The Magnetic Monopole in Theoretical Physics

A thesis presented

by

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to

The Department of Physics

in partial fullfillment of the requirements

for the degree of

Master of Science

in the subject of

Physics

University of Crete
Heraklion, Hellas

September 2011

Contents

1 Prologue

The purpose of this thesis is to give a brief review of magnetic monopoles, concentrating on gauge theories and cosmology. In Section 2 we make an introduction to the subject, while in Section 3 we present the electric-magnetic duality and the symmetry of Maxwell's equations when monopoles are included in the theory. Section 4 deals with gauge theories and especially Dirac and 't Hooft-Polyakov monopole. Sections 5 and 6 examine the cosmological point of view on the subject; Section 5 provides an introduction to General Relativity and the basic cosmological model (Friedmann models), while in section 6 we discuss some problems in the standard cosmological model, including the monopole problem, and we present the inflationary solution to these puzzles. Finally, in Section 7, we present some searches for magnetic monopoles and their influence in condensed matter physics.

I would like to thank my advisor, Prof. Nikolaos Tsamis, for suggesting the problem and his guidance, Prof. Theodore Tomaras, and Prof. Nikolaos Papanicolaou.

2 Introduction

One of the great achievements of the last few decades has been the reduction of all forces observed in nature to very few, between the basic components of matter. We currently recognize four fundamental forces or interactions between elementary particles. These forces are: gravitational, electromagnetic, strong and weak. The gravitational and electric forces are long-rage forces, their influence extending from one side of the universe to the other. The strong and weak forces, on the other hand, are short-range forces, of order 10^{-15} m and 10^{-18} m respectively.

Fundamental forces or interactions are associated with sources. These entities can be considered as both the origin of the forces or the subjects on which the forces act. Different types of force have different kinds of sources associated with different particle properties: *mass* is the source of gravitational force; *electric charge* is the source of the electric, or rather electromagnetic, force; *the color charge*, which is an attribute of the particles called quarks, is the source of the strong force, and the weak interaction has its source in the *weak charge*. Not all particles have these four attributes and this is why particles are affected differently by the four interactions.

Sources may also be *point-like* or *extended*. In the first case they are just points, without structure, while in the second case they have some sort of internal structure. Our present view of nature considers point-like sources as fundamental. Extended sources are thought of as secondary, most probably composite systems of point-like sources.

Progress in the understanding of the fundamental components of matter and their interactions has helped develop a theory, or model, of the evolution of the universe that assumes that the universe had a beginning (although it does not say anything about what might have existed before). Since then the universe has been in a continuous state of evolution toward more complex structures, although its final fate is difficult to predict. According to an idea first proposed in 1948 by Ralph Alpher, George Gamow (1904-1968) and Robert Herman, the universe began in space and time about 15×10^9 years ago in what is loosely designated as the *Big Bang*. This term was coined by the astronomer Fred Hoyle to describe graphically the magnitude and speed of the early events. There are three basic reasons in support of the Big Bang theory. One is the isotropic expansion of the universe, another is the presently cold isotropic background radiation and the third reason is the relative cosmic abundance of hydrogen, deuterium, helium and lithium.

As the average energy of the particles decreased, several phenomena occurred, called *phase transitions* or *symmetry breakings*, when certain energies were reached. These transitions resulted in important changes in the composition and structure of the universe. The transitions continued until 10^6 years after the Big Bang, when the universe finally reached a structure and composition not very different from its current form.

Any theory about the very early stages of the universe is not subject to direct experimental verification with present techniques because of the high energies involved. However, some assumptions can be verified on a much smaller scale in the laboratory by using high-energy accelerators.

It is hard to guess what the universe looked like or how it was composed at the Big Bang and immediately after, except that the average particle energy must have been extremely large, of the order of 10^{20} GeV or higher. Under those conditions all interactions were probably indistinguishable and all particles looked alike. It is assumed on theoretical grounds that, shortly after or about 10^{-42} s, the universe went through a rapid inflationary process during which its size probably increased by a factor as large as 10^{30} and conditions were established for the further "normal" expansion that allowed it to reach its present state. One result of *inflation* was that the energy of the particles dropped considerably. Another result was that gravitation became completely separated from the other interactions. However, because of its weakness, gravitation did not play a role in the universe until a much later time, about 10^6 yr.

Up to about 10^{-32} s, or for particle energies above 10^{15} GeV, all particles appeared as massless and there was no difference between quarks and leptons, as well as among the strong and electroweak interactions. During this era we may visualize the universe as a mixture of fermions and bosons, subject to two fundamental interactions, gravitation and strong-electroweak, described by what is called *Grand Unification Theory* (GUT). The GUT requires the intervention of super massive colored bosons, designated X, with rest energy of about 10^{15} GeV, to carry the interaction responsible for transitions between fermions (quarks and leptons).

From 10^6 yr up to present (a period of about 1.5×10^{10} yr), the large structures (clusters, galaxies, stars etc.) appeared under the action of gravitation. Gravity became, by default, the dominant long range interaction involving all matter, in spite of being the weakest of all forces.

Here, we will focus on the electromagnetic interaction, especially on its source. As one can see from the word "electromagnetism", it is a unification between electric and magnetic phenomena. In order to describe electromagnetism, Faraday introduced the concept of *fields* as mediators of interaction. Charges are sources of the fields. The stronger the charge, the stronger the fields. Charges are also responders to the fields. The stronger the charge, the stronger the response. An experimental result is that *a varying magnetic field requires the presence of an electric field*, and conversely *a varying electric field requires a magnetic field.* Electric charges (point-like particles) are the sources of electric field. Also accelerated charges are the sources of magnetic fields. Thats why we can say that a property of matter, called electric charge, is the source of the electromagnetic interaction.

From the above picture, it is obvious that there is a striking symmetry between electric and magnetic fields. But despite the fact that electric and magnetic phenomena are unified and there exist an electric monopole (the point-like electric charge), there is an experimental lack of finding a particle carrying magnetic charge. Which is the so-called *magnetic monopole.* All the magnets in the nature appears as dipoles (North and South). Experiments have been able to isolate positive and negative electric charges and associate a definite amount of electric charge with fundamental particles constituting matter. On the contrary, we have not been able to isolate a magnetic pole or identify a particle having only one kind of magnetism, either N or S. As is well known, cutting a bar magnet in two produces two dipole bar magnets. Maxwell's equations account for this by treating electricity and magnetism differently: there is an electric source term containing the charge e, but there is no magnetic source term. Thus free electric charges exist, but not free magnetic charges.

On the other hand, the lack of experimental evidence for magnetic monopoles didn't stop theorists from thinking about them. In 1931 P. Dirac showed the quantization condition of the electric charge, if magnetic monopoles exist. Much later the subject was revived after t' Hooft and Polyakov in 1974 proposed static magnetic monopole solutions of the classical equations for the Yang-Mills field coupled to Higgs fields. We have to stress, however, that in contrast with the Dirac monopole, which is a fundamental *point-like* magnetic charge, monopoles in gauge theories are *extended* objects. Also, all Grand Unified Theories predicts magnetic monopoles. So either one has to abandon GUTs or find a solution to this *monopole problem.* Inflationary cosmology provides us with such a solution. If the monopoles were produced in the very early universe, then a subsequent inflationary stage would drastically dilute their number density, leaving less than one monopole per present horizon scale.

3 Electric-Magnetic Duality

In classical electrodynamics, the fundamental quantities are the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$. As we mentioned in the introduction, electric and magnetic fields are unified. It was James Clerk Maxwell (1831 – 1879) who, with his intelligence, wrote down four equations and show this unification in a pure mathematical way. Maxwell also, showed the existence of electromagnetic waves which travel, in vacuum, with the speed of light. These equations, in the absence of matter (vacuum - without sources) can be written as:

$$\nabla \cdot \mathbf{E} = 0 \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{3}$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = 0 \tag{4}$$

where c is the speed of light in vacuum.

By starring at the above equations, we remark that they are symmetric under the exchange of the field $\mathbf{E}$ and $\mathbf{B}$. More precisely, they are invariant under

$$\mathbf{E} \rightarrow \mathbf{B} \qquad \text{and} \qquad \mathbf{B} \rightarrow -\mathbf{E}. \tag{5}$$

This symmetry is called the electric-magnetic duality and the exchange of electric and magnetic fields in (5) is known as the duality transformation. The above symmetry means that any theory describes a vacuum consisting only of the electric and magnetic field, $\mathbf{E}_1$ and $\mathbf{B}_1$ respectively, has the same physical interpretation as another theory describes a vacuum with the electric field $\mathbf{E}_2 = \mathbf{B}_1$ and the magnetic field $\mathbf{B}_2 = -\mathbf{E}_1$. In particular, the energy densities are the same:

$$\frac{1}{8\pi}|\mathbf{E}_1|^2 + \frac{1}{8\pi}|\mathbf{B}_1|^2 = \frac{1}{8\pi}|\mathbf{E}_2|^2 + \frac{1}{8\pi}|\mathbf{B}_2|^2. \tag{6}$$

The next step, is to search for such a symmetry when we have matter and charges in the Universe. Unfortunately, the above symmetry seems to be spoiled in nature by the fact that we clearly have electric charges but have not yet observed any magnetic charges. Now the Maxwell's equations (1) – (4) become:

$$\nabla \cdot \mathbf{E} = 4\pi\varrho_e \tag{7}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{8}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{9}$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}_e \tag{10}$$

where ϱ_e is the electric charge density and $\mathbf{j}_e$ the electric current density.

In particular, (7) shows that the electric charge ϱ_e produces the electric field while there is no magnetic charge as is obvious from (8). That's why Maxwell equations assume that there is also an electric charge and there is no magnetic charge $\mathbf{j}_m$. Also, from the above set of equations, we can see that the electric-magnetic duality breaks down. That's not a physical problem, but these four equations can be really very symmetric if we put also magnetic charges and currents in them. It seems like Maxwell's equations beg for magnetic monopoles.

By putting in the magnetic charge density ϱ_m and magnetic current density $\mathbf{j}_m$ we have that:

$$\nabla \cdot \mathbf{E} = 4\pi\varrho_e \tag{11}$$

$$\nabla \cdot \mathbf{B} = 4\pi\varrho_m \tag{12}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = \frac{4\pi}{c}\mathbf{j}_m \tag{13}$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}_e. \tag{14}$$

The above equations now look more symmetric, but the symmetry is not entirely apparent. There is again a dual symmetry and the duality transformation is:

$$\mathbf{E} \to \mathbf{B} \qquad ; \qquad \mathbf{B} \to -\mathbf{E} \tag{15}$$

$$\varrho_e, \mathbf{j}_e \to \varrho_m, \mathbf{j}_m \qquad ; \qquad \varrho_m, \mathbf{j}_m \to -\varrho_e, -\mathbf{j}_e \tag{16}$$

Physically, the duality transformation exchanges the roles of the electric and magnetic fields. Since charges are the sources and responders to the fields, we also need to exchange the electric and magnetic charge and current densities in order to leave the theory invariant.

The above picture of course is not an evidence for the existence of monopoles but as we mentioned the symmetry and beauty of Maxwell's equations is a challenge for putting magnetic monopoles there by hand.

4 Dirac Monopole and Monopoles in Gauge Theories

A gauge theory is a theory with a continuous local symmetry in its Lagrangian density. As we will see gauge theories and topology play a crucial role to the theoretical prediction of monopoles. These will be explained in more detail below.

4.1 Principle of least action

Most of classical and quantum physics can be expressed in terms of the variational principle, and it is often, when written in this form, that the physical meaning is more clearly understood. To begin with, let as consider a mechanical system whose configuration can be defined uniquely by a number of general coordinates $q^\alpha, \alpha = 1, 2, ..., n$ (usually distances and angles), together with time t. Hamilton's principle states that in moving from one configuration at time t_1 to another at time t_2 the motion of such a system is such as to make stationary the *action*

$$S = \int_{t_1}^{t_2} L(q^\alpha, \dot{q}^\alpha, t)dt. \tag{17}$$

The *Lagrangian* L is defined, in terms of the kinetic energy T and potential energy V, by $L = T - V$.

By using the least action principle, $\delta S = 0$, it can be shown that we can derive the Euler-Lagrange equations which are the system's equations of motion. These are

$$\frac{\partial L}{\partial q^\alpha} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^\alpha}\right) = 0, \qquad \alpha = 1, 2, ..., n. \tag{18}$$

By analogy, the action S for a set of fields defined on some general four-dimensional spacetime manifold[1] should take the form of an integral of some function $\mathcal{L}$, called the *Lagrangian density*, of the fields Φ^α and their first

[1] In general, a manifold is any set that can be continuously parameterised. The number of independent parameters required to specify any point in the set uniquely is the *dimension* of the manifold, and the parameters themselves are the *coordinates* of the manifold. An abstract example is the set of all rigid rotations of Cartesian coordinate systems in three-dimensional Euclidean space, which can be parameterised by the Euler angles.

derivatives over some four-dimensional region $\mathcal{R}$ of the spacetime. Thus, we can write

$$S = \int_{\mathcal{R}} \mathcal{L}\left(\Phi^{\alpha}, \partial_{\mu}\Phi^{\alpha}\right) d^4x. \tag{19}$$

The Euler-Lagrange equations now are

$$\frac{\delta \mathcal{L}}{\delta \Phi^{\alpha}} = \frac{\partial \mathcal{L}}{\partial \Phi^{\alpha}} - \partial_{\mu}\left[\frac{\partial \mathcal{L}}{\partial\left(\partial_{\mu}\Phi^{\alpha}\right)}\right] = 0. \tag{20}$$

4.2 Lagrangian formulation of classical electromagnetism without charges and currents

By using the above Lagrangian formulation for classical electromagnetism we can write Maxwell's equations in a compact form.

First of all, we must have a Lagrangian density to begin with. In this formulation this Lagrangian can be written by using the electromagnetic field tensor $F^{\mu\nu}$. Before defining this tensor we have to define the electromagnetic 4-potential. It is a potential from which the electromagnetic field can be derived. It combines both the electric scalar potential and the magnetic vector potential into a single space-time 4-vector. This 4-vector can be defined as $A^{\mu} = (\phi, \mathbf{A})$, where ϕ is the electric potential and $\mathbf{A}$ is the magnetic potential. The electric and magnetic fields associated with these 4-potentials are:

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \qquad \text{and} \qquad \mathbf{B} = \nabla \times \mathbf{A}. \tag{21}$$

Now we are ready to define the electromagnetic field tensor. It is an antisymmetric tensor and can be defined as:

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} \tag{22}$$

or

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}.$$

As from (22) we have that

$$
\begin{aligned}
F^{0i} &= \partial^0 A^i - \partial^i A^0 \\
&= \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla\phi\right)_i \\
&= -E^i
\end{aligned} \tag{23}
$$

and

$$
\begin{aligned}
F^{ij} &= \partial^i A^j - \partial^j A^i \\
&= -\varepsilon^{ijk} B^k,
\end{aligned} \tag{24}
$$

where $\varepsilon^{ijk} = \varepsilon^{ijk}$, ($\varepsilon^{123} = 1$), is the totally antisymmetric Levi-Civita symbol (tensor).

Now we have all the tools to describe the Lagrangian formulation of classical electromagnetism without charges and currents ($\varrho_e = 0$ and $\mathbf{j}_e = 0$). Classical electromagnetism and Maxwell's equations can be derived from the action:

$$
S = \int \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) d^4x \tag{25}
$$

where the Lagrangian density is

$$
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{2}\left(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\nu A_\mu \partial^\mu A^\nu\right). \tag{26}
$$

So the Euler-Lagrange equation become

$$
\begin{aligned}
\partial_\nu \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) &= 0 \\
\Rightarrow \partial_\nu F^{\mu\nu} &= 0.
\end{aligned} \tag{27}
$$

That equation is just another way of writing the two inhomogeneous Maxwell's equations. When there are charges and currents, the Lagrangian needs an extra term to account for the coupling between them and the electromagnetic field. In that case $\partial_\nu F^{\mu\nu}$ is equal to the 4-current, instead of zero, and we can write

$$
\partial_\mu F^{\mu\nu} = j^\nu, \tag{28}
$$

with

$$
j^\nu = (\varrho, \mathbf{j}). \tag{29}
$$

As we will remark now, the above representation of classical electrodynamics and especially the 4-vector potential A^μ is free of gauge. Although A^μ specifies the electric and magnetic fields in terms of **A** and ϕ, it does not do so uniquely, for under a *gauge transformation*

$$\mathbf{A} \to \mathbf{A} - \nabla f, \qquad f \to \phi + \frac{\partial f}{\partial t} \tag{30}$$

which has the covariant form

$$A^\mu \to A^\mu + \partial^\mu f, \tag{31}$$

where f is an arbitrary scalar function, **E** and **B** remain unchanged, in view of (31). Equivalently $F^{\mu\nu}$ is also unchanged, as we can see from the gauge transformation

$$F^{\mu\nu} \to F^{\mu\nu} + (\partial^\mu \partial^\nu - \partial^\nu \partial^\mu) f = F^{\mu\nu}. \tag{32}$$

We can say that classical electrodynamics as well as quantum electrodynamics (QED) is an abelian gauge theory with the symmetry group U(1), where U(1) is the group of all numbers of the form $e^{i\alpha} = cos\alpha + isin\alpha$ and since $cos^2\alpha + sin^2\alpha = 1$, the space of the group is a circle.

4.3 Dirac Monopole

The first theoretical evidence for the existence of monopoles was made by P.Dirac in 1931. He considered the existence of monopoles and by using quantum mechanics he realized that it explains the quantization of electric charge. It was the first possible explanation for the observed quantization of electric charge, although nowadays this is more commonly ascribed to the existence of quarks and non-Abelian symmetry groups. Due to the success of the above Dirac model (Dirac monopole), more and more theoretical and experimental physicists engaged in the search for magnetic monopoles. We will now describe Dirac monopole model and Dirac quantization condition.

Consider a magnetic monopole of strength g at the origin of a three dimensional space $\mathcal{R}^3$. So it produces a radial magnetic field which is given by a Coulomb-type law

$$\mathbf{B} = \frac{g}{r^3}\mathbf{r} = -g\nabla\left(\frac{1}{r}\right) \tag{33}$$

(we are using Gaussian units). Since $\nabla^2 1/r = -4\pi\delta^3 r$, we have

$$\nabla \cdot \mathbf{B} = 4\pi g \delta^3 r, \tag{34}$$

corresponding to a point magnetic charge, as desired. Since $\mathbf{B}$ is radial, the total flux through a sphere surrounding the origin is

$$\Phi = 4\pi r^2 B = 4\pi g. \tag{35}$$

He now considered a particle with electric charge e in the field of this monopole. From quantum mechanics, the wave function for a free particle is

$$\psi = \mid \psi \mid e^{\frac{i}{\hbar}(\mathbf{p}\cdot\mathbf{r} - Et)}. \tag{36}$$

In the presence of an electromagnetic field by considering also the coupling between the charge and the potential, $\mathbf{p} \rightarrow \mathbf{p} - e/c\mathbf{A}$, so

$$\psi \rightarrow \psi e^{-\frac{ie}{\hbar c}\mathbf{A}\cdot\mathbf{r}}; \tag{37}$$

or the wave function phase α changes by

$$\alpha \rightarrow \alpha - \frac{e}{\hbar c}\mathbf{A} \cdot \mathbf{r}. \tag{38}$$

Consider a closed path at fixed r, θ, with ϕ ranging from 0 to 2π. The total change in phase is

$$\begin{aligned} \triangle\alpha &= \frac{e}{\hbar c}\oint \mathbf{A} \cdot d\mathbf{l} \\ &= \frac{e}{\hbar c}\int (\nabla \times \mathbf{A}) \cdot d\mathbf{S} \\ &= \frac{e}{\hbar c}\int \mathbf{B} \cdot d\mathbf{S} \\ &= \frac{e}{\hbar c}(Flux\ through\ cap) = \frac{e}{\hbar c}\Phi(r,\theta); \end{aligned} \tag{39}$$

$\Phi(r, \theta)$ is the flux through the cap defined by a particular r and θ. As θ is varied the flux through the cap varies. As $\theta \rightarrow 0$ the loop shrinks to a point and the flux passing through the cap approaches zero:

$$\Phi(r, 0) = 0.$$

As the loop is lowered over the sphere the cap encloses more and more flux until, eventually, at $\theta \to \pi$ we should have, from (35),

$$\Phi(r, \pi) = 4\pi g. \tag{40}$$

However, as $\theta \to \pi$ *the loop* has *again shrunk to a point* so the requirement that $\Phi(r, \pi)$ is finite entails, from (39), that A *is singular at* $\theta = \pi$. We can see this as at $\theta = \pi$, $\mathbf{l} \to 0$, so if we must have $\oint \mathbf{A} \cdot d\mathbf{l}$ finite it should be that $\mathbf{A} \to \infty$.

This argument holds for all spheres of all possible radii, so it follows that $\mathbf{A}$ is singular along *the entire negative z axis.* This is known as the *Dirac string.* It is clear that by a suitable choice of coordinates the string may be chosen to be along any direction, and, in fact, need not be straight, but must be continuous.

The singularity in $\mathbf{A}$ gives rise to the so-called Dirac veto - that the wave function vanish along the negative z axis. Its phase is therefore indeterminate there and referring to (39) there is *no necessity* that as $\theta \to \pi$, $\triangle\alpha \to 0$. We must have $\triangle\alpha = 2\pi n$, however, in order for ψ to be single-valued. From (39) and (40) we then have

$$2\pi n = \frac{e}{\hbar c} 4\pi g,$$

$$eg = \frac{1}{2} n\hbar c. \tag{41}$$

This is the Dirac quantization condition. It implies that the product of *any* electric with *any* magnetic charge is given by the above. Then, in principle, if there exists a magnetic charge anywhere in the universe all electric charges will be quantized:

$$e = n\frac{\hbar c}{2g}. \tag{42}$$

Note, however, that the quantization condition has an explicit dependence on Planck's constant, and therefore on the quantum theory. In units $\hbar = c = 1$ (41) becomes

$$eg = \frac{1}{2} n. \tag{43}$$

In equation (42) contained exactly what we said in the beginning. Namely that the hypothesis of the existence of magnetic monopoles implies quantization of electric charge in units $\frac{1}{2}\hbar c/g$ and, similarly, the existence of one charge would require all poles to be quantized. Although there is a symmetry between charges and poles from the point of view of general theory, there is a difference in practice on account of the different numerical values for the quantum of charge and the quantum of pole.

If we take the experimental value for the fine-structure constant, $a \equiv \frac{e_0^2}{\hbar c} = \frac{1}{137}$ (the index 0 corresponds to the lowest value of electric charge, the electron charge), we can infer the value of g_0,

$$g_0^2 = \frac{137}{4}\hbar c. \tag{44}$$

Thus g_0 is much larger than e_0. For instance, from the Dirac quantization condition and from the electron fine-structure, we can see that $g = \frac{1}{2}\frac{\hbar c}{e}n \Rightarrow g \simeq 68.5en$. So the monopole corresponds to a fine-structure constant $\frac{137}{4}$. The forces of radiation damping must be very important for the motion of poles with an appreciable acceleration.

The great difference between the numerical values of e_0 and g_0 explains why electric charges are easily produced and not magnetic poles. Two one-quantum poles of opposite sign attract one another with a force $\frac{g_0^2}{e_0^2} = \left(\frac{137}{2}\right)^2$ times as great as that between two one-quantum charges at the same distance. It must therefore be very difficult to separate poles of opposite sign. That might be a reason for the lack of experimental results up today.

4.4 The 't Hooft-Polyakov monopole

In the above we mentioned that in the context of Maxwell's electrodynamics, with Abelian gauge group U(1), it is clear that although magnetic charges may be "added" to the theory, there is no necessity for doing this. A theory with monopoles is more symmetric between electricity and magnetism than one without, but this does not amount to a requirement that monopoles exist. They may or may not; the above considerations do not allow us to decide. When the gauge symmetry is enlarged to a non-Abelian group, however, and spontaneous symmetry breaking is introduced, the field equation yield a solution which corresponds to a magnetic charge. If such theories are correct, then, magnetic monopoles *must* exist, and should therefore be looked for.

The theoretical possibility of monopoles of this type was discovered in 1974 by 't Hooft and Polyakov. Their monopoles aren't point like particles but they are extended ones and the origin of the magnetic charge is topological.

In 1959 Y. Aharonov and D. Bohm predicted the homonym effect which show that topology and vacuum hide great physical effects which we didn't imagine until then. In classical physics the physical effect of an electromagnetic field on a charge is the Lorentz force, $\mathbf{F} = e\mathbf{E} + e\mathbf{u} \times \mathbf{B}$, and this only exists in regions where $\mathbf{E}$ and/or $\mathbf{B}$ are non-zero. The Bohm-Aharonov effect demonstrates that this is not so in quantum mechanics. There, physical effects occur in regions where $\mathbf{E}$ and $\mathbf{B}$ are both zero, but the 4-vector potential A_μ is not. Hence A_μ has more physical significance than was formerly thought.

It was realized, that the field tensor $F_{\mu\nu}$ by itself does not, in quantum theory, completely describe all electromagnetic effects on the wave function of the electron. The famous Bohm-Aharonov experiment, first beautifully performed by Chambers (1960), showed that in a multiply connected region[2] where $F_{\mu\nu} = 0$ everywhere, there are physical experiments for which the outcome depends on the loop integral

$$\frac{e}{\hbar c} \oint A_\mu dx^\mu \tag{45}$$

around an unshrinkable loop. An examination of the Bohm-Aharonov experiment indicates that in fact only *the phase factor*

$$exp\left(\frac{ie}{\hbar c} \oint A_\mu dx^\mu\right), \tag{46}$$

and *not the phase* (45), is physically meaningful.

Topological defects do not occur in the Standard Model. However, they are a rather generic prediction of theories beyond the Standard Model. They are solitonic solutions[3] of the classical equations for the scalar (and gauge) fields. They can be formed during a phase transition and since they interpolate between vacuum states they reflect the structure of the vacuum manifold.

[2]A simply connected space is one in which all closed curves may be shrunk continuously to a point. A non-simply connected space is one in which not all curves may be continuously shrunk to a point.

[3]Non-linear classical field theories possess extended solutions, commonly known as solitons, which represent stable configurations with a well-defined energy which is nowhere singular.

We have different kinds of solutions analogously the fields and dimensions. They leads to *domain walls*, *cosmic strings*, *monopoles* and *textures*. We will concentrate on monopoles and the 't Hooft-Polyakov prediction.

They proved that in models of the type described by Georgi and Glashow (1972), based on SO(3), we can construct monopoles with a mass of the order of $137M_W$, where M_W is the mass of the familiar intermediate vector boson. In the Georgi-Glashow model, $M_W < 53GeV/c^2$. Although, the trouble is that the true non-Abelian electroweak group is not SO(3), but $SU(2)\times U(1)$, given by the Weinberg-Salam model (1967), where magnetic monopoles do not exist. Nevertheless, in GUTs when we consider the SU(5) grand-unified semisimple group monopoles may exist.

Lets start with the 't Hooft-Polyakov monopole. They considered a theory with an O(3) symmetry group, containing the gauge field $F^{\alpha}_{\mu\nu}$ (α is the group index) and an isovector[4] Higgs field ϕ^{α}. The Lagrangian density is

$$\mathcal{L} = -\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu\alpha} + \frac{1}{2}\left(D_{\mu}\phi^{\alpha}\right)\left(D^{\mu}\phi^{\alpha}\right) - \frac{m^2}{2}\phi^{\alpha}\phi^{\alpha} - \lambda\left(\phi^{\alpha}\phi^{\alpha}\right)^2 \tag{47}$$

where

$$\begin{aligned} F^{\alpha}_{\mu\nu} &= \partial_{\mu}A^{\alpha}_{\nu} - \partial_{\nu}A^{\alpha}_{\mu} + e\varepsilon^{abc}A^{b}_{\mu}A^{c}_{\nu}, \\ D_{\mu}\phi^{\alpha} &= \partial_{\mu}\phi^{\alpha} + e\varepsilon^{abc}A^{b}_{\mu}\phi^{c}. \end{aligned} \tag{48}$$

Also, $F^{\alpha}_{\mu\nu}$ is a triplet of vector fields and $D_{\mu}\phi^{\alpha}$ is the covariant derivative of the triplet of scalar fields ϕ^{α} which, unlike $\partial_{\mu}\phi^{\alpha}$, it transforms covariantly under gauge transformation, i.e. like ϕ^{α} itself.

We choose the parameter m^2 to be negative so that the field ϕ gets, by technics of spontaneous symmetry breaking, a non-zero vacuum expectation value

$$\langle\phi^{\alpha}\rangle^2 = F^2, \qquad m^2 = -4\lambda F^2. \tag{49}$$

Two components of the vector field will acquire a mass

$$M_{W_{1,2}} = eF \tag{50}$$

[4]In particle physics, isovector refers to the vector transformation of a particle under the SU(2) group of isospin. An isovector state is a triplet state with total isospin 1, with the third component of isospin either 1, 0, or -1, much like a triplet state in the two-particle addition of Spin.

whereas the third component describes the surviving Abelian electromagnetic interactions. The Higgs particle has a mass

$$M_H = \sqrt{\lambda} F. \tag{51}$$

We are interested in static solutions in which the gauge potentials have the non-trivial form

$$\begin{aligned} A_i^\alpha &= -\varepsilon_{iab} \frac{r^b}{er^2} \quad (r \to \infty), \\ A_0^\alpha &= 0 \end{aligned} \tag{52}$$

and the scalar field is

$$\phi^\alpha = F \frac{r^\alpha}{r} (r \to \infty) \tag{53}$$

where, as above, $F^2 = -\frac{m^2}{4\lambda}$.

Gerard 't Hooft showed that there exist regular solutions to the field equations derived from the Lagrangian density (47), which have the asymptotic form (52) and (53). For example, the equation of motion of ϕ is

$$-\left(m^2 + 4\lambda \phi^b \phi^b\right) \phi^a = D_\mu \left(D^\mu \phi^a\right).$$

To check that, lets see that equation (53) implies $|\phi| = F$, so the left-hand side of the above equation vanishes at infinity. It is easy to see that $D_\mu \phi^a$ also vanishes; for $i = x, y, z$ we have

$$\begin{aligned} D_i \phi^a &= F \partial_i \left(\frac{r^a}{r}\right) + e \varepsilon^{abc} A_i^b F \frac{r^c}{r} \\ &= F \left(\frac{\delta^{ia}}{r} - \frac{r^i r^a}{r^3}\right) - \varepsilon^{abc} \varepsilon_{ibm} \frac{F r^m r^c}{r^3} \\ &= 0. \end{aligned}$$

Hence, at infinity, ϕ takes on its vacuum value and is covariantly constant, but has the non-trivial boundary condition (53), rather than the more usual ("Abelian") condition $\phi^{1,2} = 0$, $\phi^3 \neq 0$. On the other hand, $F_{\mu\nu}^a$ is not zero at infinity. We shall see below that there is a radial magnetic field.

Now let us generalise the definition of the electromagnetic field $F_{\mu\nu}$ so that it reduces to the usual one when the scalar field ϕ has only a third component. We put

$$F_{\mu\nu} = \frac{1}{|\phi|} \phi^a F_{\mu\nu}^a - \frac{1}{e|\phi|^3} \phi^a \left(D_\mu \phi^b\right) \left(D_\nu \phi^c\right). \tag{54}$$

It is quite clear that when

$$\begin{aligned} A^{1,2}_\mu &= 0, \quad A^3_\mu \equiv A_\mu \neq 0, \\ \phi^{1,2} &= 0, \quad \phi^3 = F \neq 0 \end{aligned} \tag{55}$$

this gives the usual $F_{\mu\nu}$, so long as $A^3_\mu = A_\mu$, the Maxwell vector potential.

Now, defining

$$A_\mu = \frac{1}{|\phi|}\phi^a A^a_\mu \tag{56}$$

a straightforward calculation gives

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{e\,|\phi|^3}\varepsilon_{abc}\phi^a \left(\partial_\mu \phi^b\right)\left(\partial_\nu \phi^c\right). \tag{57}$$

This is similar to, but more complicated than, the usual definition of the electromagnetic field, but it reduces to it when ϕ becomes fixed in isospace. Inserting the asymptotic conditions (52) and (53), it is easily seen that $A_\mu = 0$, so all the electromagnetic field is contributed by the Higgs field; and we find

$$F_{0i} = 0, \qquad F_{ij} = -\frac{1}{er^3}\varepsilon_{ijk} r^k. \tag{58}$$

This corresponds to a radial magnetic field (see (Eq. 24))

$$B_k = \frac{r^k}{er^3}. \tag{59}$$

The magnetic flux is, from (35),

$$\Phi = \frac{4\pi}{e}, \tag{60}$$

so by comparison with (40) the magnetic charge g is such that

$$eg = 1. \tag{61}$$

From (43), this is twice the Dirac unit. We conclude that the configuration of gauge and scalar fields with asymptotic from (52-53) carries a magnetic charge - that is, when viewed from infinity, there is a radial magnetic field. It has been shown by 't Hooft that this configuration is everywhere non-singular, and therefore has a finite energy. He estimates the monopole mass to be of the order $137M_W$, where M_W is a typical vector boson mass, so the monopoles are extremely heavy. The mass is inversely proportional to e^2.

Concluding, we saw that gauge theories with compact gauge groups provide for the necessary charge quantization. An advantage of 't Hooft-Polyakov monopole over Dirac monopole is that it avoids the introduction of Dirac's string. As we will show the monopole origin is topology (soliton solution) and any GUT theory predicts the existence of magnetic monopoles. In order to show the topological nature of the 't Hooft-Polyakov monopole, we will use some mathematical theorems from group theory and topology, without their proofs.

Since ϕ^a is an isovector, the unit vector $\hat{\phi}$ describes a sphere S^2 in the field space (isospace) , so the boundary describes a mapping of the sphere S^2 in coordinate space onto the $\hat{\phi}$ manifold, which is S^2. In the model considered, the non-Abelian group is SO(3), electromagnetism being represented by the Abelian subgroup U(1). If the symmetry group of the theory is G (in this case SO(3)), and the unbroken subgroup is H (in this case U(1)), then transformations belonging to H leave the vacuum manifold invariant. So the space of $\hat{\phi}$ is the set of transformations in G which are not related by transformation belonging to H. This is the definition of a coset space G/H. The existence of magnetic monopoles requires a non-trivial mapping of G/H onto S^2, the boundary in coordinate space. These mappings form a group, in this case the second homotopy group of G/H, $\pi_2(G/H)$. Magnetic monopoles will exist if this group is non-trivial.

We know that $\pi_2(G/H)$ is isomorphic to the kernel of the natural homomorphism of $\pi_1(H)$ into $\pi_1(G)$, where $\pi_1(H)$ and $\pi_1(G)$ are the first homotopy groups of H and G respectively. In this theory we have that $G = SO(3)$ and $H = U(1)$. Since SO(3) is doubly connected $\pi_1(G) = Z_2$. On the other hand, U(1) is infinitely connected, so $\pi_1(H) = Z$, the additive group of integers. So the kernel of the mapping of $\pi_1(H)$ into $\pi_1(G)$ is the additive group of *even* integers, hence

$$\pi_2(SO(3)/U(1)) = \text{additive group of even integers.} \tag{62}$$

This is consistent with the result that monopole charge was twice the Dirac quantum.

However, the correct theory for electroweak interactions, the Weinberg-Salam model, do not predict the existence of magnetic monopoles. The reason is that in this theory the electroweak group is not SO(3) but $SU(2) \times$

$U(1)$. Although the electromagnetic subgroup is again U(1), it is irregularly embedded in $SU(2)\times U(1)$ and so is non-compact. It follows that $\pi_1(H)$ does not exist (or is trivial) so $\pi_2(G/H)$ is also trivial, with the consequence that magnetic monopoles do not exist in the Weinberg-Salam model. However, if nature is "grand-unified", the electroweak group $SU(2) \times U(1)$ is a subgroup of a grand-unified semisimple group, SU(5), then this argument no longer holds and monopoles exist.

5 Monopoles and Cosmology

5.1 Big Bang Cosmology

The Newtonian theory of gravitation was the cornerstone of physics for more than two centuries, and even today it is very useful and very accurate in diverse applications, such as making calculations for the orbits of spacecraft or to describe the motion of double stars. However, the special theory of relativity of Einstein, as soon as published in 1905, began to pose problems for the Newtonian theory of gravity.

The special theory of relativity introduced a number of amazing and bizarre predictions for relations between moving observers, but perhaps the most important consequence is the existence of a global upper bound for the velocities occurring in nature: $c = 3 \times 10^8 m/s$. No subject is not moving and no interaction is not spread faster than that of light. In contrast, the Newtonian gravitational theory allows infinite speed of propagation, which clearly violate the large but still finite upper bound of speed.

Thus, Einstein has aimed to formulate a theory of gravity consistent with Special Relativity, in 1916, and finally published the *General Theory of Relativity* . The new theory is in direct break with Newtonian theory, rejecting the notion of gravity as a force, and considering the result of non-Euclidean geometry of space-time. Einstein adopted, as a basic starting point of the view, that the movements of particles are not affected by some invisible force acting on the survivors to the bodies that have mass. Instead, movements of the particles determined by the geometry of space-time around them.

But the evolution of ideas, theories and observational data for the gravity and the universe as a whole did not stop here. We are now convinced that we live in a universe that contains billions of galaxies, which are removed from one another on an expanding space-time continuum. This belief is based on Hubble observations in 1929, according to which speeds removal of galaxies is directly proportional to their distances. This is *Hubble's law* and is expressed by the relationship $\upsilon = H_0 D$, where H_0 is the Hubble parameter with a value $70 - 75 km/secMpc$, to the best current measurements and D the spatial distance between two points. The Hubble observations confirm the Friedmann's theoretical studies (1923) and formed the base of the *Big Bang* model of Gamow in the late 40's and early 50's.

Gamow made a very simple idea, as appears later. If the universe now seems to be expanding in the past should have smaller dimensions. The fact that, although simple in thought, made important theoretical progress. Smaller size, however, means higher density and higher density means higher temperature. Consequently, in the early stages of evolution, the universe must have been extremely hot and the matter in a state of complete ionization. His knowledge around issues of nuclear physics (which were not complete at the time) helped him to draw two very important conclusions of starting with a unique case of smaller dimensions in the past. The first was the proportion of so-called cosmic *light elements* of the universe and especially hydrogen and helium. According to his calculations, the ratio of these elements should be of the order of 75% hydrogen and 25% helium. The second and perhaps most impressive, Gamow's foresight, was the existence of the so-called *microwave radiation*. This is a primordial remnant of very hot initial state of the universe. According to Gamow, now we should understand that as a low frequency radiation, equivalent to that emitted from a "black body" with temperature 5 to 50 Kelvin or so. Initially, these views did not receive proper attention from the scientific community then and mistrust were widespread. However, fifteen years after their formulation, both Gamow's estimates has been confirmed observationally. Especially the detection of microwave radiation with a temperature of about 3 Kelvin (from Penzias and Wilson), is revolutionizing the field of cosmology and the steady-state / quasi-steady state cosmological model grants its position to the Big Bang model.

The Friedmann model (typically three) is the simplest cosmological solution of Einstein equations and describes an entirely uniform (homogeneous and isotropic) universe. The uniformity of Friedmann's cosmology is consistent with that of the Cosmic Microwave Background radiation (CMB), showing isotropy of about 1/10.000. However, if you focus on some local systems of galaxies or galaxy clusters can not ignore the different morphology of each and neighboring interactions. These are the two main parameters which give the galaxies a speed independent of speed due to the Hubble expansion. This speed is different from galaxy to galaxy and called peculiar velocity. The knowledge and description of galaxies peculiar velocities will enable us to understand their movement, particularly if its of the size of the Hubble velocity.

5.2 Geometry in curved spaces

According to the Theory of Relativity, space and time are not two different and independent concepts, but are linked together and form a 4-dimensional space-time continuum in which events unfold naturally. In General Relativity (which is the best theory of gravity we have so far) gravity is not considered as a force acting at a distance, but as a geometric "force" that distorts and curves the space-time.

So we have a curved space-time which to describe we need curvilinear coordinates x^α. The distances and angles on a curved space are defined with the help of the symmetric metric tensor with components

$$g_{ab} = g_{ab}(x^a) = g_{ba}. \tag{63}$$

The infinitesimal distance between two adjacent points with coordinates x^α and $x^\alpha + dx^\alpha$ is

$$ds^2 = g_{ab} dx^a dx^b, \tag{64}$$

which is supposed to apply Einstein's summation convention for repeated indices.

5.3 Ricci curvature

According to the General Theory of Relativity, the gravitational field is an expression of the non-Euclidean geometry of space-time. Consequently, the gravitational field is expressed by geometric relations. In Riemann spaces, the curvature tensor is also known as the Riemann tensor and is given by the relation

$$R^a{}_{bcd} = -\partial_d \Gamma^a{}_{bc} + \partial_c \Gamma^a{}_{bd} - \Gamma^e{}_{bc}\Gamma^a{}_{ed} + \Gamma^e{}_{db}\Gamma^a{}_{ec}, \tag{65}$$

where $\Gamma^a{}_{bc}$ are the Christoffel symbols, which with the help of the metric tensor are defined as

$$\Gamma^a{}_{bc} = \frac{1}{2} g^{ab}(\partial_c g_{db} + \partial_b g_{dc} - \partial_d g_{bc}). \tag{66}$$

With the contraction of two indices of the Riemann tensor, we are to the

Ricci tensor, which is a second order tensor with the form

$$R_{ab} = R^c{}_{acb} = g^{cd} R_{dacb}. \tag{67}$$

From the Ricci tensor and the metric tensor, we obtain the Ricci scalar quantity

$$R = g^{ab} R_{ba} = R^a{}_a. \tag{68}$$

5.4 Field Equations

It is now understood that the image of our concepts of space and time with the introduction of general relativity overturned drastically. For this reason, we had to change the equations that explain the various phenomena, but in such a way that in the classical limit we arrive back to the classical theory of Newton. Thus, Newton's equations for gravity in the context of general relativity take the form of Einstein's field equations

$$R_{ab} - \frac{R}{2} g_{ab} = \kappa T_{ab} - \Lambda g_{ab}, \tag{69}$$

where T_{ab} is the energy-momentum tensor, which is a symmetric tensor describing the energy content of space-time. Also, Λ is the cosmological constant and $\kappa = 8\pi G/c^4$ the gravitational constant. According to the above field equations, *matter bends space and the geometry of space tells the matter how to move.*

5.5 Friedmann Models

Until now, we saw some evidence of general relativity and cosmology but without making use of a certain model or assumptions. Many were those who proposed different cosmological models of the universe. The revolution, however, was made by Friedmann, with models which are the simplest cosmological models of the universe and are the basis for all cosmological study. It is easy to use and allow simple theoretical predictions, many of which have been verified experimentally.

Before writing the Friedmann equations, we will talk about the Robertson-Walker linear element. The large-scale structure of the universe today includes galaxies, galactic clusters and galactic super-clusters allocation which at present statistical homogeneity (at scales > 100Mpc). The homogeneity of the universe is also supported by the observations of the CMB, which show that the universe is exceptionally isotropic about 100,000 years after the initial explosion when it had temperature of $\sim 1000K$. The isotropy of CMB, combined with the very reasonable belief that we are not in a special position in the Universe (Copernican Principle), leads to the cosmological principle: that the universe is homogeneous and isotropic. The disparities that exist today, are small disturbances of the generally uniform image of the universe.

The metric tensor describing the Friedmann cosmological models resulting from the Robertson-Walker linear element, which is given by

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right]. \tag{70}$$

The function $a = a(t) > 0$, determines the scale of 3-dimensional space, called *scale factor* or *expansion factor*. The index of curvature, K, is normalized such that $K = 0, \pm 1$ and determines the geometry of 3-dimensional space. Euclidean geometry corresponds to the value $K = 0$, while $K = +1$ and $K = -1$ state spherical and hyperbolic geometry respectively.

Using the Robertson-Walker linear element, we can solve the Einstein equations and the result is two independent differential equations defining the evolution of the scale factor a with time. These are known as Friedmann equations and are given by relations (with $\Lambda = 0$):

$$\dot{a}^2 + K = \frac{8\pi G\rho a^2}{3} \quad \Rightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2} \tag{71}$$

and

$$\frac{2\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -8\pi Gp - \frac{K}{a^2}. \tag{72}$$

Combining these relations properly, we can find the following differential equations

$$\left(\rho a^3\right)^{\cdot} = -p\left(a^3\right)^{\cdot} \qquad \text{and} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right). \tag{73}$$

The first is known as "continuity equation"and identify the evolution of the density of matter in the universe and is independent of the index of curvature K. The second is known more as "Raychaudhuri equation".

The acceleration (or deceleration) of the expansion of the universe is determined by the sign of the quantity $\ddot{a}$. According to the second equation of (73) the universe is slowing down ($\ddot{a} < 0$) when $\rho + 3p > 0$, while otherwise we have accelerated expansion. The quantity $\rho + 3p$ defines the total gravitational energy of matter in the universe, with the contribution of pressure to the gravitational energy to be an important differentiation of general relativity from Newtonian theory. All known (conventional) forms of matter have $\rho + 3p > 0$, which means that in Friedmann-Robertson-Walker (FRW) models, accelerated expansion is achieved only with the dominance of non-conventional matter / energy with $\rho + 3p < 0$.

As we said, Hubble observationally realized that the Universe is expanding. He also introduced the Hubble parameter, which is a function of time and its equal to

$$H(t) = \frac{\dot{a}}{a}. \tag{74}$$

$H(t)$ have dimensions t^{-1} and it measures the expansion rate. The Hubble parameter decreasing with time so when we refer to it, we refer to its current value. For any value of the Hubble constant $H_0 \equiv \dot{a}(t_0)/a(t_0)$, we may define a critical present density

$$\rho_{0,crit} \equiv \frac{3H_0^2}{8\pi G}. \tag{75}$$

According to Eq.(71), whatever we assume about the constituents of the universe, the curvature constant K will be +1 or 0 or -1 according to whether the present density ρ_0 is greater than, equal to, or less than $\rho_{0,crit}$.

It is convenient to use the cosmological parameter Ω, which by definition is $\Omega \equiv \rho/\rho_{crit}$ and determines the geometry. If $\Omega > 1$, the universe is closed and has the geometry of a 3-dimensional sphere ($K = +1$); $\Omega = 1$ corresponds to a flat universe ($K = 0$); and in the case $\Omega < 1$, the universe is open and has hyperbolic geometry ($K = -1$).

More generally, for arbitrary K and a mixture of vacuum energy and relativistic and non-relativistic matter, making up fractions Ω_Λ, Ω_M, and Ω_R of the critical density, we have

$$\rho = \frac{3H_0^2}{8\pi G}\left[\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_R\left(\frac{a_0}{a}\right)^4\right], \tag{76}$$

where the present energy densities in the vacuum, non-relativistic matter and relativistic matter (i.e., radiation) are, respectively,

$$\rho_{V0} \equiv \frac{3H_0^2\Omega_\Lambda}{8\pi G}, \quad \rho_{M0} \equiv \frac{3H_0^2\Omega_M}{8\pi G}, \quad \rho_{R0} \equiv \frac{3H_0^2\Omega_R}{8\pi G}, \tag{77}$$

and, according to Eg.(71),

$$\Omega_\Lambda + \Omega_M + \Omega_R + \Omega_K = 1, \quad \Omega_K \equiv -\frac{K}{a_0^2 H_0^2}. \tag{78}$$

6 Inflationary Cosmology

As we mentioned in the previous section even from the simplest cosmological model, the FRW model, the Universe can be either accelerated or decelerated. This depends on the energy density and the pressure. Here we will mention three puzzles of Big Bang Cosmology and how inflation resolve these problems, including the monopole problem.

6.1 Classical Cosmological Problems

Standard cosmological models (Big Bang Cosmology) suffer, in particular, from the *flatness problem* and the *horizon problem.* Nevertheless, including the *monopole problem* we see that we have three cosmological puzzles. The best model which we now have to solve these problems is *inflation.* Inflation is a stage of accelerated expansion of the universe when gravity acts as a repulsive force.

The possibility of an early exponential expansion had been noticed by several authors, but at first it attracted little attention. It was Alan Guth (1981) who incited interest in the possibility of inflation by noting what it was good for. Guth noticed that, in a model of grand unification he was considering (with Henry Tye), scalar fields could get caught in a local minimum of the potential, which in his work corresponded to a state with an unbroken grand unified symmetry. The energy of empty space would then have remained constant for a while as the universe expanded, which would produce a constant rate of expansion, meaning that $a(t)$ would have grown exponentially. Eventually this inflation would be stopped by quantum-mechanical barrier penetration, after which the scalar field would start rolling down the potential toward a global minimum, corresponding to the present universe. It occurred to Guth that the existence of an area of inflation would solve one of the outstanding problems of cosmology, the "flatness problem". Soon also discovered that inflation would solve others cosmological puzzles too.

As Guth and others soon realized, his version of inflation had a fatal problem, Guth's "old inflation" was soon replaced with a "new inflation" model, due to Andrei Linde (1982) and Andreas Albrecht and Paul Steinhardt (1982). The essential element introduced by theories of new inflation was a nearly exponential expansion during the slow roll of one or more scalar fields down a potential hill. This provided a basis for "chaotic inflation" and "eternal inflation" and other variants. We will now present these three puzzles.

In FRW models the energy density of non-relativistic matter is proportional to a^{-3}, hence $\rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3}$, and for relativistic matter we have that $\rho = \rho_0 \left(\frac{a}{a_0}\right)^{-4}$, with a subscript 0 denoting a present value. So, the contribution of non-relativistic and relativistic matter to the quantity ρa^2 in Eq.(71) grows as a^{-1} and a^{-2}, respectively, as $a \to 0$, so at sufficiently early times in the expansion we may certainly neglect the constant curvature K, and Eq.(71) gives

$$\left(\frac{\dot{a}}{a}\right)^2 \to \frac{8\pi G\rho}{3}. \tag{79}$$

That is, at these early times the density becomes essentially equal to the critical density $3H^2/8\pi G$, where H is the value of the Hubble "constant" at those times. On the other hand, the observed Type Ia supernova redshift-distance relation and measurements of the ages of the oldest stars are consistent with a vanishing spatial curvature parameter Ω_K. So the total energy density of the present universe is still a fair fraction of the critical density. How is it that after billions of years, ρ is still not very different from ρ_{crit}? This is the so-called flatness problem.

The simplest solution to the flatness problem is just that we are in a spatially flat universe, in which $K = 0$ and ρ is always precisely equal to ρ_{crit}. But, as we mentioned above, a more popular solution is provided by the inflationary theories.

Secondly, with the horizon problem, we mean the fact that observationally, vastly separated regions display the same physical characteristics (eg. the nearly uniform temperature of the cosmic microwave background) when, according to standard cosmological models, these regions are causally disconnected.

The horizon problem can be illustrated by a simple example. Consider a galaxy at a proper distance of 10^9 light years away from us. Since the age of the universe is $\sim 1.5 \times 10^{10}$ years, there has been sufficient time to exchange about 15 light signals with the galaxy. At earlier times, when the scale factor a was smaller, everything was closer together and so we might have naively expected that this would improve causal contact. In a continuously decelerating universe, however, it makes the problem worse. At, for example, the epoch of recombination (when the cosmic microwave background photons were emitted) the redshift[5] z was approximately 1000, so $a(t_{rec})/a_0 \approx 10^{-3}$ and the proper distance to the "galaxy" is 10^6 light years.[6] If we assume, for simplicity, that after t_{rec} the expansion followed a matter-dominated Einstein-de Sitter universe[7], then

$$\left(\frac{t_{rec}}{t_0}\right)^{2/3} = \frac{a_{rec}}{a_0} = 10^{-3},$$

and so $t_{rec} = 1.5 \times 10^{5.5}$ years. However, assuming that prior to t_{rec} the expansion followed a radiation-dominated Einstein-de Sitter model, the proper distance to the particle (causal) horizon is $2ct_{rec} = 3 \times 10^{5.5}$ light years. Thus, by t_{rec} "we" could not have exchanged even one light signal with the other "galaxy".

6.2 Monopole Problem

As we mentioned in previous section, a particular kind of topological defect is a magnetic monopole. All GUTs theories predict the existence of monopoles. This is obviously a problem due to the lack of experimental results. We will display this problem in more details and numerically.

In grand unified theories local symmetry under some simple symmetry group is spontaneously broken at an energy $M \approx 10^{16}$ GeV to the gauge symmetry of the Standard Model under the group $SU(3) \times SU(2) \times U(1)$. In all such cases, the scalar fields that break the symmetry can be left in

[5]The redshift parameter is defined as the fractional shift in wavelength of a photon emitted by a distant galaxy at time t_{em} and observed on Earth today:

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}.$$

But $\lambda_{obs}/\lambda_{em}$ is equal to the ratio of the scale factor at the corresponding moments of time, and hence

$$1 + z = \frac{a_0}{a(t_{em})},$$

where a_0 is the present value of the scale factor.

[6]In reality the galaxy would not yet have formed, but this does not affect the main point of the argument.

[7]The Einstein-de Sitter universe is a spacetime with positive constant 4-curvature that is homogeneous and isotropic in both space and time. Hence, it possesses the largest possible symmetry group, as large as the symmetry group of Minkowski spacetime (ten parameters in the fourth dimensional case).

twisted configurations that carry non-zero magnetic charge and that cannot be smoothed out through any continuous processes. At an early time t in the standard big bang theory the horizon distance was of order $t \approx (G(k_BT)^4)^{-1/2}$ (where $G \simeq (10^{19}\text{GeV})^{-2}$ is Newton's constant), so the number density of monopoles produced at the time that the temperature drops to M/k_B would have been of order $t^{-3} \approx (GM^4)^{3/2}$, which is smaller than the photon density $\approx M^3$ at $T \approx M/k_B$ by a factor of order $(GM^2)^{3/2}$. For $M \approx 10^{16}$ GeV this factor is of order 10^{-9}. If monopoles-antimonopoles did not find each other to annihilate, then this ratio would remain roughly constant to the present, but with at least 10^9 microwave background photons per nucleon today, this would give at least one monopole per nucleon, in gross disagreement with what is observed.

At GUT time the particle horizon was only 3.8×10^{-27} cm and in the GUT phase transition there arises at least one monopole per horizon volume. The monopole number density at GUT time was then

$$N_M(t_{GUT}) = (3.8 \times 10^{-27} cm)^{-3},$$

and the linear scale factor has grown by a factor 3.1×10^{27}. Nothing could have destroyed them except monopole-antimonopole annihilation, so the monopole density today should be

$$N_M(t_0) \simeq (3.1 \times 3.8 cm)^{-3} \simeq 6.1 \times 10^{-4} cm^{-3}.$$

This is quite a substantial number compared with the photon density which is at most 1.7×10^{-7} per cm^3.

GUTs contain leptoquarks X, Y which transform quarks into leptons. By using the mass of the boson X leptoquark, monopoles are expected to be superheavy,

$$m_M \simeq \frac{m_X}{a_{GUT}} \simeq 10^{16} GeV \simeq 0.02 \mu g. \tag{80}$$

Combining this mass with the number densities of baryons and monopoles, the density parameter of monopoles becomes

$$\Omega_M = \frac{N_M}{N_B} \frac{m_M}{m_B} \Omega_B \simeq 6 \times 10^{17}. \tag{81}$$

Such a universe would be closed and its maximal lifetime would be only a fraction of the age of the present Universe, of the order of

$$t_{max} = \frac{\pi}{2H_0\sqrt{\Omega_M}} \simeq 13yr.^8$$

Monopoles have other curious properties as well. Unlike the leptons and quarks which appear to be pointlike down to the smallest distances measured ($10^{-19}m$) the monopoles have an internal structure. All their mass is concentrated within a core of about 10^{-30} m, with the consequence that the temperature in the core is of GUT scale or more. Outside that core there is a layer populated by X leptoquark vector bosons, and outside that at about 10^{-17} m there is a shell of W and Z bosons. This structure may affect the stability of matter.

6.3 How inflation solve these problems

Now we will present the main idea of inflation and the solution to the three cosmological puzzles which are given above. For this purpose we will here simply assume that the universe went through an early period of exponential expansion, without worrying about how this came about. We will concentrate on monopole problem, but the most serious of the above three cosmological problems is the horizon problem. There are also possible solutions of the flatness and monopole problems that do not rely on inflation. However, when inflation solves the horizon problem it automatically solves not only the flatness problem, but also the monopole problem.

What Guth realized was that during inflation $\dot{a}/a$ ($\equiv H$), would have been roughly constant, so $|K|/a^2H^2$ would have been decreasing more or less like a^{-2}. By having this in mind, we will give a solution to the flatness problem. Suppose that the universe began with a period of inflation during which $a(t)$ increased by some large factor $e^{\mathcal{N}}$, followed by a period of radiation dominance lasting until the time of radiation-matter equality, followed in turn by a period of matter dominance and then a period dominated by vacuum energy. If $|K|/a^2H^2$ had a value of order unity at the beginning of inflation, then at time t_I of the end of inflation $|K|/a^2H^2$ would have had a value $|K|/a_I^2H_I^2$ of order $e^{-2\mathcal{N}}$ (where a_I and H_I are the Robertson-Walker scale

[8] In a closed universe ($K = +1$) for the maximum scale we get that $a_{max}^{-2} = \frac{8\pi G\rho}{3} = \frac{8\pi G}{3}\rho_{crit}\Omega$, as $\dot{a}$, and $t_{max} = \frac{\pi}{2}a_{max}$, where we have used units with $c = 1$.

factor and expansion rate at this time), and today we will have

$$|\Omega_K| = \frac{|K|}{a_0^2 H_0^2} = e^{-2\mathcal{N}} \left(\frac{a_I H_I}{a_0 H_0} \right)^2 . \tag{82}$$

Thus, the flatness problem is avoided if the expansion during inflation has the lower bound

$$e^{\mathcal{N}} > \frac{a_I H_I}{a_0 H_0}. \tag{83}$$

To have an estimation of the above limit, we have to evaluate the number $\mathcal{N}$, the so called number of e-foldings. To evaluate this, we will make the assumption that not much happens to the cosmic scale factor and expansion rate from the end of inflation to the beginning of the radiation-dominated era, so that

$$a_I H_I \simeq a_1 H_1,$$

the subscript 1 denoting the beginning of the radiation-dominated era. By using the resent observation results we have that the condition to solve the flatness problem is $\mathcal{N} > 62$ (62 e-foldings).

For the solution to the horizon problem, we have to define the proper horizon size at the time t_L of last scattering which is

$$d_H(t_L) \equiv a(t_L) \int_{t*}^{t_L} \frac{dt}{a(t)}. \tag{84}$$

We assume that during inflation $a(t)$ increased exponentially at a rate H_I, so that

$$a(t) = a(t_*) e^{[H_I(t-t_*)]} = a_I e^{[-H_I(t_I - t)]}.$$

If we define the number of e-foldings of expansion during inflation as $\mathcal{N} \equiv H_I(t_I - t_*)$, it gives that

$$d_H(t_L) = \frac{a(t_L)}{a_I H_I} \left[e^{\mathcal{N}} - 1 \right]. \tag{85}$$

It can also be shown that the horizon problem can be solved with $\mathcal{N} > 68$ and with 68 e-foldings. Then the initial particle horizon has been blown up by a factor 10^{29} to a size vastly larger than our present Universe.

The above inflationary picture also solves the monopole problem by blowing up the size of the region required by one monopole. This exponential expansion would have greatly reduce the monopole to photon ratio. It requires the number $\mathcal{N}$ of e-foldings to be greater than 23. Of course another possible solution of the monopole problem is that inflation ends at a temperature below the grand unification scale M, so that there never was a time when the grand unification group was unbroken. An even simpler possibility, which does not rely on inflation, is that there may be no simple gauge group that is spontaneously broken to the gauge group $SU(3) \times SU(2) \times U(1)$ of the Standard Model.

The inflationary models predict that the duration of inflation is about 10^{-34}s and as we saw above when inflation solves the horizon problem, it automatically can solve not only the flatness problem, but also the monopole problem.

7 Searches for Magnetic Monopoles

As we mentioned, magnetic monopoles have been a subject of interest since Dirac established the relation between the existence of monopoles and charge quantization. Also, with the advent of "more unified" non-Abelian theories, classical composite monopole solutions were discovered. The mass of these monopoles would be of the order of the relevant gauge-symmetry breaking scale, which for grand unified theories is of order 10^{16} GeV or higher. But there are also models where the electroweak symmetry breaking can give rise to monopoles of mass ~ 10 TeV.

Since the revival of interest in monopoles in the 1970s, there have been two well-known announcements of their discovery: that of Price (1975), who found a cosmic ray track etched in a plastic detector, and that of Cabrera (1982), who reported a single event in an induction loop. The former interpretation was immediately refuted by Alvarez (1975), while the latter has never been duplicated, so is presumed spurious.

Before 2000, the best direct limit on magnetic monopoles was that obtained at Fermilab by Bertani (1990) who obtained cross section limits 2×10^{-34} cm^2 for monopole masses 850 GeV. Also, the Oklahoma experiment (2004), while not extending to as high masses, gives cross section limits some two orders of magnitude smaller.

Various experiments have also been conducted to look for cosmic monopoles. An interesting limit comes from the Rubakov-Callan mechanism for monopole catalysis of proton decay (1981, 1982),

$$M + p \to M + e^+ + \pi^0, \tag{86}$$

where MACRO (2002) found a limit on the flux of $3-8\times 10^{-16}$ cm^{-2}s^{-1}sr^{-1}.

Also, one can show theoretically, that the Large Hadron Collider (LHC) and especially the $\gamma\gamma$ channel of it, is an ideal machine to discover monopoles with masses below 1 TeV at present running energies and with less than 1 fb^{-1} of integrated luminosity. The MoEDAL experiment, installed at the LHC, is currently searching for magnetic monopoles and large supersymmetric particles using layers of special plastic sheets attached to the walls around LHCb's VELO detector. The particles it is looking for will damage the sheets along their path, with various identifying features.

Above, we present some aspects for searching directly monopoles as free particles. There has been also some indirect searches for magnetic monopoles. The indirect searches that have been proposed and carried out, rely on effects attributable to the virtual existence of monopoles. De Rújula in 1995 proposed looking at the three-photon decay of the Z boson, where the process proceeds through a virtual monopole loop. Similarly, Ginzburg and Panfil in 1982 and Ginzburg and Schiller in 1999 considered the production of two photons produced either from e^+e^- or $q\bar{q}$ collisions. Again the final photons are produced through a virtual monopole loop. Based on this theoretical scheme, an experimental limit was given by the D0 collaboration, which sets the following bounds on the monopole mass M:

$$\frac{M}{2|m'|} > \begin{cases} 610 \text{ GeV} & \text{for } S = 0 \\ 870 \text{ GeV} & \text{for } S = 1/2, \\ 1580 \text{ GeV} & \text{for } S = 1 \end{cases} \tag{87}$$

where S is the spin of the monopole and $m' = eg$ is the magnetic charge quantization number.

7.1 Monopoles and Condensed-Matter Physics

While a magnetic monopole particle has never been conclusively observed, there are a number of phenomena in condensed-matter physics where a material, due to the collective behavior of its electrons and ions, can show emergent phenomena that resemble magnetic monopoles in some respect. These should not be confused with actual monopole particles: since all known particles have zero magnetic charge, it is fundamentally impossible to find a true magnetic monopole in ordinary matter made from atoms; only quasiparticles[9] are possible. In particular, the law $\nabla \cdot \mathbf{B} = 0$ is true everywhere in these systems, which it would not be in the presence of a true magnetic monopole particle.

Recent experiments have shown strong evidence for the existence of deconfined magnetic monopoles in *spin ice*, with analogous properties to the hypothetical magnetic monopoles postulated to exist in the vacuum. A spin ice is a substance that is similar to water ice in that it can never be completely frozen. This is because it does not have a single minimal-energy state.

[9] In physics, quasiparticles (and related collective excitations) are emergent phenomena that occur when a microscopically-complicated system such as a solid behaves as if it contained different (fictitious) weakly-interacting particles in free space. Quasiparticles are most important in condensed matter physics, as it is one of the few known ways of simplifying the quantum mechanical many-body problem (and as such, it is applicable to any number of other many-body systems).

A spin ice has "spin" degrees of freedom (i.e. it is a magnet), with frustrated interactions which prevent it freezing. It shows low-temperature properties - in particular residual entropy - closely related to those of crystalline water ice. The most prominent compounds with such properties are dysprosium titanate and holmium titanate. The magnetic ordering of a spin ice resembles the positional ordering of hydrogen atoms in conventional water ice.

One example of the work on magnetic monopole quasiparticles is a paper published in the journal Science in September 2009, in which researchers Jonathan Morris and Alan Tennant along with Santiago Grigera described the observation of quasiparticles resembling magnetic monopoles. A single crystal of dysprosium titanate in a highly frustrated pyrochlore lattice was cooled to a temperature between 0.6K and 2.0K. Using observations of neutron scattering, the magnetic moments were shown to align in the spin ice into interwoven tubelike bundles resembling Dirac strings. At the defect formed by the end of each tube, the magnetic field looks like that of a monopole. Using an applied magnetic field to break the symmetry of the system, the researchers were able to control the density and orientation of these strings. A contribution to the heat capacity of the system from an effective gas of these quasiparticles was also described.

Another example of the work on magnetic monopole quasiparticles is a paper in the February 11, 2011 issue of Nature Physics which describes creation and measurement of long-lived magnetic monopole quasiparticle currents in spin ice. By applying a magnetic-field pulse to a Dy2Ti2O7 spin-ice crystal at 0.36K, the authors created a relaxing magnetic current that lasted for several minutes. They measured the current by means of the electromotive force it induced in a solenoid coupled to a sensitive amplifier, and quantitatively described it using a chemical kinetic model of point-like charges obeying the Onsager-Wien mechanism of carrier dissociation and recombination. They thus derived the microscopic parameters of monopole motion in spin ice and identified the distinct roles of free and bound magnetic charges.

8 Conclusions

In this review, we considered the possibility of existing magnetic monopoles either as point like particles or as extended objects. The point of view was especially theoretically and we also showed its influence in Cosmology. As we said, the symmetry of Maxwell's equations when monopoles exist is not a proof for the existence, but Dirac with his quantization condition incited the interest of theorists and experimental physicists to study and try to observe them. The best theoretical predictions come from Gauge Theories and GUTs, but up today none experiment observed magnetic monopoles.

Nevertheless, the lack of experimental results, is not an evidence for the absence of monopoles. As we mentioned, we have to approach very high energies to observe them, if the theories that predicts monopoles are correct. We might observe them directly or indirectly and the recently progress that made in condensed matter physics and spin ice, in the direction of searching magnetic monopoles, might give great results. Also, large accelerators and especially LHC (today) maybe detect monopoles in the future.

Until then, it remains to be seen whether we will keep writing, for the magnetic field, that $\nabla\cdot\mathbf{B} = 0$ or we will change the common form of Maxwell's equations.

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