# The Greek Geometry Textbooks from 1975 till 2019: An Anthropological Approach. 

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Master Thesis

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## Пєрíגŋчף



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#### Abstract

Euclidean Geometry is a significant branch of mathematics that concerns the mathematics community. It is a subject of the school environment attracting scientists and educators to study its history more than any other subject of school mathematics. This thesis investigates the role and the use of images and tasks in the Greek Euclidean Geometry textbooks of Lyceum from 1975 to date. The aim is to examine the ways that Geometry concepts are introduced to students in texts through three categories of images and four categories of tasks. Furthermore, it will be identified whether the priorities of authors have changed or not. To this purpose, I employ the Anthropological Theory of the Didactic (ATD). The results have shown that there is a dominant category in images which has been used mostly from the authors; nevertheless, its use declines over time. As for the tasks, from 1990 onwards, the increase in the use of a particular category modified slightly the findings. It is found that Greek Euclidean Geometry textbooks put less emphasis on the mathematical proof over time and as a consequence the categories of tasks the students engaged are influenced. With a view to gain an insight in these phenomena, the results were linked to the wider socio-cultural context of educational reformations in Greece.


Keywords: Geometry, mathematics, textbooks, images, tasks, Anthropological Theory of the Didactic

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## Abbreviations

| I | Introduction to Euclidean Geometry |
| :--- | :--- |
| T | Triangles |
| PL | Parallel Lines |
| Q | Quadrilaterals |
| IS | Inscribed Shapes |
| A and S | Analogies and similarity |
| MR | Metric Relations |
| A | Areas |
| CM | Circle's measurement |
| S | Solid Geometry |
| NC | Non-conceptual images |
| BC | Bland- conceptual images |
| C | Conceptual images |
| Qs | Questionnaires |
| E | Exercises |
| P | Problems |
| MM | Mathematical Modelling |

## List of Papers associated with this Thesis

Parts of this Thesis have been presented in $8^{\text {th }}$ Panhellenic Conference of Hellenic Association in Research in Mathematics Education, in Nicosia of Cyprus (8/12/2019) and submitted at the Research in Mathematics Education journal (27/2/2020).

1. Dakoronia, E., Anastasakis, M., (accepted for publication). Analysis of Images of Geometry textbooks from 1975 to date. Proceedings of the $8^{\text {th }}$ Panhellenic Conference of Hellenic Association in Research in Mathematics Education, Nicosia, Cyprus (In Greek).
2. Dakoronia, E., Anastasakis, M., (submitted). What kind of mathematical knowledge is promoted by the images of Greek schoolbooks of geometry from 1975 to date. Research in Mathematics Education (In Greek).

## Chapter 1 Introduction

### 1.1. Importance of this study

Geometry is a significant branch of mathematics, since it is interconnected with concepts such as space, schemas, movement and time (Koג $\dot{\zeta} \zeta \alpha, 2017$ ). Although there seems to be a decline in educational research for Euclidean Geometry since the 1970s (Inglis and Foster, 2018), it seems that mathematics educators are still interested in aspects about the teaching and learning of Euclidean Geometry (Jones, 2002). In Greek school environments, the type of Geometry that students are exposed is Euclidean Geometry, a subject characterized as "tough", due to the visual thinking required from students (Xaıpغ́tๆ, 2009). Through the study of Euclidean Geometry students are able to become familiar with the ways mathematical knowledge is organised while taking into account the comprehension of the relevant theory (Палаүعळрүíov каı Т弓єка́кๆ, 2017). To my way of thinking Geometry is a field of mathematics that does not consist of algorithms (learned by heart) and students acquire the knowledge of solving problems and constructing, thus comprehending proofs. This occurs since they are able to gain most of the information from the figure and the knowledge can be acquired from theorems and postulates with a view to achieve the complement of problems and proofs. The teaching of Geometry pertains to recognising "geometrical problems and theorems, appreciating the history and cultural context of Geometry and understanding the many and varied uses to which Geometry is put" (Jones, 2002, p.122).

Without doubt, textbooks are a dominant source of students' learning in classrooms (O'Keeffe and O'Donoghue, 2011) and the interest in the research on mathematics textbooks is expressed at an international level (Johansson, 2003). Although textbooks are considered the main source of studying mathematics in secondary education (Shield and Dole, 2012) and influence students' beliefs about "what is" and "what it means I know mathematics" (Koдદ̧́ $\alpha, 2017$ ), research on Euclidean Geometry textbooks in Lyceum remains an unexplored field. In contrary, researches relevant to notions such as "what is" and "what it means I know Geometry" are not existing in the literature. In other words, there are no researches studying what is acceptable as a knowledge in Geometry, as it is promoted by textbooks. Additionally, it is a "visual, intuitive, creative and demanding" field (Jones, 2002, p.134). Those reasons and my personal interest in Geometry throughout my secondary and tertiary education, lead me to examine what kind of knowledge is promoted by Geometry textbooks by studying the role of the images and tasks used in school textbooks from 1975 to 2019. Let me conclude by noting though that it was not my aim to demonstrate which categories of images and tasks seems appropriate or not for students learning and as a consequence should be part of textbooks.

### 1.2. Expressing the research questions

By implementing this study, I attempt to provide answers to questions related to "what is" and "what it means I know Geometry". In other words, my intention is to examine what is considered as acceptable knowledge in Greek Geometry textbooks by analysing the images and tasks used in them, as well as examining whether any changes have occurred through the years examined (1975 to 2019). In order to do this, I will attempt to answer the following research questions:

RQ 1 What categories of images can be found in Greek Geometry textbooks from 1975 to 2019 ?
RQ 2 Are there any variations in their use throughout the years?
RQ 3 Having in mind the different categories of images used in these textbooks, what can be said about the ways that geometrical concepts are presented to students?
RQ 4 What categories of tasks can be found in Greek Geometry textbooks from 1975 to 2019 ?
RQ 5 Are there any variations in the use of categories of tasks throughout the years?
RQ 6 What can be said about the ways that the tasks are presented to students?

## 1. 3 Outline of the thesis

### 1.3.1 Structure of Chapters

In Chapter 2 the review of the literature is presented. Section 2.2 introduces the reader to Geometry and Euclidean Geometry, as presented in Greek textbooks. A review on textbooks and the ways that influence the teaching and learning processes are demonstrated in Section 2.3, while the types of curriculum and which of them is about to being used in this research is presented in Section 2.4. The following sections (Sections 2.5 and 2.6) review the literature related to the role of images (Section 2.5) and tasks (Section 2.6) that examined in all textbooks of Euclidean Geometry from 1975 to 2019. Finally, Section 2.6 provides researches about textbooks in primary and secondary education and also the existing gaps that this thesis aiming at identifying answers.

In Chapter 3, the theoretical framework that guided my study, the Anthropological Theory of Didactic (ATD), is presented. Section 3.2 introduces the reader to ATD and presents briefly the methodologies usually used within this framework. The model of didactic transposition process is analysed in Section 3.3, while the most essential component of ATD, praxeology, is explained in Section 3.4. At the end of this chapter (Section 3.5), I outline the ways that certain aspects of ATD are going to be used with a view to provide answers to my research questions.

Chapter 4 describes the methodology followed throughout this research. Section 4.2 consists of four subsections. In Section 4.2.1, general information about the Geometry in Greek school environments is presented, while in Section 4.2.2 how the topics of

Geometry used in this study is explained. Specific details about every textbook of Euclidean Geometry used in Greek schools and how each textbook was prepared for analysis are presented in Sections 4.2.3 and 4.2.4 respectively. Moving on, the literature used for categorising images and tasks are presented in Sections 4.3 and 4.4. In Section 4.5, the method used for analysing my data (Qualitative Content Analysis) is presented, while the definitions about the three categories of images and the four categories of tasks are presented in Sections 4.6 and 4.7 respectively. The chapter ends with a brief presentation about the changes that occurred in the pages of topics in every textbook (Section 4.8).

In Chapter 5, the results of my study are demonstrated. The chapter is divided into two main parts, one for images (Section 5.2) and one for tasks (Section 5.3). In Section 5.2.1 overall results for the use of images in all Books are presented, while in Section 5.2.2, I examine the use of images in each topic of Euclidean Geometry. Image analysis results are summarised in Section 5.2.3. For the analysis of tasks (Section 5.3), the same structure has been used. In Section 5.3.1 overall results related to the presence of four categories of tasks in Books are presented, while their presence in each topic is analysed in Section 5.3.2. Finally, the most essential features of the analysis for tasks are summarised in Section 5.3.3.

In Chapter 6, the answers to my research questions are presented. In doing so, I interpret the results of my analysis for the presence and use of images and tasks in Greek textbooks of Euclidean Geometry, through the lens of ATD. Section 6.2 provides answers to my research questions for images from the perspective of ATD and the socio-cultural environment of Greece. Similarly, Section 6.3 identifies answers to the tasks using the terms of ATD and the ways that the results of tasks can be interpreted through other researches. Chapter 6 ends with my study's limitations and possible directions for future research.

### 1.3.2 Appendices

Appendix A provides information about the topics observed in Greek Books of Euclidean Geometry and the changes that found in the mathematical contents of those topics from 1975 until 2019 are presented.

In Appendix B, details about the $z$-test are presented. Z-test is a type of hypothesis test used in statistic in order to estimate if the differences between two groups are statistically significant or not, compared to a categorical characteristic.

Appendix C provides information about the presence of images (NC, BC C) in the selected sample of textbooks (A, B, C, D, E, F, G, H) of Euclidean Geometry in Lyceum from 1975 to 2019.

Finally, Appendix D informs about the presence of tasks (Qs, E, P, MM) in the selected sample of textbooks (A, B, C, D, E, F, G, H) of Euclidean Geometry in Lyceum from 1975 to 2019.

## Chapter 2 Literature Review

## 2. 1. Introduction

The chapter of literature review comprises several sections; Firstly, a short introduction to Geometry as a subject and the teaching of Euclidean Geometry in Greek schools is presented (Section 2.2). This is followed by literature related to textbooks (Section 2.3) and how different types of curriculum influence teachers' teaching (Section 2.4). Then, literature related to the role of images (Section 2.5) and tasks (Section 2.6) in textbooks is presented. Finally, a summary of studies about research into textbooks (internationally and in Greece) alongside with the reasons of carrying out this particular study are presented (Section 2.7).

## 2. 2. Geometry

Geometry plays a significant role in the community of mathematicians. As a school subject it is interconnected with notions such as space, geometrical figure movement and time (Koдह́ॅ $\alpha$, 2017). Taking a look into the origin of the word Geometry, it is noticeable that is divided into two sub-words "geo" and "meter", with the first meaning earth and the latter meaning measure (Jones, 2002), meaning that Geometry is not limited to the study of geometrical figures. According to Freudenthal (1973), "Geometry is a grasping space [...] in which the child lives, breaths and moves. The space that the child must learn to know, explore, conquer, in order to live, breath and move better in it" (p.403). It can be characterised as a "difficult subject" because no algorithm can be used without even being understandable from students (Xaıр ́єๆ, 2009).

Initially, Geometry counts a number of influences; It was assumed as the "ancient branch of Mathematics" with a variety of influences from cultures like Indians, Babylonians, Egyptians, Chinese and Greeks. (Jones, 2002, p.122). Euclid around 300 BC coded Geometry in text, known as Euclid's Elements, in which comprises of 13 Books. Elements consists of 10 axioms and postulates, and a variety of theorems, proven with didactive reasoning (Ibid.). Jones (2002, p.123) claims that "no other work has exercised a greater influence on scientific thinking [...], over a thousand editions of Euclid's Elements have appeared since the first printed edition in 1492". Elements are in the centre of mathematics, dominating the teaching of Geometry for over 2000 years (O' Connor and Robertson, 2002 cited at Johansson, 2005).

Euclidean Geometry is being taught in Greek schools. As a subject of the school curriculum, it captures the interest of "the historians of science and education more than any other branch of school mathematics" (Toumasis, 1990, p.491). It is the field of mathematics that provides access to students in order to study how the mathematical knowledge is organised, while understanding the relevant theory ( $\Pi \alpha \pi \alpha \gamma \varepsilon \omega \rho \gamma$ iov and Тऽєка́кๆ, 2017). In the study, А $\gamma \gamma \varepsilon \lambda \eta$ and Гаүо́тбŋऽ (2017) conclude that the understanding of geometrical figures has significant impacts on the construction of a proof; nevertheless, students tend to approach geometrical figures verbally, since they
are not familiar with the relations between geometrical objects. In terms of geometrical figures, Fischbein (1993) specified the three roles of a figure: (1) definition, (2) image and (3) figural concept. He reports that:
"(a) a geometrical figure is a mental image, the properties of which are completely controlled by a definition; (b) a drawing is not a geometrical figure itself, but a graphical or a concrete, material embodiment of it; (c) the mental image of a geometrical figure is, usually, the representation of the materialized model of it. The geometrical figure itself is only the corresponding idea that is the abstract, idealized, purified figural entity, strictly determined by its definition" (p.149).

The curriculum of Geometry in Greek schools has changed significantly since 1886 when it was first introduced. In his study, Toumasis (1990) reports that seven hours were allocated for the teaching of Geometry in 1886, while Trigonometry was a distinct subject (allocated two hours of teaching). In this syllabus no hours for Algebra were dedicated. In 1922, two hours were dedicated for the teaching of Algebra whereas Trigonometry and Geometry reduced at one and four hours respectively. Analytical Geometry was part of the curriculum of the third grade of Lyceum from 1922 until the mid of 1970's. Nowadays, Geometry can be seen as "a subdomain of Mathematics throughout the history and even today in primary schools, Geometry is given under mathematics" (Serin, 2018, p.134). In other words, Geometry in the educational system is not given the proper attention as supposed to, since it is taught for cultural reasons, according to the interviewed teachers in the study of Kuzniak and Vivier (2010). No matter what position has Geometry throughout the years, it should be kept in mind that beyond the geometrical figures or the structures that are an integral part of this field, it is accompanied by an axiomatic system. Particularly, Lemonidis (1997) pointed out that every geometrical figure drawn in the paper it is a scheme, defining by a definition, and not just a picture.

## 2. 3. Research on textbooks of Mathematics

Mathematical textbooks influence students' beliefs regarding "what is Mathematics" and what it means "I know mathematics" (Koдह́ॅa, 2017). According to Shield and Dole (2012), textbooks constitute the main source of studying mathematics in secondary education. Textbooks are considered as an "integral part" of the secondary education with a view to promote the teaching and learning of mathematics (Ibid.). The process of teaching and learning of mathematics is mostly based on textbooks and Remillard (2018) suggests that textbooks provide guidance, support and are able to enrich this process in the school environments. They can also influence teachers and students directly (Mouzakitis, 2006) This is because teachers acted as part of the curriculum and organize the teaching of mathematics, while for students it is a resource of understanding the mathematical knowledge through the theory and the activities. It could be said that textbooks can affect human notion about "what, to whom, when and how" the mathematical knowledge can be taught (M $\pi \lambda^{\prime} \lambda i ́ v \eta$, 2018, p.6).

Textbooks are a source of supporting teaching and learning process (O'Keeffe and O'Donoghue, 2011), so they act as a mediator between what is supposed to be taught in the classroom, as the curriculum provides, and what is actually taught in the enactment process between the teacher and the students in classroom. As a primary and integral part of the curriculum, Valverde, Bianchi, Wolfe, Schmidt and Houang (2002) report that textbooks "are written to serve teachers and students in this way -to work on their behalf as the links between the ideas presented in the intended curriculum and the very different world of classroom" (p.55). On the contrary of this belief, Remillard (2018, p.70) reports that
"mathematics teaching and learning resources refer to a genre of materials and tools, including, but not limited to textbooks, designed to guide, support, and enhance mathematics teaching and learning in schools. For many years, the textbook, accompanied in some countries by a teacher's guide, served as the primary instructional resource found in mathematics classroom. In 2017, the list of types of resources available is long and diverse, including, but not limited to print, digital, and online materials and tools used either periodically or over an extended period of time. Some resources are designed to be used to support and guide instruction; others are resources taken up by teachers and deployed as instructional tools."

Among the first books ever printed was "Orbis Pictus", which was for children and it contained pictures. This book was dated back in the 1658 and it was written by the educator Comenius (Qi, Zhang and Huang, 2018). Although textbooks support the teaching and learning process, Кодغ́ఢ $\alpha$ (2017) notes that the assessment of Mathematics textbooks has started only in recent times. The acquiring of new knowledge depends on the content of the textbooks and the way teachers will use them (Ibid.). They also serve another aim; teachers will rely on them to organise the structure of their teaching (Apple, 1992), and students will also rely on them to comprehend the taught knowledge. Chamblis and Calfee (1998) argued that the $75 \%-90 \%$ of teaching activities is determined by the content of the textbooks (cited in Qi et al, 2018). Another role textbooks serve is to teach mathematical concepts, enhance students skills, provide strategies for the problem solving process in a mathematical reasoning (Shuard and Rothery, 1984 as cited at O'Keeffe and O'Donoghue, 2011).

A question then arises; Is students' learning dominated by the content of textbooks? The answer Koдغ́ $\zeta \alpha$ (2017) provides underlines the role of the teacher; the way teachers use this content will have an impact on student's learning. Both textbooks and teacher guides are the main sources of covering the concepts in classroom (Pepin and Haggarty, 2001). Qi et al. (2018) report that textbooks can be seen as "supporting material" for teachers and students. Under this point of view, teachers rely on textbooks for their teaching while students rely on textbooks for their learning. Initially, textbooks were used for controlling teaching (Guo, 2001), while now they are used to support purposes. However, Vincent and Stacey (2008) conclude in their literature that "many teachers
rely on textbooks for instructional materials". To my way of thinking, this belief is explained by the way that curriculum influences the teaching and learning of mathematics, and it is going to be discussed in Section 2.4.

Mathematics textbooks provide knowledge divided into three areas (Pepin and Haggarty, 2001): (1) what kind of mathematics is represented in textbooks? (2) the beliefs pertaining to the nature of mathematics, while taking into account how they appeared in textbooks, and (3) the mathematical knowledge as it is demonstrated in textbooks (Pepin and Haggarty, 2001). According to Reys, Chaves and Reys (2004) mathematics textbooks serve three roles; Initially, the mathematical concepts are introduced to students via the sequence of chapters. This means that students should achieve the understanding of a chapter before moving on to the following, which might be more complex. Their second role is the mathematical topics supposed to be taught by teachers. These topics might be materials from previous grades or new types of topics. Lastly, activities, problems, examples and assignments are provided by textbooks in order to help teachers plan their teaching-

Rezat's (2009) attempt to summarise the ways textbooks promote the teaching and learning of mathematics can be summarised by his Tetrahedron model. In this model (Figure 1) it can be seen how the textbooks, the teacher, the student and Mathematics (in particular the mathematical knowledge) interact.


Figure 1: Tetrahedron model of textbook use (Rezat, 2009, p.1261)
Since using this model with the four basics, Rezat explained the role served by mathematics textbooks and the way those textbooks should be used by teacher and student.
> "mathematics textbooks should not be a subject to analysis detached from its use. It is an interactive part within the activities of teaching and learning mathematics [...] The mathematics textbook is implemented as an instrument at all three sides of the triangle: teachers use textbooks in the lesson and to prepare their lessons, by using the textbook in the lesson teachers also mediate textbook use to students, and finally students learn from textbooks" (p.260-261).

Additionally, pedagogy tensions and sociocultural beliefs can emerge from textbooks. Stray (1994) reports that textbooks "are designed to provide an authoritative
pedagogic version of an area of knowledge" (p.2). In sociocultural terms, they "present for each generation of students an officially sanctioned, authorized version of human knowledge and culture" (Castell, Luke and Luke, 1988, p.vii). In other words, textbooks authorize which type of knowledge is acceptable in specific time (Kод $\varepsilon$ 宂 $\alpha, 2017$ ). Specifically, what a teacher is expected to teach and what students are expected to learn. School Mathematics are considered as a "complex cultural product" (Ibid., p.344). This is a result of the coding messages that are exposed through text. Stray (1994) demonstrates that terms like "what should be taught" are combined with pedagogy tensions such as "how should be taught and learnt". Finally, Pepin and Haggarty (2001) conducted a study about Mathematics textbooks in France, Germany and England and they concluded that mathematics culture tensions should be comprehended in order to promote the process of mathematics education.

## 2. 4. Types of Curriculum

As it is already noted, mathematics textbooks act as the mediator between the curriculum and the enactment process in the classroom of a school environment (Valverde, Bianchi, Wolfe, Schmidt and Houang, 2002). In other words, curriculum and textbooks are interconnected. In order to achieve a better notion of textbooks, it seems essential to take into consideration the curriculum. Although the meaning of term curriculum varies, Remillard and Heck (2014) define mathematics curriculum as "a plan for the experiences that learners will encounter, as well as the actual experiences they do encounter, that are design to help them reach specific mathematics objectives" (p.707). In other words, the mathematics curriculum incorporates what students should do to acquire the knowledge of the new mathematical concepts, to promote and achieve deep learning, as well as in which part of the content teachers should focus in their teaching.

According to Remillard and Heck (2014) there are two types of curriculum; the official and the operational or implemented. The first is considered as the one expressing what should be taught during the teaching process. It mostly refers to the goals that educators should achieve and the activities they should engage to in the textbook. The latter takes place during the teaching, while taking into account teacher's work in classroom.

In the official type of curriculum, agencies from the governments "authorize" what is expected to be taught to students and the resources or pathways that can be used to promote deep learning (Remillard and Heck, 2014). Hence, the operational curriculum is influenced by the official, since the latter incorporates: (1) the aims and the objectives, (2) the content of consequential assessments and (3) the designated curriculum (Ibid.). Aims and objectives shape the curriculum and are referred to desired expectations about students learning. In order to test the progress that has been made by students and also measure students' knowledge and skill, the curriculum incorporates a content of consequential assessments. Finally, the designated curriculum indicates instructions which are proposed to teachers by the ministry of education in order to plan their teaching.

When the teacher counts on the official curriculum during his teaching and tries to apply it in the classroom, then the operational curriculum occurs. It could be agreed that teachers act as the "co-modulator" of the curriculum during the enactment process of
 according to the curriculum. An implemented curriculum will be referred as "the results of transformations that occur through the enactment process and exist outside the official domain" (Remillard and Heck, 2014, p.710). This type of curriculum also takes into account three parameters: (1) the teacher-intended curriculum, (2) the enacted curriculum and (3) student outcomes. Firstly, it is the teacher curriculum which consists teachers' decisions and plans with a view to apply them in the classroom. It takes place in the mind of the teacher, since they make the choices about how to perform specific mathematical contents of the teaching and they design it for specific students in a particular time. When the teacher performs the mathematical knowledge in the classroom, the interaction between them and the students is considered as the enacted curriculum. It is affected by their knowledge and their beliefs. Lastly, the results students achieve during the enacted curriculum are attributed to the term student outcomes. Students achieve learning through their interaction with the teacher, the tasks and the tools (e.g. textbooks, calculator, computers). Since addressing the two types of curriculum, the official curriculum is referring to "the goals and activities outlined by school policies or designed in textbooks" while the intended curriculum is referring to "teachers' aims" (Gehrke et al., 1992 as cited in Remillard, 2005, p.213).

This study will be focusing on the official curriculum because I did not have access into what teachers actually did while teaching Geometry in the classroom (i.e. the operational curriculum). The unit of this analysis is the textbooks of Euclidean Geometry in Lyceum, in which authors' aims about the mathematical concepts that should be taught to students are encapsulated in textbooks.

## 2. 5. The Role of Images in Mathematics Textbooks

Images are an essential part of any textbook. No mathematics textbook can exist without the inclusion of visuals. From the early school years in elementary our teachers used to claim that wherever there is a geometrical figure, plenty of information arises. In other words, geometrical figures provide mathematical ideas, especially in the solving process of tasks or in the mathematical proofs. This perception is also common in the literature. Hanna and Sidoli (2007) highlight that structure representation can provide "a wealth of information" in a proficient way (p.76). Visualization is taken to be the capacity to "represent, transform, generate, communicate, document, and reflect on visual information" (Jones and Tzekaki, 2016; Hershkowitz, 1990, p. 75), and so visual construction is a benefit to students, as they are able to facilitate their reasoning. Hanna and Sidoli (2007) support that most visual representations can usually "inspire" a theorem which has to be proven and discover the proof construction in a rigorous way step by step.

According to the theory of Duval (1998), a geometric object can be approached verbally or graphically. The first case can be achieved by theorems and postulates,
while the latter by the visualization, which corresponds to the way that an individual can gain information through images. Elia and Phillipou (2004) expand this notion by claiming that pictures act "as a mediator between the practical and the theoretical formal level of understanding [...] pictures play an important role as an aid for supporting reflection and as a means in communicating mathematical ideas" (p.327).

Studies assessing the role of images in mathematics textbooks are scarce in Greek bibliography. For instance, Elia and Phillipou (2004) analysed the role of visual representation in problems according to four categories, while Bouк\& $\lambda \alpha ́ \tau o v ~(2017) ~$ examined the gender representations in the mathematics textbooks of the second grade of Gymnasium. A $\gamma \gamma \varepsilon \lambda \eta$ and Гaүó $\tau \sigma \eta$ ( 2017 ) studied the understanding of a geometrical figure and how this has an impact on the reasoning of students through the
 analysed the intersection content of mathematics textbooks to the sixth grade of primary school and in the first grade of Gymnasium. In their research, they claim that the majority of sentences' content pertains to mathematics language while a low proportion of this content pertains to other scientific fields (i.e. Geography, etc).

## 2. 6. The Role of Tasks in Mathematics Textbooks

Tasks in this study will be defined as items assessing mathematical facts, standard methods and techniques, extra-mathematical situations and connections between mathematics and real-life situations. Tasks (e.g. exercises and problems) have an impact on the perception of students about mathematics (Mason, 2000). $\Theta \omega \mu \alpha$ and N $\alpha \rho \delta \dot{\eta}$ (2015) claim that mathematics tasks students should solve, during their schooling, are determined by their relation with the mathematics itself and their personal experience. The question arises is what really a task means? According to Niss (1993), tasks can be defined as "an orientated activity, i.e. a set of actions orientated towards undertaking certain missions such as orders, proposals, or challenges" (p.17). From tasks, it can become evident which mathematical facts (i.e. definitions, theorems) a student is or not familiar with, which standard methods and techniques (i.e. solutions to problems, display of results) have mastered to obtain results, and finally, how they can provide solutions to extra-mathematical situations (Ibid.).

Tasks do not only serve the knowing and learning of student but also, as Ridway and Passey (1993) conclude, "assessment is a murky business" (p.72). They reflect beliefs about mathematics knowledge and mathematics itself, the teaching and learning process and the way that the students, the teacher and the society are related (Ibid.). To my way of thinking, all categories of tasks represent for students the mathematical concepts that should be understandable. This can be gained through specific categories (i.e. questionnaires, exercises, problems) that student are required to solve. Their level of complexity varies from task to task.

There is a lack of studies examining the role of tasks in Greek bibliography. It should be also mentioned that studies which investigate the role of tasks, do not examine tasks from Euclidean Geometry textbooks, but general in nature problems. However, there was a research focused on the strategies being used from students during the solution
of tasks (Avסроviкíסov, $\Delta \alpha \tau \sigma o \gamma 1 \alpha ́ v v \eta$ and M $\varepsilon \lambda i ́ \delta o v, 2017$ ). The interest of the most researchers was in mathematical modelling, a trend that combines problems with the real world.

## 2. 7. Discussion

Although textbooks capture the interest of mathematics education researchers (Johansson, 2003), investigations around the theme "what is" and what it means "I know mathematics" attract researchers' attention to a lesser degree. Mathematics textbooks play a significant role in the Mathematics Education Research, since they promote students opportunities about the learning of Mathematics (M $\pi i \lambda i v \eta$, 2018). There are researches about the history of geometrical concepts in mathematics; Patsopoulos and Patronis (2006) examined the Theorem of Thales, on the aspect of the culture, mathematics and education, while Papadopoulos (2008) studied the concept of
 students about the postulates and definitions of the Euclidean Geometry. On the other hand, how Geometry can help students, unable to see, comprehend algebraic symbols is examined in the study of Tov $\tau \tau \sigma \iota \alpha \kappa \eta$ and $\Sigma \tau \alpha v \rho о \pi о$ v́ $\lambda o v$ (2015). There are also studies appertaining to references about the history of mathematics in the lower secondary textbooks of Cyprus and Greece (Xenofontos and Papadopoulos, 2015) or how frequent is the incorporation of environmental issues and topics in selected mathematics textbooks of the compulsory education in Greece (Spiropoulou, Roussos and Voutirakis, 2005).

Studies can be also categorized according to the level of education that are referring. In primary education, Zacharos and Kostourakis (2011) have studied the use of realistic problems in the first school grades, while Desli and Loukidou (2014) have examined the cases of addition and subtraction in word problems of the first and second grade in Greece. Other studies have focused on the learning of Geometry to first grade students in Greece through play (X $\alpha \rho \alpha \lambda \alpha \mu \pi i \delta o v ~ a n d ~ K \lambda \omega ́ \theta o v, ~ 2017) ~ a n d ~ o n ~ t h e ~ c o n c e p t ~ o f ~$ length measurement to fourth grade students ( $Г \kappa \varepsilon v \varepsilon ́, ~ K \alpha v \varepsilon \lambda \lambda о \pi о v ́ \lambda o v ~ a n d ~ К о \lambda \varepsilon ́ \zeta \alpha, ~$ 2015). There is also a research examined the link between spatial reasoning and creation
 and Пavaүŋ́, 2015). Finally, there is the introduction of "estimation", as it is proposed in new curriculum of school mathematics in the first, second and third grade


Among the small number of studies investigating Euclidean Geometry textbooks, in the upper secondary education, are the studies by Kuzniak and Vivier (2009),
 content of textbooks in Greece and Franc, while Mouzakitis (2009) analysed the two main Euclidean Geometry textbooks of Lyceum, in Italy and Greece. Mıх\& $\alpha \alpha \rho \alpha ́ \kappa \eta \varsigma$ (2012) investigated how the four apprehensions of Duval for a geometrical figure can affect its understanding in the second grade of Lyceum.

A question arising in the community of mathematics education is what changes should be made in order to achieve a better knowing of geometry's teaching and
learning? The existent literature recognises gaps regarding "what is" and "what it means I know Geometry". In other words, what is accepted as a mathematical knowledge and how is it promoted from the official curriculum? Having in mind the gap identified in this section, the current study aims at answering the following research questions:

RQ 1 What categories of images can be found in Greek Geometry textbooks from 1975 to 2019 ?
RQ 2 Are there any variations in their use throughout the years?
RQ 3 Having in mind the different categories of images used in these textbooks, what can be said about the ways that geometrical concepts are presented to students?
RQ 4 What categories of tasks can be found in Greek Geometry textbooks from 1975 to 2019 ?
RQ 5 Are there any variations in the use of categories of tasks throughout the years?
RQ 6 What can be said about the ways that the tasks are presented to students?

## Chapter 3 Theoretical Framework

## 3. 1. Introduction

In this chapter, I present the theoretical framework that guided my research. I have based my study on Chevallard's anthropological theory (Chevallard, 2006; Bosch and Gascon, 2014) and a number of articles related to the Anthropological Theory of the Didactic (ATD). I drew on ATD, since it provided me with a perspective for accessing the ways that mathematical terms are described in the Greek textbooks of Euclidean Geometry and how they are introduced to students, via images and tasks. My understanding of the theory is based mostly on the "Introduction to the Anthropological Theory of the Didactic" written by Bosch and Gascón in 2014.

This chapter is divided into four sections. Section 3.2 specifies the ATD and demonstrates briefly its methodologies. In Sections 3.3, the model of didactic transposition process is explained while in Section 3.4 the term "praxeology" is analysed. Lastly, in Section 3.5 I provide information on the ways that images and tasks will be explained under the light of the ATD.

## 3. 2. What is the Anthropological Theory of the Didactic?

The Anthropological Theory of the Didactic (ATD) is a significant theory in mathematics education, initially developed by Yves Chevallard in the 1980s. It is mainly utilized by French-speaking researchers in Europe, Canada, and Latin America (Bosch and Gascón, 2014) and continues its development until today. In general terms, ATD is a developing theory in which every human activity related to mathematical knowledge takes place in an institution, such as a primary/secondary school, a university department, etc. Any individual with a focus on "doing, teaching, learning, diffusing, creating and transposing mathematics" (Bosch and Gascón, 2014, p.68), as well as any other kind of didactic phenomena, leads the researchers to study them further and provide a unifying theoretical framework with an anthropological approach (Chevallard and Sensevy, 2014). ATD is a very useful methodological tool since it provides the means for examining how an institution (e.g. school or university) introduces mathematical notions via textbooks and teaching (Gonzaléz-Martin, Nardi and Biza, 2011). For instance, how images and tasks can be used as a source of introducing mathematical knowledge through the textbook of Euclidean Geometry.

The ATD belongs to the "French perspective" of mathematics education alongside with the Joint Action Theory in Didactics (JATD). JATD is an anthropological approach in mathematics education initiated by Brousseau. In JATD, no human action can be interpreted without taking into account three parameters; the teacher's action, the student's action, and the diffusion of knowledge during their interaction (Chevallard and Sensevy, 2014). Hence, the aforementioned anthropological theories share a common philosophical and scientific notion, a praxeology.

One methodology of the ATD is the didactical transposition (DT) process. Although DT was not used as a methodological tool per se (i.e. when analysing data or interpreting results), it helped me understand how mathematical knowledge is transformed before being incorporated into an institution. According to DT, there are three phases that take place during the teaching and learning process of mathematics. According to Bosch and Gascón, (2014) those three are:

- The didactical transposition process
- The didactic systems
- A hierarchy of levels of the didactic codetermination

DT can be studied under the terms of praxeologies (praxeological analysis), while in didactic systems, researchers try to provide answers about the didactic object (didactic analysis). Finally, one of the ATD's biggest concerns is to study the phenomena arising during teaching and learning processes, while taking into account the influence of the external conditions, such as the society (ecological analysis).

Every didactic system is not appropriate to be considered as an interaction only between the group of X , the team of Y and an issue that needs to be explained or taught. More specifically, there are some conditions required and appeared in the classroom, but are not mentioned throughout the teaching and learning process. Those are the teachers' and students' equipment of praxeologies, the teaching material, the disciplines, the society, etc. This means that although the didactic system contains the variants $\mathrm{X}, \mathrm{Y}$ and O , there is a number of conditions that are utilized implicitly, even though they play a significant role during the process. Chevallard used the following hierarchy (Figure 2) to define the factors that can influence a didactic system (Bosch and Gascón, 2014).

On the whole, without the scale of levels of didactic codetermination, the ATD would not have succeed the "anthropological approach", as this hierarchy shows that every didactic structure is multileveled and supported by disciplines, school environments, society and pedagogy that determine the variants X and Y (Chevallard, 2014).


Figure 2: The hierarchy of didactic codetermination

## 3. 3. The Didactic Transposition Process

The didactic transposition process (DT) follows a distinct development of the ATD. According to Chevallard and Sencevy (2014) and Bosch and Gascón (2014), the interpretation of school mathematics requires taking into consideration the fact that mathematical knowledge is introduced, constructed, modified and reconstructed at school environments. The didactic problems that are presented in a particular institution, such as a school, are related to the provided institution, and not to the personal characteristics of the individuals that are part of the institution. Therefore, the DT emphasizes the "institutional relativity of knowledge". DT is an artefact that releases the didactic analysis from the contents of the institution (Bosch \& Gascón, 2014). and it is particularly relevant to the "transformation of knowledge" between institutions (Perez, 2015, p.157).

The process of the DT can be illustrated in Figure 3 and it takes place in four stages. In the first stage, academics (scholars and other producers) produce the scholarly knowledge (Stage 1), which is transforming into Knowledge to be taught (Stage 2) by individuals who design the curriculum (noosphere). In step 2 (stage 2 to 3 ) those who design the curriculum decide which body of knowledge should be taught (Taught knowledge) in classroom by the teachers. Due to the enactment process between teachers and students in classroom during their teaching, arises the Taught knowledge (Stage 3). In other words, teachers present in classroom the knowledge that is supposed to be taught, as it is demonstrated in the curriculum. Finally, what kind of knowledge students achieve to acquire is taking place in Stage 4. It is called Learnt knowledge and it particularly corresponds to the knowledge that students gain due to their learning. To my way of thinking, the whole process (Figure 3) can be described as a journey of the knowledge; particularly, how it was decided by specific individuals (Scholarly knowledge) and what the students did finally learn.


Figure 3: The process of didactic transposition
Through didactic transposition process, there is a distinction between the "educational" and the "scholastic" mathematical knowledge (Pepin and Haggarty, 2001). The current study focuses on the second stage of the didactic transposition process (Knowledge to be taught), since the unit of analysis is the Greek Geometry textbooks. This is because I did not have access neither to the knowledge the teachers taught, nor to the knowledge that students learnt during their interaction in the school environments.

## 3. 4. A Praxeology

In order to develop an answer to the question "what is the object of knowledge", Chevallard (2006) used the term praxeology. This word has its origins in Ancient Greek times and comes from the words "praxis" and "logos", both discussed below thoroughly. This is significant since the ATD aims to provide an explanation about the kinds of knowledge that can be observed in human activities. In human societies, the term "praxeology" is defined as the study of human action. This term is related to two distinct aspects:

- What the individuals do
- How they do this action

Under the light of the anthropological approach, the term praxeology consists of two more parameters, apart from the aforementioned (Chevallard, 2006):

- What humans think
- How they think

However, the question remains the same: what exactly is a praxeology? According to Chevallard "a praxeology is, in some way, the basic unit into which one can analyse human action at large" (2006, p.23). Likewise, every human activity can be detailed under the light of a praxeology. Etymologically, praxeology is a combination of the Greek words "praxis" and "logos". The notion "praxis" is the practical part of an activity, the action, the way that someone does something. On the other hand, "logos", apart from the meaning of discourse, is associated with the noetic ability of the human mind to think and comprehend any concern or information in a rational way. So, in general terms, every human activity entails a practical part or "know-how" (praxis) and a theoretical part (logos) that justifies the action.

So, are there any connections between praxis and logos? ATD proposes that:
> "no human action can exist without being at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such an explanation or justification may be cast. Praxis thus entails logos, which, in turn, backs up praxis. For praxis needs support just because, in the long run, no human doing goes unquestioned" (Bosch and Gascón 2014, p.68; Chevallard, 2006, p.23).

This is one of ATD's essential principles. However, focusing mostly on "no human action goes unquestioned" (Bosch and Gascón 2014, p.68; Chevallard, 2006, p.23), it occurs that there is some kind of interaction between those words (praxis and logos), as both support each other. In other words, every action an individual decides to do (praxis) cannot happen without being explained. Hence, it is significant to mention that Chevallard (2006) points out that praxeology might also have an alternative, negative meaning, when the praxis and the logos are utilized irrationally. Taking everything into consideration, human praxeologies develop, change and adjust through the interaction of the individuals.

## 3. 4. 1 The four components of a praxeology

ATD acknowledges that "mathematical objects are not absolute objects but entities whose meanings arise from the practices within a given institution and the practices in which individuals are invited to engage and adopt" (Gonzaléz-Martin, Nardi and Biza, 2011, p.570). Those practices can be determined by using the notions of tasks, techniques, technologies and theories (the so called "four Ts"). In particular, tasks are considered as the way that a human being acts, when he or she is engaged in a particular activity. In order to complete tasks, individuals use some techniques i.e. how the individual will perform the tasks, or some of them. In turn, those techniques can be explained, justified and designed, by the use of technologies i.e. practises used from individuals in order to make others comprehend the way they act. It is obvious that every technology needs a background, a support. This validity is ensured by theory i.e. an often-ignored component which explains, justifies or generates all part of technology sounding unobvious or missed. ATD posits that every human activity generates a combination of tasks, techniques, technologies and theories (Gonzaléz-Martin, 2013). Chevallard and Sensevy (2014) correspond the "four Ts" with Greek letters. Explicitly, a type of tasks is equivalent to the capital T , the set of techniques is relevant to the letter t (tau), for technologies is used the letter q or $\theta$ (theta) and the theory is corresponded to Q or $\Theta$ (big theta). The quadruple, often denoted as $[\mathrm{T} / \mathrm{t} / \theta / \Theta]$, consist the praxeology (Chevallard, Bosch and Kim, 2015).

Chevallard and Sensevy (2014) refer to these four terms as the "four-component structure" of a praxeology. The first two components (tasks, techniques) are part of the block praxis. In other words, both tasks and techniques can be seen as the "ways of doing" an activity (Bosch and Gascón 2014, p.68). According to Perez (2015), praxis characterizes the "know-how" part of praxeology and thus relates to the given tasks that an individual intents to study, with the aid of a different set of techniques, that are used
to examine these problematic tasks (p.157). In contrast, logos is the theoretical part that justifies an individual's actions. Having an insight into ATD, it is subsequent to comprehend that the block of logos is structured in a way that describes, explains and justifies the block of praxis. Its components are the technological discourse, which provides an explanation of the activity, and the theory, as it is the main support of the technology (Bosch and Gascón 2014, p.68). The structure of a praxeology and the meaning of each component are presented at Figure 4 and Tables 1 and 2.


Figure 4: The components of praxeology
Table 1: Defining the praxeological part of a praxeology

| Praxis: the practical part of the activity |  |
| :--- | :--- |
| Tasks (T) | The way that a human being acts, when is engaged in a particular <br> activity. They correspond to the ways of performing a certain type <br> of tasks. |
| Techniques (t) | The ways that an individual performs a certain type of tasks. |

Table 2: Defining the theoretical part of a praxeology

| Logos: "The description and justification of the praxis" (Bosch and Gascón, 2014) |  |
| :--- | :--- |
| Technology (q) | A way of explaining and justifying or even of "designing" a <br> technique. It relates to the ways that an individual explains, <br> justifies and designs a technique. |
| Theory (Q) | The theoretical background used in determining, explaining and <br> justifying the used techniques. |

Chevallard's notion of praxeology has been extensively used in mathematics education, however, it seems that different authors assign different meanings to some components. It is useful to provide the illustrative analysis of the mathematical praxeologies involved in the works of Gonzaléz-Martin et al. (2011) and Bosch and Gascón (2014), presenting in Table 3. In both cases, the authors provide the terms of the anthropological theory in association with their study. However, they approach the component "theory" from a different perspective. For instance, Gonzaléz-Martin et al.
(2011) recognise as: (1) tasks: the problems, exercises and applications in the texts related to the concept of series, (2) techniques: the use of one of the convergent criteria, (3) technology: the process of proof of the used test, or the theoretical part that will aid students to construct the solution a series-related problem and (4) theory: the set of the principles of Analysis. On the other hand, Bosch and Gascón (2014) define in their study as: (1) tasks: the worksheets that student receive from teacher, (2) techniques: a Cabri Geometry file with graphs that students need to carry out and interpret, (3) technology: the way that Cabri can be used and the manipulation of Cabri's elements and (4) theory: the properties of the $\mathrm{y}=\mathrm{a}^{\mathrm{x}}$.

Table 3: Each component of a praxeology, as used in the studies of Gonzales-Martin et al. (2011) and Bosch and Gascon (2014)

|  | Gonzaléz-Martin et al. (2011) | Bosch and Gascón (2014) |
| :--- | :--- | :--- |
| Tasks (T) | "A task can be one of the series- <br> related problems, exercises and <br> applications in the texts" (p. <br> $569)$. | "The worksheet the teacher <br> hands out to the students" (p. 74). |
| Techniques (t) | The use of one of the <br> convergent criteria. | "A Cabri Geometry file with <br> interactive graphs elaborated by <br> the teacher, which the students <br> are asked to manipulate and <br> interpret" (p. 74). |
| Technology (q) | "The proof of a convergence <br> test or the theory behind the <br> necessary steps to solve a <br> series-related problem" (p. <br> $569)$. | The rationale beyond the use of <br> the Cabri. The interpretation of <br> the elements of the Cabri files. |
| Theory (Q) | "The set of the fundamental <br> theoretical principles of the <br> mathematical domain of <br> Analysis" (p. 570). | The properties of the y=ax can be <br> deduced from the students while <br> observing the images on the <br> screen, using the graphical and <br> numerical information provided. |

As it can be observed both writers approach the term "theory" from different perspectives. In particular, in Gonzaléz-Martin et al. (2011), the theory refers to the theoretical background of the domain of Analysis itself, which contradicts the definition providing by Chevallard. On the contrary, the original definition is more apparent in Bosch and Gascón (2014), since as "theory" they interpret the properties of the exponential function $\mathrm{a}^{\mathrm{x}}$.

Having in mind the definitions of Chevallard for the "four Ts", I will make a comment about the relation between them as it emerged from my understanding of the anthropological approach. The four terms are tasks, technique, technology and theory, and can be compared to a chain with a form as below (Figure 5):


Figure 5: My view on the 4 components
Every previous term backs up and entails the following one. When a certain type of tasks is presented, a set of techniques will be utilized from the individual with the view to present the tasks. Furthermore, a technology is required in order to approach and explain those techniques. Finally, the theory will be used with the purpose of explaining the technology. Taking for example the term "theory" that it was discussed previously, one could observe that every theory ( Q ) supports technology ( $q$ ), as it justifies and explains every part of the q that is missing. Bearing in mind the study of Chevallard et al. (2015), the $\Theta$ "governs" the $\theta$, while every $\theta$ "governs" a particular set of t .

### 3.4.2 The distinction of the praxeologies

As activities can be described according to a chain action of four components, there is a distinction between the elements of the praxeology:

- A point praxeology is called a praxeology related to a particular type of tasks.
- A local praxeology is called a praxeology that pertains to a particular technology, when a certain type of tasks is presented.
- A regional praxeology is attributed to any kind of praxeology that consists of a combination of a point and a local praxeology.

Based on the work of Bosch and Gascón (2014), I come to a conclusion that a praxeology can be classified in three distinct categories:

- The didactic praxeology is considered as any type of activity that assist the development and the utilization of other praxeologies among individuals. A striking example is the didactic process between the teacher and the students.
- The personal praxeology is considered as the personal behaviour of an individual, their personal idiosyncrasies.
- The institutional praxeology is considered as the explanation of people's behaviour collectively.

In general terms, every human being consists of its personal praxeologies. Its personal aspects, behaviour, idiosyncrasies and the way that a person acts, have the capability to enhance, modify and adapt while it enters in an institution. Bearing that in mind, it is consequent to understand that praxologies do not appear immediately and unexpectedly as Bosch and Gascón (2014) specify, but they are the result of everyday activities with an intention to be modified and expounded.

## 3. 5. Ideas incorporated in this research

In this thesis, Chevallard's notion of praxeology will be used for analysing the images (Table 4) and tasks (Table 5) found in Euclidean Geometry textbooks of Lyceum (upper high-school). To be more specific in Table 4, tasks (T) correspond to the categories of
images found in Euclidean Geometry's textbooks. To specify the type of images, the individual needs a particular set of "techniques" ( t ), in this case the categories of images. The reasons why those images are presented in such a way will be described as the "technology" (q). The theory $(\mathrm{Q})$ of praxeology in ATD includes definitions, postulates, theorems and properties. Specifically, is the set of the theoretical principles of the mathematical domain of Euclidean Geometry, as it is incorporated in textbooks.

Table 4: Use of praxeology in Images

| Tasks (T) | The images themselves. |
| :--- | :--- |
| Techniques (t) | The categories of images. |
| Technology (q) | The rationale beyond the use of those categories of images. <br> (Why are the images presented in this particular way?). |
| Theory (Q) | The theory as it is described in every Geometry textbook of <br> Lyceum. |

In terms of tasks (Table 5), the praxeology follows a similar pattern; the (unsolved) tasks of Greek Geometry textbooks are the T , while as t will be considered the categories of tasks, that I decide to divide them. The ways authors decide to present those tasks of textbooks corresponds to $q$. Once again, the theory $(\mathrm{Q})$ of praxeology in ATD includes definitions, postulates, theorems and properties and it will be considered as the set of the theoretical principles of the mathematical domain of Euclidean Geometry, incorporated in the textbooks.

Table 5: Use of praxeology in Tasks

| Tasks (T) | The tasks themselves. |
| :--- | :--- |
| Techniques (t) | The categories of tasks. |
| Technology (q) | The rationale beyond the use of those categories of tasks. <br> (Why are the tasks presented in such a way?). |
| Theory (Q) | The theory as it is described in every Geometry textbook of <br> Lyceum. |

Gaining an insight into the categories of images and the categories of tasks (techniques) preferred by the authors of the textbooks, the rationale beyond the use of images and tasks (technology) will be interpreted. For this cause, I will become aware of the ways that the geometrical terms are introduced to students via the images and the tasks of Greek Geometry's textbooks. Information on the ways that this representation serves the "Knowledge to be taught" in every textbook of Geometry will be extracted. This is my primary focus, as there is no data about the knowledge provided by the tutors of the teaching institutions (Taught knowledge), or about the teaching material that the students would be engaged (Learnt knowledge).

## Chapter 4 Methodology

## 4. 1. Introduction

In this chapter, I present the methodology followed for this study. In Section 4.2, I describe how and why the selected Geometry textbooks were merged into six Books, and the topics included in each Book. In section 4.3, the three categories of images are introduced to the readers, while in section 4.4 the four categories of tasks are presented. In section 4.5, a brief description of Qualitative Content Analysis is provided. Finally, in sections 4.6 and 4.7 I present the coding frame for analysing images and tasks respectively. The chapter ends with a brief presentation about the changes that occurred in the pages of topics in every textbook (Section 4.8).

Before moving on, I would like to remind to the reader my research questions:
RQ1 What categories of images can be found in Greek Geometry textbooks from 1975 to 2019?
RQ2 Are there any variations in their use throughout the years?
RQ3 Having in mind the different categories of images used in these textbooks, what can be said about the ways that geometrical concepts are presented to students?
RQ4 What categories of tasks can be found in Greek Geometry textbooks from 1975 to 2019 ?

RQ5 Are there any variations in the use of categories of tasks throughout the years?
RQ6 What can be said about the ways that the tasks are presented to students?

## 4. 2. Creation of Books

## 4. 2. 1. Historical background

Since the middle 1800s, the educational system in Greece has gone under a number of changes. 1975 was the last year that the educational system was divided into the primary school and the Gymnasium (the well-known sixth grade Gymnasium) according to the Greek constitution. Although there were six grades in the Gymnasium, the attendance was not compulsory. Those who managed to attend to University were able to choose between humanity and science programmes in the last three grades, as part of their graduation. Both programmes had Euclidean Geometry as part of their course. For this research, I will only analyse the textbooks of humanity programme, as it contained equivalent topics to Geometry textbook of the following years.

The educational legislation of 1975 (applied in 1976), removed grades $4^{\text {th }}, 5^{\text {th }}$ and $6^{\text {th }}$ from Gymnasium and reintroduced the institution of Lyceum (consisting of three grades). Likewise, to the sixth grade of Gymnasium, attendance in Lyceum was not compulsory. Students had to decide in which programme they should be attended. The Geometry course was removed from the $3^{\text {rd }}$ grade of Lyceum and this means that Euclidean Geometry's lectures were not delivered anymore to school leavers. In association with these changes, Geometry was not anymore a subject for a given
programmes, but rather became a subject of general education. Details can be found in Table 6.

Table 6: The teaching of Euclidian Geometry in Greek schools from 1975 to 2019

|  | $1975-1976$ | $1976-1979$ | $1979-2019$ |
| :---: | :---: | :---: | :---: |
| $4^{\text {th }}$ grade of <br> Gymnasium | $\mathrm{H}, \mathrm{S}$ |  |  |
| $5^{\text {th }}$ grade of <br> Gymnasium | $\mathrm{H}, \mathrm{S}$ |  |  |
| $6^{\text {th }}$ grade of <br> Gymnasium | H, S |  |  |
| $1^{\text {st }}$ grade of Lyceum | H, S | GL |  |
| $2^{\text {nd }}$ grade of Lyceum | H, S | GL |  |
| $3^{\text {rd }}$ grade of Lyceum | H, S | N |  |
| H: Euclidean Geometry of Humanity programme <br> S: Euclidean Geometry of Science programme <br> GL: Euclidean Geometry of General Lyceum <br> N: No course of Euclidean Geometry |  |  |  |

## 4. 2. 2. Topics of Euclidean Geometry

Since I wanted to delve deeply in analysing the Geometry textbooks used in Greek schools, for me it was essential to introduce some kind of a categorisation of the topics included in each textbook. That was far from easy since no relevant reference could be found in the literature. In consultation with my supervisor, the topics found in the Geometry textbooks were classified according to those found in the last two textbooks (textbook G and H). Both textbooks are entitled "Euclidean Geometry for the $1^{\text {st }}$ and $2^{\text {nd }}$ grade of General Lyceum", they have many similarities in content and follow the same structure.

Each of these textbooks ( G and H ) contained 13 chapters, corresponding to 13 possible different topics. After examining each chapter, those with similar content were merged into one topic. For example, chapters one "An introduction to Euclidean Geometry" and two "Basic geometric structures" were merged into the topic "Introduction to Euclidian Geometry" (I). However, most of the chapters covered different topics and they remained unchanged, with each chapter's name used as a topic name for my categorisation These were: "Triangles" (chapter three, T), "Parallel lines" (chapter four, PL), "Quadrilaterals" (chapter five, Quad) and "Inscribed shapes" (chapter six, IS). The chapters "Analogies" (chapter seven) and "Similarities" (chapter eight) were combined in a topic called "Analogies and similarity" (A and S). As A and S deal with proportions of segments and similarity in segments respectively, this combination was inevitable. The seventh chapter discusses metric relations in triangles, polygons and circles, while the next one describes the formulas to calculate areas. For
this reason, the following topics were called "Metric relations (triangles, polygons and circles)" (chapter nine, MR) and on the other one was given a more general name as "Areas" (chapter 10, A). "Circle's measurement" (chapter 11, CM) was a topic that gave students the opportunity to enrich their knowledge on polygons and in the measures pertain to circles. Finally, the last chapter was about stereometry. It contained not only the lines and segments in the three-dimensional space, but also solid figures. So, the most preferable name for this topic was "Solid Geometry" (chapters 12 and 13, S). In total, ten topics were created (Table 7).

Table 7: Topics of Euclidean Geometry

| 1. | Introduction to Euclidian Geometry (I) |
| :---: | :--- |
| 2. | Triangles (T) |
| 3. | Parallel lines (PL) |
| 4. | Quadrilaterals (Quad) |
| 5. | Inscribed shapes (IS) |
| 6. | Analogies and Similarity (A and S) |
| 7. | Metric relations (triangles, polygons circles) (MR) |
| 8. | Areas (A) |
| 9. | Circle's measurement (CM) |
| 10. | Solid Geometry (S) |

## 4. 2. 3. Gaining an insight into Textbooks ${ }^{1}$

The first Greek textbook of the selected ones was called "Euclidean Geometry" (code: Textbook A). This textbook's author is Christos Papanikolaou and it was initially used for the teaching of school Geometry from 1975 to 1976. As I have mentioned before, 1975 was the last year in which Gymnasium contained six classes. Textbook A was used for the $3^{\text {rd }}, 4^{\text {th }}, 5^{\text {th }}$ and $6^{\text {th }}$ of Gymnasium. A problem arising here; in which grade of Gymnasium each chapter was taught? For this reason, I will examine the whole textbook as a unit. Before I move on to the classification of topics, I will provide a comment about textbook A. It differed from the followings, since its contents were separated in books (VIVLION- BIB $\Lambda$ ION) and not in chapters. Consequently, every VIVLION was divided into paragraphs. As most textbooks containing the teaching material of two grades, A dealt with plane Geometry and stereometry. The Geometry of planes contained five books (VIVLION), while stereometry was divided into three books.

The first book of this textbook was called Introduction and discussed primary meanings such as angles, line-segments, planes etc. It was assorted in the first topics which contained essential geometric terms. The topic of "Parallel lines" were also described in this chapter. Going to the following chapter (BIB $\wedge$ ION ПP $\Omega$ TON) of textbook A, it is clearly observed that the topics, "Triangles" and "Quadrilaterals", were combined in the same chapter. The second book (BIBNION $\triangle$ EYTEPO) contained

[^0]parts of the circle like equal circles, symmetry, etc. and also the inscribed shapes. Although, circles were part of (I) in the final textbook, it seemed appropriate to distinct those categories and classify circle in (I). In addition, the third book (BIB 1 ION TPITO) of A contained a combination of topics. For instance, analogies, similarity, areas and metric relation (in triangles and circles) were parts of it. Finally, the fourth book (BIB $\Lambda$ ION TETAPTON) described regular polygons and it was corresponded to the topic "CM" of the above classification. Solid Geometry consisted of three books (VIVLION 5, 6 and 7). One fact that cannot be ignored is that some paragraphs were not mentioned in the future Books, as Table 8 demonstrates.

Table 8: The academic years that textbook $A$ was taught.

| Time | Grade |
| :--- | :--- |
| $1975-76$ | $3^{\text {rd }}, 4^{\text {th }}, 5^{\text {th }}$ and $6^{\text {th }}$ of Gymnasium |
| $1976-77$ | a part is taught in $3^{\text {rd }}$ grade of high-school |
| $1976-77, \ldots, 1978-79$ | a part is taught in $4^{\text {th }}, 5^{\text {th }}$ and $6^{\text {th }}$ grade of high-school |
| $1979-80, \ldots, 1986-87$ | a part is taught in $5^{\text {th }}$ grade |

As I did not have access in the teaching material of this textbook from 1976 to 1979, I examined the one referring to 1979-1987 and more specifically the edition of 1982 (Textbook B). It is worth mentioning that the 1978 (for the $4^{\text {th }}, 5^{\text {th }}$ and the $6^{\text {th }}$ grade of Gymnasium) and 1981 versions (for the $5^{\text {th }}$ grade) were also online, but the latest version was chosen. The reason was that while I was looking through the table of contents for those three textbooks, I discovered that all of them discussed the same teaching material. Even the numbering of paragraphs was the same. So, I thought it was more appropriate to analyse the latest edition of all, which was the 1982 version.

Textbook of 1982 (textbook B) acted in the same way as textbook A. To a certain extend the teaching material which was described in the table of contents was similar. The first chapter of textbook B contained a brief discussion of eight chosen subsection from A and then continued with book three. As I mentioned earlier, in textbook A all chapters were divided into books and not in chapters as it would be expected. So, some books combined the content of the ten created topics. A typical example is that book three of textbook B presented analogies, similarity, metric relations in triangles and quadrilaterals, as well as areas. In book four the students were able to improve their studying skills in regular polygons and in metric relations in circles. Eventually, the part of stereometry remained unchanged in both textbooks A and B. Taking everything into consideration, it was apparent that topics from (I) to (IS) were not included.

Having seen the textbook for $5^{\text {th }}$ grade students from 1979 to 1987, I will examine the textbook "Theoretical Geometry: Vol.1" for the $1^{\text {st }}$ grade students of Lyceum which was written by Dimitris Papamichael and Anastasios Skiadas. This textbook was coded as textbook C and it was used from 1979 until 1990. Its first version was for the $3^{\text {rd }}$
grade students of Gymnasium from 1977 to 1978. Textbook C included ten chapters and three appendices.

Table 9: The academic years that textbook C was taught.

| Time | Grade |
| :--- | :--- |
| $1977-78$ | $3^{\text {rd }}$ grade |
| $1979-80, \ldots, 1989-90$ | It was taught with changes in the $1^{\text {st }}$ grade of <br> Lyceum. |

Textbook B was replaced by the textbook called "Theoretical Geometry for 2 ${ }^{\text {nd }}$ grade students of Lyceum" (textbook D) and it was written by Nikos Varouchakis, Dimitris Papamichael, Anastasios Alibinisis and Dimitris Kontogiannis. It was used from 1986 to 1991 and the included content was similar to textbook B. It was divided into seven chapters. Analogies were included in the first one and Similarity was described in chapter two. The following chapter was referred to metrics relations in triangles and circles while areas were described in the fourth. Besides, the measurement of circle and metric relations to circles were part of chapter five. As the reader can observe chapters four and five made a reference to metric relations in circle. But why this was happening? In chapter five the student was able to learn about the length of the circle and the length of the arc which both were parts of metrics. Finally, the sixth and the seventh chapter of textbook D were referred to the three dimensional and, more specifically, in lines and planes in three values and in solid shapes. Therefore, both chapters were classified in the tenth topic, the solid elements.

Textbook C was replaced by "Theoretical Geometry for the $1^{\text {st }}$ grade of Lyceum" (textbook E), written by Anastasios Alibinisis, George Dimakos, Theodoros Exarchakos, Dimitris Kontogiannis and Goerge Tassopoulos. The first edition was released in 1990, nevertheless re-releases with improvements continued until 1996. This textbook was used for the Geometry teaching from 1990 to 1999. Looking through the analysis of textbook E, the table of contents was divided into three parts; the introduction, the five chapters and the appendix. All five chapters were parts of plane Geometry. More precisely, the first chapter was referred to basic geometric shapes which were corresponded to "Introduction to Euclidian Geometry". Chapter 2 was a combination of Triangles and Parallel Lines. The fourth chapter was referred to the topic of Quadrilaterals, while the fourth in the topic of Analogies and similarity. It is also important to mention that this chapter was stated as Thales' Theorem and its consequences and it contained the well-known Pythagorean Theorem. The fifth and the final chapters were divided in twelve sections; the first one was referred to inscribed quadrilaterals, while the other eleven were referred to circles.

In 1991 a new textbook of Euclidean Geometry appeared for the $2^{\text {nd }}$ grade students of Lyceum. It was written by Anastasios Alibinisis, George Dimakos, Michalis Drakopoulos, Athanasios Kiriazis and George Tassopoulos, and it was entitled as "Theoretical Geometry for the $2^{\text {nd }}$ grade of Lyceum" (textbook F) from 1991 to 1999.

Although 1991 was the year this textbook was released, re-releases with improvements continued until 1995. Its table of contents contained six chapters and an appendix. As for the chapters, the first-one pertained to the topic Metric relation. Chapters two and three described the Areas and regular Polygons respectively. Finally, chapters four, five and six were parts of solid Geometry.

From 1999 until 2014 the textbooks of Euclidean Geometry were merged in one for the $1^{\text {st }}$ and $2^{\text {nd }}$ grade students of Lyceum. Specifically, there were two textbooks; the first one was written by Giannis Thomaidis, Thanasis Xenos, George Pantelidis, Andrew Poulos and George Stamou. and the latter by Ilias Argiropoulos, Panagiotis Vlamos, George Katsoulis, Stilianos Markatis and Polichronis Sideris. They were published in 1999 (textbook G) and 2001 (textbook H) respectively. Both were titled "Euclidian Geometry for the $1^{\text {st }}$ and $2^{\text {nd }}$ grade of General Lyceum" ${ }^{2}$. The Geometry topics used in my thesis originated from those two textbooks. Every chapter was corresponded to the relevant topics. Textbook H was divided in two separate textbooks, one for the $1^{\text {st }}$ grade and the other for the $2^{\text {nd }}$ grade since 2014 and on. The students of the $1^{\text {st }}$ grade were able to acquire a basic knowledge in tringles, parallel lines, quadrilaterals and inscribed shapes, while the $2^{\text {nd }}$ grade students enhanced their mathematical skills in analogies, similarity, metric relations, areas, circle's measurement and solid Geometry.

I decided to rename those textbooks and titled them as textbooks A, B, C, D, E, F, G and H while I will refer to them a lot in the following sections. Furthermore, the above textbooks were sorted by chronological order, from 1975 to 2019. Apart from textbook A in which students engaged in the subject of Geometry in four grades (9th, 10th, 11th and 12th) of Gymnasium, textbooks B to H were used for Geometry's lectures in either the $1^{\text {st }}$ or the $2^{\text {nd }}$ grade of Lyceum. Specifically, two textbooks ( C and E) were utilized for the particular Geometry lessons in the $1^{\text {st }}$ grade of Lyceum, while three others (textbook B, D and F) were chosen as part of the curriculum in the $2^{\text {nd }}$ grade of Lyceum. In contrast, texts G and H provided a method that differed from the previous textbooks. Both included the teaching material, which it was used for the teaching of Geometry in the $1^{\text {st }}$ and $2^{\text {nd }}$ grade of General Lyceum. The code and the years of teaching for every textbooks are presented in Table 10.

[^1]Table 10: Providing codes to Greek textbooks of Geometry per years of teaching

| Textbook | Code | Years of teaching |
| :--- | :---: | :---: |
| Euclidian Geometry (9th, 10th, 11th and 12th grade of <br> Gymnasium) | A | $1975-1976$ |
| Euclidian Geometry (2 $2^{\text {nd }}$ Grade) | B | $1979-1986$ |
| Pure Geometry (1 ${ }^{\text {st }}$ Grade) | C | $1979-1990$ |
| Pure Geometry (2 ${ }^{\text {nd }}$ Grade) | D | $1986-1991$ |
| Pure Geometry ( $1^{\text {st }}$ Grade) | F | $1990-1999$ |
| Pure Geometry (2 ${ }^{\text {nd }}$ Grade) | F | $1991-1999$ |
| Euclidian Geometry (1 $1^{\text {st }}$ and $2^{\text {nd }}$ Grade) | H | $2001-2014^{3}$ |
| Euclidian Geometry (1 $1^{\text {st }}$ and $2^{\text {nd }}$ Grade) |  |  |

## 4. 2. 4. Generating Books

The sample contained eight textbooks (A, B, C, D, E, F, G and H). The eight textbooks were used for the teaching of Euclidian Geometry in Lyceum between the years 1975 and 2019, with a three-year gap starting from 1976. Although I did not have any access on textbook of 1976, this textbook was merely a new edition of textbook A (1975-1976) since it consisted of many mathematical concepts included originally in the textbook used in 1975. The authorship altered in all texts. For instance, a textbook might be authored by a mathematical teacher or a group of researchers who were specified in mathematics education. The selected eight textbooks were used in different Geometry lessons either in the $1^{\text {st }}$ or $2^{\text {nd }}$ grade of Lyceum or in more grades. Textbook A contained a wide variety of the mathematical concepts which were introduced to students from the $9^{\text {th }}$ until the $12^{\text {th }}$ grade of the Greek Gymnasium. Three textbooks (B, D and F) were used for the teaching of Geometry in the $2^{\text {nd }}$ grade of Lyceum, while two textbooks (C and E) were part of the curriculum in the $1^{\text {st }}$ grade of Lyceum. Textbooks G and H combined the teaching material of the $1^{\text {st }}$ and the $2^{\text {nd }}$ grade of Lyceum from 1999 to 2001 and 2001 to 2014 respectively.

The number of images in each textbook differs a lot. So, I decided to combine specific textbooks in order to have the form of G and H textbooks. One should take into account that those two textbooks ( G and H ) were complete and independent, since both satisfied a direct access in the mathematical concepts of Euclidean Geometry, for the $1^{\text {st }}$ and the $2^{\text {nd }}$ grade students of Lyceum.

[^2]Each textbook was used for a particular period. Textbook A was used from 1975 until 1976 for the $9^{\text {th }}, 10^{\text {th }}, 11$ th and 12th grade of the Greek Gymnasium. In 1979, Geometry textbooks changed. A first grade student of Lyceum used textbook C (19791990), while a second grade of Lyceum used textbook B (1979-1986). In 1986, a new textbook (textbook D) was provided for the $2^{\text {nd }}$ grade. This was used from 1986 until 1991. In 1990, the textbook for the $1^{\text {st }}$ grade was replaced by textbook E, which was used until 1999. Likewise, in $2^{\text {nd }}$ grade the provided text withdrawn from schools and its place took textbook F (1991-1999). The year 1999 was the first one in which the textbook consisted the teaching material for the $1^{\text {st }}$ and $2^{\text {nd }}$ grade. This was textbook G (1999-2001) with a short duration of time. Two years later, a new version of textbook G is provided in the educational institutions. It had many similarities with the previous Geometry textbook; however, the authors were differed in both. It was coded as H and it still is the present textbook of Euclidean Geometry lessons in the $1^{\text {st }}$ and $2^{\text {nd }}$ grade of Lyceum.

It should be kept in mind that it seemed invalid to compare the images and, therefore, the tasks in different grades. So, it was seemed more accurate to combine five textbooks in order to avoid the comparison of dissimilar chapters. I merged textbooks B (19791986) and C (1979-1990) and named them Book 2, since the new form included the teaching curriculum of school Euclidean Geometry in grades one and two of Lyceum. The following was Book 3, which combined textbooks C (1979-1990) and D (19861991). Textbook C was presented in both Books (Books 2 and 3). This is a result of its long teaching duration for the school Geometry, while in the same years, from 1979 until 1990, the textbooks of the $2^{\text {nd }}$ grade changed two times. One last vital compound was the one of textbooks E (1990-1999) and F (1991-1999), which presented n Book 4. Textbook A, G and H remained independent (did not combined to other textbooks), albeit their labelled as Book 1, Book 5 and Book 6 respectively occurred. It was not required to combine those three Books, as their mathematical concepts were appertained to two grades. The combined textbooks and the creation of Books can be seen at Table 11.

Table 11: Renaming textbooks per years of teaching

| Book | Textbook | Years of teaching |
| :--- | :--- | :--- |
| Book 1 | A | $1975-1976$ |
| Book 2 | B+C | $1979-1986$ |
| Book 3 | C+D | $1986-1990$ |
| Book 4 | E+F | $1990-1999$ |
| Book 5 | G | $1999-2001$ |
| Book 6 | H | $2001-2019$ |

In conclusion, the combination was necessary not only for the validity of the comparisons, but also for preserving the presentence of all topics in each Book. The ten topics were not part of all textbooks, since they were referred to different grades, either
the first or the second class of Lyceum. So, another factor of this combination was the importance of presentence of the Euclidean Geometry topics in all books, in order to outline my findings. For example, the topic Triangles were part of the Euclidean Geometry in the 4th grade of Gymnasium and the $1^{\text {st }}$ grade of Lyceum. So, textbooks A, C, E, G and H included triangles as a main Geometry concept, although texts B, D and F did not incorporate this topic in the teaching material. After giving textbooks the form of Books, all topics were included in the generated Books.

## 4. 3. Classification of images

According to Gonzalez-Martin et al. (2011), textbook images can be categorised as non-conceptual (NC), bland conceptual (BC) or conceptual (C). Gonzalez-Martin et al.'s (2011) classification emerged from their examination of the images used in Canadian and UK textbooks for the concept of series. The categorisation of GonzalezMartin et al. (2011) is based on the work of Elia and Philippou (2004), who analysed the role of visuals in problems.

Images that are not related to a mathematical concept (Ibid.) and serve a decorative purpose (such as portraits or photographs) are categorised as non-conceptual (NC). Images that provide "an illustration of a mathematical concept" (Ibid., p.572) and do not require any explanation, are considered as bland-conceptual (BC). Finally, images "used to explain a concept, or to illustrate one step of a proof" with an intention "to help the student understand a notion or a mathematical argument" were classified as conceptual (C) (Ibid., p.572). In Appendix 3 the whole classification of images can be viewed.

## 4. 4. Classification of tasks

In order to examine what was assessed in the textbooks of Euclidean Geometry, a combination of two approaches was used; The first is Niss' (1993) work on items of assessment, while the second is Treffert-Thomas, William, Virman, HernandezMartinez and Rogovchenco's (2017) literature review on mathematical modelling.

Specifically, the tasks that students were invited to deal with as presented in the textbooks of Euclidean Geometry are:

1. Questionnaires (Qs);
2. Exercises (E);
3. Problems (P);
4. Mathematical modelling (MM).

By definition our categories of tasks included how complex a task is because Questionnaires assessed the knowledge of mathematical tasks; Exercise assessed the mastery of standard methods and techniques; Problems involved extra-mathematical situations; and Mathematical Modelling assessed the ability of students to combine mathematical tasks with real-life situations. Lastly, Qs were used as a confirmation that the individual (student) had understand the taught theory. On the other hand, E and P were used to obtained mathematical results. Both of them had a different level of
difficulty. MM is "a discipline that attempts to describe real-world phenomena in mathematical terms and then solves them" (Upadhyay and Iyengar, 2013, p.1).

### 4.4.1 Questionnaires, Exercises, Problems

Questionnaires (Qs) assess "mathematical facts"; and they are dealing with definitions, theorems, properties, computations, based on a geometrical figure (without the use of a paper or pencil environment). Furthermore, Qs assess an individual's ability to express the aforementioned facts quickly and straightforwardly, while taking into account the combination of such. All questionnaires are introduced by imperative sentences and they are approached orally by students.

The second category is the Exercises (E) that assess "standard methods and techniques". Exercises require mathematical knowledge in a paper and pencil environment and as Niss (1993) presents, they are concerning solutions to combinations and operations to mathematical objects in a straightforward way. The level of difficulty in exercises differs per exercise. For instance, they can be introduced by the form of a questionnaire, that the individual needs to express the answer by making computations, or by the form of a problem without being complex enough to be considered in that category.

Problems ( P ) is the third category. There are different kinds of problems varying considerably by levels of complexity and difficulty. The students are required to make operations to the relevant theory or to combine parts (e.g. properties, theorems) from the taught theory with a view to solve the problem.

## 4. 4. 2 Mathematical Modelling

The final category of tasks is termed as Mathematical Modelling (MM). MM are applications of mathematics concerning real-world phenomena. In a mathematical environment the MM are introduced with the form of questionnaires, exercises and problems. Kaiser and Sriraman (2006, p.38) developed five perspectives on mathematical modelling in education. Those were:

1. Realistic or applied perspective: this perspective deals with pragmatic or utilitarian goals. For instance, a student is asked to provide solution to real world problems or to understand the real world.
2. Epistemological or theoretical perspective: the main aim originating from this type of MM is to promote theory without cantering on the realistic side of the problem
3. Socio-critical or emancipatory perspective: this perspective links the world of mathematics with the society. Student need to develop a critical understanding of the world and also how MM pertains to socio-critical decisions.
4. Contextual perspective: this perspective has subject-related and psychological goals. Its main aim is to "elicit the invention, extension and refinement of mathematical (psychological) constructs" (Treffert-Thomas et al., 2017, p.123).
5. Educational perspective: this perspective deals with pedagogical and subjectrelated goals. It is divided into two subcategories; the didactical, which structures
learner's understanding using modelling and the conceptual modelling which aims to "introduce mathematical concepts to students and develop their understanding" (Treffert-Thomas et al., 2017).

## 4. 5. Qualitative content analysis

Qualitative Content Analysis (QCA) is a method for analysing in a detailed and systematic way the meaning of qualitative in nature data (Mayring, 2000; Schreier, 2012; Schreier, 2014). Under this aspect, Basit $(2003,2010)$ highlights that the analysis of data in QCA is "a dynamic, intuitive and creative process of inductive reasoning, thinking and theorizing" (2003, 143; 2010, 182). In addition, QCA incorporates three key characteristics that differentiates it from other kinds of analysis: it is systematic, flexible and reduces the amount of data (Schreier, 2012; Schreier, 2014).

At the heart of QCA lies the coding frame. Its object is to define the categories that will provide information about a certain topic of interest. Every category is consisted of subcategories which explain and justify the main categories. According to Anastasakis (2018) those two components pertain to a mathematical system in which categories take the role of variables, while subcategories correspond to the values of the categories.

When using QCA, there is a series of steps that should be taken into consideration, when constructing a coding frame (Schreier, 2014):

1. Selecting the material. The first step in QCA is to select parts of the material that are relevant to the research question(s).
2. Structuring and generating. Structuring is considered as the process of creating the main categories that are relevant to a study's focus. Following that, the researcher constructs the subcategories for each main category. Both steps can be accomplished either in a concept-driven way or in a data-driven way or as a combination of those ways. When working in a concept driven way, categories and subcategories are based on prior research or a theory. On the other hand, when working in a data driven way, categories and subcategories are delivered throughout the analysis.
3. Defining the main categories. In this step, each category is given a name, a description (what the category refers to), indicators (signs for the presence of a phenomenon), the examples (materials illustrating each category) and the decision rules (signs to coders about which category to use).
4. Revise and expand the coding frame. In the final phase of constructing a coding frame, it is preferable to revise and expand. As many times as a researcher repeats this stage, it is more possible to join relevant categories/subcategories or even create extra, in order to cover all the gathered data.

## 4. 6. Defining the categories of Images

Schreier (2014, p.176-177) describes that the definition of each category consists of four parts: a name, a description, the indicators, the examples and the (optional) decision rules. A name, a description, the indicators and the examples are an integral
part of definition, as Schreier (2014, p.177) highlights. To my way of thinking this assists the reader to comprehend the meaning and the importance of its category. Additional aid is gained by the indicators, which support each category by incorporating signs for its recognition. Conversely, the decision rules provide information on which category the coders and the reader should be utilize in times when they are uncertain which category to use. Their inclusion in the tables is optional. In what follows I present the definitions of those four parts.

A name can be considered as a label referring to a specific category. Every name is usually generated by the researcher. For instance, in this thesis a name is one of the three categories of images, included in the analysis of Greek Geometry textbooks. A name should not be "overly long" nor "overly short and cryptic" (Ibid., p.176).

When forming the category, the primary aim of the coder is to make readers familiar with the meaning of the provided names in order to understand the research. So, the researcher should provide a detailed description of the categories and the indicators. The first component explains "what is meant by a given category" (Ibid., p.176), although the second one is considered as the features that assist the reader and the researcher to identify which category to use.

The categories can be clarified by illustrative "examples from the material" of the coding frame. Their presentence helps the reader to comprehend terms or notions of the categories that might has misled. Moreover, the examples should be substantive, "typical" and the definitions should not include more than two of them. Additionally, the last part of the definition is the decision rules. They are not a compulsory component in the qualitative content analysis, but are used as an indicator of which category is applicable. This occurs because of the overlapping of the meaning of some categories.

Having in mind the aforementioned frame, the interpretation of the four parts of definition in relevance with this thesis will be presented. Generally, as a name I labelled the three categories of images (NC, BC, C) that I will refer to. The following step was the description in which the three categories of images were specified, while in the indicators, signs about what can be considered as a specific category were presented. Additionally, the illustrations from each category of images, as providing in the selected textbook (textbooks A, B, C, D, E, F, G and H) were the examples of the definition. Finally, the variety of images, required the inclusion of the decision rules in order to guarantee that the reader will be able to understand which images can be considered as $\mathrm{NC}, \mathrm{BC}$ or C . The tables below demonstrate the three definitions of images in textbooks, following the description of Schreier (2012; 2014), according to GonzalezMartin et al. (2011, p.572).

Table 12: Definition for the category Non-Conceptual (NC) images.

| Name | NON-CONCEPTUAL (NC) |
| :--- | :--- |
| Description | "A non-conceptual (NC) image does not relate to a mathematical <br> concept. Its use is merely decorative" (Gonzales-Martin, et al, <br> 2008, p.572) |
| Indicators | A portrait, a photograph or a painting |
| Examples |  |

Table 13: Definition for the category Bland-Conceptual (BC) images.

| Name | BLAND-CONCEPTUAL (BC) |
| :--- | :--- | :--- |
| Description | "A bland-conceptual (BC) image is an illustration of a mathematical <br> concept, but it is not an integral part of an argument or an <br> explanation" (Gonzales-Martin et al., 2008, p.572). |
| Indicators | This part includes definitions, corollaries, postulates and theorems <br> with no proves when an explanation is not needed. |
| Examples |  |

Table 14: Definition for the category Conceptual (C) images.

| Name | CONCEPTUAL (C) |
| :--- | :--- |
| Description | "A conceptual(C) image is used to explain a concept or to illustrate <br> one step of a proof. It might be a part of a proving process and it is <br> explicitly intended to help the student understand a notion or a <br> mathematical argument" (Gonzales-Martin, et al., 2008, p.572). |
| Indicators | A conceptual image simplifies the mathematical concept or helps the <br> individuals understand the mathematical proofs. It also might be a <br> part of a solving task which the solution of the problem is based on <br> visual representation. It also might be an image in comments or in <br> historical reviews with explanation. |
| Examples |  |

## 4. 7. Defining the categories of Tasks

This section presents the categories of tasks that are found in the six Books between the years 1975 and 2019, with an aforementioned three-year gap (from 1976 to 1979). In this duration of time there was only a provided textbook. Although I did not have any access to it, the book of 1976 was a new edition of textbook A (1975-1976). Besides, it consisted many mathematical concepts of textbook of ' 75 .

For the classification of tasks, I used a combination of the categorisations offered by Niss (1993) and Treffert-Thomas et al. (2017). Niss' work (1993) relates to "traditional" tasks (questionnaires, exercises and problems) that are usually assessed in mathematics educations, while the work of Treffert-Thomas et al. (2017) concerns the use of mathematical modelling in higher education by lecturers.

In order to gain an insight into "what is assessed", it was my number one priority to make the readers familiar with the four categories of tasks that were examined. I approached every type of task via the definition proposed by Schreier (2014, p.176177). As a name, I chose one of the four categories of tasks (questions, exercises, problems, mathematical modelling) that were presented earlier. The part of description refers to each category of tasks, while the part of indicators provides signs about what can be considered as every category of tasks. In Examples, specific illustrations from textbooks about the four categories of tasks are presented to readers. Finally, the variety of tasks required the inclusion of decision rules; The coder will be able to make readers familiar with what category of tasks can be considered as questions, exercises, problems and mathematical modelling, as providing in textbooks.

On the whole, tasks that students were invited to deal with, as presented on the tables below are:
5. Questionnaires (Qs);
6. Exercises (E);
7. Problems (P);
8. Mathematical Modelling (MM).

Table 15: Definition for the category Questionnaires (Qs) tasks.

| Name | Questionnaires (Qs) |
| :---: | :---: |
| Description | Questionnaires are items assessing the knowledge of facts. In particular, this type of tasks pays attention to "accurate recollection of facts; students' ability to quickly, precisely and coherently express these facts; and their ability to adequately select and combine facts in various contexts" (Niss, 1993, p.16). |
| Indicators | Questionnaires contain mathematical facts pertaining to definitions, theorems, properties, formulae or results of computation that emerge straightforward from the figure of the question (Ibid.). |
| Examples | Textbook A (p.31) <br>  <br>  <br>  |
|  | Textbook D (p.123) <br>  Өعıaкá; |
|  | Textbook E (p.103) <br>  |
|  | Textbook F (p.25) <br>  <br>  ацвдиүш́vio; |
|  | Textbook G (p.78) <br> Пatín AK éva ợxotónos tns yavias tav eqa- <br>  Ar : |
|  |  |
| Decision Rules | 1. Theory-oriented questions (Textbook A, §33-49, Task B. 13 (p.31)) <br>  <br> 3. Evaluate the expressions (Textbook F, chapter 2, task 5 (p.45)) <br> 4. Sketch the figure (without providing any explanation) (Textbook G, section 6.1, <br>  <br> 5. Multiple choice (Textbook H, section 2.10, task1 (p.13)) <br> 6. Using diagrams or schemas, but simple to be solved (Textbook G, section 2.3, $\Sigma \mathrm{A}$ 3) <br> 7. True or False (Textbook G, chapter 3, task 1 (p.91)) |

Table 16: Definition for the category Exercises (E) tasks.

| Name | Exercises (E) |
| :---: | :---: |
| Description | Exercises are items assessing the mastery of standard methods and techniques in order to obtain mathematical results. Exercises deal "mostly with standardized contexts, with particular attention being paid to the range of methods and techniques students can activate, combine and flexibly employ, and to the range and complexity of contexts they can successfully deal with" (Niss, 1993, p.16). |
| Indicators | A collection of exercises appertains to "routine type considerations or operations in straightforward combinations" (Ibid., p.18). Exercises involve computations that require mathematical knowledge in a paper and pencil environment. |
| Examples | Textbook B (p.26) <br>  <br>  <br>  |
|  | Textbook C (p.177) <br>  |
|  | Textbook F (p.37) <br> 4. Ау M каı N вívaı avtí́toxa ta $\mu$ éoa $\tau \omega v$ tó $\xi \omega \nu$ AB каı ГА, va öıкаодоүи́бยเદ үıatí to عuaúүрацио гци́ца <br>  tou кúкไou. |
|  | Textbook H (p.14) <br>  <br>  <br>  <br>  vtal. |
| Decision Rules | 1. Based on diagrams or schemas and an explanation is required (Textbook G, $\Sigma \kappa \varepsilon ́ \psi о v ~ \kappa \alpha ı ~ A \pi \alpha ́ v \tau \eta \sigma \varepsilon ~(4), ~ p .37) ~(~) ~$ <br> 2. Constructions (Textbook H, Абкŋ́б\&ıऽ E $\mu \pi \varepsilon ́ \delta \omega \sigma \eta \varsigma$ (2), p.14) <br> 3. Questions pertaining to measurements (Textbook B, Task 164, p.26) <br> 4. Justifying theorem (Textbook A, §107-110, Task B. 81 (p.70)) <br> 5. Questions pertaining to relations between the points (Textbook F, chapter 1, Task 4 (p.25)) |

Table 17: Definition for the category Problems (P) tasks.

| Name | Problems (P) |
| :---: | :---: |
| Description | Problems are items assessing the performance of standard applications of mathematics in typical situations. They give close and thoughtful attention to "the range and complexity of the situations they can handle, and to the originality and depth with which they can treat these situations" (Niss, 1993, p.16). |
| Indicators | Problems involve non-routine or even extra-mathematical situations that require "considerations, operations or combinations of such" (Niss, 1993, p.18). |
| Examples | Textbook A (p.123) <br>  <br>  <br>  |
|  | Textbook B (p.42) <br>  <br>  <br>  |
|  | Textbook D (p.39) <br>  <br>  |
|  | Textbook E (p.149) <br>  <br>  $\triangle E Z H$ eivar enpabuyo of кíxNo. |
|  | Textbook F (p.116) $\qquad$ <br>  <br>  Y者 |
|  | Textbook G (p.75) <br>  <br>  <br>  үıa ta oroía to ádpoıбиа $\mathrm{AM}+\mathrm{MN}+\mathrm{NB}$ <br> عívaı to عגáxıoto ōuvató. |
| Decision Rules | 1. Multistep problem (Textbook H, इóv $\theta \varepsilon \tau \alpha$ @ $\dot{́} \mu \alpha \tau \alpha$ (3), p.245) <br> 2. Tasks pertaining to loci, as they examine the direct and indirect side of the problem (Textbook F, task 9, p.168) <br> 3. Extension of the schema, using points, lines, figures (Textbook G, Гغvıкє́s Абкŋ́бєıs (3), p.109) <br> 4. Pertain to the pattern: <br> a. Analysis- Construction- Proof- Investigate if there is an extra mathematical solution (Textbook A, Task 351, p.158) <br> 5. Improve a quality by changing the schema (Textbook C, chapter 7, task 12) |

Table 18: Definition for the category Mathematical Modelling (MM) tasks.

| Name | Mathematical Modelling (MM) |
| :---: | :---: |
| Description | 1. Mathematical modelling are items assessing the connection of the real-life situations with mathematics. Specifically, they refer to the "process of model building, leading from a real situation to a mathematical model, or to the whole applied problem-solving process, or any manner of connecting the real world with mathematics" (Blum, 1993, p.5). <br> 2. "A mathematical model consists of the extra-mathematical domain, D , of interest, some mathematical domain M , and a mapping from the extra-mathematical to the mathematical domain" (Niss, 2007, p.6) <br> "Mathematics and the rest of the world (Niss, Blum and Galbraith, 2007, p.4)" |
| Indicators | Mathematical Modelling consists of exercises and problems for open, complex, real, authentic and extra mathematical situations which belong to other subjects, or practice areas. In other words, students are asked to utilize some relations from the domain of mathematics and map properties and content from mathematics to real-life situations. |
| Examples and Decision Rules | Realistic or applied perspective: Textbook B (p.141) <br>  <br>  |
|  | Epistemological or theoretical perspective: Textbook A (p.273) <br>  |
|  | Socio-critical or emancipatory perspective: Textbook F (p.35) <br>  <br>  <br>  <br>  |
|  | Contextual perspective: Textbook D (B 6, p.79) <br>  <br>  <br>  |
|  |  <br>  $\alpha \sigma \nu \mu \beta \dot{\alpha} \tau \omega \varsigma ~ \kappa \dot{\theta} \theta \varepsilon \tau \varepsilon \varsigma$. |

## 4. 8. General information on textbooks

As noted earlier, the eight selected textbooks of Euclidean Geometry were combined into six Books. By combining specific textbooks, i.e. textbooks B and C as Book 2, textbooks C and D as Book 3 and textbooks E and F as Book 4, a total access to the topics that they were found in the Geometry course of Lyceum was achieved. Having accounted the number of images and tasks in each textbook, I consequently examined how many pages were dedicated to each of the ten topics. Most pages are referred to Solid Geometry in all Books. The number of pages dedicated to the other topics fluctuates over time. Table 19 indicates the numbers of pages for every topic, while Figure 6 demonstrates the proportion of pages found in each textbook.

Table 19: Number of pages in the particular topic

|  | Book 1 | Book 2 | Book 3 | Book4 | Book 5 | Book 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 48 | 25 | 25 | 28 | 42 | 29 |
| T | 48 | 48 | 48 | 58 | 50 | 40 |
| PL | 0 | 18 | 18 | 8 | 18 | 22 |
| Quad | 19 | 25 | 25 | 17 | 32 | 26 |
| IS | 19 | 12 | 12 | 19 | 24 | 21 |
| A and S | 36 | 32 | 32 | 17 | 40 | 38 |
| M | 17 | 17 | 17 | 20 | 24 | 22 |
| A | 25 | 23 | 19 | 21 | 26 | 26 |
| CM | 119 | 87 | 42 | 72 | 66 | 68 |
| S | 365 | 322 | 258 | 280 | 352 | 319 |



Figure 6: Proportion of pages for every topic in all Books

A closer look at the graph (Figure 6) reveals that in certain topics the proportion of pages declines, comparing Books from 1 to 6 . Specifically, those declines were found in specific topics, i.e. "Introduction to Euclidean Geometry", "Triangles", "Metric relations" and "Solid Geometry". Solid Geometry topic had the highest proportion in all Books of Euclidean Geometry, followed by Triangles. An exception was found in Book 1, in which the same proportion for the topics "Introduction to Euclidean Geometry" and "Triangles" was counted.

## Chapter 5 Results

## 5. 1 Introduction

In this chapter the findings for the image and tasks analysis are presented. The current chapter is divided into two sections. More specifically, in section 5.2 I describe the results related to the use of images in Geometry textbooks, while in section 5.3 I present my findings for the categories of tasks used in these textbooks. With regards to my research questions, Section 5.2 relates to RQ1 (What categories of images?), RQ2 (Are there any variations?) and RQ3 (What can be said about the ways that geometrical concepts are presented to students?). On the other hand, Section 5.3 relates to RQ4 (What categories of tasks are assessed?), RQ5 (Are there any variations?) and RQ6 (What can be said about the ways that the categories of tasks are presented to students?).

## 5. 2 Image Analysis

In this section results for the analysis of images are presented. Before moving on though, it would be useful to present a few details. As I have already mentioned, the selected sample included eight textbooks (textbooks A, B, C, D, E, F, G, and H). Initially, the number of NC, BC and C images per textbook was counted and then for simplicity I merged five of them (textbooks B, C, D, E and F) in order to have a combination of the content material books of the $1^{\text {st }}$ and the $2^{\text {nd }}$ grade of Lyceum, alike to texts A, G and H. For example, Book 2 consists of textbooks B and C, while Book 3 is a combination of textbooks C and D. Finally, textbooks E and F were incorporated into Book 4.

Books were constructed in that way, in order to allow direct comparisons i.e. compare the categories of images overall and per geometrical topic. The textbooks were required to be "independent" (used for teaching in the $1^{\text {st }}$ and the $2^{\text {nd }}$ grade of Lyceum concurrently) and "complete" (including all the geometrical topics). For example, an "independent" and "complete" textbook is considered a textbook such as textbooks G and H , which satisfy a direct access to the mathematical concepts of Lyceum. Besides, the creation of the Books was inevitable in terms of providing the means for comparing the images within each topic of Geometry. The created topics were based on the contents of textbooks G and H , as both included all the curriculum material of Geometry. The previous textbooks were used for the teaching of Geometry either in the $1^{\text {st }}$ or the $2^{\text {nd }}$ grade of Lyceum and they did not contain all the created topics of Geometry. By combining the textbooks, the created Books were similar to textbooks G and H in content.

### 5.2.1 Overall results



Figure 7: The use of NC, BC and C images (real numbers) for the six Books.
Figure 7 presents the total number of images found in each Book from 1975 to 2019. It should be mentioned that there is a gap between 1976 and 1979; this is because I did not have access for this period's textbook, and also its contents were similar to Book's 1. Overall, the graph indicates that Book 2 has the highest number of images (742), while Book 4 has the lowest (491). In terms of the three image categories (C, BC, NC), it was found that C images are the most frequently used category in all books, followed by BC images, while the number of NC images was consistently low across all Books.

The use of C images was over 420 images from 1975 to 1986, however from 1979 and on their use declined between Books 2 to 6 . The use of BC images varied in all Books. Explicitly, the use of BC images followed a significant downward tendency between Books 2 to 4 . Finally, the use of NC images was continually below 10, except Books 4 (36) and 6 (17).

As it can be seen from Figure 7, the use of NC, BC and C images in the Books varies considerably between the years. Bearing in mind the variation of the numbers per Book and that some of them (Books 2, 3 and 4) were combined, it seemed appropriate to make percentage comparisons. Real numbers provided substantial differences in figures and were avoided in order to guarrantee the validity of the results. For instance, Book 2 had significantly a greater number of images than the other and it might be assumed that this occurred due to my approach to combine different textbooks (i.e. textbooks B and C in Book 2). The percentages were calculated by dividing the number of each category in a particular Book by the total number of the images in this particular book. For example, in order to estimate the percentage of NC images in Book 4, I divided the number of $\mathrm{NC}(4)$, by the total number of images $(\mathrm{NC}+\mathrm{BC}+\mathrm{C}=36+131+324=491)$.


Figure 8: The use of NC, BC and C images (percentage) for the six Books.
Figure 8 presents the total number of images found in each Book from 1975 to 2019. Overall, the C images accounted for the largest proportion of the images in all Books, followed by BC images. NC images were found having the lowest proportion repeatedly.

As it can be seen, from 1975 to 1990 (Books 1, 2 and 3) C images covered over $70 \%$ of the images used, while their use is declined between the years 1990 to 2019 (Books 4, 5 and 6), with their lowest use found in Book 5 ( $60,25 \%$ ). The use of BC images followed a continual decline across the Books of period 1975 to 1999 (Books 1, 2, 3 and 4), while in Books 5 (1999-2001) and 6 (2001-2019) its proportion was over $33 \%$. Additionally, the use of NC images was constantly in low proportions (under 10\%), although in Book 4 (1990-1999) there was a climax ( $7,33 \%$ ).

According to Figure 8, C were the images used in the majority of all Books. However, from 1990 and on (Books 4, 5, 6), we can observe a decrease in their use and an increase in the use of BC images. In order to estimate whether these differences are statistically significant, the two-proportion z-test ${ }^{4}$ was used. Initially, the six Books (Books 1, 2, 3, 4, 5, 6) were divided into two groups as the $z$-test requires. The first group was consisted of Books 1, 2 and 3 and the second group of Books 4, 5 and 6. Secondly, the total number of a particular category per group was counted. The results of the z-test provided support that the decrease of C images (from $70,59 \%$ in the first group to $63,13 \%$ for the second), and the increase of BC (from $29,86 \%$ in the first group to $32,89 \%$ in the second) are statistically significant (Table 20).

[^3]Table 20: Two proportion z-test for $C$ and $B C$ images

| Category of <br> images | $1^{\text {st }}$ group (Books <br> $1,2,3)$ | $2^{\text {nd }}$ group (Books <br> $4,5,6)$ | z -test, $p<0.05$ |
| :---: | :---: | :---: | :---: |
| C | $70,59 \%$ | $63,13 \%$ | $\mathrm{z}=4.81$ |
| BC | $29,86 \%$ | $32,89 \%$ | $\mathrm{z}=-2.79$ |

## 5. 2.2 Categories of images per topic

## 5. 2. 2. 1 Introduction to Euclidian Geometry (I)



Figure 9: The use of C, BC and C images (percentage) for the topic "Introduction to Euclidean Geometry".
Figure 9 presents the proportion of NC, BC and C images used in each Book for the topic "Introduction to Euclidean Geometry" from 1975 to 2019. As it can be seen in the graph, the percentage of NC images was consistently the smallest, whereas the use of C and BC images fluctuated through time. In addition, the percentage of BC images remained at the highest levels in all Books, followed by C images.

BC images covered over $60 \%$ of the images used in the majority of Books (Books 2, 3, 5 and 6), although their use was either the same to the use of C images (Books 1) or very close (Book 4). The use of C images was under $50 \%$ across all Books from 1975 to 2019 and it can be seen that in Books 2, 3 and 5, their used varied between $21 \%$ to $24 \%$. Finally, the use of NC images was below $7 \%$ in all Books, apart from Book 4 (1990-1999) in which they presented their highest value (17,02\%).

## 5. 2. 2. 2 Triangles (T)



Figure 10: The use of NC, BC and C images (percentage) for the topic "Triangles".

Figure 10 presents the proportion of $\mathrm{NC}, \mathrm{BC}$ and C images used in each Book for the topic "Triangles" from 1975 to 2019. Overall, C images remained at the highest level in all books, in contrast to NC images that were used only in Books 4 (1990-1999) and Books 6 (2001-2019). The BC images remained in the middle level.

From 1975 to 2019, C images covered over 63\% of the images used, while periods from 1979 to 1990 (Books 2 and 3) presented the largest proportion of all Books. The use of BC images saw a decline from 1975 to 1990 (Books 1, 2 and 3), however their use increased in the period from 1990 onwards (Books 4, 5 and 6), reaching values over $32 \%$. The NC images found only in Books 4 (1990-1999) and Book 6 (20012019), reaching values below $4 \%$ respectively.

## 5. 2. 2. 3 Parallel lines (PL)



Figure 11: The use of NC, BC and C images (percentage) for the topic "Parallel Lines".

Figure 11 presents the proportion of NC, BC and C images used in each Book for the topic "Parallel Lines" from 1975 to 2019. The C images used more than any other type of images in all Books, followed by BC and NC images in popularity. As it can be seen, the three categories of images were not found in Book 1, since their content was part of the mathematical content of the topic "Introduction to Euclidean Geometry". From 1979 and on, Parallel Lines were either a separate chapter or introduced in combination with triangles.

From 1979 to 2001 (Books 2, 3, 4 and 5), C images covered over 91\% of the tasks used however their use declined in 2001 (Book 6), reaching the lowest proportion ( $57,50 \%$ ). The proportions of BC images was constantly low (below $8,40 \%$ in Books 2,3 and 5) however their highest use found in Book $6(35 \%)$. On the other hand, NC images had the highest proportion in Book 6 (7,50\%), followed by Book 4 (5,26\%), since they were not found in other Books.

### 5.2.2.4 Quadrilaterals (Quad)



Figure 12: The use of NC, BC and C images (percentage) for the topic "Quadrilaterals".
Figure 12 presents the proportion of NC, BC and C images used in each Book for the topic "Quadrilaterals" from 1975 to 2019 . Overall, C images were the most used category, followed by BC , while NC images had the smallest number of visual representation and reached their highest percentage in Book 4 (1990-1999).

Explicitly, from 1975 until 1990 (Books 1, 2 and 3), C images cover over $72 \%$ of images used, however, from 1990 to date (Books 4, 5 and 6), their use declined, while varied between $59-69 \%$. The use of BC images varied between $14-32 \%$ across all Books while from 1990 onwards (Books 4, 5 and 6) their use increased reaching proportions over $25 \%$. The NC images can only be found in two Books, specifically Book 4 (1990-1999) and 6 (2001-2019), with their use being significant different and falling from $15,38 \%$ (Book 4, highest) to 2,38\% (Book 6, lowest).

## 5. 2. 2. 5 Inscribed shapes (IS)



Figure 13: The use of NC, BC and C images (percentage) for the topic "Inscribed Shapes".
Figure 13 presents the proportion of $\mathrm{NC}, \mathrm{BC}$ and C images used in each Book for the topic "Inscribed Shapes" from 1975 to 2019. Overall, C images were the most frequently used category in all Books, while BC and NC were constantly in low proportions.

From 1975 to 1990 (Books 1, 2 and 3), C images covered over 95\% of the images used, with their highest use found in Books 2 and 3 (100\%). However, from 1990 to 2019 (Books 4, 5 and 6) their use declined reaching proportions between 73-78\%. The use of the BC images was either low (Book 1) or there was not any use (Books 2 and 3). On the other hand, in the period from 1990 onwards (Books 4,5 and 6) their use increased, counted the highest use in Book 5 (26,19\%). Finally, the proportion of NC images was either equal to the use of the BC in Book 4 (11,11\%), or under 3\% (Book 6 , lowest).

## 5. 2. 2. 6 Analogies and Similarity (A and $S$ )



Figure 14: The use of NC, BC and C images (percentage) for the topic "Analogies and Similarity".

Figure 14 presents the proportion of NC, BC and C images used in each Book for the topic "Analogies and Similarity" from 1975 to 2019. Generally, C images were the most used images across all Books, with BC and NC images following in popularity.

From 1975 to 1990 (Books 1, 2 and 3) C images covered over $85 \%$ of the images used, however their use declined in Book 4 ( $70,37 \%$, lowest) and increased in the period from 1999 onwards (Books 5 and 6), counting proportion between $82-86 \%$. The use of BC images declined from 1975 to 1999 (Books 1, 2, 3 and 4), however their proportion founded $17,50 \%$ in Book 5 (highest). Lastly, NC images were only used in Books 4 ( $18,52 \%$, highest) and 6 ( $5,71 \%$, lowest).

## 5. 2. 2. 7 Metrics Relations (M)



Figure 15: The use of NC, BC and C images (percentage) for the topic "Metrics Relations".

Figure 15 presents the proportion of NC, BC and C images used in each Book for the topic "Metrics Relations" from 1975 to 2019. Comparing the different categories of tasks, C images accounted the highest proportions in all Books, while the use of BC and NC images was very low.

From 1975 to 2019, the use of C images covered over $87 \%$, of all images used and it can be seen that there were slight differences in their proportions. The highest proportion in the use of BC images found in Book 1, followed by a decline in their use, since between the period from 1986 to 2019 (Books 3, 4, 5 and 6) its proportion varied between $5,88 \%$ to $8 \%$. Finally, the use of NC images was rare as they were used only in Books 3, 4 and 6, reaching the highest value in Books 4 and $6(4 \%)$ and the lowest in Book 3 (2,94\%).

## 5. 2. 2. 8 Areas (A)



Figure 16: The use of NC, BC and C images (percentage) for the topic "Areas".
Figure 16 presents the proportion of NC, BC and C images used in each Book for the topic "Areas" from 1975 to 2019. A glance at the graph reveals that the proportion of C images had the highest numbers constantly, followed by BC, while NC images were in low values throughout the period.

From 1975 to 1986 (Books 1 and 2), the use of C images was over 90\% (highest), however their proportion declined in Book 3 ( $50 \%$, lowest) and in the period from 1990 onwards (Books 4, 5 and 6) their proportion was over $63 \%$. As for BC images, their use was under 10\% in Books 1 and 2, while from 1986 to date (Books 3, 4, 5 and 6) their proportion fluctuate between $19 \%-47,43 \%$ (highest, Book 4). Finally, NC images were used only in Books 3, 4 and 6, counting proportion below 4\%.

## 5. 2. 2. 9 Circle's measurement (CM)



Figure 17: The use of NC, BC and C images (percentage) for the topic "Circle's Measurement".

Figure 17 presents the proportion of NC, BC and C images used in each Book for the topic "Circle's Measurement" from 1975 to 2019. Overall, C images were the most used images across all Books, with BC and NC following in popularity.

As it can be seen, from 1975 until 1990 (Books 1, 2 and 3) C images covered over $80 \%$ of the images used, however in Book 4 (1990-1999) their proportion was the lowest ( $60 \%$ ) and then increased over $71 \%$ from 1999 to date (Books 5 and 6). The use
of BC images varied between $15 \%$ and $36 \%$ (Book 4, highest), however in the period of 1986 to 1990 (Book 3), their use found to be the lowest (4\%). As for NC images, their proportion was below $7,15 \%$ throughout the years examined.

## 5. 2. 2. $10 \quad$ Solid Geometry (S)



Figure 18: The use of NC, BC and C images (percentage) for the topic "Solid Geometry".

Figure 18 presents the proportion of NC, BC and C images used in each Book for the topic "Solid Geometry" from 1975 to 2019. Overall, the C images were the ones used mostly, followed by the BC and the NC in popularity

From 1975 to 1999 (Books 1, 2, 3 and 4), C images covered over $63 \%$ to $67 \%$, of the images used, however their use declined in the period of 1999 to 2001 (Book 5) reaching the lowest proportion $(42,42 \%)$. As for the use of BC images, their proportion declined from 1975 to 1999 (Books 1, 23 and 4), while their use increased in Book 5 ( $56,82 \%$, highest). Finally, the use of NC images was below $4 \%$ from 1986 to 2019 (Books 3, 4, 5 and 6), with no NC images used from 1975 until 1986 (Books 1 and 2).

### 5.2.3 Summary

By using the absolute number of images used in each Book, the analysis revealed variations in the different categories of images used in each Book. Those variations might be a product of my approach to combine specific textbooks ( B and C as Book 2; C and D as Book 3, E and F as Book 4). So, presenting the three categories ( $\mathrm{C}, \mathrm{BC}$, NC) of images in proportion was preferable for the validity of the results. This approach was considered to be more suitable for answering RQ1 ("what categories of images can be found in Greek Geometry textbooks from 1975 to 2019?"), RQ2 ("are there any variations in their use throughout the years?") and RQ3 ("what can be said about the ways that geometrical concepts are presented to students?").

Overall, results indicated a preference in using C images in all Greek textbooks of Euclidean Geometry. BC images were used less frequently, while the use of NC was
continually in low values. The classification of images as NC, BC or C indicated varying emphasis according to the topic:

1. Percentage of $\mathbf{B C}$ images greater than the percentage of $\mathbf{C}$ images. The topic "Introduction to Euclidean Geometry" (I) (Figure 9) was the only case in which the use of BC images exceeds the use of C images. An exception observed in Book 1; both numbers were equal. In general, the introductory section comprises definitions, postulates and properties of essential schemas. There is a lack of proofs and constructions, since (I) consists mostly illustrations of mathematical concepts that do not require any explanation. The aim of this chapter is to introduce students to fundamental concepts of Euclidean Geometry.
2. Percentage of $\mathbf{C}$ images greater than the percentage of BC images (difference $\mathbf{3 0 \% - 5 0 \%}$ ). This case can be seen in the topics "Triangles", "Quadrilaterals" and "Solid Geometry", with some exceptions though. These are: (1) In Books 2 and 3 of Quadrilaterals the use of C images diverged from the BC images approximately $70 \%$ and (2) images exceeded C images (Book 5) in Solid Geometry, with the divergence being approximately $14 \%$, since many terms were presenting with the form of a definition, while in Book 6 the C images exceeded the number of BC in less than $13 \%$. Despite that, differences for C and BC images for the above-mentioned topics are not surprising. This is because the topic "Triangles" deals mostly with proofs that utilize heavily images; a typical example is the three criteria of triangles. Hence, solved examples, loci or even applications are used in this chapter in order to help students understand the mathematical concept. In "Quadrilaterals" students get familiar with specific terms from the initial grades of the elementary school. Those terms are enriched with theorems and postulates, along with the process of proof in Lyceum. Hence, there is a variety of applications related to quadrilaterals which are based on figures. Two striking examples are the criteria of parallelograms and their applications to triangles and quadrilaterals. For this cause, it is expected the conceptual images to count the highest number of images. In "Solid Geometry", it is worth mentioning that in Books 1 and 2, this topic is divided into three chapters, while the following contained two chapters. As a consequence, there is divergence in the numbers.
3. Percentage of $\mathbf{C}$ images is greater than the percentage of $\mathbf{B C}$ images (difference $\mathbf{5 0 \% - 8 0 \%}$ in the majority of Books). This case is observed in "Parallel lines" (apart from Book 6), "Inscribed shapes" (apart from Book 5), "Analogies and similarity", "Metrics relations", "Areas" (apart from Books 3, 4, 5) and "Circle's measurement" (apart from Book 4). More specifically:

- The topic of Parallel Lines constitutes C images in the highest proportion, as there are many theorems, postulates and solving tasks that require visual representations with the purpose of explaining the steps of the mathematical proofs, or to help students understand a notion of
the solving procedure. Hence, Parallel Lines deal mostly with angles that lay between line segments and parallels and so the definitions are represented in the teaching of a previous chapter. All reasons considered, it is noticeable why there is a lack of BC images and a wide variety of C images.
- Inscribed Shapes include inscriptions of angles to circle or theorems that demonstrate the inscription of different kinds of figures to circles. Apart from proofs, there are applications that cannot be solved without the aid of figures. In every step of the solution the student gathers information from the relevant schemas in order to complete the solving task.
- In Metric Relations an individual studies metrics related to triangles, quadrilaterals and circles. Those metric relations consist theorems that their proof is based on the figure and also tasks asking student to construct figures in order to understand the mathematical concept and solve the exercise.
- As for the Areas, there was a wide difference in the numbers of C and $B C$ images at the beginning. This change remained constant until book 2 , while in the following there were not such variations in proportions. This occurred from the changes on the teaching material and the addition of solving problems along with applications in the Books that enriched the knowledge supposed to be taught.
- In Circle's Measurement, it was expected to have more C images than $B C$. This chapter deals with regular polygons and terms that are related to circles. Some typical examples that the students are getting familiar with are the arc length, the length of the circle and the area of a circular disc. The images used in proofs of the theorems and in the applications are significant for the understanding of the notions of the mathematic steps.


## 5. 3 Analysis of tasks

In this section, I will present the results from the analysis of tasks. The analysis occurred after the analysis of images, so the textbooks were already merged into Books. Technically, I did not face any difficulties since the analytical approach used here, was similar to the one used for the analysis of images.

### 5.3.1 Overall results



Figure 19: The use of Qs, E, P and MM (real numbers) for the six Books.
Figure 19 indicates the total number of tasks found in each Book from 1975 to 2019 (with the aforementioned three-year gap from 1976 to 1979). In general, Book 1 had the highest number of tasks of all Books (1087), followed by Book 5 (1032). On the other hand, Book 3 counted the lowest number (518). In terms of the four task categories (Qs, E, P, MM), it was found that P tasks were the most popular category used in all Books, followed by E tasks. The use of MM tasks was continually in low values, while the use of Qs tasks increased from 1990 onwards (Books 4, 5, 6).

From 1975 until 1986 (Books 1 and 2) P tasks were over 664, however from 1986 to date their use varied between 326 to 411 . The use of $E$ tasks varied across all Books, presenting a downward tendency from 1975 to 1990 (Books 1, 2 and 3) and an upward from 1990 to 2001 (Books 4 and 5). From 1975 to 1990 (Books 1, 2 and 3) Qs tasks were rare, however their use increased in the period from 1990 onwards (Books 4, 5 and 6), with their highest use found in Book 5 (241). As for the MM tasks, their use was in low values across all Books.

As it can be observed from Figure 20, the number of the categories Qs, E, P, and MM of tasks in the Books modified considerably between the years. Having already made comparisons in proportion for images, the same approach was followed. Similar to images, the percentages for each category of tasks were calculated by dividing the number of each category in a particular book by the total number of the tasks in this specific book. For example, in order to estimate the percentage of Qs tasks in Book 1, the number of Qs (16) was divided by the total number of tasks (Qs+ E+ P+ MM = 16+ $371+696+4=1087$ ), resulting $1,47 \%$.


Figure 20: The use of Qs, E, P and MM tasks (percentage) for the six Books.
Figure 20 presents the proportion of Qs, E, P and MM tasks used in six Books from 1975 to 2019. Overall, P tasks accounted for the largest proportion of tasks in all Books, followed by E tasks. Qs tasks presented highest number from Book 4 (19901999) and on, while the MM tasks were found having the lowest proportion repeatedly.

From 1975 until 1990 (Books 1, 2 and 3), P tasks covered over $62 \%$ of the tasks used, however their use declined during the period from 1990 to date (Books 4, 5 and 6 ) with their lowest proportion found in Book 5 (34,98\%). The use of E tasks stayed almost constant throughout the years examined. On the other hand, the proportion of Qs tasks was initially low (Books 1, 2 and 3), while their use increased from 1990 onwards (Books 4, 5 and 6), reaching proportions over $16 \%$. Finally, the use of MM tasks was under 5\% across all Books, with their proportion being increased from 1999 to 2019 (Books 5 and 6).

According to Figure 20, P tasks were the most popular category in all textbooks of Geometry, apart from Book 5 (1999-2001); in this one the E tasks were slightly higher in proportion than $P$ tasks. However, a significant decrease in the use of $P$ and an increase in the use of Qs tasks can be observed from 1990 and on (Books 4, 5, 6). In order to estimate if the differences in the proportions were the same, the twoproportion z-test was used. Initially, the six Books (Books 1, 2, 3, 4, 5, 6) were divided into two groups as the z -test requires. The first group was consisted of Books 1, 2 and 3 and the second group of Books 4, 5 and 6 . Secondly, the total number of a particular category per group was counted. The results proved that the use of P tasks decreased from an average ratio of $65,89 \%$ (first group) to $42,08 \%$ (second group). On the contrary, the use of Qs tasks increased from an average ratio of $0,78 \%$ (first group) to $19,08 \%$ (second group). Similarly, the use of E tasks increased slightly from an average ratio of $32,90 \%$ (first group) to $35,53 \%$ (second group). Lastly, the use of MM tasks increased from an average ratio of $0,43 \%$ (first group) to $3,35 \%$ (second group). All the statistical differences were found to be statistically significant.

Table 21: Two proportion z-test for P, E and Qs tasks.

| Category of tasks | $1^{\text {st }}$ group (Books <br> $1,2,3)$ | $2^{\text {nd }}$ group (Books <br> $4,5,6)$ | z -test, $p<0.05$ |
| :---: | :---: | :---: | :---: |
| P | $65,89 \%$ | $42,08 \%$ | $\mathrm{z}=17.29$ |
| E | $32,90 \%$ | $35,53 \%$ | $\mathrm{z}=-2.00$ |
| Qs | $0,78 \%$ | $19,08 \%$ | $\mathrm{z}=-21.94$ |
| MM | $0,43 \%$ | $3,35 \%$ | $\mathrm{z}=-7.69$ |

### 5.3.2 Categories of tasks per topic

5.3.2.1 Introduction to Euclidian Geometry (I)


Figure 21: The use of $Q s, E, P$ and $M M$ tasks (percentage) for the topic "Introduction to Euclidean Geometry".
Figure 21 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Introduction to Euclidean Geometry" from 1975 to 2019. Overall, there is no popular category in use since specific categories ( $\mathrm{P}, \mathrm{E}$ and Qs) vary across all Books. The only type of tasks being constant and in low proportion is the MM tasks.

As it can be seen, from 1975 to 1990 (Books 1, 2 and 3) P tasks were the dominant category in use in the majority of Books, however, their use in the period from 1990 to date (Books 4, 5 and 6) was under 20\%, with their lowest use found in Book 5 (2,04\%). The use of E tasks declined across Books 1 and 2, while from 1990 to 2019 their use increased, reaching the highest proportion in Book 5 ( $65,13 \%$ ). As for Qs tasks, they were only found in Book 1 (Books 1, 2 and 3), however their use in the period from 1990 onwards (Books 4,5 and 6) increased, reaching the highest proportions of all tasks in Books 4 and 6. Finally, there were no MM tasks the years from 1975 to 1990, while there was a slight increase in their use from 1990 to 2019, with the highest proportion found in Book 6 (3,57\%).

## 5. 3. 2. 2 Triangles (T)



Figure 22: The use of Qs, E, P and MM tasks (percentage) for the topic "Triangles".
Figure 22 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Triangles" from 1975 to 2019. Overall, P tasks were the most used tasks in the majority of Books, with E, Qs and MM following in popularity.

As it can be seen, from 1975 to 1990 (Books 1, 2 and 3) P tasks covered over 55\%, however, their use declined in the period from 1990 onwards (Books 4, 5 and 6). The proportion of E tasks varied between $27 \%$ to $42 \%$, with the highest use of all tasks found in Book 5. The proportion of Qs tasks increased from 1990 to date (Books 4, 5 and 6), reaching proportions over $22 \%$. There were no MM tasks until 1999 (Books 1, 2,3 and 4 ), while their proportion found below $4 \%$ in Books 5 and 6.

## 5. 3. 2. 3 Parallel lines (PL)



Figure 23: The use of Qs, E, P and MM tasks (percentage) for the topic "Parallel Lines".

Figure 23 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Parallel Lines" from 1975 to 2019. Overall, P tasks accounted the highest
proportion in the majority of Books, with the E, Qs and MM following in popularity. As it can be seen there were no tasks in Book 1, since parallel lines were part of the topic Introduction to Euclidean Geometry.

From 1979 until 1990 (Books 2 and 3), P tasks covered over 82\%, while their use declined in the period from 1990 onwards (Books 4, 5 and 6). The use of E tasks increased from 1979 to 2001, with the highest proportion of all tasks found in Book 5 $(42,59 \%)$. Qs tasks found the period from 1990 to date (Books 4, 5 and 6), with their proportion being decreased the whole period. On the other hand, there were no MM tasks in the majority of Books from 1975 to 2019, apart from Book 4 and 6, in which their use was under $8,50 \%$.

### 5.3.2.4 Quadrilaterals (Quad)



Figure 24: The use of $Q s, E, P$ and $M M$ tasks (percentage) for the topic "Quadrilaterals."
Figure 24 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Quadrilaterals" from 1975 to 2019. Overall, P tasks were the most used tasks in the majority of Books, with the E, Qs and MM following in popularity

From 1975 until 1990 (Books 1, 2 and 3), two categories of tasks found in Books, the P and the E , with the first covered over $65 \%$ of the tasks used and the latter covered under $34 \%$. The use of P tasks declined in the period from 1990 to 2019 (Books 4, 5 and 6), with the lowest proportion found in Book 4 (35\%), while the use of E tasks increased in this period, with their highest use found in Book 4 (48,75\%). Qs tasks were only found in the period from 1990 onwards (Books 4, 5 and 6), reaching proportions over $14 \%$. On the other hand, the MM tasks only found in Book 6, with their proportion being lower than $2 \%$.

## 5. 3. 2. 5 Inscribed shapes (IS)



Figure 25: The use of Qs, E, P and MM tasks (percentage) for the topic "Inscribed Shapes".
Figure 25 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Inscribed Shapes" from 1975 to 2019. Overall, P tasks were the most used tasks the Books examined, with the E, Qs and MM following in popularity.

As it can be seen, from 1975 to 1990 (Books 1, 2 and 3), P tasks covered over 58\% of the tasks used, with their highest use found in Books 2 and 3 ( $83,33 \%$ ). On the other hand, their use declined from 1990 and on, reaching proportions lower than $55 \%$. The use of E tasks was over $16 \%$, reaching two highest proportion in Books 1 and 4. As for the Qs tasks, they found from 1990 and on (Books 4, 5 and 6), reaching proportions over $9 \%$, with their highest use found in Book $5(23,81 \%)$. Finally, there were not any MM tasks across all Books, apart Book 6 .

## 5. 3. 2. 6 Analogies and Similarity ( $A$ and $S$ )



Figure 26: The use of Qs, E, P and MM tasks (percentage) for the topic "Analogies and Similarity".

Figure 26 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Analogies and Similarity" from 1975 to 2019. In general, P tasks were the dominant type of tasks in use, with the E, Qs and MM tasks following in popularity.

Specifically, the use of P tasks declined across all Books, with their lowest proportion found in Book 5 (34,78\%). From 1975 until 1990 (Books 1, 2 and 3), E tasks covered over $42 \%$ of the tasks used, however, their use declined the period from 1990 onwards (Books 4, 5 and 6), with their lowest use found in Book 6 ( $28,28 \%$ ). Qs tasks found in Book 4, 5 and 6, reaching proportions over 12\%. Lastly, from 1986 and on (Books 3, 4, 5 and 6) the use of the MM tasks increased continually, reaching proportions between $1,20 \%$ to $4,04 \%$.

## 5. 3. 2. 7 Metrics Relations (M)



Figure 27: The use of Qs, E, P and MM tasks (percentage) for the topic "Metrics Relations".

Figure 27 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Metrics Relations" from 1975 to 2019. Overall, the dominant type of tasks across all Books were either the P or the E tasks, with the Qs and MM following in popularity.

P tasks covered over $28 \%$ across all Books, while their proportions fluctuated, with the lowest use found in Book 5 (28,13\%). The use of E tasks varied throughout the Books examined, reaching their lowest proportion at Book 6 ( $36,36 \%$ ). The period from 1990 onwards (Books 4, 5 and 6), Qs tasks were found in Books and their use increased significantly between Book 4 and 5. Finally, there were any MM tasks across the Books, apart from Book 6, in which their proportion was $1,30 \%$.

## 5. 3. 2. 8 Areas (A)

 $100,00 \% \quad$

Figure 28: The use of Qs, E, P and MM tasks (percentage) for the topic "Areas".
Figure 28 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Areas" from 1975 to 2019. Overall, the majority of tasks used were either P or E , with Qs and MM tasks following in popularity.

Between the years 1975 to 1999 (Books 1, 2, 3 and 4), P tasks covered over $46 \%$ of the tasks used, however their use declined significantly in Book 5 (30,12\%). The proportion of E tasks was between $42 \%$ and $53 \%$, apart from Book 4 in which their use was the lowest $(23,91 \%)$. As for Qs tasks, they were only found in the period from 1990 until 2019, reaching proportions over $10 \%$. Finally, MM tasks found in only three Books (Books 3, 4 and 6), with their proportions being between $2 \%$ and $4 \%$.

## 5. 3. 2.9 Circle's measurement (CM)

$\qquad$


Figure 29: The use of Qs, E, P and MM tasks (percentage) for the topic "Circle's measurement".

Figure 29 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Circle's measurement" from 1975 to 2019. Overall, P and E tasks were the most used tasks across all Books, with Qs and MM tasks following in popularity.

The years between 1975 and 2019, P tasks continually declined, falling from their highest use in Book $1(50,70 \%)$. On the other hand, the use of $E$ tasks increased in the period from 1975 to 1990 (Books 1, 2 and 3), reaching the highest use in Book 3 ( $55,56 \%$ ), although their proportion declined in the years 1990 to 2019 (Books 4, 5 and 6 ), with their lowest use found in Book $5(31,88 \%)$. Qs tasks covered under $3 \%$ of the tasks used, from 1975 until 1990 (Books 1, 2 and 3), while their use increased in the period from 1990 onwards (Books 4, 5 and 6), with their highest use found in Book 5 (24,64\%).

## 5. 3. 2. $10 \quad$ Solid Geometry (S)



Figure 30: The use of Qs, E, P and MM tasks (percentage) for the topic "Solid Geometry".

Figure 30 presents the proportion of Qs, E, P and MM tasks used in each Book for the topic "Solid Geometry" from 1975 to 2019. Overall, P tasks were the most used tasks across all Books, with E, Qs and MM tasks following in popularity.

From 1975 until 1990 (Books 1, 2 and 3), P tasks covered over $68 \%$ of the tasks used, however, their use declined in the period from 1990 onwards (Books 4, 5 and 6), with their lowest use found in Book 5 (39,44\%). The use of E tasks initially decreased after Book $2(29,79 \%)$ over time, until Book 5 ( $23,89 \%$, lowest), however in Book 6 their proportion reached their highest value ( $36,70 \%$ ). As for Qs tasks, it can be said that from 1975 to 1990 (Books 1, 2 and 3), their use was under 2\%, even though their use increased from 1990 to date (Books 4, 5 and 6), with their highest proportion found in Book 5 ( $35,56 \%$ ). Finally, the use of MM tasks covered over $0,51 \%$ in Books 1, 2, 5 and 6, while no MM tasks used in Books 3 and 4.

### 5.3.3 Summary

For the analysis of the tasks, the same pattern as in the images was found. Initially, the proportion of the four categories of tasks (Qs, E, P, MM) used in order to estimate whether or not there were variations in the total number of tasks per Book and per topic. The differences presented in proportion in order to ensure a direct access in the research questions "What categories of tasks can be found in Greek Geometry textbooks from 1975 to 2019?" (RQ4), "whether are there any variations in the use of tasks throughout the years?" (RQ5) and lastly, "what can be said about the ways that the tasks are presented to students?" (RQ6) through the tasks.

The majority of the tasks used in all Books were P tasks, ranging from $45,87 \%$ to $69,60 \%$. An exception was observed in Book 5 (1999-2001). The E tasks were used in a moderate level, with minor variations in the total proportion per Book. Whilst the total number of Questionnaires (Qs) in the years between 1975 and 1990 (Books 1, 2, 3) was extremely low (under 1,50\%), their use increased in the period from 1990 onwards (Books 4, 5, 6), reaching proportions over $16 \%$. The Books of Euclidean Geometry had continually lower proportion of MM tasks, with a slight increase the years 1999-2019 (Books 5, 6).

For the ten topics investigated, P tasks were the most used tasks in the majority of Books, with E tasks followed in popularity from 1975 to 1990 (Books 1, 2, 3). In this period (1975 to 1990) there was a lack of Qs and MM tasks. However, from 1990 to date (Books 4, 5, 6), the use of P tasks decreased while the use of Qs tasks increased reaching high proportions. It should be kept in mind that MM tasks were rare in all the created topics of Euclidean Geometry. The examination indicates that:

1. Introduction to Euclidean Geometry: During the period from 1975 until 1990, there was a high difference (almost 57\%) in the proportion of P and E (Books 2 and 3: $\mathrm{P}>\mathrm{E}$ ). In the second period (1990-2019) the E increased sharply, towards the Qs , while the values of P were extremely low.
2. Triangles: From 1975 to 1990 (Books 1, 2 and 3) P tasks and $E$ tasks were the only categories appeared in Books. On the other hand, there was a sharply increase in the use of Qs the period from 1990 onwards (Books 4, 5 and 6), towards a fall in the number of P .
3. Parallel Lines: From 1975 to 1990 (Books 1, 2 and 3) the difference of main and only categories P and E was almost at $65 \%(\mathrm{P}>\mathrm{E})$. The inclusion of Qs in the period from 1990 to 2019 had as a consequence a dramatical fall in the number of P (decline approximately $50 \%$ ) and a slight increase in the number of E tasks.
4. Quadrilaterals: between the years 1975 and 1990 (Books 1, 2, 3) the main and only categories of tasks were P tasks and E tasks, with the first being the most popular category in use and the latter being used in a moderate level (Book 1: $\mathrm{P}>\mathrm{E}$ almost at $32 \%$, Books 2 and 3: $\mathrm{P}>\mathrm{E}$ exactly $50 \%$ ). The period 1990 to 2019 the Qs presented high values, while the number of P decreased, with their difference being at 27,06 on average $(\mathrm{P}>\mathrm{Q})$.
5. Inscribed shapes: this topic indicates substantial differences between all categories of tasks. P tasks used in popularity, ranging from $58 \%$ to $83,33 \%$ (Books 1, 2, 3), despite their downfall from 1990 to 2019 (ranging from 48,81\% to $54,76 \%$ ). The E tasks used in a moderate level, reaching their lowest proportion in Books 2, 3 and 6, while the inclusion of Qs tasks in Books 4, 5 and 6 , indicated high values.
6. Analogies and Similarity: P tasks covered over $49 \%$ of all tasks used across Book, with an exception founded in Book 5 (34,78\%). The proportion of E tasks was close to the proportion of P tasks, while from 1990 to 2019 (Books 4, 5 and 6) their use declined. As for the MM tasks, included in Book from 1986 and on counting low values. The inclusion of Qs tasks was in 1990 (Books 4, 5 and 6) and they ranged from $12.07 \%$ to $29,35 \%$.
7. Metrics relations: the proportion of P tasks and E tasks indicated fluctuation according to the Books. Although their values were close, in Books 3 (19861990) and 5 (1999-2001) the use of E tasks was higher. The Qs tasks presented in 1990 (Book 4), reaching high levels between the years 1999 to 2019 (Books 5 and 6).
8. Areas: the proportions of P tasks and E tasks were very close, whilst it was observed that in 1986 (Book 3) and 1999 (Book 5) the use of E tasks was greater than the use of P tasks. The inclusion of Qs in textbooks occurred in 1990 (Books 4,5 and 6), ranging from $10,39 \%$ to $16,87 \%$.
9. Circle's measurement: the proportions of P tasks and E tasks were either close or common (Book 2) with the latter used in higher values in the period from 1986 until 1999 (Books 3 and 4). From 1975 to 1990 (Books 1, 2 and 3) the proportion of MM tasks was greater than the Qs tasks, whilst from 1990 to date (Books 4, 5 and 6) their levels remained steady. The use of Qs tasks was high from 1999 to 2019 (Books 5 and 6).
10. Solid Geometry: this topic was divided into three sections from 1975 to 1986 (Books 1 and 2), while from 1986 to 2019 (Books 3, 4, 5, and 6) it consisted of two chapters in the Books. The most popular category of all tasks was P tasks, with E following in popularity. From 1990 to 2001 (Books 4 and 5), a high use in the number of Qs tasks found, towards a decline in the number of P tasks (Book 5: $\mathrm{P}>$ Qs almost at $3 \%$ ).

## Chapter 6 Discussion

## 6. 1 Introduction

The aim of this thesis was to explore in which ways Geometry concepts are introduced to students in school textbooks by studying the categories of images and tasks used in them. This provides access into investigating whether the priorities of authors have change or not. In this chapter, the answers to my six research questions are presented. Those referring to images (RQ1, RQ2, RQ3) are presented in Section 6.2, while in Section 6.3 answers to research questions for tasks (RQ4, RQ5, RQ6) are presented. The chapter concludes with this study's limitations and possible research directions for the future (Section 6.3).

Once more, I would like to remind the reader with the research questions posed in this study:

RQ1 What categories of images can be found in Greek Geometry textbooks from 1975 to 2019 ?
RQ2 Are there any variations in their use throughout the years?
RQ3 Having in mind the different categories of images used in these textbooks, what
can be said about the ways that geometrical concepts are presented to students?
RQ4 What categories of tasks can be found in Greek Geometry textbooks from 1975 to 2019?
RQ5 Are there any variations in the use of categories of tasks throughout the years?
RQ6 What can be said about the ways that the tasks are presented to students?

## 6. 2 Images

Based on relevant literature, the images found in each Geometry Book were classified as conceptual (C), bland-conceptual (BC) and non-conceptual (NC). Overall, the performed analysis showed that C images (i.e. images related to proofs and applications), that explain the geometrical concepts is the dominant category in use across all Books. The BC images i.e. definitions, axioms, postulates and theorems (without proofs) follow in popularity. The use of NC images (i.e. portraits, photographs, paints) is extremely low in popularity.

When the same analysis was performed to each of the ten identified topics, three distinct themes were identified:

1. The topic "Introduction to Euclidean Geometry" was the only case that BC images overcame C images, apart from Book 1, where their numbers were found to be equal. As it was already mentioned, this chapter incorporates the essential terms that students are required to know in order to develop a better understanding of the mathematical concepts ahead. The already known knowledge of the previous mathematics textbooks can be found in this topic. In most textbooks there was a variety of definitions, postulates, essential
properties of geometrical figures and schemas. As a consequence, the BC images were the most popular category used from the authors in this topic.
2. In the topics of "Triangles", "Quadrilaterals" and "Solid Geometry", C images are were more frequently used than BC images, with difference varying from $30 \%$ to $50 \%$. In some Books the differences were less than $30 \%$, but the main focus was on the majority of the Books.
3. In all the other topics ("Parallel lines", "Inscribed shapes", "Analogies and Similarity", "Metrics relations", "Areas" and "Circle's measurement"), C images were used more frequently, with differences varying from $50 \%$ to $80 \%$ in the majority of the Books.

Taking into consideration Figure 8, two distinct periods can be noticed. The first one takes place from 1975 until 1990 (Books 1, 2 and 3), while the latter lasts from 1990 until 2019 (Books 4, 5 and 6). Although, C images were the most popular category used in all Books, from 1990 onwards (Books 4, 5, 6) their use decreased from an average ratio of $70,59 \%$ (in Books 1, 2, 3) to $62,13 \%$ (in Books 4, 5, 6), with the statistical difference being significant ( $\mathrm{z}=4.78, p<0.05$ ). In the use of BC images, an opposite trend can be found: from 1990 onwards (Books 4, 5, 6), their use increased from an average ratio of $29,86 \%$ in Books (Books 1, 2, 3) to $32,89 \%$ (Books 4, 5, 6) with the statistical difference being significant ( $\mathrm{z}=2.75, p<0.05$ ).

So, how can these results be interpreted according to ATD? Using ATD terminology, during the first period (Books 1, 2, 3) the tasks (images) that the students were required to engage with in order to "construct" mathematical knowledge, were done by using more frequently techniques (categories of images) emphasising to proofs and applications (i.e. C images). Thus, the authors of these textbooks were considering more important proofs and applications (C images) for the teaching and learning of Euclidean Geometry. On the contrary, during the second period (Books 4, 5, 6), the tasks (images) that students were required to engage with in order to "construct" mathematical knowledge, were done by using more frequent techniques (categories of images) emphasising to definitions, axioms postulates and theorems without containing proofs (i.e. BC images). Taking into account the decrease in the use of tasks (images), using techniques pertaining to proofs ( C images) and the concurrent increase in the use of tasks (images) using techniques referring to definitions and postulates, it is obvious that from 1975 till nowadays, there is a change in the rationale of particular categories of tasks. This means that the technology (i.e. the rationale beyond the use of those categories of images) has changed, since the authors of Books 4, 5 and 6 did not emphasise in the process of proof, opposite to the authors of Books 1, 2 and 3. Based on the discussion above and bearing in mind that textbooks are a form of knowledge accepted by the community (Castell et al., 1998) or the influence that mathematical textbooks exert on students' beliefs regarding "what is mathematics" and what it means "I know mathematics" (Koдह́丂人, 2017), it can be concluded that knowing and understanding Euclidean Geometry in the Greek educational system tends to be less related to geometrical proofs.

Mathematical proofs must be taught at all grades of teaching (Ball, Hayles, Jahnke and Movshovitz-Hadar, 2002). Proofs are essential to mathematics and can be described as the "key component" or even being "at the heart" of the mathematics education since they promote deep understanding (Ibid.). To my way of thinking, they provide a basic mathematical skill for the promotion of reasoning. Additionally, images act as the "mediator" between the theoretical and the practical part of understanding Geometry (Elia and Philippou, 2004). This occurs since many students face difficulties while trying to comprehend basic components of Geometry, such as the orology, the spatial reasoning and the steps of proofs (Xalpétๆ, 2009). Bearing in mind these difficulties, the observed decrease of C images in school textbooks, might be concerning since it leads to the conclusion that the process of proof is not an integral part of Geometry. In other words, their reduction means that from 1990 until 2019 (Books 4, 5 and 6), school textbooks tend to be less related to the process of proof (C images) and focus on images reminding the students already known facts or containing new mathematical terms (BC images).

How can this shift be explained in terms of the sociocultural environment of Greece during the studied period? Toumasis (1990) notes that from 1836 to 1985, each syllabus of Euclidean Geometry in Greece was determined by a strict interpretation of the geometrical concepts, as they were proposed in the Elements of Euclid. According to him, in all the attempts made to revise the school Geometry in Greece, "those who exercised control over the mathematics curriculum reacted strongly against the new innovations, providing educational, philosophical and ethnic arguments" (Ibid., p.491). In other words, the resistance in reforming the teaching and learning of school Geometry in Greece was influenced by certain educational, philosophical and ethnical factors.

### 6.2.1 Educational factors

According to Toumasis (1990), until the late of $19^{\text {th }}$ century "each teacher taught according to his own judgement" (p.497). This means that the teaching of Geometry was based mostly on the personal beliefs and the didactical approach of each teacher. Concequently, the education was teacher-centred, putting the teacher on the core of every teaching. Additionally, most textbooks used in teaching were translated badly from foreign textbooks (Pharsis 1868, cited by Toumasis 1990). In addition, Toumasis (1990) comments on the lack of mathematics teachers. Most of them were high-school educated, while those who had graduate from university with "sufficient knowledge of mathematics" were not enough (p.497). Going further to the $20^{\text {th }}$ century, explicitly in the 1980 and 1985, no attempt to reform the content of Euclidean Geometry took place. On the other hand, its removal from the third grade of Lyceum (the last year of secondary education) was occurred,

## 6. 2. 2 Philosophical factors

The second source of arguments have a philosophical ground. The view of Plato for mathematics was held in the society, believing that mathematics is assumed as "a means of training the mind and the judgement of the adolescent" (p.498). Another factor
proposed, was the content of Euclidean Geometry which from 1987 onwards, it was practical and, as a consequence, some difficulties occurred. The first difficulty relates to the teaching of Geometry which it was "impossible" for students who were unfamiliar with the empirical part of Geometry. Other difficulties occurred when teachers tried to engage students to the strict reasoning of Geometry (Labiris, 1907 as cited at Toumasis, 1990).

### 6.2.3 Ethical factors

In terms of ethnical arguments, most were related to the nature of Geometry (remaining theoretical) within Greece and other countries. Geometry has its origins in Ancient Greece and expanded to the West. As a consequence, it was the duty of Greeks to maintain its strict nature from theoretical and scientific aspect (Toumasis, 1990). Additionally, it was held the belief that continual changes in the content of Geometry would influence, "spoil" as proposed, and destroy its coherence (Sakellariou, 1931, cited by Toumasis, 1990). Explicitly, modifying and changing the content of Geometry of other countries' textbooks, with a view to make it more accessible, would affect the coherence of Greek Euclidean Geometry.

## 6. 2. 4 Conclusion

Bearing in mind those factors it seems that the decrease in the number of C images from 1990 to 2019 (Books 4, 5, 6) was a result of the authors perspective about the process of proof. This means that the authors of Greek school Geometry textbooks from 1975 to 1990 (Books 1, 2, 3) emphasized in high level in the process of proof due to their philosophical and ethnical beliefs. Toumasis (1990, p.499) notes:
> "The reinforcement especially, of Euclidean Geometry in all school syllabi of contemporary Greece, can be attributed to the fact that theoretical Geometry was universally considered to be the pride of the ancient Greek spirit and the standard for the scientific foundation of every mathematical branch. More Geometry in school programs meant better organizing the student's thought, according to the principles of Euclid's Elements and simultaneously stronger bonds with the tradition and national inheritance. This nationalistic role of school Geometry is a serious obstacle in the efforts for renewal and modernization of its content."

There is a possibility that this pattern (increase in the number of BC images, decline in the number of C images) might be a result of new pedagogical tensions; nevertheless having in mind the decline of the interest in research for Euclidean Geometry since the 1970s (Inglis and Foster, 2018), there is no specific nor clear conclusion. Since the BC images include figurative representations, according to Mikk (2000) and Rivers (1990) the incorporation of figures in textbooks is to make students more motivated (cited at O'Keeffe and O'Donoghue, 2011) as illustrations are noticeable from students (O'Keeffe and O'Donoghue, 2011). Under this point of view, the increased use of BC images and the decrease duse of C images are not concerning. Kuzniak and Vivier
(2009) compared Geometry textbooks in two countries (France and Greece) and concluded that even though Geometry in Greek school environments is taught only for the cultural reasons, its axiomatic theoretical system is characterised as "well-structured and stronger" (p.694) than other countries, as France. Nikoloudakis (2009) mentioned that Euclidean Geometry in Lyceum, "is taught under a theoretical framework" (p.18) and first-year students are engaged to axioms, definitions, theorems, proofs (abstract components with theoretical background), while they have to deal with "specific procedures" on Gymnasium. As a result, they will not be able to provide solutions to problems in the University. So, a paradox occurs; although the number of images referring to proofs and solved applications (C type of images) has declined, school Geometry remains strict, in terms of the axiomatic system. This might be a result of the strong bonding with tradition (Euclid's legacy), as it is reflected in the study of Kuzniak and Vivier (2009), where the interviewed Greek teachers, highlighted that "Geometry is taught for culture, for Euclid" (p.693).

In conclusion, three categories of images can be found in Greek textbooks of Euclidean Geometry from 1975 to date: (1) conceptual or C images, (2) blandconceptual or BC images and (3) non-conceptual or NC images. Images related to proofs, applications and any form of image used to make students comprehend the mathematical concepts are characterised as C images. The BC images can be seen in definitions, postulates, corollaries to remind students the figure or to provide the new mathematical figures, without any explanation. Lastly, any type of decoration as landscapes and portraits are in the category of the NC images (RQ1). Examining their presentence in Books throughout the years (between 1975 to 2019), an estimation is made; the most dominant category is the C images, following by the BC , while the NC images remain in a very low level. Nevertheless, a decline is observed in the number of C images, towards an increase in the number of BC, from 1990 and on (RQ2). For the final question of images (RQ3) using the terms of the ATD, a conclusion is made that between the years 1975 to 1990 the authors put more emphasis in the proofs and applications (C category of images), since the use of C images is continually in high levels. From 1990 to 2019, it is observed a decline in the number of C, towards an increase in the number of BC images, which means that the rationale beyond the use of the three categories has change. Specifically, the authors enrich textbooks with more images unrelated to proofs through time.

## 6. 3 Tasks

From the analysis of the tasks used in all six Books, it was found that the majority of them are P tasks, followed by E tasks in a moderate level. As for the Qs , this category was at low level from 1975 to 1990 (Books 1, 2, 3), while from 1990 and on (Books 4, $5,6)$ their use increased reaching high values ( $\mathrm{Qs}<\mathrm{P}$ between $13 \%-30 \%$ ). Finally, there is a lack of tasks that represent real life problems (MM), throughout the years examined.

The analysis of tasks for each of the ten topics, shed light on an observation; tasks vary according to the topic. In this case, the categorization according to their difference in proportion as in the images was impractical, so a division in chronological order was
decided. Two periods were observed; the first is from 1975 to 1990 (Books 1, 2, 3), while the latter takes place from 1990 to 2019 (Books 4, 5, 6). Specifically, in the years 1975 to 1990 the majority of tasks in all textbooks remain the category of P. As for the E, there are topics (Analogies and similarity, Metric Relations, Areas, Circle's measurement) in which their values are close to P tasks, with their proportions ranging from $2 \%-15 \%$, per Book. In all the other topics (Introduction to Euclidean Geometry, Triangles, Parallel Lines, Quadrilaterals, Inscribed shapes, Solid Geometry), the proportions ranges from $39 \%$ to $67 \%$ in the majority of Books. The absence of Qs and MM tasks is obvious.

On the contrary, the years from 1990 to 2019 do not follow the same pattern. Specifically, the presence and rise of Qs tasks affects the results. Apart from "Introduction to Euclidean Geometry", in which Qs are on the top or in the second level of hierarchy, all the other topics (Triangles, Parallel Lines, Quadrilaterals, Inscribed shapes, Analogies and Similarity, Metric Relations, Areas, Circle's measurement, Solid Geometry) are presented in higher proportion than these of 1975 to 1990, but still the P and the E depict higher values. The presence of Qs affects the presence of P , with the latter being decreased in these years (1990-2019), reaching lower levels than the first period (1975-1990).

Overall, in 1975 to 1990 (Books 1, 2, 3) the two categories were the P and the E categories of tasks, with the P being the dominant category. From 1990 to 2019, the P tasks remain the most popular category in use, however a decrease in their proportion is observed. Especially, the frequency of $P$ tasks is declined from an average ratio of $65,89 \%$ (Books 1, 2, 3) to $42,08 \%$ (Books 4, 5, 6) with the statistical difference being significant ( $\mathrm{z}=17.29, p<0.05$ ). Opposite results are observed in the Qs category. There is an absence in the Qs from 1975 to 1990 (Books 1, 2, 3), while from 1990 to date (Books 4, 5, 6) there is an increase in their use. This means that their use is increased from an average ratio of $0,78 \%$ (Books $1,2,3$ ) to $19,08 \%$ (Books 4, 5, 6), with the statistical difference being significant ( $\mathrm{z}=-7.69, p<0.05$ ).

Interpreting the results in ATD terms, it can be said that from 1975 to 1990 (Books $1,2,3$ ) the frequency of tasks (Tasks) using techniques that assess non-routine or extramathematical concepts ( P tasks) is higher. This means that the authors of Books 1, 2 and 3 prefer tasks that examine how students can manage the level of complexity and handle them. In contrary, the years 1990 to 2019 (Books 4, 5, 6) the use of tasks relating to non-routine situations ( P tasks) is decreased, towards an increase in the use of tasks, using techniques that assess the knowledge of facts, such as definitions, theorems, properties, formulae or results emerging straightforward from the figure of the question (Qs tasks). As a consequence, a change in the rationale of technology appears; authors prefer to include categories of tasks, assessing known mathematical facts. In the view of Vincent and Stacey (2008), that "ideally, mathematics textbooks would present a balanced view of the importance of both skills and process" (p.84), this might mean that Greek textbooks of Euclidean Geometry, in particular Books 4, 5 and 6, include
tasks assessing mathematical knowledge (Qs) and a variety of complex situations (P) in order to achieve a full range of categories of tasks.

Tasks that students are required to solve during their school years, determine their experience in mathematics and their relation to them ( $\Theta \omega \mu \alpha$ and $N \alpha \rho \delta \dot{\eta}, 2015$ ). Besides, the basic "components of mathematics" are demonstrated by tasks (Niss, 1993, p.20). As a consequence, the student's ability to comprehend and accomplish the different categories of tasks, is examined through student's responses. To my way of thinking different categories of tasks play different roles; explicitly, the comprehension of the theoretical content of mathematics can be accomplished through Qs tasks. Qs can be accomplished orally or written; it depends on the question. Most of the time, they examine if the students have achieved to understand the theory of the section. Moving on to E, they can be considered as moderate complexity. Students use the already known facts (theorems, definition, properties), presenting them written, in order to obtain mathematical results. There is a variety of methods and techniques in Geometry they should use flexibly for the solution. In the third category of tasks ( P ), the level of complexity varies. There is a range of problems that a student is required to provide solutions. In these categories of tasks, the solution method might involves subproblems, make connections between the theoretical part (i.e. combine theorems or properties) in non-routine and extra-mathematical situations. That is the main difference between E and P. Finally, there are the MM tasks which include questions, exercises or problems and they make connections between real life situations and mathematics. With the aid of MM tasks, students have the ability to provide solutions in different kind of situations by translating its content into mathematics. In particular students explore "the rest of the word outside mathematics" (Blum, 1993, p.4), making connections with mathematics.

It might sound worried that from 1990 to date, the P type of tasks tend to being declined, but similar results emerge in the study of Vincent and Stacey (2008). They categorized tasks in textbooks in a level of procedural complexity (low, moderate and high) and found out that the majority of problems were low. Linking the tasks of low procedural complexity to the Qs tasks occurs a paradox; On the one hand high procedural complexity problems (in this case P tasks) help students facilitate their reasoning, making them exposed to a range of problems and using procedures from the theoretical part of the section to provide solution, while on the other hand, their significant decline, towards an increase in the number of Qs, signifies that students need to be exposed to a full range of problem types (Ibid.), i.e. Qs, E, P and MM, in order to achieve a better notion of the mathematical content.

Linking the dominance of the P tasks with the dominance of the C images between the years 1975 to 1990 (Books 1, 2, 3), both categories attribute to a strict teaching of Geometry. As Toumasis (1990) confirms, many traditional notions were removed from the textbooks of Greek Euclidean Geometry, although its content did not change due to educational, philosophical and ethnical factors. Additionally, in the syllabus of 1985 school Geometry was based mostly on the Elements of Euclid (Ibid.). Mentioning once
more that the axiomatic theoretical system of Greek Euclidean Geometry remains "well-structured and stronger" than other countries (Kuzniak and Vivier, 2009, p.694), this has an impact in the tasks students are required to complete. Personally, I believe that the significant inclusion of Qs in the topics of Euclidean Geometry, serves a wider purpose; First and foremost, it makes students understand which concepts of the theoretical part they should comprehend in order to achieve a better knowledge of the following mathematical concepts. Additionally, more categories of tasks is a sign of how students can be eligible to a variety of different methods, since every type of tasks assess different objects (i.e. Qs assess knowledge of facts, E assess standard methods and techniques, P assess students behaviour in extra-mathematical concepts, MM assess the ways that mathematics can be used in real life situations).

All in all, for all textbooks investigated, four categories of tasks can be found in Greek textbooks of Euclidean Geometry between the years 1975 to 2019. Those are questionnaires or Qs, exercises or E, problems or P and mathematical modelling or MM. Qs deal with mathematical facts i.e. definitions, theorems, properties, formulae and computations emerging straightforward from the figure, without using a paperpencil environment. Standard methods, routine-type considerations and computations in a paper pencil environment corresponds to E tasks. The category of P tasks involves non-routine situations with a level of complexity differing from task to task. Lastly, the MM is a category consisting questions, exercises and problems, referring to real life situations (RQ4). The category used in higher proportion that the others in all Greek textbooks of Euclidean Geometry is the P tasks, following by the E tasks. Although from 1975 to 1990, the absence of Qs and MM was obvious, from 1990 to date the significant presence of Qs had as a consequence the reduction in the number of P , remaining though as the most popular category in use (RQ5). Finally, in terms of praxeology, the technology has modified, since from 1990 and on the inclusion of Qs tasks had a result in the decrease of P tasks. Enriching textbooks with different categories of tasks means that are able to become more desirable to student and serve a variety of pedagogical roles (RQ6).

## 6. 4 Limitations and future research

In this section, I would like to acknowledge the limitations of my study and present some future research directions. Initially, using the terms of the DT, my only access was in the knowledge supposed to be taught, as proposed from the textbooks of Euclidean Geometry. I did not have access to what teachers eventually managed to teach (Taught knowledge), or what students finally learn (Learnt knowledge). In other words, I was not able to collect data for the teaching and learning process and the enactment of the teachers and the students in the school environments from 1975 to 2019. Another limitation is related to students' learning. Specifically, as the examination of images and tasks was in progress, questions such as in which textbook students acquired new knowledge in a higher level and also in which textbook the best comprehension of the Geometry concepts was occurred. Those questions could not be answered, since students' age differed in textbooks from 1975 to date. Finally, data
from questionnaires or interviews to students, teachers or academics of mathematics could not be gathered in order to make comparisons in the above questions. This is happening since students of 1975 to 2000 (Books 1, 2, 3, 4, 5) will propose different perspectives (the ability to act as students will not be gained from teachers or academics) as the student of Book 6, due to their current position (teachers, academics) and their experiences.

For future research, it would be interesting to make the same study related to the images and the tasks in other mathematics textbooks of the secondary educations, i.e. algebra. This insight into both fields of mathematics will support any future improvement and update of the content of school mathematics. Moreover, it would be interesting to examine the axiomatic theoretical system, the changes in the writing style and the style of language used by authors' textbooks in order to support the teaching and learning process of mathematics. Since mathematical proofs are extremely important in the community of mathematics with a view to make students to understand why mathematics is a unique "human endeavor different from other human activities" (Chazan, 1993, p.385), an insight into the theoretical foundation of mathematics will support the student's learning and highlights the difficulties students face while trying to develop their reasoning when constructing proofs or solving tasks.

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## In Greek



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## Appendices

## Appendix A <br> Topics of Geometry

## Appendix A: topics of Geometry

Appendix A provides the topics founding in the eight selected textbooks (Textbooks A, B, C, D, E, F, G, H) of Euclidean Geometry. In particular, there is a list of the sections of the sections founding on each topic of Euclidean Geometry throughout the study of images and tasks.

Every table consists of two columns. The first-one names the number of the Book and the textbook that the particular topic corresponds. The second column gives information about all the sections founding on each textbook of Geometry, for the particular topic.

|  | Introduction to Euclidian Geometry |
| :---: | :---: |
| Book 1 <br> Textbook A | - Geometry- Fundamentals- The proposals of geometry- geometric schema <br> - Three essential categories of postulates- position's postulateequality's postulates- order's postulates <br> - Half-line - Line segment <br> - Congruence of line segments- Properties <br> - Operations and orders in the set of line segments <br> - Half-plane - plane segments <br> - Kinds of surfaces <br> - Plane geometry and stereometry <br> - Angles <br> - Congruence, operations and orders in the set of angles <br> - Supplementary angles- angle bisector- right angle <br> - Axial symmetry <br> - Perpendicular and diagonal- perpendicular bisector- Locus <br> - Property of angle bisector <br> - Central symmetry- vertically opposite angles <br> - Parallel lines <br> - In the same and opposite parallelism <br> - Angles with sides parallels or verticals <br> - Congruence and operations in the set of oriented line segments |
| Book 2 <br> Textbook C | - Introduction <br> - Fundamentals- postulates <br> - Essential proposals for the line <br> - Half-line <br> - Line segment <br> - Congruence of line segments <br> - The midpoint of line segment <br> - Operations in line segments <br> - Ratio of line segments <br> - Measurement of line segments <br> - Plane <br> - The branches of geometry <br> - Angle <br> - Broken line <br> - Perimeter of a broken line |


|  | - Convex polygons |
| :--- | :--- |
|  | - Triangle inequality for three points |
|  | - Examples- applications |
|  | - Exercises |
| - Revision of the chapter |  |


|  | - The length of a line segment- points' distance- the measurement of line segments <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Angles <br> - The definition of the angle <br> - Angles' comparison- Angles' kinds <br> - Perpendicular distance <br> - Operations in angles- other kinds of angles <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Circle <br> - The definition of a circle <br> - Central angle- relation between a central angle and a relevant arc <br> - The measure of an angle and an arc- measurement of arcs <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Sectilinear schemas <br> - Polygonal chain <br> - Polygon <br> - Polygons elements <br> - Think and answer <br> - General exercises <br> - Questions <br> - Summation |
| :---: | :---: |
| Book 6 <br> Textbook H | - An introduction to Euclidian Geometry <br> - The object of Euclidian Geometry <br> - A historical review to genesis and development of Geometry <br> - Points, lines and surfaces <br> - The plane <br> - The line <br> - The half-line <br> - The line segment <br> - The transposition in a plane <br> - Comparison of line segments <br> - Operations in line segments <br> - The length of a line segment- the distance of two points <br> - Points that are symmetric to the circle's center <br> - The half-plane <br> - The angle |


|  | - Angles' comparison |
| :--- | :--- |
| - Distance from a point to a line (or perpendicular distance) |  |
|  | - Operations in angles |
|  | - Angles' kinds and angles' relations |
|  | - The definition of circle and its elements |
|  | - Central angle- relation between a central angle and an arc |
|  | - The measure of an angle and an arc |
|  | - Polygonal chain- Polygon -Polygons' elements |


|  | Triangles |
| :---: | :---: |
| Book 1 <br> Textbook A | - Polygons <br> - The triangle- Types of triangles <br> - The sum of angles in a triangle and in a polygon <br> - Congruence of triangles <br> - Inequality relations in triangles <br> - Centers of triangle- Circumcenter- Orthocenter- Centroid-Incenter- Excenters <br> - The circle- Congruent circles- Symmetries <br> - Central angle <br> - Congruence, operations and orders in the set of arcs <br> - Midpoint of an arc- Adjacent arcs- Supplementary arcs <br> - Relative positions of a straight line and a circle <br> - Relative positions of two circles <br> - Angle of two circles- Orthogonal circles |
| Book 2 <br> Textbook C | - Types of triangles <br> - Medians, bisectors and altitudes of a triangle <br> - Congruence of triangles <br> - Congruent criteria of triangles <br> - Exterior angles of a triangle <br> - A congruent criterion <br> - Comparison of sides and angles in a triangle <br> - Comparison of sides and angles in two triangles <br> - Congruent criteria of right triangles <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter <br> - Chords and apothems <br> - Line and circle <br> - Tangent of a circle <br> - Tangent of circles from a point <br> - Intersecting circles <br> - Tangent circles <br> - Non intersecting circles <br> - Common tangent of two circles <br> - Simple geometric constructions <br> - The construction of the perpendicular bisector of a segment <br> - The circumcenter of a triangle <br> - The midpoint of a line segment <br> - The centroid of a triangle <br> - The construction of a line which is vertical to another line <br> - The orthocenter of a triangle <br> - Midpoint of an arc. The construction of angle bisector <br> - The incenters of a triangle <br> - The extenders of a triangle |


|  | - Examples- applications <br> - Exercises <br> - Revision of the chapter |
| :---: | :---: |
| Book 3 <br> Textbook | - Types of triangles <br> - Medians, bisectors and altitudes of a triangle <br> - Congruence of triangles <br> - Congruent criteria of triangles <br> - Exterior angles of a triangle <br> - A congruent criterion <br> - Comparison of sides and angles in a triangle <br> - Comparison of sides and angles in two triangles <br> - Congruent criteria of right triangles <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter <br> - Chords and apothems <br> - Line and circle <br> - Tangent of a circle <br> - Tangent of circles from a point <br> - Intersecting circles <br> - Tangent circles <br> - Non intersecting circles <br> - Common tangent of two circles <br> - Simple geometric constructions <br> - The construction of the perpendicular bisector of a segment <br> - The circumcenter of a triangle <br> - The midpoint of a line segment <br> - The centroid of a triangle <br> - The construction of a line which is vertical to another line <br> - The orthocenter of a triangle <br> - Midpoint of an arc. The construction of angle bisector <br> - The incenters of a triangle <br> - The extenders of a triangle <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter |


| Book 4 <br> Textbook E | - Polygons <br> - A triangle <br> - Congruence of triangles <br> - A perpendicular to a line through an external point <br> - Congruent criteria of right triangles <br> - The sum of angles in a triangle and in a convex n-agon <br> - The properties of right triangles <br> - The properties of isosceles triangles <br> - Inequality relations in triangles <br> - Lateral segments in a line <br> - Questions <br> - General exercises <br> - Euclid's postulate <br> - Circle- circular disk <br> - Relative positions of a straight line and a circle <br> - Circles' arcs <br> - Relations between arcs, chords and apothems <br> - Relative positions of two circles <br> - Simple geometric constructions <br> - Circles’ tangents <br> - Circles of a triangle |
| :---: | :---: |
| Book 5 <br> Textbook G | - Triangles' comparison <br> - Kinds of triangles <br> - Elements of triangles <br> - Congruent criteria of triangles <br> - Congruent criteria of right triangles <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Essential loci <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Symmetric schemas <br> - Central symmetry <br> - Axial symmetry <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Inequality relations <br> - Relations between an internal angle and an external angle <br> - Inequality relations between triangle's sides and triangle's angles <br> - Triangle inequality <br> - Vertical and diagonal lines <br> - The trace and the projection of a literal line-segment |


|  | - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Relative positions of a straight line and a circle <br> - The tangent line and line segments that are tangent to a circle <br> - Relative positions of two circles <br> - Circles with no common points <br> - Tangent circles <br> - Intersecting circles <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Simple geometric constructions <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |
| :---: | :---: |
| Book 6 <br> Textbook H | - Kinds of triangles and elements of triangles <br> - First Congruent criterion of triangles <br> - Second Congruent criterion of triangles <br> - Third congruent criterion of triangles <br> - The existence and uniqueness of a perpendicular <br> - Congruent criteria of right triangles <br> - Circle- perpendicular bisector- bisector <br> - Central symmetry <br> - Axial symmetry <br> - Relations between an internal angle and an external angle <br> - Inequality relations between triangle's sides and triangle's angles <br> - Triangle inequality <br> - Vertical and diagonal lines <br> - Relative positions of a straight line and a circle <br> - Tangent segments <br> - Relative positions of two circles <br> - Simple geometric constructions <br> - Essential constructions of triangles |


| Parallel lines |  |
| :---: | :---: |
| Book 1 <br> Textbook A | Part of Topic (I) |
| Book 2 <br> Textbook C | - The perpendicular theorems of two lines <br> - Distance between a point and a line <br> - Comparison of diagonal segments <br> - Points that are equidistant from the endpoints of a line segment <br> - Points that are equidistant in two intersecting lines <br> - Parallel lines <br> - Angles of parallel lines that intersect to another line <br> - Construction of parallel lines <br> - Angles with parallel sides <br> - Angles with vertical sides <br> - Sum of angles of a triangle <br> - Sum of angles of a polygon <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter |
| Book 3 <br> Textbook C | - The perpendicular theorems of two lines <br> - Distance between a point and a line <br> - Comparison of diagonal segments <br> - Points that are equidistant from the endpoints of a line segment <br> - Points that are equidistant in two intersecting lines <br> - Parallel lines <br> - Angles of parallel lines that intersect to another line <br> - Construction of parallel lines <br> - Angles with parallel sides <br> - Angles with vertical sides <br> - Sum of angles of a triangle <br> - Sum of angles of a polygon <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter |
| Book 4 <br> Textbook E | - Parallel lines <br> - The secant of two lines <br> - Criteria of parallelism |
| Book 5 <br> Textbook G | - The definition of parallel lines <br> - Angles of parallel lines that intersect of another line <br> - The existence of parallel lines' Theorem <br> - Postulate of parallelism <br> - Properties of parallel lines <br> - Construction of parallel lines <br> - Angles with parallel sides or angles with vertical sides <br> - Sum of angles of a triangle and sum of angles of a convex polygon <br> - Applications <br> - Think and answer |


|  | • First group of tasks <br> • Second group of tasks |
| :--- | :--- |
|  | - General exercises |
|  | - Questions |
|  | - Summation |
| Book 6 | - The definition of parallel lines |
| Textbook H | - The tangent of two lines- Euclid's postulate |
|  | - Construction of parallel line |
|  | - Angles with parallel sides |
|  | - Remarkable triangle's circles |
|  | - Sum of angles of a triangle |
|  | - Angles with vertical sides |
|  | - Sum of angles of a convex n-agon |


| Quadrilaterals |  |
| :---: | :---: |
| Book 1 Textbook A | - Quadrilaterals- Parallelogram <br> - Rectangle- Rhombus- Square <br> - Parallel transport <br> - Trapezium- Isosceles trapezium <br> - Applications of properties of parallelograms |
| Book 2 <br> Textbook C | - Parallelogram <br> - Criteria of parallelograms <br> - Applications of parallelograms <br> - The division of a line segment in $n$ equal segments <br> - Rectangle <br> - Distance between two parallel lines <br> - The mid parallel of two parallel lines <br> - A property of a right triangle <br> - Rhombus <br> - Square <br> - Trapezium <br> - Isosceles trapezium <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter |
| Book 3 <br> Textbook C | - Parallelogram <br> - Criteria of parallelograms <br> - Applications of parallelograms <br> - The division of a line segment in $n$ equal segments <br> - Rectangle <br> - Distance between two parallel lines <br> - The mid parallel of two parallel lines <br> - A property of a right triangle <br> - Rhombus <br> - Square <br> - Trapezium <br> - Isosceles trapezium <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter |
| Book 4 <br> Textbook E | - The parallelogram <br> - Applications of parallelograms <br> - The rectangle <br> - The rhombus <br> - The square <br> - The trapezium <br> - Questions <br> - General exercises <br> - Non-Euclidian geometries |
| Book 5 <br> Textbook G | - Definition and the properties of parallelograms <br> - In general |


|  | - Properties of parallelograms <br> - Criteria of parallelograms <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Kinds of parallelograms <br> - Classification of parallelograms <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Applications of parallelograms' properties <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Trapezium <br> - Properties of trapezium <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Remarkable part of a triangle <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |
| :---: | :---: |
| Book 6 <br> Textbook H | - Introductory section of quadrilaterals <br> - Parallelograms <br> - Rectangle <br> - Rhombus <br> - Square <br> - Applications in triangles <br> - The barycenter of a triangle <br> - The orthocenter of a triangle <br> - A property of right triangle <br> - Trapezium <br> - Isosceles trapezium <br> - Remarkable lines and triangle's circles |


| Inscribed shapes |  |
| :---: | :---: |
| Book 1 <br> Textbook A | - Relation of central and inscribed angle <br> - Angle between a chord and a tangent <br> - Angle of secant chords <br> - Inscribed quadrilaterals <br> - Circumscribed quadrilaterals <br> - Escribed polygons |
| Book 2 <br> Textbook C | - Inscribed angles <br> - Angle between a chord and a tangent <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter <br> - Inscribed quadrilateral <br> - Inscribable quadrilateral <br> - Properties of circumcircle quadrilateral <br> - Circumscribable quadrilaterals <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter |
| Book 3 <br> Textbook C | - Inscribed angles <br> - Angle between a chord and a tangent <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter <br> - Inscribed quadrilateral <br> - Inscribable quadrilateral <br> - Properties of circumcircle quadrilateral <br> - Circumscribable quadrilaterals <br> - Examples- applications <br> - Exercises <br> - Revision of the chapter |
| Book 4 <br> Textbook E | - Inscribed angles <br> - Angle between a chord and a tangent <br> - Inscribed and inscribable quadrilaterals <br> - Circumscribed and circumscribable quadrilaterals <br> - Questions <br> - General exercises <br> - Geometric constructions |
| Book 5 <br> Textbook G | - Inscribed angles <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Inscribed and inscribable quadrilaterals |


|  | - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Utilization of Loci and geometrical constructions <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |
| :---: | :---: |
| Book 6 <br> Textbook H | - Introduction- Definitions <br> - Relation of central and inscribed angle <br> - Angle between a chord and a tangent <br> - Essential geometric constructions in circle <br> - Inscribed quadrilateral <br> - Inscribable quadrilateral <br> - Loci and geometric constructions with the aid of loci |


| Analogies and Similarity |  |
| :---: | :---: |
| Book 1 <br> Textbook A | - Metric geometry- shapes of geometry- Units of measurement <br> - Analogies and their properties <br> - Mid Average ratio- Analogy level 4 <br> - Thali's theorem <br> - Construction of analogy level 4 <br> - Similar triangles <br> - Similar polygons <br> - Homothety <br> - Geometric constructions <br> - Pencil of lines- Pencil's theorems <br> - Orthogonal projections <br> - Apollonius circle |
| Book 2 <br> Textbook B | Metric geometry <br> - Shapes of geometry <br> - Ratio of uniforms in geometric shapes <br> - Measure of geometric shapes <br> - Units of measurement <br> - Symmetric geometric shapes <br> - Ratio of uniforms in geometric shapes <br> - Analogies and their properties <br> - Mid Average ratio <br> - Analogy level 4 <br> - Thali's theorem <br> - Construction of analogy level 4 <br> - Division of a segment of a given ratio Similar triangles <br> - Definition <br> - Triangles similarity theorems Similar polygons <br> - Definition <br> - Polygons similarity theorems Homothety <br> - Definitions <br> - Dilation Theorem Geometric constructions <br> - Examples Pencil of lines <br> - Definition <br> - Pencil's theorems Orthogonal projections <br> - Definitions <br> - Projection of a line segment |
| Book 3 <br> Textbook D | Introduction <br> - The measurement of segments. The product of a segment with a number. |


|  | - The ratio of two segments. Internal and external division of segments. <br> - Proportional segments- Analogies Thali's Theorem <br> - Generalization. Secants of a trapezium or a triangle. <br> - Geometric constructions. The theorem of bisector. Central Pencil of lines <br> - Pencil's Theorem. Concurrent lines Exercise Homothety <br> - Definition. Homothetic sectilinear schemes. Similar polygons <br> - Similar polygons. Similar triangles. Similar right triangles Exercises |
| :---: | :---: |
| Book 4 <br> Textbook E | - Introductory terms <br> - Thali's Theorem <br> - Similar polygons <br> - The Pythagorean Theorem <br> - Questions <br> - General exercises <br> - Thalis and Pythagoras |
| Book 5 <br> Textbook G | - The definition of ratio <br> - Product of a line segment with a number- symmetric and nonsymmetric line segments <br> - The ratio of two line segments- The length of a line segment <br> - Analogies- The properties of analogies <br> - Partitioning line segments in a given ratio (internal and external) <br> - Thali's Theorem <br> - Properties of segments that lie between parallels <br> - Thali's theorem in a triangle and the converse of Thali's theorem in a triangle <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - The circle of Apollonius <br> - Theorems of bisectors of a triangle- Conjecture harmonic points <br> - Locus of points that the ratio of their distances is equal to $\mathrm{m} / \mathrm{n}$, where $\mathrm{n} \neq 1$ <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |


|  | - Similar sectilinear schemas <br> - Diminution- Enlargement and similarity <br> - Construction of similar polygons- Construction of similar triangles- Construction of two similar schemas- Measurement of inaccessible points' distance <br> - Criteria of similar triangles <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |
| :---: | :---: |
| Book 6 <br> Textbook H | - Introduction <br> - Partitioning line segments in $n$ congruent segments <br> - Product of a line segment with a number- Line segment's ratio <br> - Proportional line segments- Analogies <br> - The length of a line segment <br> - Partitioning line segments in a given ratio (internal and external) <br> - Thali's Theorem <br> - Theorems of bisectors of a triangle <br> - The circle of Apollonius <br> - Similar sectilinear schemas <br> - Criteria of similarity |


| Areas |  |
| :---: | :---: |
| Book 1 <br> Textbook A | - Areas of simple closed curves <br> - Area of rectangle and area of parallelogram <br> - Area of triangle <br> - Area of trapezium <br> - Areas of polygons <br> - Transformation of a polygon <br> - Heron's formula <br> - Calculating the radii of circles in a triangle <br> - Ratio of areas of similar polygons |
| Book 2 <br> Textbook B | Areas of simple closed curves <br> - Definition <br> - Equal areas or equivalent schemes <br> - The postulates of areas of schemes <br> - Area of rectangle <br> - Area of parallelogram <br> - Area of triangle <br> - Area of trapezium <br> - Areas of polygons <br> - Transformation of a polygon <br> - The product of two line segments <br> - Triangle's area from its sides <br> - Calculating the radii of circles in a triangle <br> - Ratio of areas of similar polygons |
| Book 3 <br> Textbook D | The term of an area <br> - Polygonal surface. The term of an area Areas of polygons <br> - Area of rectangle. Essential formulas of areas <br> - Formulas of the area of a triangle Comparison between areas <br> - Ratio of the areas of two triangles. Ratio of the areas of similar polygons <br> Squaring a polygonal surface <br> - The problem of squaring. Polygon's squaring. <br> - Squaring a polygonal surface Exercises |
| Book 4 <br> Textbook F | - Polygonal surfaces <br> - The term of an area <br> - Area of essential polygons <br> - Formulas of the area of a triangle <br> - Comparison between areas <br> - Squaring a polygon <br> - Questions <br> - General exercises <br> - The Pythagorean Theorem in the Euclid's elements |
| Book 5 <br> Textbook G | - The definition of area <br> - Introductory section |


|  | - Areas of polygons <br> - Areas of familiar schemas <br> - Essential theorems <br> - Different methods of calculating the area of triangle <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Relations of areas of similar schemas <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Squaring the schemas <br> - The definition of squaring <br> - The proof of Pythagorean Theorem with the utilization of areas <br> - Exercises <br> - General exercises <br> - Questions <br> - Summations |
| :---: | :---: |
| Book 6 <br> Textbook H | - Polygonal regions <br> - The area of sectilinear segment- Equivalent sectilinear schemas <br> - The area of essential line segment <br> - Different types of areas in a triangle <br> - Areas' ratio of similar triangles and similar polygons <br> - Transformation of a polygon into itself |


| Metric relations |  |
| :---: | :---: |
| Book 1 Textbook A | - Metric relations in right triangles <br> - Pythagoras's theorem <br> - Geometric constructions <br> - Metric relations in triangles <br> - Theorems of medians <br> - Metric relations in quadrilaterals <br> - Theorems of angle bisector in a triangle <br> - Harmonic division of a line segment <br> - Power of a point to a circle <br> - Quadratic equations in geometry <br> - The division of a segment in an average ratio and... (golden ratio) <br> - Radical axis- radical center |
| Book 2 <br> Textbook B | Metric relations in a Triangle <br> - Metric relation <br> - Metric relations in right triangles <br> - Pythagoras's theorem <br> - Theorems for right triangles <br> - Diagonal of orthogonal <br> - The altitude of an equilateral triangle <br> - Geometric constructions <br> - Metric relations in triangles <br> - The first theorem of median <br> - The second theorem of median <br> - A core criterion for the kind of angle in a triangle Metric relations in Quadrilaterals <br> - The first theorem of Ptolemy <br> - The second theorem of Ptolemy <br> - The theorem of internal bisector <br> - The theorem of external bisector <br> - Harmonic division of a line segment <br> - Apollonius circle <br> - Power of a point to a circle <br> - Quadratic equations in geometry <br> - Golden ratio <br> - Radical axis <br> - Radical center |
| Book 3 <br> Textbook D | Metric relations in a right triangle <br> - Projections of vertical lines in hypotenuse <br> - Pythagoras's theorem. Geometric constructions Metric relations in triangles <br> - Generalization of the Pythagorean Theorem. Theorems of medians <br> - Formulas of medians <br> Metric relations in circles <br> - Secants of a circle from a point. Power of a point to a circle |


|  | - Secant and tangent of a circle. Geometric construction Exercises |
| :---: | :---: |
| Book 4 <br> Textbook F | - The generalization of Pythagorean Theorem <br> - Theorems of medians <br> - Theorems of triangle's bisectors <br> - Secants of circles <br> - Secant and tangent of a circle <br> - Geometrical constructions <br> - Questions <br> - General exercises <br> - The trisection of an angle and the duplication of the cube |
| Book 5 <br> Textbook G | - Metric relations in a Triangle <br> - Metric relations in right triangles <br> - Metric relations in triangles <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Metric relations in a circle <br> - Power of a point to a circle <br> - The geometric construction of positive root of Quadratic equations- the problem of golden ratio <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |
| Book 6 <br> Textbook H | - Orthogonal projections <br> - The Pythagorean Theorem <br> - Geometrical constructions <br> - The generalization of Pythagorean Theorem <br> - Theorems of medians <br> - Essential loci <br> - Secants of circles |


| Circle's measurement |  |
| :---: | :---: |
| Book 1 Textbook A | - Regular polygons- general theorems and notations <br> - Square <br> - Regular hexagon <br> - Regular (equilateral) triangle <br> - Regular decagon <br> - Regular pentagon <br> - Regular pentadecagon <br> - Measurement of a circle- relevant theorems <br> - Calculating $\operatorname{Pi}(\pi)$ <br> - Area of a circle- Sector of a circle- Circular segment- Meniscus |
| Book 2 <br> Textbook B | - Definition of regular polygons <br> - Regular polygonal line <br> - Calculating the angle of a regular polygon <br> - Theorems and general notations <br> - Area of a regular polygon <br> - Symmetry in regular polygons <br> - Similarity in regular polygons <br> - Useful relations and calculations in regular polygons <br> - Regular polygons inscribed in a circle <br> - Relevant theorems <br> - Calculating $\operatorname{Pi}(\pi)$ <br> - Arc length of a circle <br> - Area of a circle <br> - Sector of a circle <br> - Circular segment <br> - Meniscus |
| Book 3 <br> Textbook D | Introduction <br> - Elements of regular polygon Metric relations <br> - Similarity. Essential formulas Inscription of regular $n$-agons in circles <br> - Rectangle. Hexagon and equilateral triangle. Decagon and pentagon. <br> Circle's measurement <br> - Length of circle. Calculating the length of a circle. <br> - Arc length. Area of circular disc Exercises |
| Book 4 Textbook F | - The definition of a regular polygon <br> - The properties and the elements of regular polygons <br> - The inscription of regular polygons in a circle <br> - The length of a circle <br> - Arc length <br> - Area of circular disc <br> - The area of a circular sector and the area of a circular segment |


|  | - Exercises <br> - General exercises <br> - The paradox in Mathematics |
| :---: | :---: |
| Book 5 <br> Textbook G | - Regular polygons <br> - The definition of a regular polygon and its elements <br> - Essential properties of regular polygons <br> - The inscription of a square, a regular hexagon and an equilateral triangle in a circle <br> - The inscription of a regular decagon <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - The length of circle <br> - Approaching the length of a circle with the aid of regular polygons <br> - The length of a circle and the arc length <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Area of circular disc <br> - Approaching the area of a circular disc with the aid of regular polygons <br> - The area of a circular sector and the area of a circular segment <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |
| Book 6 <br> Textbook H | - The definition of a regular polygon <br> - The properties and the elements of regular polygons <br> - The inscription of regular polygons in a circle <br> - Approaching the length of a circle with the aid of regular polygons <br> - Arc length <br> - Approaching the area of a circle with the aid of regular polygons <br> - The area of a circular sector and the area of a circular segment <br> - Squaring the circle |


| Solid Geometry |  |
| :---: | :---: |
| Book 1 <br> Textbook A | - The plane- Postulates <br> - Determination of a plane <br> - Planes in three dimensions <br> - Lines and planes in three dimensions- A line perpendicular to a plane <br> - Theorems of three perpendiculars <br> - Mediator plane <br> - Parallel lines <br> - Perpendicular and diagonal segment to plane <br> - Parallel lines and plane <br> - Parallel lines- Thali's Theorem <br> - Inconsistent lines- common perpendicular <br> - Orthogonal projections <br> - Axial symmetry <br> - Symmetry for a plane <br> - Central symmetry <br> - Dihedral angles- Corresponding plane angle- Bisector plane- Kinds of dihedral angles- Vertical planes <br> - Solid angles- Trihedral solid angles <br> - Orientation of trihedral solid angle <br> - Supplementary of trihedral solid angle <br> - Theorems of congruence in solid angles <br> - Inequality relations in triangles <br> - Polyhedral- Tetrahedral- Kinds of tetrahedral <br> - Center of mass of a tetrahedron <br> - Pyramid- regular pyramid <br> - Truncated pyramid- regular truncated pyramid <br> - Prism <br> - Parallelepiped- orthogonal parallelepiped <br> - Prismatoid <br> - Measurement of polyhedral- surfaces <br> - Volumes of polyhedral <br> - Similar polyhedral <br> - Surfaces and solids of revolution- Definitions <br> - Cylinder <br> - Cone <br> - Truncated cone <br> - Rotation of a triangle around an axis <br> - Sphere- Definitions- Symmetries <br> - Relative positions of a line and a sphere <br> - Relative positions of a sphere and a plane <br> - Relative positions of two spheres <br> - Determination of a sphere <br> - Loci <br> - Graphic applications <br> - Spherical zone- Spherical surfaces |


|  | - Spherical sector- Volume of a sphere- Spherical ring- Spherical segment <br> - Spherical polygons |
| :---: | :---: |
| Book 2 <br> Textbook B | - The plane- Postulates |
|  | - Determination of a plane |
|  | - Lines in three dimensions |
|  | - Planes in three dimensions |
|  | - Lines and planes in three dimensions |
|  | - Theorems of three perpendiculars |
|  | - Mediator plane |
|  | - Parallel lines |
|  | - Perpendicular and diagonal segment to plane |
|  | - Parallel lines and plane |
|  | - Parallel lines- Thali's Theorem |
|  | - Inconsistent lines- common perpendicular |
|  | - Orthogonal projections |
|  | - Axial symmetry |
|  | - Symmetry for a plane |
|  | - Central symmetry |
|  | - Dihedral angles- Corresponding plane angle |
|  | - Bisector plane- Vertical planes |
|  | - Solid angles- Trihedral solid angles |
|  | - Orientation of trihedral solid angle |
|  | - Supplementary of trihedral solid angle |
|  | - Theorems of congruence in solid angles |
|  | - Inequality relations in triangles |
|  | - Polyhedral- Tetrahedral- Kinds of tetrahedral |
|  | - Center of mass of a tetrahedron |
|  | - Pyramid- regular pyramid |
|  | - Truncated pyramid- regular truncated pyramid |
|  | - Prism |
|  | - Parallelepiped- orthogonal parallelepiped |
|  | - Prismatoid |
|  | - Measurement of polyhedral- surfaces |
|  | - Volumes of polyhedral |
|  | - Similar polyhedral |
|  | - Surfaces and solids of revolution- Definitions |
|  | - Cylinder |
|  | - Cone |
|  | - Truncated cone |
|  | - Rotation of a triangle around an axis |
|  | - Sphere- Definitions- Symmetries |
|  | - Relative positions of a line and a sphere |
|  | - Relative positions of a sphere and a plane |
|  | - Relative positions of two spheres |
|  | - Determination of a sphere |
|  | - Loci |


|  | - Graphic applications <br> - Spherical zone- Spherical surfaces <br> - Spherical sector- Volume of a sphere <br> - Spherical ring- Spherical segment |
| :---: | :---: |
| Book 3 <br> Textbook D | Lines and planes in three dimensions <br> - Postulates. Determination of a plane <br> - Positions of lines and planes. Half-spaces Parallelism in space <br> - Parallel lines. Aline parallel to a plane <br> - Parallel planes. Transitivity of parallelism <br> - Thali's Theorem Perpendicularity in space <br> - Angle of two lines. Perpendicularity of a line and plane <br> - Orthogonal lines and perpendicularity. Mediator plane <br> - Theorems of three perpendiculars. The construction of a vertical line to a plane <br> - The distance between a point and a plane. The distance of parallel planes <br> - Common perpendicular of two inconsistent lines. Dihedral angle. Vertical planes <br> - Projection in a plane. Angle of a line and a plane <br> - Exercises <br> Prism and cylinder <br> - Surface of a cylinder. Prism. Vertical intersection of a prism. <br> - Area of lateral surface. <br> - Parallelepiped. Orthogonal parallelepiped- cube <br> - Cylinder <br> Sphere <br> - Definitions. Relative positions of a line and a sphere. Relative positions of a sphere and a plane <br> - Spherical tholos- pole of a sphere. Spherical zone Exercises |
| Book 4 <br> Textbook F | - Introduction <br> - The planes in three dimensions(space) <br> - The lines in three dimensions(space) <br> - Half space <br> - Constructions in the space <br> - The perpendicularity of a line and a circle <br> - The parallelism of lines <br> - The parallelism of a line and a circle <br> - The parallelism of planes <br> - Orthogonal lines <br> - Dihedral angle <br> - Vertical planes <br> - The projection in a plane |


|  | - Questions <br> - General exercises <br> - The Plato <br> - Convex polyhedral <br> - The tetrahedron <br> - The pyramid <br> - The truncated pyramid <br> - The prism <br> - Parallelepiped <br> - Questions <br> - General exercises <br> - Platonic solids <br> - The cylinder <br> - The cone <br> - The truncated cone <br> - The rotation of a polygonal chain and a polygonal region <br> - The sphere <br> - The measurement of the sphere <br> - Questions <br> - General exercises <br> - The discovery of irrational numbers |
| :---: | :---: |
| Book 5 <br> Textbook G | - Relative relations between lines and planes in space <br> - Introduction <br> - The plane in space <br> - Relative relation of a line and a plane <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - The perpendicularity and the parallelism between lines and planes in space <br> - Lines that lie vertically on the plane <br> - The parallelism of lines in space <br> - The parallelism of planes <br> - The projection of a schema on the plane <br> - The angles between lines and planes <br> - Vertical planes <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |


|  | - Prisms and their elements <br> - The properties of prisms <br> - The area of a right prism <br> - The volume of solids <br> - The volume of a right prism <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - Pyramids <br> - The area of a regular pyramid and the area of a regular truncated pyramid <br> - The volume of pyramids and the volume of truncated pyramids <br> - Application <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - The cylinder <br> - The area and the volume of a cylinder <br> - The cone and the truncated cone <br> - The area and the volume of a truncated cone <br> - The sphere <br> - Sphere's elements <br> - Relative positions of a line and a plane on the sphere <br> - The measurement of the sphere- Pappus's theorems <br> - Platonic solids <br> - Applications <br> - Think and answer <br> - First group of tasks <br> - Second group of tasks <br> - General exercises <br> - Questions <br> - Summation |
| :---: | :---: |
| Book 6 <br> Textbook H | - The introductory section of lines and planes in space <br> - The definition of plane <br> - Relative positions of lines and planes <br> - The parallelism of lines and planes- Thali's theorem <br> - The angle of two lines- orthogonal lines <br> - The distance of a point and a plane- the distance of two parallel planes <br> - Dihedral angles- Corresponding plane angle of a dihedral angleVertical planes <br> - The projection of a point on the plane- the angle of a line and a plane |


| - Polyhedral |  |
| :--- | :--- |
| - The definition of the prism and prism's elements |  |
| - Parallelepiped, cube |  |
| - The measurement of the prism |  |
| - The definition of a pyramid and pyramid's elements |  |
| - Regular pyramid- the tetrahedron |  |
| - The measurement of the pyramid |  |
| - The definition of the truncated pyramid and the elements of a |  |
|  | - Truncated pyramid |
| - The measurement of the truncated pyramid |  |
| - The definition of a cylinder and cylinder's elements |  |
| - The measurement of a cylinder |  |
| - The definition of a cone and cone's elements |  |
| - The measurement of a cone |  |
| - Truncated cone |  |
| - The definition of a sphere and sphere's elements |  |
| - Relative positions of a line and a plane on the sphere |  |
| - The measurement of the sphere |  |
| - Regular polyhedral |  |


| Extra Topics |  |
| :---: | :---: |
| Book 1 <br> Textbook A | - Geometric constructions- Elementary problems of geometry <br> - Simple constructions of triangles <br> - Analytical method- examples <br> - Elementary loci- examples |
| Book 2 <br> Textbook B | Geometric constructions and loci of the first and the second book <br> - Definitions <br> - Elementary problems of geometry <br> - Simple constructions of triangles <br> - Analytical method <br> - Loci <br> - Elementary loci |
| Book 2 <br> Textbook C | Essential terms in circle <br> - Circle <br> - Circular disk <br> - Incenter angles and arcs <br> - Chords of an angle. Semi-circle <br> - Congruence of arcs <br> - The midpoint of an arc <br> - Incongruent arcs <br> - Operations in arcs <br> - Measurement of arcs <br> - Expansion of arc's term <br> - Examples and applications <br> - Exercises <br> - Revision of the chapter <br> Relations and operations of angles <br> - Congruence of angle <br> - Construction of an angle equal to a given angle <br> - Angle bisector <br> - Incongruent angles <br> - Unequal angles <br> - Sum of angles <br> - Expansion of angle's term <br> - Subtraction of angles <br> - Ratio of two angles <br> - Angles' measurement <br> - Supplementary angles <br> - Adjacent angles <br> - Consecutive angles with sum of two or for right angles <br> - Opposite vertical angles <br> - The right angle. Complementary angles <br> - Vertical lines <br> - Examples and applications <br> - Exercises <br> - Revision of the chapter |

Loci and geometrical constructions

- The term of locus
- The characteristic property
- Fundamental proposals in loci
- Discover loci
- Geometric constructions through analytical and complex method
- Simple constructions of a triangle
- Examples
- Solving problems involving loci
- Exercises
- Revision of the chapter


# Appendix B <br> The z-test 

## Appendix B: the z-test: Definition and calculator for two population proportions

A test of two proportions or z-test is a type of hypothesis test. It is used in Statistics in order to estimate if the differences between two groups are statistically significant or not from each other, compared to a categorical characteristic. There are two ways of running a z-test; any researcher can use a test statistic formula or with the aid of the internet calculator for z-score.

There are some steps that an individual has to follow while running this test using a statistic formula:

1. Find the two different proportions ( $\boldsymbol{p}_{1}, \boldsymbol{p}_{2}$ ). This is gaining while dividing Size $\left(n_{1}\right)$ from the total number of group one and size $\left(n_{2}\right)$ from the total number of group two.
2. Find the overall sample proportion. In this step the numerator is considered as the total sum of samples $n_{1}$ and $n_{2}$, while the denominator is considered the total sum of group 1 and group 2 .
3. Estimate the test statistic formula. This number $(\boldsymbol{Z})$ is called z -score:

$$
Z=\frac{\left(\hat{p}_{1}-\hat{p}_{2}\right)-0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_{1}}+\frac{1}{n_{2}}\right)}}
$$

4. Find the $z$-score compared to the alpha (a) value, by using the known values.

| Confidence Level | Alpha | Alpha/2 | z alpha/2 |
| :---: | :---: | :---: | :---: |
| $90 \%$ | $10 \%$ | $5.0 \%$ | 1.645 |
| $95 \%$ | $5 \%$ | $2.5 \%$ | 1.96 |
| $98 \%$ | $2 \%$ | $1.0 \%$ | 2.326 |
| $99 \%$ | $1 \%$ | $0.5 \%$ | 2.576 |

5. Determine the significant difference. If the z score is greater than z alpha/2 or lesser than z alpha/ 2 then there is a significant difference, otherwise there is not.

# Appendix C <br> Classifying Images 

## Appendix C: the presence of images ( $\mathrm{NC}, \mathrm{BC}, \mathrm{C}$ ) in the selected sample of textbooks (A, B, C, D, E, F, G, H) of Euclidean Geometry in Lyceum from 1975 to 2019.

Appendix C illustrates the presence of images (NC, BC, C) in the textbooks of Euclidean Geometry in Lyceum, between the years 1975 and 2019. This classification of images emerged from the analysis of the visuals of the concept of series in Canada's and UK's textbooks, on the study of González-Martín et al (2011, p.572). The selected sample included eight textbooks (textbooks A, B, C, D, E, F, G and H) which they used for the teaching of Geometry from 1975 to date. Every textbook is divided into four columns, each one pertains to a different component. The first column is associated with the titles of Chapters in accordance with the topics that are presented in Geometry textbooks. Some textbooks did not contain any title (e.g. Textbook A) or they were partitioned in extra similar sections such as "similar triangles" and "similar polygons" (e.g. Textbook B). Columns two, three and four indicate the use of the three categories of images ( $\mathrm{NC}, \mathrm{BC}$ and C ) found on each textbook.

The table of Textbook A is divided into four columns and nine rows containing figures. Column one pertains to the name of chapter, while columns two to four on the categories of images ( $\mathrm{NC}, \mathrm{BC}, \mathrm{C}$ ). A closer look on the table reveals numbers which are referred to the name of the images. For instance, number 93 in the row of Triangles' chapter corresponds to scheme 93 of textbook $\mathbf{A}$. As it is found, this textbook contains 594 images and it was used for the teaching of Geometry in 1975.

| TEXTBOOK A |  |  |  |
| :---: | :---: | :---: | :---: |
| Chapter | NC | BC | C |
| Introduction (Parallel lines) | - | $\begin{aligned} & 1-3,6-10,20-25,27-30,33-40,42,43,47,48 \mathrm{a}, \\ & 48 \mathrm{~b}, 48 \mathrm{c}, 49,50,55,56,59,70-72,74-76,(81,85, \\ & 90,91) \end{aligned}$ | 4, 5, 11-19, 26, 31, 32, 41, 44, 45, 46a, 46b, 51-$54,57,58,60-69,73,(77-80,82-84,86-89)$ |
| Triangles | - | 93, 94, 95, 96, 97, 98-101, 103-105, 108 | 92, 102, 106, 107, 109-123, 151-156 |
| Quadrilaterals | - | $124,127,131,132,138,139,143 b, 144$ | $\begin{aligned} & 125,126,128-130,133-137,140-142,143 \mathrm{a}, \\ & 143 \mathrm{~b}, 145-150 \end{aligned}$ |
| Circle | - | $\begin{aligned} & \text { 157, 158, 163, 166-168, 172, 173, 178, 179, 181, } \\ & 191-193 \end{aligned}$ | $\begin{aligned} & 159-162,164,165,169-171,174-177,180,182- \\ & 190,194,195 \end{aligned}$ |
| Inscribed shapes | - | 200 | 196-199, 201-218 |
| Constructions and loci for $10^{\text {th }}$ and $11^{\text {th }}$ grade students*5 | - | $247,248,249,250,251$ | 219-246, 252-257 |
| Analogies \& Similarity | - | 258, 265, 266, 284, 285 | 259-264, 267-283, 286-293, 338 |
| Metrics relation | - | 296, 297, 353-355 | 294, 295, 298-309, 332-337, 339-352, 356, 357 |
| Areas | - | 310, 323 | 311-315, 316a, 316b, 317-322, 324-331 |
| Circle's measurement | - | 358, 380, 382, 383a, 383b | 359-379, 381 |

[^4]| Solid Geometry | - | $384,385,387,390,393,395-398,400,418,422$, | $386,388,389,391,392,394,399,401-417,419-$ |
| :---: | :---: | :--- | :--- |
|  |  | $430,435,439,444,447-449,455,457,459,460$, | $421,423-429,431-434,436-438,440-443,445$, |
|  |  | $462,465,469,475,479,488,489,491,492,501$, | $446,450-454,456,458,461,463,464,466-468$, |
|  |  | $503,507,508,522,523,528-531,533-535,537-540$, | $470-474,476-478 \mathrm{a}, 478 \mathrm{~b}, 480-487,490,493-500$, |
|  |  | $542-545,547,551,560-573,578,583,585,586,588$ | $502,504-506,509-521,524-527,532,536,541$, |
|  |  | $546,548-550,552-559,574-577,579-582,584$, |  |
|  |  | $587,589-594$ |  |

Textbook B is parted in four columns (title of each chapter, NC, BC, C) and six rows consisting numbers. Numbers are associated with the title of every scheme, as provided in textbook. In some cases, a scheme might consist a combination of graphs, such schemas 98a and 98b (Chapter of Areas, C). This textbook used for Geometry's teaching of the $2^{\text {nd }}$ Grade of Lyceum between the years 1979 and 1986.

| TEXTBOOK B |  |  |  |
| :---: | :---: | :---: | :---: |
| Chapter | NC | BC | C |
| Constructions and loci for $1^{\text {st }}$ and $2^{\text {nd }}$ grade students* ${ }^{6}$ | - | 29-33 | 1-28, 34-39 |
| Analogies \& similarity | - | 40, 48, 55, 66, 67, 135-137 | 41-47, 49-54, 56-65, 68-75, 115-134, 138, 139 |
| Metrics relations | - | 78, 79 | 76, 77, 80-91, 114 |
| Areas | - | 92, 105 | 93-98a, 98b, 99-104, 106-113 |
| Circle's measurement | - | 140, 162, 164, 165a, 165b | 141-161 |
| Solid Geometry | - | 166, 167, 169, 172, 175, 177-180, 182, 200, 204, 212, 217, 221, 226, 229-231, 237, 239, 241, 242, 244, 247, 251, 257, 261, 270, 271, 273, 274, 283, $285,289,290,304,305,310-313,315,316,317$, 319-322, 324-327, 329, 333, 342-355, 360, 365, 367, 368, 370 | 168, 170, 171, 173, 174, 176, 183-199, 201-203, 205-211, 213-216, 218-220, 222-225, 227, 228, 232236, 238, 240, 243, 245, 246, 248-250, 252-256, 258260a, 260b, 262-269, 272, 275-282, 284, 286-288, 291-303, 306-309, 314, 318, 323, 328, 330-332, 334341, 356-359, 361-364, 366, 369, 371-373 |

[^5]This table depicts the number of $\mathrm{NC}, \mathrm{BC}$ and C as enumerated in the eight chapters of textbook C . As evident in the table, in the front part of every number there is a symbol " p ", which is an abbreviation of the word "page", since images were not enumerated in schemas as in the textbooks A and B. For simplicity, the images of every page were enumerated. It is noted that when there is a wide number of images in a page, then they are enumerated vertically or horizontally, starting from figure 1 (e.g. p.31(1, 2, 3, 4, 5)). The textbook below was taught in class of Geometry of the $1^{\text {st }}$ grade of Lyceum from 1979 until 1990.

| TEXTBOOK C |  |  |  |
| :---: | :---: | :---: | :---: |
| Chapter | NC ${ }^{7}$ | BC | C |
| Introduction to Euclidian Geometry | $\begin{gathered} \text { p. } 5 \\ \text { p. } 7 \end{gathered}$ | $\begin{aligned} & \text { p.8, p.9(1, 2, 3, 4), p.10(2, 3, 4, 5, 6), p.11(1, 2, } \\ & 3,4), \text { p.12(1, 2, 3, 4), p.13(1, 2, 3, 4), p.14(1, 2, 3), } \\ & \text { p.17(1, 2), p.18(3, 4), p.19(1,2,3, 4), p.20(1,2,3, } \\ & 4), \text { p.21(1,2,3, 4, 5, 6, 7), p.22(1, 2), p.23(1,2,3) } \end{aligned}$ | $\begin{aligned} & \text { p.10(1), p.14(4), p.15(1), p.17(3), p18(1, 2), p.22(4), } \\ & \text { p.23(4), p.24(1, 2), p.25(1, 2), p.26(1, 2) } \end{aligned}$ |
| Circle's terms* | - | $\begin{aligned} & \text { p.30(1, 2, 3), p.31(1, 2, 3, 4, 5), p.33(1, 2), p.34(2), } \\ & \text { p35(1,2,3,4,5,6), p.38(1, 2, 3) } \end{aligned}$ | p.32(1, 2), p.34(1), p.36(1, 2, 3), p.39(1, 2, 3), p.40(1) |
| Relations and operations of angles* | - | $\begin{aligned} & \text { p.45(1, 2), p.46(1), p.47(1), p.50(1a, 1b, 1c, 2), } \\ & \text { p.53(2, 3, 4), p.54(1) } \end{aligned}$ | p.43(1a, 1b), p.44(1, 2), p.45(1), p.49(1), p.51(1, 2, 3, 4), p.52(1), p.53(1), p.54(2), p.55(1, 2, 3) |
| Triangles | - | $\begin{aligned} & \text { p.58(1, 2, 3, 4), p.59(1, 2, 3), p.119(3, 4, 5), } \\ & \text { p.121(1), p.126(5), p.137(1), p.138(1, 2, 3, 4, 5), } \\ & \text { p.139(2), p.142(1, 2, 3) } \end{aligned}$ | p.59(4, 5), p.60(1, 2), p.61(1, 2), p.62(1, 2), p.63(1, 2, 3, 4), p.64(1, 2), p.66(1, 2, 3), p.67(1), p.68(1), p.69(1), p.70(1, 2, 3, 4), p.71(1), p.118(1), p.119(1, 2), p.121(2, 3), p.122(1, 2), p.123(1, 2), p.124(1, 2), p.125(1, 2), p.126(1, 2, 3, 4), p.130(1, 2), p.131(1, 2, 3), p.132(1, 2) p.137(2, 3), p.139(1), p.140(1, 2, 3, 4), p.141(1, 2), p.142(4, 5), p.143(1), p.144(1, 2), p.145(1, 2, 3, 4, 5), p.146(1, 2, 3, 4), p.147(1, 2, 3) |
| Parallel lines | - | p.76(1, 2), p.83(2) | $\begin{aligned} & \text { p.75(1, 2), p.76(3), p.77(1, 2, 3), p.78(1, 2, 3), p.79(1), } \\ & \text { p.80(1, 2), p.81(1,2,3), p.82(1), p.83(1), p.84(1, 2, 3), } \end{aligned}$ |

[^6]|  |  |  | $\begin{aligned} & \text { p. } 85(1,2,3,4), \text { p. } 86(1,2), \text { p. } 87(1,2,3), \text { p.88(1, 2, 3), } \\ & \text { p. } 89(1) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Quadrilaterals | - | $\begin{aligned} & \text { p.94(1), p.98(1), p.100(2), p.103(1), p.104(1), } \\ & \text { p.106(1) } \end{aligned}$ | $\begin{aligned} & \text { p.93(1, 2), p.94(2,3, 4), p.95(1), p.96(1, 2), p.97(1, 2), } \\ & \text { p.98(2), p.99(1, 2, 3), p.100(1), p.101(1), p.102(1, 2), } \\ & \text { p.105(1, 2), p.106(2, 3, 4), p.107(1, 2), p.108(1, 2), } \\ & \text { p.109(1,2,3), p.110(1, 2), p.111(1,2,3) } \end{aligned}$ |
| Constructions and loci* | - | p.151(1, 2, 3), p.158(3, 4) | $\begin{aligned} & \text { p.152(1, 2, 3, 4, 5), p.153(1,2), p.155(1, 2, 3, 4, 5), } \\ & \text { p.156(1), p.157(1, 2, 3), p.158(1, 2), p.160(1, 2), } \\ & \text { p.161(1, 2, 3, 4), p.162(1,2, 3, 4), p.163(1,2, 3), } \\ & \text { p.164(1,2), p.165(1, 2, 3, 4, 5), p.166(1,2), p.167(1, 2) } \end{aligned}$ |
| Inscribed quadrilaterals | - | - | $\begin{aligned} & \text { p.128(1,2,3), p.129(1, 2), p.171(1,2,3), p.172(1,2, } \\ & 3,4), \text { p.173(1), p.174(1,2), p.175(1,2,3), p.176(1,2,3) } \end{aligned}$ |

Textbook D illustrates the number of three categories images (NC, BC and C), found on the chapters of Geometry's books, which they used in the $2^{\text {nd }}$ grade of Lyceum for the Geometry lectures during 1986 and 1991. Textbook D acts in a similar way as textbooks A and B, since the numbers below pertain to the numbered images, as depicted in the textbook.

| TEXTBOOK D ${ }^{8}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Chapter | NC | BC | C |
| Analogies \& Similarity | $1^{9}(\mathrm{p} .27)$ | $1,2,15 \mathrm{a}, 15 \mathrm{~b}$ $9$ | $3 \mathrm{a}, 3 \mathrm{~b}, 4,5 \mathrm{a}, 5 \mathrm{~b}, 5 \mathrm{c}, 6 \mathrm{a}, 6 \mathrm{~b}, 6 \mathrm{c}, 7-14,16 \mathrm{a}$, 16b, 17, 18a, 18b <br> 1a, 1b, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13 |
| Metrics relations | 1(p.41) | 3, 4 | $1,2,5-11,12 a, 12 b, 12 c, 13 a, 13 b, 14,15$, 16a, 16b, 17a, 17b, 18a, 18b, 19, 20a, 20b, 21$24,25 \mathrm{a}, 25 \mathrm{~b}$ |
| Areas | 1(p.63) | 1a, 1b, 1c, 1d, 4a, 4b, 5, 6, 7, 8, 9, 14, 15 | 2a, 2b, 3a, 3b, 3c, 10-13, 16-20 |
| Circle's measurement | 1(p.81) | 15 | 1-14, 16-21 |
| Solid Geometry | $\begin{aligned} & \text { 2(p.101, } \\ & \text { p.127) } \end{aligned}$ | 7-11, p.109(I, II, III), 30, 31, 39, 43a, 43b, 44, <br> 3a, 3b, 4a, 4b, 5, 6, 8a, 8b, 15, 18 | $\begin{gathered} 1-6,12-24^{10}, 26-29,32-36 \mathrm{a}, 36 \mathrm{~b}, 36 \mathrm{c}, 36 \mathrm{~d}, \\ 37-42,45-47 \mathrm{a}, 47 \mathrm{~b}, 48-50, \\ 1,2,7,9-14,16,17,19 \mathrm{a}, 19 \mathrm{~b}, 19 \mathrm{c}, 20 \mathrm{a}, 20 \mathrm{~b}, \\ 20 \mathrm{c}, 21,22 \end{gathered}$ |

[^7]Textbook E was used for the teaching of Geometry in the $1^{\text {st }}$ grade of Lyceum from 1990 to 1999. It is parted in seven rows with numbers and four columns for the numbers of chapters and the NC, BC and C images. E follows a similar pattern as textbooks A, B and D. Every visual representation is entitled with a number, and the numeration of the visuals is starting from number 1 in all chapters. Hence, there were pages that consisted untitled figures; In these cases, I provide the number of the page (e.g. p.9). This book consisted solved tasks, in which the students were supposed to understand the graph in order to gain information and complete the exercise. When there is a figure of this category, then the letter " $t$ " is used, as an abbreviation of the schemas related to "tasks".

| TEXTBOOK E |  |  |  |
| :---: | :---: | :---: | :---: |
| Chapter | NC | BC | C |
| Introduction to Euclidian Geometry | $\begin{gathered} \text { p.9, p.11, p.12, } \\ \text { p.13, p.17, p. } 20, \\ \text { p. } 32, \text { p. } 35 \end{gathered}$ | p.14, p.15, 1, 2, 4, 6, 7a, 7b, 8, 11, 12, <br> $13,14,15,16,19,20,21,23,24$ | $\begin{gathered} 3,5,6,7,9,10, \text { p. } 23(\mathrm{t}), \text { p.25(t1(1, 2)), p.27(1), } 17, \\ 18,22,25,26,27,28, \text { p.32(t1, t2) } \end{gathered}$ |
| Triangles | p.37, p. 44 | $1-7,9-21,23,24,30-33,51$ | $\begin{gathered} 8,22,25, \text { p. } 41(3), 26,27,28 \mathrm{a}, 28 \mathrm{~b}, 28 \mathrm{c}, 29,34,35, \\ \text { p. } 47(\mathrm{t} 1, \mathrm{t} 2), \text { p. } 48(\mathrm{t} 3), 43,44,45, \text { p. } 57(1,3,4), 52-58, \\ \mathrm{p} .63(1, \mathrm{t} 1, \mathrm{t} 2, \mathrm{t} 3), 59,60 \mathrm{a}, 60 \mathrm{~b}, 61-65, \text { p. } 67(\mathrm{t} 1, \mathrm{t} 2) \end{gathered}$ |
| Parallel lines | p. 49 |  | $\begin{gathered} 36-42, \text { p.51(a, b), p.52(c, t2), 46, 47, 48a, 48b, 49, } \\ 50, \text { p.57(2) } \end{gathered}$ |
| Quadrilaterals | $\begin{aligned} & \text { p.72, p.74, p.83, } \\ & \text { p. } 84, \text { p. } 88(1,2) \end{aligned}$ | 1, 4, 5, 6, 20, 21, p.88-90(1, 2, 3, 4) | $\begin{gathered} 2,3,7-15, \text { p.78(t1, t2), 16-19, p.82(t1, t2), } 22, \\ \text { p. } 85(\mathrm{i}, \mathrm{ii}, \mathrm{iii}) \end{gathered}$ |
| Thali's Theorem | $\begin{gathered} \text { p. } 91, \text { p. } 95, \text { p. } 100, \\ \text { p. } 105(1,2) \end{gathered}$ | p.91, 3, p.98(1) | $\begin{gathered} 1,2 \mathrm{a}, 2, \mathrm{p} .94(\mathrm{t} 1, \mathrm{t} 2), 4,5,6, \mathrm{p} .98(\mathrm{t} 1, \mathrm{t} 2), 7,8,9,10, \\ 11,12, \mathrm{p} \cdot 107(1,2,3) \end{gathered}$ |
| Circle ${ }^{11}$ | $\begin{aligned} & \text { p.108, p.136, } \\ & \text { p. } 141 \end{aligned}$ | 2-4, 10, 11, 16, p.138(1, 2), 47-51 | $\begin{gathered} 1,5-9, \text { p.112(tt }, \mathrm{t} 2), 12-14 \mathrm{a}, 14 \mathrm{~b}, 14 \mathrm{c}, 15,17-19 \\ \mathrm{p} .118(\mathrm{t} 1, \mathrm{t} 2), 20-22^{12}, 24,25, \mathrm{p} .124(\mathrm{t} 1, \mathrm{t} 2), \text { p.126(t)), } \\ 27-31,44-46, \mathrm{p} .139(\mathrm{t} 1), \mathrm{p} .140(\mathrm{t} 2), 52,53, \mathrm{p} .142(\mathrm{t} 1) \\ \hline \end{gathered}$ |
| Inscribed shapes | $\begin{gathered} \text { p.129, p.132, } \\ \text { p.144, p.147, p. } 153 \end{gathered}$ | 32, 41, 54, 59 | $\begin{aligned} & 33-40 \mathrm{a}, 40 \mathrm{~b}, 40 \mathrm{c}, 40 \mathrm{~d}, 41-43, \mathrm{p} .134(\mathrm{t} 1, \mathrm{t} 2), 55-59, \\ & 146(\mathrm{t} 1(\text { scheme } 59)), \text { p. 147(t2), 60-63, p.149(t1) } \end{aligned}$ |

[^8]Textbook F was used for the teaching of Geometry for the 2nd grade students of Lyceum between the years 1991 and 1999. This table is divided into four columns, each one represents a different component (Chapter, $\mathrm{NC}, \mathrm{BC}$ and C ) and four rows with numbers, related to the name of every chapter. The numeration of graphs is starting in the beginning of the chapter and the figures of the tasks are representing with the letter " $t$ '. Furthermore, this textbook includes pages with unnumbered images that pertain to the non-conceptual visual

| TEXTBOOK F |  |  |  |
| :---: | :---: | :---: | :---: |
| Chapter | NC | BC | C |
| Metrics relations | p. 7 | 1, 8 | $\begin{gathered} 2 \mathrm{a}, 2 \mathrm{~b}, 3, \mathrm{p} .9(\mathrm{t} 1), 4, \mathrm{p} .13(\mathrm{t} 1(1,2), \mathrm{t} 2), 5,6,7, \\ \text { p. } 17(\mathrm{t} 1, \mathrm{t} 2), 9,10,11,12,13,14, \text { p. } 27, \text { p. } 28(1,2) \end{gathered}$ |
| Areas | p. 29 | 1, 3a, 3b, 3c, $4^{13}$, p.47(1, 2, 3), p.48(1) | 2, 7-9i, 9ii, 10, 11, p.34(t1), p.36(a, b), 12, p.37(3), $13, \mathrm{p} .38-39(\mathrm{t} 1, \mathrm{t} 2), 14,15 \mathrm{a}, 15 \mathrm{~b}, \mathrm{p} .41(\mathrm{t} 1, \mathrm{t} 2), 16-18$, |
| Circle's measurement | p. 49 | 8a, 8b, 8c, 9-13, p.62(Note) | $\begin{gathered} 1-4, \text { p. } 53(\mathrm{t} 1), 5-7, \text { p. } 55(\mathrm{t} 1), \text { p. } 56(\mathrm{t} 2), \text { p. } 63(\mathrm{t} 1), \text { p. } 68- \\ 69(1,2,3,4) \end{gathered}$ |
| Solid Geometry | $\begin{gathered} \hline \text { p. } 71, \\ \text { p.100, } \\ \text { p.101, } \\ \text { p. } 119 \end{gathered}$ | $\begin{gathered} 1-8,10-13, p .82(1), p .89(1), 38,41,45, \\ 47,48 \\ 1,2,4,9,10,13 \\ 1,3-5,19,22-24 \end{gathered}$ | 9, p.75(t1), 14-22, p.80(t1), p.81(t2/1, 2), 23-28, p. $84-85(\mathrm{t} 1, \mathrm{t} 2), 29-34, \mathrm{p} .88(\mathrm{t}), 36,37, \mathrm{p} .90(\mathrm{t} 1), 39$, 40-44, 46, 49, p.96(t1/1, 2), <br> 3, p.103(t1), 5-8, p.109(t1), $11^{14}, 14$, p.115(t1) 2, p.121, 6, p.125(t1), 7-1015, 12, p.130(1, 2), 20a, 20b, 20c, 21a, 21b, 21c, 25-27, p.137(t1, t2, t3), p.138(Note) |

[^9]Textbook G used for the teaching of Geometry of the grades $1^{\text {st }}$ and $2^{\text {nd }}$ of Lyceum from 1999 to 2001. Figures were unnumbered in order to present the figures of each page. The enumeration starts from number one in every page.

| TEXTBOOK G |  |  |  |
| :---: | :---: | :---: | :---: |
| Chapter | NC | BC | C |
| Introduction to Euclidian Geometry | $\begin{aligned} & \text { p.4(2), } \\ & \text { p.5(1,2), } \\ & \text { p.6(1), } \\ & \text { p.8(1) } \end{aligned}$ | $\begin{aligned} & \text { p.2, p.4(1,3), p.10(1), p.11(1, 2, 3, 4), p.12(1, } \\ & \text { 2,3), p.13(1,2,3, 4), p.14(1), p.18(1, 2), } \\ & \text { p.19(1,2,3,4,5), p.20(1,2,3, 4, 5), p.21(1, 2, } \\ & 3,4,5), \text { p.22(1), p.23(1,2,3), p.25(1,2,3), } \\ & \text { p.30(1,2,3), p.31(1,2), p.32(1), p.39(1,2, } \\ & 4), \text { p.40(1,2,4), p.41(1) } \end{aligned}$ | $\begin{aligned} & \text { p.16(t1, t2), p.19(6, 7), p.24(1, 2), p.26(1, 2), } \\ & \text { p.27(1), p.28(1, t1), p.32(2), p.33(1,2), p.34(1), } \\ & \text { p.36(1/Comments), p.37(t1), p.40(3) } \end{aligned}$ |
| Triangles | - | $\begin{aligned} & \text { p.43(1,2,3, 4, 5, 6, 7), p.44(2, 3, 4), p.45(1, 2, } \\ & 3), \text { p. } 48(1), \text { p. } 51(1), \text { p. } 53(1), \text { p. } 60(1,2,3,4), \\ & \text { p. } 61(2,3,4,5,6), \text { p.76(1,2), p.77(3), p.79(1), } \\ & \text { p. } 80(1,2,3,4,5,6), \text { p.81(1) } \end{aligned}$ | p.44(1), p.46(1, 2), p.47(1, 2, 3), p.48(2, 3), <br> p.49(1, 2), p.50(1, 2), p.51(2, 3), p.52(1, 2, 3), p.53- <br> 54(t1, t2, t3, t4), p.59(t1), p.60(5), p.61(1), p.62(t1, <br> t2), p.64(1, 2), p.65(1), p.66(1), p.67(1, 2, 3, 4), <br> p.68(1, 2), p.69(1), p.70(1, 2), p.71-73(t1, t2, t3, t4, <br> t5(1, 2), t6), p.77(1, 2, 4), p.81(2(Note), t1), <br> p.84(t1(1, 2)), p.85(t2, t3(2, 3)), p.86(t4, t5), <br> p.87(t6, t7), p88-89(t1, t1.a, t1.b, t1.c) |
| Parallel lines | - | p.93(1), p.94(1) | $\begin{aligned} & \text { p. } 94(2,3), \text { p. } 95(1,2, \text { Historical reference(1)), } \\ & \text { p. } 96(\text { Historical reference( } 1,2)), \text { p. } 97(1,2), \text { p. } 98(1, \\ & 2,3), \text { p. } 99(\mathrm{t} 1,1 \mathrm{a}, 1 \mathrm{~b}), \text { p. } 100(1 \mathrm{c}, 2 \mathrm{a}, 2 \mathrm{~b}, 2 \mathrm{c}), \\ & \text { p.101(1,2), p.102(1, 2), p.103(1, 2), p.104-105(t1, } \\ & \mathrm{t} 2 . \mathrm{a}, \mathrm{t} 2 . \mathrm{b}, \mathrm{t} 3) \end{aligned}$ |
| Quadrilaterals | - | $\begin{aligned} & \text { p.111(1, 2, 3, 4), p.112(3, 4), p.119(1, 2, 3), } \\ & \text { p.120(2), p.130(1, 2, 3), p.134(2), p.135(2) } \end{aligned}$ | $\begin{aligned} & \text { p.112(1, 2), p.113(1), p.114(1, 2, 3), p.115(1), } \\ & \text { p.116(t1, t2), p.119(4), p.120(1), p.121(t1, t2), } \\ & \text { p.124(1, 2), p.125(1,2,t1), p.126(t2(1, 2), t3), } \\ & \text { p.127(t1), p.130(4), p.131(1, t1), p.132(t2), } \\ & \text { p.134(1), p.135(1), p.136(1), p.137(1,2), p.138(t1) } \end{aligned}$ |


| Inscribed shapes | - | $\begin{gathered} \text { p.143(1, 2, 3), p.144(3), p.145(1a, 1b), } \\ \text { p.146(1), p.147(2), p.152(1), p.153(2, 3) } \end{gathered}$ | $\begin{aligned} & \text { p.144(1, 2), p.145(2), p.146-147(t1(1, 2, 3, 4, 5)), } \\ & \text { p.147-148(t1, t2, t3.a, t3.b, t3.c, t3.d), p.152(2), } \\ & \text { p.153(1,4), p.154(t1, t2), p.158(1, 2, 3, 4), p.159(1, } \\ & 2,3), \text { p.160(1, 2, 3), p.161(t1, t2) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Analogies \& Similarity | - | $\begin{aligned} & \text { p.168(1), p.169(1), p.173(3), p.184(1), p.193, } \\ & \text { p.194(1, 2) } \end{aligned}$ | $\begin{aligned} & \text { p.167-168(t1(1, 2)), p.173(1, 2), p.174(1), } \\ & \text { p.175(1), p.176(1.a, 1.b), p.177-178(1, 2, 3.a, 3.b), } \\ & \text { p.179(t1), p.182(1,2, 3), p.183(1), p.184-185(1, 2), } \\ & \text { p.186(1), p.187(t1), p.194(3), p.195(1), p.196(1), } \\ & \text { p.197(1, 2), p.198(1), p.199(1(Note), 2), p.200(1, } \\ & \text { t1), p.201(t2, t3) } \end{aligned}$ |
| Metrics relations | - | p.209(1, 2), p.217(1) | $\begin{aligned} & \text { p.210(1, 2), p.211(1, 2, 3), p.212(1, t1 (1, 2)), } \\ & \text { p.213-214(1,2,3, 4, 5), p.216(1a, 1b, 2), 217(2.a, } \\ & \text { 2.b), p.218(1), p.219(1), p.220(t1, t2), p.221-222(t3, } \\ & \text { t4.a, t4.b), p.227(1, 2), p.228(1), p.229-231(1, 2), } \\ & \text { p.232(tl, t2), p.233(t3(1, 2, 3)) } \end{aligned}$ |
| Areas | - | $\begin{array}{\|c} \mathrm{p} .239(1), \text { p.240(1, 2, 3, 4), p.241(1, 2), } \\ \mathrm{p} .242(2), \text { p.243(1), p.244(2), p.252(1, 2) } \end{array}$ | $\begin{aligned} & \text { p.241(3), p.242(1,3), p.243(2, 3), p.244(1), } \\ & \text { p. } 245(1,2), \text { p. } 246(1), \text { p. } 247(\mathrm{t} 1(1,2), \mathrm{t} 2), \text { p. } 248(\mathrm{t} 3), \\ & \text { p. } 251(1,2), \text { p.252(3), p.253(t1), p.256(1), p.257- } \\ & 258(\mathrm{t} 1(1,2)), \text { p.259(1) } \end{aligned}$ |
| Circle's measurement | p.283(1) | 265(1, 2), p.266(1), p.277(2), p.281(1) | $\begin{aligned} & \text { p.263(1), p.265(3, 4), p.266(2), p.267(1), p.267- } \\ & 270(\mathrm{t} 1, \mathrm{t} 2, \mathrm{t} 3, \mathrm{t} 4(1,2)), \text { p.268(1), p.269(1, 2), } \\ & \text { p.270(1), p.272(t1, t2), p.275(1), 277(1), p.279(t1), } \\ & \text { p.282(1, 2), p.284(Exercise(1, 2, 3), t1), p.285(t2) } \end{aligned}$ |
| Solid Geometry | p. 290 | $\begin{aligned} & \mathrm{p} .289(1), \text { p.290(1, 2, 3, 4, 5), p.291(1, 2, 3), } \\ & \text { p.293(1.a, 1.b, 1.c), p.296(1, 2), p.298(2), } \\ & \text { p.300(2), p.302(1, 2), p.304(1, 2, 3), p.305(1, } \\ & 2), \text { p.316(1, 2, 3), p.317(1, 2, 3, 4, 5, 6), } \\ & \text { p.318(1), p.319(2), p.320(1), p.321(1, 2, 3), } \\ & \text { p.322(1, 2), p.326(1, 2, 3), p.327(1, 2, 3, 4), } \end{aligned}$ | $\begin{aligned} & \text { p.291(4), p.292(1, 2a, 2b, 2c), p.293(t1, t2), } \\ & \text { p.294(t3), p.296(3), p.297(1, 2), p.298(1), p.299(1, } \\ & 2,3), \text { p.300(1, 3), p.301(1, 2), p.303(1, 2), p.305(3), } \\ & \text { p.306(1, 2a, 2b), p.307-309(t1, t2, t3.a, t3.b, t4, t5.a, } \\ & \text { t5.b, t5.c), p.318(2), p.319(1), p.320(2), p.322(3), } \\ & \text { p.323(1), p.324(t1), p.328(1), p.329(1, 2, 3), } \end{aligned}$ |


|  |  | $\begin{aligned} & \text { p.332(1, 2, 3, 4, 5, 6), p.333(1, 2), p.334(1), } \\ & \text { p.335(1,2,3), p.336(1, 2), p.337(1), p.339(1, } \\ & 2), \text { p.340(1), p.341(1,2,3), p.343(1), p.345(1, } \\ & 2,3,4,5), \text { p.346(1) } \end{aligned}$ | $\begin{aligned} & \text { p.330(t1), p.336(3), p.340(2), p.342(1, 2, 3, 4), } \\ & \text { p.346(t1), p.347(t2.a, t2.b, t2.c), p.348(t3.a, t3.b) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |

Textbook H is the present book of Geometry for the $1^{\text {st }}$ and $2^{\text {nd }}$ grade of Lyceum's students. Until 2014 this book combined the teaching material of both grades. At the school year of 2014, this book was replaced by two separate books, volume 1 and 2 for grades one and two respectively. The teaching material, the images and the exercises did not show any modification. The unnumbered images are representing with the letter " $p$ " in the frond part (e.g. p.2)

| TEXTBOOK H |  |  |  |
| :---: | :---: | :---: | :---: |
| Chapter | NC | BC | C |
| Introduction to Euclidian Geometry | p.2, p. 8 | $\begin{gathered} 1-7,9,11-20,22-25,27,28,30-34,39- \\ 44,48,51-54 \end{gathered}$ | $\begin{gathered} 8,10,21 \mathrm{a}, 21 \mathrm{~b}, 21 \mathrm{c}, 21 \mathrm{~d}, 26,29,35-38, \\ 45-47,49,50,55-61 \end{gathered}$ |
| Triangles | p. 34 | $\begin{gathered} 1-10 \mathrm{a}, 10 \mathrm{~b}, 24,25,37-39,42,43,54,61 \mathrm{~b}, \\ 62 \mathrm{a}, 62 \mathrm{~b}, 62 \mathrm{c}, 62 \mathrm{~d}, 62 \mathrm{e}, 63 \end{gathered}$ | $\begin{gathered} \text { 11-23, 26-36, 40, 41, 44, 45a, 45b, 45c, } \\ 45 \mathrm{~d}, 45 \mathrm{e}-60,61 \mathrm{a}, 64-75 \end{gathered}$ |
| Parallel lines | $\begin{gathered} \text { p. } 74, \text { p. } 90(1, \\ 2) \end{gathered}$ | 1, 11-13 | 2-10, 14-17a, 17b-24, p.86(t1, t2) |
| Quadrilaterals | p. 96 | $1-4,7-9,13,16,19,34,37$ | $\begin{gathered} \hline 5,6,10-12,14,15,17,18,20-23,24 \mathrm{a}, \\ 24 \mathrm{~b}, 25-33,35,36,38-40 \\ \hline \end{gathered}$ |
| Inscribed shapes | p. 122 | 1, 2, 3, 5, 10, 18, 30 | 4a, 4b, 4c, 6-9a, 9b, 11-17, 19-29, 31 |
| Analogies \& Similarity | p.144, p. 170 | $2,3,7$ | $\begin{gathered} 1,4,5,6,8 \mathrm{a}, 8 \mathrm{~b}-22, \\ 1-10 \end{gathered}$ |
| Metric relations | p. 182 | 18, 20 | 1-9, p.188, 10a, 10b-16, 17a, 17b, 19, 21 |
| Areas | p. 210 | $1-4, \mathrm{p} .212^{16}(1,2)$ | $\begin{gathered} 5-17 \mathrm{a}, 17 \mathrm{~b}-26, \mathrm{p} .228(1 / \text { Historical } \\ \text { reference }) \end{gathered}$ |
| Circle's measurement | p.232, p. 256 | $6 \mathrm{a}, 6 \mathrm{~b}, 15,17,18,20$ | $\begin{gathered} 1-5,7-12,13,14,16,19,21,22, \text { p. } 253- \\ 254(1,2,3) \end{gathered}$ |
| Solid Geometry | $\begin{gathered} \text { p.258, p.286, } \\ \text { p.288, } \end{gathered}$ | $\begin{gathered} 1,2,5,8-11,13,26-29,36-38,40,41 \\ 1-3,5-9,14 \mathrm{a}, 15 \mathrm{a}, 15 \mathrm{~b}, 22,23 \mathrm{a}, 23 \mathrm{~b}, 31, \\ 42,43,45 \mathrm{a}, 45 \mathrm{~b}, 45 \mathrm{c}, 46 \mathrm{a}, 46 \mathrm{~b}, 46 \mathrm{c}, 50 \\ \text { p. } 324 \text { (Historical reference) } \\ \hline \end{gathered}$ | $\begin{gathered} 3,4,6,7,12,14-25,30-35,39,42,42 \mathrm{a}- \\ 49 \\ 4,10-13,14 \mathrm{~b}, 16 \mathrm{a}, 6 \mathrm{~b}-21,24-30,32-41, \\ 44,47-49,49 \mathrm{a} \end{gathered}$ |

[^10]
## Appendix D <br> Classifying tasks

# Appendix D: the presence of tasks (Qs, $E, P, M M$ ) in the selected sample of textbooks (A, B, C, D, E, F, G, H) of Euclidean Geometry in Lyceum from 1975 to 2019. 

Appendix D provides information about the presence of tasks (Qs, E, P, MM) in the selected sample of textbooks (A, B, C, D, E, F, G, H) of Euclidean Geometry in Lyceum from 1975 to 2019. The textbooks are divided in chronological order and the way they present tasks has changed through the years. The enumeration of tasks was valid for the progress of this study, since I focused on the ways that Geometry concepts are introduced to students and whether or not the authors priorities have changed from 1975 to 2019 , using four categories of tasks.

Textbook A was used for the teaching of Euclidean Geometry from 1975 to 1976 in the $9^{\text {th }}, 10^{\text {th }}, 11^{\text {th }}$ and $12^{\text {th }}$ grade of Gymnasium. In textbook A students needed to provide solutions in tasks according to its level of complexity. There were two groups presenting the tasks, Group A and Group B. Every set of paragraphs contained in majority E and P tasks.



|  |  | B |  | 234. 235, 236, 237 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Inscribed shapes | §203-207 | A | 241, 242, 244 | 238, 239, 240, 243, 245 |  |
|  |  | B | 246, 247 | 248, 249, 250, 251, 252 |  |
|  | §208-213 | A | 253, 254, 255, 256, 257 |  |  |
|  |  | B | 259, 261 | $\begin{gathered} 258,260,262,263,264, \\ 265 \end{gathered}$ |  |
|  | §214-216 | A | 266, 267, 268, 269, 270 |  |  |
|  | §217-218 | B |  | $\begin{gathered} 271,272,273,274,275, \\ 276 \end{gathered}$ |  |
|  | ```Абк\etá\sigma&ı\varsigma \pi\rhoо\varsigma \varepsilon\pi\alphav\alphá\lambda\eta\eta\psi\eta``` | A | 277, 278, 279, 281, 283 | $\begin{gathered} 280,282,284,285,286, \\ 287 \\ \hline \end{gathered}$ |  |
|  |  | B | $\begin{gathered} \hline 288,289,290,295,296, \\ 300,201,302,308,310, \\ 311 \end{gathered}$ | $\begin{gathered} \hline 291,292,293,294,297, \\ 298,299,303,304,305, \\ 306,307,309,312,313, \\ 314,315,316,317 \end{gathered}$ |  |
| Constructions and loci for $10^{\text {th }}$ and $11^{\text {th }}$ grade students* ${ }^{17}$ | §219-231 | A | 322, 326 | $\begin{gathered} 318,319,320,321,323, \\ 324,325,327,328,329, \\ 330,331,332 \end{gathered}$ |  |
|  | §232-239 | A |  | 333, 334, 335, 336 |  |
|  | §240-245 | A |  | $\begin{gathered} 337,338,339,340,341, \\ 342,343,344,345,346, \\ 347,348,349,350,351 \\ \hline \end{gathered}$ |  |
|  |  | B |  | $\begin{gathered} 352,353,354,355,356, \\ 357,358,359,360 \end{gathered}$ |  |

[^11]|  | §246-252 | A |  | $\begin{gathered} 361,362,363,364,365, \\ 366,367,368,369,370, \\ 371,372,373 \\ \hline \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | B |  | $\begin{gathered} 374,375,376,377,378, \\ 379,280 \end{gathered}$ |  |
|  | Абкŋ́бєıऽ $\pi \rho \circ \varsigma$ $\varepsilon \pi \alpha v \alpha ́ \lambda \eta \psi \psi \eta$ | A |  | $\begin{gathered} \hline 381,382,383,384,385, \\ 386,387,388,389,390, \\ 391,392,393,394,395, \\ 396,397 \\ \hline \end{gathered}$ |  |
|  |  | B |  | $\begin{gathered} 398,399,400,401,402, \\ 403 \end{gathered}$ |  |
|  |  | - |  | $\begin{gathered} 404,405,406,407,408, \\ 409,410,411,412,413 \end{gathered}$ |  |
| Analogies \& similarity | §253-263 | A | $\begin{gathered} \hline 414,415,416,417,418, \\ 419,420 \end{gathered}$ |  |  |
|  | §264-267 | A | $\begin{gathered} 421,422,423,424,425, \\ 426,427,428,429,430, \\ 431 \end{gathered}$ |  |  |
|  | §268-279 | A | $\begin{gathered} \hline 432,433,434,435,436, \\ 438,439,440,441,442, \\ 443,444,445,447 \\ \hline \end{gathered}$ | 437, 446 |  |
|  |  | B | 448, 449, 454, 456 | $\begin{gathered} 450,451,452,453,455, \\ 457 \end{gathered}$ |  |
|  | §280-288 | B |  | $\begin{gathered} \hline 458,459,460,461,462, \\ 463,464,465,466,467, \\ 468,469 \\ \hline \end{gathered}$ |  |
|  | §289-292 | B | 470 | $\begin{gathered} 471,472,473,474,475, \\ 476 \end{gathered}$ |  |
|  | §293-294 | A | 477 | 478, 479, 480 |  |


|  |  | B | 483, 484 | 481,482 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | §341 | A |  | $\begin{gathered} 596,597,598,599,600, \\ 601 \end{gathered}$ |  |
|  |  | B |  | 602, 603, 604 |  |
|  | §342-349 | A | 605, 606, 607, 608 | 609, 610 |  |
|  |  | B |  | 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622 |  |
|  | §350-352 | B |  | 623, 624, 625, 626, 627 |  |
| Metrics relations | §295-305 | A | $\begin{gathered} 485,486,487,488,489, \\ 490,491,492 \end{gathered}$ | 493, 494, 495, 496 |  |
|  |  | B | 501 | $\begin{aligned} & 497,498,499,500,502, \\ & 503,504,505,506,507 \end{aligned}$ |  |
|  | §306-310 | A | $\begin{gathered} 508,509,510,511,512, \\ 513,514 \\ \hline \end{gathered}$ |  |  |
|  |  | B | 515, 516 | $\begin{gathered} 517,518,519,520,521, \\ 522,523,524,525,526,527 \end{gathered}$ |  |
|  | §334-335 | A | 575, 576, 577 |  |  |
|  |  | B |  | 578, 579 |  |
|  | §336-337 | A | 580, 581, 582, 583 | 584 |  |
|  |  | B |  | 585, 586, 587, 588 |  |
|  | §338-340 | B | 589, 590, 591, 592 | 593, 594, 595 |  |
| Areas | §311-318 | A | 528, 529, 530, 531, 532 |  |  |
|  | §319-327 | A | $\begin{gathered} 533,534,535,540,541, \\ 547,549 \end{gathered}$ | $\begin{gathered} 536,537,538,539,542, \\ 543,544,545,546,548,550 \end{gathered}$ |  |
|  |  | B | 551, 552, 553 | $\begin{gathered} 554,555,556,557,558, \\ 559,560,561,562,563, \\ 564,565,566,567,568,569 \end{gathered}$ |  |
|  | §328-333 | A | 570, 571, 572, 573, 574 |  |  |


| Circle's measurement | §353-364 | A | 631 | $\begin{gathered} 628,629,630,632,633, \\ 634,635 \end{gathered}$ | 636,637 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | §365-370 | A |  | 639, 640, 643, 644, 645 | 638, 641, 642, 646 |  |
|  |  | B |  | 647, 654, 659 | $\begin{gathered} 648,649,650,651,652, \\ 653,655,656,657,658 \\ \hline \end{gathered}$ |  |
|  | §371-385 | A |  | 660, 664, 665, 666, 667, 668, 670, 671, 672, 674, 675, 676, 679 | 663, 669, 673, 677, 678 | $\begin{aligned} & 661, \\ & 662, \end{aligned}$ |
|  |  | B |  | 680, 681, 682, 683 | 684, 685, 686, 687, 688, 689, 690, 691, 692, 693 , 694, 695, 696, 697, 698 |  |
| Solid Geometry | §386-399 | A |  |  | 699, 700, 701, 702, 703 |  |
|  |  | B |  |  | 704, 705, 706, 707 |  |
|  | §400-411 | A |  | 708 | 709, 710, 711, 712, 713 |  |
|  |  | B |  |  | 714, 715, 716, 717, 718, $719,720,721,722,723$, 724, 725 |  |
|  | §412-419 | A |  |  | 726, 727, 728, 729, 730 |  |
|  |  | B |  |  | 731, 732, 733, 734 |  |
|  | §420-424 | A |  | 736, 737, 738, 739 | 735 |  |
|  |  | B |  |  | $\begin{gathered} 740,741,742,743,744, \\ 745 \end{gathered}$ |  |
|  | §425-436 | A |  | 746, 748, 750, 751 | 747, 749 |  |
|  |  | B |  | 752, 753 | $\begin{gathered} 754,755,756,757,758, \\ 759 \end{gathered}$ |  |
|  | §437-444 | A |  | 760, 761 | 762, 763, 764 |  |
|  |  | B |  |  | $\begin{gathered} 765,766,767,768,769, \\ 770,771,772,773 \\ \hline \end{gathered}$ |  |


|  | §445-461 | A |  | 774 | $\begin{gathered} 775,776,777,778,779, \\ 780 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | B | 788, 792 |  | $\begin{gathered} \hline 781,782,783,784,785, \\ 786,787,789,790,791, \\ 793,794,795,796,797 \\ \hline \end{gathered}$ |  |
|  | §462-478 | A |  |  | $\begin{aligned} & 798,799,800,801,802, \\ & 803,804,805,806,807 \end{aligned}$ |  |
|  |  | B |  |  | $\begin{aligned} & 808,809,810,811,812, \\ & 813,814,815,816,817 \\ & \hline \end{aligned}$ |  |
|  | §479-492 | A |  | 821, 825, 826, 827 | $\begin{gathered} 818,819,820,822,823, \\ 824 \end{gathered}$ |  |
|  |  | B |  |  | $\begin{gathered} \hline 828,829,830,831,832, \\ 833,834,835,836,837, \\ 838,839,840 \end{gathered}$ |  |
|  | §493-496 | B |  |  | $\begin{gathered} 841,842,843,844,845, \\ 846,847,848,849,850, \\ 851,852,853 \end{gathered}$ |  |
|  | §497-501 | A |  | 854, 856, 857, 858, 859 | 855, 860 |  |
|  |  | B |  |  | 861, 862, 863, 864, 865 |  |
|  | §502-505 | A |  | 866 | 867, 868, 869, 870, 871 |  |
|  | §506-512 | A |  |  | $\begin{gathered} \hline 872,873,874,875,876, \\ 877,878,879,880,881,882 \end{gathered}$ |  |
|  |  | B |  |  | 883, 884, 885, 886 |  |
|  | §513-519 | A |  | $\begin{gathered} 887,888,889,890,891, \\ 892,893,894,895,896 \\ \hline \end{gathered}$ |  |  |
|  |  | B |  | 900 | 897, 898, 899, 901 |  |
|  | §520-529 | A |  | $\begin{gathered} 902,903,904,905,906, \\ 907 \\ \hline \end{gathered}$ |  |  |


|  |  | B | 908, 909 | $\begin{gathered} 910,911,912,913,914, \\ 915,916,917,918 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | $\begin{gathered} \hline 919,920,921,922,925, \\ 926,927,928,929 \end{gathered}$ | 923, 924 |  |
|  |  | B | 930, 931, 932 | 933, 934, 935 |  |
|  |  | A | 936, 937, 938 | 939 |  |
|  |  | B | 940 | 941, 942, 943 |  |
|  | §530-534 | A | 944, 945, 946, 947, 948 |  |  |
|  |  | B |  | 949, 950, 951, 952, 953 |  |
|  | §535-543 | A | $\begin{gathered} \hline 954,955,956,957,958, \\ 959,960 \end{gathered}$ |  |  |
|  |  | B | 961, 962 | 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974 |  |
|  | §544-550 | A | $\begin{gathered} 975,976,977,978,979, \\ 982,983,985,986 \end{gathered}$ | 980, 981, 984 |  |
|  |  | B | 987 | $\begin{gathered} 988,989,990,991,992, \\ 993,994,995 \\ \hline \end{gathered}$ |  |
|  | §551-552 | A | 996, 997 | $\begin{gathered} 999,1000,1001,1002, \\ 1003,1004,1005 \end{gathered}$ | 998 |
|  |  | B | 1006 | 1007 |  |
|  | §553-554 |  | 1008, 1010, 1011 | 1009, 1012, 1013, 1014 |  |
|  | §555-565 | A | 1015, 1016, 1018 | $\begin{aligned} & 1017,1019,1020,1021, \\ & 1022,1023,1024,1025 \end{aligned}$ |  |
|  |  | B | 1033 | $\begin{gathered} 1026,1027,1028,1029, \\ 1030,1031,1032,1034, \\ 1035,1036,1037,1038, \\ 1039,1040 \end{gathered}$ |  |
|  | §566-569 | A |  | 1041, 1042, 1043, 1044 |  |


|  | B |  |  | $\begin{gathered} 1045,1046,1047,1048 \\ 1049,1050,1051 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| §570-573 | A |  | 1052, 1053, 1054, 1055 |  |  |
|  | B |  |  | $\begin{gathered} 1056,1057,1058,1059, \\ 1060 \end{gathered}$ |  |
| §574-579 | A |  | $\begin{gathered} \hline 1061,1062,1063,1064, \\ 1065,1066,1067,1068, \\ 1069,1070,1071,1072, \\ 1073,1074,1075 \end{gathered}$ |  |  |
|  | B |  | 1079 | $\begin{gathered} \text { 1076, 1077, 1080, 1081, } \\ 1082 \\ \hline \end{gathered}$ | 1078 |
| §580-582 |  | $\begin{gathered} \hline 1083,1084, \\ 1085,1086, \\ 1087 \end{gathered}$ |  |  |  |

Textbooks B was used for the teaching of Euclidean Geometry of the $2^{\text {nd }}$ Grade of Lyceum between the years 1979 and 1986 . Once more the majority of tasks was the categories $E$ and $P$. Students needed to provide solution according to the level of complexity of tasks, since tasks were divided into two groups A and B .

| TEXTBOOK B |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Paragraph | Item | Qs | E | P | MM |
| Constructions and loci for $1^{\text {st }}$ and $2^{\text {nd }}$ grade students* | §1-2 | A |  | 5, 9 | $\begin{gathered} 1,2,3,4,6,7,8,10,11,12,13, \\ 14,15,16,17 \end{gathered}$ |  |
|  | §3-4 | A |  |  | 18, 19, 20, 21 |  |
|  | §5 | A |  |  | $\begin{gathered} 22,23,24,25,26,27,28,29,30, \\ 31,32,33,34,35,36 \end{gathered}$ |  |
|  |  | B |  |  | 37, 38, 39, 40, 41, 42, 43, 44, 45 |  |
|  | §6-7 | A |  |  | $\begin{gathered} 46,47,48,49,50,51,52,53,54, \\ 55,56,57,58 \end{gathered}$ |  |
|  |  | B |  |  | 59, 60, 61, 62, 63 |  |
|  | Абкŋ́бєıऽ $\pi \rho о \varsigma$ $\varepsilon \pi \alpha \nu \alpha ́ \lambda \eta \psi \eta$ | A |  |  | $\begin{gathered} 64,65,66,67,68,69,70,71,72, \\ 73,74,75,76,77,78,79, \\ \hline \end{gathered}$ |  |
|  |  | B |  |  | 80, 81, 82, 83 |  |
|  |  | - |  |  | 84, 85, 86, 87, 88, 89, 90, 91, 92 |  |
| Analogies \& similarity | §9-17 | A |  | 93, 94, 95, 96, 97, 98, 99 |  |  |
|  | §18-20 | A |  | $\begin{gathered} 100,101,102,103,104, \\ 105,106,107,108,109,110 \end{gathered}$ |  |  |
|  | §21-33 | A |  | $\begin{gathered} 111,112,113,114,115, \\ 117,118,119,120,121,122, \\ 123,124,126 \end{gathered}$ | 116, 125 |  |
|  |  | B |  | 127, 128, 133, 134 | 129, 130, 131, 132, 135, 136 |  |
|  | §34-40 | B |  |  | $137,138,139,140,141,142,143$, $144,145,146,147,148$ |  |
|  | §41-43 | B |  | 149 | 150, 151, 152, 153, 154, 155 |  |


|  | §44 | A | 156 | 157, 158, 159 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | B | 162, 163 | 160, 161 |  |
|  | §87-88 | A | 259, 260, 261, 262 | 263 |  |
|  |  | B |  | 264, 265, 266, 267 |  |
|  | §89-91 | B | 268, 269, 270, 271 | 272, 273 |  |
|  | §92 | A |  | 274, 275, 276, 277, 278, 279 |  |
|  |  | B |  | 280, 281, 282 |  |
|  | §93-100 | A | 283, 284, 285, 286 | 287, 288 |  |
|  |  | B |  | $\begin{gathered} 289,290,291,292,293,294,295, \\ 296,297,298,299,300 \end{gathered}$ |  |
|  | §101-103 | B |  | 301, 302, 303, 304, 305 |  |
| Metrics relations | §46-56 | A | $\begin{gathered} \hline 164,165,166,167,168 \\ 169,170,171 \end{gathered}$ | 172, 173, 174, 175 |  |
|  |  | B | 180 | $\begin{gathered} 176,177,178,179,181,182,183, \\ 184,185,186 \end{gathered}$ |  |
|  | §57-61 | A | $\begin{gathered} 187,188,189,190,191, \\ 192 \end{gathered}$ |  |  |
|  |  | B | 194, 195 | $\begin{gathered} 196,197,198,199,200,201,202, \\ 203,204,205,206 \end{gathered}$ |  |
|  | §85-86 | A | 254, 255, 256 |  |  |
|  |  | B |  | 257, 258 |  |
| Areas | §62-69 | A | 207, 208, 209, 210, 211 |  |  |
|  | §70-78 | A | $\begin{gathered} 212,213,214,219,220, \\ 226,228 \end{gathered}$ | $\begin{gathered} 215,216,217,218,221,222,223, \\ 224,225,227,229 \end{gathered}$ |  |
|  |  | B | 230, 231, 232 | $\begin{gathered} 233,234,235,236,237,238,239, \\ 240,241,242,243,244,245,246, \\ 247,248 \end{gathered}$ |  |
|  | §78-84 | A | 249, 250, 251, 252, 253 |  |  |


| Circle's measurement | §104-115 | A | 309 | $\begin{gathered} 306.307,308,310,311, \\ 312,313 \end{gathered}$ | 314, 315 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | §116-121 | A |  | 317, 318, 321, 322, 323 | 316, 319, 320, 324 |  |
|  |  | B |  | 325, 331, 333 | 326, 327, 328, 329, 330, 332 |  |
|  | §122-136 | A |  | $\begin{gathered} 334,338,339,340,341, \\ 342,344,345,346,348,349 \\ 350,353 \end{gathered}$ | 337, 343, 347, 351, 352 | 335, |
|  |  | B |  | 354, 355, 356, 357 | $\begin{gathered} 358,359,360,361,362,363,364, \\ 365,366,367,368,369,370,371, \\ 372 \\ \hline \end{gathered}$ |  |
| Solid Geometry | §137-150 | A |  |  | 373, 374, 375, 376, 377 |  |
|  |  | B |  |  | 378, 379, 380, 381 |  |
|  | §151-162 | A |  | 382, | 383, 384, 385, 386, 387 |  |
|  |  | B |  |  | $\begin{gathered} 388,389,390,391,392,393,394, \\ 395,396,397,398,399 \end{gathered}$ |  |
|  | §163-170 | A |  |  | 400, 401, 402, 403, 404 |  |
|  |  | B |  |  | 405, 406, 407, 408 |  |
|  | §171-175 | A |  | 410, 411, 412, 413 | 409 |  |
|  |  | B |  |  | 414, 415, 416, 417 |  |
|  | §176-187 | A |  | 418, 419, 420, 423 | 421, 422 |  |
|  |  | B |  | 424, 425 | 426, 427, 428, 429 |  |
|  | §188-195 | A |  | 430, 431 | 432, 433, 434 |  |
|  |  | B |  |  | 435, 436, 437, 438, 439, 440, 441 |  |
|  | §196-212 | A |  | 446 | 442, 443, 444, 445, 447, 448 |  |
|  |  | B | 456, 460 |  | $\begin{aligned} & 449,450,451,452,453,454,455, \\ & 457,458,459,461,462,463,464 \\ & \hline \end{aligned}$ |  |
|  | §213-229 | A |  |  | $\begin{gathered} 465,466,467,468,469,470,471, \\ 472,473,474 \end{gathered}$ |  |


|  |  | B |  |  | $475,476,477,478,479,480,481$, |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |$)$


|  |  | B | 620, 621 | $\begin{gathered} 622,623,624,625,626,627,628 \\ 629,630,631,632,633 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | §296-302 | A | $\begin{gathered} \hline 634,635,636,637,638, \\ 639,642,644,645 \end{gathered}$ | 640, 641, 643 |  |
|  |  | B | 646 | $647,648,649,650,651,652,653$ |  |
|  | §303-304 | A | 655, 656 | 658, 659, 660, 661, 662, 663, 664 | 657 |
|  |  | B | 665 | 666 |  |
|  | §305-306 |  | 667, 669, 670 | 668, 671, 672, 673 |  |
|  | §307-317 | A | 674, 675, 677 | $\begin{gathered} 676,678,679,680,681,682,683 \\ 684 \end{gathered}$ |  |
|  |  | B | 692 | $\begin{gathered} 685,686,687,688,689,690,691 \\ 693,694,695,696,697,698 \\ \hline \end{gathered}$ |  |
|  | §318-321 | A |  | 699, 700, 701, 702 |  |
|  |  | B |  | 703, 704, 705, 706, 707, 708 |  |
|  | §322-325 | A | 709, 710, 711, 712 |  |  |
|  |  | B |  | 713, 714, 715, 716, 717 |  |
|  | §326-331 | A | $\begin{gathered} 718,719,720,721,722, \\ 723,724,725,726,727,728, \\ 729,730,731 \end{gathered}$ |  |  |
|  |  | B | 735 | 732, 733, 736, 737, 738 | 734 |

For the teaching of Euclidean Geometry in the $1^{\text {st }}$ grade of Lyceum from 1979 to 1990, textbooks C was used. The tasks were divided into two groups using the symbolism * (i.e. group of tasks * and group of tasks $* *$ ) and introduced to students after the mathematical concepts of particular chapter. As it can be seen, no Q and MM found in textbook C .


Textbooks D was used for Euclidean Geometry's lectures between the years 1986 and 1991 and it was taught in the $2^{\text {nd }}$ grade of Lyceum. Every chapter included at the end of it two groups of tasks (group A and group B). Most of the tasks found in the categories E and P.

| Textbook D |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Chapter | Item | Qs | E | P | MM |
| Analogies \& Similarity | A |  | $\begin{gathered} 1,2,3,5,6,9,16,20,23,24, \\ 25,26 \end{gathered}$ | $\begin{aligned} & 4,7,8,10,11,12,13,14,15,17, \\ & 18,19,21,22,27,28,29,30,31 \end{aligned}$ |  |
|  | B |  | $1,2,3,4,6,7,8,9,13,15$ | $\begin{gathered} 5,10,11,12,14,16,17,18,19 \\ 20,21 \end{gathered}$ |  |
|  | A |  | 1, 2, 6, 7, 8, 11, 14, 16, 17, 18 | $3,4,5,10,12,13,15$ | 9 |
|  | B |  | $3,4,5,6,8,9,10,11$ | 1,2, 7, 12, 13 |  |
| Metric Relations | A |  | $\begin{gathered} 1,2,3,4,5,6,7,8,9,10,13, \\ 14,16,17,18,19,21,22,23,25, \\ 26,31,32,33,34 \end{gathered}$ | $\begin{gathered} 11,12,15,20,24,27,28,29,30, \\ 35,36,37 \end{gathered}$ |  |
|  | B |  | $\begin{gathered} 1,2,3,5,7,13,15,17,22,23, \\ 24,26 \end{gathered}$ | $\begin{gathered} 4,6,8,9,10,11,12,14,16,18, \\ 19,20,21,25 \end{gathered}$ |  |
| Areas | A |  | $\begin{gathered} 1,2,6,7,9,13,14,16,17,18 \\ 19,20,21,22,24,26,27 \\ \hline \end{gathered}$ | $\begin{gathered} 3,4,5,8,10,11,12,15,23,25 \\ 26,28,29 \end{gathered}$ |  |
|  | B |  | 1, 3, 8, 9, 12, 15 | 2, 4, 5, 7, 10, 11, 13, 14 | 6 |
| Circle's measurement | A |  | $\begin{gathered} 3,6,7,8,9,10,11,12,15,18 \\ 19,20,21,24 \end{gathered}$ | $1,2,4,5,13,14,16,22,23$ | 17 |
|  | B |  | 1, 2, 3, 4, 6, 10 | 5, 7, 8, 9, 11, 12 |  |
| Solid Geometry | A | 4 | $2,3,11,15,19,21,23$ | $\begin{gathered} 1,5,6,7,8,9,10,12,13,14,16 \\ 17,18,19,20,22,24,25,26,27 \\ 28,29,30 \end{gathered}$ |  |
|  | B |  | 1, 4, 5 | $2,3,6,7,8,9, \underset{15}{10,11,12,13,14}$ |  |
|  | A |  | $2,3,7,8,14,20,23,24,25$ | $\begin{gathered} 1,4,5,6,9,10,11,12,13,15 \\ 16,17,18,19,21,22,26 \end{gathered}$ |  |
|  | B |  | 2, 3, 6, 12 | $1,4,5,7,8,9,10,11$ |  |

Textbook E was part of the curriculum in the $1^{\text {st }}$ grade of Lyceum the years 1990 to 1999. Every chapter was divided into sections containing questions and revised exercises at the end of it.

| Textbook E |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Chapter | Paragraph | Qs | E | P | MM |
| Introduction to Euclidean Geometry | §1.1-1.4 |  | 1,3 | 2 |  |
|  | §1.5-1.6 |  |  | 1,2,3 |  |
|  | §1.7 |  | 1,2,3,5 | 4 |  |
|  | §1.8-1.12 |  | 2, 3, 4, 5, 6 | 1 |  |
|  | Ерютŋ́б\&ı¢ | $\begin{array}{r} 1,2,3,4,5,6,7,8, \\ 9,11,12,13,15,16, \\ 17,18,19,20,21,22 \end{array}$ | 10, 14 |  |  |
|  | Гєviкと́ऽ $\alpha \sigma \kappa \eta ́ \sigma \varepsilon!\varsigma$ |  | 1,2,3,6, 7, 8 | 4, 5, 9, 11 | 10 |
| Triangles | §2.1-2.5 |  | 1,2,3,4, 7, 8, 11, 13 | 5, 6, 9, 10, 12, 14 |  |
|  | §2.8-2.9 |  | 1,2, 5, 6, 7, 8, 9, 10, 11, 15 | 3,12, 13, 14, 17 |  |
|  | §2.10-2.11 |  | 1,5, 6, 8,9 | 2, 3, 4, 7 |  |
|  | §2.12-2.13 |  | 2, 3, 4, 6, 7, 10 | $\begin{gathered} 1,5,8,9,11,12,13,14, \\ 15,16,17,18,19,20,21 \end{gathered}$ |  |
|  | Ерюти́бєı¢ | $\begin{array}{r} 1,2,3,4,5,6,7,8, \\ 9,10,11,16,17,18 \\ 19,20,21,22,23,24 \\ \hline \end{array}$ |  |  |  |
|  | Гعทıќя $\alpha \sigma \kappa \eta ́ \sigma \varepsilon!\varsigma$ |  | 3, 4, 8, 9, 12, 17 | $\begin{gathered} 1,2,5,6,7,10,13,14 \\ 15,16 \end{gathered}$ |  |
| Parallel lines (Parallel lines' chapter is introduced towards the Triangles' chapter) | §2.6-2.7 |  | 1,3 | 2, 4, 5 |  |
|  | §2.8-2.9 |  |  | 4 | 16 |
|  |  | 12, 13, 14, 15 |  |  |  |
|  | Гєvıкє́я $\alpha \sigma \kappa \eta ́ \sigma \varepsilon!\varsigma$ |  | 11 |  |  |


| Quadrilaterals | §3.1-3.2 |  | $\begin{gathered} 1,2,3,4,7,8,9,10,11,12, \\ 18,19,20,21,22,23 \\ \hline \end{gathered}$ | $5,6,13,14,15,16,17,24$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | §3.3-3.5 |  | 2, 5, 9, 10, 11, 16, 17, 18 | $\begin{gathered} 1,3,4,6,7,8,12,13,14, \\ 15 \end{gathered}$ |  |
|  | §3.6 |  | $1,2,3,5,6,7,8,10,13$ | 4, 9, 11, 12 |  |
|  | Eр¢ти́б\&ı¢ | $\begin{aligned} & 1,2,3,4,5,6,7,8, \\ & 9,10,11,12,13 \end{aligned}$ |  |  |  |
|  | Гєvıкє́s $\alpha \sigma \kappa \eta ́ \sigma \varepsilon \varsigma$ |  | 3, 5, 7, 8, 10, 11 | 1, 2, 4, 6, 9, 12 |  |
| Analogies \& Similarity | §4.1-4.2 |  | 1, 3, 4, 5 | 2, 6, 7, 8, 9 |  |
|  | §4.3 |  | 5, 6, 7, 10 | 2, 3, 4, 8, 9, 11, 12, 13 | 1 |
|  | §4.4 |  | 1,2,3, 4, 5, 7, 8 | 6, 9, 10 |  |
|  | Ерюти́б\&ı¢ | 1,2, 3, 4, 5, 6, 7 |  |  |  |
|  | Гєvıкє́ऽ $\alpha \sigma \kappa \eta َ \sigma \varepsilon เ \varsigma$ |  | 3, 5, 6, 8, 17 | $\begin{aligned} & 1,2,4,7,9,10,11,12, \\ & 13,14,15,16,18,19 \end{aligned}$ |  |
| Circle (part of theTriangles' chapter) | §5.1-5.2 |  | 1,2, 3, 4, 5, 8 | 6,7 |  |
|  | §5.3-5.4 |  | 1, 3, 4, 5, 6, 7 | 2 |  |
|  | §5.5 |  | 1,3,4 | 2, 5, 6, 7, 8 |  |
|  | §5.6 |  | 1,2,3,4, 7 | 5,6,8 |  |
|  | §5.9 |  |  | 1,2, 3, 4, 5, 6, 7, 8 |  |
|  | §5.10 |  | 2, 5 | $1,3,4,6,7,8,9,10$ |  |
|  |  | $\begin{gathered} 1,2,3,4,5,6,7,8 \\ 9,10,11,12,13,14 \\ 15,19 \end{gathered}$ |  |  |  |
|  | Гعvikés $\alpha \sigma \kappa \eta ́ \sigma \varepsilon เ ร$ |  | 4 | 1, 6 |  |
| Inscribed shapes | §5.7-5.8 |  | $1,2,4,5,8,9,14$ | $3,6,7,10,11,12,13,15$ |  |
|  | §5.11-5.12 |  | 1,2, 7, 8 | $3,4,5,6,9,10,11,12$ |  |


|  | Eр@tท́osıs | 16, 18, 20, 21 | 17 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Гєvıкと́ऽ $\alpha \sigma \kappa \eta ́ \sigma \varepsilon \iota \varsigma$ |  | 2, 3, 11 | $5,7,8,9,10,12$ |  |

Textbook F was used for the teaching of Geometry in the $2^{\text {nd }}$ grade of Lyceum the years 1991 to 1999. The tasks were provided to students either at the end of the section or after a group of sections. At the end of every chapter students were engaged to extra tasks with the form of Questions and General exercises.

| TEXTBOOK F |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Chapter | Paragraph | Q | E | P | MM |
| Metric Relations | §1.1 |  | 1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13 | 6,10 |  |
|  | §1.2 |  | 1, 2, 3, 4, 8, 11, 12, 13 | 5, 6, 7, 9, 10 |  |
|  | §1.3 |  | $1,2,3,5,7,9,11,12$ | 4, 6, 8, 10 |  |
|  | §1.4-1.6 |  | 1,2,3, 4, 5 | $\begin{gathered} 6,7,8,9,10,11,12,13, \\ 14,15 \end{gathered}$ |  |
|  | Ep¢tท́б\&ıs | 1,3,6 | 2, 4, 5 |  |  |
|  | Гєviкє́s $\alpha \sigma \kappa \eta ́ \sigma \varepsilon \iota \varsigma$ |  |  | $\begin{gathered} 1,2,3,4,5,6,7,8,9,10 \\ 11,12,13,14,15,16,17, \\ 18,19,20 \end{gathered}$ |  |
| Areas | §2.1-2.3 |  | 1,2, 4, 9 | 3, 5, 6, 7, 8, 9, 10 | 12 |
|  | §2.4-2.6 |  | 1,2, 3, 4, 7 | $\begin{gathered} 5,6,8,9,10,11,12,13, \\ 14,15 \end{gathered}$ |  |
|  | Eр¢тท́бغıs | $\begin{gathered} 1,3,4,5,6,7, \\ 8 \end{gathered}$ | 2 |  |  |
|  | Гєvıкє́s $\alpha \sigma \kappa \eta$ бєıऽ |  | 7 | 1, 2, 3, 4, 5, 6, 8, 9, 10, 11 |  |
| Circle's measurement | §3.1-3.2 |  | 1, 2, 3, 4, 5, 6, 8 | 7, 9, 10, 11, 12 |  |
|  | §3.3 |  | 1,2,3, 7, 8, 9 | 4, 5, 6, 10 |  |
|  | §3.4-3.5 |  | 4, 6 | 5 | 1,2,3 |
|  | §3.6-3.7 |  | 1, 2, 4, 5, 13 | $3,6,7,8,9,10,11,12$ |  |
|  | Epotíocis | 1,2, 8, 10, 12 | $3,4,5,6,7,9,10,11$ |  |  |
|  | ГЕทıкє́s $\alpha \sigma \kappa \eta ́ \sigma \varepsilon \iota$ |  |  | $1,2,3,4,5,6,7,8,9$ |  |


| Solid Geometry | §4.1-4.5 |  | 1,2,3, 4, 5, 7, 8 | 6, 9, 10, 11 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | §4.6 |  |  | 1, 2, 3, 4, 5, 6, 7, 8, 9 |  |
|  | §4.7-4.8 |  | 3, 6 | 1, 2, 4, 5, 7, 8 |  |
|  | §4.9 |  |  | 1,2, 3, 4 |  |
|  | §4.10 |  |  | 1, 2, 3, 4, 5, 6 |  |
|  | §4.11-4.13 |  | 7 | $\begin{gathered} 1,2,3,4,5,6,8,9,10,11, \\ 12,13 \end{gathered}$ |  |
|  | Ерютŋ́бغıs | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8,9,10,11, \\ 12,13,14,15, \\ 16,17,18 \end{gathered}$ |  |  |  |
|  | ГЕvikés абкท́б\&ıs |  | 3, 4 | 1,2, 5, 6, 7 |  |
|  | §5.1-5.2 |  | 1, 2, 5, 8 | 3, 4, 6, 7, 9, 10 |  |
|  | §5.3-5.4 |  | 1, 4, 7 | 2, 3, 5, 6, 8, 9 |  |
|  | §5.5-5.6 |  | 1,3, 5, 7 | $2,4,6,8,10,11,12$ |  |
|  |  | 1,2,3, 4, 5, 6 |  |  |  |
|  | Гєขıќ́s $\alpha \sigma \kappa \eta ́ \sigma \varepsilon!\varsigma$ |  | 5 | 1,2,3, 4, 6, 7, 8, 9, 10, 11 |  |
|  | §6.1 |  | 1, 2, 4 | 3, 5, 6 |  |
|  | §6.2-6.3 |  | 1,3 | 2, 4, 5, 6, 7 |  |
|  | §6.4-6.5 |  | 1, 5, 9 | 2, 3, 4, 6, 7, 8, 10 |  |
|  | §6.6-6.7 |  | 2, 3, 4, 5, 7, 11, 12 | 1, 6, 8, 9, 10, 13 |  |
|  | Ep¢tท́б\&1s | 1,3, 4, 5 | 2 |  |  |
|  | Гєviкと́ऽ $\alpha \sigma \kappa \eta$ бєıऽ |  |  | 1, 2, 3, 4 |  |

Textbooks G contained mathematical concepts for both grades, one and two of Lyceum for the years 1999 and 2001. Tasks were introduced to students either at the end of the section or after a combination of sections. They had the form of "Think and Answer", group A and group B. At the end of the chapter student had the opportunity to provide solutions in General exercises and Questions, in order to revise their mathematical knowledge.

| TEXTBOOK G |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Chapter | Paragraph | Item | Q | E | P | MM |
| Introduction to Euclidean Geometry | §2.1-2.2 | $\Sigma \mathrm{A}$ | 1,2, 5, 6, 7 | 3, 4, |  |  |
|  |  | A |  | 1,2,3, 4, 5, 6 |  |  |
|  |  | B |  | 2, 3, 4, 5 | 1 |  |
|  | §2.3 | EA | 1,2,3 |  |  |  |
|  | §2.4 | $\Sigma \mathrm{A}$ | 1,2 | 4, 5 |  | 3 |
|  |  | A |  | 1,2, 3, 4, 5, 6 |  |  |
|  |  | B |  | 1,2,3,4 |  |  |
|  | §2.5 | EA |  | 1,2, 3, 4 |  |  |
|  |  |  |  | 1, 2, 3, 4 |  |  |
|  |  |  | 1,2, 3, 4, 5 |  |  |  |
| Triangles | §3.1 | $\Sigma \mathrm{A}$ | 1,2, 3, 4, 6, 7 | 5 |  | 8, 9, 10 |
|  |  | A |  | $\begin{aligned} & 1,2,3,4,5,6, \\ & 7,8,9,10,11, \\ & 12,13,14,15 \end{aligned}$ |  |  |
|  |  | B |  | $\begin{gathered} 4,5,6,7,11, \\ 13 \end{gathered}$ | $\begin{gathered} 1,2,3,8,9 \\ 10,12 \end{gathered}$ |  |
|  | §3.2 | $\Sigma \mathrm{A}$ |  | 1,2,3 |  |  |
|  |  | A |  |  | 1,2 |  |
|  | §3.3 | $\Sigma \mathrm{A}$ | 1,2,3 |  |  |  |
|  |  | A |  | 1,3 | 2 |  |
|  | §3.4 | EA | 1,2, 3, 4, 5, 6 |  |  |  |


|  |  | A |  | $\begin{aligned} & 1,2,4,5,7,8, \\ & 9,10,11,12 \end{aligned}$ | 3, 6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | B |  | $\begin{gathered} 6,12,13,14, \\ 15 \end{gathered}$ | $\begin{gathered} 1,2,3,4,5,7, \\ 8,9,10,11,16, \\ 17 \end{gathered}$ |  |
|  | §3.5 | LA | 1,2 | 3 |  |  |
|  |  | A |  | 1,2,3 | 4 |  |
|  |  | B |  |  | 1,2,3 |  |
|  | §3.6 | $\Sigma \mathrm{A}$ | 1,2 |  |  |  |
|  |  | A |  | 1,3 | 2 |  |
|  |  | B |  |  | 1,2 |  |
|  | §3.7 | EA |  | 1, 3, 5, 4 |  | 2 |
|  |  | A |  | 1,3 | 2 |  |
|  |  | B |  |  | 1,2,3 |  |
|  | Г¢v |  |  | 6 | 1,2,3, 4, 5 |  |
|  | Ep¢rท́ | $\eta$ Пท | $\begin{gathered} 1,2,3,4,5,6 \\ 7,8,9,10 \end{gathered}$ |  |  |  |
| Parallel lines | §4.1-4.2 | $\Sigma \mathrm{A}$ | $\begin{gathered} 1,3,4,5,6,7, \\ 12 \end{gathered}$ | 2, 8, 9, 10, 11 |  |  |
|  |  | A |  | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8,9,10,11, \\ 12,13 \\ \hline \end{gathered}$ |  |  |
|  |  | B |  | 2, 3, 5, 6, 7 | $\begin{gathered} 1,4,8,9,10 \\ 11,12,13,14 \\ 15,16 \end{gathered}$ |  |
|  | $\Gamma \varepsilon v$ |  |  |  | $1,2,3,4,5,6,$ |  |
|  | Epotí |  | 1, 2, 3, 4, 5, 6 |  |  |  |
| Quadrilaterals | §5.1 | EA | 1, 3, 4, 6, 7, 8 | 2, 5 |  |  |



|  | §6.2 | EA | 1, 2, 3, 4, 5, 6 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A |  | $1,2,3,4,5,6 \text {, }$ |  |  |
|  |  | B |  | 2, 3, 5, 6, 8 | 1,4,7 |  |
|  | §6.3 | EA | 1,2,3 |  |  |  |
|  |  | A |  |  | $1,2,3,4,5,6,$ |  |
|  |  | B |  |  | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8,9,10 \end{gathered}$ |  |
|  | Гєvıќ¢ Абкท́бөıऽ |  |  | 7 | $\begin{array}{r} 1,2,3,4,5,6, \\ 8,9,10,11,12 \end{array}$ |  |
|  | Ep@tท́ |  | 1, 2, 3, 4, 5, 6 |  |  |  |
| Analogies \& Similarity | §7.1-7.2 | EA | 1,3, 5, 7, 8 | 2, 4, 6 |  |  |
|  |  | A |  | 1, 2, 4, 6, 7, 8 | 3, 5 |  |
|  |  | B |  | 2, 3, 5, 6 | 1, 4, 7, 8 |  |
|  | §7.3 | EA | 1, 2, 3, 4, 5, 6 |  |  |  |
|  |  |  |  |  | $1,2,3,4,5,6 \text {, }$ |  |
|  |  |  | 1,2,3, 4, 5 |  |  |  |
|  | §8.1-8.2 | EA | 1,2, 5, 6, 7, 8 | 10,11 |  | 3, 4, 9 |
|  |  | A |  | $\begin{gathered} 1,2,3,4,7,8, \\ 9,11,12 \end{gathered}$ | 5, 6, 10 |  |
|  |  | B |  | 1,2,3 | $\begin{gathered} 4,5,6,7,8,9, \\ 10 \end{gathered}$ |  |
|  | Гєvıќ¢ Абкท́бөı¢ |  |  | 3, 9 | $\begin{gathered} 1,2,4,5,6,7, \\ 8,10,11 \end{gathered}$ | 8 |
|  |  |  | 1,2,3, 4, 5 |  |  |  |


| Metric Relations | §9.1 | $\Sigma \mathrm{A}$ | $\begin{gathered} 2,3,4,6,10 \\ 11 \end{gathered}$ | $\begin{gathered} 1,5,7,8,9 \\ 12,13,14,15, \\ 16,17,18 \\ \hline \end{gathered}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A |  | $\begin{gathered} 1,2,3,4,5,7, \\ 8,9,10,11,12, \\ 13,14,15,16, \\ 17,18 \\ \hline \end{gathered}$ | 6 |  |
|  |  | B |  | 1,2, 5, 6, 7, 15 | $\begin{gathered} 3,4,8,9,10 \\ 11,12,13,14 \\ 16,17,18 \end{gathered}$ |  |
|  | §9.2 | ᄃA | 1,2, 3, 4 | 5, 6, 7, 8 |  |  |
|  |  | A |  | 1,2, 4, 5, 6 | 3 |  |
|  |  | B |  | 1,2, 3, 4, 5, 6 | 7, 8, 9 |  |
|  |  |  |  |  | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8,9,10 \end{gathered}$ |  |
|  |  |  | 3, 4, 7, 9 | 1,2, 5, 6, 8 |  |  |
| Areas | §10.1-10.2 | $\Sigma \mathrm{A}$ | 1,3, 5, 6, 8, 9 | 2, 4, 7 |  |  |
|  |  | A |  | $\begin{array}{r} 1,4,5,6,7,8 \\ 9,10,12,13,14 \end{array}$ | 2, 3, 11 |  |
|  |  | B |  | 7, 8, 9, 12 | $\begin{gathered} 1,2,3,4,5,6 \\ 10,11,13,14 \\ 15 \end{gathered}$ |  |
|  | §10.3 | IA | 9 | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8 \end{gathered}$ |  |  |
|  |  | A |  | 1,2,3, 4, 5 |  |  |
|  |  | B |  | $\begin{gathered} 1,2,3,4,5,6, \\ 8,9 \end{gathered}$ | 7 |  |
|  | §10.4 | Абкฑ́б\&ı¢ |  | 1,2, 4 | 3 |  |



|  |  |  |  | 1 | $\begin{gathered} 2,3,4,5,6,7 \\ 8,9,10 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Eрळтท́бєı¢ K $\alpha \tau \alpha v o ́ \eta \sigma \eta \varsigma$ |  | $1,2,3,4,5,6,$ | 7, 9 |  |  |
|  | §13.1 | $\Sigma \mathrm{A}$ | $\begin{gathered} 1,2,3,4,5,7, \\ 8,9 \end{gathered}$ |  |  | 6 |
|  |  | A |  | 1,2,3, 4, 5 | 6 |  |
|  |  | B |  | 2,3 | 1, 4, 5, 6 |  |
|  | §13.2 | $\Sigma \mathrm{A}$ | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8 \end{gathered}$ |  |  |  |
|  |  | A |  | 1,2 | 3, 4, 5 |  |
|  |  | B |  | 1,2 | 3, 4, 5, 6 |  |
|  | §13.3-13.5 | EA | $\begin{gathered} 1,2,5,6,7, \\ 10,11,13,14, \\ 15,16,17,18, \\ 19,20,21,22, \\ 23 \end{gathered}$ | 3, 4, 8, 9, 12 |  |  |
|  |  | A |  | $\begin{gathered} 1,2,4,5,6,7, \\ 9,10,11 \end{gathered}$ | 8 | 3 |
|  |  | B |  | 1,2, 5, 8, 13 | $\begin{array}{r} 3,4,6,7,9, \\ 10,11,12,14 \end{array}$ |  |
|  | Гєvıкદ́¢ Абкท́бєıऽ |  |  | 10 | $\begin{gathered} 1,2,3,4,5,6 \\ 7,8,9,11 \end{gathered}$ |  |
|  |  |  | 1,2, 3, 4, 5, 6 |  |  |  |

Textbook H was used for the teaching of Euclidean Geometry for the $1^{\text {st }}$ and $2^{\text {nd }}$ grade students of Lyceum. From 2014 and on, two separate textbooks provided to students for the $1^{\text {st }}$ and $2^{\text {nd }}$ grade respectively. Tasks have the form of Questions, Knowing, Proving and Complex themes and students were able to engage with them at the end of a section or a combination of sections. Finally, General Exercise were part of every chapter, requiring students to solve them with a view to enhance their Geometry skills.

| Textbook H |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Chapter | Paragraph | Item | Q | E | P | MM |
| Introduction toEuclidean Geometry | §2.1-2.10 | E.K. | 1,2, 3, 4, 5 |  |  |  |
|  |  | A.E. | , | 2, 3, 4 |  |  |
|  |  | A.A. |  | 1,2 | 3 |  |
|  |  | $\Sigma . \Theta$. |  |  | 1 | 2 |
|  | §2.11-2.16 | E.K. | 1,2,3,4, 5 |  |  |  |
|  |  | A.E. |  | 1,2 |  | 3 |
|  |  | A.A. |  | 1,2,3 |  |  |
|  |  | $\Sigma . \Theta$ |  |  | 1,2 |  |
|  | §2.17-2.18 | E.K. | 1,2,3,4,5,6,7,8 |  |  |  |
|  |  | A.E. |  | 1,2 |  |  |
|  |  | A.A. |  |  | 1,2 |  |
|  | §2.19 | E.K. | 1,2,3, 4, 5 |  |  |  |
|  |  | A.E. |  | 1,2, 3, 4 |  |  |
|  |  | A.A. |  | 1,2,3 |  |  |
|  | Гєvıкદ́¢ Абкฑ́бєıऽ |  |  | 1,2,3,5 | 4 |  |
| Triangles | §3.1-3.2 | A.E. |  | 1,2,3, 4 |  |  |
|  |  | A.A. |  | 2,3 | 1 |  |
|  | §3.3-3.4 | E.K. | 1,2,3 |  |  |  |
|  |  | A.E. |  | 1,2 | 3 |  |
|  |  | A.A. |  | 1 | 2,3 |  |
|  |  | $\Sigma . \Theta$ |  | 1,3 | 2 |  |
|  | §3.5-3.6 | E.K. | 1,2,3,4,5,6,7,8 |  |  |  |


|  |  | A.E. |  | 1, 2, 4 | 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A.A. |  |  | 1,2, 3, 4, 5 |  |
|  |  | $\Sigma . \Theta$ |  |  | 1,2 |  |
|  | §3.7 | E.K. | 1 |  |  |  |
|  |  | A.E. |  |  | 1,2 |  |
|  | §3.8-3.9 | A.E. | 1 | 2, 3, 4, 5, 6 |  |  |
|  | §3.10-3.12 | E.K. | 1,2 | 3 |  |  |
|  |  | A.E. |  | 1,2, 5, 7, 8, 9 | 3, 4, 6 | 10 |
|  |  | A.A. |  |  | $1,2,3,4,5,6 \text {, }$ |  |
|  |  | $\Sigma . \Theta$ |  |  | 1,2,3, 4 |  |
|  | §3.13 | E.K. | 1,2 |  |  |  |
|  |  | A.E. |  | 1,2 | 3 |  |
|  | §3.14-3.15 | E.K. | 1,2,3 |  |  |  |
|  |  | A.E. |  | 1,2 | 3 |  |
|  |  | A.A. |  | 1 | 2,3 |  |
|  | §3.16 | E.K. | 1,2 |  |  |  |
|  |  | A.E. | 1,2,3 |  |  |  |
|  |  | A.A. |  |  | 1, 2, 3, 4 |  |
|  | §3.17-3.18 | E.K. | 1,2, 3, 4 |  |  |  |
|  |  | A.E. |  | 1,2 | 3, 4, 5 |  |
|  | Гعvik |  |  |  | 1,2,3,4,5,6,7,8 |  |
| Parallel Lines | §4.1-4.5 | E.K. | 1, 2, 3, 4, 5 |  |  |  |
|  |  | A.E. |  | 1,2,3, 4, 5, 6 |  |  |
|  |  | A.A. |  |  | 1,2, 3, 4, 5 |  |
|  |  | Г. $\Theta$ |  |  | 1,2,3, 4 |  |
|  | §4.6-4.8 | E.K. | 4, 5 | 1,2,3 |  |  |
|  |  | A.E. |  | 2, 3, 4, 5, 6, 7 | 1 |  |
|  |  | A.A. |  | 1,2 | 3, 4, 5, 6, 7 |  |


|  |  | $\Sigma . \Theta$ |  |  | 1,2, 3, 4, 5, 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Гєvıкє́s Абкŋ́бєıऽ |  |  |  | $1,2,3,4,5,6,$ |  |
| Quadrilaterals | §5.1-5.2 | E.K. | 1,2, 5 | 3, 4 |  |  |
|  |  | A.E. |  | 1,2, 3, 4 |  |  |
|  |  | A.A. |  |  | 1,2,3, 4, 5 |  |
|  |  | $\Sigma . \Theta$ |  |  | 1,2,3,4 | 5 |
|  | §5.3-5.5 | E.K. | 1,2,3, 4, 5 |  |  |  |
|  |  | A.E. |  | 1,2,3, 5, 6 | 4 |  |
|  |  | A.A. |  | 1,2,3 | 4 |  |
|  |  | $\Sigma . \Theta$ |  |  | 1,2,3 |  |
|  | §5.6-5.9 | E.K. | 3, 4, 5 | 1,2 |  |  |
|  |  | A.E. |  | $1,2,3,4,5,6 \text {, }$ |  |  |
|  |  | A.A. |  | 1, 7, 8 | 2, 3, 4, 5, 6, 9 | 10 |
|  |  | $\Sigma . \Theta$ |  |  | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8 \end{gathered}$ |  |
|  | §5.10-5.11 | E.K. | 1,2, 3, 4 |  |  |  |
|  |  | A.E. |  | 1,2, 3, 4, 5 | 6 |  |
|  |  | A.A. |  | $1,2,4,5,6,8,$ | 3, 7, 10 |  |
|  |  | $\Sigma . \Theta$ |  |  | 1,2, 3, 4, 5 |  |
|  |  |  |  |  | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8,9,10 \\ \hline \end{gathered}$ |  |
| Inscribed shapes | §6.1-6.4 | E.K. | 1,2, 3, 4 |  |  |  |
|  |  | A.E. |  | 1,2,3, 4, 5, 6 |  | 7 |
|  |  | A.A. |  |  | 1,2,3 | 4 |
|  |  | $\Sigma . \Theta$ |  |  | 1,2,3 |  |
|  | §6.5-6.6 | E.K. | 1, 2, 3, 4, 5, 6 |  |  |  |



| Metrics relation | §9.1-9.2 | E.K. | 2 | 1, 3, 4 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A.E. |  | 1,2 | 3 |  |
|  |  | A.A. |  | 1,5 | 2, 3, 4 |  |
|  |  | Г. $\Theta$. |  | 4 | 1,2,3, 5, 6 |  |
|  | §9.4 | E.K. | 1,2, 3, 4 |  |  |  |
|  |  | A.E |  | 1,2, 3, 4 |  |  |
|  |  | A.A. |  | 3, 4 | 1, 2, 5, 6 |  |
|  |  | $\Sigma . \Theta$. |  |  | 1,2,3 |  |
|  | §9.5-9,6 | E.K. | 1,2,3 |  |  |  |
|  |  | A.E. |  | 1,2, 3, 4 |  |  |
|  |  | A.A. |  | 6 | 1,2, 3, 4, 5 |  |
|  |  | $\Sigma . \Theta$ |  | 2,3 | 4,5 | 1 |
|  | §9.7 | E.K. | 1,2,3 |  |  |  |
|  |  | A.E. |  | 1,2, 3, 4 |  |  |
|  |  | A.A. |  | 1,3 | 2, 4, 5 |  |
|  |  | $\Sigma . \Theta$. |  | 4 | 1,2,3 |  |
|  | Гєvıкє́¢ Абкฑ́бєıऽ |  |  |  | 1,2, 3, 4, 5, 6, |  |
|  |  |  | 1,2, 4 |  | 7,8 | 6 |
| Areas | §10.1-10.3 |  | 1,2, 4 |  | 13 | 6 |
|  |  | A.A. |  | 2 | 1,3,4,5,6,7 | 8 |
|  |  | $\Sigma . \Theta$. |  |  | 1,2,3,4,5 |  |
|  | §10.4 | E.K. | 3 | 1,2 |  |  |
|  |  | A.E. |  | 1,3,4 | 2 |  |
|  |  | A.A. |  | 1,3,5 | 2, 4 |  |
|  |  | $\Sigma . \Theta$. |  |  | 1,2,3 |  |
|  | §10.5 | E.K |  | 1,2,3 |  |  |
|  |  | A.E. |  | 1,3, 4, 5 | 2 |  |
|  |  | A.A. |  | 6 | 1, 2, 3, 4, 5 |  |


|  |  | इ. $\Theta$. |  |  | 1, 2, 3, 4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | §10.6 | E.K. | 1, 2, 3, 4 |  |  |  |
|  |  | A.E. |  | 1,2 | 3,4 |  |
|  |  |  |  | 1,2, 3, 4, 6, 8 | 5, 7, 9, 10, 11 |  |
| Circle's measurement | §11.1-11.2 | E.K. | 1,2, 3, 4, 5, |  |  |  |
|  |  | A.E. |  | 1,2, 3, 4, 5, 6 | 7 |  |
|  |  | A.A. |  | 5,6 | 2, 3, 4 | 1 |
|  |  | $\Sigma . \Theta$. |  |  | 2, 3 | 1 |
|  | §11.3 | E.K. | 1 | 2, 3, 4 |  |  |
|  |  | A.E. |  | 1,2,3 | 4 |  |
|  |  | A.A. |  | 1 | 2, 3, 4 |  |
|  |  | $\Sigma . \Theta$. |  | 2 | 1,3 |  |
|  | §11.4 | E.K. | 1,2 |  |  |  |
|  |  | A.E. |  | 1,2, 3, 5 |  | 4 |
|  |  | A.A. |  | 3 | 1,2 |  |
|  |  | $\Sigma . \Theta$. |  | 3 | 1,2 |  |
|  | §11.6-11.8 | E.K. | 1,2,3 |  |  |  |
|  |  | A.E. |  | 1,2,3, 4, 5 |  |  |
|  |  | A.A. |  |  | 1,2,3, 4, 5 |  |
|  |  | $\Sigma . \Theta$. |  | 1 | 2, 3, 4 |  |
|  | Гєvıкє́s Абкŋ́бєıऽ |  |  | 6 | $\begin{gathered} 1,2,3,4,7,8, \\ 9,10 \end{gathered}$ | 5 |
| Solid Geometry | §12.1-12.2 | E.K. | 1,2,3, 4, 5 |  |  |  |
|  | §12.3 | E.K. |  |  |  | 1,2,3 |
|  |  | A.E. |  | 1, 2, 3, 4 |  |  |
|  |  | A.A. |  | 3 | 1,2, 4, 5 |  |
|  | §12.4 | A.E. |  | 1,2, 8 | $\begin{gathered} 3,4,5,6,7,9 \\ 10 \end{gathered}$ |  |
|  |  | A.A. |  |  | 1,2,3,4 |  |


|  |  | $\Sigma . \Theta$. |  |  | 1,2, 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | §12.5 | E.K. |  |  |  | 1,2,3 |
|  |  | A.E. |  | 2 | 1,3,4 |  |
|  |  | A.A. |  | 1, 2, 3 |  |  |
|  | §12.6 | A.E. |  | 10 | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8,9 \end{gathered}$ |  |
|  |  | A.A. |  |  | 1,2,3 |  |
|  |  | $\Sigma . \Theta$. |  | 6 | $\begin{gathered} 1,2,3,4,5,7, \\ 8 \end{gathered}$ |  |
|  | §12.7 | E.K. | 1 | 2 |  |  |
|  |  | A.E. |  | 1, 2, 5 | 3, 4 |  |
|  | §12.8 | E.K. | 1,2,3,4 |  |  |  |
|  |  | A.E. |  | 2, 3, 8 | $\begin{gathered} 1,4,5,6,7,9, \\ 10 \end{gathered}$ |  |
|  |  | A.A. |  | 2, 3 | 1, 4, 5, 6 |  |
|  | Г¢vıкє́¢ |  |  |  | $\begin{aligned} & 1,2,3,4,5,6, \\ & 7.8 \end{aligned}$ |  |
|  | §13.1-13.4 | A.E. |  | $\begin{gathered} 1,2,3,4,5,8, \\ 9,10 \end{gathered}$ | 6 | 7 |
|  |  | A.A. |  | 2, 5 | 1,3, 4, 6, 7 |  |
|  |  | $\Sigma . \Theta$. |  | 1,4 | 2,3 |  |
|  | §13.5-13.9 | A.E. |  | $\begin{gathered} 1,3,4,6,7,8, \\ 9 \end{gathered}$ | 2 | 5 |
|  |  | A.A. |  |  | $\underset{7}{1,2,3,4,5,6,}$ |  |
|  |  | $\Sigma . \Theta$. |  |  | 1,2,3 |  |
|  | §13.10-13.12 | A.E. |  | 1,2,3,5 |  | 4 |
|  |  | A.A. |  | 1 | 2,3 |  |


|  | §13.13-13.15 | A.E. | $1,2,4,5,6,7,$ |  | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A.A. | 4, 5, 6 | 1,2,3 |  |
|  | §13.16-13.18 | A.E. | $\begin{gathered} 1,2,3,5,6,7, \\ 8,9 \end{gathered}$ |  | 4, 10 |
|  |  | A.A. | 1,3 | 2 |  |
|  |  | $\Sigma . \Theta$. | 1,4 | 2, 3, 5 |  |
|  | Гєvıкદ́¢ Абкך์б\&ıऽ |  |  | $\begin{gathered} 1,2,3,4,5,6, \\ 7,8 \end{gathered}$ |  |


[^0]:    1 Information for older textbooks were collected from the blog: https://parmenides52.blogspot.com/p/school.html

[^1]:    ${ }^{2} 1^{\text {st }}$ and $2^{\text {nd }}$ grade of general Lyceum are corresponded to the $4^{\text {th }}$ and $5^{\text {th }}$ grade of secondary high-school or more specifically to the $10^{\text {th }}$ and $11^{\text {th }}$ grade respectively.

[^2]:    ${ }^{3}$ Textbook H is divided into two schoolbooks (1 $1^{\text {st }}$ and $2^{\text {nd }}$ grade), from 2014 and on.

[^3]:    ${ }^{4}$ Z-test are used when the access in all the population examined is not achievable (i.e. research about all students of the country). Although z-test for the comparison here are significant, I did have access to all books (thus, I had access to the population and not a sample of it). Nevertheless, the use of z-test supports further that the differences between Books $1,2,3$ and Books 4, 5, 6 are statistically significant. For more details about the nature and the use of $z$-test see Appendix 2.

[^4]:    $5 *$ : This symbolism defines the extra chapters which are used in the charts.

[^5]:    $6 *$ : This symbolism defines the extra chapters which are used in the charts.

[^6]:    ${ }^{7}$ The images of those pages are part of the introduction.

[^7]:    ${ }^{8}$ Analogies and similarity are two separate chapters and so the scheme's numbering starts from figure 1. In a similar way acts "Solid Geometry" which is a combination of two chapters. The first-one starts after page 101, while the second one after page 127.
    ${ }^{9}$ Pages 2-6 are missing. Hence, this image belongs to the chapter of similarity and in this point of view I assume that there is also a visual representation for analogies, though it is not appropriate to include it.
    ${ }^{10}$ Figure 25 does not exist.

[^8]:    ${ }^{11}$ Circle's chapter is part of triangles' chapter in charts.
    ${ }^{12}$ Figure 23 does not exist in this chapter.

[^9]:    ${ }^{3}$ Figures 5 and 6 do not exist in this chapter.
    ${ }_{14}^{14}$ Figure 12 does not exist in this chapter.
    ${ }^{15}$ Figure 11 does not exist in this chapter.

[^10]:    ${ }^{16}$ Unnumbered images

[^11]:    17 *: This symbolism defines the extra chapters which are used in the charts.

