CROSS LAYER RATE ADAPTATION IN WIRELESS AD-HOC NETWORKS

Ph.D. Dissertation

by

Emmanouil G. Spanakis

Advisor

Prof. Apostolos Traganitis

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CROSS LAYER RATE ADAPTATION IN WIRELESS AD-HOC NETWORKS

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Writer: Emmaouil G. Spanakis

Computer Science Department – University of Crete

Supervisor: Prof. Apostolos Traganitis

Computer Science Department – University of Crete

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ABSTRACT

Wireless ad hoc networks serve as the transport mechanism among devices or between devices and traditional backbone networks allowing users to access information from anywhere a wireless connection is feasible, without relying on any kind of infrastructure. In a wireless network, with nodes sharing the same spectrum, each transmission is affected from, and affects, all other transmissions in range. When multiple uncoordinated links share a common medium the effect of interference is a crucial limiting factor for network performance. A medium access mechanism should be able to control the amount of interference experienced by receivers and, in certain cases, to enforce concurrent transmissions, by tightly coupling both physical and medium access layers to maximize network performance.

The general objective of this thesis is to present an in-depth analysis of how cross layer techniques can be used in the design and study of wireless ad hoc networks. More specifically, we focus on finding how adapting various parameters of the telecommunication system can allow concurrent transmissions, minimize interference, enhance network throughput, maximize individual link data rates, and optimally utilize the network resources for all competing transmissions.

Initially, we define and study the transmission rate regions for a simplified wireless network with a given degree of interference, considered as noise, and individual power constrains. We define the necessary conditions that maximize the system’s aggregate rate for a simplified two-link interference wireless channel and provide criteria under which simultaneous link operation outperforms timesharing. We identify critical points in the rate region where higher aggregate rates can be achieved at the expense of higher power expenditure. In case of light interference the relation between the maximum achieved rate and transmission power is shown to be almost linear, but in case of strong interference, there is need for disproportionally high total power. Finally, for higher order modulations,
we give the condition on the maximum individual transmission power for switching to the next higher modulation level in order to achieve higher aggregate rate.

Then, we study the interference exhibited at the center of a circular networking area when interfering nodes are randomly distributed or have specific network topology. We define the interference limited communication range $d_{ILR}$ to be the critical communication region around a receiver, with a large number of surrounding interfering nodes, within which a successful communication link can be formed. By adapting the transmission rate we can allow more transmitters to have a chance for a successful communication. We study how interference levels and the interference limited communication range $d_{ILR}$ values are affected by the number of the surrounding interfering transmitters, the path loss exponent and most importantly on the selected mode and rate of operation. The results obtained through extensive simulations indicate that our proposed model for the estimation of the interference and of the interference limited communication range is very accurate, even when we relax our assumption on the receiver's position at the center of the networking area.

Finally, for a multicast group we study the multicast throughput which is the product of the transmission rate and the number of the successful multicast receptions. We show that the multicast throughput, for a specific topology of the networking environment, does not depend on the transmission rate when the path loss exponent is equal to two but it increases with the transmission rate when the path loss exponent is greater than two. By changing the transmission rate we can control the number of receivers able to receive a multicast transmission. For the simple case of two multicast transmitters our results demonstrate that there exists a specific transmission rate and a corresponding percentage of successful receivers that maximizes the multicast throughput.
Τα ad hoc ασύρματα δίκτυα παρέχουν το μηχανισμό μεταφοράς δεδομένων μεταξύ συσκευών ή μεταξύ συσκευών και δικτών κορμού, έτσι ώστε οι χρήστες να έχουν πρόσβαση σε πληροφορίες από οποιοδήποτε σημείο είναι εφικτή η δημιουργία μίας ασύρματης σύνδεσης, χωρίς να απαιτείται η ύπαρξη εγκατεστημένης υποδομής.

Οι κόμβοι ενός ασύρματου δικτύου διαμοιράζονται το ίδιο τμήμα του φάσματος και κατά συνέπεια κάθε μετάδοση επηρεάζεται και επηρεάζεται από όλες τις υπόλοιπες μεταδόσεις που λαμβάνουν χώρα. Οι παρεμβολές που εμφανίζονται σε ένα διαμοιραζόμενο μέσο εξαιτίας των ασυντόνιστων προσπελάσεων των κόμβων του δικτύου είναι ένας σημαντικός περιοριστικός παράγοντας της απόδοσης του δικτύου.

Ένας μηχανισμός προσπέλασης μέσου θα πρέπει να είναι σε θέση να ελέγχει τις παρεμβολές στους δέκτες και σε ορισμένες περιπτώσεις να επιβάλει τις ταυτόχρονες μεταδόσεις μέσω της εισχυρισμένης σύζευξης των επιπέδων του φυσικού μέσου και της προσπέλασης μέσου, έτσι ώστε να επιτευχθεί η μεγιστοποίηση της απόδοσης του δικτύου.

Ο γενικός στόχος της παρούσας διδακτορικής διατριβής είναι η ανάλυση των τρόπων χρήσης διαστρωματικών τεχνικών στη μελέτη και σχεδίαση αποδοτικών ασύρματων δικτών ad-hoc. Ειδικότερα, επεξεργάζομαστε στην επίδραση που έχουν διάφορες παράμετροι του τηλεπικοινωνιακού συστήματος, ώστε να επιτυγχάνονται ταυτόχρονες μεταδόσεις, να μειώνονται οι παρεμβολές, να αυξάνεται η διαπερατότητα του δικτύου, να αυξάνεται η απόδοση του συστήματος καθώς και να ελεγχόμενη επιβάλεται με βέλτιστο τρόπο το ασύρματο μέσο μετάδοσης από όλους τους κόμβους που ανταγωνίζονται για την πρόσβαση σε αυτό.

Αρχικά μελετάμε τις περιοχές μεταβολής του ρυθμού μετάδοσης σε ένα απλό ασύρματο δίκτυο δυο ζεύξεων οι οποίες αλληλοσυνεπώς επαναδιακόπτονται. Καθορίζουμε τις αναγκαίες συνθήκες, τα κριτήρια επιτυχίας και τη μέθοδο διαδικασίας συνδέσμων των κορμού και της κομμάτων. Παρέχουμε ιστορία
για το πότε η ταυτόχρονη λειτουργία των ζεύξεων είναι προτιμότερη από την ενεργοποίηση τους σε διαφορετικές χρονικές στιγμές. Αποδεικνύουμε ότι η σχέση μεταξύ του μέγιστου επιτυγχανόμενου ρυθμού μετάδοσης και της ισχύος εκπομπής είναι σχεδών γραμμική στην περίπτωση παρεμβολής χαμηλής ισχύος ενώ στην περίπτωση ισχυρής παρεμβολής απαιτείται να δαπανήσουμε δυσανάλογα υψηλή ισχύς εκπομπής.

Στη συνέχεια μελετάμε την παρεμβολή που υφίσταται ένας δέκτης στο κέντρο μιας κυκλικής δικτυακής περιοχής, όταν οι κόμβοι που δημιουργούν παρεμβολές είναι τυχαία κατανεμημένοι στο χώρο. Ορίζουμε την «περιορισμένη λόγω παρεμβολών εμβέλεια» (Interference Limited communication Range, d<sub>ILR</sub>) ως την κρίσιμη περιοχή γύρω από το δέκτη μέσα στην οποια μπορούμε να έχουμε επιτυχώς μία ζεύξη επικοινωνίας παρά την παρεμβολή ενός μεγάλου αριθμού γειτνιαζόντων κόμβων που εισάγουν παρεμβολές. Προτείνουμε ένα μοντέλο, το οποίο περιγράφει τις επιδράσεις που έχουν το πλήθος των πομπών που δημιουργούν παρεμβολές, ο εκθέτης απωλειών και ο ρυθμός μετάδοσης στα επίπεδα παρεμβολής και στο μέγεθος της περιοχής d<sub>ILR</sub>. Αποδεικνύουμε με εκτενείς προσομοιώσεις ότι το προτεινόμενο μοντέλο είναι ιδιαίτερα ακριβές στις εκτιμήσεις του, ακόμη και όταν χαλαρώσουμε την υπόθεση ότι ο δέκτης μη βρίσκεται στο κέντρο της περιοχής.

Τέλος μελετάμε στην περίπτωση επικοινωνίας πολυεκπομπής τη διαπερατότητα πολυεκπομπής που ορίζεται ως το γινόμενο όλων των επιτυχών λήψεων επί τον ρυθμό μετάδοσης. Αποδεικνύουμε ότι η διαπερατότητα του δικτύου δεν εξαρτάται από το ρυθμό μετάδοσης όταν ο εκθέτης απωλειών είναι ίσος με δύο και οτι αυξάνεται με συγκεκριμένο τρόπο σε συνάρτηση με τον ρυθμό μετάδοσης όταν ο εκθέτης απωλειών είναι μεγαλύτερος του δύο. Επιπλέον, δείχνουμε ότι προσαρμόζοντας το ρυθμό μετάδοσης θα ελέγξουμε το πλήθος των δεκτών που λαμβάνουν επιτυχώς μία μετάδοση πολυεκπομπής. Για την απλή περίπτωση δύο πολυεκπομπών που αλληλοπαρεμβάλλονται βρίσκουμε ότι υπάρχει ένας συγκεκριμένος ρυθμός μετάδοσης και ένα αντίστοιχο ποσοστό δεκτών τα οποία μεγιστοποιούν τη διαπερατότητα πολυεκπομπής.
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Dedicated to
my loving Parents
and Brother
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Chapter 1

INTRODUCTION

1. Introduction

1.1. Wireless ad-hoc networks

Wireless telecommunication technologies are becoming increasingly popular in our everyday personal and business activities. They serve as the transport mechanism among devices or between devices and the traditional backbone networks allowing users to access information from anywhere at any time and any place a wireless connection is feasible. The recent advances especially in microelectronics are opening up a huge list of possibilities for the future of wireless networking. The main advantages, compared with conventional wired networks, include the reduction in infrastructure requirements but most importantly the support for mobile communications by anyone from anywhere at any time and any place. The goal for the future is wireless technologies to be able to allow communication without relying on any kind of fixed infrastructure.

Ad-hoc networking is one of the most active fields of research in wireless networking environments today. Significant research has been going on for more than 35 years. Within the past few years the field has seen a rapid expansion mainly due to wide availability of inexpensive wireless devices and the increasing interest of the networking community in mobile computing.

Ad-hoc networks are consisting of wireless nodes able to communicate without the need for any pre-existing infrastructure or centralized administration or control. Each node is responsible for discovering all other nodes for communication. Not all network nodes are able to communicate directly with each other so intermediate nodes must act as relays in order to
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route the data to the designated destination across the network; thus the problem of routing arises which has attracted great attention during the last years.

1.1.1. Properties of wireless ad-hoc networks

Ad hoc networks have become a major research domain [Murthy 2004]. This trend is enforced by recent advances in microelectronics and wireless communications, with the emergence of a new generation of communication devices and with the appearance of many new applications. In ad-hoc network application scenarios the interconnected devices have to spontaneously form a network and maintain network connectivity to support all underlined services and ensure the exchange of information. In some cases pre-deployment might be required if the formed network is not user-driven. Interactivity also influences possible solutions for controlling the network (no interactivity: follow a desired pre-defined behavior, low interactivity: network partially controlled through user interactions and high interactivity: system behavior depends entirely on user behavior and demands). There are many application domains for wireless ad-hoc networks: military tactical operations, law enforcement, search and rescue, disaster relief operations, commercial use and many more.

Some of the unique characteristics and limitations of wireless ad hoc networks are: i) unreliable channel, ii) dynamic topology changes, iii) node mobility, iv) limited bandwidth, and v) limited energy resources. The protocols designed for ad-hoc networks must be able to deal with a set of requirements within the environment of operation, such as:

- Self healing: detect, localize, and repair any failures automatically.
- Self configuration: regenerate adequate configurations depending on the current situation in terms connectivity, quality of service and other network parameters.
- Self management: maintain and manage devices/offered services.
- Self optimization: optimal choice of operation modes given system behavior and status.
- Self Adaptation: adaptation to changing networking environmental conditions.
The challenges in the design of protocol architectures for wireless ad-hoc networks is to efficiently convey information using an unreliable physical channel within a highly dynamic set of mobile, limited-range, limited-energy, half-duplex or full-duplex radios without the support of any infrastructure.

An efficient network protocol should be able to optimize specific network characteristics like throughput, delay, and energy consumption without sacrificing fairness among users, robustness, and Quality of Service (QoS). This set of design goals, in such a highly dynamic and vast changing networking environment, suggests that tradeoffs are required in the design of any protocol architecture for ad-hoc networks.

1.1.2. Open Problems

There are many unresolved issues for wireless ad hoc networks; we briefly describe some of the most important open problems:

*Scalability:* The network’s ability to continue normal operation without any performance degradation when the number of active nodes increases dramatically. The problem of scalability affects link connectivity and packet delivery ratio, due to the increase of the network size and the overhead of controlling a larger number of possible active links and requests for transmissions.

*Energy efficiency:* Mobile nodes in ad-hoc networks possibly operate on limited power resources. Energy efficiency focuses on the management of the available power capabilities in the best possible way, in order to prolong the node lifetime and its presence in the network. Some of the techniques involve variable transmission power schemes (given the network condition), selecting routes that minimize the total consumed transmission power over a routing path, entering sleeping or hibernation mode whenever a node can remain idle, etc.
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Quality of Service Requirements: In a heterogeneous and extensively changing networking environment it is very difficult and complicated, at the same time, to satisfy the quality of communication services among individual network links. A promising approach is cross layer or vertical design, allowing different parts of information, of the traditional network stack, to be shared among network layers, for better improvement of overall performance.

Security: Wireless network nodes operate in a very unsecure environment, where links share the same medium for transmissions and are exposed to a number of security hazards that can dramatically tamper the transmitted data and affect network performance.

Channel characteristics: The physical processes of reflection, refraction and scattering of a radio signal are relatively well understood but difficult to quantify in detail when a large number of nodes and obstacles are included in the network terrain.

Node mobility: Refer to the pattern of movement of network nodes. Despite the research work in this area the modern mobility models fail to accurately capture realistic mobility scenarios.

Application scenarios: Until today there is not any dominant ("killer") application for ad-hoc networks and all proposed models and protocols only describe hypothetical scenarios.

1.1.3. Performance Metrics

In the literature for the research work in ad-hoc networks a number of different metrics are used in order to evaluate network performance. Some of the most important used for studying this kind of networks are briefly summarized below:

- Throughput - the fraction of the raw bandwidth used exclusively for data transmission.
- Quality of Service (QoS) – delay, high packet delivery ratio, and guaranteed bandwidth describe QoS at the application layer
- Energy - crucial for limited energy operated devices to avoid power starvation
• Fairness – assigning the network resources in a balanced fashion among the nodes.
• Stability – Wireless network protocols are dynamic systems, their performance can become unstable if certain conditions are not satisfied.
• Robustness – the capability to continue network operation when network conditions change over time (i.e. mobility or node failures cause link failures but protocol performance must not be tampered).

1.2. Layered vs. Cross Layer Architecture

The layering architecture is the building block of networking for more than thirty years and is still extensively used especially in wired networks. Its primary principle is based on isolating network functions into modular units with minimum interactions, through specified structures and interfaces. The objective is to easily manage a complex entity by creating and maintaining disjoint and well integrated separable modules. Each module is able to perform a specific task, in every possible way, but cannot intervene with another module’s task.

The open system interconnection model (OSI) was designed by the International Organization for Standardization (ISO) to enable interaction among all kind of devices using different types of communication technologies. Today it is used as a reference model to describe and outline modular networking systems. The OSI model divides network related functions into seven different layers: the **physical layer** must be able to adapt to changes in link characteristics; the **multiple access control (MAC) layer** must minimize collisions, allow fair access of the channel and ensure reliable transport of data; the **network layer** must establish multi-hop routing paths, given specific requirements in transmission rate, transmission power and delay and hide interaction with traditional networks (i.e. mesh networking); the **transport layer** must handle end to end packet delivery delay and losses; lastly the **application layer** must be able to cope with the characteristics of the underline networking system and end user application special requirements.
Introduction

Even though this layering approach was widely adopted it seems unable to follow the growth of wireless technologies and the new types of services offered to users. The use of layered architectural design, in ad hoc wireless networks, did not fulfill the expectations in performance, because of the lack of infrastructure, limited resources, mobility, topology of nodes, and unstable link conditions. A new promising direction for research in wireless ad-hoc network is cross layer design that gives new perspectives towards understanding network functionality and to meet essential performance demands.

Cross-layer design is a broad concept and includes various design alternatives and implementations. An extreme implementation would be to collapse the entire network stack; a different approach would be to keep the layers intact and just share information between them accordingly or even decide to merge some of the layers into a single one given the application specific demands. The research on cross layering has been mainly deployed into two different directions. The first approach focuses on compatibility more than performance, by vertical cutting on the traditional layers of the network protocol stack. Usually not more than two or three layers are involved and the rest are not affected at all. This method can provide optimization and enhancements through a simple and effective solution, which just extends parts of the traditional layering structure allowing inter-exchange and sharing of information throughout the participating layers. An important advantage is that it can be deployed even in already existing networks by just enhancing the existing protocols used. The other direction provides new definitions of the functional entities and their interactions for a networking system making. This approach tries to maximize performance and doesn’t make any compromise to maintain compatibility. Despite the major weaknesses of compatibility and the cost for deployment this approach reveals new ways in network design by incorporating new concepts from information theory, topology control, cooperative networking, smart networks and other [Aune 2004] [Conti 2004].
A protocol architecture, for wireless ad-hoc networks, that coordinates combined information from multiple layers, can provide an optimal and a global solution in terms of performance by taking into account specific service requirements of the application and tailoring the protocol stack accordingly in order to achieve design goals and network efficiency. Today it is widely discussed and argued how efficient and how optimal for achieving specific design goals in a heterogeneous networking environment would be to use such an architectural model in protocol design. In this dissertation we apply cross layer design techniques such as rate adaptation and power control and examine their advantages in dealing with the problem of interference channel.

1.3. Interference Channel

Interference is a central phenomenon in wireless communication when multiple uncoordinated links share a common communication medium. Most state-of-the-art wireless systems deal with interference in one of two ways even though both of them might be sub-optimal:

- Orthogonalize the communication links in time or frequency, so that they do not interfere with each other at all;
- Allow the communication links to share the same degrees of freedom, but treat each other’s interference as adding to the noise floor.

The first approach entails an \textit{a priori} loss of degrees of freedom in both links, no matter how weak the potential interference is. The second approach treats interference as pure noise while it actually carries information and has structure that can potentially be exploited in mitigating its effect. These considerations lead to the natural question of what is the best performance one can achieve without making any \textit{a priori} assumptions on how the common resource is shared. A basic information theory model to study this question is the two-user Gaussian interference channel, where two point-to-point links with additive white Gaussian noise interfere with each other. In Chapter 3 we examine some cooperation issues for this channel.
Introduction

A new model for interference calculation in wireless multi-hop ad-hoc networks is presented in [Hekmat 2006] by taking into account the number of nodes, the density of nodes, radio propagation aspects, multi-hop characteristics, and the amount of relay traffic. The expected values are used to determine the network capacity and data throughput per node, assuming that all nodes are placed on a regular lattice (enabling the calculation of interference without having detailed information about movement patterns and exact location of nodes). The effects of network size variations, network density and traffic load are evaluated. Interference is shown to be upper bounded, in carrier sensing wireless networks, limiting throughput optimization. In Chapter 4 and 5, we study some aspects of the interference issue on a randomly distributed ad-hoc wireless network.

In [Ganti 2008] the tail properties of interference for stationary and isotropic spatial distribution of transmitting nodes is studied. Previously the properties of interference were only known for a homogeneous Poisson node distribution on a plane. It is shown that when the path loss function has a singularity at the origin, the interference has a heavy-tailed distribution under very mild conditions. On the other hand when the path loss is bounded, the distribution of the interference is predominantly dictated by fading.

1.4. Capacity of Wireless networks

One of the longest outstanding problems in multiuser information theory is the capacity region of the two-user Gaussian interference channel. The multiuser channel consists of two point-to-point links interfering with each other through crosstalk. Each transmitter has a message intended only for a corresponding receiver. The capacity region of this channel is the set of all simultaneously achievable rates, and characterizes the fundamental tradeoff between the performances achievable over the links in the face of interference.

The problem of characterizing the capacity of wireless ad-hoc networks has been studied thoroughly at the Information-theoretic level but the characterization of the capacity region
and the rate adaptation techniques in wireless ad-hoc networks have been open issues for years. Han and Kobayashi [1] have studied the information theoretic bounds of the achievable rate region of interference channels; specifically they have shown the capacity region in the case of strong interference. This approach involves splitting the transmitted information of the users into two parts: private information to be decoded only at own receiver and common information that can be decoded at both receivers. By decoding the common information, part of the interference can be canceled off, while the remaining private information from the other user is treated as noise. The method allows arbitrary splits of each user’s transmit power into the private and common information portions as well as time sharing between multiple such splits. Unfortunately, the optimization among such myriads of possibilities is not well-understood, and it is also not clear how close to capacity can such a scheme get and whether there will be other strategies that can do significantly better. The best known achievable region, [Han 1981] is hard to compute and is unclear if it is optimal. Related to this is our study of the achievable rate region of a simplified two-user interference Channel presented in Chapter 3.

Gupta and Kumar [Gupta 2000] have studied the throughput of a wireless network showing that as the number of nodes increases, the throughput per source-to-destination pair decreases approximately like $1/\sqrt{n}$, where $n$ is the number of nodes in the network. A feasible throughput of $\lambda(n)$ bits per second for each node means that there is a spatial and temporal scheme, such that a multi-hop operational scheme would allow every node to send at $\lambda(n)$ bits per second on average to its chosen destination node. The throughput $\lambda(n)$ obtainable by each node for a randomly chosen destination is $\Theta(W/\sqrt{n\log n})$ bits per second under a noninterference protocol. It is shown that throughput diminishes to zero as the number of users is increased and that under optimal conditions and the assumption that data are delivered through a multihop path where the node of a source-destination pair for each hop are not vanishingly far away, the per node throughput goes to zero at least as $1/\sqrt{n}$ bits per second.
Introduction

In [Ashish 2004] the upper and lower bounds of the capacity region where presented by identifying exclusion regions\(^1\). These results, much more generally can be applied to arbitrary wireless topologies, including those arising from directional antennas, obtaining much higher bounds for the capacity.

When a number of pairs of transmitters and receivers wish to communicate they are subject to interference from all other transmissions in range. The problem of throughput scaling as the number of users grows in [Sanjeev 2004] is been addressed using a deterministic approach and a stronger version of the results obtained by Gupta and Kumar for random node locations for non-i.i.d. node distributions is finally proven. The key contribution of this work is how specific properties of the set of node locations affect throughput growth and how this can be easily calculated for different distributions of random node locations.

For multi-access, broadcast, and additive Gaussian interference channels, the growth of TDMA-achievable rates, with the energy per bit, was shown to be suboptimal in the low-power regime, except in some special cases: multi-access channels where the users' energy per bit are identical and broadcast channels where the receivers have identical signal-to-noise ratios [Caire 2004]. Especially for the additive Gaussian noise interference channel, and outside of a small region of interference parameters, TDMA is proven to be suboptimal.

The effect of transmission power on the throughput capacity considering a scheduling time division multiple access protocol and an interference model is analyzed in [Behzad 2006]. It shows the effect on capacity of the increase of the transmission power; network capacity can be maximized independent of node topology distribution and traffic patterns. In contrast with

\(^1\) An exclusion region is a region around each receiver such that nodes inside this region are not allowed to transmit.
previous results, the use of common minimum transmission power for maintaining connectivity and maximizing throughput capacity is shown to be optimum, in terms of maximizing the network’s capacity, when every node can directly communicate with every other.

The achievable rate regions for the Gaussian interference channel are analyzed in [Sason 2006], based on reducing one user’s transmission power such that its message can be decoded to both receivers. The main contribution of this study is that time-sharing can be used in order for the convex closure to outperform the sub-region of the Han and Kobayashi case for symmetric interference channel under moderate interference. The maximum sum-capacity is achieved when the stronger user transmits at its maximum rate, while the weaker user treats stronger user as interference.

A very important study for a simplified Han-Kobayashi scheme [Etkin 2008] presented new outer bounds for the Gaussian interference channel that can be achieved within a single bit. This scheme is shown to be asymptotically optimal at high SNR regimes and a generalization of the point-to-point to interference-limited scenarios is presented. Following this work in [Bresler 2008] a two-user Gaussian interference channel for a natural deterministic channel model is presented. The results are that the deterministic channel can approximate the Gaussian channel and the capacity regions are shown to differ by a universal constant. Because of the simplicity of the deterministic channel model, it provides a lot of insight to the structure of the various near-optimal schemes for the Gaussian interference channel.

The effort of establishing the capacity region of a Gaussian interference network has led to the characterization of the capacity region of a general two-user Gaussian interference channel within one bit. In [Sreekanth 2008] new improved outer bounds on the capacity region are presented. It is shown that treating interference as noise achieves optimum sum capacity of the two-user Gaussian interference channel in a low interference regime, where the interference parameters are below certain thresholds.
Introduction

Recently a network utility maximization problem is presented in [Yang 2009], taking into account a multiple-hop sensor network with respect to energy consumption in a data relaying process. A joint power and rate adaptation cooperation technique was presented and a price-based algorithm was proposed to solve this problem when nodes were relaying data along a path.

1.5. Contribution of this dissertation

Wireless ad hoc networks are becoming increasingly popular serving as the transport mechanism among or between devices and traditional backbone networks allowing users to access information, from anywhere a wireless connection could be feasible, without relying on any kind of fixed infrastructure. Due to the special characteristics, the fast changing channel conditions and the lack of uncertainty for network connectivity, serious challenges exist in the design of protocols in such networking environments. A layered approach is argued if it is the most profitable architectural solution for an ad hoc wireless network. Cross layering approaches, where protocol layers can interact, exchange information, and fine tune their parameters according to network status, are becoming increasingly popular and promising. This dissertation presents an analysis of how cross-layer techniques can be used in the design and study for wireless ad hoc networks, supported by extensive simulations in order to quantify and verify the results and the impact on performance of the proposed methods.

In a wireless network, with nodes sharing the same spectrum, the medium access mechanism should be able to control the amount of interference experienced by receivers and, in certain cases, to enforce concurrent transmissions, in order to maximize network performance by tightly coupling both physical and medium access layers and possibly higher layers. In our study, using cross layer techniques, we are interested in finding how we can adapt various parameters of the telecommunication system, (i.e. transmission rate, bandwidth, BER, power), in order to: allow concurrent transmissions, enhance network’s total throughput by maximizing, when possible, individual link data rates, minimize transfer time of a data packet.
and allow the best possible usage of network’s resources for all competing transmissions. As expected, the problem of finding the performance tradeoffs between throughput and energy efficiency becomes a difficult challenge in the design of joint rate and power adaptation.

First we define and study the transmission rate regions, sets of the achievable transmission rates of all active links of the network, for a simplified wireless network given the degree of interference (considered as noise) and individual maximum power constraints. We consider a simplified real-world wireless telecommunication system where the network nodes use a specific modulation scheme with a specific BER and a constant bandwidth. When links operate simultaneously they are subject to interference from the thermal noise and the other concurrent transmissions. In this case we establish a linear relationship between the achieved rate and the SINR, as long as the symbol transmission rate does not exceed the available bandwidth, assuming the absence of any error control coding or any multiuser decoding scheme nor any cooperation between transmitters and/or receivers. Our results show the achievable rate region with respect to the network topology, the degree of interference (considered as additional noise) and the individual maximum power constrains of each transmitter. We are well aware that the rates achieved can never get close to the capacity bounds described in previous works, but our model reflects the situation of many, practical, low power and low complexity communication networks such as sensor networks, and it enables us to make some useful observations, with practical significance, for the operation of these networks. We describe the necessary conditions that maximize the system’s total aggregate rate for a simplified 2-link interference wireless network channel, showing criteria under which simultaneous link operation outperforms timesharing solutions. We identify critical points, in the rate regions, where we can achieve higher aggregate rates in practical band limited interfering channels and we study and compare the aggregate rates achieved for simultaneous and timesharing operation, as a function of the transmission power. Finally, for the case of higher order modulations, we investigate the conditions on the maximum individual
transmission power for switching to the next higher order modulation in order to achieve higher aggregate rate.

Next we study the interference exhibited at the center of a circular networking area when interfering nodes are randomly distributed or have specific network topology. We define the *interference limited communication range* $d_{ILR}$ to be the critical communication region around a receiver, given the number of surrounding interfering nodes, within which if a transmitter is present, a successful communication link can be formed. By adapting the transmission rate we can adjust this critical interference limited region allowing more transmitters to be able to overcome interference and thus have the opportunity to successfully communicate with the receiver of interest. We model and calculate interference when the interfering transmitters are randomly placed using a uniform distribution and when they are placed on a regular lattice. We demonstrate that the theoretical calculation results are in a very good agreement with the mean interference values obtained through simulations. We show how the value of $d_{ILR}$ is affected by the number of the surrounding interfering transmitters, the path loss exponent and the selected mode and rate of operation. We verify all our theoretical results with simulations and show that even when we relax our assumption on the receiver’s position the expected values of the interference levels and the $d_{ILR}$ remain almost constant for a 60% or 70% offset from the center of the circle.

Last in our work we study a multicast transmission in an interference environment. We define the multicast throughput to be the product of the transmission rate and the number of successful receptions. For a specific network model, we show that this multicast throughput does not depend on the transmission rate when the path loss exponent is equal to two. For a path loss exponent greater than two the multicast throughput increases with the transmission rate. In both cases we determine the dependence of the percentage of the successful receivers on the transmission rate. For a simple network of two multicast transmitters, with a specific placement, we describe the conditions for which the multicast rate is maximized. Our results
indicate that a multicast transmitter must operate using a specific transmission rate in order to maximize the multicast throughput of its group. The percentage of nodes that can successfully receive a multicast transmission is also found to be constant, in this case.

1.6. Thesis Outline

Following this introduction the rest of this thesis is organized in 4 chapters as follows. In Chapter 2 we present our network model of a simplified wireless ad-hoc network build from unstructured activation of devices able to interconnect, when in proximity, without relying on any coordination or centralized control. All network nodes share a single channel, cannot receive and transmit at the same time, and are subject to interference from the thermal noise and the other concurrent transmissions. We show that, for the commonly used modulation schemes, there is a linear relationship between the achieved rate and the Signal to Interference plus Noise Ratio, as long as the symbol transmission rate does not exceed the system’s bandwidth. In Chapter 3 we define and study the rate regions of a simplified wireless network given the degree of interference and specific power constrains. We define the necessary conditions that maximize the system’s aggregate rate and provide criteria under which simultaneous link operation outperforms timesharing. We also identify critical points in the rate region where higher aggregate rates can be achieved at the expense of power. In Chapter 4 we study the interference exhibited at the center of a circular networking area when interfering nodes are randomly distributed or have specific network topologies and we define and calculate the interference limited communication range $d_{ILR}$ as the radius of the region around a receiver within which a successful communication is possible. We compare our analytical results with simulation results, and find to be in a very good agreement. In chapter 5 we study the multicast throughput and show its dependence on the basic transmission rate and the value of the path loss exponent. Finally for a simple multicast network of two transmitters we provide the conditions under which the multicast throughput is maximized. Chapter 6 contains conclusions and discussion on the application of our results to scheduling and routing problems.
Chapter 2

SYSTEM MODEL

2. System Model

In most wireless ad hoc networks the nodes compete to access the shared wireless medium. A medium access mechanism, using cross layer cooperative communication techniques, can be used for interference compensation at a receiving node from all other interfering transmissions in range and allow successful reception of the signal of interest and thus improve the overall network performance. Transmission rate adaptation can help maximize channel capacity and power control can help lower the power consumption, two contradictory goals in an interference wireless networking environment where communication links share a common communication medium.

2.1. Network Model

We assume a wireless network where \( N \) nodes are distributed over a circular area of radius \( r_{\text{max}} \). All nodes in the network share a single channel and cannot receive and transmit at the same time. We will assume two different models of operation in the general description of our model. First for point to point communication at a given time slot we assume that \( K \) links can be formed with \( K \) nodes acting as transmitters and another \( K \) as receivers (\( K \leq N/2 \)). Second, multicast communication where a number of transmitters wish to multicast their data to their disjoint sets of receivers. In both of these cases, when many transmitters are operating simultaneously, all receiving nodes are subject to interference from the thermal noise and the concurrent transmissions.
2.2. Interference Channel

The interference channel arises naturally in many communication scenarios. In a wireless network, with nodes sharing the same spectrum, each communication is taking place over the so called interference channel. Interference mitigation is critical for optimum network performance when a number of uncoordinated links share a common medium. The difficulty in controlling interference is to optimally choose how to adapt the various characteristics, among the many possibilities, an ad-hoc wireless network has (many of which aren’t clearly understood even today).

The study of the interference channel was initiated by Shannon in 1961 and since then this problem has been thoroughly elaborated at the Information-theoretic level but the characterization of the capacity region and the rate adaptation techniques in wireless ad-hoc networks remain still open issues.

Most of the known studies either orthogonalize links, in time or in frequency, so that they do not interfere with each other, or allow links to operate simultaneously assuming that the interference from all other operating links behaves like additional Gaussian noise and adjusting the operational characteristics of the link to compensate with this interference. Both approaches can be suboptimal for the network performance; the first avoids concurrent transmission without taking in to account any knowledge for the underlying communication system, the topology of the links and the level of interference; the second one makes the crude approximation that interference behaves like Gaussian noise because in reality interfering transmissions carry structured information that could be exploited for successful reception of data.

It is critical to compare the performance of these methods and find out which are these distinct special cases that we can observe an improvement in performance. The basic and simplest theoretical model is the Gaussian interference channel, where two point-to-point
System Model

links with additive white Gaussian noise (AWGN) interfere with each other, as is illustrated in the figure below.

Figure 2.1: Two users Interference Channel

In a more general setting, the interference channel refers to a network consisting of an equal number (N) of transmitting and receiving nodes. A one to one correspondence between senders and receivers exists and each transmitter communicates only with its corresponding receiver, and each receiver is only interested in decoding the information from its corresponding transmitter. When many transmitter - receiver pairs share a wireless channel (common medium), each transmission is affected from the interference caused by all the other (N-1) links and at the same time interferes with the operation of all other active links. In an interference channel at each time we have N primary links and N*(N-1) interfering links.

In summary, interference remains a fundamental problem in wireless communications when multiple uncoordinated links share a common communication medium. The network layer must be able to sense the amount of interference at a receiver, during the
reception of a packet, and cope with it using power and/or rate adaptation (transmission rate adaptation can help maximize channel usage and power control can help lower the power consumption and interference). In the remaining of this chapter, we model how interference can affect specific characteristics (transmission rate and power), of a wireless network.

2.3. Signal to Interference plus Noise Ratio (SINR)

The Signal to Interference plus Noise Ratio (SINR) is a performance metric that measures the ratio of the received signal power to the power of the undesired signals in the receiver. We calculate the Signal-to-Interference-plus-Noise ratio (SINR) at each receiver as the ratio of the power of the signal of interest (meaningful information of a link) to the combined power of the additive white Gaussian noise and the interference caused by the other active transmissions in range. Thus:

\[ \text{SINR}_{(i)} = \frac{P_i G_{ii}}{(\sum_{j \neq i} P_j G_{ij}) + n_i} \]  

(2.1)

where \( G_{ij} \) is the path loss from transmitter \( j \) to receiver \( i \) and is equal to \( G_{ij} = \left( \frac{\lambda}{4\pi} \right)^2 \cdot d_{(ij)}^{-\alpha} \), where \( \lambda \) is the wavelength, \( \alpha \) is the path loss exponent and has typical values from 2 to 5, \( d_{(ij)} \) is the distance between nodes \( i \) and \( j \), and \( P_i \) is the transmitted power. White Gaussian noise is denoted as \( n_i \).

Another way of looking at the definition of the SINR, is to combine the transmission power of each transmitter with the quantity \( (\lambda/4\pi)^2 \) and consider \( P_i \) as the transmitted power at the distance of 1 meter and \( G_{ij} = d_{(ij)}^{-\alpha} \). The product of \( P_i G_{ii} \) is the received power of interest for the signal for interest and \( \sum_{j \neq i} P_j G_{ij} \) is the total interfering power of all other transmissions received at the receiver \( i \). In the general case \( G_{ij} \) could be
System Model

formulated to include fading effects, in this case it would have the expression \( G_{ij} = (\lambda/4\pi)^2 \cdot d_{(ij)}^{-a} \cdot f_{(i,j)} \), where \( f_{(i,j)} \) is the fading coefficient (a non negative random variable).

A specific transmission \( i \) over a wireless link is considered successful when condition (2.2), known as the **SINR criterion**, is true for a given threshold value \( \gamma_{(i)} \) and for a positive vector \( P=(P_1,\ldots,P_k) \) of transmission powers.

\[
\frac{P_i G_{ii}}{(\sum_{j \neq i} P_j G_{ij}) + n_i} \geq \gamma_{(i)} \tag{2.2}
\]

For each transmission link \( i \), the value of the SINR threshold \( \gamma_{(i)} \) depends on many design parameters and properties of the telecommunication system, such as the target BER, the modulation scheme, the error correction coding employed and the desired transmission bit-rate for each link. A set of simultaneous transmissions is considered successful when condition (2.2) is true for all the transmissions of this set. Given the telecommunication system and the application specific characteristics, each transmission might require different values for the SINR threshold \( \gamma \) for successful communication.

2.4. Link Model

2.4.1. Shannon Capacity Links

Shannon formulated the basic problem of reliable point-to-point transmission of information in statistical terms, using probabilistic models for information sources and communication channels. Based on such a statistical formulation a bandwidth constraint and additive noise can be associated with the channel and can be incorporated into a single parameter, that was named channel capacity. When that interference is considered as
additive white Gaussian noise an ideal band limited channel of bandwidth $BW$ has a capacity $C$ given by:

$$C = BW \log_2 \left(1 + \text{SINR}_{(i)} \right) \frac{\text{bits}}{\text{sec}}$$

The meaning of this formulation is the following: If the information rate $R$ from the source is less than $C$ ($R < C$) then it is theoretically possible to achieve reliable (error-free) transmission through the channel by appropriate (but not specified) coding. On the other hand, if $R > C$, reliable transmission is not possible regardless of the amount of signal processing performed at the transmitter and receiver. Thus Shannon established the basic limits on communication of information and gave birth to a new field that is now called information theory. Under the Shannon assumption, bits transmitted with this rate are received with asymptotically small probability of error.

### 2.4.2. Real-world links

Unfortunately meeting such a high standard, as set by Shannon in his seminal work, proved to be, after almost 50 years, a very difficult problem and a far more difficult one when attempting to extent Shannon’s theory to networks.

Because of this difficulty in our network model we consider mainly (but not always) a wireless telecommunication system where the network nodes use a specific modulation scheme with a specific BER and a constant bandwidth. We assume the absence of any error control coding or any multiuser decoding scheme or any cooperation between transmitters and/or receivers and make the crude approximation that the interference is white Gaussian noise, which combined with the thermal noise results in a combined total power spectral density $N_o/2$ in bandwidth $BW$. In this case, as will be shown below, there is a linear relationship between the achieved rate and the SINR [see also Wieselthier 2007],
System Model

as long as the symbol transmission rate does not exceed the available bandwidth (or, equivalently, as long as the symbol duration has a minimum value of $1/BW$).

We assume that the system’s bandwidth does not depend on the symbol duration of either the desired signal or the interfering signals. Thus the total power spectral density would be:

$$\frac{N_0}{2} = \frac{(P_{\text{int}} + P_{\text{noise}})}{2BW} \quad (2.4)$$

The energy per symbol for a link would be equal to $E_s = P_i G_{ii}/R_s$ which results in an energy per symbol to interference-plus-noise density ratio equal to:

$$\frac{E_s}{N_0} = \frac{P_i G_{ii}}{(\sum_{j\neq i} P_j G_{ij}) + n_i} \cdot \frac{BW}{R_s} \quad (2.5)$$

The values for the $E_s/N_0$ can be found from the Symbol Error Rate (SER) requirements. If we assume Gray symbol coding, the relationship between SER and BER is $SER = k*BER$, where $k = \log_2 M$ is the number of bits per symbol. The relationship between BER and $E_s/N_0$ is given below for various values of $M$ for MPSK modulation [Proakis 2000]:

$$\text{BER}_{\text{BPSK}} = Q\left(\sqrt{2 \frac{E_s}{N_0}}\right) \quad (2.6)$$

$$\text{BER}_{\text{QPSK}} = Q\left(\sqrt{\frac{E_s}{N_0}}\right) \quad (2.7)$$
where \( Q \) is the error function defined as:

\[
Q(z) = P(X \geq z) = \int_{z}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx
\]

In general for an M-PSK modulation scheme [Proakis 2000]:

\[
\text{BER}_{M-PSK} = \frac{2}{\log_2 M} \cdot Q \left( \sqrt{2 \frac{E_s}{N_0} \cdot \sin \frac{\pi}{M}} \right) \quad (2.8)
\]

The actual bit transmission rate for each link would then be given by:

\[
R_b = \frac{\text{# bits}}{\text{symbol}} \cdot \frac{\text{# symbols}}{\text{second}} \cdot \frac{\text{bits}}{\text{sec}} = k \cdot R_s \quad (2.9)
\]

From the above we can define the analytical formula for the transmission rate in bits per second:

\[
R_{b,\text{(BER,k)}}^{i} = k \cdot \frac{P_i G_{ii}}{(\sum_{j \neq i} P_j G_{ij}) + n_i} \cdot \frac{\text{BW}}{E_s/N_0} = k \cdot \text{SINR}_i \cdot \frac{\text{BW}}{E_s/N_0} \quad (2.10)
\]

The quantity \( E_s/N_0 \) in Equation (2.3) is calculated by solving Equation (2.8) for the given value of BER and the level \( k \) of the M-PSK modulation. We assume that the channel bandwidth is constant, and the symbol transmission data rate is bounded by the system’s bandwidth, that is \( R_{b,\text{(BER,k)}}^{i} < k \cdot BW \).

Therefore the rate function that corresponds to the specific telecommunication system, the quality of the channel, the desired target BER and the measured SINR is given by the following relation:
This formula has been derived under the assumption that the nodes use a specific modulation scheme (namely M-PSK) and taking into account the well known [Proakis 2000] relationship between the BER and the energy per symbol to interference ratio ($E_s/N_0$). The above analysis can be extended to any other communication system that uses a different modulation scheme resulting in a similar expression for the rate function of the transmission links.
Chapter 3

RATE REGION AND POWER CONSIDERATIONS IN A SIMPLE WIRELESS INTERFERENCE CHANNEL

3. Rate Regions and Power Considerations in a simple Wireless Interference Channel

3.1. Introduction

In this Chapter we define and study the transmission rate regions for a simplified wireless network. The rate regions describe all the sets of the achievable transmission rates of all active links of the network given the degree of interference, considered as noise, and individual maximum power constrains. The goal is to define the necessary conditions that maximize the system’s total aggregate rate for a simplified 2-link interference wireless network channel, showing criteria under which simultaneous link operation outperforms timesharing solutions. To this end, it is necessary to identify critical points, in the rate regions, where we can achieve higher aggregate rates in practical band limited interfering channels. Herein, we study and compare the aggregate rates achieved for simultaneous and timesharing operation, as a function of the transmission power. Finally, for the case of higher order of modulations, we investigate the conditions on the maximum individual transmission power for switching to the next higher order modulation in order to achieve higher aggregate rate.

In a wireless network, with nodes sharing the same spectrum, it is evident that each transmission is affected from, and affects, all other transmissions in range. The medium access mechanism should be able to control the amount of interference experienced by receivers and, in certain cases, to enforce concurrent transmissions, in order to maximize network performance by tightly coupling both physical and medium access layers. If the total amount of
Rate Regions and Power Considerations in a simple Wireless Interference Channel

interference at a receiver, during a reception of a packet, is high the network layer should
decrease the transmission rate and/or increase the power to cope with it. Here we study the
rate and power adaptation for an ad-hoc wireless network in order to allow concurrent
transmissions, enhance network capacity by maximizing, when possible, individual link data
rates, minimize transfer time of a data packet and allow the best possible usage of network’s
resources for all competing transmissions. Transmission rate adaptation can help maximize
channel capacity and power control can help lower the power consumption.

Throughput enhancement and energy saving are two contradictory goals in an interference
wireless networking environment where multiple links share a common communication
medium. As was previously mentioned, most of the known studies either orthogonalize links, in
time or frequency, so interference does not affect each other at all, or allow links to operate
simultaneously and assume that the interference from all other operating links behaves like
additional Gaussian noise. Higher transmission rates can be achieved with higher levels of
modulation (M-ary modulation), but higher transmission power is required to maintain
acceptable bit error rate performance. Therefore, the problem of finding the performance
tradeoffs between throughput and energy efficiency becomes a difficult challenge in the design
of joint rate and power adaptation.

This is a problem that has been studied thoroughly at the Information-theoretic level but the
characterization of the capacity region and the rate adaptation techniques in wireless ad-hoc
networks have been open issues for years (an analytical account of related work is presented in
Chapter 2). One of the most influential works considered is the pioneer research of Han and
Kobayashi [Han 1981] on the information theoretic bounds of the achievable rate region of
interference channels and the derivation of the capacity region in the case of strong
interference. The best known achievable region [Han 1981], is hard to compute and is unclear
if it is optimal. Other outer bounds exist but it is unclear how tight they are with respect to
capacity [Sato 1978], [Costa 1985], [Etkin 2006]. In [Kramer 2004] new outer bounds for the
capacity have been shown for a simplified Han and Kobayashi scheme. Recent work on the interference channel [Charafeddine 2007] has shown the frontiers for the achievable rate regions for a n-user interference channel determining the shape and size of the rate regions. The usual assumptions in previous works are that interference is considered as additional noise and that the maximum achievable rate depends logarithmically on the Signal to Interference and Noise Ratio (SINR).

We consider a simplified wireless telecommunication system (along the lines of Chapter 2) where the network nodes use a specific modulation scheme with a specific BER and a constant bandwidth. When links operate simultaneously they are subject to interference from the thermal noise and the other concurrent transmissions. The value of the SINR threshold $\gamma$ is supposed to be constant and the same for all networking links. Wieselthier and Ephremides in [Wieselthier 2007] have shown that in this case there is a linear relationship between the achieved rate and the SINR, as long as the symbol transmission rate does not exceed the available bandwidth. In addition we assume the absence of any error control coding or any multiuser decoding scheme nor any cooperation between transmitters and/or receivers. We are well aware that the rates achieved can never get close to the capacity bounds described in previous works, but our model reflects the situation of many, practical, low power and low complexity communication networks such as sensor networks, and it enables us to make some useful observations, with practical significance, for the operation of these networks. Our results show the achievable rate region with respect to the network topology, the degree of interference (considered as additional noise) and the individual maximum power constrains of each transmitter. We study and define the necessary conditions that maximize the system’s total aggregate rate and provide conditions under which simultaneous link operation is better than the timesharing operation.

The chapter is organized as follows: In section 4.2 we present our study for the case of a simplified 2-link interference channel, when nodes use BPSK modulation, and determine the
conditions under which simultaneous link operation outperforms timesharing solutions. In section 4.3 we extend the results to higher order MPSK modulation schemes. The conclusions are presented in the last section.

3.2. Two Link Rate Region (BPSK Modulation)

This section presents the achievable transmission rate region for a simplified wireless network model formed by two links sharing a single channel and assuming that at a given time slot they are either activated simultaneously or one by one. To simplify the presentation, for this section, we will consider a BPSK modulation scheme (parameter \( k \) will be equal to 1) and in most cases we will assume symmetrical links. In later section we will study cases for higher modulation order (M-PSK where \( K > 1 \)) and compare the results.

3.2.1. TDMA (Time Division Multiple Access) case

When links are activated in a time division fashion each one uses only a portion of the available slot time. Using Equation (2.11) and since there is no mutual interference we get (for simplicity we assume that \( n_1 = n_2 = n \)):

\[
R_i^{\text{TDMA}} = BW \cdot \min \left\{ 1, \frac{P_i G_i}{n} / \frac{E_b}{N_0} \right\}
\]  

We can see from Equation (3.1) that transmission rates are monotonically increasing with transmission power \( P_i \) as long as they are below the maximum rate \( BW \). It is wasteful to use more power than the necessary \( P_i^{\text{max}} \) to achieve this maximum rate.

\[
P_i^{\text{max}} = \frac{n}{G_i} \cdot \frac{E_b}{N_0}
\]
Rate Regions and Power Considerations in a simple Wireless Interference Channel

Obviously the maximum transmission rate achieved with timesharing is: \( R_{\text{TDMA}}^{\text{max}} = BW \).

### 3.2.2. Interference Channel

When the two links operate simultaneously we obtain the normalized transmission rates from Equation (3.5):

\[
\frac{R_1}{BW} = \min \left\{ 1, \frac{aP_1}{bP_2 + 1} \right\}
\]

\[
\frac{R_2}{BW} = \min \left\{ 1, \frac{cP_2}{dP_1 + 1} \right\}
\]

where: \( a = \frac{G_{11}E_{\text{dB}}}{N_0} \), \( b = \frac{G_{12}E_{\text{dB}}}{N_0} \), \( c = \frac{G_{22}E_{\text{dB}}}{N_0} \), \( d = \frac{G_{21}E_{\text{dB}}}{N_0} \).

We observe that \( R_i \) is monotonically increasing with \( P_i \) and monotonically decreasing with \( P_j \). For any specific power pair we get exactly one rate pair, by solving Equation (3.3). Each link can, under certain conditions, achieve maximum rate equal to \( BW \), but in that case the required power \( P_{\text{MAX}} \) that maximizes the rate of the link of interest, depends on the transmission power of the other link as well, and is certainly higher than the maximum power \( P_i^{\text{max}} \) calculated in (3.2). In fact it is possible to have \( R_1 = BW \) and \( R_2 = BW \) with simultaneous transmissions if:

\[
ac > bd \Rightarrow \frac{E_b}{N_0} \leq \frac{G_{11} \cdot G_{22}}{G_{12} \cdot G_{21}}
\]

in which case: \( P_{\text{MAX}}_{1_{\text{Intf}}} = \frac{c+b}{ac-bd} \) and \( P_{\text{MAX}}_{2_{\text{Intf}}} = \frac{a+d}{ac-bd} \).
We see that inequality (3.4) involves only the network topology (the $G_{ij}$'s of each link) and the desired BER ($E_b/N_0$). If (3.4) is satisfied we say that we are in the light interference regime and it is possible to achieve maximum aggregate rate twice as much as the one achieved with timesharing (notice, however that the $P_{\text{intf}}^\text{MAX}$ can be very high). If inequality (3.4) is not satisfied, then strong interference is present and we have to resort to a time division scheme in order to achieve the maximum possible aggregate rate, or operate one link at full rate and the other with lower rate (discussed in later section).

### 3.2.3. Achievable Rate Region using $P_i^{\text{max}}$: TDMA vs Interference operation

We compute now the rate regions, using Equation (3.3), of two links operating simultaneously under the assumption that their maximum individual transmission powers are those calculated in (3.2). In Figures 3.2 and 3.3 we present the rate regions for strong interference ($ac < bd$) and light interference ($ac > bd$) respectively.

![Figure 3.1: 2-link rate region, Strong Interference.](image)

(a=c=1, b=d=1.5, $P_{\text{intf}}^{\text{max}} = 1$mW)
In Figures 3.2 and 3.3 we can identify point B, where both transmitters use their maximum power. If this point is above the TDMA line (Figure 3.3) the aggregate interference rate achieved can be higher than the TDMA rate.

The condition under which the normalized aggregate interference rate can be higher than the normalized TDMA rate (equal to 1) when $P_1^{\text{max}} = 1/a$ and $P_2^{\text{max}} = 1/c$ is:

$$R_1(P_1^{\text{max}}) + R_2(P_2^{\text{max}}) \geq 1 \Rightarrow$$

$$\frac{1}{bP_2^{\text{max}}} + \frac{1}{dP_1^{\text{max}}} \geq \frac{bd}{ac} \leq 1 \Rightarrow \frac{E_b}{N_0} \leq \frac{G_{11} \cdot G_{22}}{\sqrt{G_{12} \cdot G_{21}}}$$

which is exactly inequality (3.4) obtained above.
3.2.4. Achievable Rate Region with Excess Power

Next in our analysis we are studying the achievable rate regions for the 2-link case when the individual maximum transmission powers are in excess of $P_i^\text{max}$.

3.2.4.1. Light Interference case

In case of light interference, where inequality (3.4) is satisfied, using transmission powers greater than those determined from Equation (3.2), we obtain the rate region shown in Figure 3.4 (maximum transmission powers are $P_{\text{Intf}}^\text{max} = 1.2 \text{ mW}$).

Comparing the rate regions shown in Figure 3.2 and Figure 3.3 we can see that higher aggregate rates can be achieved at the expense of higher power expenditure. This is depicted in Figure 3.4. Observe that the relation between maximum achieved rate and maximum transmit power is almost linear in this case.
3.2.4.2. **Strong Interference case**

In the strong interference case we have shown in previous section, that, restricting the maximum individual transmission powers according to Equations (3.2) and (3.3), the maximum achieved aggregate interference rate is always less than the TDMA rate. When this restriction on the transmission power is removed, the aggregate interference rate can be greater than the TDMA rate as shown in Figure 3.5 below where the maximum individual transmission powers are $p_{\text{int}}^{\text{max}} = 4mW > p_i^{\text{max}}$. 

---

Figure 3.4: Light interference: Maximum normalized aggregate rate vs. power
Figure 3.5: $P_{\text{2intf}}^{\text{max}} = 4\text{mW}$, Maximum Aggregate Interference rate is equal to 1.31

Compare with the rate region in Figure 3.2 (where $P_{i}^{\text{max}} = 1\text{mW}$). We observe that point A, in Figure 3.6, is the point where we have the maximum aggregate rate. At this point the 2\textsuperscript{nd} link operates with the maximum power: $P_{\text{2intf}}^{\text{max}} = 4\text{mW}$ whereas the other link operates with such a power so that link 2 achieves the maximum rate supported by the bandwidth. The normalized aggregate rate at operating point A in the above plot is calculated as follows:

$$R_{\text{aggr}} = \frac{c-P_{\text{2intf}}^{\text{max}}}{d-P_{1}+1} + \frac{a-P_{1}}{b-P_{\text{2intf}}^{\text{max}}+1}, R_{2} = \frac{c-P_{\text{2intf}}^{\text{max}}}{d-P_{1}+1} = 1 \text{ with } P_{1} = \frac{c-P_{\text{2intf}}^{\text{max}}-1}{d}$$

For operation at the point A, the plot in Figure 3.7, below, shows that in the case of strong interference it is possible to achieve aggregate rates higher than the TDMA rate, but at the cost of a disproportionally high total power.
Figure 3.6: Strong interference case: Maximum aggregate rate vs. maximum individual power

### 3.2.5. Cognitive Interpretation

Cognitive radio is defined to be an intelligent communication system able to adapt its internal state to any variation in the surrounding networking environment by making, in real time (on the spot), corresponding changes in certain operating parameters (e.g., transmit power, carrier frequency, and modulation strategy). The goal is to provide reliable and high quality communication services when required and efficiently utilize available radio spectrum.

A “cognitive” transmitter or receiver is aware of its environment and can combine this knowledge, for specific application needs and set the various characteristics for each transmission in an appropriate fashion that would optimize network performance. Such a system is able to learn through experience and capable of generating solutions for communications problems unforeseen by its designers. Cognitive terminals are intelligent enough to detect user communications needs as a function of user context, and to share available radio resources and wireless services, among networking nodes, appropriately.
In wireless communication the paradigm of cognitive radio describes that the network, in a centralized manner, or the wireless nodes themselves, when ad-hoc or mess or sensor networks are formed, is able to change the various critical transmission or reception parameters to adapt to particular needs and ensure successful communication. This parameter alteration technique is based on the observations of several factors from the external and internal radio environment, such as radio frequency spectrum, user behavior, and network state.

In our wireless network model, each node can communicate with any other node over the wireless medium. Groups of nodes may cooperate amongst one another to jointly encode or decode the transmitted signals correctly. We consider the scenario where there are two transmitters and two receivers, with each transmitter intending to send an independent message to a different receiver. When nodes do not cooperate, the communication model is that of the interference channel discussed extensively before and even though its capacity remains an open problem in information theoretic level we have presented our results for a simplified wireless system.

In this context our cooperative or cognitive interpretation for the region of achievable data rates is presented here. Previously in Figures 3.3 and 3.5 we have seen that there are cases where a transmitter is able to operate at its full rate while the other is also active. In Figures 3.7 and 3.8 we denote these cases with points C and A respectively on the left side plots.

Our cognitive interpretation suggests a method that allows links to efficiently use and reuse the radio spectrum when operating simultaneously just by selecting an appropriate power level for transmission. Higher aggregate rates can be achieved at the expense of higher power expenditure for the primary and the secondary link. The relation for the primary and the secondary link power expenditure is shown in the figures below for the case of light and strong interference.
In Figures 3.7 and 3.8 we allow simultaneous transmission of a primary and a secondary user and show the power expenditure for both users. When the primary link operates at its full rate the secondary link is still able to operate successfully by adapting its power in such a way (as the figure on the right shows for the case of strong and light interference) that will not affect the primary user's transmission, at the expense of extra power expenditure. In the plots, on the right side, for the critical point C in Figure 3.7 and point A in Figure 3.8 we see the expense in power for both links given the transmission rate of the second link (Figure 3.7) and the achievable rate of the second user given the transmission power of the second link (Figure 3.8).

Figure 3.7: Cognitive interpretation for light interference case
3.2.6. Achievable Rate with Low Power

3.2.6.1. Two link case

In the previous section we have studied the rate region when the transmission power exceeds the maximum power \( P^{\text{max}}_i \) determined from Equation (3.2). In this section we study the achievable transmission rates when the nodes use lower transmission powers. We show that there is a power threshold value below which, when links operate simultaneously, the achieved aggregate rate is higher than the rate achieved when the links operate in a timesharing fashion.

We assume, for simplicity, that the nodes are symmetrically placed (so that \( a=c \) and \( b=d \)) and use the same transmission power \( P < P^{\text{max}}_i =1/a \). When nodes transmit in timesharing mode the achieved rate is:

\[
R_{\text{TDMA}} = BW \cdot a \cdot P \leq BW
\]  

(3.5)
When the links operate simultaneously, the maximum aggregate rate can be as high as two times the BW depending on the level of the interference. With our assumptions and the rates from Equation (3.3) we get:

\[ R_{\text{Intf}} = 2 \cdot BW \cdot \frac{a \cdot P}{b \cdot P + 1} \leq 2 \cdot BW \]  

(3.6)

From Equations (3.5) and (3.6) we find that:

\[ R_{\text{Intf}} \geq R_{\text{TDMA}} \Leftrightarrow 2 \cdot \frac{a \cdot P}{b \cdot P + 1} \geq a \cdot P \Leftrightarrow \Leftrightarrow b \cdot P \leq 1 \Leftrightarrow \]

\[ 0 \leq P \leq \frac{1}{b} = n/G_{ij} \]  

(3.7)

That is, if the power is less than \( 1/b = n/G_{ij} \) then the simultaneous operation of the links results in higher aggregate rate compared to the timesharing operation. This can be explained by the fact that when the transmission power is small (so that \( PG_{ij} < n \), that is when the interference is small compared to noise), the operation of the links is noise limited and therefore it is better to have two such links operating simultaneously than just one at a time.
From Figure 3.8, where Equations (3.5) and (3.6) are plotted, it is clear that when the available power is low, simultaneous operation is better, even in the strong interference case as long as condition (3.7) is satisfied.

3.2.6.2. **N-links case (for specific topologies)**
Similar results can be obtained for a more general setting, where we assume that we have N wireless links active in the network. If parameters $a$ and $b$ are the same for all links (as for example the topologies shown in Figure 3.8) then condition (3.7) holds for the general case. If we assume that the interfering parameter $b$ is the same for all links then:

$$R_{\text{Intf}} \geq R_{\text{TDMA}} \iff \sum_{N} \frac{a \cdot P}{b \cdot P + 1} \geq a \cdot P \iff$$

$$N \cdot \frac{a \cdot P}{(N - 1) \cdot b \cdot P + 1} \geq a \cdot P \iff$$

$$0 \leq P \leq \frac{1}{b} = \frac{n}{G_{ij}}$$

which is exactly inequality (3.7) obtained above.
3.3. Two Link Rate Region (M-PSK Modulation)

3.3.1. Operation with maximum individual powers

In this section we investigate the achievable rate regions when we use higher order modulation schemes for the simplified two link case. The bit rate per link is given by Equation (2.11), where the value of $E_s/N_0$ is different for each modulation index $k$ and can be calculated using the BER formulas from Equations (2.3), that is:

$$R^k_i = k \cdot BW \cdot \min \left\{ 1, \frac{a_k \cdot P_i}{b \cdot P_j + 1} \right\}$$

(3.9)

where: $a_k = \frac{g_{ii}}{n_i} \left( \frac{E_s}{N_0} \right)_k$ has values decreasing with $k$ and $b = \frac{g_{ij}}{n_i}$ is constant for every $k$.

Figures 3.12 and 3.13 show the achievable rate regions for various PSK modulation levels ($k=2, 4, 8, 16, 32, 64$) with strong interference and light interference respectively. The rate region shapes are similar to the ones presented in the previous section for the BPSK. For the strong interference case we can see that, in order to achieve the highest possible aggregate rate for given maximum individual transmission powers, the preferred operating mode is the TDMA with the highest possible modulation index $k$ allowed by the maximum transmit power. In the low interference case, since $(E_s/N_0)_k$ increases with $k$ for a given BER, inequality (3.4) will cease to be true above some $k$, and the shape of the corresponding rate regions will change to shape consistent with strong interference (Figure 3.2). In this situation, it is not always clear which is the preferred mode of operation. For the specific example in Figure 3.13 it is easy to see that QPSK is the preferred mode of operation.
Figure 3.11: Rate Region for higher order PSK modulation

Strong Interference case

Figure 3.12: Rate Region for higher order PSK modulation

Light Interference case
The maximum achievable capacity of the i-th link (considering the interference as noise) would be given by the Shannon formula [Charafeddine 2007], where $P^*$ is a positive power vector:

$$c_i(P^*) = BW \cdot \log_2 \left( 1 + \frac{P_i G_{ii}}{G_{ij} \cdot P_j + n_i} \right) = BW \cdot \log_2 \left( 1 + \frac{a \cdot E_b \cdot P_i}{b \cdot P_j + 1} \right)$$ (3.9)

Of course this rate cannot be achieved by our simplified wireless network and for reasonable values of BER, and this is shown in Figure (3.10) (where, for comparison reasons, we have assumed BER=10^{-6} i.e. $E_b/N_0 \approx 11.3$).
Figure 3.13: Rate Region vs. Shannon capacity region in strong (top) and light (bottom) interference cases.
3.3.2. Operation in the low power regime and with various levels of power

It is easy to see that the analysis in Section 3.2.6 holds true for higher order modulations and therefore if the power satisfies the inequality (3.7), that is if $P < \frac{1}{b}$ then the operation of the links is noise limited and their simultaneous operation results in higher aggregate rate than their timesharing operation. In Figure 3.15 we plot the aggregate rates vs. power of a symmetrical 2-link network with the two links operating with the same power and with various levels of modulation. We can see that if the maximum individual transmission power is less than $P_A$, then the preferred mode of operation is BPSK with simultaneous activation. If the available power is between $P_A$ and $P_B$ then it is better to operate simultaneously with QPSK modulation. Similarly if the available power is between $P_B$ and $P_C$ then it is better to operate simultaneously with 8-PSK modulation, etc.

The threshold powers $P_A$, $P_B$, $P_C$ etc. can be calculated as follows: Let $P_{8PSK}$ be the lowest power for which the simultaneous operation with 8-PSK has higher aggregate rate than the maximum achievable rate with QPSK which is $4BW$.

This means that for $P \geq P_{8PSK}$ we have:

$$2 \cdot 3 \cdot BW \cdot \frac{a_3 \cdot P}{b \cdot P + 1} \geq 4BW \Rightarrow a_3 \cdot P \geq \frac{2}{3} (b \cdot P + 1) \Rightarrow$$

$$P \geq \frac{3}{(3 \cdot a_3 - 2 \cdot b)} \quad (3.10)$$

The condition for operating with 8-PSK, if $(3 \cdot a_3 - 2 \cdot b) > 0$, is $P_{8PSK} = \frac{3}{(3a_3 - 2b)} = P_B$. In the general case:
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\[ P_{M-PSK} = \frac{k}{\{ka_k - (k-1)b\}} \]  \hspace{1cm} (3.11)

where \( k = \log_2 M \).

![Figure 3.14: Aggregate Rate for M-PSK modulation](image)

3.4. Conclusion

In this section, for a simple wireless network which uses a specific modulation scheme with a specific BER and a constant bandwidth, we studied the transmission rate regions given the degree of interference, considered as noise, and individual maximum power constrains. Conditions that maximize the system’s total aggregate rate for a simple 2-link interference wireless network channel are given establishing criteria under which simultaneous link operation outperforms timesharing solutions. We identify critical points, in the rate regions presented, where we can achieve higher aggregate rates in practical band limited interfering
channels, at the expense of higher power expenditure. We show that the relation between maximum achieved rate and power consumption is almost linear in the case of light interference, but there is need for disproportionally high total power in the case of strong interference. We studied the aggregate rates achieved with simultaneous and with timesharing operation when operating in the low power regime and found that up to a certain power level the simultaneous operation results in higher aggregate rates. Finally, for higher order modulations, we gave the condition on the maximum individual transmission power for switching to the next higher order modulation in order to achieve higher aggregate rate.
CHAPTER 4

INTERFERENCE LIMITED COMMUNICATION RANGE

4. Interference Limited Communication Range

4.1. Introduction

A wireless ad hoc network is typically formed by randomly deployed nodes that have specific capabilities and limited resources. In such systems interference is experienced among communication links (many transmitters try to communicate with a common receiver; many receivers listen a common transmitter; between different transmitter-receiver pairs) and plays a dominant role in determining the network performance, the achievable transmission rate of links, the total aggregate throughput, the power consumption, etc.

It has been extensively reported that the performance of wireless system is limited by the effect of interference. In [Renato 2007, Moraes 2009] an analytical model for interference calculation is presented which allows the assessment of the signal to noise plus interference ratio, at a receiver node, at any point in the network when nodes communicate with a close neighbor. In [Gupta 2000] upper and lower capacity order-bounds are derived using the concept of the exclusion region\(^2\) to limit interference caused to the receiver. It is shown that the network capacity decreases when the size of the exclusion region increases. In [Hasan 2004], the size of the exclusion region for CDMA networks is studied. It is

\(^2\) An exclusion region is a region around each receiver such that nodes inside this region are not allowed to transmit.
suggested that there exists an optimal size for the exclusion region that maximizes capacity. In [Rehka 2006] it is shown that the optimal size for the exclusion region radius, for maximizing network capacity, depends on the density of the network nodes.

Despite the increasing research work on the analysis of ad-hoc networks in the recent years, the effect of interference has not been studied exhaustively given the difficulty in modeling the random distribution and the mobility of nodes. Interference modeling can be used to provide useful insights for further analysis of important quantities in ad-hoc networks, such as: routing, energy consumption, achievable rate, network throughput, and end-to-end delay in data transmission. Given the amount of interference exhibited at a receiver, the network layer should adapt the transmission rate and/or power levels to cope with it or select a cancelation method to avoid interference effects on data transmissions.

In this chapter we study the interference exhibited at the center of a circular networking area when interfering nodes are randomly distributed or have specific network topologies. Going a step further from previous works, we define the \textit{interference limited communication range} \( d_{ILR} \) to be the critical communication region around a receiver, surrounded by a large number of randomly placed interfering nodes, within which if a transmitter is present, a successful communication link can be established. The value of \( d_{ILR} \), as we will show in the following, depends on a number of parameters: the number and the density of the surrounding interfering transmitters, the exclusion region around each receiver, the interference power level, and most importantly on the selected rate or mode of operation of the receiver. We illustrate the concept in Figure 4.1. The receiver of interest is at the center of the networking area around which many interfering transmitters are randomly deployed. Given the amount of interference exhibited at the receiver’s site the interference limited communication range \( d_{ILR} \) is shown (the light shaded disk). Every transmitter located within this critical area could successfully communicate with the receiver despite all other active interfering transmissions. By changing the rate we can adjust the
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communication range allowing more or fewer transmitters to be able to overcome interference and be able to successfully communicate with the receiver of interest (R_x).

In the rest of the chapter we first present the network model and our analysis on interference modeling for two cases: a) when the interfering transmitters are randomly placed using a uniform distribution and b) when they are placed on a regular lattice. We then calculate the critical interference limited communication range (d_{ILR}) and present our results. We verify the results of the theoretical calculations (based on the proposed model) for the mean interference power and the interference limited communication range, using simulations.

![Diagram](image-url)

Figure 4.1: Interference Limited Communication Range
4.2. Interference modeling

The exact calculation of the interference in a wireless ad hoc network is a difficult task even when special algorithms are used to share information of power and distances among network nodes. Many known studies try to model and calculate interference levels for a wireless multi-hop ad-hoc network, taking into account the number of nodes, density of nodes, radio propagation, and the amount of relay traffic [Hekmat 2004], [Rekha 2006]. In [Rekha 2006] interference power at the receiver site is calculated when transmitting nodes are distributed according to a Matern hard-core process\(^3\) in an annular exclusion region around the receiver.

In our model we make the approximation that interference behaves like Gaussian noise which is equivalent to the applicability of the SINR criterion (Equation 2.1) and is equal to the sum of the received power, from all active transmitters \(I_i = \left(\sum_{j \neq i} P_j G_{ij}\right)\), at the receiver of interest \(i\). \(P_j\) is the transmitted power and the path loss from transmitter \(j\) to receiver \(i\) is 

\[
G_{ij} = \frac{\lambda}{4\pi} \cdot d_{(ij)}^{-a} = D \cdot d_{(ij)}^{-a},
\]

\(a\) is the path loss exponent and has typical values from 2 to 5 and \(d_{(ij)}\) is the distance between nodes \(i\) and \(j\). In the following sections we describe our approach for interference modeling when the transmitting nodes are positioned within the networking area using a random uniform distribution or a static honey-grid topology.

\(^3\) A Matern hard-core process [Stoyan 1995] describes patterns produced by the locations of centers of non-overlapping circles of radius \(r_{min}/2\) describing exclusion regions that constituent nodes are forbidden to be closer than a certain minimum distance \(r_{min}\).
4.2.1. Random Uniform Topology

We assume that a receiver of interest is placed at the center of a circular area and $N_t$ transmitters, with equal transmission powers $P$, are placed randomly with uniform distribution inside an area bounded by the circles with radius $r_{\text{max}}$ and $r_{\text{min}}$ ($r_{\text{min}}$ is the radius of the exclusion region around the receiver where no transmitters are allowed to operate).

Inside the ring from $r$ to $r + \text{dr}$ there are on average

\[
N(r) = \frac{N_t \cdot 2\pi r \text{dr}}{\pi (r_{\text{max}}^2 - r_{\text{min}}^2)} = \left(\frac{2N_t r}{r_{\text{max}}^2 - r_{\text{min}}^2}\right) \text{dr}
\]

transmitters, causing interference with power equal to:
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\[ I(r) = \sum_{N(r)} P_j G_{ij} = PD \left( \frac{2N_tr}{r_{\max}^2 - r_{\min}^2} \right) dr = \]

\[ I(r) = \left( \frac{2PDN_t}{(r_{\max}^2 - r_{\min}^2) r^a - 1} \right) dr \quad (4.1) \]

The total mean interference at the center of the circle then would be equal to:

\[ I = \int_{r_{\min}}^{r_{\max}} \left( \frac{2PDN_t}{(r_{\max}^2 - r_{\min}^2) r^a - 1} \right) dr = \frac{2PDN_t}{r_{\max}^2 - r_{\min}^2} \int_{r_{\min}}^{r_{\max}} \frac{dr}{r^{a-1}} \quad (4.2) \]

We distinguish two cases (for the calculation of the integral) depending on the path loss exponent \( a \). For path loss exponent \( a \) equal to 2 we get:

\[ I = \frac{2PDN_t}{r_{\max}^2 - r_{\min}^2} (\ln r_{\max} - \ln r_{\min}) \quad (4.3) \]

For \( r_{\max} >> r_{\min} \) we have

\[ I \approx \frac{2PDN_t}{r_{\max}^2} (\ln r_{\max} - \ln r_{\min}) \]

that is, the interference has a strong dependence on \( r_{\max} \) (and consequently on the transmitter density \( \frac{N_t}{\pi r_{\max}^2} \)) and a much weaker (logarithmic) dependence on \( r_{\min} \) (the size of exclusion region)

and for \( a>2 \) we get:
\[ I = \frac{2PDN_t}{(a - 2)(r_{\text{max}}^2 - r_{\text{min}}^2)} \left( \frac{1}{r_{\text{min}}^{a-2}} - \frac{1}{r_{\text{max}}^{a-2}} \right) \] (4.4)

Again for \( r_{\text{max}} >> r_{\text{min}} \) we have

\[ I \approx \frac{2PDN_t}{(a - 2) r_{\text{max}}^2} \left( \frac{1}{r_{\text{min}}^{a-2}} \right) \]

that is, the interference has a strong dependence on \( r_{\text{max}} \) (and consequently on the transmitter density \( \frac{N_t}{\pi r_{\text{max}}^2} \)) and an almost equal degree of dependence on \( r_{\text{min}} \).

4.2.2. **Honey Grid topology: Simplified estimation of interference**

In this section we assume that all transmitter nodes’ positions are on a regular lattice, forcing a specific granularity on a two-dimensional plane where each node has a specific number of immediate neighboring nodes (same for all nodes not placed at the edges of the networking area) and the distance between any two neighboring nodes is the same.

The topology that satisfies these restrictions is a hexagonal lattice (commonly called honey-grid) [Hekmat 2006b]. As we can see in Figure 4.3, all transmitting nodes are placed in co-centered hexagons around the central receiving node placed in the middle of the networking area. In this point the maximum interference is exhibited (worst case scenario) since it allows the maximum number of interfering signals.
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The network size is related to the number of these hexagons and the number of nodes placed on each one of these hexagons, as shown in the following formula:

\[
N = 1 + \sum_{i=1}^{L} 6i = 1 + 3L(L + 1) \quad (4.5)
\]

where \( L \) is the number of hexagons and is given by:

\[
L = \left\lfloor \frac{1}{4} + \frac{N-1}{3} - \frac{1}{2} \right\rfloor
\]

To calculate the amount of interference we use a simplification and assume that the distance between the interfering nodes, on each hexagon ring, and the central receiving node is equal to the distance of the hexagon corner nodes. Thus in ring \( i \), located at distance \( d \cdot i \) we have \( 6 \cdot i \) interfering nodes and the expected value of interference is equal to:

Figure 4.3: Honey-Grid placement of nodes. Receiver is placed at the center of the honey grid.
Interference Limited Communication Range

\[ E(I)' = \sum_{i=1}^{L} P \cdot D \cdot \frac{6 \cdot i}{(d \cdot i)^\eta} = 6 \cdot P \cdot D \cdot \sum_{i=1}^{L} \frac{i}{d^\eta \cdot (d \cdot i)^\eta} = P \cdot D \cdot \sum_{i=1}^{L} \frac{1}{i^{\eta-1}} \quad (4.5) \]

where \[ L = \left\lfloor \frac{1}{\sqrt{4 + \frac{N-1}{3}} - \frac{1}{2}} \right\rfloor \] and \[ D = (L_{\text{wavelength}}/4\pi)^2 \]

When network size increases then \( L \to \infty \) and the sum \( \sum_{i=1}^{L} \frac{1}{i^{\eta-1}} \) is equal to \( \zeta(\eta - 1) \) : the Riemann-zeta function [Rieman 1953]). When the path loss exponent is greater than 2 then we know that the above expression is upper-bounded by \( \sum_{i=1}^{\infty} \frac{1}{i^{\eta-1}} \leq 1 + \int_1^\infty \frac{1}{x^{\eta-1}} \, dx = \frac{\pi^{\eta-1}}{\eta-2} \) [14]. We can thus get the various values of the zeta function \( \zeta(\eta - 1) \) obtained from integral arguments called zeta constants. For any positive even number \( z \), the representation of \( \zeta(z) \) can be obtained using the following procedure. First, we define \( \phi \) by \( \phi(z) = z \cot(\pi z) \) on \( U - \{0\} \), where \( U \) is a sufficiently small closed disk around \( 0 \). Then, using well-known property about residue, we obtain \( \text{Res} \left( \frac{\cot(\pi z)}{z^m} ; 0 \right) = \frac{1}{(2\pi)^{m}} \lim_{z \to 0} \frac{z^m}{\cot(\pi z)} \), and the meaning of the above formula is that the right hand side of the equation converges and the value is equal to that of the left hand side. Next, it is known that \( \zeta \) function has the following property for arbitrary integer \( m \geq 1 \): \( \zeta(2m) = -\frac{\pi}{2} \text{Res} \left( \frac{\cot(\pi z)}{z^{2m}} ; 0 \right) \). The following table shows that most commonly used values of the Riemann zeta function.

<table>
<thead>
<tr>
<th>( \zeta(0) )</th>
<th>( \zeta(0.5) )</th>
<th>( \zeta(1) )</th>
<th>( \zeta(1.5) )</th>
<th>( \zeta(2) )</th>
<th>( \zeta(2.5) )</th>
<th>( \zeta(3) )</th>
<th>( \zeta(4) )</th>
<th>( \zeta(6) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1/2</td>
<td>-1/463</td>
<td>( \infty )</td>
<td>2.612</td>
<td>1.64</td>
<td>1.341</td>
<td>1.20</td>
<td>1.12</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Based on the above we can calculate the exact amount of interference power level. These results describe a worst case scenario and an upper bound for the interference level.
The above analysis shows the method for calculation the interference in the center of a honey grid topology but will not be used in the rest of the thesis.

4.3. Interference Limited Communication range

The *Interference Limited Communication Range* ($d_{ILR}$) is defined to be the maximum distance, from a receiver, within which a transmitter similar to the other $N_t$ transmitters and given the number of interfering nodes, can successfully send its data to the receiver with rate $R_b$. Our analysis, presented here, extents the results presented in [Renato 2007] where only interference is studied, by introducing the interference limited communication range around a receiver.

4.3.1. Wireless System with real-world links

In this section, we consider the receiver at the center of the circular area to be part of a wireless network with real-life links that use a specific modulation scheme, a specific target BER and constant bandwidth. Again, we assume the absence of any error control coding or any multiuser decoding scheme or any cooperation between transmitters and/or
receivers and make the approximation that the interference behaves like white Gaussian noise, which combined with the thermal noise results in a combined total noise power spectral density $N_0/2$ in bandwidth $BW$. From formula 2.5 we know that the energy per symbol for a link is given by:

$$\frac{E_s}{N_0} = \frac{P_i G_{ii}}{\left(\sum_{j \neq i} P_j G_{ij}\right) + n_i} \cdot \frac{BW}{R_s}$$

As explained in section 2.3, if condition (2.2) (the SINR criterion) is true for a given threshold value $\gamma$ a communication can be successful, thus:

$$\frac{P_i G_{ii}}{\left(\sum_{j \neq i} P_j G_{ij}\right) + n_i} \geq \gamma \iff$$

$$\frac{P_i \cdot D \cdot d_{ii}^{-a}}{\left(\sum_{j \neq i} P_j \cdot D \cdot d_{ij}^{-a}\right) + n_i} \geq \frac{E_s}{N_0} \cdot \frac{R_b}{k \cdot BW}$$  \hspace{1cm} (4.6)

where the white Gaussian noise is denoted by $n_i$, $k=\log_2 M$ is the number of bits per symbol, the path loss from transmitter $j$ to receiver $i$ is $G_{ij} = D \cdot d_{ij}^{-a}$, where $a$ is the path loss exponent and has typical values from 2 to 5 and $D = (\lambda/4\pi)^2$. The transmission bit rate is given by $R_b = k \cdot R_s$ and the values for the $E_s/N_0$ can be found from the Symbol Error Rate (SER) or BER requirements (see Chapter 2 where all the network model details are described).

From the analysis presented in Section 4.2.1 for the interference calculation, Equation 4.6, assuming that all transmitters in the area operate with transmission power $P$ and that the interference power is much more greater than the background noise ($I >> n$) we see that the “interference limited range” $d_{ILR}$, as defined above, is the distance that satisfies the equality in (4.6). Therefore:
Interference Limited Communication Range

\[ d_{ILR} = \sqrt{\frac{P_i \cdot D}{\sum_{j \neq i} P_j \cdot D \cdot d_{(i,j)}^{-a}} + n_i} \]

\[ d_{ILR} \approx \sqrt{\frac{P \cdot D}{\sum_{j \neq i} P \cdot D \cdot d_{(i,j)}^{-a}}} \]

\[ d_{ILR} = \sqrt{\frac{P D}{E_s R_b N_0 k \cdot BW}} \]

\[ d_{ILR} = \sqrt{\frac{1}{E_s R_b N_0 k \cdot BW}} \]

(4.7)

Again we distinguish two cases:

for \( a = 2 \)

\[ d_{ILR} = \sqrt{\frac{1}{R_b N_t}} \cdot \frac{k \cdot BW \cdot (r_{max}^2 - r_{min}^2)}{2 E_s N_0} \cdot \left( \ln r_{max} - \ln r_{min} \right) \]

(4.8)

for \( a > 2 \)

\[ d_{ILR} = \sqrt{\frac{1}{R_b N_t}} \cdot \left( a - 2 \right) \cdot \frac{k \cdot BW \cdot (r_{max}^2 - r_{min}^2)}{2 E_s N_0} \cdot \left( \frac{1}{r_{min}^{a-2}} - \frac{1}{r_{max}^{a-2}} \right) \]

(4.9)
In Figure 4.5 we see that the theoretical value of the Interference Limited communication range is a decreasing function of the number of interfering nodes and asymptotically falls to zero. When we have a higher value for the path loss exponent \(a>2\) we see that we obtain lower values for the \(d_{ILR}\) when the number of transmitting nodes is relatively small. As \(N_t\) increases, the rate of convergence of \(d_{ILR}\) to zero is: \(\frac{1}{2}\) for \(a=2\) and \(\frac{1}{a}\) for \(a > 2\).

4.3.2. Wireless System with Shannon capacity links

If the receiver at the center of the circular area is part of a wireless network with Shannon capacity links, then \(R_b = BW \cdot \log_2(1 + \gamma)\) and thus the SINR threshold \(\gamma\), for rate \(R_b\) is equal to \(\gamma = \frac{R_b}{2BW} - 1\). Therefore the criterion for successful communication is:
Interference Limited Communication Range

\[
\frac{P_i G_{ii}}{\left( \sum_{j \neq i} P_j G_{ij} \right) + n_i} \geq 2^{\frac{R_p}{BW}} - 1 \quad (4.7)
\]

Following the analysis presented before, and taking into account the interference calculations (Section 4.2.1) and the criterion for successful communication (Equation 4.7), the interference limited communication region is given by:

\[
d_{\text{ILR}} = a \sqrt{\frac{1}{2^{\frac{R_p}{BW}} - 1}} \cdot \frac{P_i \cdot D}{\left( \sum_{j \neq i} P_j \cdot D \cdot d_{(ij)}^{-a} \right) + n_i} \approx a \sqrt{\frac{1}{2^{\frac{R_p}{BW}} - 1}} \cdot \frac{P \cdot D}{\left( \sum_{j \neq i} P \cdot D \cdot d_{(ij)}^{-a} \right)} \Rightarrow
\]

\[
d_{\text{ILR}} = \frac{a}{\sqrt{2^{\frac{R_p}{BW}} - 1}} \cdot \frac{PD}{I} \quad (4.10)
\]

We have two cases:

for \( a = 2 \)

\[
d_{\text{ILR}} = \sqrt{\frac{r_{\text{max}}^2 - r_{\text{min}}^2}{2(\ln r_{\text{max}} - \ln r_{\text{min}})}} \cdot \frac{1}{\sqrt{2^{\frac{R_p}{BW}} - 1}} \cdot \frac{1}{N_t} \quad \Rightarrow (4.11)
\]

for \( a > 2 \)

\[
d_{\text{ILR}} = a \sqrt{\frac{1}{2^{\frac{R_p}{BW}} - 1}} \cdot \frac{1}{N_t} \cdot \frac{1}{\sqrt{2(\ln r_{\text{max}} - \ln r_{\text{min}})}} \cdot \frac{(a - 2)(r_{\text{max}}^2 - r_{\text{min}}^2)}{2 \left( \frac{1}{r_{\text{min}}^{a-2}} - \frac{1}{r_{\text{max}}^{a-2}} \right)} \quad (4.12)
\]
Figure 4.6: Interference Limited Communication Range for Shannon capacity link.

In Figure 4.6 we see that the $d_{ILR}$ has the same behavior as before (Figure 4.5) but its values are much higher (as is expected because of the much better performance of the link). For small number of transmitters the $d_{ILR}$ with $a = 2$ is larger than the $d_{ILR}$ with $a > 2$. The rate of convergence to zero is the same as before.

4.4. Simulation Results

In order to validate our theoretical results, for the computation of the interference values and the interference limited communication range, we contacted extensive Monte Carlo simulations [Robert 2004] using the network model presented above.

4.4.1. Interference measurements

In Figure 4.7 we see the results obtained from our simulation compared with the theoretical results as presented in Section 4.2. We use Equations 4.3 and 4.4 in order to calculate the theoretical interference values when the number of the interfering nodes increases from 1 to 100 and nodes are randomly placed (using uniform distribution) inside
the circular area bounded by the circles with \( r_{\text{min}} \) (with values from 1 to 10) and \( r_{\text{max}} = 100 \) meters. The receiver of interest is positioned at the center of the area where the highest amount of interference is exhibited. The transmission power is assumed the same for all transmitting nodes and is equal to 1mW. In the simulations we calculate the distance of each transmitting node from the center of the networking area where the receiver of interest is located. Path losses due to fading and shadowing are not considered. The path loss exponent \(-\alpha\) varies from 2 (free space) to 4. The channel bandwidth is assumed constant and equal to \(10^7\)Hz and all transmitting nodes employ a B-PSK modulation scheme and thus \(k\) in Equations 4.8 and 4.9 is equal to 1.

\[ a = 2 \]

\[ a = 4 \]
The results from the proposed theoretical model are in a very good agreement with the values of the mean interference levels obtained by simulations. Figure 4.7(a) shows the effect of the path loss exponent on interference and Figure 4.7(b) shows the effect of $r_{\text{min}}$ (the distance around the receiver where a transmitter cannot be placed, i.e. the exclusion region). As expected a higher value for the path loss exponent results in lower levels of interference, and a larger $r_{\text{min}}$ leads to lower interference.

In Figure 4.8 we present the mean interference levels using simulations, for path loss exponent equal to 2 and 4 (plot (a) and plot (b) respectively) when we relax our assumption on the receiver’s position and assume that it is placed at distance $x$ from the center of the networking area, where the offset $x$ is $x \in [0, r_{\text{max}}]$. When a receiver is close to the borders of the networking area the interference level decreases. This is only natural,
because the number of the nearby interfering transmitters decreases near the borders, whereas the distance of many transmitters becomes larger than $r_{\text{max}}$ and thus their interference diminishes. The difference between the interference level at the center of the networking area (calculated with Equations 4.3 and 4.4) and at a quite large offset from the center is remarkably small as is evident from Figure 4.8. In fact the approximation is quite good with even a 60% or 70% (for $a=2$) and 90% (for $a = 4$) offset from the center of the circle. The interference level on the border of the circular area (100% offset) is approximately half of the level at the center. Therefore we can assume that, for the largest part of the networking area, the interference level as given by Equations 4.3 and 4.4 is a quite accurate approximation. The same holds true for the values of the $d_{\text{ILR}}$ (Equations 4.8 and 4.9).

If more accuracy is desired it is possible to calculate the interference at any offset $x$ using the method described in [Renato 2007], but in this case a much more complicated analysis would be required.
In this section we present our simulation results for the $d_{ILR}$ which is the maximum distance from which a transmitter can successfully send data to the centrally positioned receiver with a specific rate.

For each simulation experiment we randomly place the transmitting nodes, using a uniform distribution, around the receiver located in the center of the networking area, calculate the exhibited interference levels at the receiver and using formula (4.7) or (4.8) we find a
specific $d_{IL,R}$ value. From all the values collected we calculate the mean $d_{IL,R}$ and the confidence intervals for various numbers of interfering nodes. This case corresponds to a situation where the receiver measures each time the interference level and calculates the value of $d_{IL,R}$.

We compare our simulation results with the results from calculations based on the theoretical analysis presented in Section 4.3. In Figure 4.9 we present the mean interference limited communication range with the confidence intervals (standard deviation) versus the number of interfering nodes for path loss exponent equal to 2 and 4 in plots (a) and (b) respectively. We observe (Figure 4.9) that the theoretical calculations give a lower value but are within the deviation interval of the simulation results, for $a = 2$, plot (a), but not for $a = 4$, plot (b). The theoretical and simulation results are in better agreement as the number of interfering nodes increases.
The reason of this discrepancy lies in the way we use the interference measurements to calculate $d_{ILR}$. A different approach for measuring the value of $d_{ILR}$ is to first measure the mean interference level, repeating the experiment (random placement of transmitting nodes) a large number of times, and then using formulas (4.7) or (4.8) compute the $d_{ILR}$ using this mean value of interference. In this case as we see in Figure (4.10) the results are in excellent agreement with the ones found from the theoretical calculations (formulas (4.8) and (4.9)). This corresponds to a situation where a receiving node monitors the varying (due to changes in the network) interference for a period of time and calculates the mean interference levels in order to find the $d_{ILR}$.
Interference Limited Communication Range

In the next set of simulations the procedure described at the beginning of this section 4.4.2 is employed (that is, we measure the value of the $d_{ILR}$ for each placement of the transmitting nodes and then calculate its mean value).

Changing the distance $r_{\text{min}}$ (exclusion region) the interference limited communication range is not significantly affected, as shown in Figure 4.10.
As was noted before, for a higher path loss exponent there is a discrepancy between the simulation and calculation results for the $d_{ILR}$, which is reduced as the number of interfering nodes increases. This agrees with the results presented in [Renato 2007] where it is noted that for small number of interfering nodes the theoretical and simulation results don’t seem to converge. In Figure 4.12 we see that for path loss exponent $a$ equal to 4 the calculated $d_{ILR}$ values are smaller than those for $a$ equal to 2, where as the opposite happens for the simulated values.
In Figure 4.13 we compare the calculated values of the mean interference limited communication region and the results from our simulations, for path loss exponent equal to 2 and 4 (plot (a) and plot (b) respectively), when we relax our assumption on the receiver’s position and assume a specific offset $x$ from the center of the networking area, where $x \in [0, r_{max}]$, as we have done previously for the interference study. We can see that when the receiving node moves to the border of the networking area the value of the $d_{ILR}$ is increasing (since the interference is decreasing) but it remains very close to its value at the center for a 60% or 70% (for $a = 2$) and for a 80% or 90% (for $a = 4$) offset from the center of the circle. The calculated values (black line) are always smaller than the simulation results; that is they correspond to a worst case scenario.
Figure 4.13: Mean $d_{ILR}$ values calculated (black line) vs. simulation results (light blue lines) for path loss exponent $\alpha$ equal to 2, plot (a), and equal to 4, plot (b), for different positions of the receiving node (r_{max}=100 meters, and r_{min}=2 meters).
4.5. Conclusion

In this chapter we presented our analysis on interference calculation for a receiving node placed in the vicinity of the center of the networking area when interfering transmitters are randomly placed around it using a uniform distribution and when they are placed on a regular lattice. We found that the theoretical analysis gives a very good approximation of the mean interference found through simulations. We extended previous works by introducing the interference limited communication range \(d_{\text{ILR}}\) as the critical region around a receiver within which a transmitter, despite the presence of the other interfering nodes, can successfully send its data to the receiver of interest with a specific rate. We verified our results with simulations and shown that when we relax our assumption on the receiver’s position and assume a specific offset \(x\) from the center of the networking area the values of the \(d_{\text{ILR}}\) remain almost constant even for 60\% or 70\% offset from the center of the circle.
CHAPTER 5

CROSS LAYER WIRELESS MULTICAST

5. Cross Layer Wireless Multicast

A wireless network is typically formed by randomly deployed nodes and is inherently broadcast in its nature; a transmission can be potentially received by all nodes within the transmission range of a sender. Multicast over wireless networks is an efficient mechanism for point to multi-point communication using shared network resources. Multicast communication is important especially due to the widespread deployment of wireless networks, fast-improving capabilities of mobile devices and the fast deployed services provided to wireless users.

The problem of multicast is challenging because it enables efficient communication not only between nodes but between groups of nodes and at the same time it maximizes spectrum usage and reduces power consumption (a single transmission may result in transferring data packets to more than one receiver). Multicast wireless applications include: military command and control, distance education, intelligent transportation systems, e-commerce, mobile auctions, and other.

Multicast communications have been supported for many years by the wired networks. Existing multicast protocols can be extended to wireless networks; however, there are many difficulties and drawbacks that must be taken into account: limited energy and bandwidth, mobility of nodes, loss of packets and inefficient routing, and many advantages due to the broadcast nature of the wireless channel. In a wireless network, every transmission can be potentially received by all nodes that lie within a node’s communication range and thus interference (which affects the communication range) plays an important role for successful multicast communication. Similar to the unicast paradigm,
Cross Layer Wireless Multicast

the medium access layer should be able to adjust transmission rate and/or power to adapt to the amount of interference exhibited at each receiver. A medium access mechanism can schedule transmissions from multicast or point-to-point transmitters so that interference does not affect each other or allow simultaneous operation under the assumption that interference from all operating transmitters behaves like white Gaussian noise. In this last case, we have seen in Chapter (4) that transmission rate adaptation can help control the communication range in a wireless environment with interference coming from the transmissions of many transmitters.

In this section we study the multicast throughput (or multicast aggregate rate) of a multicast group which is the product of the number of the successful multicast receptions times the transmission rate. A successful reception is one where the SINR criterion (section 2.4) is satisfied given the simultaneous interfering transmissions, the transmission power, and the surrounding noise levels.

We will show that for specific network geometries and specific multicast link characteristics, the multicast throughput (as defined above) does not depend on the transmission rate when the path loss exponent is equal to two. Changing the transmission rate in this case modifies only the number of nodes that receive successfully the multicast transmission and not the total amount of transferred data. When the path loss exponent is greater than two the multicast throughput increases with the transmission rate. In both cases, as the transmission rate increases, fewer receivers, in the multicast group, will be able to correctly receive the message. In addition we show that in the theoretical case where the link rates are determined by the Shannon capacity theorem, we can find a transmission rate which maximizes the multicast throughput.

For a simple multicast network consisting of two transmitters and their corresponding groups of receivers, employing BPSK modulation, we provide the conditions under which the multicast throughput is maximized. Our results indicate that there is a value for the
transmission rate, for which the multicast throughput is maximized. This transmission rate depends mainly on the induced interference; by moving the transmitting multicast nodes further away the multicast throughput is increased as expected. When the receiving nodes are randomly placed within a circular networking area using a uniform distribution, the percentage of nodes that can successfully receive a multicast transmission, for maximum multicast rate, is found to be close to 30%-35%. Similar results are given when the multicast links operate with their Shannon capacity.

5.1. Related Work

Gupta and Kumar [Gupta 2000] studied the asymptotic unicast capacity of random multi-hop wireless networks for two different models. Grossglauser and Tse [Grossglauser 2001] showed that mobility can improve the unicast capacity for arbitrary large delay. The broadcast capacity of an arbitrary network has been studied in [Keshavarz 2006] showing that the capacity of a given network is $\Theta(W)$ for single source broadcast and the achievable broadcast capacity per node is only $\Theta\left(\frac{W}{N}\right)$ if each of the $N$ nodes will serve as source node. The upper bound $\Theta(W)$ on broadcast capacity holds since each node can receive at most $W$ bits/sec. The capacity $\Theta(W)$ is achieved by constructing a connected dominating set in which we can schedule every node to transmit at least once in constant time slots. These capacity bounds also apply to random networks. The work in [Jacquet 2005] studied the scaling properties of multicast for random wireless networks. They showed that the maximum rate at which a node can transmit multicast data is $O\left(\frac{W}{\sqrt{kN\log N}}\right)$. Kyasanur and Vaidya [Kyasanur 2005] studied the capacity region on a given multi-hop multi-radio multi-channel wireless network when there are a number of channels available and each node has a number of wireless interfaces. In [Alicherry 2005, Kodialam 2003] it has shown how to satisfy certain traffic demands from wireless nodes by jointly routing, link scheduling, and channel assignment under certain wireless interference models. In [Pantelidou 2009] the
problem of scheduling is addressed through rate and power control for multicast traffic. The objective is to maximize the total user utility in terms of the average received rates considering that accurate knowledge of the wireless channel conditions may not be always available. An on-line, stationary scheduling and rate control policy is introduced. The authors establish its optimality among all policies with the same knowledge regarding the current channel state through stochastic approximation arguments.

5.2. System Model

We consider first a wireless telecommunication system where network nodes use a specific modulation scheme with specific BER requirements for successful communication and a constant bandwidth, and that the network nodes do not employ any coding or cooperation among them, similarly to the model discussed in Chapter 2. We then consider the case when the nodes communicate with rates close to their Shannon capacity. We assume that in the center of the networking area we have a multicast transmitter with $N_r$ receivers placed randomly around the transmitter using a uniform distribution. We also assume that there are $N_t$ other transmitters deployed randomly in the same area, again using a uniform distribution.
Similarly with our previous analysis, presented in chapter 2, we use the Signal-to-Interference-plus-Noise ratio (SINR) to describe the ratio of the signal of interest for each simultaneous transmission to the additive white Gaussian noise and the interference caused by the other multicast or unicast transmissions. A receiver is able to successfully decode a multicast transmission if condition (5.1), the SINR criterion (2.1), is true for a given threshold value $\gamma_i$ for a positive vector $P = (P_1, \ldots, P_N)$ of transmission powers.
The left side of inequality (5.1) is the measured SINR for each one of the $i$ receivers. The path loss from transmitter $j$ to receiver $i$ is $G_{ij} = L \cdot d_{ij}^{-a}$, where $a$ is the path loss exponent and has typical values from 2 to 5, $d_{ij}$ is the distance between nodes $i$ and $j$ and $L$ is the propagation loss at 1 meter. White Gaussian noise is denoted as $n_i$. The value of the SINR threshold $\gamma_i$ depends (as we have seen in Chapter 2) on the design parameters and properties of the telecommunication system, such as the target BER, the modulation scheme and the desired transmission bit-rate for each link.

5.3. Multicast throughput of a multicast group

The multicast throughput of a multicast group is the product of the number of the successful multicast receptions and the transmission rate. The condition that must be satisfied for a successful reception is given by the SINR criterion in (5.1).

In Figure 5.1 we see an example where we have a multicast transmitter ($T_{i}$) placed in the center of the circular networking area, a number of interfering transmitters (light blue triangles) and the assigned set of receivers (yellow circles) for ($T_{i}$). We assume that all the transmitters including ($T_{i}$) are transmitting simultaneously with equal powers $P$. We are interested in finding the multicast throughput of the multicast transmitter ($T_{i}$). To this end we need to calculate the number of successful receptions when the multicast transmitter transmits with a specific rate. This transmission rate determines the communication range and therefore the number of the receivers with successful reception, as is shown in Figure 5.2 where the “un-successful” receivers are depicted in grey.
From our analysis, in Chapter 5, if the receiving nodes around each transmitter are placed randomly using a uniform distribution, the *interference limited communication range* $d_{ILR}$ describes the critical region around a receiver within in which if a transmitter is located a successfully data transmission may occur. In Figure 5.2 we can see this region as the light grey circle around each receiver.

From our analysis, in Chapter 4, if the transmitting nodes around a centrally positioned receiver are placed randomly using a uniform distribution, then its reception range can be determined by the *interference limited communication range* $d_{ILR}$ which can be calculated using Equations 4.8 or 4.9 and depends on the transmission rate. Moreover we have shown, through simulations (Figure 4.13), that throughout the networking area (with the exception of the area close to the perimeter), the nodes have an almost constant *interference limited communication range* $d_{ILR}$ which can be calculated using also Equations 4.8 or 4.9. In Figure 5.2 we show this region as a dotted circle around receiver Rx₁.

Therefore we can argue that the multicast transmitter (Tr₁) will be able to communicate successfully with one of the receivers of its multicast group if it is located within the interference limited communication region of this receiver (as shown in Figure (5.2) for Rx₁). This condition determines the “multicast transmission range”, shown with the thick line around transmitter T₁, within which, given the other interfering transmissions, the SINR criterion is satisfied and all nodes inside are able to receive the multicast transmission.

The multicast transmission range is affected mainly by the characteristics of the underlying telecommunication system: the number of the surrounding interfering transmitters, the power level and most importantly on the selected bit rate for operation.
We have defined the multicast throughput to be equal to product of the selected rate ($R_{T_m}$) for the multicast transmission and the number of receivers ($Q(d)$) for which the SINR criterion is satisfied, thus:

$$R^m_b = Q(d) \cdot R_{T_m}$$  \hspace{1cm} (5.2)
Since we assumed that the receivers of the multicast group are randomly placed with a uniform distribution around their transmitter, we have:

\[ Q(d) = \frac{\pi(d^2)}{\pi(r_{max}^2)} N_r \]

(5.3)

where \( N_r \) is the number of receivers of the multicast group and \( d \) is the multicast transmission range which (as we argued above) is equal to the \( d_{ILR} \) and is given by the formulas found in section 4.3 and repeated in equations 5.4 and 5.5 under the assumption that there are \( N_t \) surrounding transmitters operating simultaneously. That is:

\[
d = d_{ILR} = \sqrt{\frac{1}{R_{Tm} N_t}} \cdot \sqrt{\frac{k \cdot BW}{2 \frac{E_s}{N_o}}} \cdot \frac{(r_{max}^2 - r_{min}^2)}{(ln r_{max} - ln r_{min})} \quad \text{(for } a = 2) \]  

(5.4)

\[
d = d_{ILR} = a \sqrt{\frac{1}{R_{Tm} N_t}} \cdot \sqrt{\frac{k \cdot BW}{2 \frac{E_s}{N_o}}} \cdot \frac{(r_{max}^2 - r_{min}^2)}{r_{min}^{a-2} - r_{max}^{a-2}} \quad \text{(for } a > 2) \]  

(5.5)

When the multicast transmitter selects a lower rate then it will have higher values for \( d_{ILR} \) and more receivers will be able to successfully decode the multicast transmission satisfying the SINR requirements. Thus the natural question arises: is there an optimum transmission rate that maximizes the multicast throughput?

For a path loss exponent equal to 2 we have:

\[ R_t^m = N_r \pi d_{ILR}^2 R_{Tm} = \]
The multicast throughput, as we can see from Equation 5.6, does not depend on the transmission rate for \( a = 2 \). The transmission rate determines only the multicast transmission range and therefore the number of the receivers with successful reception.

For a path loss exponent greater than 2 we have:

\[
R_{m}^B = N_r \frac{\pi d_{IR}^2}{\pi r_{max}^2} R_{T_m} = \frac{k \cdot BW \cdot (r_{max}^2 - r_{min}^2)}{2 E_s N_0 \cdot (ln r_{max} - ln r_{min})} \cdot \frac{1}{R_{T_m} N_t} \Rightarrow
\]

\[
R_{B}^m = \frac{k \cdot BW \cdot (r_{max}^2 - r_{min}^2)}{2 E_s N_0 (ln r_{max} - ln r_{min}) r_{max}^2} \frac{N_r}{N_t}
\]  

(5.6)
\[
\begin{align*}
A &= \left( \frac{a}{(a-2)} \cdot \frac{k \cdot \text{BW} \cdot (r_{\text{max}}^2 - r_{\text{min}}^2)}{2 N_0 \left( \frac{1}{r_{\text{min}}^{a-2}} - \frac{1}{r_{\text{max}}^{a-2}} \right)} \right)^2 \cdot \frac{N_r}{N_t \bar{a}} \\
R_b &= A \cdot \left( \frac{k \cdot \text{BW}}{2 N_0} \right)^{\frac{2}{a}} \cdot R_{T_m}^{1 - \frac{2}{a}} \\
\end{align*}
\]

Where

In this case, the multicast throughput increases with the transmission rate (which can take values up to the systems’ bandwidth BW since \( R_{T_m} \leq \text{BW} \)). In Figure 5.3 below we show the results of simulations in the case of a multicast group of 40 receivers operating in an environment with 10 interfering transmitters. It is clear that the theoretical model is quite accurate in determining the dependence of the multicast throughput on the transmission rate.
If we assume the existence of Shannon capacity links for the multicast transmission we can use Equations 4.11 and 4.1.2 to calculate the $d_{ILR}$ and we get:

- For path loss exponent equal to 2:

$$R_t^n = N_r \frac{\pi d_{ILR}^2}{\pi r_{max}^2} \quad R_{Tm} =$$

$$= R_{Tm} N_r \frac{1}{r_{max}^2} \left( \frac{r_{max}^2 - r_{min}^2}{2(ln r_{max} - ln r_{min})} \cdot \frac{1}{\sqrt{\frac{R_{Tm}}{2BW}} - 1} \cdot \frac{1}{N_t} \right)^2$$

Figure 5.3: 10 transmitters and 40 receivers placed randomly following a uniform distribution.
Cross Layer Wireless Multicast

\[ R_{T_m} = N_r \frac{r_{\text{max}}^2 - r_{\text{min}}^2}{2(\ln r_{\text{max}} - \ln r_{\text{min}})} \frac{1}{r_{\text{max}}} \frac{1}{N_t} \Rightarrow \]

\[ R_b^m = \frac{r_{\text{max}}^2 - r_{\text{min}}^2}{2(\ln r_{\text{max}} - \ln r_{\text{min}}) r_{\text{max}}^2} \frac{R_{T_m}}{N_r} \frac{N_r}{2BW - 1} \]

(5.8)

It is easy to see that for \( a = 2 \) the \( R_b^m \) has a maximum value for \( R_{T_m} = 0 \). This maximum value is:

\[ R_{b_{\text{max}}}^m = \frac{r_{\text{max}}^2 - r_{\text{min}}^2}{2(\ln r_{\text{max}} - \ln r_{\text{min}}) r_{\text{max}}^2} \frac{N_r \cdot BW}{N_t \cdot \ln 2} \]

For path loss exponent greater than 2 we have:

\[ R_b^m = N_r \frac{\pi d_{LR}^2}{\pi r_{\text{max}}^2} R_{T_m} = \]

\[ = N_r \frac{1}{r_{\text{max}}^2} \frac{1}{N_t} \frac{(a - 2)(r_{\text{max}}^2 - r_{\text{min}}^2)}{2 \left( \frac{1}{r_{\text{min}}^{a-2}} - \frac{1}{r_{\text{max}}^{a-2}} \right) ^2} R_{T_m} = \]

\[ = N_r \frac{1}{r_{\text{max}}^2} \frac{1}{N_t \cdot \frac{2}{a}} \left( \frac{R_{T_m}}{2BW - 1} \right) \]

\[ \Rightarrow \]
Cross Layer Wireless Multicast

\[ R_b^m = A \cdot \frac{R_{Tm}}{\left( \frac{R_{Tm}}{2 BW - 1} \right)^{\frac{1}{a}}} \]  \quad (5.9)

where \( A = \frac{\left( a - 2 \right) \cdot \left( r_{\text{max}}^2 - r_{\text{min}}^2 \right)}{r_{\text{max}}^2 - r_{\text{min}}^2} \cdot \frac{N_r}{N_t^a} \).

In this case \( R_b^m \) has a maximum value for \( R_{Tm} = R^* \) which is the solution of the equation

\[ \frac{R^*}{2 BW \left( \alpha - 2 \ln(2BW) \right)} = \alpha \]

The calculated values of the \( R^* \), for different path loss exponent \( a \), are very close to the simulation values shown in Figure 5.4.
5.4. The case of two multicast transmitters

In the previous section we have seen the analysis on the multicast throughput of a multicast transmitter when there are many other interfering transmitters in the same area. In this section we study, through simulation experiments, the simple case of two multicast transmitters placed at a specific distance from each other. We assume a circular networking area of 100 meters radius ($r_{\text{max}}$). All receiving nodes are placed randomly with uniform distribution around both transmitters. The multicast transmitters are placed symmetrically with respect to the center of the networking area. For each multicast transmission we assume first real-world links with a BPSK modulation scheme, $10^{-6}$ target Bit Error Rate, constant bandwidth and no cooperation among the nodes. Then we examine the case where we have Shannon capacity links.
In Figure 5.5 the light colors (green and yellow) indicate the area where the multicast receivers of Tx2 (placed in position [20, 0] and [40, 0] respectively) can successfully receive its multicast transmission. The results look very much alike and the only difference is that, for the same transmission rate, when the distance between the transmitters increases the area of successful multicast reception becomes larger.

In Figure 5.6 we present the mean normalized multicast throughput (log-scale), versus the normalized transmission rate. The transmitters are placed 40 meters apart of each other and the receivers are randomly placed with a uniform distribution around them. In the first
plot each line corresponds to a different number of receivers (ranging from 1 to 100 receivers) for each multicast transmitter. The black vertical line shows the transmission rate of the multicast transmitter for which the multicast throughput is maximized. As we can see this transmission rate is independent of the number of the receivers. It depends only on the placement of the transmitters. The second plot shows the percentage of nodes that satisfy the SINR criterion and successfully receive the multicast data. The percentage of the receiving nodes for which the multicast rate is maximized is the same and independent of their number; the corresponding curves are overlapping.

![Figure 5.6: Two Transmitters placed at distance of 40m. Receivers randomly placed around them. BPSK links](image)

Similar results, Figure 5.7, are obtained when we assume that we have Shannon capacity links. The behavior is the same. The only difference is that the maximum multicast
throughput is much higher, and the percentage of successful receptions when the maximum throughput is achieved is lower, and the corresponding transmission rate higher.

In Figure 5.8 we see the maximum multicast throughput that is increasing almost linearly when the number of receiving nodes grows. The percentage of the receivers able to successfully receive a multicast transmission and the transmission rate that maximizes the multicast throughput is almost constant, as was noted before.
By changing the distance between the two transmitters we see that the transmission rate, for which the multicast throughput is maximized, is changing. If we increase the distance between the two transmitters the interference will be reduced and the maximum multicast throughput is achieved when the transmitters operate with increased rate (see Figure 5.9). In Figure 5.10 we show the simulation results for the same experimental setup, for the case of Shannon capacity links.
Figure 5.9: Two Transmitters placed at distances (20, 40, 80 meters) apart. 100 Receivers randomly placed. Multicast throughput with BPSK links.
Figure 5.10: Two Transmitters placed at distances (20, 40 and 80 meters) apart. 100 Receivers randomly placed. Multicast throughput using Shannon capacity links
5.5. Conclusion

In this section we studied the multicast throughput of a multicast group (the sum of the rates of successful receptions) operating in an environment with many other interfering transmitters. We have shown that the multicast throughput does not depend on the transmission rate when the path loss exponent is equal to 2. In this case a lower transmission rate allows more receivers to be able to receive the multicast transmission but the multicast throughput is constant. For a path loss exponent greater than two the multicast throughput increases with the transmission rate.

For the simple case of two multicast transmitters our simulation results suggest that for a given placement of the transmitters, there exist a specific transmission rate and a corresponding percentage of successful receivers that maximize the multicast throughput. Placing the transmitters further apart reduces the mutual interference in the multicast group and increases the multicast throughput, as expected.
CONCLUSIONS & DISCUSSION

6. Conclusions & Discussion

6.1. Conclusions

In this dissertation we presented an analysis of how cross layer techniques can be used in the study of wireless ad hoc networks. When multiple wireless links share a common medium the effect of interference, a central phenomenon in wireless communication, is a dominant limiting factor for network performance. Most state-of-the-art systems deal with interference either by orthogonalizing the communication links in time or frequency or by allowing links to operate simultaneously, sharing the same wireless spectrum, and treat interference as noise. Both cases might be sub-optimal for network performance.

Our objective is to combine information from multiple layers and provide an optimal solution, in terms of performance, by taking into account specific service requirements and tailor the protocol stack accordingly in order to achieve network efficiency. In our study we focus on finding how adapting various parameters of the telecommunication system we can optimally utilize network resources, for all competing transmissions, by allowing concurrent transmissions, minimizing interference, enhancing throughput and maximizing individual link data rates. In our thesis we present three important topics in the context of cross layer rate and power adaptation for a simple wireless ad-hoc network which uses a specific modulation scheme, a specific target BER and a constant bandwidth.

First we studied the transmission rate regions, given the degree of interference and individual maximum power constrains and demonstrated the conditions for maximizing the system’s aggregate rate. We identified critical points, in the rate regions, where we can
Conclusions & Discussion

have higher aggregate rates, in practical band limited interfering channels, at the expense of higher power. We also established criteria under which simultaneous link operation outperforms timesharing solutions and showed that the relation between maximum achieved rate and power consumption in case of light and strong interference. For light interference this relation is found to be almost linear but there is need for disproportionally high total power consumption to compensate strong interference. In the low power regime we found the power levels where simultaneous link operation outperforms timesharing solutions. Finally, in this section, we gave the condition on the maximum individual transmission power for switching to the next higher order of modulation to achieve higher aggregate rates.

Next we presented an analysis for the estimation of the interference at a receiving node which is placed in the vicinity of the center of a circular networking area and interfering transmitters are randomly placed around it using a uniform distribution or placed on a regular lattice. It is shown that the theoretical calculation of the interference is in a very good agreement with the simulation results. We introduced the interference limited communication range ($d_{\text{ILR}}$) to be the critical region around a receiver within which a transmitter, despite the presence of the other interfering nodes, can successfully send its data to the receiver of interest. We derived a formula for calculating $d_{\text{ILR}}$ for various values of the path loss exponent and verified its accuracy through simulations and shown that even when we relax our assumption on the receiver's central position the values of the $d_{\text{ILR}}$ remain almost constant.

Finally, for a multicast group we studied the multicast throughput which is the product of the transmission rate and the number of the successful multicast receptions. We’ve shown for a specific network topology that the multicast throughput does not depend on the transmission rate when the path loss exponent is equal to two but it increases with the transmission rate when the path loss exponent is greater than two. By changing the
transmission rate we can control the number of receivers able to receive a multicast transmission. For the simple case of two multicast transmitters our results reveal that there exists a specific transmission rate and a corresponding percentage of successful receivers that maximizes the multicast throughput.

The future plan is to extend our work towards investigating a more general model and estimating the achievable rate regions for an interference channel when more than two links are present, when fading effects are considered in the wireless channel path loss model and when the nodes are able to use different types of modulation schemes. In this context we are planning to find the conditions that maximize the system’s aggregate rate and the criteria where simultaneous link operation outperforms timesharing solutions. We also plan to study the achievable rate regions and find the relation between the maximum achieved rate and the transmission power in these cases. We want to extend our results on the interference limited communication range around a receiver when nodes are able to use different modulation schemes and when nodes are placed using different distributions and find the relation of this critical region with the transmission rate and the path loss exponent and compare it with the results presented in this thesis. For the multicast case we plan to extend our results under imperfect knowledge of the channel state, where a transmission is successful with a given probability even when the SINR criterion is satisfied, when more than two multicast transmitters are present and when the transmitters can have common receivers and receiving nodes are assumed to have Multi-packet Reception Capabilities (MPR).

6.2. Discussion on Possible Application of our Presented Results on Cross Layer Scheduling in wireless ad-hoc networks

When wireless nodes share the same medium, interference is the most important limiting factor of network performance. Efficient scheduling can be used to minimize interference,
Conclusions & Discussion

avoid unsuccessful transmissions, retransmission of data packets and high energy consumption, improve transmission link rates, and enhance network performance.

In [Moscib 2006] topology control in the context of the physical Signal-to-Interference-plus-Noise-Ratio (SINR) model is studied focusing on how, and how fast, links can be activated over time, presenting theoretical upper bounds on the scheduling complexity of arbitrary topologies in wireless networks. The problem of allocating data rates and finding a stabilizing scheduling policy in a multi-hop wireless network is studied in [Lin 2004]. The proposed mechanism can fully utilize the capacity of the network, maintain fairness, and improve quality of service. This approach comes with a significant cost in computational power and scheduling time especially for finding these scheduling schemes that maximize the total weighted link capacity. Following this work the performance impact when imperfect scheduling is used is studied in [Lin 2006] showing that in many network configurations imperfect scheduling can perform closely enough to an optimal solution and at the same time reduce the computational overhead required. In [Pantelidou 2008] a joint scheduling and rate control policy is introduced identifying a set of transmitters that can access the wireless medium using specific transmission rates for static and time-varying wireless networks. Borbash and Ephremides [Borbash 2006] studied the problem of determining the minimal scheduling length that satisfies specific demands in a wireless network where links are allowed to be activated simultaneously given that Signal-To-Interference plus Noise Ratio (SINR). In this work it is proven that the problem of scheduling is at least as hard as the MAX-SIR-MATCHING problem, which is easier to describe and when the demands have a super increasing property the problem can be tractable. The problem of determining the minimum-length schedule that satisfies specific traffic demands is studied in [Kompella 2008]. A cross layer model is adopted using the Signal to Interference plus Noise ratio constrain considering dynamic power and rate control to generate feasible matching. Each such matching consists of a set of links that can be simultaneously active.
6.2.1. Scheduling for arbitrary topologies

The problem of scheduling for arbitrary topologies can be easily appreciated with the following example. In figure 6.1 we calculate the achievable aggregate rates of all the possible scheduling schemes on a network of 200 nodes randomly placed using a uniform distribution in a circular area of 100 meters. For the underlying telecommunication system we make the assumptions presented in Chapter 2. We calculate the aggregate rate, for two different types of modulation schemes BPSK and QPSK, for all the disjoint sets of links that can be formed. Links within each set are said to operate simultaneously, and each set is scheduled to operate in a time division manner with each other. When links are scheduled to operate in a simple time division fashion each link is activated alone and thus the maximum aggregate rate is bounded by the system’s bandwidth. We see that there is an optimum grouping of the links where the aggregate rate is maximized. Unfortunately after much extensive simulations we were not able to arrive at some rules of general applicability for determining the optimum scheduling. The optimum scheduling scheme, for maximizing the aggregate rate, depends on the network conditions the node topology, the surrounding interference, the transmission power levels, the path loss characteristics, and the requirements in transmission rate for each link.
To fully utilize network resources, efficient scheduling is required. Even though effective scheduling can be used to maximize the network’s throughput, it might have significant cost in computational power and scheduling latency for successfully scheduling all network links (worst case scenarios). The problem of minimum-length scheduling is thus of great importance in order to satisfy specific traffic demands [Borbash 2006].

We have seen that throughput enhancement and energy saving are two contradictory goals. Thus the problem of finding the performance tradeoffs becomes a difficult task in the design of a scheduling algorithm able to jointly optimize transmission rate and power. Previous works have given great attention to time division solutions (for their simplicity and the ability to improve throughput and provide fairness among users) and to power control in order to decrease the interference footprint for reliable communication. Scheduling in wireless networks is a task that involves solving a complex, difficult, and computationally expensive, for real time implementation, problem. It is critical for the optimal selection of a scheduling scheme to be able to deliver the best possible solution in reasonable time and with reasonable computational demands.
In this section we discuss on how our results on interference power, interference limited communication region calculations, and rate control can be used by a scheduling scheme in order to maximize network’s performance. In section 4.3 we studied the interference limited communication range $d_{ILR}$ to be the critical communication region around a receiver, given the number of surrounding interfering nodes, within which if a transmitter is present, a successful communication link can be formed. We propose to use $d_{ILR}$ as the criterion for cross layer scheduling. The interference limited communication range incorporates important characteristics and properties of the telecommunication system (transmission power, interference power, modulation scheme, target BER, SINR requirements, path loss characteristics, and most importantly the achievable transmission rate) that could be used for efficient scheduling. As we have described before the value of $d_{ILR}$ depends on a number of parameters: the number and the density of the surrounding interfering transmitters, the exclusion region, the interference power level, and most importantly on the selected rate of operation. Adapting the transmission rate of each one of the links we can change the $d_{ILR}$ accordingly making links to be feasible or infeasible given the surrounding interference, affecting the number of possible matchings in the network and adapting to specific scheduling needs. The problem of efficient scheduling, in terms of maximizing the network’s aggregate rate, depends on the selection of a transmission rate, possibly different for each link, that allows each receiving node to have such a $d_{ILR}$ value that ensures successful operation, and allows the scheduling scheme to form the minimum number of sets, where in each set the maximum possible number of links exist and is able to operate simultaneously, in order to maximize the aggregate transmission rate.

6.2.2. Multicast routing

In multicast communication a source node conveys information to the members of a multicast group. The objective of multicast routing, within a mobile ad hoc network, is to multicast data packets with acceptable Quality of Service (QoS) requirements, to maximize
spatial reuse efficiency (number of retransmissions required to deliver each generated data packet to all members of a multicast group), minimize energy dissipation, and maximize multicast rate. There are many multicast routing protocols designed for mobile ad hoc networks, and they can be categorized into two groups: (i) tree-based approaches and (ii) mesh-based approaches. Tree-based approaches create trees originating at the source and terminating at multicast group members with the objective of minimizing a cost function. Mesh-based multicasting is suited to highly dynamic topologies where more than one path between the source and multicast group members exist; even if one of the paths is broken, i.e. due to mobility the others are available to be used.

When wireless nodes transmit over the wireless channel, the signal arrives not only at the intended receiver, but also to all other receiving nodes in vicinity, hence the problem of interference. We have studied the properties in an interference wireless environment where multicast communication takes place in chapter 5. We have shown that the multicast rate depends on a variety of factors, such as the transmission power, the modulation scheme, the propagation environment and the transmission rate. We have also demonstrated that the transmission rate can be used in order to define the multicast range and the number of users that can successfully receive a multicast transmission when other interfering transmissions are present. A routing protocol able to exploit the dependency of the multicast rate and the number of users able to successfully decode a multicast transmission on the transmission rate, can use this information to find the optimum balance between keeping interference levels at a manageable level and to utilize the available bandwidth efficiently for finding these routes that maximize routing and network performance.
7. Bibliography


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