mainly due to the facts that multiple photons are emitted during bremsstrahlung and that electrons lose energy much faster than photons do.

![Heitler's model](image.png)

Figure 1.6: Heitler’s model describing the development of the EM and hadronic shower. Taken from [45].

### 1.3.2 Heitler-Matthews model for hadronic showers

Heitler’s model can be adapted to describe hadronic showers. As indicated before, we can consider an EAS as the sum of three components: hadronic (nucleons, mesons), muonic and electromagnetic.

For the hadronic shower, the relevant parameter is the hadronic interaction length \(\lambda_I\). At each step of thickness \(d = \lambda_I \ln 2\), hadronic interactions produce \(2N_\pi\) charged pions and \(N_\pi\) neutral pions. While neutral pions decay immediately due to the very small decay length, charged pions interact further. The hadronic shower keeps growing, feeding the EM part at each step, until the energy of charged pions drops to a level where decay into muons is more likely than a new interaction. This critical energy \(E_c^\pi\) is 20 GeV in air. A schematic of a hadronic cascade can be seen in Fig. 1.7.

The longer it takes for charged pions to reach \(E_c^\pi\), the larger will be the EM component, and therefore, in long developing showers, the energy of the muons from the decaying pion will be smaller, which means that deep showers will have a
Figure 1.7: Schematic evolution of an hadronic cascade. At each step roughly 1/3 of the energy is transferred from the hadronic cascade to the electromagnetic one. From [51].

smaller number of muons reaching ground while primaries with higher cross sections with air (i.e. higher mass) will have a larger muon to electron ratio at ground.

The number of muons in the shower can be obtained by assuming that all pions decay into muons when they reach the critical energy $N_\mu = (2N_\pi)^n_c$, where $n_c = \ln(E_0/E_c^\gamma)/\ln 3N_\pi$ is the number of steps needed for the pions to reach $E_c^\gamma$. Using $\beta = \ln 2N_\pi/\ln 2N_\pi$ we have

$$N_\mu = \left(\frac{E_0}{E_c^\pi}\right)^{\beta}$$

Unlike the electron number, the muon multiplicity does not grow linearly with the primary energy, but at a slower rate. The value $\beta$ depends on the average pion multiplicity used and on the inelasticity of the hadronic interactions.

The determination of depth of the shower maximum is more complicated in the case of hadronic cascades in comparison to the purely EM one. The larger cross section and the larger multiplicity at each step reduces the value of $X_{max}$ while the energy evolution of those quantities will change the elongation rate. Moreover, the inelasticity of the interaction will also modify both the $X_{max}$ and the elongation rate.

As a first approximation, we can obtain the elongation rate when introducing the cross section and multiplicity energy dependence. Using a proton air cross section of 550 mb at 1018 eV and a rate of change of about 50 mb per decade of energy we
get
\[ \lambda_I \approx 90 - 9 \log(E_0/EeV) \text{g/cm}^2 \] (1.20)
and assuming that the first interaction initiates \(2N_\pi\) cascades of energy \(E_0/6N_\pi\) with \(N_\pi \propto (E_0/PeV)^{1/5}\) for the evolution of the first interaction multiplicity with energy, one can calculate the elongation rate

\[ D_{10}^p = \frac{dX_{\text{max}}}{d \log E_0} = \frac{d(\lambda_I \ln 2 + \lambda_r \ln [E_0/(6N_\pi E^c_\gamma)])}{d \log E_0} \] (1.21)

or

\[ D_{10}^p = \frac{4}{5} D_{10}^\gamma - 9 \ln 2 \simeq 62 \text{ g/cm}^2. \] (1.22)

This is a robust result as it only depends on the cross section and multiplicity evolution with energy and it is in good agreement with simulation codes.

A direct consequence of the larger hadronic multiplicity, which increases the rate of conversion of the primary energy into secondary particles is the so called elongation rate theorem which states that the elongation rate for electromagnetic showers \((D_{10}^\gamma)\) is an upper limit to the elongation rate of hadronic showers.

An extension of the above description to nuclear primaries can finally be done using the superposition model. In this framework, the nuclear interaction of a nucleus with atomic number \(A\) is simply viewed as the superposition of the interactions of \(A\) nucleons of individual energy \(E_0 = A\). Showers started by heavy nuclei will thus develop faster, and with less shower to shower fluctuations than showers initiated by lighter nuclei. The faster development implies that pions in the hadronic cascade will reach their critical energy sooner and therefore augment the relative number of muons with respect to the electromagnetic component. From these simple assumptions, one can directly see that

1. Showers initiated by nuclei with atomic number \(A\) will develop higher in the atmosphere. The offset with respect to proton showers is simply

\[ X_{\text{max}}^A = X_{\text{max}}^p - \lambda_r \ln A \] (1.23)

2. Showers from heavy nuclei with atomic number \(A\) will have a larger number of muons at ground

\[ N^A_\mu = N^p_\mu A^{1-\beta} \] (1.24)

3. The evolution of the primary cross-section with energy is the same for protons and heavier nuclei. Therefore different nuclei will have identical elongation rates and will show up as parallel lines in an \(X_{\text{max}}\) versus energy plot (see Fig. 1.8).

4. The shower-to-shower fluctuations of \(X_{\text{max}}\) are smaller for heavy nuclei than for light ones.

All the above results have been confirmed by simulations and all interaction models share those basic principles. The offset in \(X_{\text{max}}\) from iron to proton showers is more than 100 \text{ g/cm}^2, and iron showers have 1.8 times the muon content of proton showers. The reproduction of these trends is of particular importance in the attempt to relate experimentally measured quantities to mass composition.
1.3.3 Air shower simulations

In order to derive the characteristics of the primary particle from an air shower, we need a theoretical description of the EAS. The simplest analytical description is the Heitler’s model (for electromagnetic showers) and the Heitler-Matthews’s model (for hadron-initiated showers), and they can be used to extract a lot of properties of the air shower. However, these models predict only the average development of EASs, giving no insight in the fluctuations in shower evolution which are often very important. Due to that, complex Monte Carlo simulation codes have to be used to make reliable predictions of the EAS properties.

In this work, the simulation of EAS is performed with CORSIKA (COsmic Ray SImulations for KAscade), a Monte-Carlo software to study the evolution of EAS in the atmosphere initiated by any particle, using FORTRAN routines. [42]. Its applications range from Cherenkov telescope experiments \( E \approx 10^{12} \text{ eV} \) up to the highest energies observed \( E > 10^{20} \text{ eV} \).

By using Monte-Carlo simulations, naturally extra uncertainties come into play. The most significant ones come from the cross-sections between the hadrons and the air molecules, the fact that the CR energies can be far above the ones available in the LHC (so we have to rely on the theoretical predictions when we extrapolate towards those energies) and the fact that in collider experiments which are used to calibrate the interaction models the very forward particles are not accessible, while those particles carry most of the hadronic energy, and they deposit a significant energy fraction into the atmosphere.

The central part of CORSIKA consists of a high-energy particle interaction model. Currently the most popular models are: EPOS LHC [60, 59], QGSJetII-04 [56], and Sibyll 2.3c [34, 63], which have been updated to take into account LHC data.
for particle collisions at 7 TeV. In our work we use the first two, QGSJetII is based on quark-gluon string model while EPOS is based on parton and hydrodynamical models. The differences in the results that these hadronic models give are usually not negligible, however the general trends are qualitatively similar in both models.

The longitudinal profile of the shower, $N(X) = \frac{dE}{dX}(X)$, is usually parametrized by the Gaisser-Hillas’ function

$$N(X) = N_{\text{max}} \left( \frac{X - W_0}{X_{\text{long}} - W_0} \right)^{\frac{X_{\text{long}} - W_0}{\lambda}} \cdot \exp \left( \frac{X_{\text{long}} - X}{\lambda} \right) \quad (1.25)$$

Here $X$ stands for the atmospheric slant depth at altitude $x$

$$X(x) = \int_{-\infty}^{x} \frac{\rho(l)}{\cos \theta} \, dl \quad (1.26)$$

where $\rho(l)$ is the density of air at altitude $l$ and $\theta$ is the local zenith angle of the shower axis. In Eq. (1.25), $X_{\text{long}}$ denotes the depth of the maximum of the longitudinal part of the shower (after the first interaction), $W_0$ and $\lambda$ are shape parameters and $N_{\text{max}}$ is the size of EAS in its maximum.

### 1.4 Composition

Composition measurements can be made directly only in the low energy region, up to $\sim 10^{13}$ eV, via space-based experiments (see, e.g., [14]), as in this region the flux is yet large enough. These direct measurements reveal that the CR abundances at those low energies are similar to those of the interstellar medium. However, composition measurements at the highest energies are crucial to separate the different scenarios of origin and propagation of CRs. For these energies, composition measurements are done through the analysis of the profile and particle content of the extensive air shower, created by the primary CR when it enters the atmosphere. These are measured by both fluorescence and ground array detectors, such as those used by the Auger experiment.

Composition studies through EAS analyses are challenging because of the intrinsic shower to shower fluctuations which characterize shower properties. These fluctuations originate from the random nature of the interaction processes, e.g. the position of the first interaction, and from the discrete sampling at ground. However, showers initiated by different primary types can, at least statistically, be differentiated, due to their different cross section with air molecules.

Presently, the best indicator of the composition of the primary particle is the atmospheric depth of the shower maximum, $X_{\text{max}}$, given in $g/cm^2$, which carries information about the primary mass and the hadronic interaction properties at very high energy. Because of the random nature of the air shower, instead of $X_{\text{max}}$ we use the average shower maximum, $\langle X_{\text{max}} \rangle$. From what we said in Sections 1.3.1, 1.3.2, $\langle X_{\text{max}} \rangle$ scales approximately as $\ln(E/A)$, where $E$ is the energy and $A$ is the atomic mass of the primary cosmic ray which generated the shower, and therefore showers
generated by heavier primaries develop faster than those generated by lighter ones. This means that on average, the shower maximum for protons happens deeper in the atmosphere than that for the same energy iron nucleus, $\langle X_{\text{max}}^p \rangle > \langle X_{\text{max}}^\text{Fe} \rangle$. In addition, proton showers fluctuate more about $\langle X_{\text{max}} \rangle$ (for a given primary energy) providing another measure of composition: the root mean square fluctuations about $\langle X_{\text{max}} \rangle$, or $\sigma_{X_{\text{max}}}$.

Another effective indicator of composition is the particle content of the shower, such as the number of muons: showers initiated by protons contain a smaller number of muons and a larger number of electrons than those started by heavier nuclei with the same energy. This phenomenon is also easily understood through the Heitler model.

In practice, shower maxima and particle numbers measured in observatories are compared with Monte Carlo air shower simulations which involve an extrapolation to higher energies of hadronic interactions known at energies of laboratory accelerators ($\lesssim$ TeV).

### 1.4.1 $X_{\text{max}}$ and $\sigma_{X_{\text{max}}}$ measurements

Observations of shower properties from the knee to just below the ankle suggest a transition from light primaries dominating at the knee towards heavier primaries dominating up to $\sim 10^{17}$ eV (e.g. [27]). This is in accordance with the expectations that the knee is the result of a rigidity dependent end of Galactic CRs which may be due to maximum acceleration at the sources and/or containment in the Galactic magnetic field, as we mentioned in Section 1.1.1. Just before the ankle, the trend seems to reverse back toward a lighter composition, being consistent with light primaries at $10^{18}$ eV, and light nuclei seem to be dominant around few EeV [7].

The energy evolution of $\langle X_{\text{max}} \rangle$ and $\sigma_{X_{\text{max}}}$ in the region $10^{17} - 10^{20}$ eV from the Pierre Auger Observatory data can be observed in Fig. 1.9. From the analysis of $X_{\text{max}}$, it follows that the mean mass of the UHECR is getting lighter up to $10^{18.3}$ eV. Above this energy, the trend reverses and the composition becomes heavier, which as we will see in the next section, is very unexpected and poses a serious problem. Additionally, there is no straightforward way to reconcile the two Auger datasets in detail, and the Auger Collaboration reports strained fits to the observed $X_{\text{max}}$ distribution in more energy bins than what expected from random fluctuations alone. No primary composition can fully reproduce the observed distributions [71].

### 1.4.2 Composition Problem

As we explained above, the composition-sensitive observables $\langle X_{\text{max}} \rangle$ and $\sigma_{X_{\text{max}}}$ seem to suggest that above $10^{18.3}$ eV a transition towards heavier primaries takes place. Moreover, additional composition-sensitive quantities obtained from the surface water-Cherenkov detectors, when interpreted using Standard Model EAS simulations, also point toward a heavy composition. This result however is highly unexpected, as it is in contradiction with some strong astrophysical arguments suggesting that the composition at those energies should be light:

- Since this energy region corresponds to the ankle area, the CR spectrum is
transitioning to a shallower slope, not a steeper one, which means that there is no corresponding spectral indication that the UHECR accelerators are reaching their maximum energy [5].

- At these energies cosmic-ray arrival directions start exhibiting dipole anisotropies [30, 31, 9]. For example, Auger Collaboration reports that at energies $> 10^{18.9}$ eV a dipole of amplitude $6.6^{+1.2}_{-0.8}$% is observed. This problem might not be so serious if the Galactic magnetic field is overall as weak as indicated, e.g., by [46]. However, recent studies of the Galactic magnetic field have shown that it is approximately an order of magnitude stronger than previously thought [72] in a small region near the reported hotspot from Telescope Array [8, 9]. Since heavy nuclei are strongly deflected by magnetic fields, if indeed the average Galactic magnetic field is proven to be just a few times stronger than the existing models, it would lead to heavy primaries spreading out over all the sky erasing any trace of anisotropy. This means that to display the observed dipole anisotropy, UHECR primaries must be light.

- All heavier nuclei except iron photodissociate fast during propagation (e.g., [15, 61, 70], so unless it starts out as pure iron, the composition quickly becomes lighter during propagation. However, as we already saw in Fig. 1.9, iron is far from a best-fit to Auger composition-sensitive observables. Instead, a better fit for the observations is a mix of intermediate-mass nuclei, requiring an astrophysically contrived composition of the accelerated particles at the source (e.g., [67, 38]). On the other hand, models that are more natural astrophysically can not reproduce the composition-sensitive observables well [74, 36].

Besides these astrophysical arguments, there are also particle-physics considerations that contribute to the composition problem. The main composition-sensitive quantity obtained from the surface water-Cherenkov detectors, that we mentioned above, is the number of muons of the shower that reach the ground. Though com-
pared against the EAS simulation predictions it also suggests heavy composition, as do the $X_{\text{max}}$ observations, the predictions of these two observables are not in agreement. The number of muons of reaching the ground is too large, suggesting a far heavier composition than the best-fit composition from $X_{\text{max}}$ (heavier even than iron) [66].

These two problems suggest that the air-shower simulations (or rather, the hadronic collision simulation models on which these are based) may not be representing the evolution of showers correctly. This does not come completely as a surprise, since Auger probes interactions above 100 TeV center of mass, while hadronic interactions are only known around a few TeV (LHC reaches 13.6 TeV or $\sim 10^{13.1}$ eV). So already the first collision of a $10^{17}$ eV cosmic ray with a stationary atmospheric proton is far above the LHC energies. Our simulations are based simply on theoretical extrapolations of hadronic models to higher energies.

This has led several authors to consider that the problem may lie in the hadronic collision models themselves. The solution that has been proposed in this context is that, instead of the composition getting heavier, there may be a new phenomenon beyond the Standard Model, yet unseen in man-made accelerators, that happens in the first collision of UHECR primaries above a threshold energy $E_{\text{th}}$ and alters the air-shower’s development. This scenario is widely recognized both by the Auger Collaboration [1, 2] and other authors [32]. Such a solution has also been proposed in [58] (hereafter PT19), and expanded upon in [64] (hereafter RPT22) and this work.
Chapter 2

Addressing the Composition Problem with New Physics

In this work, we expand upon the possible solution to the composition problem which was suggested in PT19. As in PT19, we consider the scenario that the apparent transition to heavy primaries above $10^{18.3}$ eV which the composition-sensitive observables seem to suggest is in reality a consequence of a new physical effect which appears at super-LHC energies. In this model, the effect takes place in the first collision of an UHECR with a molecule of the atmosphere, when the UHECR energy exceeds a threshold energy $E_{th}$. The phenomenology of this new effect is such that a collision of this kind produces a state whose decay increases the particle multiplicity early in air shower and thus leads to light primaries appearing heavy-like. Such an effect can be a result of several possible new physics mechanisms, which are reviewed in [53, 54]. They are based either on the possible existence of yet undiscovered particles (mini black holes, “strangelets”) or on special phases of QCD, such as the disoriented chiral condensate (DCC).

In PT19, it was assumed that for energies above $10^{18.3}$ eV:

1. The flux of cosmic rays is dominated by a single primary type.
2. Extragalactic primaries remain light.
3. The growth of $\langle X_{\text{max}} \rangle$ with energy in observational data (which is abnormal for light primaries) is exclusively a result of the new effect and reflects its phenomenology.

Under these assumptions, the phenomenological constraints on any such new effect (the interactions and decay properties of the state produced in above-threshold collisions) were determined analytically. It was demonstrated that if the multiplicity of first-collision products increased over the SM predictions at a certain rate, the Auger data on $\langle X_{\text{max}} \rangle$ can be fully reproduced (without the composition getting heavier). It was also shown qualitatively that the agreement of $\sigma_{X_{\text{max}}}$ with the Auger data can be improved by a simultaneous increase in the proton-air cross-section over the SM prediction. However, the optimal behavior of the cross-section to best match Auger observations was not calculated.
In RPT22, the analytic formulation of PT19 was studied using EAS simulations, under the assumption that Galactic cosmic rays consist of a single species of nuclei, specifically helium. It was demonstrated that the simulation results for $\langle X_{\text{max}} \rangle$ implementing the aforementioned modifications to hadronic interactions at energies above $10^{18}$ eV (while keeping the extragalactic CR composition light) generally reproduced the Auger data, however small deviations from this data at low energy were observed. One possible source of these deviations is the assumption of a single-species Galactic component, as it known that at these energies CRs consist of multiple types of nuclei, with energy-dependent ratios [6, 40].

In this work, we address the problem by considering a mixed composition of Galactic cosmic rays in agreement with observational data in [40, 6]. We extend the analytic formulation of PT19 as in RPT22 using air-shower simulations with the CORSIKA\textsuperscript{1} software. As we mentioned in Section 1.3.3, we use the EPOS LHC and QGSJETII-04 high-energy particle interaction models, as well as FLUKA\textsuperscript{2} – a Monte-Carlo package used at CERN – for low-energy interactions. The main goal of this work is to

1. Model a mixed Galactic CR composition that is in agreement with the observational data, at the energies where the transition to extragalactic CRs occurs.

2. Test whether a mixed Galactic CR composition along with an increase in the multiplicity of first-collision products can produce better results for the $X_{\text{max}}$ distribution in comparison to RPT22.

3. Calculate the optimal change in cross-section that best matches Auger $X_{\text{max}}$ data.

2.1 Mathematical Formulation

Our implementation of new physics above $E_{\text{th}}$ does not rely on a particular new physics mechanism, and is purely phenomenological. As in PT19, we assume that the new physics phenomenon impacts the phenomenology of the first collision of the above-threshold-energy CR with the atmosphere in two ways:

1. it modifies the cross-section between the CR and air molecules; and

2. it increases the particle multiplicity of the collision’s products

Since the products of this first collision have, on average, energies below $E_{\text{th}}$, they are not affected by the new physics effect.

In Section 2.1.1, we give a quantitative description of these phenomenological changes and in Section 2.1.2 we show how they affect the shower maximum. Then, in Section 2.1.3, we explain how these changes are implemented in our EAS simulations. Next, in Section 2.1.4 we describe our model for the transition from Galactic to extragalactic CRs.

\textsuperscript{1}CORSIKA version 7.7402

\textsuperscript{2}FLUKA version 2020.0.3
2.1.1 Parametrization of changes in cross section and multiplicity

Both cross section and particle multiplicity depend on the energy of the incoming particle. Up to the moment we have not yet specified the value of the threshold energy, \( E_{th} \) where the new physics emerge. This energy must be beyond the reach of LHC, so \( E_{th} \gtrsim 10^{17} \text{ eV} \) which corresponds to a CM collision energy \( E_{CM,th} \gtrsim 14 \text{ TeV} \). Moreover, as discussed in PT19, the requirement that \( \langle X_{\text{max}} \rangle_{EG} \) (the \( \langle X_{\text{max}} \rangle \) of extragalactic CRs) must not at any energy exceed the SM prediction for protons is automatically met by this criterion. On the other hand, \( E_{th} \) must be such that the new physics effect emerges before the break that suggests transition to heavy primaries in Auger data, thus \( E_{th} \lesssim 10^{18.3} \text{ eV} \). We use \( E_{th} = 10^{18} \text{ eV} \), which corresponds to a CM collision energy \( E_{CM,th} \sim 50 \text{ TeV} \) for a collision of an incoming CR proton with a stationary proton of the atmosphere, although good fits to the Auger data can be produced using other threshold energies in this range as well.

Cross-section

At high energies, the proton-Air cross section increases logarithmically \[57\] and thus we can parametrize it, for high energies below \( E_{th} \) as

\[
\sigma_{p-Air} = \sigma_0 + \beta \log \varepsilon \quad (2.1)
\]

where \( \sigma_0, \beta \) are constants and \( \varepsilon = E/E_{th} \). This parametrization therefore holds for \( \varepsilon \leq 1 \). The values of \( \sigma_0 \) and \( \beta \) can be calculated from the SM-extrapolation hadronic interaction models. Our results from QGSJetII-04 and EPOS-LHC are displayed in the first two lines of Table 2.1.

Above the energy threshold, so for \( \varepsilon > 1 \), the new effect is likely to alter the proton-Air cross-section behavior. Assuming that the effect will only affect the coefficient \( \beta \) of the energy-dependent term in our parametrization, and that the cross-section function is continuous at \( \varepsilon = 1 \),

\[
\sigma_{p-Air,\text{new}} = \sigma_0 + \beta' \log \varepsilon \quad (2.2)
\]

If we define the fractional change of new physics \( \beta' \) in terms of Standard Model \( \beta \):

\[
\delta = \frac{\beta'}{\beta} - 1 \quad (2.3)
\]

we get the parametrization

\[
\sigma_{p-Air,\text{new}} = \sigma_{p-Air} + \delta \cdot \beta \log \varepsilon \quad (2.4)
\]

Therefore, for \( \varepsilon > 1 \), the cross section will exhibit a logarithmic deviation from the SM value as the energy grows, with the deviation determined by the coefficient \( \delta \beta \). No deviation from the SM value means that \( \delta = 0 \), however values of \( \delta \) up to 3.5 can also be within the uncertainties in the SM predictions for the above-LHC energies, and thus may be consistent with SM as well.
<table>
<thead>
<tr>
<th></th>
<th>EPOS LHC</th>
<th>QGSJETII-04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$ (mb)</td>
<td>527.61</td>
<td>499.35</td>
</tr>
<tr>
<td>$\beta$ (mb)</td>
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<tr>
<td>$\alpha$ (gr/cm$^2$)</td>
<td>63.74</td>
<td>60.37</td>
</tr>
</tbody>
</table>

Table 2.1: Parameters for cross section and shower maximum. These parameters were calculated by performing linear fit in the SM data from EAS simulation with proton as a projectile.

**Multiplicity**

We parametrize the increase in the multiplicity of the first collision products, i.e. the increase in the number of the secondary particles produced by it for $\varepsilon > 1$ as

$$n(\varepsilon) = N(\varepsilon)/N_{SM}(\varepsilon), \quad n(\varepsilon) \geq 1$$

(2.5)

where $N(\varepsilon)$ is the actual number of secondary particles produced under new physics, while $N_{SM}(\varepsilon)$ is the number of secondaries predicted by SM (through EAS simulations) at this energy.

### 2.1.2 Shower maximum

As we discussed in Section 1.3.1, $X_{\text{max}}$ can be divided into two terms:

$$X_{\text{max}} = X_{\text{int}} + X_{\text{long}}$$

(2.6)

where $X_{\text{int}}$ is the depth of the first interaction and $X_{\text{long}}$ the additional column density required for the shower to reach its maximum, corresponding to its "longitudinal" development. So for the average slant depth we have,

$$\langle X_{\text{max}} \rangle = \langle X_{\text{int}} \rangle + \langle X_{\text{long}} \rangle$$

(2.7)

from which we get its variance,

$$\text{Var}(X_{\text{max}}) = \text{Var}(X_{\text{int}}) + \text{Var}(X_{\text{long}})$$

(2.8)

**First interaction**

The depth of the first interaction $X_{\text{int}}$ is affected by the new phenomenon through the change of the cross-section, $\sigma_{p-\text{Air}}$. The probability that a CR primary has not collided with the atmosphere until a height $x$ is given by

$$\exp \left( -\frac{\sigma_{\text{CR-Air}}}{m} \int_{l=\infty}^{x} \rho(l) dl \right)$$

(2.9)

where $m$ is the average mass of the air particles and $\rho(l)$ is the density of the atmosphere at height $l$. Thus the average column density traversed by the primary until the first interaction is given by

$$\langle X_{\text{int}} \rangle = \frac{m}{\sigma_{\text{CR-Air}}(\varepsilon)}$$

(2.10)
and so the new-physics-induced increase in $\sigma_{p-Air}$ will expectantly reduce it. Since the primary traveling through the atmosphere until its first interaction is a process described by Poisson statistics, the variance of $X_{\text{int}}$ is given by

$$\text{Var}(X_{\text{int}}) = \langle X_{\text{int}} \rangle^2 = \frac{m^2}{\sigma_{\text{CR-Air}}^2(\varepsilon)}$$  \hspace{1cm} (2.11)

For $\varepsilon > 1$ the cross section deviates from the SM predicted value for each energy as described in Eq. (2.4), and so the $X_{\text{int}}$ moments change to,

$$\langle X_{\text{int, new}} \rangle = \frac{m}{\sigma_{p-Air}(\varepsilon) + \delta \beta \log \varepsilon}$$  \hspace{1cm} (2.12)

and

$$\text{Var}(X_{\text{int, new}}) = \frac{m^2}{[\sigma_{p-Air}(\varepsilon) + \delta \beta \log \varepsilon]^2}$$  \hspace{1cm} (2.13)

**Longitudinal development**

As we described in detail in Sections 1.3.1 and 1.3.2, after the first interaction, the shower develops in the atmosphere and the number of its particles increases, until the energy of secondary particles is reduced to a level where ionization losses dominate, as there is not enough energy for new particle production. $X_{\text{long}}$ is the depth in the atmosphere (expressed as column density), starting from the first collision height, where the energy loss rate from the shower components into the atmosphere reaches its maximum.

When we described the simplified Heitler’s model of air-showers, we showed that it bears the result that the depth of the longitudinal cascade maximum increases logarithmically with the energy (Eq. (1.17)). As we mentioned there, despite the simplifications of the Heitler’s model, this result is accurate for real air-showers, although for an accurate estimation of the parameters, we have to implement numerical simulations. So we write $\langle X_{\text{long}} \rangle$ as

$$\langle X_{\text{long}} \rangle = X_0 + \alpha \log \varepsilon$$  \hspace{1cm} (2.14)

where $X_0$ and $\alpha$ are the constant parameters that need to be determined. We calculated the best-fit parameters, which are presented in the last two lines of Table 2.1, using CORSIKA EAS simulations, implementing again the two hadronic interaction models QGSJetII-04 and EPOS LHC. The two models produce slightly different values for the parameters, however their results are qualitatively similar.

Since we consider that (on average) all the products of the first collision have energy below $E_{th}$, the change in the longitudinal evolution of the shower is affected by the new physics only via the change in the multiplicity. To quantify the change in the $X_{\text{long}}$ distribution, we empirically model the shower as $n(\varepsilon)$ "component showers" of energy, on average, $\varepsilon/n(\varepsilon)$, developing independently. This way, for $\varepsilon > 1$, $\langle X_{\text{long}} \rangle$ changes to

$$\langle X_{\text{long, new}} \rangle = X_0 + \alpha \log \varepsilon \frac{\varepsilon}{n(\varepsilon)}$$  \hspace{1cm} (2.15)

To determine the variance of $X_{\text{long}}$, we consider that a reasonable estimation of the overall $X_{\text{long}}$ is the average of the individual "component shower" maxima,
This way, $X_{\text{long}}$ will be the "sample mean" of $n$ "draws" from the underlying distribution of $X_{\text{long},i}$ and the distribution of these "sample means" has a variance that is given by the "error in the mean" formula,

$$\text{Var}(X_{\text{long,new}}) = \frac{\text{Var}(X_{\text{long},i})}{n(\varepsilon)}$$

(2.16)

where $\text{Var}(X_{\text{long},i})$ is the variance of $X_{\text{long},i}$. Since each "component shower" starts from a particle with energy below $E_{\text{th}}$, the variance of each $X_{\text{long},i}$ is unaffected by the new physics and follows the SM predictions.

As we see, the new physics effect:

- Lowers the depth of the first interaction $\langle X_{\text{int}} \rangle$ and its variance $\text{Var}(X_{\text{int}})$ via the increase of the CR-Air cross section $\sigma_{\text{CR-Air}}$; and,

- Lowers the shower longitudinal-development maximum $\langle X_{\text{long}} \rangle$ as well as its variance $\text{Var}(X_{\text{long}})$ via the increase of the multiplicity of the first-interaction products.

resulting in a decrease of the total $\langle X_{\text{max}} \rangle$ and its variance $\text{Var}(X_{\text{max}})$. As we discussed in Section 1.4 this is exactly the behavior that heavier particles exhibit, and thus, at least qualitatively, we see that our new physics scenario can mimic the observed transition to heavy primaries.

**Constraining $n(\varepsilon)$ and $\delta$**

There are two free parameters in our phenomenological model: $n(\varepsilon)$ and $\delta$. As we assume that the change of slope in the Auger data for $\langle X_{\text{max}} \rangle$ which suggests transition to heavy primaries is entirely a result of new physics, we take all primaries at those energies to be protons\textsuperscript{3}. This allows us to determine $n(\varepsilon)$ as a function of $\delta$ and thus $\delta$ becomes the only free parameter. However, as we will see in Section 2.2 $\delta$ can also be optimized, using the $\sigma_{X_{\text{max}}}$ distribution.

For energies $E \geq 10^{18.3}$ eV, the Auger Collaboration reports that $\langle X_{\text{max}} \rangle$ behaves as

$$\langle X_{\text{max, Auger}} \rangle = X_{0, \text{Auger}} + \alpha_{\text{Auger}} \log \varepsilon$$

(2.17)

with $X_{0, \text{Auger}} = 749.74$ gr/cm$^2$ and $\alpha_{\text{Auger}} = 23.98$ gr/cm$^2$. This result is obviously different to the one produced by proton-primary showers, as simulations based on SM extrapolation models demonstrate. To derive the dependence of $n(\varepsilon)$ on $\delta$ so that our new physics effect makes the proton-primary result for $\langle X_{\text{max}} \rangle$ consistent with $\langle X_{\text{max, Auger}} \rangle$, we shall simply equate the behavior of Auger data to the new physics result, which we get from Eqs. (2.7), (2.12), (2.15). So,

$$\langle X_{\text{max, Auger}} \rangle(\varepsilon) = \langle X_{\text{max, new}} \rangle(\varepsilon)$$

(2.18)

\textsuperscript{3}Although, as we will see in Section 2.1.4, there are heavier (Galactic) CRs with energies above $10^{18}$ eV, their per-nucleon kinetic energy never exceeds this threshold.
or

\[ X_{0, \text{Auger}} + \alpha_{\text{Auger}} \log \varepsilon = \frac{m}{\sigma_{p-\text{Air}}(\varepsilon) + \delta \beta \log \varepsilon} + X_0 + \alpha \log \varepsilon \frac{\varepsilon}{n(\varepsilon)} \]  \hspace{1cm} (2.19)

Solving for \( n(\varepsilon) \) we get

\[ \log n(\varepsilon) = \frac{X_0 - X_{0, \text{Auger}}}{\alpha} + \frac{\alpha - \alpha_{\text{Auger}}}{\alpha} \log \varepsilon + \frac{1}{\alpha} \frac{m}{\sigma_{p-\text{Air}}(\varepsilon) + \delta \beta \log \varepsilon} \]  \hspace{1cm} (2.20)

where \( \sigma_{p-\text{Air}}(\varepsilon) \) given by Eq. (2.1).

The variance of \( X_{\text{max, new}} \) is then obtained from Eqs. (2.8), (2.13), (2.16)

\[ \text{Var}(X_{\text{max, new}}) = \sigma^2_{X_{\text{max}}} = \frac{m^2}{[\sigma_{p-\text{Air}}(\varepsilon) + \delta \beta \log \varepsilon]^2} + \frac{\text{Var}(X_{\text{long}})}{n(\varepsilon)} \]  \hspace{1cm} (2.21)

where again the only free parameter is \( \delta \), as \( n(\varepsilon) \) is given by Eq. (2.20).

### 2.1.3 CORSIKA simulations

We now come to the EAS simulations part of this work. Our goals here are

1. to study if the implementation of the new physics effects that we postulated reproduce the behavior of our analytic results; and,

2. to find the optimal phenomenological values of the two parameters in our formulation – cross section (quantified by \( \delta \)) and multiplicity (quantified by \( n(\varepsilon) \)) – so that the new proton-Air interactions reproduce the Auger data in simulations if the CR composition at high energies remains light.

To do that, we ran simulations of air-showers initiated by primaries with energies in the range \( 10^{17} - 10^{20} \) eV, with step 0.1 in \( \log E \). At each energy bin we simulated 1000 EASs. For proton primaries with energy \( E < E_{\text{th}} = 10^{18} \) eV we ran SM EAS simulations, as their first-collision energy is below the new physics and thus can be treated simply with SM extrapolations. We also performed the same simulations separately for each of the heavier Galactic primary species, as their per-nucleon kinetic energy is always below \( 10^{18} \) eV. These heavier primaries are the 10 elements mentioned in Section 2.1.4. On the other hand, for all proton primaries with energy \( E > 10^{18} \) eV we simulated EAS treating the first-collision with new physics as per our phenomenological implementation.

The SM results were produced from CORSIKA EAS simulations using the EPOS-LHC or QGSJETII-04 high-energy particle interaction models, together with FLUKA for low-energy interactions. In addition, we used the CONEX, an efficient hybrid scheme for one-dimensional extensive air shower simulation [25] which significantly reduces the simulation time. Three output files are created for each simulation.

In the first file the energy deposited as a function of atmospheric depth is recorded. As it is usually done, we fitted a Geisser-Hillas function

\[ \frac{dE}{dX}(X) = N_{\text{max}} \left( \frac{X - W_0}{X_{\text{long}} - W_0} \right)^{X_{\text{long}}-W_0} \lambda X_{\text{long}}-X \]  \hspace{1cm} (2.22)
(where, as we discussed in Section 1.3, \( N_{\text{max}} \) is the maximum number of particles observed at depth \( X_{\text{long}} \), and \( W_0 \) and \( \lambda \) are primary mass and energy dependent parameters) to the simulated data to determine \( X_{\text{long}} \) for each shower. We next determined the average value of \( X_{\text{long}} \) and its variance for each energy bin.

The second output file contains data on the primary’s cross section with the atmosphere, from which we were able to determine \( \langle X_{\text{int}} \rangle \). Finally, information on the secondary particles created following the first interaction (which are the results from the SM extrapolated models) is recorded in the third output file (stack file).

In order to implement the new physics effects in our simulations, we first determined the multiplicity \( n(\varepsilon) \) at each energy bin, for every different value of \( \delta \), using Eq. (2.20) and calculated the new proton-Air cross-section from Eq. (2.2). As we view our shower as splitting into \( n(\varepsilon) \) "component showers" after the first collision, we represent this behavior in the simulations by combining stack files from the same energy bin (generated by SM simulations) in accordance with the multiplicity calculated. Specifically, we rounded \( n(\varepsilon) \) to the nearest integer and combined as many stack files, taking into consideration the conservation of energy and momentum. This was done by dividing each particle’s energy and momentum by the number of stacked files. The stack files were then used as an input to CORSIKA, which simulated the air-showers and produced the data files for the energy deposition as a function of depth in the atmosphere.

2.1.4 Modelling the Galactic and extragalactic components

At energies above \( 10^{18.3} \) eV, both Auger and TA report a dipole distribution of cosmic rays uncorrelated with the galactic plane [4, 10]. This suggests that above this energy, CR are of extra-galactic origin. Therefore, in our energy area of interest \( 10^{17} - 10^{20} \) eV there is both a Galactic and an extragalactic component in the CR flux, and to get our results, we first need to model both of them.

Galactic CR primaries consist of several components. These primaries are particles accelerated at astrophysical sources and so nuclei that are not abundant products of stellar nucleosynthesis, such as lithium, beryllium, and boron are not part of their composition. Nuclei with odd atomic numbers (\( Z = 5, 7, 9, 11, 13, 15, 17 \) and 19) are also usually disregarded in experiments, because they are secondary nuclei of the cosmic radiation copiously produced by spallation reactions in the interstellar medium [29].

In the low-energy region \( 10^{12} - 10^{15} \) eV, primary cosmic rays can be measured directly by experiments in space or on balloons (as at these energies there is sufficient flux), and these experiments have provided data for the fluxes of different nuclei. Figure 2.1 shows this experimental data for the major nuclear components, which as we see are: H, He, C, O, Ne, Mg, Si, S, Ar, Ca and Fe. All other primary nuclei are extremely rare. As we can see in the Figure, all elements follow power laws with a common spectral index of \( 2.67 \pm 0.05 \), as was expected due to Fermi acceleration (see Section 1.2.1).

However, in our energy range of interest \( 10^{17} - 10^{20} \), direct measurements are not available, and the composition studies are done indirectly, through simulation
analysis. Between $10^{15}$ eV and this energy region, the power law behavior of the CR spectrum goes through changes: at the first and second knee its spectral index is increased, and although around the ankle region it returns back to $\sim 2.7$, the relative abundances of its different components are not the same as the ones at $10^{12} - 10^{15}$ eV.

We assume that above $10^{17}$ eV each component of the Galactic CR flux follows a power law of slope $-\gamma_G$ (again, due to Fermi acceleration), with an exponential cutoff at a characteristic energy $E_{G,i}$, where $i$ refers to the nucleus type. The cutoff is created by Galactic accelerators reaching their maximum energy, and since from the Hillas criterion this maximum energy is $E_{max} \propto Z$ (see Section 1.2.2), the cutoff is charge dependent: $E_{G,i} = Z_i \cdot E_{G,H}$ ($H$ refers to hydrogen).

Thus, we can write the differential flux for each nucleus type $i$ with atomic
Table 2.2: Parameters $J_{G,0}$ for each component, expressed in (eV km$^2$ yr sr)$^{-1}$ units.

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>He</th>
<th>C</th>
<th>O</th>
<th>Ne</th>
<th>Mg</th>
<th>Si</th>
<th>S</th>
<th>Ar</th>
<th>Ca</th>
<th>Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_{G,0} \times 10^{12}$</td>
<td>5.00</td>
<td>0.79</td>
<td>0.08</td>
<td>0.08</td>
<td>0.016</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.016</td>
<td>0.0002</td>
<td>0.16</td>
</tr>
</tbody>
</table>

number $Z$ as:

$$J_G^i(\varepsilon) = J_{G,0}^i \left(\frac{\varepsilon}{\varepsilon_{17.5}}\right)^{-\gamma_G} \exp \left(-\frac{\varepsilon}{Z_i \cdot \varepsilon_{G,H}}\right)$$  \hspace{1cm} (2.23)

where $\varepsilon_{17.5} = 10^{17.5} \text{eV}/E_{th} = 10^{-0.5}$. The common spectral index for each nucleus type is taken to be $\gamma_G = 2.76$, according to [39]. For the other parameters $J_{G,0}^i$ and $\varepsilon_{G,H}$ we have additional constraints. At $E = 10^{17.0}$ eV we assume that the flux is completely dominated by Galactic CRs, and thus we tuned the parameters so that at this energy the relative abundances of the different nuclei are in agreement with the predictions of Telescope Array [6]. Moreover, we assume that at energies lower than those where losses (either $e^+e^-$ or photopion production) become important, the extragalactic CR spectrum $J_{EG}(\varepsilon)$ is a single powerlaw, which also constrains the cutoff energy $\varepsilon_{G,H}$. For the extragalactic CRs, we assume that they consist solely of protons.

Under these constraints, we get the values for $J_{G,0}^i$ that are shown in Table 2.2. The cutoff energy for Fe was set to $\varepsilon_{G,Fe} = 10^{17.9}/E_{th}$, which means that for $H$ it is $\varepsilon_{G,H} = 10^{16.5}/E_{th}$ and for the other elements $\varepsilon_{G,i} = (Z_i/26) \cdot (10^{17.9}/E_{th})$. From these results, we get the extragalactic spectrum $J_{EG}(\varepsilon) = J_{\text{total, Auger}} - J_G(\varepsilon)$ to be consistent with $J_{EG}(\varepsilon) \propto \varepsilon^{-3.0}$ between $10^{17.5}$ and $10^{18.2}$ eV, which is consistent with our assumption that the extragalactic flux is of light composition and remains that way until the highest energies. This result is also in general agreement with [16].

In Figure 2.2 we present the ratios of the flux of each Galactic component to the total CR flux. We can see that while the proton flux falls very quickly, there is still a significant contribution to the total flux from Galactic components above $E_{th}$. However these are heavy particles and thus their energy per nucleon will be below the $E_{th}$, and so we do not need to apply any new physics corrections to their EAS simulations. In Figure 2.3 we show the CR spectrum with the Galactic and extragalactic components, as we modelled them, along with the total flux observed by Auger (and its errors $^4$).

Under the assumptions we chose for the modelling of the Galactic and extragalactic components, we see that extragalactic CRs dominate already at $10^{18}$ eV. Thus the composition at $10^{18.5}$ is light, and the ankle must be an $e^+e^-$ "dip". In this case, the probability density function of $X_{max}$ will be

$$p(X_{max}) = \sum_i f_i p_G(X_{max}) + (1 - f)p_{EG}(X_{max})$$  \hspace{1cm} (2.24)

where $f(\varepsilon)$ is the fraction of the Galactic component over the total CR flux, and is analyzed into $f_i$ which correspond to each species of the Galactic CRs. Obviously $f = \sum_i f_i$. On the other hand, the extragalactic components has a single component,
Figure 2.2: Ratios of the flux of each Galactic species to the total CR flux. At $10^{17}$ eV H, He, Fe, and the combined intermediate nuclei have roughly the same abundances, in agreement with [6]. The lighter nuclei fall faster than the heavier due to the charge-dependent cutoff.

and its fraction to the total CR flux is $1 - f(\varepsilon)$. The subscripts $G$ and $EG$ refer to the Galactic and extragalactic populations respectively. From this we get the average shower maximum:

$$\langle X_{\text{max}} \rangle = \sum f_i \langle X_{\text{max}}^i \rangle_G + (1 - f) \langle X_{\text{max}} \rangle_{EG}$$ (2.25)

and its variance

$$\text{Var}(X_{\text{max}}) = \sum f_i \text{Var}(X_{\text{max},G}^i) + (1 - f) \text{Var}(X_{\text{max},EG}) +$$

$$+ f(1 - f) \langle X_{\text{max}} \rangle_{EG}^2 + \sum f_i \langle X_{\text{max}}^i \rangle_G^2 - \left( \sum f_i \langle X_{\text{max}}^i \rangle_G \right)^2 -$$

$$- 2(1 - f) \langle X_{\text{max}} \rangle_{EG} \sum f_i \langle X_{\text{max}}^i \rangle_G$$ (2.26)

2.2 Results

We performed CORSIKA simulations as described in Section 2.1.3 for $\delta = 0, 2.9, 3.5, 4, 6, 8$ and $10$, treating the Galactic-to-extragalactic transition as described in Section 2.1.4. For all values of $\delta$, the simulated results for $\langle X_{\text{max}} \rangle$ are in perfect agreement with the Auger data (Fig. 2.3), even better than in RPT22, as our results show no deviation from the Auger data at lower energies.
Figure 2.3: CR energy spectrum flattened by $E^3$. Blue: Auger Collaboration data for the total CR flux at those energies. Green: Galactic CR flux model, assuming mixed composition power laws at low energies with charge-dependent exponential cutoffs, e.g. for Fe at $10^{17.9}$ eV. Red: extragalactic CR flux, obtained as the difference between observed data and Galactic CR.

Our simulation results for $\sigma_{\text{max}}$ exhibit a higher dependence on the value of $\delta$. In order to find the value of $\delta$ that produces the optimal fit of our new-physics $\sigma_{\text{max}}$ results to the Auger data, we quantified their agreement using the reduced $\chi^2$ statistic. In Fig. 2.4 we plot the reduced $\chi^2$ as a function of $\delta$ for our simulation results. We fit second order polynomials (dashed lines) to the datapoints and from them, we calculate the position of the minimum $\chi^2$. The minimum for EPOS-LHC is $\chi^2 = 1.9$ at $\delta = 3.2$, while the minimum for QGSJET-04 is $\chi^2 = 4.6$ at $\delta = 7.1$. Therefore, in our plots we show the results for the $\delta$ values that are closest to those minima: $\delta = 3.5$ and $\delta = 8$ (we did not plot results for all simulated values of $\delta$ in order to keep the figures legible). At their minimum $\chi^2$ the cross-section at an energy of $10^{19}$ eV is 737.4 mb (803.0 mb) for EPOS-LHC (QGSJET-04), contrary to the SM extrapolated result 577.56 mb (536.84 mb). At the same energy, the multiplicity has increased by a factor of 2.845 (1.339).

We see from Fig. 2.5 that EPOS-LHC produces the better result. The result of QGSJET-04 is similar to the ones in RPT22 and its deviations from the Auger data are covered by the uncertainties, however the result of EPOS-LHC is in excellent agreement with the observations.

Overall, we see that the results we get with a mixed composition Galactic component are better that the ones in RPT22, therefore it is a fair assessment that the discrepancies with the observational data present in RPT22 are due to the assumption of a Galactic component consisting purely of helium used there.
2.3 Summary and conclusions

In this work, we used a new physics phenomenological model to address the composition problem of UHECRs. We first described the effects of our new physics on the first interaction which essentially were (a) an increase in the cross-section and (b) an increase in the multiplicity of the first collision products.

To quantify the increase in cross-section due to the new effect, we introduced a parameter $\delta$, defined as the fractional change of the energy dependent term coefficient of the proton-Air cross-section in the new physics scenario with respect to its SM value. The change in the multiplicity was parametrized with a function $n(\varepsilon)$, defined as the ratio of the number of the first interaction products under new physics over the one predicted by SM. We described the way these changes in cross-section and multiplicity affect the development of the air showers, and specifically our main observable: the atmospheric slant depth $X_{\text{max}}$, at which the shower reaches its maximum development, and its first two moments. By assuming that the change in $\langle X_{\text{max}} \rangle$ slope at high energies observed by Auger (which indicates a transition to heavy primaries) is entirely a result of new physics and that all extragalactic primaries at those energies are protons, we expressed $n(\varepsilon)$ as a function of $\delta$, thus leaving $\delta$ as the only free parameter in our model.

We then constructed a model for the transition from Galactic to extragalactic primaries in the ankle region. We considered a mixed composition Galactic component, consisting of H, He, C, O, Ne, Mg, Si, S, Ar, Ca and Fe nuclei and assumed a single composition extragalactic component, consisting entirely of protons. We assumed that the fluxes of the Galactic component species around $10^{17}$ eV follow power laws with a common spectral index, but with a charge-dependent exponential cutoff. Constraining the different parameters with observational data, we obtained the flux of every CR component in the energy range $10^{17} - 10^{20}$ eV. We then calculated the contribution of each component to the total average shower maximum and its variance.

We simulated EAS initiated by each component of the total CR flux with CORSIKA, applying the modifications dictated by our new physics effect for the interactions of energy $E > E_{\text{th}} = 10^{18}$ eV and obtained the results for $\langle X_{\text{max}} \rangle$ and $\sigma_{X_{\text{max}}}$. The results demonstrate that these adjustments to hadronic interactions at energies above $10^{18}$ eV, along with our Galactic-to-extragalactic transition model reproduce the observations of $X_{\text{max}}$. The Auger data for $\langle X_{\text{max}} \rangle$ is reproduced for any value of $\delta$ (by construction), while $\sigma_{X_{\text{max}}}$ is best reproduced for $\delta$ between 3 and 8. When the SM modeling of high-energy hadronic interactions is done using the EPOS-LHC model, the optimal value for $\delta$ is 3.2, while when QGSJETII-04 is implemented, the optimal value for $\delta$ is 7.1. Overall, for the values of $\delta$ we used, the best fit to the Auger data for $\sigma_{X_{\text{max}}}$ was provided by EPOS-LHC with $\delta = 3.5$. This $\delta$ is translated to an increase cross-section given by Eq.(2.4) and to an increase in multiplicity given by Eq.(2.20).

These results provide phenomenological constraints on the properties (cross-section and multiplicity alteration) of any new effect beyond the SM that may describe interactions at CM energies above $\sim 50$ TeV, if the change of behavior of the $X_{\text{max}}$ distribution observed by Auger above $10^{18.3}$ eV is accredited entirely to it.
Figure 2.3: $\langle X_{\text{max}} \rangle$ as a function of energy. SM extrapolations (grey) for proton primaries through EPOS LHC (left) and QGSJET-II-04 (right) do not agree with observations from Auger Observatory (blue) above $E_{\text{th}} = 10^{18.5}$ GeV. When we alter the way the cross section and the first-interaction product multiplicity scale with energy as described in our new physics model, EAS simulations with protons as a primaries (green and red) are in total agreement with the observed data even at the highest energies. At lower energies, our mixed Galactic composition model also results in excellent agreement with the Auger data.
Figure 2.4: Agreement between new-physics EAS simulations and Auger data for $\sigma_{X_{\text{max}}}$ and $E > 10^{18.5}$ eV, quantified through the reduced $\chi^2$ statistic, as a function of the value of the $\delta$ parameter. In order to find the position of the minimum $\chi^2$, we perform a parabolic fit to the datapoints. The two last datapoints of EPOS-LHC are ignored for the fit. The locations of the minima are at $\delta = 3.2$ for EPOS-LHC and at $\delta = 7.2$ for QGSJETII-04. EPOS-LHC produces results closer to Auger data.
Figure 2.5: $\sigma_{X_{\text{max}}}$ as a function of energy. SM extrapolations (grey) for proton primaries do not agree with observations from Auger Observatory (blue). After implementing our new physics model, EAS simulations with protons as a primaries (green and red) are in total agreement with the observed data at the highest energies. At lower energies, our mixed Galactic composition model results in excellent agreement with the Auger data for EPOS-LHC, while for QGSJETH-04 the discrepancies are covered by the uncertainties.
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