# Network-Level Cooperation: Throughput, Stability, and Energy Issues

Nikolaos Pappas

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University of Crete

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Doctoral Thesis Committee:

Apostolos Traganitis, Professor, University of Crete, (Advisor)
Panagiotis Tsakalides, Professor, University of Crete
Vasilios Siris, Assistant Professor, Athens University of Economics and Business
Anthony Ephremides, Professor, University of Maryland
Marios Kountouris, Assistant Professor, Supelec
Stavros Toumpis, Assistant Professor, Athens University of Economics and Business
Athanasios Mouchtaris, Assistant Professor, University of Crete

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UNIVERSITY OF CRETE
DEPARTMENT OF COMPUTER SCIENCE

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Dissertation submitted by

**Nikolaos Pappas**

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Author:

Nikolaos Pappas

Examination Committee:

Apostolos Traganitis, Professor, University of Crete

Panagiotis Tsakalides, Professor, University of Crete

Vasilios Siris, Assistant Professor, Athens University of Economics and Business

Anthony Ephremides, Professor, University of Maryland

Marios Kountouris, Assistant Professor, Supelec

Stavros Toumpis, Assistant Professor, Athens University of Economics and Business

Athanasios Mouchtaris, Assistant Professor, University of Crete

Departmental approval:

Panos Trahanias, Professor Head of the Department

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Η παρούσα έρευνα έχει συγχρηματοδοτηθεί από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο - ΕΚΤ) και από εθνικούς πόρους μέσω του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» του Εθνικού Στρατηγικού Πλαισίου Αναφοράς (ΕΣΠΑ) – Ερευνητικό Χρηματοδοτούμενο Έργο: Ηράκλειτος ΙΙ. Επένδυση στην κοινωνία της γνώσης μέσω του Ευρωπαϊκού Κοινωνικού Ταμείου.

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# Περίληψη


Η συνεργασία στις ασύρματες επικοινωνίες, συμβάλλει στο να αντιμετωπιστούν τα προβλήματα της εξασθένισης και των διαλείψεων στο ασύρματο κανάλι. Ο κύριος στόχος είναι να αυξηθεί ο ρυθμός διαμεταγωγής σε ένα δίκτυο καθώς και η αξιοπιστία των χρονικά μεταβαλλόμενων ζεύξεων. Είναι γνωστό ότι οι ασύρματες επικοινωνίες μπορούν να επωφεληθούν από την συνεργασία ανάμεσα στους κόμβους λόγω της φύσης της ασύρματης μετάδοσης. Προς το παρόν η πλειοψηφία των τεχνικών συνεργασίας περιορίζεται στο φυσικό επίπεδο. Η συνεργασία σε επίπεδο δικτύου ορίζεται ως η απλή αναμετάδοση χωρίς καμιά ανάμιξη του φυσικού επιπέδου. Υπάρχουν ενδείξεις ότι η συνεργασία σε επίπεδο δικτύου μπορεί να έχει παρόμοια οφέλη με την συνεργασία στο φυσικό επίπεδο, ταυτόχρονα όμως είναι πιο απλή στην υλοποίηση.

Στο πρώτο μέρος της διατριβής, μελετάμε την λειτουργία ενός κόμβου που αναμεταδίδει πακέτα από πολλούς χρήστες σε έναν προορισμό. Θεωρήσαμε την περίπτωση όπου ο αναμεταδότης δεν έχει δικά του πακέτα και απλά αναμεταδίδει πακέτα από τους χρήστες Οι χρήστες έχουν κορεσμένες ουρές, η πρόσβαση στο ασύρματο κανάλι είναι τυχαία ενώ επιτρέπεται η ταυτόχρονη μετάδοση πολλαπλών πακέτων. Μελετήσαμε αναλυτικά την ευστάθεια στην ουρά του αναμεταδότη (δηλαδή τις συνθήκες για τις οποίες είναι φραγμένη). Μελετήσαμε αναλυτικά το ρυθμό διαμεταγωγής ανά χρήστη καθώς και τον συνολικό ρυθμό και βρήκαμε τις συνθήκες κάτω από τις οποίες ο αναμεταδότης προσφέρει όφελος στο δίκτυο. Ένα επίσης χρήσιμο αποτέλεσμα που προέκυψε από αυτήν την μελέτη είναι ο αριθμός των χρηστών που μεγιστοποιεί τον συνολικό ρυθμό διαμεταγωγής του δικτύου με τον αναμεταδότη. Το προηγούμενο είναι ένα θεωρητικό αποτέλεσμα αλλά με πολλές πρακτικές εφαρμογές, ειδικά για του παρόχους ασύρματης πρόσβασης. Μελετήσαμε τις περιπτώσεις όπου ο αναμεταδότης μπορεί να μεταδίδει και να λαμβάνει ταυτόχρονα αλλά και την περίπτωση που δεν έχει αυτή την δυνατότητα. Στην περίπτωση όπου ο αναμεταδότης μπορεί να λαμβάνει και να στέλνει πακέτα ταυτόχρονα προκύπτει το πρόβλημα της αυτό-παρεμβολής (self-interference). Λαμβάνοντας υπόψη την αυτό-παρεμβολή μελετήσαμε αναλυτικά την ευστάθεια στην ουρά του αναμεταδότη, το ρυθμό διαμεταγωγής ανά χρήστη καθώς και τον συνολικό

ρυθμό. Ορίσαμε την έννοια της μερικής συνεργασίας (με πιθανοκρατικούς όρους) σε επίπεδο δικτύου, δηλαδή όταν ένας κόμβος δεν συνεργάζεται πλήρως αλλά μερικώς με τους άλλους κόμβους του δικτύου και, αποδείξαμε ότι υπό συνθήκες η μερική συνεργασία είναι η βέλτιστη επιλογή όσον αφορά την περιοχή ευστάθειας και κατά συνέπεια την μέγιστη διαμεταγωγή. Η τοπολογία που μελετάμε αποτελείται από έναν χρήστη, έναν αναμεταδότη ο οποίος έχει την δική του κίνηση και έναν κοινό δέκτη. Ο αναμεταδότης εξαρτώμενος από διάφορες συνθήκες του δικτύου αποφασίζει αν θα συνεργαστεί ή όχι την τρέχουσα χρονική στιγμή ώστε να βοηθήσει τον χρήστη να μεταδώσει τα πακέτα που δεν φτάνουν απευθείας στον δέκτη. Οι συνθήκες που έχουν σημασία είναι η ένταση της κίνησης στους χρήστες καθώς και η ποιότητα των καναλιών μεταξύ των χρηστών και του δέκτη. Αποδείξαμε ότι ανάλογα με τις συνθήκες, η βέλτιστη στρατηγική συνεργασίας για τον αναμεταδότη είναι η μερική συνεργασία.

Το δεύτερο μέρος της διατριβής εστιάζει στις ασύρματες επικοινωνίες με ανανεώσιμες πηγές ενέργειας (πράσινες επικοινωνίες). Πιο συγκεκριμένα μελετήσαμε την επίδραση της συνεργασίας σε επίπεδο δικτύου στην περιοχή ευστάθειας σε δίκτυα με ανανεώσιμες πηγές ενέργειας. Επίσης μελετήσαμε την επίδραση των ανανεώσεων πηγών ενέργειας σε γνωστικά δίκτυα επικοινωνιών (cognitive networks) (όπου οι κόμβοι έχουν διαφορετικούς ενεργειακούς περιορισμούς και μοιράζονται ένα ασύρματο κανάλι). Μελετήσαμε ένα δίκτυο αποτελούμενο από δύο ζεύγη πηγής-προορισμού, όπου στο πρωτεύον (υψηλής προτεραιότητας) ζευγάρι ο πομπός έχει ενεργειακούς περιορισμούς ενώ στο δευτερεύον όχι. Ο πομπός με την χαμηλή προτεραιότητα μεταδίδει όταν ο άλλος παραμένει ανενεργός, ενώ όταν είναι ενεργός μεταδίδει με μια δοσμένη πιθανότητα. Επιλέγουμε αυτήν την πιθανότητα ώστε να μεγιστοποιηθεί ο ρυθμός διαμεταγωγής του δευτερεύοντος ζευγαριού και ταυτόχρονα να παραμένει ευσταθής ο κύριος μεταδότης. Επίσης καθορίζουμε πλήρως την περιοχή ευστάθειας του δικτύου.

Στο τελευταίο μέρος της διατριβής, προτείναμε μία τεχνική δρομολόγησης που συνδυάζει την Κωδικοποίηση Δικτύου (ΚΔ) με πλεονασμό σε πολλαπλά μονοπάτια, και ερευνήσαμε την απόδοση και την αξιοπιστία που μπορεί να επιτευχθεί με αυτήν την τεχνική. Πιο συγκεκριμένα μελετήσαμε το ισοζύγιο ανάμεσα στην καθυστέρηση λήψης πακέτου και την ταχύτητα διαμεταγωγής και έγινε σύγκριση με άλλες κλασσικές μεθόδους δρομολόγησης όπως: βέλτιστου μονοπατιού, πολλαπλών μονοπατιών, πολλαπλών μονοπατιών αλλά με την ίδια πληροφορία σε όλα (μέγιστη δυνατή πλεονάζουσα πληροφορία). Η μελέτη έγινε σε ασύρματα δίκτυα με μονοπάτια που δεν έχουν ούτε κοινές ζεύξεις αλλά ούτε και κοινούς κόμβους. Επιπλέον η αναμετάδοση των εσφαλμένων πακέτων γίνεται από την αρχή του κάθε μονοπατιού. Επίσης, μελετήσαμε την ΚΔ με πλεονασμό σε πολλαπλά μονοπάτια και σε πιο ρεαλιστικές τοπολογίες για ασύρματα δίκτυα. Πιο συγκεκριμένα στις τοπολογίες αυτές τα πολλαπλά μονοπάτια μπορούν να έχουν κοινούς κόμβους, και επιπλέον όταν συμβαίνει σφάλμα στη μετάδοση ενός πακέτου σε ένα μονοπάτι τότε η αναμετάδοση λαμβάνει χώρα από τον προηγούμενο κόμβο και όχι από τον αρχικό κόμβο του μονοπατιού.

# Abstract

Cooperative communications help overcome fading and attenuation in wireless networks. Its main target is to increase the communication rates across the network and the reliability of time-varying links. It is known that wireless communications can benefit from the cooperation of nodes that overhear the transmissions. Most cooperative techniques studied so far have been on physical layer cooperation. The Network-Level cooperation is plain relaying without any physical layer considerations. There is evidence that network-level cooperation can achieve similar gains with physical-layer cooperation, and at the same time is simpler to implement.

In the first part of the thesis, we study the impact of a relay node to a network with a finite number of users-sources and one destination node. We assume that the users have saturated queues and the relay node does not have packets of its own; we have random access of the medium and the time is slotted. The relay node stores a source packet that it receives successfully in its queue when the transmission to the destination node has failed. The relay and the destination nodes have multi-packet reception capabilities. We obtain analytical equations for the characteristics of the relays queue such as average queue length, stability conditions etc. We also study the throughput per user and the aggregate throughput for the network. We study both the cases of a half and a full-duplex relay. For the full-duplex relay, we also study the impact of self interference on the stability, the throughput per user-source as well as the aggregate throughput. Furthermore, we evaluate the benefits of using one user of a two-user random access system to relay traffic of the other user. We introduce the notion of Network-Level Partial Relay Cooperation, and we prove that under certain conditions the optimum cooperation

strategy for the relay is to partially cooperate.

The second part of the thesis is devoted to energy harvesting wireless networks. We study the impact of energy constraints on a network with a source-user, a relay and a destination. This part studies the impact of energy harvesting on network-level cooperation. Specifically, we provide an exact characterization of the stability region. We also consider the concept of cognitive radio communication (with nodes with different energy constraints) in sharing a common wireless channel. Specifically, we give high-priority to the energy-constrained source-destination pair, i.e., primary pair, and low-priority to the pair which is free from such constraint, i.e., secondary pair. In contrast to the traditional notion of cognitive radio, in which the secondary transmitter is required to relinquish the channel as soon as the primary is detected, the secondary transmitter not only utilizes the idle slots of the primary pair but also transmits along with the primary transmitter with a given probability. We choose that probability to maximize the secondary pairs throughput. We obtain the two-dimensional maximum stable throughput region. The region is obtained for both cases in which the capacity of the battery at the primary node is limited or unlimited.

Finally, we investigate the performance that can be achieved by exploiting path diversity through multipath forwarding together with redundancy through linear network coding, in wireless mesh networks with directional links. We capture the tradeoff between packet delay and throughput achieved by combining multipath forwarding and network coding, and compare this tradeoff with that of simple multipath routing where different flows follow different paths, the transmission of multiple copies of packets over multiple paths, and single path routing.

***Στους γονείς μου***

***Αμαλία, Δημήτρη***

***Στη Μαρία***

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# Chapter 1

# Introduction

Traditional analysis of wireless networks still considers the network as a collection of point-to-point links. However this approach in most cases is not appropriate. In a wireless multiple access network, the users can share the common medium, overhearing each other's transmissions and "pushed" to consider cooperating in order to deliver their messages more efficiently. As a result the study of cooperation techniques such as the classical relay channel or the multi-access wireless channel is of great importance.

## 1.1 Cooperative Communications

Cooperative communications help overcome fading and attenuation in wireless networks. Its main target is to increase the communication rates across the network and to increase the reliability of time-varying links. It is known that wireless communications from a source to a destination can benefit from the cooperation of nodes that overhear the transmission. The classical relay channel was originally introduced by van der Meulen [1] and exemplifies this situation. Earlier works on the relay channel were based on information theoretical formulations as in [2], [3] and [4].

Most cooperative techniques studied so far have been on physical layer cooperation, including decode-and-forward (DF), amplify-and-forward (AF) and compress-and-forward (CF). The physical layer cooperation enables non-trivial benefits. However, there is evidence that similar gains can be achieved with "network-layer" cooperation (or packet-level cooperation), that is plain relaying without any physical layer considerations [4] and [5].

Our work in this dissertation focuses at the network-level cooperation, taking into account the physical layer properties and realizations as well as the MAC layer.

### 1.1.1 Network-Level Cooperation

The work in [6] investigated the network-level cooperation in a network consisting of a source and a relay by considering the cases of either full or no cooperation at the relay.

In this part of our work for the network-level cooperation we consider a network with $N$ users-sources, one pure relay node and a single destination. We assume random access to the channel, time is considered slotted, and each packet transmission takes one time slot. The wireless channel between the nodes in the network is modeled by a Rayleigh narrowband flat-fading channel with additive Gaussian noise. The relay and the destination are equipped with multiuser detectors, so that they may decode packets successfully from more than one transmitter at a time (MPR capability). A user's transmission is successful if the received signal to interference plus noise ratio ($SINR$) is above a threshold $\gamma$. We also assume that acknowledgements (ACKs) are instantaneous and error free. The relay does not have packets of its own and the sources are considered saturated with unlimited amount of traffic. The sources transmit packets to the destination with the cooperation of the relay. The relay node stores a source packet that it receives successfully in its queue when the direct transmission to the destination node has failed signified by the absence of ACK from the destination. The queue in the relay has infinite capacity. Our study includes both cases for the relay, half and full duplex (receives and transmits simultaneously).

In wireless networks when a node transmits and receives simultaneously the problem of self interference arises. Information theoretic aspects of this problem can be found at the work of Shannon on [7], although the capacity region of the two-way channel is not known for the general case [8]. There are some techniques that allow the possibility of perfect self interference cancelation [8]. In practice though, there are technological limitations [9]- [10] which can limit the accuracy of the self interference cancelation. Various methods for performing self interference cancelation at the nodes' receivers can be found in [11] and [12]. The conclusion is that there is a trade off between transceiver complexity and the accuracy of the self interference cancelation. However in this dissertation we do not consider any specific self interference cancelation mechanism, because it is out of the scope of this work. The self interference cancelation at the relay is modeled as a variable power gain.

### 1.1.2 Network-Level Partial Relay Cooperation

In [6], it was shown that the stability region of full cooperation under random-access does not always strictly contain the non-cooperative stability region. A major contribution in this dissertation, is to introduce the notion of partial network-level cooperation (or probabilistic cooperation). By probabilistic cooperation we mean that under certain conditions in the network, the cooperating node may accept a packet from the source with a certain probability. We consider the collision channel with erasures and random access of the medium. The network consists of a source and a relay node. The source and the relay node have external arrivals; furthermore, the relay is forwarding part of the source node's traffic to the destination. Unlike the work in [6], the relay node is equipped with a flow controller that regulates the internal arrivals from the source based on the conditions in the network to ensure the stability of the queues. The flow controller regulates the rate of endogenous arrivals by randomly accepting the incoming packets with a probability; that is, it controls the *amount of cooperation* that it is willing to provide. We prove that the system is always better than or at least equal to the system without the flow controller.

A key difference between physical-layer and network-layer cooperation ideas is that the objective rate function that is maximized is the so-called stable throughput region which captures the bursty nature of traffic from the source. Another major difference is that network-level cooperation is simple to implement.

## 1.2 Energy harvesting wireless networks

Exploiting renewable energy resources from the environment (there are various forms of energy that can be harvested including thermal, vibration, solar, acoustic and wind), often termed as the energy harvesting, offers unattended operability of infrastructureless wireless networks.

In [13], the capacity of the additive white Gaussian noise channel with stochastic energy harvesting at the source was shown to be equal to the capacity with an average power constraint given by the energy harvesting rate. However, like most of information-theoretic research, the result is obtained for point-to-point communication with an always backlogged source. In [14], the slotted ALOHA protocol was considered for a network of nodes having energy harvesting capability and the maximum stable throughput region was obtained for bursty traffic. An exact characterization of the region was given in the paper for a two-node case over a collision channel.

### 1.2.1 The effect of energy harvesting in network-level cooperation

In this dissertation we study among others the impact of energy constraints in a network with a source-user, a relay and a destination. The source and the relay node have external arrivals; furthermore, the relay is forwarding part of the source node's traffic to the destination. The source and the relay nodes have energy harvesting capability and a battery to store the harvested energy.

### 1.2.2 Cognitive Channel

Cognitive radio communication provides an efficient means of sharing radio spectrum between users having different priorities [15]. The high-priority user, called primary, is allowed to access the channel whenever it needs, while the low-priority user, called secondary, is required to make a decision on its transmission based on what the primary user does. The system considered in this part of the dissertation is comprised of nodes that are either subject to energy availability constraint imposed by the battery status and stochastic recharging process or are free from such constraint by assuming that they are connected to a constant power source.

We consider the simple cognitive system of two source-destination pairs and derive the *maximum stable throughput region* for a cognitive access protocol on the general multipacket reception channel model[1] in which a transmission may succeed even in the presence of interference [16–18]. The secondary node can take advantage of such an additional reception capability by transmitting simultaneously with the primary. We adopt a similar cognitive access protocol proposed in [19], and also studied in [20], in which the secondary node not only utilizes the idle periods of the primary node, but also competes with the primary by randomly accessing the channel to increase its own throughput. However, the secondary user is still required to coordinate its transmission in order not to hamper the required throughput level of the primary link given the energy harvesting rate and this is done by appropriately choosing the random access probability.

## 1.3 Network Coding for Wireless Mesh Networks

The core notion of network coding introduced in [21] is to allow and encourage mixing of data at intermediate network nodes. Network coding is a generalization of the traditional store and forward technique. Most of the theoretical results in network coding are for multicast but the vast majority of Internet traffic is unicast. An application of network coding to wireless environments has to address multiple unicast flows, if it has any chance of being used. In particular, with multicast, all receivers want all packets. Thus intermediate nodes can encode any packets together, without worrying about decoding which will happen eventually at the

[1]When compared to collision channel model, it better captures the effects of fading, attenuation and interference at the physical layer.

destinations.

We consider unicast flows in a multi-hop wireless mesh network with lossy directional links. In such networks the largest percentage of uplink traffic is destined for or originates from a gateway interconnecting the mesh network to a wired network. Moreover, a mesh node can provide access to multiple clients. Hence, the uplink traffic from these clients that is destined to the same gateway can be coded at the mesh node, and decoded at the gateway. Similarly, downlink traffic destined for the clients of the same mesh node can be coded at the gateway and decoded at the mesh node.

In this dissertation we investigate the performance that can be achieved by exploiting path diversity through multipath forwarding and redundancy through network coding. The analytical framework presented in this part considers the case of end-to-end and hop-by-hop retransmission for achieving reliability, and is generalized for an arbitrary number of paths and hops. We consider both end-to-end and hop-by-hop coding. The application of linear network coding results in the considerable reduction of the computational complexity at the nodes.

## 1.4 Performance Measures

In Chapters 2 and 3, we obtain analytical expressions for the characteristics of the relay's queue such as arrival and service rate of the relay's queue, the stability condition and the average length of the queue as functions of the probabilities of transmissions and the outage probabilities of the links. We study the impact of the relay node on the throughput per user-source and the aggregate throughput. We show that the throughput per user-source does not depend on the probability of the relay transmissions and that there is an optimum number of users that maximizes the aggregate throughput. Furthermore, in Chapter 3, we study the stability condition and the average length of the queue as functions of the self interference coefficient (because the relay can transmit and receive simultaneously). We study the impact of the relay node and the self interference coefficient on the throughput per user-source and the aggregate throughput.

In the chapters 4, 5, and 6, the emphasis will be given on the stable throughput region, also called the stability region. The stability region is a rate measure based on the networking perspective under bursty arrivals. It quantifies the maximum rates sustainable by the network while ensuring that all queues remain stable. In Chapter 4 we characterize the stable throughput region under conditions of no cooperation at all, full cooperation, and probabilistic (opportunistic) cooperation. We prove that the system with the flow controller is always better than or at least equal to the system without the flow controller.

In Chapter 5 (cooperation and energy harvesting) we provide the stability region of a cooperative network under energy harvesting capabilities. In Chapter 6, the stability region is obtained for a cognitive access on the general multipacket reception channel model, and the nodes have different energy constraints.

In Chapters 7 and 8 we compare the performance and tradeoff in terms of packet delay and throughput achieved by combining multipath forwarding and network coding, with that of simple multipath routing of different flows (which achieves the highest throughput), the transmission of multiple copies of a single flow over multiple paths (which achieves the highest redundancy and the least delay), and traditional single path routing.

## 1.5 Outline of the Dissertation

The work we present in this dissertation is organized in three parts. The first part consisting of the chapters 2, 3 and 4 is about network-level cooperation. In Chapters 2 and 3, we study the impact of a relay node to a network with a finite number of users-sources and a destination node. We assume that the users have saturated queues and the relay node does not have packets of its own; we have random access of the medium and the time is slotted. The relay node stores a source packet that it receives successfully in its queue when the transmission to the destination node has failed. The relay and the destination nodes have multi-packet reception capabilities. We obtain analytical equations for the characteristics of the relay's queue such as average queue length, stability conditions etc. We also study the throughput per user and the aggregate throughput for the network. In chapter 2 the relay is half-duplex, in 3 the relay can transmit and receive at the same time, thus in 3, we also study the impact of self interference on the stability, the throughput per user-source as well as the aggregate throughput. In 4, we evaluate the benefits of using one user of a two-user random access system to relay traffic of the other user. Furthermore, we define the notion of Network-Level Partial Relay Cooperation.

The second part consisting of the chapters 5 and 6 is devoted to energy harvesting wireless networks. In Chapter 5 we study the impact of energy constraints on a network with a source-user, a relay and a destination. Specifically, we provide an exact characterization of the stability region of a network consisting of a source, a relay and a destination node. This chapter studies the impact of energy harvesting on network-level cooperation. In Chapter 6, we consider two source-destination pairs and apply the concept of cognitive radio communication in sharing the common channel. Specifically, we give high-priority to the energy-constrained source-destination pair, i.e., primary pair, and low-priority to the pair which is free from such constraint, i.e., secondary pair. In contrast to the traditional notion of cognitive radio, in which the secondary transmitter is required to relinquish the channel as soon as the primary is detected, the secondary transmitter not only utilizes the idle slots of primary pair but also transmits along with the primary transmitter with probability $p$. This is possible because we consider the general multi-packet reception model. Taking into account the requirement on the primary pair's throughput, the probability $p$ is chosen to maximize the secondary pair's throughput. To this end, we obtain the two-dimensional maximum stable throughput region which describes the theoretical limit on rates that we can push into the network while maintaining the queues in the network to be stable. The result is obtained for both cases in which the capacity of the battery at the primary node is infinite and also finite.

The last part, including the Chapters 7 and 8, is about network coding and path diversity in wireless mesh networks. We investigate the performance that can be achieved by exploiting path diversity through multipath forwarding together with redundancy through linear network coding, in wireless mesh networks with directional links. A key contribution of this part is to capture the tradeoff between packet delay and throughput achieved by combining multi-path forwarding and network coding, and compare this tradeoff with that of simple multipath routing where different flows follow different paths, the transmission of multiple copies of packets over multiple paths, and single path routing. The analytical framework in Chapter 7 considers the case of end to end retransmissions, hop-by-hop case is considered in chapter 8. Additionally in chapter 8, we consider the case of hop-by-hop coding process.

A summary of our contributions and a description of future work are included in Chapter 9.

# Chapter 2

# Stability and Performance Issues of a Relay Assisted Multiple Access Scheme with MPR Capabilities

The material in this chapter was presented in [22].

## 2.1 Introduction

The classical relay channel was originally introduced by van der Meulen [1]. Earlier works on the relay channel were based on information theoretical formulations as in [2] and [4]. Recently several works have investigated relaying capability at the MAC layer [4], [23], [5], [24]. More specifically, in [4], the authors have studied the impact of cooperative communications at the multiple-access layer with TDMA. They introduced a new cognitive multiple-access protocol in the presence of a relay in the network. The relay senses the channel for idle channel resources and exploits them to cooperate with the terminals in forwarding their packets. Most cooperative techniques studied so far have been on physical layer cooperation, however there are evidences (as in [4]) that the same gains can be achieved with network layer cooperation, that is plain relaying without any physical layer considerations.

The classical analysis of random multiple access schemes like slotted ALOHA [25] has focused on the so called collision model, the collision channel however is not the appropriate for wireless networks. Random access with multi-packet reception (MPR) has attracted attention recently [26], [27]. The authors in [28] consider the effect of MPR on stability and delay of slotted ALOHA based random-access system and it is shown that the stability region undergoes a phase transition from a concave region to a convex polyhedral region as the MPR capability improves. All these previous approaches come together in the model that we consider in this work.

In this work we examine the operation of a node relaying packets from a number of users-sources to a destination node as shown in Fig. 2.1, and is an extension of our work in [29] (in that work we assumed random access scheme with collision channel model with erasures). We assume MP) capability for the relay and the destination node.

We assume random access to the channel, time is considered slotted, and each packet transmission takes one time slot. The wireless channel between the nodes in the network is modeled by a Rayleigh narrowband flat-fading channel with additive Gaussian noise. The relay and the destination are equipped with multiuser detectors, so that they may decode packets successfully from more than one transmitter at a time (MPR capability). A user's transmission is successful if the received signal to interference plus noise ratio ($SINR$) is above a threshold $\gamma$. We also assume that acknowledgements (ACKs) are instantaneous and error free. The relay does not have packets of its own and the sources are considered saturated with unlimited amount of traffic.

We obtain analytical expressions for the characteristics of the relay's queue such as arrival and service rate of the relay's queue, the stability condition and the average length of the queue as functions of the probabilities of transmissions and the outage probabilities of the links. We study the impact of the relay node on the throughput per user-source and the aggregate throughput. We show that the throughput per user-source does not depend on the probability of the relay transmissions and that there is an optimum number of users that maximizes the aggregate throughput.

Section 2.2 describes the system model, in Section 2.3 we study the characteristics of the relay's queue and we derive the equations for the throughput per user and the aggregate throughput. We present the arithmetic and simulation results in Section 2.4 and, finally, our conclusions are given in Section 2.5.

## 2.2 System Model

### 2.2.1 Network Model

We consider a network with $N$ users-sources, one relay node and a single destination node. The sources transmit packets to the destination with the cooperation of the relay; the case of $N = 2$ is depicted in Fig. 2.1. We assume that the queues of the two sources are saturated (i.e. there are no external arrivals); the relay does not have packets of its own, and just forwards the packets that it has received from the two users. The relay node stores a source packet that it receives successfully in its queue when the direct transmission to the destination node has failed. We assume that we have random access of the medium. Each of the receivers (relay and destination) is equipped with multiuser detectors, so that they may decode packets successfully from more than one transmitter at a time. Nodes cannot transmit and receive at the same time. The queue in the relay has infinite capacity.

It is important to note that the relay node must be easier accessible than the destination, meaning that the user - relay channel has to be more reliable than the user-destination one. At the same time the relay - destination channel must be more reliable than the user - destination channel. Otherwise the presence of the relay degrades the performance of the whole network. In the following subsection we present all the details about the physical layer model assumed in this work.

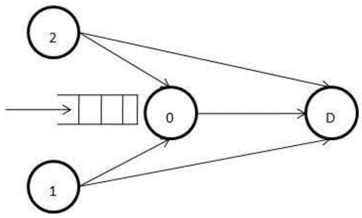



Figure 2.1: The simple network model

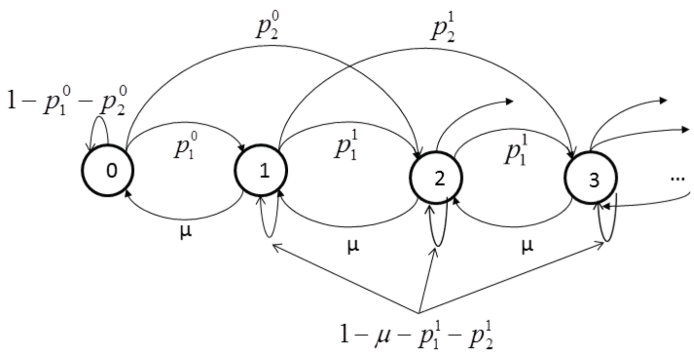



Figure 2.2: Markov Chain model

### 2.2.2 Physical Layer Model

The MPR channel model used in this work is a generalized form of the packet erasure model. In the wireless environment, a packet can be decoded correctly by the receiver if the received $SINR$ exceeds a certain threshold. More precisely, suppose that we are given a set $T$ of nodes transmitting in the same time slot. Let $P_{rx}(i,j)$ be the signal power received from node $i$ at node $j$ (when $i$ transmits), and let $SINR(i,j)$ be the $SINR$ determined by node $j$, i.e.,

$$SINR(i,j) = \frac{P_{rx}(i,j)}{\eta_j + \sum_{k \in T \setminus \{i\}} P_{rx}(k,j)}$$

where $\eta_j$ denotes the receiver noise power at $j$. We assume that a packet transmitted by $i$ is successfully received by $j$ if and only if $SINR(i,j) \geq \gamma_j$, where $\gamma_j$ is a threshold characteristic of node $j$. The wireless channel is subject to fading; let $P_{tx}(i)$ be the transmitting power at node $i$ and $r(i,j)$ be the distance between $i$ and $j$. The power received by $j$ when $i$

transmits is $P_{rx}(i,j) = A(i,j)g(i,j)$ where $A(i,j)$ is a random variable representing channel fading. We assume that the fading model is slow, flat fading, constant during a timeslot and independently varying from timeslot to timeslot. Under Rayleigh fading, it is known [30] that $A(i,j)$ is exponentially distributed. The received power factor $g(i,j)$ is given by $g(i,j) = P_{tx}(i)(r(i,j))^{-\alpha}$ where $\alpha$ is the path loss exponent with typical values between 2 and 4. The success probability of link $ij$ when the transmitting nodes are in $T$ is given by

$$P^{j}_{i/T} = \exp\left(-\frac{\gamma_j \eta_j}{v(i,j)g(i,j)}\right) \prod_{k \in T \setminus \{i,j\}} \left(1 + \gamma_j \frac{v(k,j)g(k,j)}{v(i,j)g(i,j)}\right)^{-1} \tag{2.1}$$

where $v(i,j)$ is the parameter of the Rayleigh random variable for fading. The analytical derivation for this success probability can be found in [31].

## 2.3 Analysis

In this section we will derive the equations for the characteristics of the relay's queue, such as the arrival and service rates, the stability conditions, and the average queue length. We will provide an analysis for two cases: first, when the network consists of two users (non-symmetric) and the second is for $n > 2$ symmetric users.

### 2.3.1 Two-user case

The service rate is given by

$$\mu = q_0(1-q_1)(1-q_2)P^{d}_{0/0} + q_0 q_1 (1-q_2) P^{d}_{0/0,1} + q_0 q_2 (1-q_1) P^{d}_{0/0,2} + q_0 q_1 q_2 P^{d}_{0/0,1,2} \tag{2.2}$$

where $q_0$ is the transmission probability of the relay given that it has packets in its queue, $q_i$ for $i \neq 0$ is the transmission probability for the $i$-th user.The term $P^{j}_{i/i,k}$ is the success probability of link $ij$ when the transmitting nodes are $i$ and $k$ and is given by (2.1). If the queue of the relay node is empty, the arrival rate is denoted by $\lambda_0$ and if it is not by $\lambda_1$; thus we have the following equation for the average arrival rate $\lambda$ (where $Q$ is the size of the relay's queue)

$$\lambda = P\left(Q=0\right)\lambda_0 + P\left(Q>0\right)\lambda_1 \tag{2.3}$$

If the queue of the relay is empty then the relay, naturally, does not attempt to transmit, thus the probability of arrival is $\lambda_0 = p_1^0 + 2p_2^0$, where $p_i^0$ is the probability of receiving $i$ packets given that the queue is empty. The expressions for the $p_i^0$ are:

$$\begin{aligned} p_1^0 = q_1(1-q_2)(1-P_{1/1}^d)P_{1/1}^0 + q_2(1-q_1)(1-P_{2/2}^d)P_{2/2}^0 + \\ + q_1q_2(1-P_{1/1,2}^d)P_{1/1,2}^0\left[P_{2/1,2}^d + (1-P_{2/1,2}^d)(1-P_{2/1,2}^0)\right] + \\ + q_1q_2(1-P_{2/1,2}^d)P_{2/1,2}^0\left[P_{1/1,2}^d + (1-P_{1/1,2}^d)(1-P_{1/1,2}^0)\right] \end{aligned} \tag{2.4}$$

$$p_2^0 = q_1q_2(1-P_{1/1,2}^d)(1-P_{2/1,2}^d)P_{1/1,2}^0P_{2/1,2}^0 \tag{2.5}$$

If the queue is not empty then the arrival rate is given by $\lambda_1 = p_1^1 + 2p_2^1$, where $p_i^1$ is the probability of receiving $i$ packets given that the queue is not empty and $p_i^1 = (1-q_0)p_i^0$; thus $\lambda_1 = (1-q_0)\lambda_0$, this is because the relay cannot receive and transmit at the same time. In Fig. 2.2, we present the discrete time Markov Chain that describes the queue evolution. Each state, denoted by an integer, represents the queue size at the relay node. The transition matrix of the above DTMC is a lower Hessenberg matrix and is given by:

$$P = \begin{pmatrix} a_0 & b_0 & 0 & 0 & \cdots \\ a_1 & b_1 & b_0 & 0 & \cdots \\ a_2 & b_2 & b_1 & b_0 & \cdots \\ 0 & b_3 & b_2 & b_1 & \cdots \\ 0 & 0 & b_3 & b_2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{2.6}$$

Where $a_0 = 1 - p_1^0 - p_2^0, a_1 = p_1^0, a_2 = p_2^0$ and $b_0 = \mu, b_1 = 1 - \mu - p_1^1 - p_2^1, b_2 =$

$p_1^1, b_3 = p_2^1$.

Since the $P$ is an infinite-dimension matrix, we are going to obtain the expression for the steady-state distribution vector $s$ using difference equation technique. The difference equations are given by:

$$Ps = s \Rightarrow s_i = a_i s_0 + \sum_{j=1}^{i+1} b_{i-j+1} s_j \tag{2.7}$$

We apply Z-transform technique to compute the steady-state distribution:

$$A(z) = \sum_{i=0}^{2} a_i z^{-i}, B(z) = \sum_{i=0}^{3} b_i z^{-i}, S(z) = \sum_{i=0}^{\infty} s_i z^{-i} \tag{2.8}$$

It is known that [32]:

$$S(z) = s_0 \frac{z^{-1} A(z) - B(z)}{z^{-1} - B(z)} \tag{2.9}$$

The probability that the queue in the relay is empty is given by the following formula [32]:

$$P\left(Q = 0\right) = s_0 = \frac{1 + B^{'}(1)}{1 + B^{'}(1) - A^{'}(1)} \tag{2.10}$$

Where $A^{'}(1) = -p_1^0 - 2p_2^0$ and $B^{'}(1) = \mu - p_1^1 - 2p_2^1 - 1$. Then the probability that the queue in the relay is empty is given by:

$$P\left(Q = 0\right) = \frac{\mu - \lambda_1}{\mu - \lambda_1 + \lambda_0} \tag{2.11}$$

From the above equations we can compute the average arrival rate $\lambda$:

$$\lambda = P\left(Q = 0\right)\lambda_0 + P\left(Q > 0\right)\lambda_1 = \frac{\mu\lambda_0}{\mu - \lambda_1 + \lambda_0} \tag{2.12}$$

Note that the average arrival rate does not depend on $q_0$ (the proof is straightforward and thus is omitted). An important tool to determine stability is Loyne's criterion [33], which states that if the arrival and service processes of a queue are jointly strictly stationary and ergodic, the queue is stable if and only if the average arrival rate is strictly less than the average service rate. If the queue is stable, the departure rate (throughput) is equal to the arrival rate. The queue is stable if $q_0$ satisfies $q_{0min} < q_0 < 1$. The expression for $q_{0min}$ is given by (2.13), in order to obtain $q_{0min}$ we have to solve the inequality $\lambda_1 < \mu$.

$$q_{0min} = \frac{\lambda_0}{\lambda_0 + (1-q_1)(1-q_2)P^d_{0/0} + q_1(1-q_2)P^d_{0/0,1} + q_2(1-q_1)P^d_{0/0,2} + q_1 q_2 P^d_{0/0,1,2}} \tag{2.13}$$

Notice that the conditions $\frac{\lambda}{\mu} < 1$ and $\frac{\lambda_1}{\mu} < 1$ are equivalent in our model.

$$\frac{\lambda}{\mu} < 1 \Leftrightarrow \lambda < \mu \Leftrightarrow \frac{\mu\lambda_0}{\mu - \lambda_1 + \lambda_0} < \mu \Leftrightarrow \frac{\lambda_0}{\mu - \lambda_1 + \lambda_0} < 1 \Leftrightarrow \lambda_0 < \mu - \lambda_1 + \lambda_0 \Leftrightarrow \frac{\lambda_1}{\mu} < 1$$

It is known [32] that the average queue size is $\overline{Q} = -S^{'}(1)$, where $S^{'}(1) = s_0 \frac{K^{''}(1)}{L^{''}(1)}$. The expression for $K(z)$ is given by

$$K(z) = \left(-z^{-2}A(z) + z^{-1}A^{'}(z) - B^{'}(z)\right)\left(z^{-1} - B(z)\right) - \left(z^{-1}A(z) - B(z)\right)\left(-z^{-2} - B^{'}(z)\right) \tag{2.14}$$

and $L(z) = \left(z^{-1} - B(z)\right)^2$.

After some algebra the average queue size is given by:

$$\overline{Q} = \frac{(\lambda_1 - \mu)(2p_1^0 + 5p_2^0) + \lambda_0(\mu - 2p_1^0 - 5p_2^0)}{(\mu - \lambda_1 + \lambda_0)(\lambda_1 - \mu)} \tag{2.15}$$

The throughput rate $\mu_i$ for the user $i$ is given by the (2.16) and (2.17).

$$\mu_1 = q_0 P\left(Q>0\right) q_1 \left((1-q_2)P^d_{1/0,1} + q_2 P^d_{1/0,1,2}\right) + \\ + \left[1 - q_0 P\left(Q>0\right)\right] q_1 \left[(1-q_2)\left(P^d_{1/1} + (1-P^d_{1/1})P^0_{1/1}\right) + q_2\left(P^d_{1/1,2} + (1-P^d_{1/1,2})P^0_{1/1,2}\right)\right] \tag{2.16}$$

$$\mu_2 = q_0 P\left(Q>0\right) q_2 \left((1-q_1)P^d_{2/0,2} + q_1 P^d_{2/0,1,2}\right) + \\ + \left[1 - q_0 P\left(Q>0\right)\right] q_2 \left[(1-q_1)\left(P^d_{2/2} + (1-P^d_{2/2})P^0_{2/2}\right) + q_1\left(P^d_{2/1,2} + (1-P^d_{2/1,2})P^0_{2/1,2}\right)\right] \tag{2.17}$$

In the (2.16) and (2.17) we assume that the queue is stable, hence the arrival rate from each user to the queue is the contributed throughput from it. The aggregate throughput is $\mu_{total} = \mu_1 + \mu_2$. Notice that the throughput per user is independent of $q_0$ as long as it is in the stability region. This is explained because the product $q_0 P\left(Q>0\right)$ is constant. If we consider the previous network without the relay node then the throughput rates for the users are the following:

$$\mu_1 = q_1(1-q_2)P^d_{1/1} + q_1 q_2 P^d_{1/1,2}$$
$$\mu_2 = q_2(1-q_1)P^d_{2/2} + q_1 q_2 P^d_{2/1,2}$$

### 2.3.2 N-symmetric users

We now generalize the above for the case of a symmetric $n$-users network. Each user attempts to transmit in a slot with probability $q$; the success probability to the relay and the destination when $i$ nodes transmit are given by $P_{0,i}$, $P_{d,i}$ respectively. There are two cases for the $P_{d,i}$, $P_{d,i,0}$, $P_{d,i,1}$ denoting success probability when relay remains silent or transmits respectively. Finally $P_{0d,i}$ is the link probability of success from the relay to the destination when $i$ nodes transmit. The above success probabilities for the symmetric case are given by $P_{0,i} = P_0\left(\frac{1}{1+\gamma_0}\right)^{i-1}$, $P_{d,i,j} = P_d\left(\frac{1}{1+\gamma_d}\right)^{i-1}\left(\frac{1}{1+\beta\gamma_0}\right)^{j}$, $j = 0,1$ and $\beta = \frac{v_{0d}g_{0d}}{v_d g_d} > 1$.

$P_{0d,i} = P_{0d} \left( \frac{1}{1+\frac{1}{\beta}\gamma_d} \right)^i$, $P_0 = \exp\left(-\frac{\gamma_0 \eta_0}{v_0 g_0}\right)$, $P_d = \exp\left(-\frac{\gamma_d \eta_d}{v_d g_d}\right)$, $P_{0d} = \exp\left(-\frac{\gamma_0 \eta_0}{v_0 g_0}\right)$.

The service rate is given by the following equation:

$$\mu = \sum_{k=0}^{n} \binom{n}{k} q_0 q^k (1-q)^{n-k} P_{0d,k} \tag{2.18}$$

The average arrival rate $\lambda$ of the queue is given by:

$$\lambda = P\left(Q = 0\right)\lambda_0 + P\left(Q > 0\right)\lambda_1 \tag{2.19}$$

Where $\lambda_0 = \sum_{i=0}^{n} i p_i^0$ and $\lambda_1 = (1 - q_0)\lambda_0$. $p_i^0$ is the probability of receiving $i$ packets given that the queue is empty, the expression for $p_i^0$ is given by ( 2.20). $p_i^1$ is the probability of receiving $i$ packets given that the queue is not empty and $p_k^1 = (1 - q_0)p_k^0$.

$$p_k^0 = \sum_{i=k}^{n} \binom{n}{i} \binom{i}{k} q^i (1-q)^{n-i} P_{0,i}^k \left(1 - P_{d,i,0}\right)^k \left[1 - P_{0,i}(1 - P_{d,i,0})\right]^{i-k},\ 1 \le k \le n \tag{2.20}$$

The elements of the transition matrix are $a_0 = 1 - \sum_{i=1}^{n} p_i^0$, $a_i = p_i^0\ \forall i > 0$ and $b_0 = \mu$, $b_1 = 1 - \mu - \sum_{i=1}^{n} p_i^1$,$b_{i+1} = p_i^1\ \forall i > 1$. The Z-transforms are:

$$A(z) = \sum_{i=0}^{n} a_i z^{-i}, B(z) = \sum_{i=0}^{n+1} b_i z^{-i}, S(z) = \sum_{i=0}^{\infty} s_i z^{-i} \tag{2.21}$$

Following the same methodology as in the two-user case and applying the above to (2.10) we obtain the probability that the queue in the relay is empty is given by:

$$P\left(Q = 0\right) = \frac{\mu - \lambda_1}{\mu - \lambda_1 + \lambda_0} \tag{2.22}$$

The queue is stable if $q_0$ satisfies $q_{0min} < q_0 < 1$. The expression for $q_{0min}$ is given by the (2.23).

$$q_{0min} = \frac{C}{C + \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} P_{0d,k}} \tag{2.23}$$

where

$$C = \sum_{k=1}^{n} \sum_{i=k}^{n} k \binom{n}{i} \binom{i}{k} q^i (1-q)^{n-i} P_{0,i}^k \left(1 - P_{d,i,0}\right)^k \left[1 - P_{0,i}(1 - P_{d,i,0})\right]^{i-k}$$

Following the same methodology as in the two-user case, we obtain that the average queue size is given by:

$$\overline{Q} = \frac{(\lambda_1 - \mu) \sum_{i=1}^{n} i(i+3) p_i^0 + \lambda_0 \left(2\mu - \sum_{i=1}^{n} i(i+3) p_i^1\right)}{2(\mu - \lambda_1 + \lambda_0)(\lambda_1 - \mu)} \tag{2.24}$$

The throughput per user for the network without the relay is given by

$$\mu = \sum_{k=0}^{n-1} \binom{n-1}{k} q^{k+1} (1-q)^{n-1-k} P_{d,k+1}$$

The throughput per user for the network with the relay is given by ( 2.25). The aggregate throughput is $\mu_{total} = n\mu$.

$$\mu = q_0 P\left(Q > 0\right) \sum_{k=0}^{n-1} \binom{n-1}{k} q^{k+1} (1-q)^{n-1-k} P_{d,k+1,1} + \\ + \left[1 - q_0 P\left(Q > 0\right)\right] \sum_{k=0}^{n-1} \binom{n-1}{k} q^{k+1} (1-q)^{n-1-k} \left[P_{d,k+1,0} + \left(1 - P_{d,k+1,0}\right) P_{0,k+1}\right] \tag{2.25}$$

The throughput per user as a function of $q$ is given by (2.26). In order to maximize $\mu(q)$, we need to find $q^*$ such that $\mu(q^*) \geq \mu(q)\ \forall\ 0 < q < 1$. The analysis for finding the optimum is straight forward, has some complex calculations and will not add new insights to the results.

We will present a numerical evaluation of this problem in the next section.

$$\mu(q) = (1-q)^n D \sum_{k=0}^{n-1} \binom{n-1}{k} \left(\frac{q}{1-q}\right)^{k+1} P_{d,k+1,1} + \\ +(1-q)^n(1-D) \sum_{k=0}^{n-1} \binom{n-1}{k} \left(\frac{q}{1-q}\right)^{k+1} \left[P_{d,k+1,0} + (1-P_{d,k+1,0})P_{0,k+1}\right] \tag{2.26}$$

where

$$D = \frac{\sum_{k=1}^{n}\sum_{i=k}^{n} A_{i,k}\left(\frac{q}{1-q}\right)^{i}}{\sum_{k=0}^{n} B_k \left(\frac{q}{1-q}\right)^{k} + \sum_{k=1}^{n}\sum_{i=k}^{n} A_{i,k}\left(\frac{q}{1-q}\right)^{i}}$$

where $A_{i,k} = k\binom{n}{i}\binom{i}{k} P_{0,i}^{k}\left(1-P_{d,i,0}\right)^{k}\left[1-P_{0,i}(1-P_{d,i,0})\right]^{i-k}$ and $B_k = \binom{n}{k} P_{0d,k}$

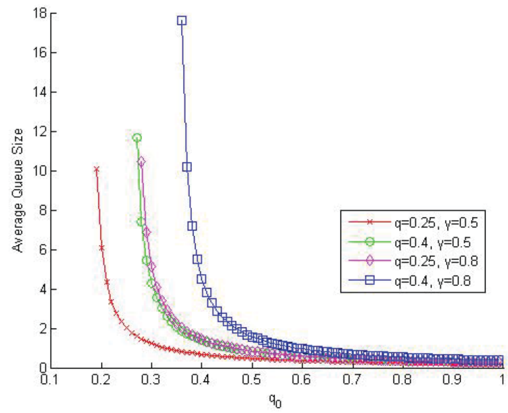
## 2.4 Numerical Results

In this section we present numerical results for the analysis presented above. The results presented below have been verified by simulations which confirmed the accuracy of the analysis in the previous section. To simplify the presentation we consider the case where all the users have the same link characteristics and transmission probabilities. The parameters used in the numerical results are as follows. The distances in meters are given by $r(i,d) = r_d = 130$, $r(i,0) = r_0 = 60\ \forall i \geq 1$ and $r(0,d) = r_{0d} = 80$. The path loss is $\alpha = 4$ and the receiver noise power $\eta = 10^{-11}$. The transmit power for the relay is $P_{tx}(0) = 10$ mW and for the i-th user $P_{tx}(i) = 1$ mW.

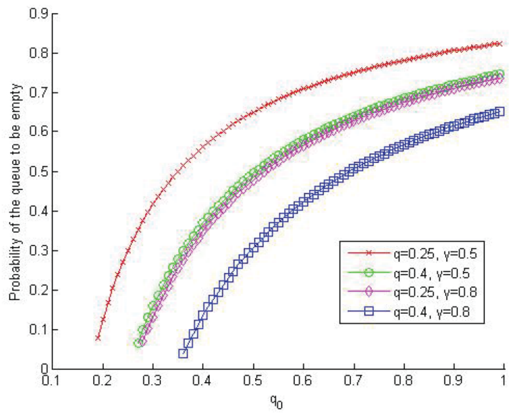
### 2.4.1 Properties of the queue of the relay for the case of $n = 2$ users

Fig. 2.3(a) and 2.3(b) present the average queue size and the probability of the queue to be empty as the $q_0$ varies for various values of $q$ and $\gamma$. As the relay transmission probability $q_0$

increases then the queue is more likely to be empty. Equally expected is the decrease of the average queue size as $q_0$ increases.




(a) Average Queue Size




(b) Probability of the queue to be empty

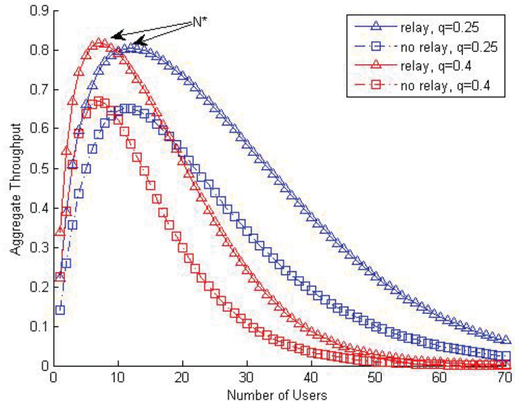
Figure 2.3: Properties of the relay's queue for the case of two-users

### 2.4.2 The impact of the number of users

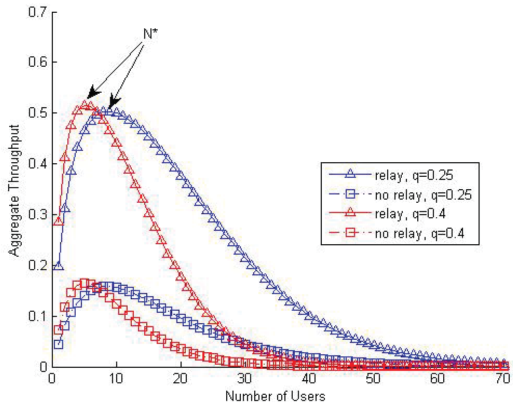
Fig. 2.4(a) - 2.4(b) and Fig. 2.5(a) - 2.5(b) show the aggregate throughput versus the number of users for $\gamma < 1$ and $\gamma > 1$ respectively. Notice that with small values of $\gamma$ is more likely to have more successful simultaneous transmissions comparing to larger $\gamma$. For $\gamma < 1$ it is possible for two or more users to transmit successfully at the same time, comparing to $\gamma > 1$

which that probability tends to zero.

The figures show that the relay offers a significant advantage compared to the network without the relay. When the threshold $\gamma$ increases the gain in term of percentage it is greater. Another interesting observation is that given the link characteristics and the transmission probabilities, there is an optimum number of users $N^*$ that maximizes the aggregate throughput. This number could be used as a criterion for finding the optimum size of a subset of users that a relay can serve.



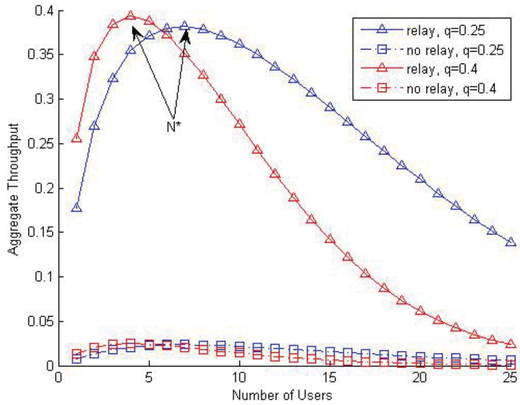

(a) $\gamma = 0.5$



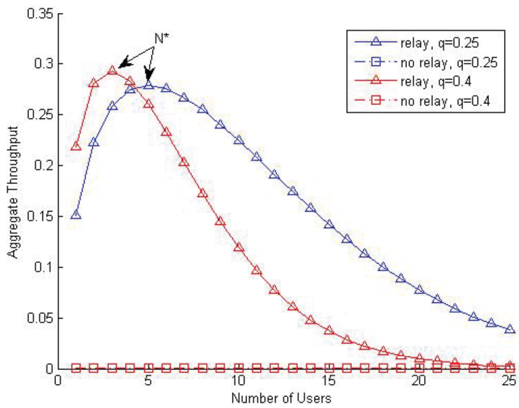

(b) $\gamma = 0.8$

Figure 2.4: Aggregate throughput vs number of users for $\gamma < 1$

Fig. 2.6(a) and 2.6(b) show the aggregate throughput versus the number of the users served




(a) $\gamma = 1.2$



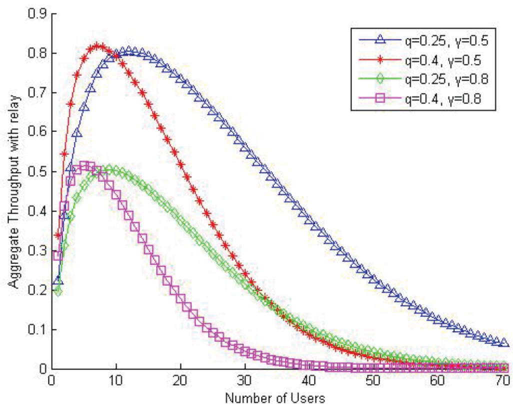

(b) $\gamma = 2.5$

Figure 2.5: Aggregate throughput vs number of users for $\gamma > 1$

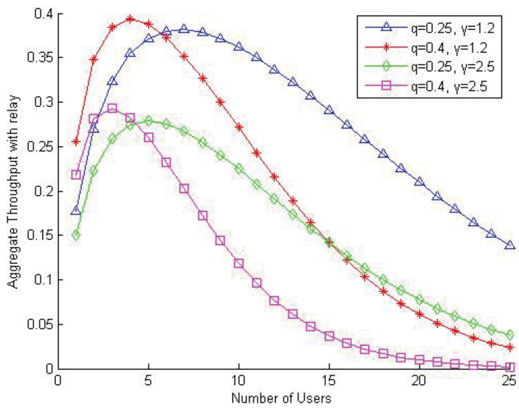
by the relay for several values of $q$ and $\gamma$. As $\gamma$ increases the number of users that achieves the maximum aggregate throughput is decreasing. The same conclusion comes for the values of $q$, as the $q$ increases.

Fig. 2.7(a) and 2.7(b) show the throughput per user versus the user's transmission probability $q$ for several values of $n$ and $\gamma$. An intuitive result for the $q^*$ (the value of $q$ that maximizes the throughput per user), is that as $n$ increases then the $q^*$ decreases.

Fig. 2.8(a) and 2.8(b) present the $q_{0min}$ threshold versus the number of users for $\gamma < 1$ and $\gamma > 1$ respectively.




(a) $\gamma < 1$



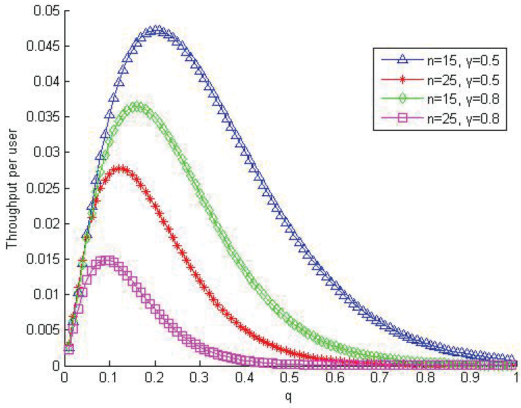

(b) $\gamma > 1$

Figure 2.6: Aggregate throughput with relay vs number of users

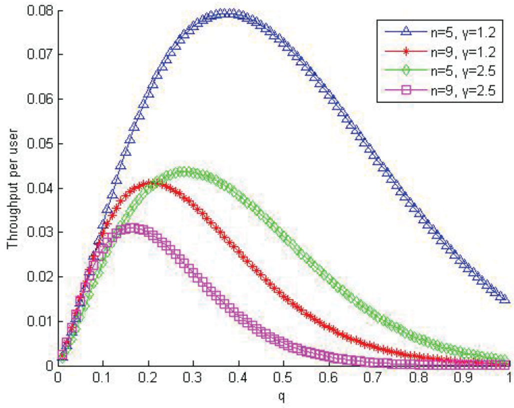
The advantage that the relay offers is more obvious when the number of users is large. This is expected and feasible because of the MPR capabilities and the capture effect of the channel comparing to the collision channel in our previous work.

## 2.5 Conclusions

In this work, we examined the operation of a node relaying packets from a number of users to a common destination node. We assumed MPR capability for the relay and for the destination



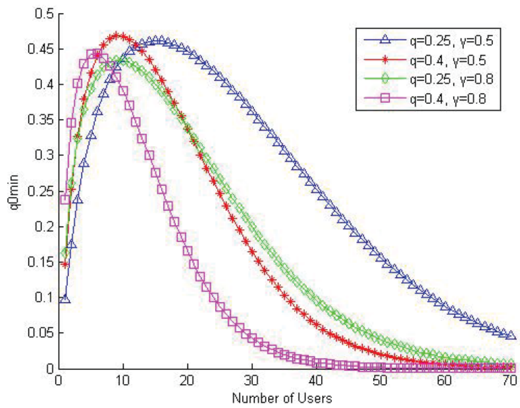

(a) $\gamma < 1$



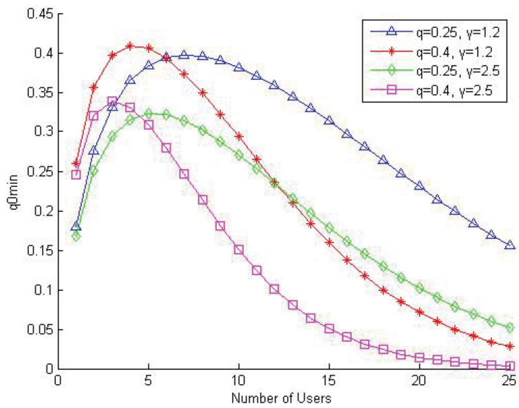

(b) $\gamma > 1$

Figure 2.7: Throughput per user vs q

node. We studied a multiple capture model, where a user's transmission is successful if the received $SINR$ is above a threshold $\gamma$. We obtained analytical expressions for the relay's queue characteristics such as the stability condition, the values of the arrival and service rates, the average queue size. We showed that the arrival rate at the queue is independent of the relay probability of transmission, when the queue is stable. We studied the throughput per user and the aggregate throughput, and found that, under stability conditions, the throughput per user does not depend on the relay probability of transmission. The analytical results have been verified with simulations. In Section 2.4 we have given the conditions under which the




(a) $\gamma < 1$




(b) $\gamma > 1$

Figure 2.8: $q_{0min}$ vs Number of users

utilization of the relay offers significant advantages. An interesting result is that, given the link characteristics and the transmission probabilities, there is an optimum number of users that maximizes the aggregate throughput. These results could be useful in a network with many users and multiple relays for determining the way to allocate the users among the relays. With the MPR and the capture effect the advantages from deploying a relay node are more pronounced.

An extension of the present work is the case of the relay node which is capable of transmitting and receiving packets at the same time (full duplex), the case of multiple relays (with possible cooperation among them) it is interesting too. Another possible extension is the case of dynamic adjustment of the transmission probabilities depending on the network conditions. Future extensions of this work will include users with non-saturated queues i.e. sources with external random arrivals, a relay node with its own packets and different priorities for the users. Another interesting extension of this work concerns the energy consumption in the total network, and in particular at the relay node.

# Chapter 3

# Relay-Assisted Multiple Access with MPR Capability and Simultaneous Transmission and Reception

The material in this chapter was presented in [34].

## 3.1 Introduction

In wireless networks when a node transmits and receives simultaneously the problem of self interference arises. Information theoretic aspects of this problem can be found at the work of Shannon on [7], although the capacity region of the two-way channel is not known for the general case [8]. There are some techniques that allow the possibility of perfect self interference cancelation [8]. In practice though, there are technological limitations [9]- [10] which can limit the accuracy of the self interference cancelation. Various methods for performing self interference cancelation at the nodes' receivers can be found in [11] and [12]. The conclusion is that there is a trade off between transceiver complexity and the accuracy of the self interference cancelation.

In this work we examine the operation of a node relaying packets from a number of users-sources to a destination node as shown in Fig. 3.1, and is an extension of [29] and [22]. We assume MPR capability for the relay and the destination node. The relay node can transmit and receive at the same time. We assume random access to the channel, time is considered slotted, and each packet transmission takes one time slot. The wireless channel between the nodes in the network is modeled by a Rayleigh narrowband flat-fading channel with additive Gaussian noise. A user's transmission is successful if the received signal to interference plus noise ratio ($SINR$) is above a threshold $\gamma$. We also assume that acknowledgements (ACKs) are instantaneous and error free. The relay does not have packets of its own and the sources are considered saturated with unlimited amount of traffic. We do not consider any specific self interference cancelation mechanism, because it is out of the scope of this work. The self interference cancelation at the relay is modeled as a variable power gain.

We obtain analytical expressions for the characteristics of the relay's queue (such as arrival and service rates), we study the stability condition and the average length of the queue as functions of the probabilities of transmission, the self interference coefficient and the outage probabilities of the links. We study the impact of the relay node and the self interference coefficient on the throughput per user-source and the aggregate throughput.

Section 3.2 describes the system model, in Section 3.3 we study the characteristics of the relay's queue and we derive the equations for the throughput per user and the aggregate throughout. We present the numerical results in Section 3.4 and, finally, our conclusions are given in Section 3.5.

## 3.2 System Model

### 3.2.1 Network Model

We consider a network with $N$ sources, one relay node and a single destination node. The sources transmit packets to the destination with the cooperation of the relay; the case of $N = 2$

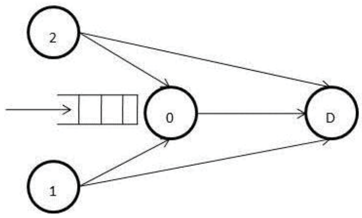



Figure 3.1: The simple network model

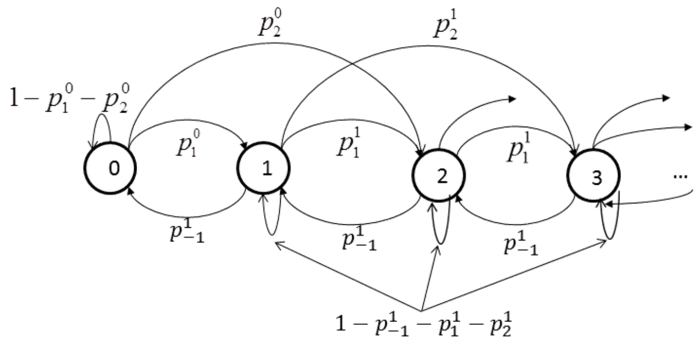



Figure 3.2: Markov Chain model for the two-user case

is depicted in Fig. 3.1. We assume that the queues of the two sources are saturated (i.e. there are no external arrivals but unlimited packet volume in the buffers); the relay does not have packets of its own, and just forwards the packets that it has received from the two users. The relay node stores a source packet that it receives successfully in its queue when the direct transmission to the destination node has failed. We assume random access to the channel. Each of the receivers (relay and destination) is equipped with multiuser detectors, so that they may decode packets successfully from more than one transmitter at a time. The relay node can receive and transmit packets simultaneously.

### 3.2.2 Physical Layer Model

The MPR channel model used in this work is a generalized form of the packet erasure model. We assume that a packet transmitted by $i$ is successfully received by $j$ if and only if $SINR(i,j) \geq \gamma_j$, where $\gamma_j$ is a threshold characteristic of node $j$. The wireless channel is subject to fading; let $P_{tx}(i)$ be the transmitting power at node $i$ and $r(i,j)$ be the distance between $i$ and $j$. The received power by $j$ when $i$ transmits is $P_{rx}(i,j) = A(i,j)g(i,j)$ where $A(i,j)$ is a random variable representing channel fading. Under Rayleigh fading, it is known [30] that $A(i,j)$ is exponentially distributed. The received power factor $g(i,j)$ is given by $g(i,j) = P_{tx}(i)(r(i,j))^{-\alpha}$ where $\alpha$ is the path loss exponent with typical values between 2 and 4. We model the self interference by a scalar $g \in [0,1]$. We refer to the $g$ as the self interference coefficient. When $g = 1$, no self interference cancelation technique is used and $g = 0$ when there is perfect self interference cancelation. The success probability in the link $ij$ is given by:

$$P_{i/T}^{j} = \exp\left(-\frac{\gamma_j \eta_j}{v(i,j)g(i,j)}\right)(1+\gamma_j(r(i,j))^{\alpha}g)^{-m} \prod_{k\in T\setminus\{i,j\}} \left(1+\gamma_j \frac{v(k,j)g(k,j)}{v(i,j)g(i,j)}\right)^{-1} \tag{3.1}$$

where T is the set of transmitting nodes at the same time, $v(i,j)$ is the parameter of the Rayleigh random variable for fading; $m = 1$ when $j \in T$ and $m = 0$ else. The analytical derivation for this success probability can be found in [31] and [35].

## 3.3 Analysis

In this section we derive the equations for the characteristics of the relay's queue, such as the arrival and service rates, the stability conditions, and the average queue length. We will provide an analysis for two cases: first, when the network consists of two users (non-symmetric) and the second is for $n > 2$ symmetric users.

### 3.3.1 Two-user case

**Computation of the average arrival and service rate**

The service rate is given by:

$$\mu = q_0(1-q_1)(1-q_2)P^d_{0/0} + q_0q_1(1-q_2)P^d_{0/0,1} + q_0q_2(1-q_1)P^d_{0/0,2} + q_0q_1q_2P^d_{0/0,1,2} \tag{3.2}$$

where $q_0$ is the transmission probability of the relay given that it has packets in its queue, $q_i$ for $i \neq 0$ is the transmission probability for the $i$-th user. The term $P^j_{i/i,k}$ is the success probability of link $ij$ when the transmitting nodes are $i$ and $k$ and can be calculated based on (3.1).

The average arrival rate $\lambda$ of the queue is given by:

$$\lambda = P\left(Q=0\right)\lambda_0 + P\left(Q>0\right)\lambda_1 \tag{3.3}$$

Where $\lambda_0$ is the average arrival rate at the relay's queue when the queue is empty and $\lambda_1$ when it's not. $\lambda_0 = r^0_1 + 2r^0_2$, where $r^0_i$ is the probability of receiving $i$ packets given that the queue is empty. The expressions for $r^0_i$ are given by:

$$\begin{aligned} r^0_1 = {} & q_1(1-q_2)(1-P^d_{1/1})P^0_{1/1} + q_2(1-q_1)(1-P^d_{2/2})P^0_{2/2} + q_1q_2(1-P^d_{1/1,2})P^0_{1/1,2}P^d_{2/1,2} + \\ & + q_1q_2(1-P^d_{2/1,2})P^0_{2/1,2}P^d_{1/1,2} + q_1q_2(1-P^d_{1/1,2})P^0_{1/1,2}(1-P^d_{2/1,2})(1-P^0_{2/1,2}) + \\ & + q_1q_2(1-P^d_{2/1,2})P^0_{2/1,2}(1-P^d_{1/1,2})(1-P^0_{1/1,2}) \end{aligned} \tag{3.4}$$

$$r^0_2 = q_1q_2(1-P^d_{1/1,2})(1-P^d_{2/1,2})P^0_{1/1,2}P^0_{2/1,2} \tag{3.5}$$

Accordingly, $\lambda_1 = r^1_1 + 2r^1_2$, where $r^1_i$ is the probability of receiving $i$ packets when the queue is not empty. The expressions for the $r^1_i$ are lengthy and given by:

$$r_1^1 = (1-q_0)q_1(1-q_2)(1-P^d_{1/1})P^0_{1/1} + q_0q_1(1-q_2)(1-P^d_{1/0,1})P^0_{1/0,1} + (1-q_0)q_2(1-q_1)(1-P^d_{2/2})P^0_{2/2}+$$
$$+q_0q_2(1-q_1)(1-P^d_{2/0,2})P^0_{2/0,2} + (1-q_0)q_1q_2(1-P^d_{1/1,2})P^0_{1/1,2}(1-P^d_{2/1,2})(1-P^0_{2/1,2})+$$
$$+q_0q_1q_2(1-P^d_{1/0,1,2})P^0_{1/0,1,2}(1-P^d_{2/0,1,2})(1-P^0_{2/0,1,2}) + (1-q_0)q_1q_2(1-P^d_{1/1,2})P^0_{1/1,2}P^d_{2/1,2}+$$
$$+q_0q_1q_2(1-P^d_{1/0,1,2})P^0_{1/0,1,2}P^d_{2/0,1,2} + (1-q_0)q_1q_2(1-P^d_{2/1,2})P^0_{2/1,2}(1-P^d_{1/1,2})(1-P^0_{1/1,2})+$$
$$+q_0q_1q_2(1-P^d_{2/0,1,2})P^0_{2/0,1,2}(1-P^d_{1/0,1,2})(1-P^0_{1/0,1,2}) + (1-q_0)q_1q_2(1-P^d_{2/1,2})P^0_{2/1,2}P^d_{1/1,2}+$$
$$+q_0q_1q_2(1-P^d_{2/0,1,2})P^0_{2/0,1,2}P^d_{1/0,1,2} \quad (3.6)$$

$$r_2^1 = (1-q_0)q_1q_2(1-P^d_{1/1,2})P^0_{1/1,2}(1-P^d_{2/1,2})P^0_{2/1,2} + q_0q_1q_2(1-P^d_{1/0,1,2})P^0_{1/0,1,2}(1-P^d_{2/0,1,2})P^0_{2/0,1,2} \quad (3.7)$$

In Fig. 3.2 we present the discrete time Markov Chain (DTMC) that describes the queue evolution. Each state is denoted by an integer and represents the queue size at the relay node. The transition matrix of the above DTMC is is a lower Hessenberg matrix given by:

$$P = \begin{pmatrix} a_0 & b_0 & 0 & 0 & \cdots \\ a_1 & b_1 & b_0 & 0 & \cdots \\ a_2 & b_2 & b_1 & b_0 & \cdots \\ 0 & b_3 & b_2 & b_1 & \cdots \\ 0 & 0 & b_3 & b_2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (3.8)$$

Where $a_0 = 1 - p_1^0 - p_2^0, a_1 = p_1^0, a_2 = p_2^0$, $b_0 = p_{-1}^1$ and $b_{i+1} = p_i^1$ $i = 0, 1, 2, 3$. The quantity $p_i^0$ ($p_i^1$) is the probability that the queue size increases by $i$ packets when the queue is empty (not empty). Note that $p_i^0 = r_i^0$, because when the queue is empty the probability of $i$ packets arriving is the same with the probability that the queue size increases by $i$ packets; when the queue is not empty however, this is not true. For example the probability of 2 packets arriving is not the same with the probability of increasing the queue size by 2; this is because both arrivals and departures can occur at the same time. The expressions for the $p_i^1$ are also given by lengthy expressions listed below:

$$p_{-1}^1 = q_0(1-q_1)(1-q_2)P_{0/0}^d + q_0(1-q_1)q_2P_{0/0,2}^dP_{2/0,2}^d + q_0(1-q_1)q_2P_{0/0,2}^d(1-P_{2/0,2}^d)(1-P_{2/0,2}^0)+$$
$$q_0q_1(1-q_2)P_{0/0,1}^dP_{1/0,1}^d + q_0q_1(1-q_2)P_{0/0,1}^d(1-P_{1/0,1}^d)(1-P_{1/0,1}^0) + q_0q_1q_2P_{0/0,1,2}^dP_{1/0,1,2}^dP_{2/0,1,2}^d+$$
$$+q_0q_1q_2P_{0/0,1,2}^d(1-P_{1/0,1,2}^d)(1-P_{1/0,1,2}^0)(1-P_{2/0,1,2}^d)(1-P_{2/0,1,2}^0) + q_0q_1q_2P_{0/0,1,2}^dP_{1/0,1,2}^d(1-P_{2/0,1,2}^d)(1-P_{2/0,1,2}^0)+$$
$$q_0q_1q_2P_{0/0,1,2}^d(1-P_{1/0,1,2}^d)(1-P_{1/0,1,2}^0)P_{2/0,1,2}^d \tag{3.9}$$

$$p_0^1 = 1 - p_{-1}^1 - p_1^1 - p_2^1 \tag{3.10}$$

$$p_1^1 = (1-q_0)q_1(1-q_2)(1-P_{1/1}^d)P_{1/1}^0 + (1-q_0)q_1q_2(1-P_{1/1,2}^d)P_{1/1,2}^0P_{2/1,2}^d+$$
$$(1-q_0)q_1q_2(1-P_{1/1,2}^d)P_{1/1,2}^0(1-P_{2/1,2}^d)(1-P_{2/1,2}^0) + (1-q_0)(1-q_1)q_2(1-P_{2/2}^d)P_{2/2}^0+$$
$$(1-q_0)q_1q_2(1-P_{2/1,2}^d)P_{2/1,2}^0P_{1/1,2}^d + (1-q_0)q_1q_2(1-P_{2/1,2}^d)P_{2/1,2}^0(1-P_{1/1,2}^d)(1-P_{1/1,2}^0)+$$
$$+q_0q_1q_2P_{0/0,1,2}^d(1-P_{1/0,1,2}^d)P_{1/0,1,2}^0(1-P_{2/0,1,2}^d)P_{2/0,1,2}^0 + q_0q_1(1-q_2)(1-P_{0/0,1}^d)(1-P_{1/0,1}^d)P_{1/0,1}^0+$$
$$+q_0q_1q_2(1-P_{0/0,1,2}^d)(1-P_{1/0,1,2}^d)P_{1/0,1,2}^0P_{2/0,1,2}^d + q_0q_1q_2(1-P_{0/0,1,2}^d)(1-P_{1/0,1,2}^d)P_{1/0,1,2}^0(1-P_{2/0,1,2}^d)(1-P_{2/0,1,2}^0)+$$
$$+q_0q_2(1-q_1)(1-P_{0/0,2}^d)(1-P_{2/0,2}^d)P_{2/0,2}^0 + q_0q_1q_2(1-P_{0/0,1,2}^d)(1-P_{2/0,1,2}^d)P_{2/0,1,2}^0P_{1/0,1,2}^d+$$
$$+q_0q_1q_2(1-P_{0/0,1,2}^d)(1-P_{2/0,1,2}^d)P_{2/0,1,2}^0(1-P_{1/0,1,2}^d)(1-P_{1/0,1,2}^0) \tag{3.11}$$

$$p_2^1 = (1-q_0)q_1q_2(1-P_{1/1,2}^d)P_{1/1,2}^0(1-P_{2/1,2}^d)P_{2/1,2}^0 + q_0q_1q_2(1-P_{0/0,1,2}^d)(1-P_{1/0,1,2}^d)P_{1/0,1,2}^0(1-P_{2/0,1,2}^d)P_{2/0,1,2}^0 \tag{3.12}$$

The difference equations that govern the evolution of the states are given by:

$$Ps = s \Rightarrow s_i = a_i s_0 + \sum_{j=1}^{i+1} b_{i-j+1} s_j \tag{3.13}$$

We apply the Z-transform technique to compute the steady state distribution, i.e. we let

$$A(z) = \sum_{i=0}^{2} a_i z^{-i}, B(z) = \sum_{i=0}^{3} b_i z^{-i}, S(z) = \sum_{i=0}^{\infty} s_i z^{-i} \tag{3.14}$$

It is known that [32]:

$$S(z) = s_0 \frac{z^{-1}A(z) - B(z)}{z^{-1} - B(z)} \tag{3.15}$$

It is also known that the probability of the queue in the relay is empty is given by [32]:

$$P\left(Q=0\right) = \frac{1+B^{'}(1)}{1+B^{'}(1)-A^{'}(1)} \tag{3.16}$$

The expressions of $A^{'}(1)$ and $B^{'}(1)$ are:

$$A^{'}(z) = \left(\sum_{i=0}^{2} a_i z^{-i}\right)^{'} = -\sum_{i=1}^{2} i a_i z^{-(i+1)} \Rightarrow A^{'}(1) = -\sum_{i=1}^{2} i a_i \Rightarrow A^{'}(1) = -\sum_{i=1}^{2} i p_i^0 = -\lambda_0 \tag{3.17}$$

$$B^{'}(z) = \left(\sum_{i=0}^{3} b_i z^{-i}\right)^{'} = -\sum_{i=0}^{3} i b_i z^{-(i+1)} \Rightarrow B^{'}(1) = -\sum_{i=0}^{3} i b_i = -b_1 - 2b_2 - 3b_3 = -1 + p_{-1}^1 - p_1^1 - 2p_2^1 \tag{3.18}$$

Then the the probability of the queue in the relay is empty is

$$P\left(Q=0\right) = \frac{p_{-1}^1 - p_1^1 - 2p_2^1}{p_{-1}^1 - p_1^1 - 2p_2^1 + \lambda_0} \tag{3.19}$$

So, the average arrival rate $\lambda$ is given by:

$$\lambda = \frac{p_{-1}^1 - p_1^1 - 2p_2^1}{p_{-1}^1 - p_1^1 - 2p_2^1 + \lambda_0}\lambda_0 + \frac{\lambda_0}{p_{-1}^1 - p_1^1 - 2p_2^1 + \lambda_0}\lambda_1 \tag{3.20}$$

#### Condition for the stability of the queue

An important tool to determine stability is Loyne's criterion [33], which states that if the arrival and service processes of a queue are jointly strictly stationary and ergodic, the queue is stable if and only if the average arrival rate is strictly less than the average service rate. If the queue is stable, the departure rate (throughput) is equal to the arrival rate. $\lambda_1 < \mu \Leftrightarrow r_1^1 + 2r_2^1 < \mu$ where $r_1^1 = (1-q_0)A_1 + q_0 B_1$, $r_2^1 = (1-q_0)A_2 + q_0 B_2$ and $\mu = q_0 A$. The expressions for $A, A_i, B_i$ are given by:

$$
\begin{aligned}
A_1 = {} & q_1(1-q_2)(1-P^d_{1/1})P^0_{1/1} + q_2(1-q_1)(1-P^d_{2/2})P^0_{2/2} + q_1q_2(1-P^d_{1/1,2})P^0_{1/1,2}(1-P^d_{2/1,2})(1-P^0_{2/1,2}) + \\
& + q_1q_2(1-P^d_{1/1,2})P^0_{1/1,2}P^d_{2/1,2} + q_1q_2(1-P^d_{2/1,2})P^0_{2/1,2}(1-P^d_{1/1,2})(1-P^0_{1/1,2}) + q_1q_2(1-P^d_{2/1,2})P^0_{2/1,2}P^d_{1/1,2} \\
B_1 = {} & q_1(1-q_2)(1-P^d_{1/0,1})P^0_{1/0,1} + q_2(1-q_1)(1-P^d_{2/0,2})P^0_{2/0,2} + q_1q_2(1-P^d_{1/0,1,2})P^0_{1/0,1,2}(1-P^d_{2/0,1,2})(1-P^0_{2/0,1,2}) + \\
& + q_1q_2(1-P^d_{1/0,1,2})P^0_{1/0,1,2}P^d_{2/0,1,2} + q_1q_2(1-P^d_{2/0,1,2})P^0_{2/0,1,2}(1-P^d_{1/0,1,2})(1-P^0_{1/0,1,2}) + \\
& + q_1q_2(1-P^d_{2/0,1,2})P^0_{2/0,1,2}P^d_{1/0,1,2}
\end{aligned} \tag{3.21}
$$

$$
\begin{aligned}
A_2 &= q_1q_2(1-P^d_{1/1,2})P^0_{1/1,2}(1-P^d_{2/1,2})P^0_{2/1,2}, \\
B_2 &= q_1q_2(1-P^d_{1/0,1,2})P^0_{1/0,1,2}(1-P^d_{2/0,1,2})P^0_{2/0,1,2}
\end{aligned} \tag{3.22}
$$

$$
A = (1-q_1)(1-q_2)P^d_{0/0} + q_1(1-q_2)P^d_{0/0,1} + q_2(1-q_1)P^d_{0/0,2} + q_1q_2P^d_{0/0,1,2} \tag{3.23}
$$

Then the values of $q_0$ for which the queue is stable is given by $q_{0min} < q_0 < 1$, where:

$$
q_{0min} = \frac{A_1 + 2A_2}{A + A_1 + 2A_2 - B_1 - 2B_2} \tag{3.24}
$$

### Average queue size

The average queue size is given by [32]: $\overline{Q} = -S^{'}(1)$ where $S^{'}(1) = s_0 \frac{K^{''}(1)}{L^{''}(1)}$. The expressions for $K(z)$ and $L(z)$ are given by:

$$K(z) = \left(-z^{-2}A(z) + z^{-1}A^{'}(z) - B^{'}(z)\right)\left(z^{-1} - B(z)\right) - \left(z^{-1}A(z) - B(z)\right)\left(-z^{-2} - B^{'}(z)\right) \tag{3.25}$$

$$L(z) = \left(z^{-1} - B(z)\right)^2 \tag{3.26}$$

Then $K^{''}(1)$ and $L^{''}(1)$ are given by:

$$\Rightarrow K^{''}(1) = \left(2A(1) - 2A^{'}(1) + A^{''}(1) - B^{''}(1)\right)\left(-1 - B^{'}(1)\right) - \left(2 - B^{''}(1)\right)\left(-A(1) + A^{'}(1) - B^{'}(1)\right) \tag{3.27}$$

$$L^{''}(z) = \left[2\left(z^{-1} - B(z)\right)\left(-z^{-2} - B^{'}(z)\right)\right]^{'} \Rightarrow L^{''}(1) = 2\left(-1 - B^{'}(1)\right)^2 \tag{3.28}$$

The values of $A^{''}(1)$ and $B^{''}(1)$ are:

$$A^{''}(z) = \left(-\sum_{i=1}^{2} i a_i z^{-(i+1)}\right)^{'} = \sum_{i=1}^{2} i(i+1) a_i z^{-(i+2)} \Rightarrow A^{''}(1) = 2p_1^0 + 6p_2^0 \tag{3.29}$$

$$B^{''}(z) = \left(-\sum_{i=1}^{3} i b_i z^{-(i+1)}\right)^{'} = \sum_{i=1}^{3} i(i+1) b_i z^{-(i+2)} \Rightarrow B^{''}(1) = 2 - 2p_{-1}^1 + 4p_1^1 + 10p_2^1 \tag{3.30}$$

The average queue size is given by:

$$\overline{Q} = \frac{(p_1^1 + 2p_2^1 - p_{-1}^1)(4p_1^0 + 10p_2^0) + \lambda_0(2p_{-1}^1 - 4p_1^1 - 10p_2^1)}{2(p_1^1 + 2p_2^1 - p_{-1}^1)(p_{-1}^1 - p_1^1 - 2p_2^1 + \lambda_0)} \tag{3.31}$$

**The throughput per user and the aggregate throughput**

The throughput rates $\mu_1, \mu_2$ for the users $1, 2$ are given by:

$$\mu_1 = q_0 P\left(Q > 0\right) q_1 \left\{(1 - q_2)\left[P^d_{1/0,1} + (1 - P^d_{1/0,1})P^0_{1/0,1}\right] + q_2\left[P^d_{1/0,1,2} + (1 - P^d_{1/0,1,2})P^0_{1/0,1,2}\right]\right\} + \\ + \left[1 - q_0 P\left(Q > 0\right)\right] q_1 \left\{(1 - q_2)\left[P^d_{1/1} + (1 - P^d_{1/1})P^0_{1/1}\right] + q_2\left[P^d_{1/1,2} + (1 - P^d_{1/1,2})P^0_{1/1,2}\right]\right\} \tag{3.32}$$

$$\mu_2 = q_0 P\left(Q > 0\right) q_2 \left\{(1 - q_1)\left[P^d_{2/0,2} + (1 - P^d_{2/0,2})P^0_{2/0,2}\right] + q_1\left[P^d_{2/0,1,2} + (1 - P^d_{2/0,1,2})P^0_{2/0,1,2}\right]\right\} + \\ + \left[1 - q_0 P\left(Q > 0\right)\right] q_2 \left\{(1 - q_1)\left[P^d_{2/2} + (1 - P^d_{2/2})P^0_{2/2}\right] + q_1\left[P^d_{2/1,2} + (1 - P^d_{2/1,2})P^0_{2/1,2}\right]\right\} \tag{3.33}$$

In the equations above we assume that the queue is stable, hence the arrival rate from each user to the queue is a contribution to its overall throughput. The aggregate throughput is $\mu_{total} = \mu_1 + \mu_2$. Notice that the throughput per user is independent of $q_0$ as long as it is in the stability region. This is explained because the product $q_0 P\left(Q > 0\right)$ is constant. The proof is straightforward and thus is omitted.

When the queue is unstable however, the aggregate throughput is the summation of all the direct throughput between the users and the destination plus the service rate of the relay.

### 3.3.2 N-symmetric users

We now generalize the above for the case of a symmetric $n$-users network. Each user attempts to transmit in a slot with probability $q$; the success probability to the relay and the destination when $i$ nodes transmit are given by $P_{0,i}$, $P_{d,i}$ respectively. There are two cases for the $P_{d,i}$ , $P_{d,i,0}$, $P_{d,i,1}$ denoting success probability when relay remains silent or transmits respectively. The above success probabilities for the symmetric case are given by $P_{d,i,j} = P_d\left(\frac{1}{1+\gamma_d}\right)^{i-1}\left(\frac{1}{1+\beta\gamma_0}\right)^j$, $j = 0,1$ and $\beta = \frac{v_{0d}g_{0d}}{v_d g_d} > 1$. $P_{0d,i} = P_{0d}\left(\frac{1}{1+\frac{1}{\beta}\gamma_d}\right)^i$, $P_0 = \exp\left(-\frac{\gamma_0\eta_0}{v_0 g_0}\right)$, $P_d = \exp\left(-\frac{\gamma_d\eta_d}{v_d g_d}\right)$, $P_{0d} = \exp\left(-\frac{\gamma_0\eta_0}{v_0 g_0}\right)$. There are two cases for the $P_{0,i}$, $P_{0,i,0}, P_{0,i,1}$ denoting success probability when relay remains silent or transmits respectively. The success probabilities are given by $P_{0,i,0} = P_0\left(\frac{1}{1+\gamma_0}\right)^{i-1}$ and $P_{0,i,1} = P_0\left(1+\gamma_0 r_0^{\alpha} g\right)^{-1}\left(\frac{1}{1+\gamma_0}\right)^{i-1}$ where $r_0$ is the distance between the users and the relay, $\alpha$ is the path loss exponent and $g$ is the self interference coefficient.

**Computation of the average arrival and service rate**

The service rate is given by the following equation:

$$\mu = \sum_{k=0}^{n}\binom{n}{k} q_0 q^k (1-q)^{n-k} P_{0d,k} \tag{3.34}$$

The average arrival rate $\lambda$ of the queue is given by:

$$\lambda = P\left(Q=0\right)\lambda_0 + P\left(Q>0\right)\lambda_1 \tag{3.35}$$

$\lambda_0 = \sum_{k=1}^{n} k r_k^0$ where the $r_k^0$ is the probability that the relay received $k$ packets when the queue is empty, the expression for $r_k^0$ is given by:

$$r_k^0 = \sum_{i=k}^{n}\binom{n}{i}\binom{i}{k} q^i (1-q)^{n-i} P_{0,i,0}^k \left(1-P_{d,i,0}\right)^k \left[1-P_{0,i,0}(1-P_{d,i,0})\right]^{i-k},\ 1 \le k \le n \tag{3.36}$$

$\lambda_1 = \sum_{k=1}^{n} k r_k^1$ where the $r_k^1$ is the probability that the relay received $k$ packets when the queue is not empty and is given by:

$$r_k^1 = (1-q_0)\sum_{i=k}^{n}\binom{n}{i}\binom{i}{k}q^i(1-q)^{n-i}P_{0,i,0}^k\left(1-P_{d,i,0}\right)^k\left[1-P_{0,i,0}(1-P_{d,i,0})\right]^{i-k}+$$
$$+q_0\sum_{i=k}^{n}\binom{n}{i}\binom{i}{k}q^i(1-q)^{n-i}P_{0,i,1}^k\left(1-P_{d,i,1}\right)^k\left[1-P_{0,i,1}(1-P_{d,i,1})\right]^{i-k},\ 1\le k\le n \tag{3.37}$$

The elements of the transition matrix are given by: $a_k = p_k^0$, $b_0 = p_{-1}^1$, $b_1 = p_0^1$ and $b_{k+1} = p_k^1\ \forall k > 0$ where:

$$p_k^0 = \sum_{i=k}^{n}\binom{n}{i}\binom{i}{k}q^i(1-q)^{n-i}P_{0,i,0}^k\left(1-P_{d,i,0}\right)^k\left[1-P_{0,i,0}(1-P_{d,i,0})\right]^{i-k},\ 1\le k\le n \tag{3.38}$$

$$p_{-1}^1 = q_0\sum_{k=0}^{n}\binom{n}{k}q^k(1-q)^{n-k}P_{0d,k}\left[1-P_{0,k,1}(1-P_{d,k,1})\right]^k \tag{3.39}$$

$$p_k^1 = (1-q_0)\sum_{i=k}^{n}\binom{n}{i}\binom{i}{k}q^i(1-q)^{n-i}P_{0,i,0}^k\left(1-P_{d,i,0}\right)^k\left[1-P_{0,i,0}(1-P_{d,i,0})\right]^{i-k}+$$
$$+q_0\sum_{i=k}^{n}\binom{n}{i}\binom{i}{k}q^i(1-q)^{n-i}(1-P_{0d,i})P_{0,i,1}^k\left(1-P_{d,i,1}\right)^k\left[1-P_{0,i,1}(1-P_{d,i,1})\right]^{i-k}+$$
$$+q_0\sum_{i=k+1}^{n}\binom{n}{i}\binom{i}{k+1}q^i(1-q)^{n-i}P_{0d,i}P_{0,i,1}^{k+1}\left(1-P_{d,i,1}\right)^{k+1}\left[1-P_{0,i,1}(1-P_{d,i,1})\right]^{i-k-1} \tag{3.40}$$

$$p_0^1 = 1 - p_{-1}^1 - \sum_{i=1}^{n} p_i^1 \tag{3.41}$$

The probability that the queue in the relay is empty is given by (3.16), the expressions for $A'(1)$ and $B'(1)$ are:

$$A^{'}(z)=\left(\sum_{i=0}^{n}a_iz^{-i}\right)^{'}=-\sum_{i=1}^{n}ia_iz^{-(i+1)}\Rightarrow A^{'}(1)=-\sum_{i=1}^{n}ia_i\Rightarrow A^{'}(1)=-\sum_{i=1}^{n}ip_i^0=-\lambda_0 \tag{3.42}$$

$$B^{'}(z)=\left(\sum_{i=0}^{n+1}b_iz^{-i}\right)^{'}=-\sum_{i=i}^{n+1}ib_iz^{-(i+1)}\Rightarrow B^{'}(1)=-\sum_{i=i}^{n+1}ib_i=-b_1-\sum_{i=2}^{n+1}ib_i= \\ =-1+p_{-1}^1-\sum_{i=1}^{n}ip_i^1 \tag{3.43}$$

Then the probability that the queue in the relay is empty is given by:

$$P\left(Q=0\right)=\frac{p_{-1}^1-\sum\limits_{i=1e}^{n}ip_i^1}{p_{-1}^1-\sum\limits_{i=1}^{n}ip_i^1+\lambda_0} \tag{3.44}$$

**Condition for the stability of the queue**

$\lambda_1<\mu\Leftrightarrow\sum_{k=1}^{n}kr_k^1<\mu$ where $r_k^1=(1-q_0)A_k+q_0B_k$ and $\mu=q_0A$. The expressions for $A, A_k, B_k$ are :

$$A_k=\sum_{i=k}^{n}\binom{n}{i}\binom{i}{k}q^i(1-q)^{n-i}P_{0,i,0}^k\left(1-P_{d,i,0}\right)^k\left[1-P_{0,i,0}(1-P_{d,i,0})\right]^{i-k} \tag{3.45}$$

$$B_k=\sum_{i=k}^{n}\binom{n}{i}\binom{i}{k}q^i(1-q)^{n-i}P_{0,i,1}^k\left(1-P_{d,i,1}\right)^k\left[1-P_{0,i,1}(1-P_{d,i,1})\right]^{i-k} \tag{3.46}$$

$$A=\sum_{k=0}^{n}\binom{n}{k}q^k(1-q)^{n-k}P_{0d,k} \tag{3.47}$$

The values of $q_0$ for which the queue is stable is given by $q_{0min} < q_0 < 1$, where:

$$q_{0min} = \frac{\sum_{k=1}^{n} kA_k}{A + \sum_{k=1}^{n} kA_k - \sum_{k=1}^{n} kB_k} \tag{3.48}$$

**Average queue size**

As we showed in the Section 3.3.1, the average queue size is given by: $\overline{Q} = -S^{'}(1)$ where $S^{'}(1) = s_0 \frac{K^{''}(1)}{L^{''}(1)}$. The expressions for $K^{''}(1)$ and $L^{''}(1)$ are given by (3.27) and (3.28). The expressions for $A^{''}(1)$ and $B^{''}(1)$ are:

$$A^{''}(z) = \left(-\sum_{i=1}^{n} ia_i z^{-(i+1)}\right)^{'} = \sum_{i=1}^{n} i(i+1)a_i z^{-(i+2)} \Rightarrow A^{''}(1) = \sum_{i=1}^{n} i(i+1)a_i = \\ = \sum_{i=1}^{n} i(i+1)p_i^0 \tag{3.49}$$

$$B^{''}(z) = \left(-\sum_{i=i}^{n+1} ib_i z^{-(i+1)}\right)^{'} = \sum_{i=1}^{n+1} i(i+1)b_i z^{-(i+2)} \Rightarrow B^{''}(1) = \sum_{i=1}^{n+1} i(i+1)b_i = \\ = 2 - 2p_{-1}^1 + \sum_{i=1}^{n} i(i+3)p_i^1 \tag{3.50}$$

Following the same methodology as in Section 3.3.1 we obtain that the average queue size is given by:

$$\overline{Q} = \frac{\left(\sum_{i=1}^{n} ip_i^1 - p_{-1}^1\right)\sum_{i=1}^{n} i(i+3)p_i^0 + \lambda_0\left(2p_{-1}^1 - \sum_{i=1}^{n} i(i+3)p_i^1\right)}{2\left(\sum_{i=1}^{n} ip_i^1 - p_{-1}^1\right)\left(p_{-1}^1 - \sum_{i=1}^{n} ip_i^1 + \lambda_0\right)} \tag{3.51}$$

**The throughput per user and the aggregate throughput**

The throughput per user for the network with the relay when the queue is stable is given by:

$$\mu = q_0 P\left(Q>0\right)\sum_{k=0}^{n-1}\binom{n-1}{k}q^{k+1}(1-q)^{n-1-k}\left[P_{d,k+1,1}+(1-P_{d,k+1,1})P_{0,k+1,1}\right]+ \\ +\left[1-q_0P\left(Q>0\right)\right]\sum_{k=0}^{n-1}\binom{n-1}{k}q^{k+1}(1-q)^{n-1-k}\left[P_{d,k+1,0}+\left(1-P_{d,k+1,0}\right)P_{0,k+1,0}\right] \tag{3.52}$$

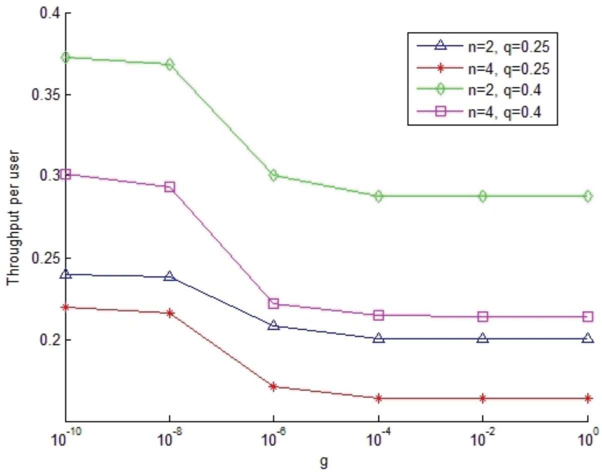
When the queue is unstable though, the throughput per user is given by the summation of the direct to the destination throughput plus the service rate of the relay divided by the number of the users $n$. The aggregate throughput is $\mu_{total} = n\mu$.

## 3.4 Numerical Results

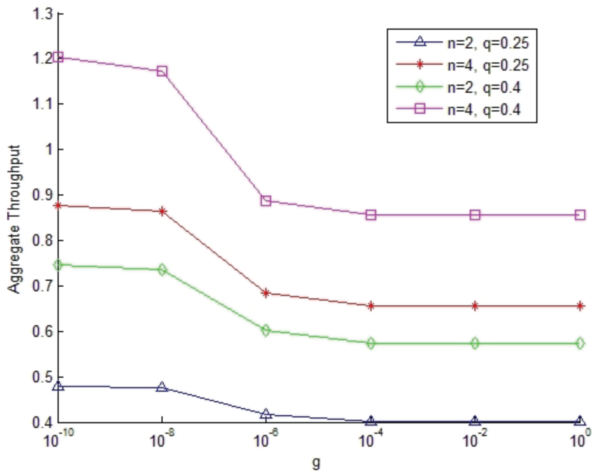
In this section we present numerical results for the analysis presented above. To simplify the presentation we consider the case where all the users have the same link characteristics and transmission probabilities. The parameters used in the numerical results are as follows. The distances in meters are given by $r_d = 130$, $r_0 = 60$ and $r_{0d} = 80$. The path loss is $\alpha = 4$ and the receiver noise power $\eta = 10^{-11}$. The transmit power for the relay is $P_{tx}(0) = 10$ mW and for the i-th user $P_{tx}(i) = 1$ mW. We used $\gamma < 1$ because it is possible for two or more users to transmit successfully at the same time.

The figures 3.3(a) and 3.5(a) present the throughput per user versus $g$ (the self-interference coefficient) for various values of $q$,$\gamma$ and $n$. The figures 3.3(b) and 3.5(b) show the aggregate throughput versus $g$.

The figures 3.4(a) and 3.6(a) present the throughput per user versus the number of the users in the network for various values of $q$,and $g$ for $\gamma = 0.2$ and $\gamma = 0.6$ respectively. The figures 3.4(b) and 3.6(b) show the aggregate throughput versus the number of the users.



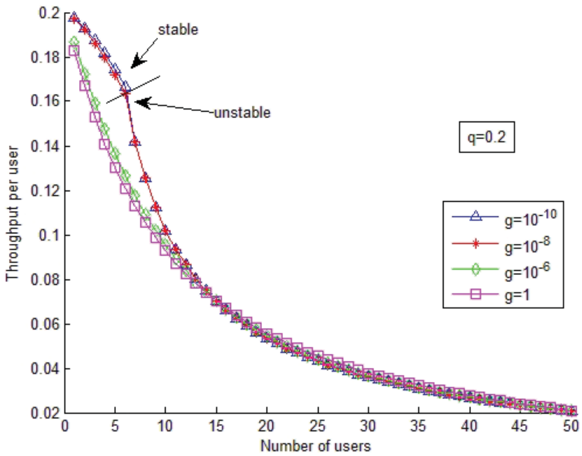

(a) Throughput per user vs self interference coefficient



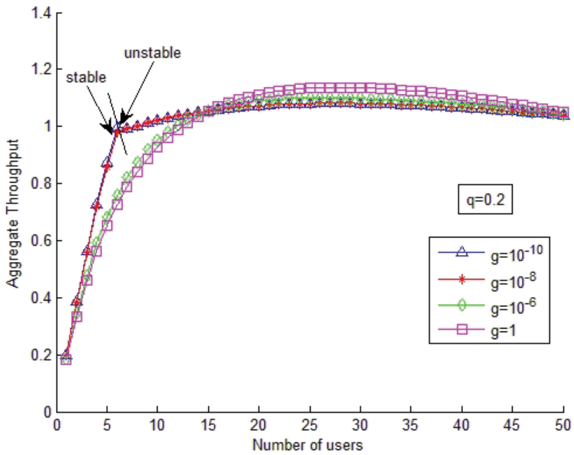

(b) Aggregate throughput vs self interference coefficient

Figure 3.3: Throughput per user and aggregate throughput vs the self interference coefficient for $\gamma = 0.2$

When $\gamma = 0.2$ we observe that for $g = 10^{-10}$ and $g = 10^{-8}$ (almost perfect self-interference cancelation) the relay's queue is unstable for relative small number of users. The previous result is because the small value of $\gamma$ is more likely more transmissions from the users to the relay to be successful, but at the same time the relay can transmit at most one packet per time slot. For $\gamma = 0.6$ the queue is never unstable for the parameters described in the figures, and




(a) Throughput per user vs the number of the users

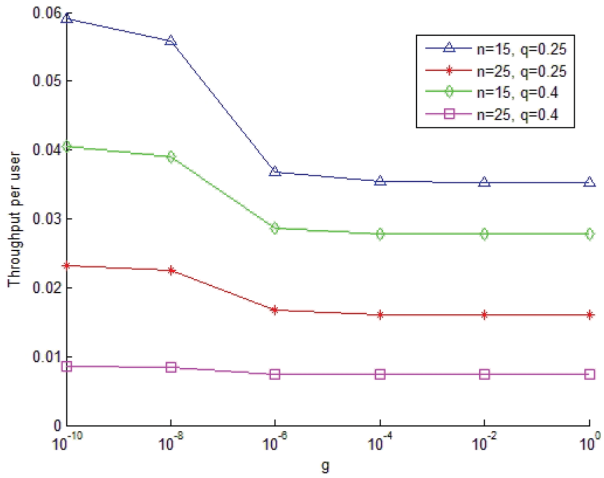



(b) Aggregate throughput vs the number of the users

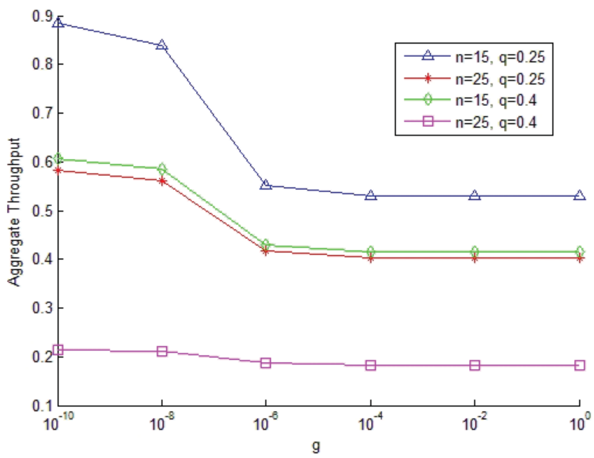
Figure 3.4: Throughput per user and aggregate throughput vs the number of the users for $\gamma = 0.2$

for $g = 10^{-10}$ and $g = 10^{-8}$ the advantages in term of throughput are obvious compared to no self interference cancelation.

The Fig. 3.7(a) and Fig. 3.7(b) show the $q_{0min}$ vs $n$ for $\gamma = 0.2$ and $\gamma = 0.6$ respectively. Note that $q_{0min} < q < 1$, so when $q_{0min} \geq 1$ then the queue is unstable as in case of $\gamma = 0.2$ for $g = 10^{-10}$ and $g = 10^{-8}$.




(a) Throughput per user vs self interference coefficient

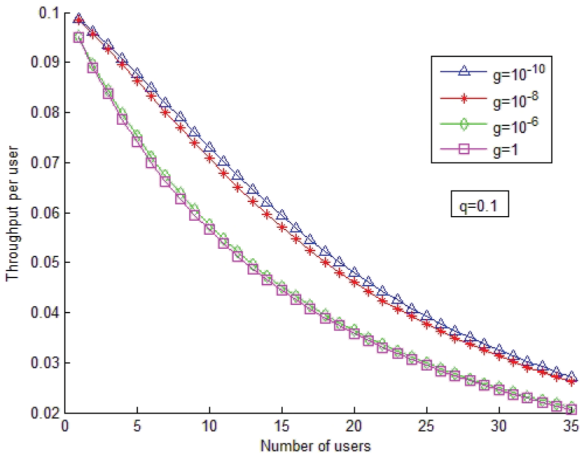



(b) Aggregate throughput vs self interference coefficient

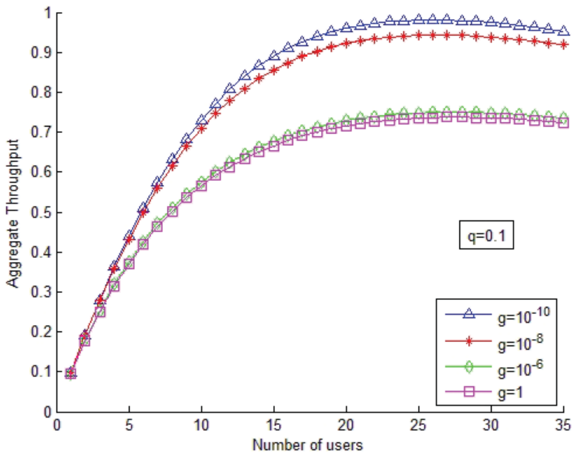
Figure 3.5: Throughput per user and aggregate throughput vs the self interference coefficient for $\gamma = 0.6$

## 3.5 Conclusions

In this work, we examined the operation of a node relaying packets from a number of users to a common destination node. We assumed MPR capability for the relay and for the destination node. We studied a multiple capture model, where a user's transmission is successful if the received $SINR$ is above a threshold $\gamma$. The relay node can also receive and transmit




(a) Throughput per user vs the number of the users

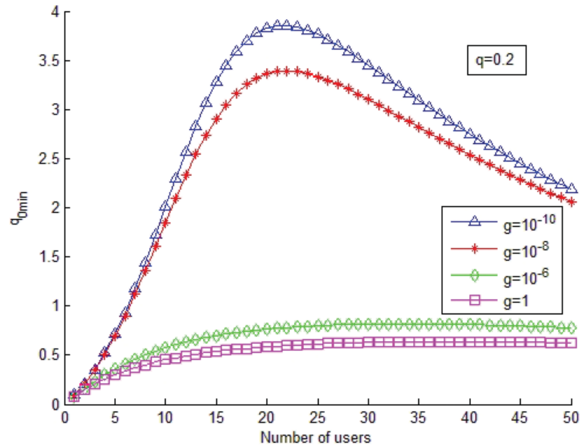



(b) Aggregate throughput vs the number of the users

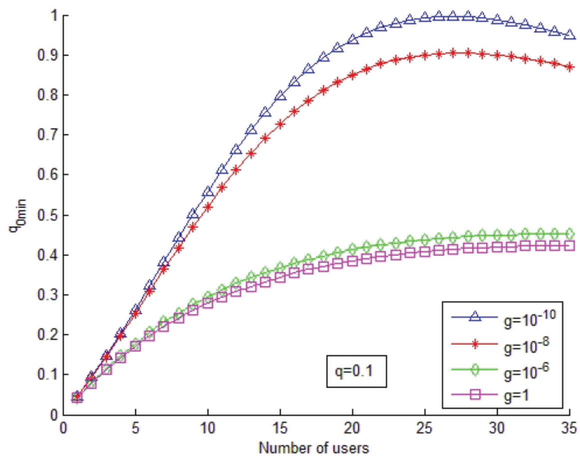
Figure 3.6: Throughput per user and aggregate throughput vs the number of the users for $\gamma = 0.6$

simultaneously, so the problem of self interference arises.

We obtained analytical expressions for the relay's queue characteristics such as the stability condition, the values of the arrival and service rates, the average queue size. We studied the throughput per user and the aggregate throughput, and found that, under stability conditions, the throughput per user does not depend on the relay probability of transmission. We




(a) Stability threshold vs the number of the users for $\gamma = 0.2$




(b) Stability threshold vs the number of the users for $\gamma = 0.6$

Figure 3.7: Stability threshold vs the number of the users

studied the impact of self interference coefficient on the throughput per user and the aggregate throughput of the network.

We showed that for perfect self-interference cancelation, the advantages are obvious. Another interesting result is that the self interference coefficient plays a crucial role when $\gamma$ is small (and $g$ tends to zero) because it can easily cause an unstable queue.

Future extensions of this work should include users with non-saturated queues i.e. sources with external random arrivals, a relay node with its own packets and different priorities for the users.

# Chapter 4

# Wireless Network-Level Partial Relay Cooperation

The material in this chapter was presented in [36].

## 4.1 Introduction

Cooperative communication helps overcome fading and attenuation in wireless networks. Its main purpose is to increase the communication rates across the network and to increase reliability of time-varying links. It is known that wireless communication from a source to a destination can benefit from the cooperation of nodes that overhear the transmission. The classical single relay channel [1] exemplifies this situation. Further work on the relay channel in [2] and [3] has enabled substantial performance improvement.

However, there is evidence that additional gains can be achieved with "network-layer" cooperation (or packet-level cooperation), that is plain relaying without any physical layer considerations [4] and [5]. In this work, we focus on this type of cooperation. The work in [6] investigated the network-level cooperation in a network consisting of a source and a relay by considering the cases of full or no cooperation at the relay. A key difference between physical-layer and network-layer cooperation ideas is that the objective rate function that is maximized is the so-called stable throughput region which captures the bursty nature of traffic from the source. In [6], it was shown that the stability region of full cooperation under random-access does not always strictly contain the non-cooperative stability region.

The main contribution in this work is to introduce the notion of partial network-level cooperation by adding a flow controller for the traffic coming to the relay from the source. We prove that the system is always better than or at least equal to the system without the flow controller. Specifically, we provide an exact characterization of the stability region of a network consisting of a source, a relay and a destination node as shown in Fig. 4.1. We consider the collision channel with erasures and random access of the medium. The source and the relay node have external arrivals; furthermore, the relay is forwarding part of the source node's traffic to the destination. Unlike the work in [6], the relay node is equipped with a flow controller that regulates the internal arrivals from the source based on the conditions in the network to ensure the stability of the queues. We characterize the stable throughput region under conditions of no cooperation at all, full cooperation, and probabilistic (opportunistic) cooperation. By probabilistic cooperation we mean that under certain conditions in the network, the relay may accept a packet from the source. The characterization of the stability regions is known to be challenging because the queues of the users are coupled (i.e., the service process of a queue depends on the status of the other queues). A tool that bypasses this difficulty is the stochastic dominance technique [37].

## 4.2 System Model

We consider a time-slotted system in which the nodes are randomly accessing a common receiver as shown in Fig. 4.1. We denote with $S$, $R$, and $D$ the source, the relay and the destination, respectively. Packet traffic originates from $S$ and $R$. Because of the wireless broadcast nature, $R$ may receive some of the packets transmitted from $S$ and then relay those

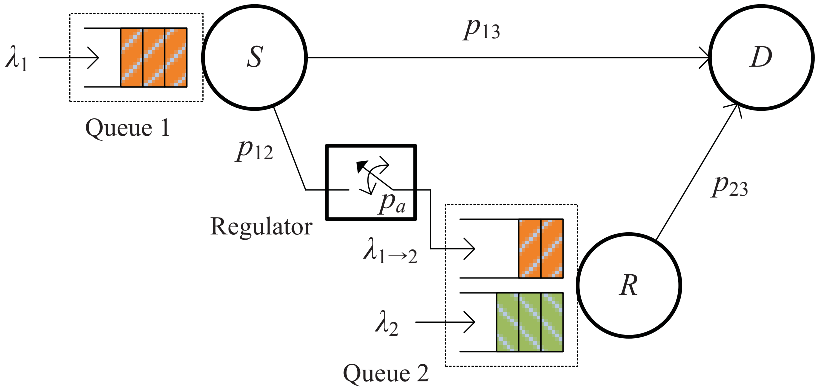



Figure 4.1: Network model with regulator at the relay

packets to $D$. The packets from $S$ which failed to be received by $D$ but were successfully received by $R$ are relayed by $R$. As we impose half-duplex constraint, $R$ can overhear $S$ only when it is idle. Each node has an infinite size buffer for storing incoming packets, and the transmission of each packet occupies one time slot. Node $R$ has separate queues for the exogenous arrivals and the endogenous arrivals that are relayed through $R$. But, we can let $R$ to maintain a single queue and merge all the arrivals into a single queue as the achievable stable throughput region is not affected [6]. This is because the link quality between $R$ and $D$ is independent of which packet is selected for transmission.

The packet arrival processes at $S$ and $R$ are assumed to be Bernoulli with rates $\lambda_1$ and $\lambda_2$, respectively, and are independent of each other. Node $R$ is equipped with a flow controller that regulates the rate of endogenous arrivals from $S$ by randomly accepting the incoming packets with probability $p_a$; that is, it controls the *amount of cooperation* that it is willing to provide. In each time slot, nodes $S$ and $R$ attempt to transmit with probabilities $q_1$ and $q_2$, respectively, if their queues are not empty. Decisions on transmission are made independently among the nodes. We assumed collision channel with erasures in which, if both $S$ and $R$ transmit in the same time slot, a collision occurs and both transmissions fail. The probability that a packet transmitted by node $i$ is successfully decoded at node $j(\neq i)$ is denoted by $p_{ij}$ which is the probability that the signal-to-noise-ratio (SNR) over the specified link exceeds a certain threshold for the successful decoding. These erasure probabilities capture the effect of random fading at the physical layer. The probabilities $p_{13}$, $p_{23}$, and $p_{12}$ denote the success probabilities over the link $S-R$, $R-D$, and $S-R$, respectively. Node $R$ has a better channel to $D$ than $S$, that is $p_{23} > p_{13}$.

The cooperation is performed at the protocol level as follows. When $S$ transmits a packet, if $D$ decodes the packet successfully, it sends an ACK and the packet exits the network; if $D$ fails to decode the packet but $R$ does and the flow controller decides to relay the packet, then $R$ sends an ACK and takes over the responsibility of delivering the packet to $D$ by placing it in its queue. If neither $D$ nor $R$ decode (or if $R$ does not store the packet), the packet remains in $S$'s queue for retransmission. The ACKs are assumed to be error-free, instantaneous and broadcasted to all relevant nodes.

Denote by $Q_i^t$ the length of queue $i$ at the beginning of time slot $t$. Based on the definition in [38], the queue is said to be *stable* if

$$\lim_{t\to\infty} Pr[Q_i^t < x] = F(x) \text{ and } \lim_{x\to\infty} F(x) = 1$$

Loynes' theorem [33] states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, then the queue is stable. If the average arrival rate is greater than the average service rate, then the queue is unstable and the value of $Q_i^t$ approaches infinity almost surely. The stability region of the system is defined as the set of arrival rate vectors $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$ for which the queues in the system are stable.

## 4.3 Main Results

This section describes the stability region for the system presented in the previous section and depicted in Fig. 4.1. The relay node $R$ is equipped with a flow controller, and the parameter $p_a$ of the flow controller is the probability to accept for the received packet from the source

$S$. So, our objective is to find the optimum value of $p_a$ denoted by $p_a^*$ which maximizes the stability region. This value reflects the cooperation degree that maximizes the stability region.

**Theorem 4.3.1.** *The stability region of the opportunistic cooperative network depicted in Fig. 4.1 is described by:*

$$\mathcal{R} = \mathcal{R}_1 \bigcup \mathcal{R}_2 \tag{4.1}$$

- *The subregion $\mathcal{R}_1$ is described as follows:*
  - *if $q_1 < \frac{p_{23}}{p_{13}+p_{23}}$, then $p_a^* = 1$ and the region is given by Eq.(4.2).*
  - *if $q_1 \geq \frac{p_{23}}{p_{13}+p_{23}}$, then $p_a^* = 0$ and the region is given by Eq.(4.3).*
- *The subregion $\mathcal{R}_2$ is described as follows:*
  - *if $q_2 \geq \frac{p_{13}}{p_{13}+p_{23}}$, then $p_a^* = 1$ and the region is given by Eq.(4.4).*
  - *if $q_2 < \frac{p_{13}}{p_{13}+p_{23}}$, then the subregion $\mathcal{R}_2$ is $\mathcal{R}_2 = \mathcal{R}_2^{'} \bigcup \mathcal{R}_2^{''}$ where:*
    - *if $\lambda_1 < q_1(1-q_2)p_{13}$, then $p_a^* = 0$ and the region is given by Eq.(4.5).*
    - *if $\lambda_1 \geq q_1(1-q_2)p_{13}$, then $p_a^* = \frac{\lambda_1 - q_1(1-q_2)p_{13}}{q_1(1-q_2)(1-p_{13})p_{12}}$ and the region is given by Eq.(4.6).*

*Proof.* The proof is given in Section 4.4. □

As seen in the theorem, there are three possible optimal values of $p_a$. When $p_a^*$ equals to 0 or 1, the relay rejects or accepts all the incoming traffic from the source, respectively. The more interesting case is when $q_2 < \frac{p_{13}}{p_{13}+p_{23}}$ (the relay transmission probability is less than a threshold which is a function of the channel success probabilities) and at the same time the average arrival rate at the source is $\lambda_1 \geq q_1(1-q_2)p_{13}$; in this case the optimum cooperation strategy is probabilistic routing by the relay. The incoming traffic from the source is relayed in part, meaning that the relay accepts a packet from the source with probability $p_a^*$, where $p_a^* = \frac{\lambda_1 - q_1(1-q_2)p_{13}}{q_1(1-q_2)(1-p_{13})p_{12}}$ $(0 < p_a^* < 1)$. The intuition behind this result is that when the relay is not attempting to transmit "very often" and at the same time, the arrival rate at the source is greater than a certain value, then the relay is cooperating only partially. *Thus, $p_a^*$ controls the amount of cooperation.*

$$\mathcal{R}_1 = \Bigg\{(\lambda_1, \lambda_2) : \frac{q_1 p_{12}(1-p_{13}) + (1-q_1)p_{23}}{q_1[p_{13} + (1-p_{13})p_{12}]}\lambda_1 + \lambda_2 < (1-q_1)p_{23}, \\ \frac{p_{12}(1-p_{13})}{p_{12}(1-p_{13}) + p_{13}}\lambda_1 + \lambda_2 < q_2(1-q_1)p_{23}\Bigg\} \quad (4.2)$$

$$\mathcal{R}_1 = \Bigg\{(\lambda_1, \lambda_2) : \frac{\lambda_1}{q_1 p_{13}} + \frac{\lambda_2}{(1-q_1)p_{23}} < 1, \;\; \lambda_2 < q_2(1-q_1)p_{23}\Bigg\} \quad (4.3)$$

$$\mathcal{R}_2 = \Bigg\{(\lambda_1, \lambda_2) : \frac{(1-q_2)p_{12}(1-p_{13}) + q_2 p_{23}}{(1-q_2)\left[p_{13} + (1-p_{13})p_{12}\right]}\lambda_1 + \lambda_2 < q_2 p_{23}, \\ \lambda_1 < q_1(1-q_2)\left[p_{13} + (1-p_{13})p_{12}\right]\} \quad (4.4)$$

$$\mathcal{R}_2^{'} = \Bigg\{(\lambda_1, \lambda_2) : \frac{\lambda_1}{(1-q_2)p_{13}} + \frac{\lambda_2}{q_2 p_{23}} < 1, \;\; \lambda_1 < q_1(1-q_2)p_{13}\Bigg\} \quad (4.5)$$

$$\mathcal{R}_2^{''} = \{(\lambda_1, \lambda_2) : \lambda_1 + \lambda_2 < q_1(1-q_2)p_{13} + q_2 p_{23}(1-q_1), \\ q_1(1-q_2)p_{13} \leq \lambda_1 < q_1(1-q_2)[p_{13} + (1-p_{13})p_{12}]\} \quad (4.6)$$

## 4.4 Stability Analysis using Stochastic Dominance

The expressions for the average service rates seen by source $S$ and relay $R$ are given by:

$$\mu_1 = \{(1-q_2)\Pr(Q_2 \neq 0) + \Pr(Q_2 = 0)\}\, q_1 \left(p_{13} + (1-p_{13})p_{12}p_a\right) \tag{4.7}$$

and

$$\mu_2 = q_2 \left[1 - q_1 \Pr(Q_1 \neq 0)\right] p_{23} \tag{4.8}$$

Since the average service rate of each queue $\mu_1$ and $\mu_2$ depends on the queue size of the other queue, they cannot be computed directly. We bypass this difficulty by utilizing the idea of stochastic dominance [37]; that is, we first construct hypothetical dominant systems, in which one of the nodes transmits dummy packets even when its packet queue is empty. Since the queue sizes in the dominant system are, at all times, at least as large as those of the original system, the stability region of the dominant system inner-bounds that of the original system. It turns out, however, that the stability region obtained using this stochastic dominance technique coincides with that of the original system which will be discussed in detail later in this section. Thus, the stability regions for both the original and the dominant systems are the same.

### 4.4.1 The first dominant system: source node transmits dummy packets

In this sub-section we obtain the region $\mathcal{R}_1$ of Theorem 4.3.1. We consider the first dominant system, in which node $S$ transmits dummy packets with probability $q_1$ whenever its queue is empty, while node $R$ behaves in the same way as in the original system. All other assumptions remain unaltered in the dominant system. Thus, the service rate at the relay node is given by:

$$\mu_{Q_2} = q_2 \left(1 - q_1\right) p_{23} \tag{4.9}$$

To derive the stability condition for the queue in the relay node, we need to calculate the total arrival rate. There are two independent arrival processes at the relay: the exogenous traffic with arrival rate $\lambda_2$ and the endogenous traffic from $S$. In the dominant system, when $R$ receives a dummy packet from $S$, it simply discards that packet. When the dominant system is stable, the queue at $S$ is stable, so the departure rate of the source packets (excluding the dummy ones) is equal to the arrival rate $\lambda_1$. Denote by $S_A$ the event that $S$ transmits a packet and the packet leaves the queue, then:

$$\Pr(S_A) = [(1-q_2)\Pr(Q_2 \neq 0) + \Pr(Q_2 = 0)]\,[p_{13} + (1-p_{13})p_{12}p_a] \tag{4.10}$$

Among the packets that depart from the queue of $S$, some will exit the network because they are decoded by the destination directly, and some will be relayed by $R$. Denote by $S_B$ the event that the transmitted packet from $S$ will be relayed from $R$, then:

$$\Pr(S_B) = [(1-q_2)\Pr(Q_2 \neq 0) + \Pr(Q_2 = 0)]\,(1-p_{13})p_{12}p_a \tag{4.11}$$

The conditional probability that a transmitted packet from $S$ (dummy packets excluded) arrives at $R$ given that the transmitted packet exits node $S$'s queue is given by:

$$\Pr(S_B|S_A) = \frac{(1-p_{13})p_{12}p_a}{p_{13} + (1-p_{13})p_{12}p_a} \tag{4.12}$$

The total arrival rate at the relay node is:

$$\lambda_{Q_2} = \lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13} + (1-p_{13})p_{12}p_a}\lambda_1 \tag{4.13}$$

By Loyne's Theorem, the stability condition for queue 2 at node $R$ is given by $\lambda_{Q_2} < \mu_{Q_2}$ and, thus:

$$\lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13} + (1-p_{13})p_{12}p_a}\lambda_1 < q_2\,(1-q_1)\,p_{23} \tag{4.14}$$

The probability that the queue is not empty can be computed by Little's theorem and is given by:

$$\Pr(Q_2 \neq 0) = \frac{\lambda_{Q_2}}{\mu_{Q_2}} = \frac{\lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13}+(1-p_{13})p_{12}p_a}\lambda_1}{q_2\left(1-q_1\right)p_{23}} \tag{4.15}$$

Thus, after substituting Eq.(4.15) into Eq.(4.7), the average service rate seen by $S$ is

$$\mu_1 = \frac{q_1}{(1-q_1)p_{23}}\left\{\left[p_{12}p_a(1-p_{13})+p_{13}\right](1-q_1)p_{23} \right. \\ \left. - p_{12}(1-p_{13})p_a\lambda_1 - \left[p_{12}(1-p_{13})p_a + p_{13}\right]\lambda_2\right\} \tag{4.16}$$

The stability condition for queue 1 at the source node is $\lambda_1 < \mu_1$, and after some algebra, we obtain:

$$\left[1 + \frac{p_{12}p_a(1-p_{13})q_1}{(1-q_1)p_{23}}\right]\lambda_1 + \frac{q_1\left[p_{12}p_a(1-p_{13})+p_{13}\right]}{(1-q_1)p_{23}}\lambda_2 \\ < q_1\left[(1-p_{13})p_{12}p_a + p_{13}\right] \tag{4.17}$$

An important observation made in [37] is that the stability conditions obtained by using the stochastic dominance technique are not merely sufficient conditions for the stability of the original system but are sufficient and necessary conditions. The *indistinguishability* argument applies to our problem as well. Based on the construction of the dominant system, it is easy to see that the queues of the dominant system are always larger in size than those of the original system, provided they are both initialized to the same value. Therefore, given $\lambda_2 < \mu_2$, if for some $\lambda_1$, the queue at $S$ is stable in the dominant system, then the corresponding queue in the original system must be stable; conversely, if for some $\lambda_1$ in the dominant system, the queue at node $S$ saturates, then it will not transmit dummy packets, and as long as $S$ has a packet to transmit, the behavior of the dominant system is identical to that of the original system because the dummy packet transmissions are increasingly rare as we approach the stability boundary. Therefore, we can conclude that the original system and the dominant system are indistinguishable at the boundary points.

Now we will find the value of $p_a$ that maximizes $\lambda_1$. After replacing $\lambda_1$ with $y$ and $\lambda_2$ with $x$ we have:

$$y = \frac{-q_1\left[p_{13} + (1-p_{13})p_{12}p_a\right]}{q_1 p_{12}(1-p_{13})p_a + (1-q_1)p_{23}} x + \frac{q_1\left[p_{13} + (1-p_{13})p_{12}p_a\right](1-q_1)p_{23}}{q_1 p_{12}(1-p_{13})p_a + (1-q_1)p_{23}} \tag{4.18}$$

when

$$0 \leq x \leq q_2(1-q_1)p_{23} - \frac{p_{12}p_a(1-p_{13})}{p_{13} + (1-p_{13})p_{12}p_a} y \tag{4.19}$$

After differentiating $y$ with respect to $p_a$, we have

$$\frac{dy}{dp_a} = \left(\frac{A}{B}\right)' = \frac{A'B - AB'}{B^2} \tag{4.20}$$

where $B = q_1 p_{12}(1-p_{13})p_a + (1-q_1)p_{23}$ and

$$A'B - AB' = (1-p_{13})p_{12}q_1(x - p_{23} + p_{23}q_1)(p_{13}q_1 - p_{23} + q_1 p_{23}) \tag{4.21}$$

From Eq.(4.9), it is obvious that $x - p_{23} + p_{23}q_1 < 0$. If $p_{13}q_1 - p_{23} + p_{23}q_1 < 0$, then we have that $q_1 < \frac{p_{23}}{p_{13}+p_{23}}$. Then, $\frac{dy}{dp_a} > 0$ and $y$ is an increasing function of $p_a$ and, thus $p_a^* = 1$. Then, Eq.(4.19) becomes

$$0 \leq x \leq q_2(1-q_1)p_{23} - \frac{p_{12}(1-p_{13})}{p_{13} + (1-p_{13})p_{12}} y \tag{4.22}$$

and Eq.(4.18) becomes

$$y = \frac{-q_1\left[p_{13} + (1-p_{13})p_{12}\right]}{q_1 p_{12}(1-p_{13}) + (1-q_1)p_{23}} x + \frac{q_1\left[p_{13} + (1-p_{13})p_{12}\right](1-q_1)p_{23}}{q_1 p_{12}(1-p_{13}) + (1-q_1)p_{23}} \tag{4.23}$$

The stability region for this case is given by Eq.(4.2). If $q_1 > \frac{p_{23}}{p_{13}+p_{23}}$, it follows that $\frac{dy}{dp_a} < 0$

and, thus, $y$ is a decreasing function of $p_a$ and $p_a^* = 0$. Then Eq.(4.19) becomes

$$0 \leq x \leq q_2(1 - q_1)p_{23} \tag{4.24}$$

and Eq.(4.18) becomes

$$y + \frac{q_1 p_{13}}{(1 - q_1)p_{23}} x = q_1 p_{13} \tag{4.25}$$

The stability region is given by Eq.(4.3).

### 4.4.2 The second dominant system: relay node transmits dummy packets

In this sub-section we obtain the region $\mathcal{R}_2$ of Theorem 4.3.1. We consider the second dominant system, in which node $R$ transmits dummy packets with probability $q_2$ whenever its queue is empty, while node $S$ behaves in the same way as in the original system. All the other assumptions remain unaltered in the dominant system. The service rate for the source node is

$$\mu_1 = q_1(1 - q_2)\left[p_{13} + (1 - p_{13})p_{12}p_a\right] \tag{4.26}$$

Thus, queue $1$ is stable if

$$\lambda_1 < q_1(1 - q_2)\left[p_{13} + (1 - p_{13})p_{12}p_a\right] \tag{4.27}$$

The probability that the queue is not empty is:

$$\Pr(Q_1 \neq 0) = \frac{\lambda_1}{\mu_1} = \frac{\lambda_1}{q_1(1 - q_2)\left[p_{13} + (1 - p_{13})p_{12}p_a\right]} \tag{4.28}$$

The total arrival rate at the relay node is given by:

$$\lambda_{Q_2} = \lambda_2 + \Pr(S_B|S_A)\lambda_1 \tag{4.29}$$

where $S_A$ and $S_B$ are defined in the previous sub-section.

Note that $\Pr(S_A) = (1-q_2)\left(p_{13} + (1-p_{13})p_{12}p_a\right)$, $\Pr(S_B) = (1-q_2)(1-p_{13})p_{12}p_a$ and, thus, we have $\Pr(S_B|S_A) = \frac{(1-p_{13})p_{12}p_a}{p_{13}+(1-p_{13})p_{12}p_a}$. From the above it follows that the total arrival rate at the relay node is:

$$\lambda_{Q_2} = \lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13}+(1-p_{13})p_{12}p_a}\lambda_1 \tag{4.30}$$

The service rate for the relay node is:

$$\mu_{Q_2} = q_2\left[1 - q_1\Pr(Q_1 \neq 0)\right]p_{23} \tag{4.31}$$

Thus, from Loyne's stability criterion, it follows that the queue is stable if $\lambda_{Q_2} < \mu_{Q_2}$ and, thus:

$$\lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13}+(1-p_{13})p_{12}p_a}\lambda_1 < q_2\left[1 - q_1\Pr(Q_1 \neq 0)\right]p_{23} \tag{4.32}$$

After some algebra, we obtain:

$$\lambda_2 + \frac{(1-q_2)(1-p_{13})p_{12}p_a + q_2p_{23}}{(1-q_2)\left[p_{13}+(1-p_{13})p_{12}p_a\right]}\lambda_1 < q_2p_{23} \tag{4.33}$$

The indistinguishability argument at saturations holds here as well. Next we find the value of $p_a$ that maximizes $\lambda_2$. After replacing $\lambda_1$ with $x$ and $\lambda_2$ with $y$ we have:

$$y + \frac{(1-q_2)p_{12}(1-p_{13})p_a + q_2p_{23}}{(1-q_2)\left[p_{13}+(1-p_{13})p_{12}p_a\right]}x = q_2p_{23} \tag{4.34}$$

After differentiating $y$ with respect to $p_a$ we have

$$\frac{dy}{dp_a} = \left(\frac{A}{B}\right)' = \frac{A'B - AB'}{B^2} \tag{4.35}$$

where:

$$A^{'}B - AB^{'} = xp_{12}(1-p_{13})(1-q_2)(p_{13}q_2 - p_{13} + q_2p_{23}) \tag{4.36}$$

If $p_{13}q_2 - p_{13} + q_2p_{23} > 0$, it follows that $\frac{p_{13}}{p_{13}+p_{23}} < q_2 < 1$ and $y$ increases; thus $p_a^* = 1$ and, therefore:

$$x < q_1(1-q_2)\left[p_{13} + (1-p_{13})p_{12}\right] \tag{4.37}$$

and

$$y + \frac{(1-q_2)p_{12}(1-p_{13}) + q_2p_{23}}{(1-q_2)\left[p_{13} + (1-p_{13})p_{12}\right]}x = q_2p_{23} \tag{4.38}$$

The stability region is then given by Eq.(4.4). If $q_2 < \frac{p_{13}}{p_{13}+p_{23}}$, it follows that $y$ decreases and thus $p_a^* = 0$, hence:

$$x < q_1(1-q_2)p_{13} \tag{4.39}$$

and

$$y + \frac{q_2p_{23}}{(1-q_2)p_{13}}x = q_2p_{23} \tag{4.40}$$

The stability region is then given by Eq.(4.5).

If $x \geq q_1(1-q_2)p_{13}$ and $x \leq q_1(1-q_2)\left[p_{13} + (1-p_{13})p_{12}p_a\right]$, it follows from Eq.(4.27) that $p_a \geq \frac{x - q_1(1-q_2)p_{13}}{q_1(1-q_2)(1-p_{13})p_{12}}$ and, thus we obtain that:

$$p_a^* = \frac{x - q_1(1-q_2)p_{13}}{q_1(1-q_2)(1-p_{13})p_{12}} \tag{4.41}$$

and

$$x + y = p_{23}q_2 + q_1(1-q_2)p_{13} - q_1q_2p_{23} \tag{4.42}$$

Finally, since $0 \leq p_a \leq 1$ we have that

$$x \leq q_1(1-q_2)\left[p_{13} + (1-p_{13})p_{12}\right] \tag{4.43}$$

and the stability region is given by Eq.(4.6).

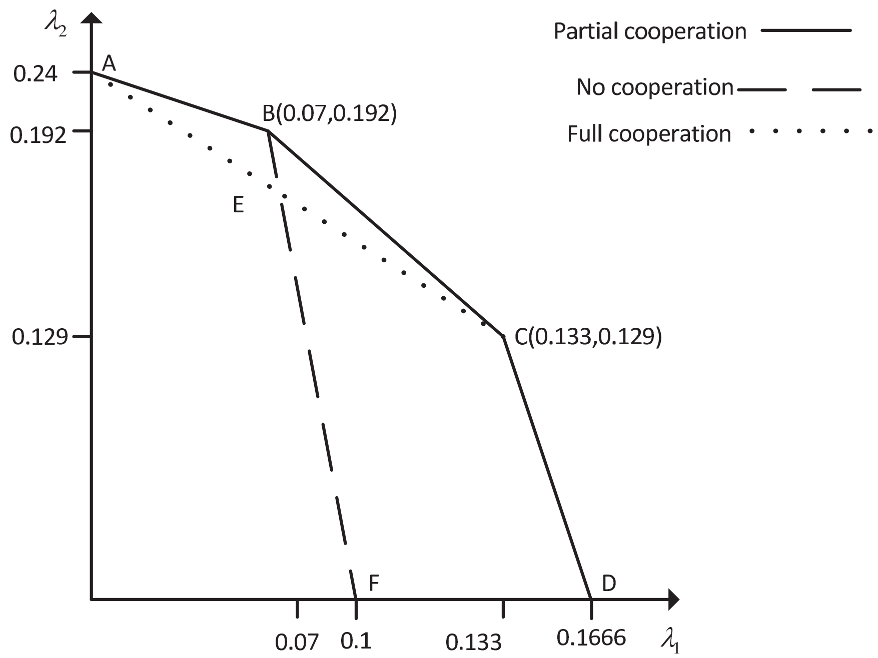



Figure 4.2: Illustration of the stability region ($q_1 = 0.2$, $q_2 = 0.3$, $p_{13} = 0.5$, $p_{12} = 0.9$ and $p_{23} = 0.8$)

## 4.5 Numerical Results

In this section, we obtain the stability region for the three cases of no-cooperation, full cooperation and partial cooperation and we compare them in a numerical illustration where $p_a < 1$. We let $q_1 = 0.2$, $q_2 = 0.3$, $p_{13} = 0.5$, $p_{12} = 0.9$, and $p_{23} = 0.8$. In Fig. 4.2, we show the stability regions for the three cases. The region of partial cooperation contains the regions of the other cases. The boundaries of the stability region for the partial cooperation scheme is described by the line segments $ABCD$, and contains the region of non-cooperation ($ABF$) and the full cooperation ($ACD$). The triangular area $BEC$ in Fig. 4.2 is achieved only by the partial cooperation scheme, showing that this scheme is superior compared to the rest schemes.

The line segment $AB$ belongs to the stability region of both no-cooperation and partial

Table 4.1: The values of $p_a^*$

| Line | $p_a^*$ |
|---|---|
| $AB$ | 0 |
| $BF$ | 0 |
| $AC$ | 1 |
| $CD$ | 1 |
| $BC$ | $\frac{\lambda_1 - 0.07}{0.063}$ |

cooperation schemes. It is the boundary when $\lambda_1 \leq 0.07$ (which is the average arrival rate at the source) and, corresponds to the scheme of no-cooperation or when $p_a^* = 0$. The line segment $CD$ is the boundary for the stability region for full cooperation and partial cooperation with $p_a^* = 1$ schemes when $0.133 \leq \lambda_1 < 0.1666$. The most interesting case is the $BC$ segment. This boundary is achieved only by the partial cooperation scheme. The value of $p_a^*$ that achieves the boundary is $p_a^* = \frac{\lambda_1 - 0.07}{0.063}$, as $0.07 < \lambda_1 < 0.133$. In this case the relay, through the flow controller, regulates the endogenous traffic from the source by randomly accepting (with $p_a^*$) the packets from the source. Note that as $\lambda_1$ increases (in the interval $(0.07, 0.133)$) so does $p_a^*$. The values of $p_a^*$ that achieve the boundaries of the regions are given in Table 4.1.

The intuition behind these results, is that when the traffic level at the source is relatively low, the optimal scheme for the relay is not to cooperate at all. When the traffic level at the source is high, the best scheme is to fully cooperate, Finally, when the source has an intermediate level of traffic, the optimal scheme is to partially offer relay services.

## 4.6 Conclusion

We introduced the notion of partial network-level cooperation by assuming a flow controller for the endogenous traffic to the relay from the source node of the network in Fig. 4.1. We provided an exact characterization of the stability region for this network. We proved that the system with the flow controller is always better than or at least equal to the system without the flow controller. The flow controller regulates the degree of cooperation offered by the relay.

# Chapter 5

# Wireless Network-Level Cooperation with Energy Harvesting Capabilities

## 5.1 Introduction

Exploiting renewable energy resources from the environment, often termed energy harvesting, permits unattended operability of infrastructure-less wireless networks. There are various forms of energy that can be harvested including thermal, vibration, solar, acoustic, wind, and even ambient radio power. The additional functionality of harvesting energy, however, permits our assessment of the system long-term performance such as in terms of the throughput, fairness and stability. In [14], the slotted ALOHA protocol was considered for a network of nodes having energy harvesting capability and the maximum stable throughput region was obtained for bursty traffic. An exact characterization of the region was given in the paper for a two-node case over a collision channel.

In this chapter, we study the impact of energy constraints on a network with a source-user, a relay and a destination. Specifically, we provide an exact characterization of the stability region of a network consisting of a source, a relay and a destination node as shown in Fig. 5.1. We consider the collision channel with erasures and random access of the medium. The source and the relay node have external arrivals; furthermore, the relay is forwarding part of the source node's traffic to the destination.

The analysis is not trivial even for such a simple network because the service process of a node not only depends on the status of its battery but also on the idleness or not of the other node. Note that the reason why the exact region is known only for the two-node and the three-node cases (even without energy availability constraints) is the *interaction* between the queues of the nodes [37–40]. In addition, we use the stochastic dominance technique and Loynes' theorem [33] for the stability of stationary system to solve the problem. Also, as pointed out in [14], it is important to note that the "service process" of the battery, i.e., the use of its energy, is independent of whether the transmission is successful or not.

The rest of this chapter is organized as follows. In section 5.2, we define the stability region, describe the channel model, and explain the packet arrival and energy harvesting models. In 5.3, we present the stability region, the proof of the result is given in 5.4. Finally, we conclude our work in 5.5.

## 5.2 System Model

We consider a time-slotted system in which the nodes are randomly accessing a common receiver as shown in Fig. 5.1, furthermore the nodes are powered from randomly time-varying renewable energy sources. Each node stores the harvested energy in a battery. We denote with $S$, $R$, and $D$ the source, the relay and the destination, respectively. Packet traffic originates from $S$ and $R$. Because of the wireless broadcast nature, $R$ may receive some of the packets transmitted from $S$ and then relay those packets to $D$. The packets from $S$ which failed to be received by $D$ but were successfully received by $R$ are relayed by $R$. As we impose half-duplex constraint, $R$ can overhear $S$ only when it is idle.

Each node has an infinite size buffer for storing incoming packets, and the transmission of each packet occupies one time slot. Node $R$ has separate queues for the exogenous arrivals and the endogenous arrivals that are relayed through $R$. But, we can let $R$ to maintain a single queue and merge all the arrivals into a single queue as the achievable stable throughput region is not affected [6]. This is because the link quality between $R$ and $D$ is independent of which packet is selected for transmission.

The packet arrival and energy harvesting processes at $S$ and $R$ are assumed to be Bernoulli with rates $\lambda_S$, $\delta_S$ and $\lambda_R$, $\delta_R$, respectively, and are independent of each other. A node is called active if both its packet queue and the battery are nonempty at the same time, and idle otherwise. In each time slot, nodes $S$ and $R$ attempt to transmit with probabilities $q_S$ and $q_R$, respectively, if their are active. Decisions on transmission are made independently among the nodes. We assumed collision channel with erasures in which, if both $S$ and $R$ transmit in the same time slot, a collision occurs and both transmissions fail. The probability that a packet transmitted by node $i$ is successfully decoded at node $j(\neq i)$ is denoted by $p_{ij}$ which is the probability that the signal-to-noise-ratio (SNR) over the specified link exceeds a certain threshold for the successful decoding. These erasure probabilities capture the effect of random fading at the physical layer. The probabilities $p_{SD}$, $p_{RD}$, and $p_{SR}$ denote the success probabilities over the link $S-R$, $R-D$, and $S-R$, respectively. Node $R$ has a better channel to $D$ than $S$, that is $p_{RD} > p_{SD}$.

The cooperation is performed at the protocol level as follows. When $S$ transmits a packet, if $D$ decodes the packet successfully, it sends an ACK and the packet exits the network; if $D$ fails to decode the packet but $R$ does, then $R$ sends an ACK and takes over the responsibility of delivering the packet to $D$ by placing it in its queue. If neither $D$ nor $R$ decode (or if $R$ does not store the packet), the packet remains in $S$'s queue for retransmission. The ACKs are assumed to be error-free, instantaneous and broadcasted to all relevant nodes.

Denote by $Q_i^t$ the length of queue $i$ at the beginning of time slot $t$. Based on the definition in [38], the queue is said to be *stable* if

$$\lim_{t\to\infty} Pr[Q_i^t < x] = F(x) \text{ and } \lim_{x\to\infty} F(x) = 1$$

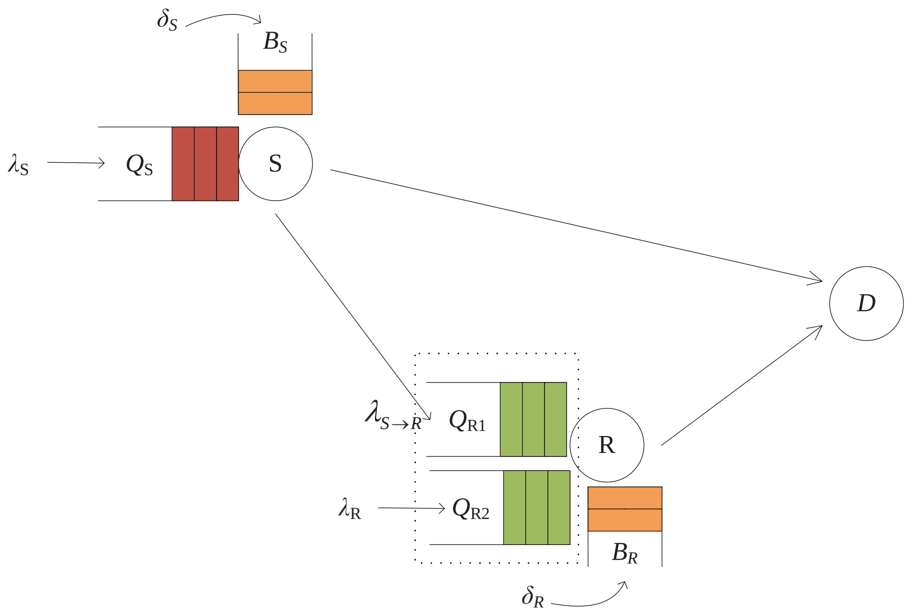



Figure 5.1: System Model

Loynes' theorem [33] states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, then the queue is stable. If the average arrival rate is greater than the average service rate, then the queue is unstable and the value of $Q_i^t$ approaches infinity almost surely. The stability region of the system is defined as the set of arrival rate vectors $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$ for which the queues in the system are stable.

## 5.3 Main Results

This section describes the stability region of a network consisting of a source, a relay and a destination as depicted in Fig. 5.1. Notice that the service process of a queue not only depends on the status of its associated energy source but also on the status of other node's queue and energy source. Denote by $Q_i$ and $B_i$ the steady state number of packets and energy units in the queue and the energy source at node $i$, respectively. Then, $Q_i$s and $B_i$s, form a four dimensional Markov chain which makes the analysis daunting.

### 5.3.1 The stability region

**Theorem 5.3.1.** *The stability region of the network in Fig. 5.1 is described by:*

$$\mathcal{R} = \mathcal{R}_1 \bigcup \mathcal{R}_2 \tag{5.1}$$

*where*

$$\mathcal{R}_1 = \Bigg\{(\lambda_1, \lambda_2) : \left[1 + \frac{\min(\delta_S, q_S)(1-p_{SD})p_{SR}}{(1-\min(\delta_S, q_S))p_{RD}}\right]\lambda_S + \\ + \frac{\min(\delta_S, q_S)\left[p_{SD} + (1-p_{SD})p_{SR}\right]}{(1-\min(\delta_S, q_S))p_{RD}}\lambda_R < \min(\delta_S, q_S)\left[p_{SD} + (1-p_{SD})p_{SR}\right], \\ \lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD} + (1-p_{SD})p_{SR}}\lambda_S < \min(\delta_R, q_R)(1-\min(\delta_S, q_S))p_{RD}\Bigg\} \tag{5.2}$$

*and*

$$\mathcal{R}_2 = \Bigg\{(\lambda_1, \lambda_2) : \lambda_R + \frac{(1-\min(\delta_R, q_R))(1-p_{SD})p_{SR} + \min(\delta_R, q_R)p_{RD}}{\left[1-\min(\delta_R, q_R)\right]\left[p_{SD} + (1-p_{SD})p_{SR}\right]}\lambda_S < \min(\delta_R, q_R)p_{RD}, \\ \lambda_S < \min(\delta_S, q_S)(1-\min(\delta_R, q_R))\left[p_{SD} + (1-p_{SD})p_{SR}\right]\Bigg\} \tag{5.3}$$

*Proof.* The proof is given in Section 5.4. □

Fig. 5.2 and 5.3 illustrate the subregions $\mathcal{R}_1$ and $\mathcal{R}_2$ described in Theorem 5.3.1.

## 5.4 Analysis using Stochastic Dominance

The stochastic dominant technique is essential in order to decouple the interaction between the queues, and thus to characterize the stability region. That is we first construct parallel

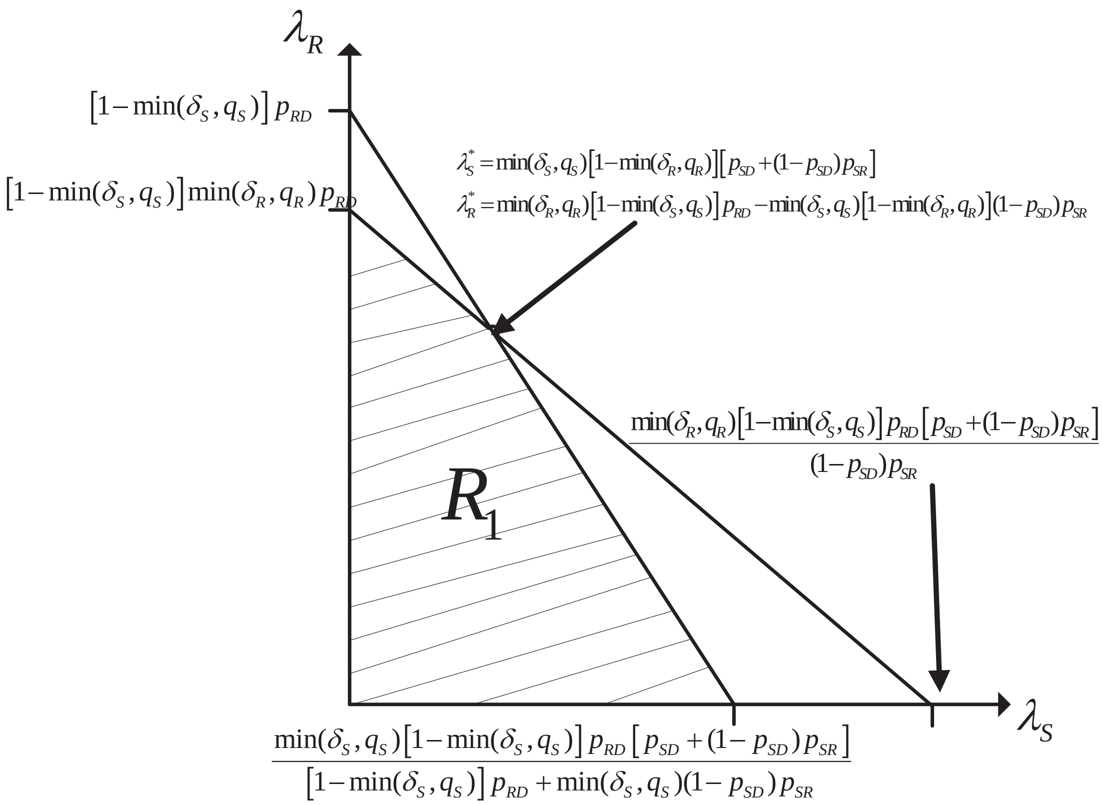



Figure 5.2: The stability region $\mathcal{R}_1$

dominant systems in which one of the nodes transmits dummy packets even when its packet queue is empty. Note, that even in the dominant system, a node cannot transmit it the energy source is empty (because even the dummy packet consumes one energy unit).

The essence of the dominant system is to make the analysis tractable by decoupling the interaction between the queues. Since the queue sizes in the dominant system are, at all times, at least as large as those of the original system, the stability region of the dominant system inner bounds that of the original system. It turns out however that the stability region obtained using this stochastic dominance technique coincides with that of the original system which will be discussed in detail later in this section. Thus, the stability regions for both the original and the dominant systems are the same.

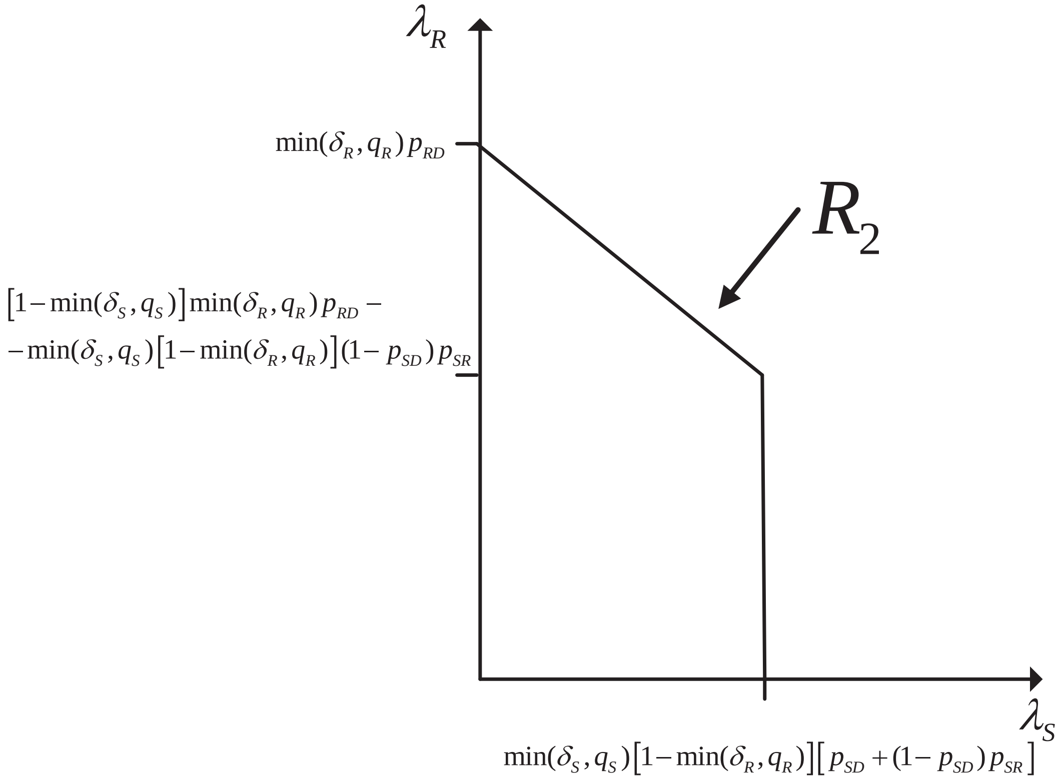



Figure 5.3: The stability region $\mathcal{R}_2$

To derive the stability condition for the queue in the relay node, we need to calculate the total arrival rate. There are two independent arrival processes at the relay: the exogenous traffic with arrival rate $\lambda_R$ and the endogenous traffic from $S$. Denote by $S_A$ the event that $S$ transmits a packet and the packet leaves the queue, then:

$$\Pr(S_A) = [1 - q_R \Pr(B_R \neq 0, Q_R \neq 0)]\, [p_{SD} + (1 - p_{SD}) p_{SR}] \tag{5.4}$$

Among the packets that depart from the queue of $S$, some will exit the network because they are decoded by the destination directly, and some will be relayed by $R$. Denote by $S_B$

the event that the transmitted packet from $S$ will be relayed from $R$, then:

$$\Pr(S_B) = [1 - q_R \Pr(B_R \neq 0, Q_R \neq 0)]\,(1 - p_{SD}) p_{SR} \tag{5.5}$$

The conditional probability that a transmitted packet from $S$ (dummy packets excluded) arrives at $R$ given that the transmitted packet exits node $S$'s queue is given by:

$$\Pr(S_B|S_A) = \frac{(1 - p_{SD}) p_{SR}}{p_{SD} + (1 - p_{SD}) p_{SR}} \tag{5.6}$$

The arrivals from the source to the relay is

$$\lambda_{S \to R} = \Pr(S_B|S_A) \lambda_S \tag{5.7}$$

The total arrival rate at the relay node is:

$$\lambda_{R,total} = \lambda_R + \frac{(1 - p_{SD}) p_{SR}}{p_{SD} + (1 - p_{SD}) p_{SR}} \lambda_S \tag{5.8}$$

We will begin the analysis with the case when $\delta_S < q_S$ and $\delta_R < q_R$, and we will construct hypothetical dominant systems. In the first dominant system the source node transmits dummy packets when its queue is empty, and each transmission occupies one energy unit. The service rate at the relay is:

$$\mu_R = \{q_R(1 - q_S) \Pr(B_S \neq 0, B_R \neq 0) + q_R \Pr(B_S = 0, B_R \neq 0)\}\, p_{RD} \tag{5.9}$$

$$\mu_R = \{q_R(1 - q_S) \Pr(B_S \neq 0) \Pr(B_R \neq 0) + q_R \Pr(B_S = 0) \Pr(B_R \neq 0)\}\, p_{RD} \tag{5.10}$$

However, $B_i$ forms a discrete $M/M/1$ which is decoupled from the remaining system and its arrival and service rate is $\delta_i$ and $q_R$ respectively. From Little's theorem we obtain that

$$\Pr(B_i \neq 0) = \frac{\delta_i}{q_i} \tag{5.11}$$

Thus,

$$\mu_R = \delta_R(1-\delta_S)p_{RD} \tag{5.12}$$

From Loyne's criterion, the relay is stable if $\lambda_{R,total} < \mu_R$.

$$\lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD}+(1-p_{SD})p_{SR}}\lambda_S < \delta_R(1-\delta_S)p_{RD} \tag{5.13}$$

The average number of packets per active slot for $R$ is $(1-\delta_S)q_R p_{RD}$ The fraction of active slots is given by

$$\Pr\left(B_R \neq 0, Q_R \neq 0\right) = \frac{\lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD}+(1-p_{SD})p_{SR}}\lambda_S}{(1-\delta_S)q_R p_{RD}} \tag{5.14}$$

The service rate of the source is given by

$$\mu_S = \Pr\left(B_S \neq 0\right)\left\{q_S(1-q_R)\Pr\left(Q_R \neq 0, B_R \neq 0\right) + q_S\left[1-\Pr\left(Q_R \neq 0, B_R \neq 0\right)\right]\right\}\left[p_{SD}+(1-p_{SD})p_{SR}\right] \tag{5.15}$$

$$\mu_S = \delta_S\left[p_{SD}+(1-p_{SD})p_{SR}\right]\left[1-\frac{\lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD}+(1-p_{SD})p_{SR}}\lambda_S}{(1-\delta_S)p_{RD}}\right] \tag{5.16}$$

The queue in $S$ is stable if $\lambda_S < \mu_S$ then:

$$\lambda_S < \delta_S\left[p_{SD}+(1-p_{SD})p_{SR}\right]\left[1-\frac{\lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD}+(1-p_{SD})p_{SR}}\lambda_S}{(1-\delta_S)p_{RD}}\right] \tag{5.17}$$

after some manipulations we have

$$\left[1+\frac{\delta_S(1-p_{SD})p_{SR}}{(1-\delta_S)p_{RD}}\right]\lambda_S+\frac{\delta_S\left[p_{SD}+(1-p_{SD})p_{SR}\right]}{(1-\delta_S)p_{RD}}\lambda_R<\delta_S\left[p_{SD}+(1-p_{SD})p_{SR}\right] \tag{5.18}$$

An important observation made in [37] is that the stability conditions obtained by using stochastic dominance technique are not merely sufficient conditions for the stability of the original system but are sufficient and necessary conditions. The *indistinguishability* argument applies to our problem as well. Based on the construction of the dominant system, it is easy to see that the queues of the dominant system are always larger in size than those of the original system, provided they are both initialized to the same value.

Therefore, given $\lambda_R < \mu_R$, if for some $\lambda_S$, the queue at $S$ is stable in the dominant system then the corresponding queue in the original system must be stable; conversely, if for some $\lambda_S$ in the dominant system, the node $S$ saturates, then it will not transmit dummy packets, and as long as $S$ has a packet to transmit, the behavior of the dominant system is identical to that of the original system because the action of dummy packet transmissions is employed increasingly rarely as we approach the stability boundary. Therefore, we can conclude that the original system and the dominant system are indistinguishable at the boundary points.

In the second dominant system, the relay node transmits dummy packets. The service rate for the source is

$$\mu_S=\left\{q_S(1-q_R)\Pr\left(B_S\neq 0, B_R\neq 0\right)+q_S\Pr(B_S\neq 0)\left[1-\Pr\left(B_R\neq 0\right)\right]\right\}\left[p_{SD}+(1-p_{SD})p_{SR}\right] \tag{5.19}$$

and

$$\mu_S=\left\{q_S(1-q_R)\Pr\left(B_R\neq 0\right)+q_S\left[1-\Pr\left(B_R\neq 0\right)\right]\right\}\Pr(B_S\neq 0)\left[p_{SD}+(1-p_{SD})p_{SR}\right] \tag{5.20}$$

$$\mu_S=\delta_S(1-\delta_R)\left[p_{SD}+(1-p_{SD})p_{SR}\right] \tag{5.21}$$

Thus from Loynes the queue in source is stable if $\lambda_S < \mu_S$ thus

$$\lambda_S < \delta_S(1-\delta_R)\left[p_{SD} + (1-p_{SD})p_{SR}\right] \tag{5.22}$$

The average number of packets per active slot for $S$ is $(1-\delta_R)q_S\left[p_{SD} + (1-p_{SD})p_{SR}\right]$ The fraction of active slots for the source $S$ is

$$\Pr\left(B_S \neq 0, Q_S \neq 0\right) = \frac{\lambda_S}{(1-\delta_R)q_S\left[p_{SD} + (1-p_{SD})p_{SR}\right]} \tag{5.23}$$

The service rate for the relay is given by

$$\mu_R = \delta_R\left[1 - q_S \Pr\left(B_S \neq 0, Q_S \neq 0\right)\right] p_{RD} \tag{5.24}$$

$$\mu_R = \delta_R\left[1 - \frac{\lambda_S}{(1-\delta_R)\left[p_{SD} + (1-p_{SD})p_{SR}\right]}\right] p_{RD} \tag{5.25}$$

Thus from Loynes the queue in $R$ is stable if $\lambda_{R,total} < \mu_R$

$$\lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD} + (1-p_{SD})p_{SR}}\lambda_S < \delta_R\left[1 - \frac{\lambda_S}{(1-\delta_R)\left[p_{SD} + (1-p_{SD})p_{SR}\right]}\right] p_{RD} \tag{5.26}$$

after some algebra we obtain

$$\lambda_R + \frac{(1-\delta_R)(1-p_{SD})p_{SR} + \delta_R p_{RD}}{(1-\delta_R)\left[p_{SD} + (1-p_{SD})p_{SR}\right]}\lambda_S < \delta_R p_{RD} \tag{5.27}$$

The indistinguishability argument at saturations holds here as well.

The next case that we will consider is where $\delta_S > q_S$ and $\delta_R < q_R$. The source operates like no having any energy constraints (see [14]). The service rate at the source is

$$\mu_S = \{q_S(1-q_R)\Pr\left(Q_R \neq 0, B_R \neq 0\right) + q_S\left[1 - \Pr\left(Q_R \neq 0, B_R \neq 0\right)\right]\}\left[p_{SD} + (1-p_{SD})p_{SR}\right] \tag{5.28}$$

The service rate at the relay is

$$\mu_R = \{q_R(1-q_S)\Pr\left(Q_S \neq 0, B_R \neq 0\right) + q_R\Pr(B_R \neq 0)\Pr\left(Q_S = 0\right)\}\, p_{RD} \tag{5.29}$$

As the previous case, we construct the dominant systems. In the first dominant system the source keeps transmitting dummy packets and we are following the same methodology. Given that node $R$ is active its probability of success is $q_R(1-q_S)p_{RD}$. Thus

$$\Pr\left(B_R \neq 0, Q_R \neq 0\right) = \frac{\lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD}+(1-p_{SD})p_{SR}}\lambda_S}{(1-q_S)q_R p_{RD}} \tag{5.30}$$

$$\left[1 + \frac{q_S(1-p_{SD})p_{SR}}{(1-q_S)p_{RD}}\right]\lambda_S + \frac{q_S\left[p_{SD}+(1-p_{SD})p_{SR}\right]}{(1-q_S)p_{RD}}\lambda_R < q_S\left[p_{SD}+(1-p_{SD})p_{SR}\right] \tag{5.31}$$

when

$$\lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD}+(1-p_{SD})p_{SR}}\lambda_S < \delta_R(1-q_S)p_{RD} \tag{5.32}$$

In the second dominant system, the relay transmits dummy packets.

$$\lambda_R + \frac{(1-\delta_R)(1-p_{SD})p_{SR} + \delta_R p_{RD}}{(1-\delta_R)\left[p_{SD}+(1-p_{SD})p_{SR}\right]}\lambda_S < \delta_R p_{RD} \tag{5.33}$$

when

$$\lambda_S < q_S(1-\delta_R)\left[p_{SD}+(1-p_{SD})p_{SR}\right] \tag{5.34}$$

The next case is where $\delta_S < q_S$ and $\delta_R > q_R$. The relay operates like no having any energy constraints. The service rate at the source is

$$\mu_S = \{q_S(1-q_R)\Pr(B_S \neq 0, Q_R \neq 0) + q_S\Pr(B_S \neq 0)[1-\Pr(Q_R \neq 0)]\}[p_{SD} + (1-p_{SD})p_{SR}] \tag{5.35}$$

The service rate at the relay is

$$\mu_R = \{q_R(1-q_S)\Pr(B_S \neq 0, Q_S \neq 0) + q_R[1-\Pr(Q_S \neq 0, B_S \neq 0)]\}p_{RD} \tag{5.36}$$

In the first dominant system (source transmit dummy packets), we have:

$$\left[1+\frac{\delta_S(1-p_{SD})p_{SR}}{(1-\delta_S)p_{RD}}\right]\lambda_S + \frac{\delta_S[p_{SD}+(1-p_{SD})p_{SR}]}{(1-\delta_S)p_{RD}}\lambda_R < \delta_S[p_{SD}+(1-p_{SD})p_{SR}] \tag{5.37}$$

when

$$\lambda_R + \frac{(1-p_{SD})p_{SR}}{p_{SD}+(1-p_{SD})p_{SR}}\lambda_S < q_R(1-\delta_S)p_{RD} \tag{5.38}$$

from the second dominant we obtain

$$\lambda_R + \frac{(1-q_R)(1-p_{SD})p_{SR}+q_Rp_{RD}}{(1-q_R)[p_{SD}+(1-p_{SD})p_{SR}]}\lambda_S < q_Rp_{RD} \tag{5.39}$$

when

$$\lambda_S < \delta_S(1-q_R)[p_{SD}+(1-p_{SD})p_{SR}] \tag{5.40}$$

The remaining case is when $\delta_R > q_R$ and $\delta_S > q_S$. The analysis is the same as the case without energy harvesting constraints. The stability region is given in [6].

We obtained the stability regions for all possible cases separately and they are summarized in Theorem 5.3.1.

## 5.5 Conclusion

In this chapter, we studied the impact of energy constraints on a network with a source-user, a relay and a destination. The source and the relay node have external arrivals; furthermore, the relay is forwarding part of the source node's traffic to the destination. We provided an exact characterization of the stability region of the network depicted in Fig. 5.1.

# Chapter 6

# Optimal Utilization of a Cognitive Shared Channel with a Rechargeable Primary Source Node

The material in this chapter was presented in [41] and [42].

## 6.1 Introduction

Cognitive radio communication provides an efficient means of sharing radio spectrum between users having different priority [15]. The high-priority user, called primary, is allowed to access the channel whenever it needs, while the low-priority user, called secondary, is required to make a decision on its transmission based on what the primary user does. The system considered in this chapter is comprised of nodes that are either subject to energy availability constraint imposed by the battery status and stochastic recharging process or free from such constraint by assuming that they are connected to a constant power source.

In this chapter, we consider the simple cognitive system of two source-destination pairs as shown in Fig. 6.1 and derive the *maximum stable throughput region* for a cognitive access

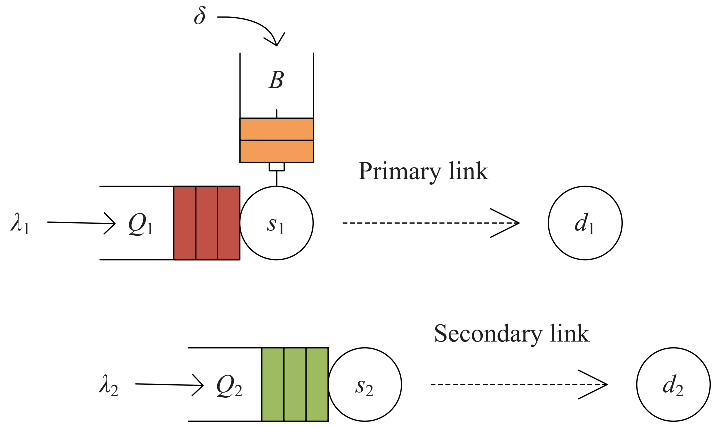



Figure 6.1: An example cognitive communication system

protocol on the general multipacket reception channel model[1] in which a transmission may succeed even in the presence of interference [16–18]. The secondary node can take advantage of such an additional reception capability by transmitting simultaneously with the primary. We adopt a similar cognitive access protocol proposed in [19], and also studied in [20], in which the secondary node not only utilizes the idle periods of the primary node, but also competes with the primary by randomly accessing the channel to increase its own throughput. However, the secondary user is still required to coordinate its transmission in order not to hamper the required throughput level of the primary link given the energy harvesting rate and this is done by appropriately choosing the random access probability.

To position our contribution with respect to the recent literature, we start a brief background review. In [13], the capacity of the additive white Gaussian noise channel with stochastic energy harvesting at the source was shown to be equal to the capacity with an average power constraint given by the energy harvesting rate. However, like most of information-theoretic research, the result is obtained for point-to-point communication with an always backlogged source. In [14], the slotted ALOHA protocol was considered for a network of nodes having energy harvesting capability and the maximum stable throughput region was obtained for

[1]When compared to collision channel model, it better captures the effects of fading, attenuation and interference at the physical layer.

bursty traffic. An exact characterization of the region was given in the paper for a two-node case over a collision channel. The analysis is not trivial even for such a simple network because the service process of a node not only depends on the status of its battery but also on the idleness or not of the other node. Note that the reason why the exact region is known only for the two-node and the three-node cases (even without energy availability constraints) is the *interaction* between the queues of the nodes [37–40].

The initial study of a simple model involving only two source-destination pairs is not only instructive but also necessary. The reason is that the interaction between nodes causes considerable difficulties at the analytical level, and yet, reveals major insights at the conceptual level. In addition, we use the stochastic dominance technique and Loynes' theorem [33] for the stability of stationary system to solve the problem. Also, as pointed out in [14], it is important to note that the "service process" of the battery, i.e., the use of its energy, is independent of whether the transmission is successful or not.

The rest of the chapter is organized as follows. In Section 6.2, we define the stability region, describe the channel model, and explain the packet arrival and energy harvesting models. In Section 6.3, we present the conditions for stability of the considered cognitive access protocol when the capacity of the battery at the primary node is assumed to be infinite. The proof of the result is given in Section 6.4 which utilizes the stochastic dominance technique and arguments similar to those used in [14] and [37]. In Section 6.5, we extend the result to the case when the capacity of battery is finite. As will be shown, the stability region for the case with finite capacity battery is a subset of that for the case with infinite capacity battery. For comparison's sake, in Section 6.6, the result obtained in Section 6.3 is derived again for the case without multipacket reception capability, i.e., for a collision channel with additional probabilistic erasures. Finally, we draw some conclusions in Section 6.7.

## 6.2 System Model

We consider a time-slotted communication system consisting of two primary and secondary source-destination pairs of nodes, $(s_1, d_1)$ and $(s_2, d_2)$, respectively, as shown in Fig. 6.1. Each source node has an infinite capacity buffer $Q_i$ $(i \in \{1, 2\})$ for storing arriving packets of fixed length. The secondary node is plugged to a reliable power supply, whereas the primary node is powered through a random time-varying renewable energy process and has a battery $B$ for storing energy which is assumed to be harvested in a certain unit from the environments. The capacity of the battery is denoted by $c$. We first consider the case with $c = \infty$ and, after that, we relax $c$ to take any finite integer value. The slot duration is equal to the transmission time of a single packet and one unit of energy is consumed in each transmission. The packet arrival and energy harvesting processes are all modeled as independent Bernoulli processes of rate $\lambda_i$ and $\delta$ per slot, respectively[2]. The primary node is considered *active* if both $Q_1$ and $B_1$ are nonempty at the same time. Similarly, the secondary is called *active* if $Q_2$ is nonempty. Otherwise, they are called *idle*.

A shared channel is assumed and a transmission is said to be successful if the received signal-to-interference-plus-noise-ratio (SINR) exceeds a certain threshold which depends on the modulation scheme, the target bit-error-rate, and the number of bits in the packet (i.e., the transmission rate for a fixed packet duration). Denote by $q_{i/\boldsymbol{I}}$ the probability that the transmission by source $i$ succeeds given that the sources in $\boldsymbol{I}$ are transmitting simultaneously. Specifically, in our cognitive communication system in Fig. 6.1, the following success probabilities are of interest:

$$q_{1/1},\ q_{2/2},\ q_{1/1,2},\ q_{2/1,2}$$

and it is assumed that $q_{1/1} \geq q_{1/1,2}$ and $q_{2/2} \geq q_{2/1,2}$. Define $\Delta_1 = q_{1/1} - q_{1/1,2}$ and $\Delta_2 = q_{2/2} - q_{2/1,2}$. In case that the simultaneous transmissions always fail, we have $q_{i/1,2} = 0$

[2]In practice, statistics of the energy harvesting models are time varying. However they can be approximated by piecewise stationary processes. For example, energy harvesting by solar cells could be taken as being stationary over one hour periods. Thus, our results could be used over these time periods. See [43]

for all $i$.

Denote by $Q_i^t$ the length of $Q_i$ at the beginning of time slot $t$, the queue is said to be *stable* if

$$\lim_{x\to\infty}\lim_{t\to\infty}\Pr[Q_i^t < x] = 1 \tag{6.1}$$

Loynes' theorem [33] states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, then the queue is stable. If the average arrival rate is greater than the average service rate, then the queue is unstable and the value of $Q_i^t$ approaches to infinity almost surely. The stability region of the system is defined as the set of arrival rate vectors $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$ for which the queues in the system are stable.

## 6.3 Main Results

This section describes the cognitive access protocol and presents our main results concerning its stability. The proofs of the results are presented in the next section.

### 6.3.1 Description of the cognitive access protocol

The opportunistic cognitive access protocol proposed in [19] and also used in [20] is modified and studied again in the context of the energy harvesting environment. The energy-constrained primary node $s_1$ (see Fig. 6.1) transmits a packet whenever it is active. Note that the transmission by the primary node $s_1$ is independent of the secondary node $s_2$. On the other hand, the transmission by the secondary node $s_2$ must be chosen in a careful manner in order not to impede the primary's performance guarantees. Under our cognitive access protocol, node $s_2$ observes the status of $s_1$ and if $s_1$ is idle, i.e., either $Q_1$ or $B_1$ is empty, it transmits with probability 1 if its own packet queue $Q_2$ is nonempty. Otherwise, if $s_1$ is active, $s_2$ transmits with probability $p$ to take advantage of the multipacket reception capability by transmitting along with the primary node although at the same time it risks impeding the primary node's

$$\mathcal{R}_1' = \left\{(\lambda_1, \lambda_2) : \frac{\Delta_2}{q_{1/1,2}q_{2/2}}\lambda_1 + \frac{\lambda_2}{q_{2/2}} < 1, \ \ 0 \leq \lambda_1 \leq \delta q_{1/1,2}\right\} \tag{6.2}$$

$$\mathcal{R}_1'' = \left\{(\lambda_1, \lambda_2) : \frac{q_{2/1,2}\lambda_1 + \Delta_1\lambda_2}{\delta q_{1/1}q_{2/1,2} + \Delta_1 q_{2/2}(1-\delta)} < 1, \ \ \delta q_{1/1,2} < \lambda_1 < \delta q_{1/1}\right\} \tag{6.3}$$

success. The design objective is to choose the transmission probability $p$ such that the secondary's throughput is maximized while maintaining the stability of primary's packet queue at given packet arrival and energy harvesting rates.

### 6.3.2 Stability Criteria

Denote by $\mathcal{R}$ the stability region of the system by considering all possible values of $p$ and define $\eta = q_{1/1}q_{2/1,2} + q_{2/2}q_{1/1,2} - q_{2/2}q_{1/1}$. Note that $\eta$ reflects the degree of multipacket reception capability. In the case of a collision channel in which $q_{1/1} = q_{2/2} = 1$ and $q_{1/1,2} = q_{2/1,2} = 0$, $\eta = -1$. It is clear that $\eta$ increases as the multipacket reception capability improves.

**Theorem 6.3.1.** *The stability region of the cognitive multiaccess system is described by*

$$\mathcal{R} = \mathcal{R}_1 \bigcup \mathcal{R}_2 \tag{6.5}$$

*where the subregion $\mathcal{R}_1$ is described as follows:*

- *If $\eta > 0$, $\mathcal{R}_1 = \mathcal{R}_1' \bigcup \mathcal{R}_1''$ where $\mathcal{R}_1'$ and $\mathcal{R}_1''$ are given by* (6.2) *and* (6.3).
- *If $\eta \leq 0$,*

$$\mathcal{R}_1 = \left\{(\lambda_1, \lambda_2) : \frac{\lambda_1}{q_{1/1}} + \frac{\lambda_2}{q_{2/2}} < 1, \ \ \lambda_1 < \delta q_{1/1}\right\} \tag{6.6}$$

$$\mathcal{R}_2'' = \left\{(\lambda_1, \lambda_2) : \frac{q_{2/1,2}\lambda_1 + \Delta_1\lambda_2}{\delta q_{1/1}q_{2/1,2} + \Delta_1 q_{2/2}(1-\delta)} < 1, \ \ (1-\delta)q_{2/2} < \lambda_2 < (1-\delta)q_{2/2} + \delta q_{2/1,2}\right\} \tag{6.4}$$

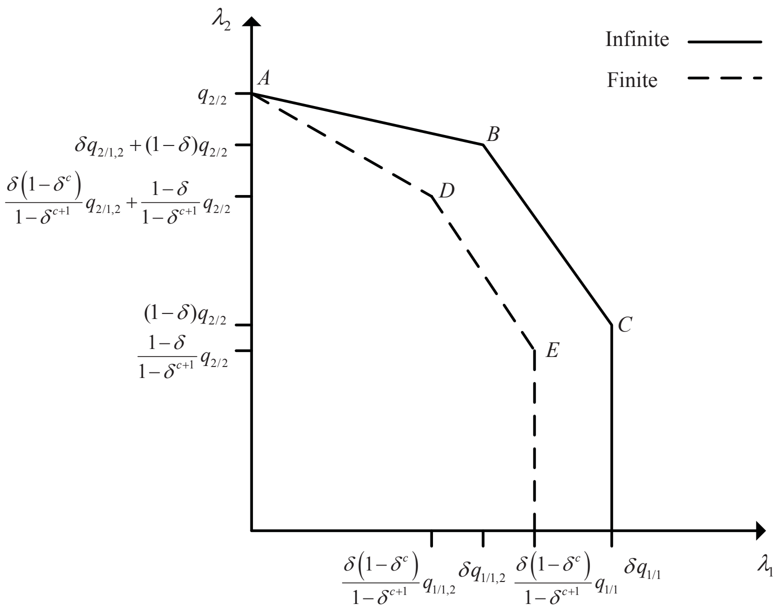
*and the subregion $\mathcal{R}_2$ is described as $\mathcal{R}_2 = \mathcal{R}_2' \bigcup \mathcal{R}_2''$ with*

$$\mathcal{R}_2' = \left\{(\lambda_1, \lambda_2) : \lambda_1 < \delta q_{1/1}, 0 \leq \lambda_2 \leq (1-\delta) q_{2/2}\right\} \tag{6.7}$$

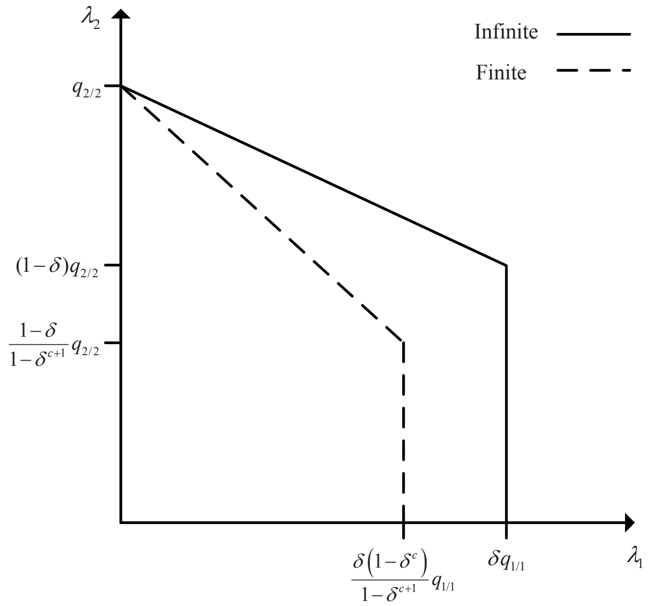
*and $\mathcal{R}_2''$ as given by* (6.4).

*Proof.* The proof is given in Section 6.4. □

The optimal $p^*$ achieving the boundary of the stability region is explicitly given in the following section. The subregion $\mathcal{R}_1$ is depicted in Fig. 6.2 with solid line. Specifically, if $\eta > 0$, the line segments $AB$ and $BC$ correspond to the boundaries due to the inequalities (6.2) and (6.3), respectively. The subregion $\mathcal{R}_2$ is also illustrated in the Fig. 6.3 with solid line. Note that when $\eta > 0$, $\mathcal{R}_2$ is always contained in $\mathcal{R}_1$, i.e., $\mathcal{R}_2 \subset \mathcal{R}_1$, which is not necessarily true if $\eta \leq 0$.




(a) The case with $\eta > 0$




(b) The case with $\eta \leq 0$

Figure 6.2: The subregion $\mathcal{R}_1$ with multipacket reception capability (solid and dotted lines depict the case when the capacity of the primary node is infinite and finite, respectively.)

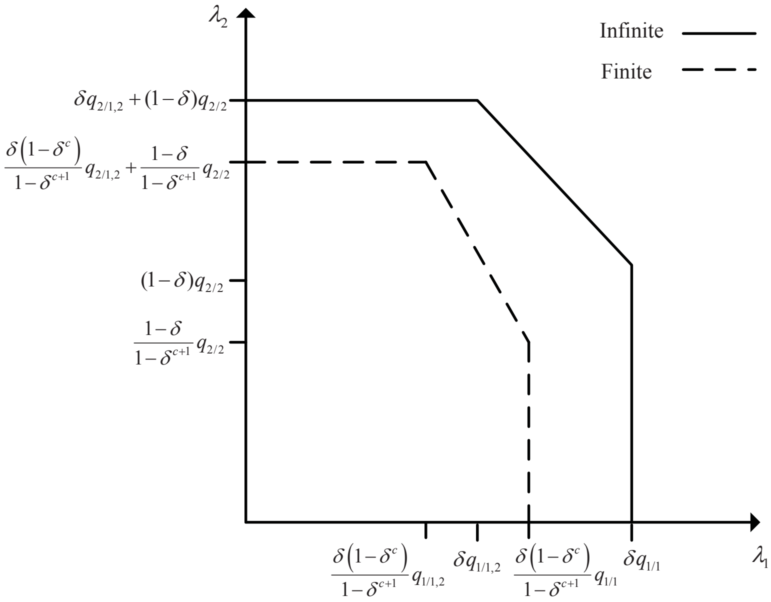



Figure 6.3: The subregion $\mathcal{R}_2$ with multipacket reception capability (solid and dotted lines depict the case when the capacity of the primary node is infinite and finite, respectively.)

## 6.4 Analysis using Stochastic Dominance

Under the cognitive access protocol described in Section 6.3.1, the expressions for the average service rates seen by $s_1$ and $s_2$ are given by

$$\mu_1 = q_{1/1}\Pr[B_1 \neq 0, Q_2 = 0] + q_{1/1,2}\Pr[B_1 \neq 0, Q_2 \neq 0]p + q_{1/1}\Pr[B_1 \neq 0, Q_2 \neq 0](1-p) \tag{6.8}$$

and

$$\mu_2 = q_{2/2}(1 - \Pr[B_1 \neq 0, Q_1 \neq 0]) + q_{2/1,2}\Pr[B_1 \neq 0, Q_1 \neq 0]p \tag{6.9}$$

Note that computing the average service rates $\mu_1$ and $\mu_2$ requires the specifications of a joint probability of doublets $(B_1, Q_2)$ and $(B_1, Q_1)$, respectively. Since, however, $Q_1$, $Q_2$, and $B_1$ are all *interacting*, it is difficult to track them. We bypass this difficulty by utilizing the idea of stochastic dominance [37]. That is we first construct parallel dominant systems in which one of the nodes transmits dummy packets even when its packet queue is empty. The essence of the dominant system is to make the analysis tractable by decoupling the interaction between the queues. Since the queue sizes in the dominant system are, at all times, at least as large as those of the original system, the stability region of the dominant system inner bounds that of the original system. It turns out however that the stability region obtained using this stochastic dominance technique coincides with that of the original system which will be discussed in detail later in this section. Thus, the stability regions for both the original and the dominant systems are the same.

### 6.4.1 The first dominant system: secondary node transmits dummy packets

Construct a hypothetical system in which the secondary node $s_2$ transmits dummy packets when its packet queue is empty. Hence $s_2$ transmits with probability 1 whenever $s_1$ is idle and with probability $p$ if $s_1$ is active. As a result, the average service rate of $s_1$ in (6.8) reduces to

$$\mu_1 = q_{1/1,2}\Pr[B_1 \neq 0]p + q_{1/1}\Pr[B_1 \neq 0](1-p) \tag{6.10}$$

Since $s_1$ transmits with probability 1 whenever it is active, if $Q_1$ is saturated[3], $B_1$ is modeled as a decoupled discrete-time $M/D/1$ system with arrival and service rates $\delta$ and 1, respectively. It follows from Little's theorem that $B_1$ is nonempty for a fraction of time $\delta$ [44]. Consequently, we have

$$\mu_1 = \delta(q_{1/1,2}p + q_{1/1}(1-p)) \tag{6.11}$$

For $\lambda_1$ satisfying $\lambda_1 < \mu_1$, i.e., when $Q_1$ in this dominant system is stable, we now obtain

[3]Note that in describing the service rates in (6.8) and (6.9), it is assumed that the corresponding packet queue is nonempty. This is simply because if the queue is empty, the ”server” becomes idle.

the average service rate of $s_2$. We note from (6.9) that the probability of $s_1$ being active, i.e., $\Pr[B_1 \neq 0, Q_1 \neq 0]$, needs to be specified beforehand. For this, we take an approach similar to the one used in [14]. The approach utilizes a simple property of a stable system, that is the rate of what comes is equal to the rate of what goes out. Given the fact that $s_1$ is active, the average number of packets out of $Q_1$ is given by $q_{1/1,2}p + q_{1/1}(1-p)$. Because the average number of packets into $Q_1$ is $\lambda_1$ and, because it satisfies $\lambda_1 < \mu_1$, the fraction of active slots must be

$$\Pr[B_1 \neq 0, Q_1 \neq 0] = \frac{\lambda_1}{q_{1/1,2}p + q_{1/1}(1-p)} \tag{6.12}$$

After some manipulation, the average service rate of $s_2$ can be obtained from (6.9) as

$$\mu_2 = \frac{q_{2/1,2}p - q_{2/2}}{q_{1/1,2}p + q_{1/1}(1-p)}\lambda_1 + q_{2/2} \tag{6.13}$$

By applying Loynes' theorem, we find that the stability condition for the dominant system is given by

$$\lambda_2 < \frac{q_{2/1,2}p - q_{2/2}}{q_{1/1,2}p + q_{1/1}(1-p)}\lambda_1 + q_{2/2} \tag{6.14}$$

when

$$\lambda_1 < \delta(q_{1/1,2}p + q_{1/1}(1-p)) \tag{6.15}$$

An important observation made in [37] is that the stability conditions obtained by using stochastic dominance technique are not merely sufficient conditions for the stability of the original system but are sufficient and necessary conditions. The *indistinguishability* argument applies to our problem as well. Based on the construction of the dominant system, it is easy to see that the queues of the dominant system are always larger in size than those of the original system, provided they are both initialized to the same value.

Therefore, given $\lambda_1 < \mu_1$, if for some $\lambda_2$, the queue at $s_2$ is stable in the dominant system then the corresponding queue in the original system must be stable; conversely, if for some $\lambda_2$ in the dominant system, the node $s_2$ saturates, then it will not transmit dummy packets, and as long as $s_2$ has a packet to transmit, the behavior of the dominant system is identical to that of the original system because the action of dummy packet transmissions is employed increasingly rarely as we approach the stability boundary. Therefore, we can conclude that the original system and the dominant system are indistinguishable at the boundary points.

The portion of the stable throughput region by the first dominant system is given by the closure of the rate pairs $(\lambda_1, \lambda_2)$ described by (6.14) and (6.15) as $p$ varies over $[0, 1]$. To obtain the closure of the rate pair, we first fix $\lambda_1$ and maximize $\lambda_2$ as $p$ varies over $[0, 1]$. By replacing $\lambda_1$ by $x$ and $\lambda_2$ by $y$, the boundary of the stability region for fixed $p$ can now be written as

$$y = \frac{q_{2/1,2}p - q_{2/2}}{q_{1/1,2}p + q_{1/1}(1-p)}x + q_{2/2} \tag{6.16}$$

for $0 \leq x \leq \delta(q_{1/1,2}p + q_{1/1}(1-p))$. Differentiating $y$ with respect to $p$ yields,

$$\frac{dy}{dp} = \frac{\eta x}{\left(q_{1/1} + p(q_{1/1,2} - q_{1/1})\right)^2} \tag{6.17}$$

where $\eta$ is defined in Section 6.3.2. It can be observed that the denominator is strictly positive and the numerator can be positive or negative depending on the value of $\eta$.

- If $\eta > 0$, the first derivative is strictly positive, and $y$ is an increasing function of $p$. Therefore $p^* = 1$. However, this is valid only if $0 \leq x \leq \delta(q_{1/1,2}p + q_{1/1}(1-p))$. Thus, $p^*$ can take a value of 1 only if $0 \leq x \leq \delta q_{1/1,2}$. Substituting $p^* = 1$ into (6.16) gives the boundary of the subregion characterized by (6.2).

- If $\eta > 0$ and $x > \delta q_{1/1,2}$, then $p^* = \frac{\delta q_{1/1} - x}{\delta(q_{1/1} - q_{1/1,2})}$. By substituting $p^*$ into (6.16) and after some simple algebra, we obtain the boundary of the subregion characterized by (6.3).

- If $\eta \leq 0$, the derivative is non-positive for all feasible $p$ and, thus, $y$ is a decreasing function of $p$ in the range of all possible values of $x$. Therefore, $p^* = 0$ and the stability region is given in (6.6).

Note that for the first dominant system the value of $\lambda_1$ is upper bounded by the term $\delta q_{1/1}$.

### 6.4.2 The second dominant system: primary node transmits dummy packets

In the previous section, we obtained the stability region of the first dominant system which yields one part of the stability region of the original system. To finalize the analysis, consider the complementary dominant system in which the primary node $s_1$ transmits dummy packets whenever its packet queue is empty, and the secondary node $s_2$ behaves exactly as in the original system. Even in the dominant system, however, $s_1$ cannot transmit if its battery is empty. Therefore, the average service rate of $s_2$ in (6.9) reduces to

$$\mu_2 = q_{2/2}\left(1 - \Pr[B_1 \neq 0]\right) + q_{2/1,2}\Pr[B_1 \neq 0]p \tag{6.18}$$

Since $s_1$ transmits with probability 1 whenever its battery is nonempty, $B_1$ is modeled as a decoupled discrete-time $M/D/1$ system with arrival rate $\delta$ and service rate 1. Consequently, (6.18) becomes

$$\mu_2 = q_{2/2}\left(1 - \delta\right) + q_{2/1,2}\delta p \tag{6.19}$$

From Little's theorem, the probability that $Q_2$ is nonempty for some $\lambda_2 < \mu_2$ is given by

$$\Pr[Q_2 \neq 0] = \frac{\lambda_2}{q_{2/2}\left(1 - \delta\right) + q_{2/1,2}\delta p} \tag{6.20}$$

Because in this dominant system $B_1$ is decoupled, i.e., independent, from the rest of the system, we can rewrite the average service rate of $s_1$ in (6.8) as

$$\begin{aligned}\mu_1 = \Pr[B_1 \neq 0]\{q_{1/1}\left(1 - \Pr[Q_2 \neq 0]\right) \\ + q_{1/1,2}\Pr[Q_2 \neq 0]p + q_{1/1}\Pr[Q_2 \neq 0](1-p)\}\end{aligned} \tag{6.21}$$

Plugging (6.20) into (6.21) and, after some manipulations, we find the stability condition for this dominant system is given by

$$\lambda_1 < \mu_1 = \frac{\delta p(q_{1/1,2} - q_{1/1})}{(1-\delta)q_{2/2} + \delta p q_{2/1,2}}\lambda_2 + \delta q_{1/1} \tag{6.22}$$

for

$$\lambda_2 < (1-\delta)\, q_{2/2} + \delta p q_{2/1,2} \tag{6.23}$$

The indistinguishability argument at saturations holds here as well.

To specify the boundary of the stability region which is the closure of the rate pairs $(\lambda_1, \lambda_2)$ over feasible $p$, we follow the same methodology as in the previous section. By replacing $\lambda_1$ and $\lambda_2$ by $y$ and $x$, respectively, the boundary for fixed $p$ is written as

$$y = \frac{\delta p(q_{1/1,2} - q_{1/1})}{(1-\delta)q_{2/2} + \delta p q_{2/1,2}}x + \delta q_{1/1} \tag{6.24}$$

for $0 \leq x \leq (1-\delta)\, q_{2/2} + \delta p q_{2/1,2}$. It is not difficult to see that its first derivative with respect to $p$ is given as

$$\frac{dy}{dp} = -\frac{\theta x}{\left((1-\delta)q_{2/2} + \delta p q_{2/1,2}\right)^2} \tag{6.25}$$

where $\theta = \delta(1-\delta)q_{2/2}(q_{1/1} - q_{1/1,2})$. Since $\theta$ is always non-positive under our assumption, $y$ is a non-increasing function of $p$. Therefore, the optimal value of $p^*$ maximizing $y$ is 0 but this is valid only if the condition $0 \leq x \leq (1-\delta)\, q_{2/2} + \delta p q_{2/1,2}$ is met. At $p = 0$, it becomes $0 \leq x \leq (1-\delta)\, q_{2/2}$. Substituting $p^* = 0$ into (6.24) yields (6.7). If $x > (1-\delta)\, q_{2/2}$, we obtain $p^* = \frac{x-(1-\delta)q_{2/2}}{\delta q_{2/1,2}}$. By substituting $p^*$ into (6.24), we obtain (6.4). Note that in obtaining the stability region for this dominant system, it is assumed that $\lambda_2 < \mu_2$. At $\lambda_1 = 0$, the optimal transmission probability of the secondary node is $p = 1$ which gives the upper bound on $\lambda_2$ in (6.4).

$$\mathcal{R}_1' = \left\{(\lambda_1, \lambda_2) : \frac{\Delta_2}{q_{1/1,2}q_{2/2}}\lambda_1 + \frac{\lambda_2}{q_{2/2}} < 1,\ \ 0 \leq \lambda_1 \leq \frac{\delta(1-\delta^c)}{1-\delta^{c+1}}q_{1/1,2}\right\} \tag{6.26}$$

$$\mathcal{R}_1''=\left\{(\lambda_1, \lambda_2) : \frac{q_{2/1,2}\lambda_1 + \Delta_1\lambda_2}{\frac{\delta(1-\delta^c)}{1-\delta^{c+1}}q_{1/1}q_{2/1,2} + \Delta_1 q_{2/2}\frac{1-\delta}{1-\delta^{c+1}}} < 1,\ \ \frac{\delta(1-\delta^c)}{1-\delta^{c+1}}q_{1/1,2} < \lambda_1 < \frac{\delta(1-\delta^c)}{1-\delta^{c+1}}q_{1/1}\right\} \tag{6.27}$$

## 6.5 The Case with Finite Capacity Battery

We now consider a realistic scenario in which the primary node is equipped with a battery whose capacity is finite. The harvested energy units can be stored only if the battery is not fully charged.

**Theorem 6.5.1.** *The stability region of the cognitive multiaccess system with finite battery is described by*

$$\mathcal{R} = \mathcal{R}_1 \bigcup \mathcal{R}_2 \tag{6.29}$$

*where the subregion $\mathcal{R}_1$ is described as follows:*

- *If $\eta > 0$, $\mathcal{R}_1 = \mathcal{R}_1' \bigcup \mathcal{R}_1''$ where $\mathcal{R}_1'$ and $\mathcal{R}_1''$ are given by* (6.26) *and* (6.27). *The optimal probabilities $p*$ achieving the boundaries of the subregions $\mathcal{R}_1'$ and $\mathcal{R}_1''$ are obtained as $p^* = 1$ and $p^* = \left(\frac{\delta(1-\delta^c)}{1-\delta^{c+1}}q_{1/1} - \lambda_1\right) / \left(\frac{\delta(1-\delta^c)}{1-\delta^{c+1}}\Delta_1\right)$, respectively.*

- *If $\eta \leq 0$,*

$$\mathcal{R}_1 = \left\{(\lambda_1, \lambda_2) : \frac{\lambda_1}{q_{1/1}} + \frac{\lambda_2}{q_{2/2}} < 1,\ \ \lambda_1 < \frac{\delta(1-\delta^c)}{1-\delta^{c+1}}q_{1/1}\right\} \tag{6.30}$$

$$\mathcal{R}_2'' = \left\{(\lambda_1, \lambda_2) : \frac{q_{2/1,2}\lambda_1 + \Delta_1\lambda_2}{\frac{\delta(1-\delta^c)}{1-\delta^{c+1}}q_{1/1}q_{2/1,2} + \Delta_1 q_{2/2}\frac{1-\delta}{1-\delta^{c+1}}} < 1,\right.$$
$$\left.\frac{1-\delta}{1-\delta^{c+1}}q_{2/2} < \lambda_2 < \frac{1-\delta}{1-\delta^{c+1}}q_{2/2} + \frac{\delta(1-\delta^c)}{1-\delta^{c+1}}q_{2/1,2}\right\} \tag{6.28}$$

*The optimal $p*$ achieving the boundary is zero.*

*The subregion $\mathcal{R}_2$ is described as $\mathcal{R}_2 = \mathcal{R}_2' \bigcup \mathcal{R}_2''$ with*

$$\mathcal{R}_2' = \left\{ (\lambda_1, \lambda_2) : \lambda_1 < \frac{\delta(1-\delta^c)}{1-\delta^{c+1}} q_{1/1}, 0 \leq \lambda_2 \leq \frac{1-\delta}{1-\delta^{c+1}} q_{2/2} \right\} \tag{6.31}$$

*and $\mathcal{R}_2''$ as given by* (6.28). *The optimal $p^*$ achieving the boundaries of the subregions $\mathcal{R}_2'$ and $\mathcal{R}_2''$ are obtained as $p^* = 0$ and $p^* = \left( \lambda_2 - q_{2/2} \frac{1-\delta}{1-\delta^{c+1}} \right) / \left( \frac{\delta(1-\delta^c)}{1-\delta^{c+1}} q_{2/1,2} \right)$, respectively.*

*Proof.* In the dominant system in which the primary node transmits dummy packets when its queue is empty, $B$ is decoupled from the remaining of the system and modeled as a discrete-time $M/D/1/c$ system with arrival and service rates $\delta$ and $1$, respectively. We know in that case that $B$ is always ergodic and nonempty with

$$\Pr[B \neq 0] = \frac{\delta(1-\delta^c)}{1-\delta^{c+1}} \tag{6.32}$$

with $\delta$ strictly less than $1$. If $\delta = 1$, $B$ is nonempty with probability 1 which is not of our interest since we can rule out the role of the battery in the steady-state. The rest of the proof is similar to that of Theorem 6.3.1.

□

The subregion $\mathcal{R}_1$ is depicted in Fig. 6.2 with dotted line. Specifically, if $\eta > 0$, the line segments $AD$ and $DE$ correspond to the boundaries due to the inequalities (6.26) and (6.27), respectively. The subregion $\mathcal{R}_2$ is also plotted in Fig. 6.3 with dotted line. One can easily observe from the figures that the stability region for the case with finite capacity battery is always a subset of that for the case with infinite capacity battery. Also, note that as $c \to \infty$, the stability region for the finite battery case approaches to that for the infinite battery case.

## 6.6 Collision Channel with Probabilistic Erasures

For the completeness of our discussion, we present the stability conditions for the collision channel case with probabilistic erasures.

The stability region is given by:

$$\mathcal{R} = \mathcal{R}_1 \bigcup \mathcal{R}_2 \tag{6.33}$$

where

$$\mathcal{R}_1 = \left\{ (\lambda_1, \lambda_2) : \frac{\lambda_1}{q_{1/1}} + \frac{\lambda_2}{q_{2/2}} < 1, 0 \leq \lambda_1 \leq \delta q_{1/1} \right\} \tag{6.34}$$

and

$$\mathcal{R}_2 = \left\{ (\lambda_1, \lambda_2) : \lambda_1 < \delta q_{1/1}, 0 \leq \lambda_2 \leq (1-\delta) q_{2/2} \right\} \tag{6.35}$$

The proof is omitted for brevity.

It is trivial to observe that $\mathcal{R}_2 \subset \mathcal{R}_1$ and, thus, $\mathcal{R} = \mathcal{R}_1$. The optimal $p^*$ achieving the boundaries is always $p^* = 0$. It is intuitive that the well-designed cognitive access protocol will not allow the secondary node to access the channel when the primary node is transmitting. This is because such simultaneous transmissions would definitely result in a collision. The stability region is depicted in the Fig. 6.4. Since the stability region is identical with the subregion $\mathcal{R}_1$ for the case of $\eta \leq 0$ with multipacket reception capability in Fig. 6.2(b), the stability region for the collision case is a subset of that for the case with the multipacket reception capability.

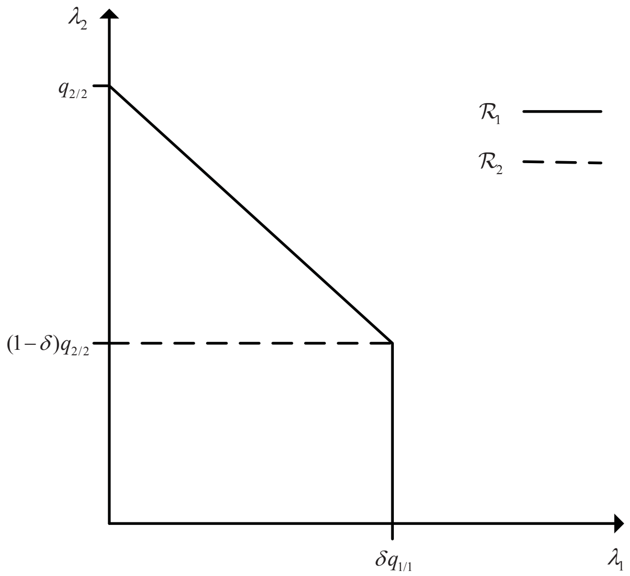



Figure 6.4: The stability region for the case of collision channel with probabilistic erasures (solid and dotted lines depict the subregions $\mathcal{R}_1$ and $\mathcal{R}_2$, respectively.)

## 6.7 Conclusion

We employed an opportunistic multiple access protocol that observes the priorities among the users to better utilize the limited energy resources. Owing to the multipacket reception capability, the secondary node not only utilizes the idle slots but also can take advantage of such an additional reception by transmitting along with the primary node by randomly accessing the channel in a way that does not adversely affect the quality of the communication over the primary link. Consequently, at a given input rate of the primary source, we could choose the optimal access probability by the secondary transmitter to maximize its own throughput and this maximum was also identified. The result is obtained for both cases when the capacity of the battery at the primary node is infinite and also finite. This initial research provides some insights on how to run such a network of nodes having different energy constraints. The channel model we assumed in this chapter is time invariant, we plan to study the impact of channel state information in future work. Extending the approach proposed here to more realistic environments with multiple set of source-destination pairs, although highly desirable, presents serious difficulties due to the interaction between the nodes.

# Chapter 7

# Path Diversity Gain with Network Coding and Multipath Transmission in Wireless Mesh Networks

The material in this chapter was presented in [45].

## 7.1 Introduction

The core notion of network coding introduced in [21] is to allow and encourage mixing of data at intermediate network nodes. Network coding is a generalization of the traditional store and forward technique. Most of the theoretical results in network coding are for multicast but the vast majority of Internet traffic is unicast. An application of network coding to wireless environments has to address multiple unicast flows, if it has any chance of being used. In particular, with multicast, all receivers want all packets. Thus intermediate nodes can encode any packets together, without worrying about decoding which will happen eventually at the destinations.

We consider unicast flows in a multi-hop wireless mesh network with lossy directional links. In such networks the largest percentage of uplink traffic is destined for or originates from a gateway interconnecting the mesh network to a wired network. Moreover, a mesh node can provide access to multiple clients. Hence, the uplink traffic from these clients that is destined to the same gateway can be coded at the mesh node, and decoded at the gateway. Similarly, downlink traffic destined for the clients of the same mesh node can be coded at the gateway and decoded at the mesh node.

The goal of this work is to investigate the performance that can be achieved by exploiting path diversity through multipath forwarding and redundancy through network coding. Specifically, we compare the performance and tradeoff in terms of packet delay and throughput achieved by combining multipath forwarding and network coding, with that of simple multipath routing of different flows (which achieves the highest throughput), the transmission of multiple copies of a single flow over multiple paths (which achieves the highest redundancy and the least delay), and traditional single path routing.

The idea of using redundancy is central in channel coding theory. In this work we use redundant paths to send coded packets in order to recover the loss of information using packets from another path, thus decreasing the delay. The work in [46] uses path diversity for fast recovery from link outages. The work in [47] introduces error correcting network coding as a generalization of classical error correcting codes. The work in [48] studies the coding delay in packet networks that support network coding. The authors in [49] propose efficient algorithms for the construction of robust network codes for multicast connections. The goal of this work is to to provide instantaneous recovery from single edge failures. The work in [50] presents an approach for designing network codes by considering path failures in the network instead of edge failures. There is a lot of work for opportunistic routing in wireless mesh networks, with or without network coding. COPE [51], MORE [52] and $MC^2$ [53] investigate network coding with opportunistic routing in wireless networks with broadcast transmissions, focusing exclusively on the throughput improvements. ExOR [54] and ROMER [55] investigate opportunistic routing in broadcast wireless networks without network coding. Moreover, these works also focus on the throughput improvements, except [55] which also considers the packet delivery ratio. The work of [56] considers diversity coding, and investigates the allocation of data to multiple paths that maximizes the probability of successful reception. The work of [57] extends the previous work, in the case where the failure probabilities are different for different paths, and when the paths are not necessarily independent.

Our contribution and a key difference with the previous works is that we study the delay and throughput tradeoff and compare network coding with other transmission schemes such as single path, multipath and multicopy. We study the average delay per packet and the throughput achieved, disregarding the queueing delay at the sender, the encoding and decoding delays, and the ACK transmission delays.

The model we assume is a one-source unicast acyclic network with lossy links. The nodes inside the network (except the source and the destination) act as relays, do not decode the information but simply forward coded packets that have been previously received from the source or the previous node. This allows for uncoordinated, low-complexity processing at the nodes.

The analytical framework presented in this work considers the case of end-to-end retransmission for achieving reliability, and is generalized for an arbitrary number of paths and hops. We also derive the minimum and maximum number of coded packets that are needed at the receiver to retrieve all packets sent by the sender; this can be used to obtain a lower and upper bound for the delay in the case of linear network coding with multipath forwarding. The application of linear network coding results in the considerable reduction of the computational complexity at the nodes. An interesting conclusion that comes out from this work is that network coding gives us an advantage in terms of delay-throughput trade-off. We will see that when the number of paths increasing network coding unfolds it advantages comparing to other routing schemes.

The chapter is organized as follows: Section 7.2 presents the analytical model for the throughput and delay in the case of end-to-end retransmissions, for three and for $2^k - 1$ path

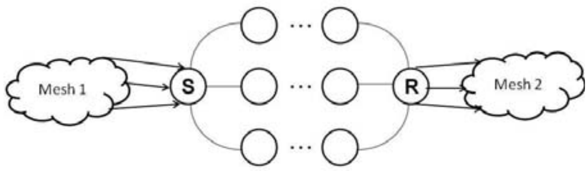



Figure 7.1: Simple three path network

networks. Section 7.3 presents numerical results based on the previous models, and section 7.4 concludes the chapter.

## 7.2 Analytical Model

In this section we are presenting the analytical model for the throughput and delay in the case of end-to-end retransmissions. We initially consider the simple model shown in figure 7.1, which contains three paths from node S to node R, then we extend to $2^k - 1$ paths.

Recall from the previous section that node S can be a wired network gateway and node R can be a mesh node connecting multiple wireless clients or vice versa. Each path has $n$ hops.

We consider the following approaches for transmitting packets from S to R: With single path routing, packets from all flows follow the same single path, leaving the other paths unutilized. With simple multipath routing, all the paths are used to transmit packets from different flows; the packets belonging to the same flow follow the same path. Another alternative is to transmit copies of packets belonging to a single flow on all the links; we call this scheme multicopy.

Finally the last scheme combines multipath with network coding. For the three path network shown in figure 7.1 node S sends three linear combinations $x_i a + y_i b$ for $i = 1, 2, 3$, of packets $a$ and $b$ along the three paths; the receiver needs to receive at least two linear independent of these combinations in order to decode the packets, and retrieve the original two packets $a$ and $b$.

We assume that the probability of a packet error on each link is the same, and equal to $e$. If a path consists of $n$ hops, then the probability of a packet correctly reaching node R is $P_c = (1-e)^n$; then $P_e = 1 - P_c$ is the probability of a packet error along the whole path.

Next we compute the packet delay and throughput achieved by each of the forwarding schemes mentioned above. The packet delay $D$ is the delay for transmitting a packet from the S to R, when the packet is at the head of the transmission queue at S, i.e., we do not include in $D$ the queuing delay at S, and we assume there is no congestion, hence no queuing delay, in the intermediate nodes. We also assume that the transmission delay of each hop is one. Moreover, if a packet is not correctly received by R, it is retransmitted by node S; we disregard the delay for transmitting ACKs back from R to S.

### 7.2.1 Analysis for a three-path network with $n$ hops

**Single Path**

The average delay is given by

$$D_{sp} = (1-P_e)n + P_e(n + D_{sp}) \Leftrightarrow D_{sp} = \frac{n}{1-P_e}\,,$$

and the throughput is

$$Thr_{sp} = \frac{1}{D_{sp}} = \frac{1-P_e}{n}\,.$$

**Multipath**

Multipath has the same delay as single path

$$D_{mp} = D_{sp}\,,$$

and its throughput is three times the throughput of single path

$$Thr_{mp} = 3Thr_{sp} \,.$$

**Multicopy**

The delay and throughput are

$$D_{mcop} = \frac{n}{1 - P_e^3}\,, \quad Thr_{mcop} = \frac{1}{D_{mcop}}\,.$$

**Multipath with Network Coding**

The delay $D_{nc}$ is the average delay to receive at least two of the three independent linear combinations sent by node S:

$$\begin{aligned} D_{nc} &= (1 - P_e)^3 n + 3P_e(1 - P_e)^2 n + \\ &+ 3P_e^2(1 - P_e)(n + D_1) + P_e^3(n + D_{nc}) \end{aligned}$$

where

$$D_1 = (1 - P_e^2)n + P_e^2(n + D_1)\,.$$

The first term in the expression for $D_{nc}$ corresponds to the case of correct transmission on all three paths. The second term corresponds to the case of an error in one of the three paths. The third term corresponds to the case of errors in two of the three paths, hence there is an additional delay $D_1$ to receive one more linear combination. The last term corresponds to the case where there were errors on all three paths. Since in the time interval $D_{nc}$ node R receives two data packets, the average throughput is given by

$$Thr_{nc} = \frac{2}{D_{nc}}\,.$$

### 7.2.2 Generalization for $2^k - 1$ paths

Here we extend the previous model to the case of $2^k - 1$ paths,

**Single Path**

$$D_{sp} = \frac{n}{1 - P_e}\,, \quad Thr_{sp} = \frac{1}{D_{sp}}\,.$$

**Multipath**

$$D_{mp} = D_{sp}\,, \quad Thr_{mp} = (2^k - 1)Thr_{sp}\,.$$

**Multicopy**

$$D_{mcop} = \frac{n}{1 - P_e^{2^k-1}}\,, \quad Thr_{mcop} = \frac{1}{D_{mcop}}\,.$$

**Multipath with Network Coding**

We have $k$ packets to transmit through $2^k - 1$ paths with $n$ hops each. We calculate the delay for the reception of at least $k$ from a set of $2^k - 1$ packets. The delay to receive $k$ packets when we have already received $j$ packets is denoted by $D_{k,j}$. The delay we are interested in is $D_{nc} = D_{k,0}$.

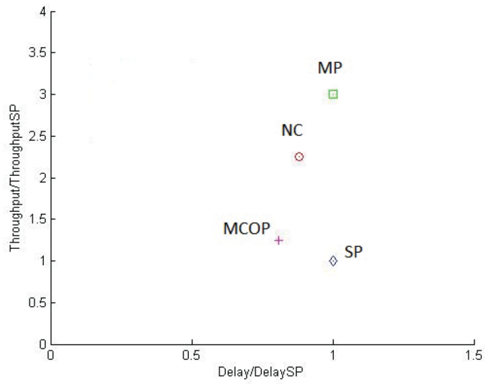
$$D_{k,j} = \sum_{i=k-j}^{2^k-j-1} \binom{2^k - j - 1}{i} P_c^i P_e^{2^k-i-j-1} n +$$
$$+ \sum_{i=0}^{k-j-1} \binom{2^k - j - 1}{i} P_c^i P_e^{2^k-i-j-1} (n + D_{k,i})\,,$$

$$Thr_{nc} = \frac{k}{D_{nc}}\,.$$

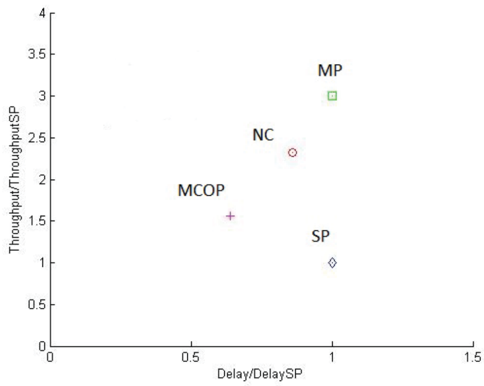
## 7.3 Numerical Experiments

In this section we present arithmetic results based on the models in section 7.2. Figures 7.2(a) and 7.2(b) show the delay - throughput tradeoff for error probabilities $0.2$ and $0.4$ respectively for networks with three paths of one hop each. Multipath with network coding achieves delay which is smaller than single and multipath, but worst than multi-copy forwarding. The throughput achieved by multipath with network coding is better than that achieved by multi-copy forwarding.

Error probabilities are assumed to be between $0.1$ and $0.8$ in the figures shown in current section. We want to compare the presented routing schemes in environments with heavy noise. Heavy noise in the wireless medium can be explained by interference, path losses and fading as it is well known.



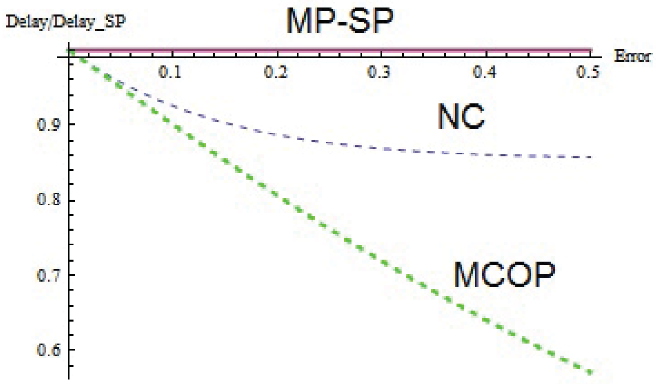

(a) $e = 0.2$



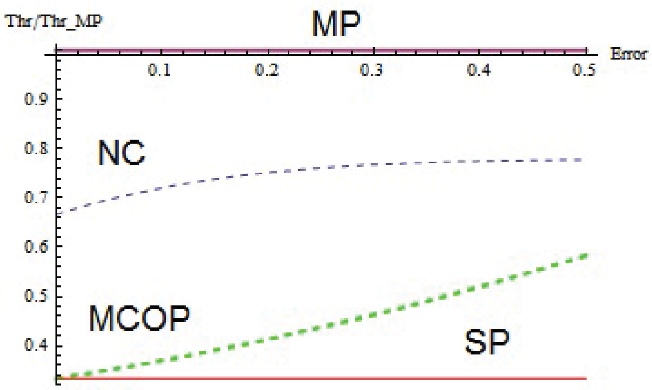

(b) $e = 0.4$

Figure 7.2: Delay-throughput tradeoff in the case of three paths with one hop each

Figure 7.3(a) shows that, as expected, the improvement in terms of lower delay which is achieved by multipath with network coding and multi-copy increases with increasing error probability. Regarding throughput, observe that a higher loss probability does not significantly affect the gains of multipath with network coding over single path forwarding, as much as it does for multicopy forwarding; this is also shown in figure 7.3(b).

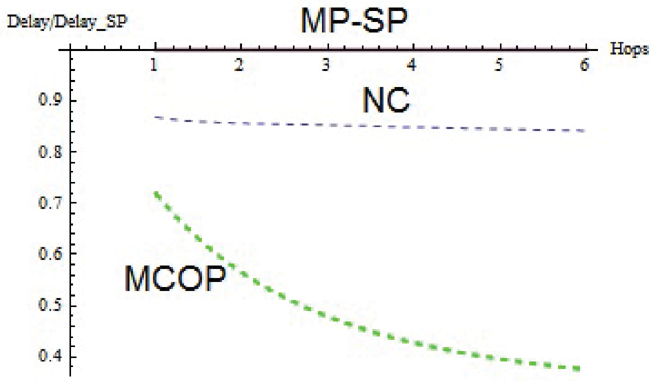



(a) $D/D_{sp}$ vs error probability

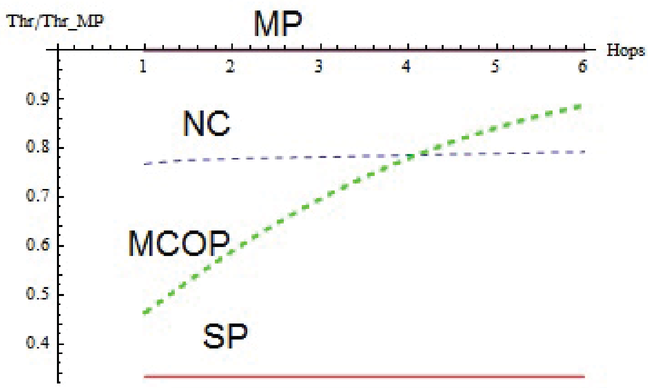



(b) $Thr/Thr_{mp}$ vs error probability

Figure 7.3: Delay and throughput for different errors probabilities, in the case of three paths with one hop each




(a) $D/D_{sp}$ vs number of hops

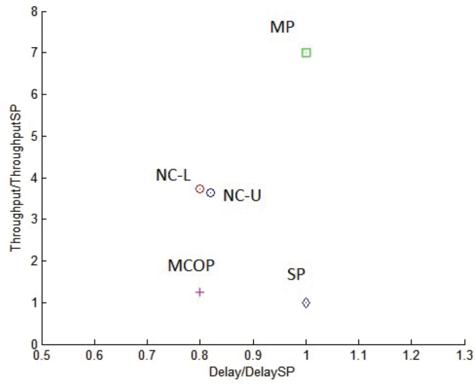



(b) $Thr/Thr_{mp}$ vs number of hops

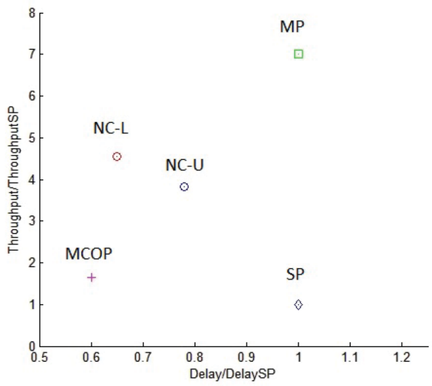
Figure 7.4: Delay and throughput for different number of hops, in the case of three paths and $e = 0.3$

Figures 7.4(a) and 7.4(b) show how the number of hops affects the throughput and delay. In particular, figure 7.4(a) shows that the improvement in terms of lower delay compared to single path forwarding increases with the number of hops, for both multipath forwarding with network coding and multicopy forwarding. Moreover, figure 7.4(b) shows that whereas for multipath forwarding with network coding, the throughput improvement compared to simple multipath forwarding remains relatively constant as the number of hops increase, for multicopy forwarding the throughput gain increases and after some number of hops, the gain with multicopy forwarding is higher compared to the gain with multipath forwarding with network coding. The above results for three paths indicate that the network coding delay gain over the single path and multipath schemes is about $15 - 20\%$. On the other hand, multicopy forwarding is superior when the loss becomes large and for a large number of hops because of its higher redundancy.

Figures 7.5(a) and 7.5(b) show the delay - throughput tradeoff for error probabilities $0.2$ and $0.4$ respectively for a network with seven paths, each with one hop. These figures include two graphs for network coding, one corresponding to the case of decoding after receiving three linear combinations (which is denoted by NC-L) and one for decoding after receiving four (which is denoted by NC-U); These numbers represent the lower and upper bound of the number of coded packets required to retrieve all packets at the receiver, as indicated by lemma 7.5. Multipath with network coding achieves delay, which is better than single and multipath, but approaches the delay of multicopy forwarding. Comparison of Figures 7.5(a) and 7.5(b), with Figures 7.2(a) and 7.2(b) shows that the improvements of network coding increase as the number of paths increases. Also, the throughput achieved by multipath with network coding is better than that achieved by multicopy forwarding.



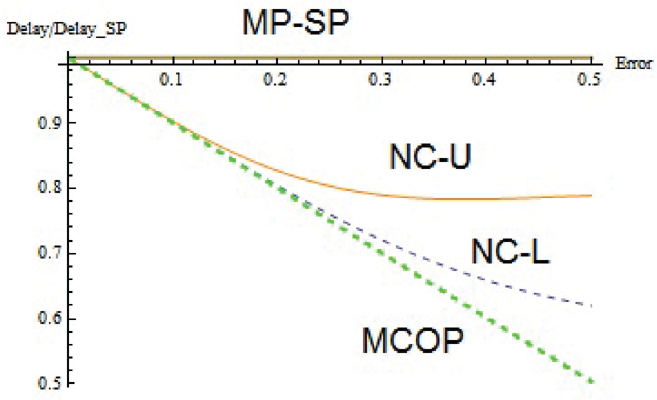

(a) $e = 0.2$



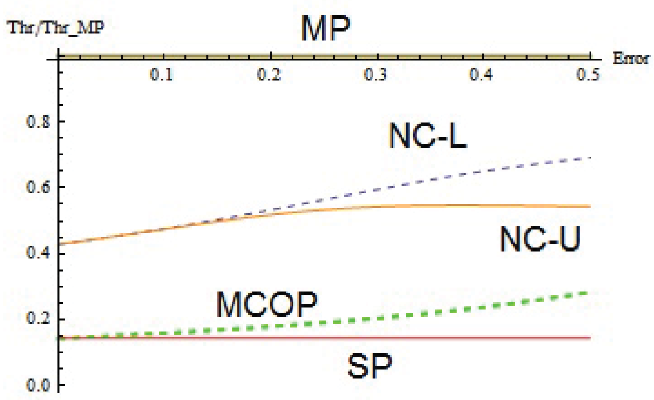

(b) $e = 0.4$

Figure 7.5: Delay-thoughput tradeoff in the case of seven paths

Figure 7.6(a) shows that, as in 7.3(a), the improvement in terms of lower delay which is achieved by multipath with network coding and multicopy increases with increasing error probability for the case of seven paths.

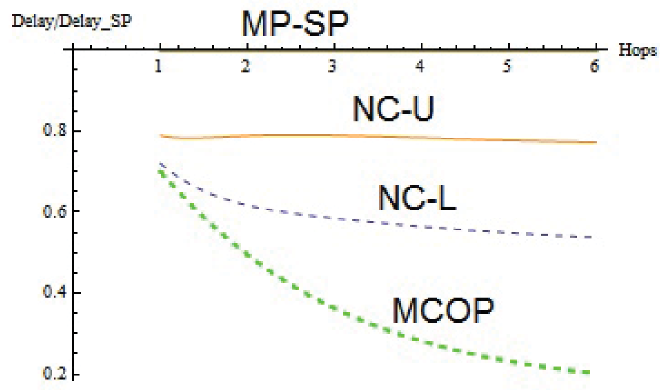



(a) $D/D_{sp}$ vs error probability

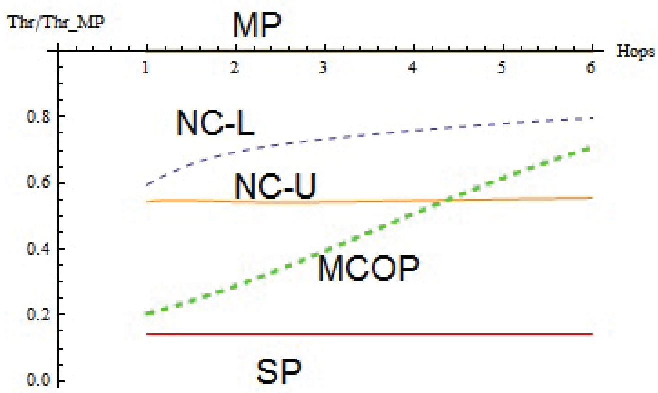



(b) $Thr/Thr_{mp}$ vs error probability

Figure 7.6: Delay and throughput in the case of seven paths with one hop each




(a) $D/D_{sp}$ vs number of hops




(b) $Thr/Thr_{mp}$ vs number of hops

Figure 7.7: Delay and throughput for a different number of hops, in the case of seven paths and $e = 0.3$

Figures 7.7(a) and 7.7(b) show how the number of hops affects the throughput and delay for a network with seven paths. In particular, figure 7.7(a) shows that the improvement in terms of lower delay compared to single path forwarding increases with the number of hops, for both multipath forwarding with network coding and multicopy forwarding. Moreover, figure 7.7(b) shows that whereas for multipath forwarding with network coding, the throughput improvement compared to simple multipath forwarding remains relatively constant as the number of hops increases, for multicopy forwarding the throughput gain increases and after some number of hops, the gain with multicopy forwarding is higher compared to the gain with multipath forwarding with network coding. The above results for seven paths suggest that the network coding delay gain compare to single path and multipath forwarding is about $20 - 40\%$. On the other hand, in the case of seven paths, the delay with network coding is close to the delay with multicopy forwarding.

## 7.4 Conclusion

In this chapter we investigated the performance and reliability that can be achieved by exploiting path diversity through multipath forwarding together with redundancy through network coding, when end-to-end retransmissions are used for achieving reliable packet transmission. The work in this chapter is at very fundamental level and it is not supposed to provide blueprints for a real network, however it helps understanding about network coding and its impact on redundancy and the trade-off among other routing schemes. We compared the performance and tradeoff in terms of packet delay and throughput achieved by combining multipath forwarding and network coding, with that of simple multipath routing of different flows, transmission of multiple copies of a single flow over multiple paths, and single path routing. We saw that network coding decreases the delay that is needed for the transmission of a packet compared with multipath and traditional single path forwarding, achieving a delay-throughput balance that lies between the corresponding performance of simple multipath and multicopy forwarding, which sends the same packet across all available paths. Another important result is that as the number of available paths increases, the gain from network coding also increases.

Future work will investigate the delay - throughput tradeoffs in the case of hop-by-hop retransmissions. Another important issue is the correlation of losses among the paths, it is interesting also the study of paths that contain links with bursty errors. Initial results indicate that in the case of networks that have paths with common links, the advantages of network coding are more pronounced. The analysis done in the present chapter will serve as a guideline for a more general network model including the previous considerations.

## 7.5 Lemma

In this appendix we give a Lemma that identifies the minimum and the maximum number of linear combinations needed in order to retrieve all the packets when we use network coding.

**Lemma 7.5.1.** *Consider a linear $n$ dimensional vector space $GF(2)^n$. We need exactly $n$ linear independent vectors and at most $\lceil (2^n - 1)/2 \rceil = \lceil 2^{n-1} - 1/2 \rceil = 2^{n-1}$ different vectors(not independent) (excluding the zero vector) in order to reconstruct the vector space.*

*Proof.* It is obvious that we need exactly $n$ linear independent vectors, but with a random selection of $n$ linear combinations there is a possibility that the vectors cannot span the space due to linear dependency. The vectors in $GF(2)^n$ have $n$ coordinates and there are $2^n - 1$ vectors (excluding the zero one). If we choose $2^{n-1} - 1$ we can span an $n - 1$ dimensional subspace of $GF(2)^n$ space; this means that we have a collection of vectors that in total have $n - 1$ coordinates with value $1$. So, we need one more coordinate to be different than zero in the previous collection of $2^{n-1} - 1$, we choose another one from the pool of the $2^n - 1$ vectors. Now it is obvious from Pigeonhole principle that we are able to span the $n$ dimensional space. □

For example, if $n = 3$, we need three linear independent vectors in order to construct the vector space, and any collection of $4 = 2^{3-1}$ different vectors spans the three dimensional vector space.

## 7.6 Generalization for $2^k - 1$ paths with different error probabilities

In this appendix we extend the model in section 7.2.2 to the case of paths with different error characteristics. With this extension we are able to encapsulate to our study notions as congestion that exist in real networks. We denote $P_{e,i}$ the probability of a transmission error at path $i$.

**Single path**

$$D_{sp} = \frac{n}{1 - \min_i P_{e,i}}$$

$$Thr_{sp} = \frac{1}{D_{sp}}$$

**Multipath**

$$D_{mp} = \frac{1}{2^k - 1} \sum_{i=1}^{2^k-1} \frac{n}{1 - P_{e,i}}$$

$$Thr_{mp} = 2^k - 1 - \sum_{i=1}^{2^k-1} P_{e,i}$$

**Multicopy**

$$D_{mcop} = n/(1 - \prod_{i=1}^{2^k-1} P_{e_i})$$

$$Thr_{mc} = \frac{1}{D_{mc}}$$

**Multipath with Network Coding**

If $N = 2^k - 1$ is the number of paths and $K$ is the number of linear combinations necessary to decode correctly the original $k$ packets, then from lemma 7.5 we must have $k \leq K \leq 2^{k-1}$.

The delay to receive these $K$ linear combinations is

$$
\begin{aligned}
D_{nc} = \prod_{i=1}^{N}(1-P_{e,i})n + \sum_{i=1}^{N} P_{e,i} \prod_{\substack{j=1 \\ j\neq i}}^{N}(1-P_{e,i})n+ \\
+\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} P_{e,i}P_{e,j} \prod_{\substack{k=1 \\ k\neq i,j}}^{N}(1-P_{e,k})n + ... \\
+\sum_{i_1=1}^{N} ... \sum_{\substack{i_{N-K}=1 \\ i_{N-K}\neq i_1,...,i_{N-K-1}}}^{N} \prod_{k=1}^{N-K} P_{e,i_k} \prod_{\substack{j=1 \\ j\neq i_1,...,i_{N-K}}}^{N} (1-P_{e,j})n+ \\
+\sum_{i=1}^{N}(1-P_{e,i}) \prod_{\substack{j=1 \\ j\neq i}}^{N} P_{e,j}(n+D_i)+ \\
+\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N}(1-P_{e,i})(1-P_{e,j}) \prod_{\substack{k=1 \\ k\neq i,j}}^{N} P_{e,k}(n+D_{i,j}) + ... \\
+\sum_{i_1=1}^{N} ... \sum_{\substack{i_{K-1}=1 \\ i_{K-1}\neq \\ i_1,...,i_{K-2}}}^{N} \prod_{k=1}^{K-1} \prod_{\substack{j=1 \\ j\neq \\ i_1,...,i_{K-1}}}^{N} P_{e,j}(1-P_{e,i_k})(n+D_{i_1,...,i_{K-1}})+ \\
+\prod_{i=1}^{N} P_{e,i}(n+D_{nc})
\end{aligned}
$$

where $D_{i_1,...,i_j}$ is the delay to receive the additional $K-j$ linear combinations after receiving $j$ linear combinations. The calculation of this delay is similar to the calculations in the preceding sections. Note that when a packet needs to be retransmitted, it follows the same path as the path of the initial transmission attempt. An interesting extension is to retransmit packets using the paths with the smallest error probability. Finally, the throughput is given by

$$Thr_{nc} = \frac{k}{D_{nc}}.$$

# Chapter 8

# Delay and Throughput of Network Coding with Path Redundancy for Wireless Mesh Networks

The material in this chapter was presented in [58].

## 8.1 Introduction

In this chapter, we extend our work in [45]. In that work we investigated the performance that can be achieved by exploiting path diversity through multipath forwarding for end to end retransmissions. We saw that network coding decreases the delay that is needed for the transmission of a packet compared with multipath and traditional single path forwarding, achieving a delay-throughput balance that lies between the corresponding performance of simple multipath and multicopy forwarding, which sends the same packet across all available paths. Another result was that as the number of available paths increases, the gain from network coding also increases.

We consider unicast flows in a multi-hop wireless (mesh) network with lossy directional links. In such networks the largest percentage of uplink traffic is destined for or originates from a gateway interconnecting the mesh network to a wired network. Moreover, a mesh node can provide access to multiple clients. Hence, the uplink traffic from these clients that is destined to the same gateway can be coded at the mesh node, and decoded at the gateway. Similarly, downlink traffic destined for the clients of the same mesh node can be coded at the gateway and decoded at the mesh node. In real-world wireless scenarios, end-to-end connectivity is often intermittent, limiting the performance of end-to-end transport protocols. For this reason hop by hop retransmission is preferred [59] and this work will focus on hop by hop retransmission in the presence of link losses with either end to end or hop by hop network coding process.

The goal of this work is to investigate the performance that can be achieved by exploiting path diversity through multipath forwarding and redundancy through network coding in a multihop network using hop by hop retransmission. Specifically, we compare the performance and tradeoff in terms of packet delay and throughput achieved by combining multipath forwarding and network coding, with that of simple multipath routing of different flows, the transmission of multiple copies of a single flow over multiple paths (which achieves the least delay due to the highest redundancy), and traditional single path routing.

The idea of using redundancy is central in channel coding theory. In this work we use redundant paths to send coded packets in order to recover the loss of information using packets from another path, thus decreasing the delay. The work in [46] uses path diversity for fast recovery from link outages. The work in [47] introduces error correcting network coding as a generalization of classical error correcting codes. The work of [56] considers diversity coding, and investigates the allocation of data to multiple paths that maximizes the probability of successful reception. The work of [57] extends the previous work, in the case where the failure probabilities are different for different paths, and when the paths are not necessarily independent.

Our contribution and a key difference with the previous works is that we study the delay and throughput tradeoff and compare network coding with other transmission schemes such as single path, multipath and multicopy. We study the average delay per packet and the throughput achieved, disregarding the queueing delay at the sender, the encoding and decoding delays, and the ACK transmission delays. The model we assume is a one-source unicast acyclic network with lossy directional links. The analytical framework presented in this work considers the case of hop by hop retransmission for achieving reliability, and is generalized for an arbitrary number of paths and hops. The coding process we study includes end to end and hop by hop coding.

The rest of the chapter is organized as follows: Section 8.2 presents the network models assumed in the present work. Sections 8.3 and 8.4 presents the analytical model for the throughput and delay in the case of hop by hop retransmissions where the coding is end to end and hop by hop respectively. Section 8.5 presents the case of a network with three paths and different error probabilities. Section 8.6 presents numerical results based on the previous models, and finally section 8.7 concludes the chapter.

## 8.2 The model of the network

The model we assume is a one-source unicast acyclic network with lossy directional links. We consider the case of hop by hop retransmission. When an error occurs at the transmission between two nodes for example node $i$ to $i+1$, node $i$ re-sends the information to $i+1$. Figure 8.1 shows a network with node-disjoint paths where the coding process is end to end. Figure 8.2(b) presents a network with paths having nodes in common where the coding process is hop by hop. When the network has more than one hop, the inner nodes can decode the information and then re-encode it. In this work we study the average delay per packet and the throughput achieved, disregarding the queueing delay at the sender, the encoding and decoding delays, and the ACK transmission delays. For the sake of simplicity we assume that the number of hops is the same for every path in the network and every link has the same error probability $e$. In section 8.5 we relax this assumption and present the analysis of a network

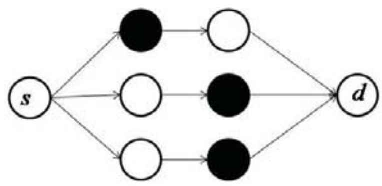
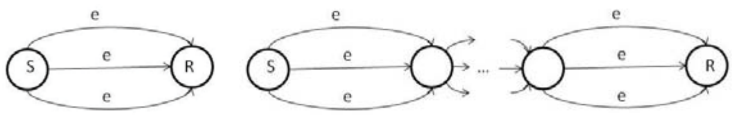



Figure 8.1: An instance of a network with node-disjoint paths, with $n = 3$ and $m = 3$, the corresponding state is $S = (1, 2, 2)$.

with three paths each with a different error probability.

## 8.3 Analytical model for node-disjoint paths (End to end coding)

Consider a source s and its receiver d. The network we study here has $n$ paths, each path having $m$ hops. The original packets are $k$ (where $k \leq n$). In order to find the average time that is needed for d to receive the packets, we model our problem using absorbing Markov Chains [60]. The chain is absorbed when the receiver d has received $k$ packets. A state of this chain is denoted by $S$. $S$ is a $n$- tuple: $S = (s_1, s_2, ..., s_n)$, where $s_i$ is the number of hops traversed by a packet on path $i$, note that $0 \leq s_i \leq m$ and $1 \leq i \leq n$. For example in Figure 8.1, the nodes with black color are the ones that have received already the packet.




(a) One hop (b) n hops

Figure 8.2: Simple network with three paths having nodes in common

The state space denoted by $V_S$ contains all the $(m+1)^n$ states of the Markov Chain. $V_S$ is divided into two sub-spaces $V_T$ and $V_A$, $V_S = V_T \cup V_A$. $V_T$ and $V_A$ are the spaces that contain the transient and absorbing states respectively. There are $|V_S| = (m+1)^n$ states in total. The absorbing ones are:

$$|V_A| = \sum_{i=k}^{n} \binom{n}{i} \tag{8.1}$$

The transient states are:

$$|V_T| = (m+1)^n - \sum_{i=k}^{n} \binom{n}{i} \tag{8.2}$$

The transition matrix $T$ of the Markov Chain has the following canonical form [60]:

$$T = \begin{pmatrix} P & R \\ 0 & I \end{pmatrix} \tag{8.3}$$

$P$ is an $|V_T| \times |V_T|$ matrix, $R$ is $|V_T| \times |V_A|$ and $I$ is $|V_A| \times |V_A|$ matrix. It is known that for an absorbing Markov Chain the matrix $I - P$ has an inverse [60]. Also it is known that:

$$t = (I - P)^{-1}\mathbf{1}_{|V_T|\times 1} \tag{8.4}$$

where $t$ is the expected number of steps before the chain is absorbed and $\mathbf{1}_{|V_T|\times 1}$ is the all-ones column vector. The first element of $t$ is the expected time for the chain to be absorbed starting from the initial state, that is the delay we want to compute. The rest of this section presents the procedure in order to compute the matrix $P$. We assign indices for the transient states, the initial state $S_0 = (0, 0, ..., 0)$ being the first one. This indexing facilitates the computation of the elements of matrix $P$, for example $P_{ij}$ is the probability of transition from $S_i = (s_1^i, ..., s_n^i)$ to $S_j = (s_1^j, ..., s_n^j)$. The elements of $P$ can be computed by the following:

$$P_{ij} = \begin{cases} 0, & \text{if } \exists k \text{ s.t. } s_k^j < s_k^i \text{ or } s_k^j - s_k^i > 1 \\ e^{n-correct-final}(1-e)^{correct}, & \text{otherwise.} \end{cases} \tag{8.5}$$

$$final = \sum_{k=1}^{n} \lfloor \frac{s_k^i}{m} \rfloor \tag{8.6}$$

$$correct = \sum_{k=1}^{n} (s_k^j - s_k^i) \tag{8.7}$$

The chain is absorbed when the receiver has received at least $k$ packets, which means $final \geq k$.

Next we show how the previous procedure can be applied for the computation of the delay and throughput for single path, multipath, multicopy and multipath with network coding.

### 8.3.1 Single Path

For this case, we apply the previous procedure with $n := 1$ and $k := 1$, to calculate the delay $D_{sp}$. The throughput is given by $Thr_{sp} = \frac{1}{D_{sp}}$.

### 8.3.2 Multipath

The delay for multipath is equal to $D_{sp}$. The throughput is given by $Thr_{mp} = \frac{n}{D_{sp}}$.

### 8.3.3 Multicopy

Multicopy is the technique for maximum redundancy, we send the same symbol to all paths. We apply the previous procedure with $n := n$ and $k := 1$, to calculate the delay for multicopy $D_{mcop}$. The throughput is given by $Thr_{mcop} = \frac{1}{D_{mcop}}$.

### 8.3.4 Multipath with Network Coding

There are $n$ paths and we send $k$ original(uncoded) symbols through $n$ linear combinations (redundancy), the procedure is applied with parameter $n := n$ and $k := k$, to calculate the delay for network coding $D_{nc}$. The throughput is given by $Thr_{nc} = \frac{k}{D_{nc}}$.

In section 8.6 we will present the arithmetic results derived from the previous procedure for various numbers of paths and hops.

## 8.4 Analytical model for paths with nodes in common (Hop by hop coding)

The derivation of the equations in this section is based on [45] section II. There is a small change for the case of network coding.

### 8.4.1 Three paths

In this part we will present the equations corresponding to network depicted in Figure 8.2(a). The probability of error in each path is $e$.

#### Single Path

The average delay is given by $D_{sp} = \frac{1}{1-e}$ and the throughput is $Thr_{sp} = \frac{1}{D_{sp}} = 1 - e$ .

#### Multipath

Multipath has the same delay as the single path $D_{mp} = D_{sp}$ and its throughput is three times the throughput of single path $Thr_{mp} = 3Thr_{sp}$ .

#### Multicopy

The delay and throughput are $D_{mcop} = \frac{1}{1-e^3}$ and $Thr_{mcop} = \frac{1}{D_{mcop}}$ respectively.

#### Multipath with Network Coding

The delay $D_{nc}$ is the average delay to receive at least two of the three independent linear combinations sent by node S: $D_{nc} = \frac{(1-e)^3+3e(1-e)^2+3e^2(1-e)(1+D_1)+e^3}{1-e^3}$ where $D_1 = D_{mcop} = \frac{1}{1-e^3}$. The additional delay $D_1$ is to receive one more linear combination when we have already received one. Since in the time interval $D_{nc}$ node R receives two data packets, the average throughput is given by $Thr_{nc} = \frac{2}{D_{nc}}$.

### 8.4.2 Seven paths

#### Single Path

The average delay is given by $D_{sp} = \frac{1}{1-e}$ and the throughput is $Thr_{sp} = \frac{1}{D_{sp}} = 1 - e$.

#### Multipath

Multipath has the same delay as the single path $D_{mp} = D_{sp}$ and its throughput is seven times the throughput of the single path $Thr_{mp} = 7Thr_{sp}$.

#### Multicopy

The delay and throughput are $D_{mcop} = \frac{1}{1-e^7}$ and $Thr_{mcop} = \frac{1}{D_{mcop}}$ respectively.

#### Multipath with Network Coding

We have 3 packets to transmit through $2^3 - 1 = 7$ paths. According to lemma in appendix A in [45] we need at least 3 and at most 4 linear packet combinations to be able to decode the initial packets. The delay for receiving 3 or 4 linear combinations is denoted by $D_{nc-L}$,$D_{nc-U}$ respectively.

$$D_{nc-L} = \frac{1}{1-e^7}[\sum_{i=3}^{7}\binom{7}{i}(1-e)^i e^{7-i} + \sum_{i=1}^{2}\binom{7}{i}(1-e)^i e^{7-i}(1+D_{3,3-i}) + e^7],$$

where $D_{3,i}$ is the delay to receive $i = 1, 2$ encoded packets when 3 needed, $D_{3,1} = \frac{1}{1-e^7}$, $D_{3,2} = \frac{1}{1+e^7}[1 - e^7 + (1 + \frac{1}{1-e^7})(e^3(1-e^4) + e^4(1-e^3)]$ The average delay to receive 4 linear combinations is given by:

$$D_{nc-U} = \frac{1}{1-e^7}[\sum_{i=4}^{7}\binom{7}{i}(1-e)^i e^{7-i} + \sum_{i=1}^{3}\binom{7}{i}(1-e)^i e^{7-i}(1+D_{4,4-i}) + e^7],$$

where $D_{4,i}$ is the delay to receive $i = 1, 2, 3$ encoded packets when 4 needed, $D_{4,1} = D_{3,1}$, $D_{4,2} = D_{3,2}$, $D_{4,3} = D_{nc-L}$. The throughput is given by: $Thr_{nc} = \frac{3}{D_{nc}}$ .

**Note:** If the network topology has $n$ hops as in figure 8.2(b), then in order to find the total delay with the previous models we just need to add the delays for all the hops. In the case where all links have the same error probabilities then the total delay is $n$ times the delay for one hop.

## 8.5 Analytical model for the network with three paths and one hop each with different link errors

In this section we will give the equations for the delay and throughput for the above routing schemes when then paths have different error probabilities. The derivation of the equations in this section is based on [45] Appendix B. There is a small change for the case of network coding.

### 8.5.1 Single Path

The single path routing scheme selects the best available path from the three available. Thus the delay is $D_{sp} = \frac{1}{1-\min_i e_i}$ and the throughput is $Thr_{sp} = \frac{1}{D_{sp}}$.

### 8.5.2 Multipath

In this routing scheme different data flows follow different paths, so the average delay per packet and the throughput are: $D_{mp} = \frac{1}{3}\sum_{i=1}^{3} \frac{1}{1-e_i}$, $Thr_{mp} = \frac{3}{D_{mp}}$ respectively.

### 8.5.3 Multicopy

The multicopy scheme uses all available paths to forward the same flow, in this way achieves the maximum redundancy (but wasting resources). The average delay is: $D_{mcop} = 1/(1 - \prod_{i=1}^{3} e_i)$ and the average throughput is: $Thr_{mc} = \frac{1}{D_{mc}}$.

### 8.5.4 Multipath with Network Coding

Multipath with Network Coding uses all available paths sending linear combinations of initial packets to each of them. In this case with three paths available, we encode two packets and there are three linear combinations. In order to decode the initial packets we have to receive two linear independent combinations. The average delay is given by:

$$D_{nc} = \frac{1}{1-\prod_{i=1}^{3} e_i}[\prod_{i=1}^{3}(1-e_i) + \sum_{i=1}^{3} e_i \prod_{j=1,j\neq i}^{3}(1-e_j) + \\ + \sum_{i=1}^{3}(1-e_i)(1+D_1)\prod_{j=1,j\neq i}^{3} e_j + \prod_{i=1}^{3} e_i]$$

where $D_1 = \frac{1}{1-\prod_{i=1}^{3} e_i}$.

Notice that for the Multipath with Network Coding scheme we are not able to compute the average delay per packet (because of the linear combinations) thus we calculate the delay needed to receive at least two linear independent combinations. This means that the above delay is the delay to receive all the uncoded packets. The throughput is: $Thr_{mc} = \frac{2}{D_{nc}}$.

## 8.6 Numerical Experiments

In this section we present arithmetic results based on the models described in the sections 8.3, 8.4 and 8.5.

### 8.6.1 Results for networks with node disjoint paths (End to end coding)

Table 8.1 shows the delay - throughput tradeoff for networks with node disjoint pats. Multipath with network coding achieves delay which is smaller than single and multipath, but worst than multi-copy forwarding. The throughput achieved by multipath with network coding is better than this achieved by multicopy forwarding. The gain from network coding is not so much, about $7-9\%$ in terms of delay for the errors $e=0.2$, $e=0.4$ and three paths with two hops each.

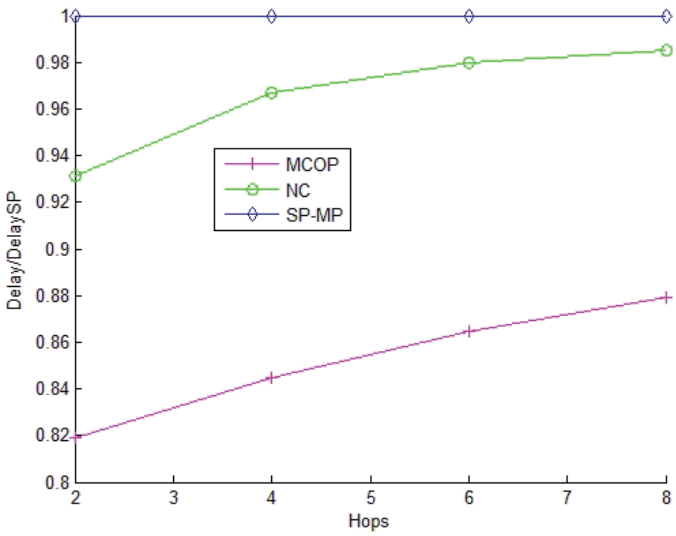
| Scheme | Error | Paths | Hops | Delay/DelaySP | Thr/ThrSP |
|---|---|---|---|---|---|
| NC | 0.2 | 3 | 2 | 0.9312 | 2.148 |
| SP | 0.2 | 3 | 2 | 1 | 1 |
| MP | 0.2 | 3 | 2 | 1 | 3 |
| MCOP | 0.2 | 3 | 2 | 0.819 | 1.221 |
| NC | 0.2 | 3 | 4 | 0.967 | 2.07 |
| SP | 0.2 | 3 | 4 | 1 | 1 |
| MP | 0.2 | 3 | 4 | 1 | 3 |
| MCOP | 0.2 | 3 | 4 | 0.845 | 1.184 |
| NC | 0.4 | 3 | 2 | 0.93 | 2.15 |
| SP | 0.4 | 3 | 2 | 1 | 1 |
| MP | 0.4 | 3 | 2 | 1 | 3 |
| MCOP | 0.4 | 3 | 2 | 0.694 | 1.44 |
| NC | 0.4 | 3 | 4 | 0.967 | 2.07 |
| SP | 0.4 | 3 | 4 | 1 | 1 |
| MP | 0.4 | 3 | 4 | 1 | 3 |
| MCOP | 0.4 | 3 | 4 | 0.761 | 1.31 |
| NC-L | 0.2 | 7 | 2 | 0.825 | 3.64 |
| NC-U | 0.2 | 7 | 2 | 0.888 | 3.38 |
| SP | 0.2 | 7 | 2 | 1 | 1 |
| MP | 0.2 | 7 | 2 | 1 | 7 |
| MCOP | 0.2 | 7 | 2 | 0.8 | 1.25 |
| NC-L | 0.4 | 7 | 2 | 0.771 | 3.89 |
| NC-U | 0.4 | 7 | 2 | 0.903 | 3.32 |
| SP | 0.4 | 7 | 2 | 1 | 1 |
| MP | 0.4 | 7 | 2 | 1 | 7 |
| MCOP | 0.4 | 7 | 2 | 0.613 | 1.63 |



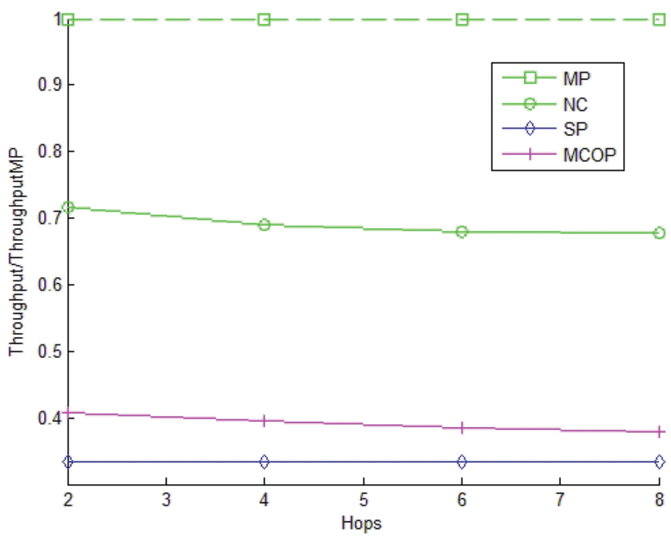
Table 8.1: Delay-Throughput Tradeoff for node disjoint paths

Multipath with network coding achieves delay, which is slightly better than single and multipath (about $4\%$), but worst than multi-copy forwarding for error probabilities $0.2$ and $0.4$ for the network with three paths and four hops. In comparing with two hops we observe that the gain for network coding is decreased. We see that network coding approaches multipath in term of delay, this is expected because of the relatively small number of paths and packets.

Figures 8.3(a) and 8.3(b) show how the number of hops affects the delay and throughput compared to delay for single path and throughput for multipath respectively.




(a) $D/D_{sp}$ vs number of hops



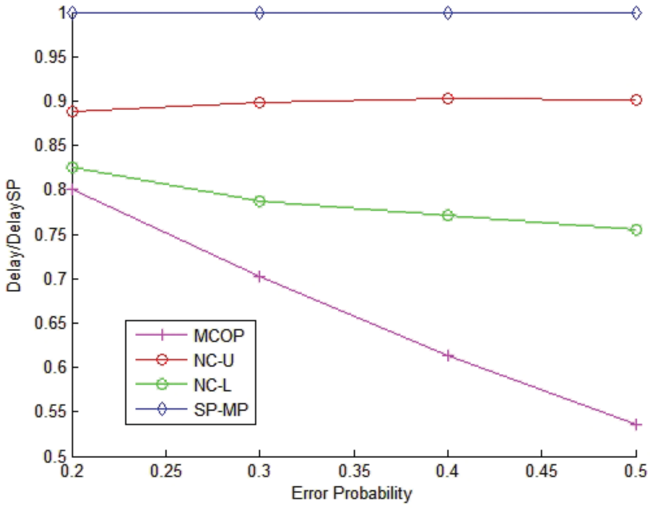

(b) $Thr/Thr_{mp}$ vs number of hops

Figure 8.3: Delay and throughput for a different number of hops, in the case of three paths and $e = 0.2$ (node disjoint paths)

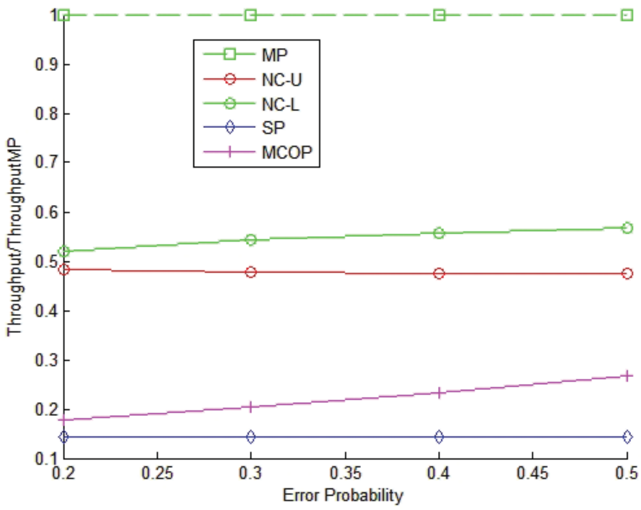
In the following there are plots for the network with seven paths and two hops. Table 8.1

includes two lines for network coding, one corresponding to the case of decoding after receiving three linear combinations (which is denoted by NC-L) and one for decoding after receiving four (which is denoted by NC-U); These number represent the lower and upper bound of the number of coded packets required to retrieve all packets at the receiver, as indicated by lemma [45]. Multipath with network coding achieves delay, which is better than single and multipath (about $20\%$), but worst than multi-copy forwarding. In term of throughput, network coding is much better($150\%$) than multicopy. Multicopy is superior when the loss become large and for a large number of hops because of its higher redundancy.

Throughput achieved by multipath with network coding is better than that achieved by multi-copy routing. Figure 8.4(a) shows that, as expected, the improvement in terms of lower delay which is achieved by multipath with network coding and multi-copy increases not so much with increasing error probability. Regarding throughput, we observe that a higher loss probability does affect the gains of multipath with network coding over single-path forwarding, as much they do in the case of multi-copy transmission; this is also shown in figure 8.4(b).




(a) $D/D_{sp}$ vs $e$




(b) $Thr/Thr_{mp}$ vs $e$

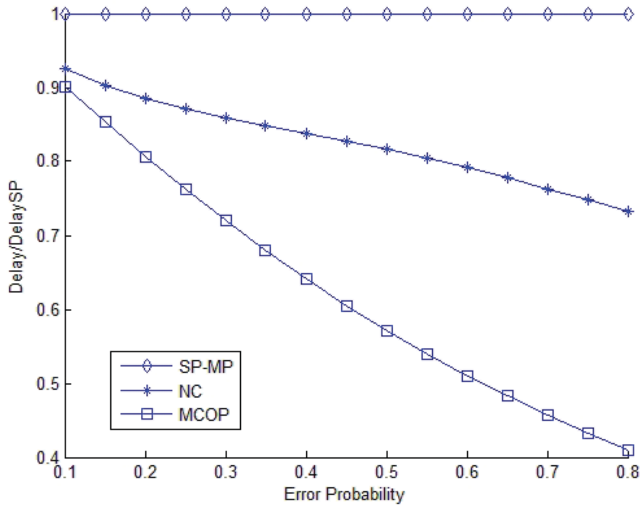
Figure 8.4: Delay and throughput vs $e$, in the case of seven paths and two hops each (node disjoint paths)

### 8.6.2 Results for networks with paths having common nodes (hop by hop coding)

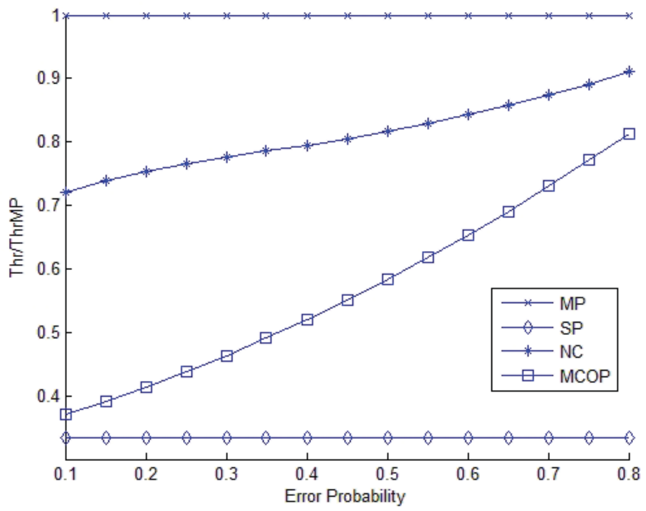
| Scheme | Error | Paths | Delay/DelaySP | Thr/ThrSP |
|---|---|---|---|---|
| NC | 0.2 | 3 | 0.8845 | 2.261 |
| SP | 0.2 | 3 | 1 | 1 |
| MP | 0.2 | 3 | 1 | 3 |
| MCOP | 0.2 | 3 | 0.807 | 1.24 |
| NC | 0.4 | 3 | 0.838 | 2.386 |
| SP | 0.4 | 3 | 1 | 1 |
| MP | 0.4 | 3 | 1 | 3 |
| MCOP | 0.4 | 3 | 0.641 | 1.56 |
| NC-L | 0.2 | 7 | 0.804 | 3.733 |
| NC-U | 0.2 | 7 | 0.827 | 3.629 |
| SP | 0.2 | 7 | 1 | 1 |
| MP | 0.2 | 7 | 1 | 7 |
| MCOP | 0.2 | 7 | 0.8 | 1.25 |
| NC-L | 0.4 | 7 | 0.656 | 4.573 |
| NC-U | 0.4 | 7 | 0.777 | 3.862 |
| SP | 0.4 | 7 | 1 | 1 |
| MP | 0.4 | 7 | 1 | 7 |
| MCOP | 0.4 | 7 | 0.601 | 1.664 |

Table 8.2: Delay-Throughput Tradeoff for paths with node in common

Figures 8.5(a) and 8.5(b) show how the error probability $e$ affects the delay and throughput compared to delay for single path and throughput for multipath respectively for the network with three paths and hop by hop coding process. Figures 8.6(a) and 8.6(b) show how the error probability $e$ affects the delay and throughput compared to delay for single path and throughput for multipath respectively for the network with seven paths. In these plots we see the advantage of network coding as error increases. In heavy noise the network coding outperforms even multipath in terms of throughput and it has only a fraction of delay of the singlepath (and multipath) scheme.



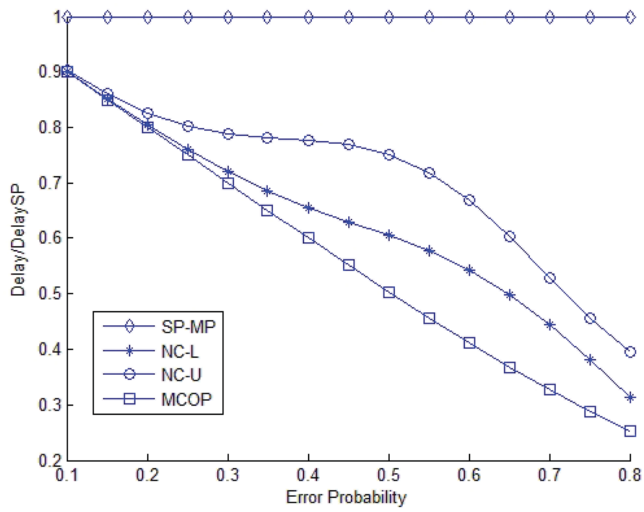

(a) $D/D_{sp}$ vs $e$



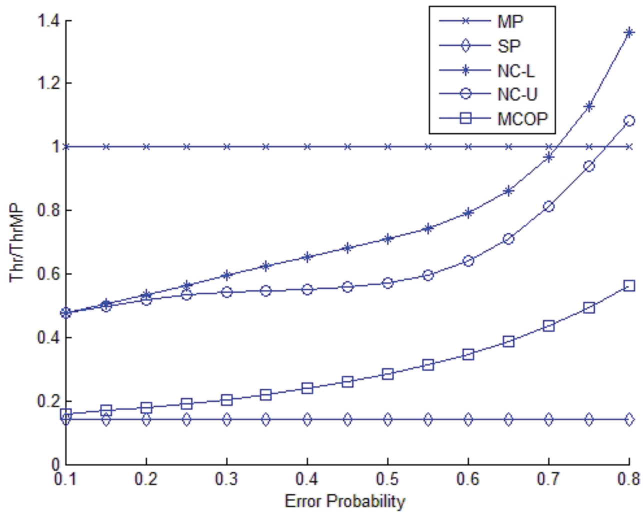

(b) $Thr/Thr_{mp}$ vs $e$

Figure 8.5: Delay and throughput vs $e$, in the case of three paths (paths with common nodes)

Table 8.2 shows the delay - throughput tradeoff the networks with paths having nodes in common for error probabilities $e = 0.2$ and $e = 0.4$. For the case of three paths multipath with network coding achieves delay, which is better than single and multipath (about $13-18\%$), but worst than multi-copy forwarding. In term of throughput network coding is much better($90\%$) than multicopy. For the case of seven paths multipath with network coding achieves delay, which is better than single and multipath (about $22-40\%$), but slightly worse than multi-copy forwarding. In term of throughput network coding is much better($200-350\%$) than multicopy.




(a) $D/D_{sp}$ vs $e$




(b) $Thr/Thr_{mp}$ vs $e$

Figure 8.6: Delay and throughput vs $e$, in the case of seven paths (paths with common nodes)

The above results indicate that the network coding in a network with paths with nodes in common has profound advantages compared to topologies with node-disjoint paths.

### 8.6.3 Results for Network with three paths with different error probabilities

Table 8.3 shows the delay-throughput trade-off for two different scenarios.

| Scheme | $e_1$ | $e_2$ | $e_2$ | Delay/DelaySP | Thr/ThrSP |
|---|---|---|---|---|---|
| NC | 0.3 | 0.4 | 0.5 | 0.974 | 2.053 |
| SP | 0.3 | 0.4 | 0.5 | 1 | 1 |
| MP | 0.3 | 0.4 | 0.5 | 1.189 | 2.523 |
| MCOP | 0.3 | 0.4 | 0.5 | 0.745 | 1.343 |
| NC | 0.5 | 0.6 | 0.8 | 1.056 | 1.894 |
| SP | 0.5 | 0.6 | 0.8 | 1 | 1 |
| MP | 0.5 | 0.6 | 0.8 | 1.583 | 1.895 |
| MCOP | 0.5 | 0.6 | 0.8 | 0.658 | 1.52 |

Table 8.3: Delay-Throughput Tradeoff for three paths with different error probabilities

In the case of $e_1 = 0.5$, $e_2 = 0.6$ and $e_2 = 0.8$ the multipath with network coding is the superior routing scheme, has almost the same delay as the singlepath but the double throughput. Multipath has the same throughput with network coding but $60\%$ more delay than single path.

Summarizing the above we can state that network coding offers significant advantages as the number of paths increases, when the nodes inside the network are able to decode and encode the received packets and finally under heavy noise environments.

## 8.7 Conclusion

In this chapter we investigated the performance and reliability that can be achieved by exploiting path diversity through multipath forwarding together with redundancy through network coding, when hop by hop retransmissions are used for achieving reliable packet transmission with end to end and hop by hop coding. We compared the performance and tradeoff in terms of packet delay and throughput achieved by combining multipath forwarding and network coding, with that of simple multipath routing of different flows, transmission of multiple copies of a single flow over multiple paths, and single path routing. We saw that network coding decreases the delay that is needed for the transmission of a packet compared with multipath and traditional single path forwarding, achieving a delay-throughput balance that lies between the corresponding performance of simple multipath and multicopy forwarding, which sends the same packet across all available paths. We saw that as the number of hops increases the gain for delay decreases for the network with node disjoint paths (end to end coding). Another important result is that as the number of available paths increases, the gain from network coding also increases. The significant advantages of network coding with redundancy appeared when hop by hop coding (paths with nodes in common) applied. Under heavy noise though the network coding scheme outerforms all the other routing schemes. This is obvious from the arithmetic results in the network with paths having different error probabilities.

The hop by hop coding process is not computationally expensive due to the linearity of the network coding technique and for this reason the delay from decoding and encoding is not so important.

The conclusion is that network coding offers significant advantages as the number of paths increases, when the nodes inside the network are able to decode and encode the received packets and finally under heavy noise environments.

Future work will investigate the delay - throughput tradeoff in the presence of bursty errors for hop by hop retransmissions. Another extension of this work should be the study of networks with different error probability for each hop for more complex topologies. Our future work involves the impact of interference and congestion to schemes described above.

# Chapter 9

# Conclusions

## 9.1 Summary of Contributions

In this dissertation, we focused on the wireless network-level cooperation.

We first examined the operation of a node relaying packets from a number of users to a common destination node. We assumed MPR capability for the relay and for the destination node. We studied a multiple capture model, where a user's transmission is successful if the received $SINR$ is above a threshold $\gamma$. We obtained analytical expressions for the relay's queue characteristics such as the stability condition, the values of the arrival and service rates and the average queue size. We showed that the arrival rate at the queue is independent of the relay probability of transmission, when the queue is stable. We studied the throughput per user and the aggregate throughput, and found that, under stability conditions, the throughput per user does not depend on the relay probability of transmission. We also have given the conditions under which the utilization of the relay offers significant advantages. An interesting result is that, given the link characteristics and the transmission probabilities, there is an optimum number of users that maximizes the aggregate throughput. These results could be useful in a network with many users and multiple relays for determining the way to allocate the users among the relays. With the MPR and the capture effect the advantages from deploying a relay node are more pronounced.

In Chapter 3 we extend the analysis of Chapter 2 by assuming that the relay node is capable of transmitting and receiving packets at the same time (full duplex) thus, the problem of self interference arises. We studied the impact of the self interference coefficient on the throughput per user and the aggregate throughput of the network. We showed that for perfect self-interference cancelation, the advantages are more pronounced. Another interesting result is that the self interference coefficient plays a crucial role when $\gamma$ is small because it can easily cause an unstable queue at the relay.

In Chapter 4, we introduced the notion of partial network-level cooperation by assuming a flow controller for the endogenous traffic to the relay from the source node. The flow controller regulates the degree of cooperation offered by the relay. The network was consisting of a source, a relay and a destination node. We provided an exact characterization of the stability region for this network. We proved that the system with the flow controller is always better than or at least equal to the system without the flow controller.

In Chapter 5, we studied the impact of energy constraints on a network with a source-user, a relay and a destination. The source and the relay node have external arrivals; furthermore, the relay is forwarding part of the source node's traffic to the destination. We provided an exact characterization of the stability region.

In Chapter 6, we employed an opportunistic multiple access protocol that observes the priorities among the users to better utilize the limited energy resources. Owing to the multipacket reception capability, the secondary node not only utilizes the idle slots but also can take advantage of such an additional reception by transmitting along with the primary node by randomly accessing the channel in a way that does not adversely affect the quality of the communication over the primary link. Consequently, at a given input rate of the primary source, we could choose the optimal access probability by the secondary transmitter to maximize its own throughput and this maximum was also identified. The result is obtained for both cases when the capacity of the battery at the primary node is infinite and also finite. This initial research provides some insights on how to run such a network of nodes having different energy constraints.

In Chapters 7 and 8, we investigated the performance and reliability that can be achieved by exploiting path diversity through multipath forwarding together with redundancy through network coding, when end-to-end hop-by-hop retransmissions are used for achieving reliable packet transmission. The work in these chapters is at a very fundamental level and it is not supposed to provide blueprints for a real network, however it helps our understanding about network coding and its impact on redundancy and the trade-off among other routing schemes. We compared the performance and tradeoff in terms of packet delay and throughput achieved by combining multipath forwarding and network coding, with that of simple multipath routing of different flows, transmission of multiple copies of a single flow over multiple paths, and single path routing. We saw that network coding decreases the delay that is needed for the transmission of a packet compared with multipath and traditional single path forwarding, achieving a delay-throughput balance that lies between the corresponding performance of simple multipath and multicopy forwarding, which sends the same packet across all available paths. Another important result is that as the number of available paths increases, the gain from network coding also increases. The significant advantages of network coding with redundancy appeared when hop by hop coding (paths with nodes in common) was applied. Under heavy noise though the network coding scheme outerforms all the other routing schemes. The hop by hop coding process is not computationally expensive due to the linearity of the network coding technique and for this reason the delay from decoding and encoding is not so important. The conclusion is that network coding offers significant advantages when the number of paths increases, when the intermediate nodes of the network are able to decode and encode the received packets and finally when operating in heavy noise environments.

## 9.2 Future Work

In Chapters 2 and 3, we assumed saturated queues of the users. A very interesting extension is to assume sources with external random arrivals (bursty traffic). However, this extension will present analytical difficulties because of the interactions between the queues. Another extension could be the application of different priorities for the users in accessing the relay and the impact on the throughput per user and the aggregate throughput. The case of dynamic adjustment of the transmission probabilities depending on the network conditions is very interesting.

In Chapter 4, we introduced the notion of partial network-level cooperation in a network with one relay. Extending this type of cooperation in networks with more users and many relays with possible cooperation among them would be very interesting.

In Chapter 6, the channel model we assumed is time invariant, a future extension could study the impact of channel state information. Extending the approach proposed in that chapter to more realistic environments with multiple set of source-destination pairs, presents serious difficulties due to the interaction between the nodes.

In Chapters 7 and 8, we investigated the performance and reliability that can be achieved by exploiting path diversity through multipath forwarding together with redundancy through network coding. An important issue is the correlation of losses among the paths, it is interesting also the study of paths that contain links with bursty errors. Initial results indicate that in the case of networks that have paths with common links, the advantages of network coding are more pronounced. The analysis done in these chapters will serve as a guideline for a more general network model including the previous considerations. Another extension of this work should be the study of networks with different error probability for each hop for more complex topologies. Our future work involves the impact of interference and congestion to schemes described above.

# Bibliography

[1] E. C. V. D. Meulen, “Three-terminal communication channels,” *Advances in Applied Probability*, vol. 3, no. 1, pp. pp. 120–154, 1971.

[2] T. Cover and A. Gamal, “Capacity theorems for the relay channel,” *Information Theory, IEEE Transactions on*, vol. 25, no. 5, pp. 572 – 584, Sep. 1979.

[3] A. Gamal and M. Aref, “The capacity of the semideterministic relay channel (corresp.),” *Information Theory, IEEE Transactions on*, vol. 28, no. 3, p. 536, may 1982.

[4] A. Sadek, K. Liu, and A. Ephremides, “Cognitive multiple access via cooperation: Protocol design and performance analysis,” *Information Theory, IEEE Transactions on*, vol. 53, no. 10, pp. 3677 –3696, 2007.

[5] B. Rong and A. Ephremides, “Protocol-level cooperation in wireless networks: Stable throughput and delay analysis,” in *Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, 2009. WiOPT 2009. 7th International Symposium on*, 2009, pp. 1 –10.

[6] ——, “On stability and throughput for multiple access with cooperation,” in *under Submission*.

[7] C. E. Shannon, “Two-way communication channels,” *Proceedings of the 4th Berkeley Symposium Mathematical Statistics and Probability*, 1961.

[8] T. M. Cover and J. A. Thomas, *Elements of Information Theory 2nd Edition*, 2nd ed., ser. Wiley Series in Telecommunications and Signal Processing. Wiley-Interscience, July 2006.

[9] S. Nikolaou, R. Bairavasubramanian, J. Lugo, C., I. Carrasquillo, D. Thompson, G. Ponchak, J. Papapolymerou, and M. Tentzeris, "Pattern and frequency reconfigurable annular slot antenna using pin diodes," *Antennas and Propagation, IEEE Transactions on*, vol. 54, no. 2, pp. 439 – 448, 2006.

[10] O. Eliezer, R. Staszewski, I. Bashir, S. Bhatara, and P. Balsara, "A phase domain approach for mitigation of self-interference in wireless transceivers," *Solid-State Circuits, IEEE Journal of*, vol. 44, no. 5, pp. 1436 –1453, May 2009.

[11] H. Suzuki, K. Itoh, Y. Ebine, and M. Sato, "A booster configuration with adaptive reduction of transmitter-receiver antenna coupling for pager systems," in *Vehicular Technology Conference, 1999. VTC 1999 - Fall. IEEE VTS 50th*, 1999.

[12] D. Halperin, T. Anderson, and D. Wetherall, "Taking the sting out of carrier sense: interference cancellation for wireless lans," in *In Proceedings of the 14th ACM international conference on Mobile computing and networking*, 2008.

[13] O. Ozel and S. Ulukus, "Information-theoretic analysis of an energy harvesting communication system," in *Proceedings of IEEE PIMRC*, Sep. 2010.

[14] J. Jeon and A. Ephremides, "The stability region of random multiple access under stochastic energy harvesting," *To appear in the proceedings of IEEE ISIT 2011*.

[15] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Processing Magazine*, vol. 24, no. 3, pp. 79–89, May 2007.

[16] S. Ghez and S. Verdú, “Stability property of slotted aloha with multipacket reception capability,” *IEEE Transactions on Automatic Control*, vol. 33, no. 7, pp. 640 – 649, Jul. 1988.

[17] Q. Z. L. Tong and G. Mergen, “Multipacket reception in random access wireless networks: from signal processing to optimal medium access control,” *IEEE Communications Magazine*, vol. 39, no. 11, pp. 108–112, Nov. 2001.

[18] V. Naware, G. Mergen, and L. Tong, “Stability and delay of finite-user slotted aloha with multipacket reception,” *IEEE Transactions on Information Theory*, vol. 51, no. 7, pp. 2636–2656, Jul. 2005.

[19] B. Rong and A. Ephremides, “On opportunistic cooperation for improving the stability region with multipacket reception,” *Proceedings of NET-COOP, LNCS*, vol. 5894, pp. 45–59, 2009.

[20] S. Kompella, G. D. Nguyen, J. E. Wieselthier, and A. Ephremides, “Stable throughput tradeoffs in cognitive shared channels with cooperative relaying,” *Proceedings of IEEE INFOCOM 2011*.

[21] R. Ahlswede, N. Cai, S. R. Li, and R. W. Yeung, “Network information flow,” in *IEEE Transactions on Information Theory*, 2000, pp. 1204–1216.

[22] N. Pappas, A. Ephremides, and A. Traganitis, “Stability and performance issues of a relay assisted multiple access scheme with mpr capabilities,” in *Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, 2011. WiOPT 2011. 9th International Symposium on*, May 2011.

[23] O. Simeone, Y. Bar-Ness, and U. Spagnolini, “Stable throughput of cognitive radios with and without relaying capability,” *Communications, IEEE Transactions on*, vol. 55, no. 12, pp. 2351 –2360, 2007.

[24] B. Rong and A. Ephremides, “Cooperation above the physical layer: The case of a simple network,” in *Information Theory, 2009. ISIT 2009. IEEE International Symposium on*, 28 2009.

[25] D. Bertsekas and R. Gallager, *Data networks (2nd ed.)*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1992.

[26] S. Ghez, S. Verdu, and S. Schwartz, “Stability properties of slotted aloha with multi-packet reception capability,” *Automatic Control, IEEE Transactions on*, vol. 33, no. 7, pp. 640 –649, Jul. 1988.

[27] G. del Angel and T. L. Fine, “Optimal power and retransmission control policies for random access systems,” *IEEE/ACM Trans. Netw.*, vol. 12, pp. 1156–1166, December 2004. [Online]. Available: http://dx.doi.org/10.1109/TNET.2004.838605

[28] V. Naware, G. Mergen, and L. Tong, “Stability and delay of finite-user slotted aloha with multipacket reception,” *Information Theory, IEEE Transactions on*, vol. 51, no. 7, pp. 2636 – 2656, 2005.

[29] N. Pappas, A. Traganitis, and A. Ephremides, “Stability and performance issues of a relay assisted multiple access scheme,” in *Global Telecommunications Conference, 2010. GLOBECOM 2010. IEEE*, 2010.

[30] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. New York, NY, USA: Cambridge University Press, 2005.

[31] G. Nguyen, S. Kompella, J. Wieselthier, and A. Ephremides, “Optimization of transmission schedules in capture-based wireless networks,” in *Military Communications Conference, 2008. MILCOM 2008. IEEE*, 2008, pp. 1 –7.

[32] F. Gebali, *Analysis of Computer and Communication Networks*. Springer, 2010.

[33] R. Loynes, “The stability of a queue with non-independent inter-arrival and service times,” *Proc. Camb. Philos.Soc*, vol. 58, no. 3, pp. 497–520, 1962.

[34] N. Pappas, A. Ephremides, and A. Traganitis, “Relay-assisted multiple access with multi-packet reception capability and simultaneous transmission and reception,” in *Information Theory Workshop (ITW), 2011 IEEE*, oct. 2011, pp. 578 –582.

[35] P. Weeraddana, M. Codreanu, M. Latva-aho, and A. Ephremides, “The benefits from simultaneous transmission and reception in wireless networks,” in *Information Theory Workshop (ITW), 2010 IEEE*, 30 2010-sept. 3 2010, pp. 1 –5.

[36] N. Pappas, J. Jeon, A. Ephremides, and A. Traganitis, “Wireless network-level partial relay cooperation,” in *IEEE International Symposium on Information Theory (ISIT)*, July 2012.

[37] R. Rao and A. Ephremides, “On the stability of interacting queues in a multi-access system,” *IEEE Transactions on Information Theory*, vol. 34, no. 5, pp. 918–930, Sep. 1988.

[38] W. Szpankowski, “Stability conditions for some distributed systems: Buffered random access systems,” *Advances in Applied Probability*, vol. 26, no. 2, pp. 498–515, Jun. 1994.

[39] B. S. Tsybakov and V. A. Mikhailov, “Ergodicity of a slotted aloha system,” *Problems of Information Transmission*, vol. 15, no. 4, pp. 301–312, 1979.

[40] W. Luo and A. Ephremides, “Stability of $N$ interacting queues in random-access systems,” *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1579–1587, Jul. 1999.

[41] N. Pappas, J. Jeon, A. Ephremides, and A. Traganitis, “Optimal utilization of a cognitive shared channel with a rechargeable primary source node,” in *Information Theory Workshop (ITW), 2011 IEEE*, oct. 2011, pp. 598 –602.

[42] ——, “Optimal utilization of a cognitive shared channel with a rechargeable primary source node,” *Journal of Communications and Networks (JCN) Special Issue on Energy Harvesting in Wireless Networks*, vol. 14, no. 2, 2012.

[43] V. Sharma, U. Mukherji, and V. Joseph, “Efficient energy management policies for networks with energy harvesting sensor nodes,” in *Communication, Control, and Computing, 2008 46th Annual Allerton Conference on*, sept. 2008, pp. 375 –383.

[44] L. Kleinrock, *Queueing Theory, Volume I: Theory*. New York: Wiley, 1975.

[45] N. Pappas, V. A. Siris, and A. Traganitis, “Path diversity gain with network coding and multipath transmission in wireless mesh networks,” in *The Second IEEE WoWMoM Workshop on Hot Topics in Mesh Networking*, 2010.

[46] Ayanoglu, C.-L. I, R. Gitlin, and J. Mazo, “Diversity coding for transparent self-healing and fault-tolerantcommunication networks,” in *IEEE Transactions on Communications*, 1993, pp. 1677 – 1686.

[47] R. W. Yeung and N. Cai, “Network error correction, part i: Basic concepts and upper bounds,” in *Communications in Information and Systems*, 2006, pp. 19 – 36.

[48] D. Lun, M. Medard, R. Koetter, and M. Effros, “Further results on coding for reliable communication over packet networks,” in *ISIT*, 2005.

[49] G. Booker and A. Sprintson, “Design of efficient robust network codes for multicast connections,” in *ISIT*, 2009.

[50] H. Bahramgiri and F. Lahouti, “Robust network coding against path failures,” in *IET Communications*, 2010, pp. 272–284.

[51] S. Katti, D. Katabi, W. Hu, H. Rahul, and M. Medard, “The importance of being opportunistic: Practical network coding for wireless environments,” in *Allerton*, 2005.

[52] S. Chachulski, M. Jennings, S. Katti, and D. Katabi, "Trading structure for randomness in wireless opportunistic routing," in *ACM SIGCOMM*, 2007.

[53] C. Gkantsidis, W. Hu, P. Key, B. Radunovic, S. Gheorghiu, and P. Rodriguez, "Multipath code casting for wireless mesh networks," in *CoNext*, 2007.

[54] S. Biswas and R. Morris, "Opportunistic routing in multi-hop wireless networks," in *ACM SIGCOMM*, 2005.

[55] Y. Yuan, S. H. Y. Wong, S. Lu, and W. Arbuagh, "Romer: Resilient opportunistic mesh routing for wireless mesh networks," in *Proceedings of IEEE WiMesh*, 2005.

[56] A. Tsirigos and Z. Haas, "Analysis of multipath routing, part 1: The effect of packet delivery ratio," in *IEEE Transactions on Wireless Communications vol.3 no. 1*, 2004, pp. 138–146.

[57] ——, "Analysis of multipath routing, part 2: Mitigation of the effects of frequently changing topologies," in *IEEE Transactions on Wireless Communications vol.3 no. 2*, 2004, pp. 500–511.

[58] N. Pappas, V. Siris, and A. Traganitis, "Delay and throughput of network coding with path redundancy for wireless mesh networks," in *Wireless and Mobile Networking Conference (WMNC), 2010 Third Joint IFIP*, oct. 2010, pp. 1 –6.

[59] S. Heimlicher, M. Karaliopoulos, H. Levy, and M. May, "End-to-end vs. hop-by-hop transport under intermittent connectivity," in *Autonomics '07: Proceedings of the 1st international conference on Autonomic computing and communication systems*, 2007.

[60] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*. McGraw-Hill Higher Education, 2002.