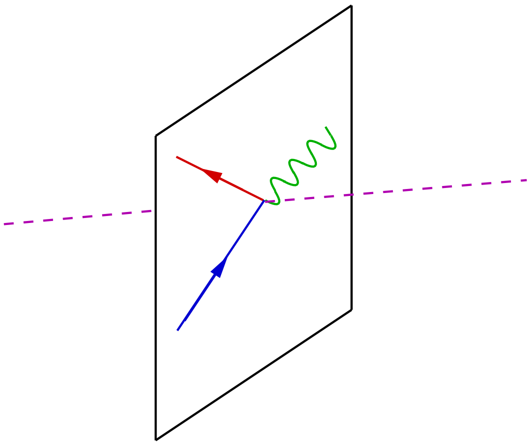
University of Crete
Department of Physics

# Cosmological evolution in brane-worlds with large transverse dimensions: Inflation and dark matter

A Doctoral Dissertation

G. Panotopoulos



Heraklion, Greece, April 2006

Grigoris Panotopoulos

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submitted to the Department of Physics, University of Crete
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# Cosmological evolution in brane-worlds with large transverse dimensions: Inflation and dark matter


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# Abstract

In the present work we discuss inflation, dark matter and cosmological evolution in the context of the brane-world scenario. Being string theory inspired, the brane-world models provide corrections to the General Relativity, which is considered to be the low-energy limit of string theory. We find that novel cosmologies are obtained, which potentially can provide answers to some of the longstanding problems of modern cosmology, such as the origin and nature of dark energy. At the same time the successes of the standard four-dimensional cosmology are preserved, and in some cases the treatment in the framework of brane cosmology is even more satisfactory.

# Chapter 1

# Introduction

The standard electroweak model is a mathematically-consistent renormalizable field theory which predicts or is consistent with all experimental facts. It successfully predicted the existence and form of the weak neutral current, the existence and masses of the $W$ and $Z$ bosons, and the charm quark, as necessitated by the GIM mechanism. The charged current weak interactions, as described by the generalized Fermi theory, were successfully incorporated, as was quantum electrodynamics. When combined with quantum chromodynamics for the strong interactions and general relativity for classical gravity, the standard model is almost certainly the approximately correct description of nature down to at least $10^{-16}$cm, with the possible exception of the Higgs sector. However, the theory has far too much arbitrariness to be the final story. For example, the minimal version of the model has 21 free parameters, assuming massless neutrinos and not counting electric charge ($Y$) assignments. Most physicists believe that this is just too much for the fundamental theory. The complications of the standard model can also be described in terms of a number of problems.

## 1.1 Gauge Problem

The standard model is a complicated direct product of three sub-groups, $SU(3) \times SU(2) \times U(1)$, with separate gauge couplings. There is no explanation for why only the electroweak part is chiral (parity-violating). Similarly, the standard model incorporates but does not explain another fundamental fact of nature: charge quantization, namely why all particles have charges which are multiples of $e/3$. This is important because it allows the electrical neutrality of atoms ($|q_p| = |q_e|$). Possible explanations include: grand unified theories, the existence of magnetic monopoles, and constraints from the absence or cancellation of anomalies.

## 1.2 Fermion Problem

All matter under ordinary terrestrial conditions can be constructed out of the fermions $(\nu_e, e^-, u, d)$ of the first family. Yet we know from laboratory studies that there are $\geq 3$ families: $(\nu_\mu, \mu^-, c, s)$ and $(\nu_\tau, \tau^-, t, b)$ are heavier copies of the first family with no obvious role in nature. The standard model gives no explanation for the existence of these heavier families and no prediction for their number. Furthermore, there is no explanation or prediction of the fermion masses, which vary over at least five orders of magnitude, or of the CKM mixings. There are many possible suggestions of new physics that might shed light on this, including composite fermions; family symmetries; radiative hierarchies, in which the fermion masses are generated at the loop-level, with the lighter families requiring more loops; and the topology of extra space-time dimensions, such as in superstring models. Despite all of these ideas there is no compelling model and none of these yields detailed predictions. The problem is just too complicated. Simple grand unified theories don't help very much with this, except for the prediction of $m_b$ in terms of $m_\tau$ in the simplest versions.

## 1.3 Higgs/hierarchy Problem

In the standard model one introduces an elementary Higgs field into the theory to generate masses for the $W$, $Z$, and fermions. For the model to be consistent the Higgs mass should not be too different from the $W$ mass, $M_H^2 = \mathcal{O}(M_W^2)$. If $M_H$ were to be larger than $M_W$ by many orders of magnitude there would be a hierarchy problem, and the Higgs self-interactions would be excessively strong. Combining theoretical arguments with laboratory limits one obtains $M_H \lesssim 1$ TeV.

However, there is a complication. The tree-level (bare) Higgs mass receives quadratically-divergent corrections from loop diagrams. One finds

$$M_H^2 = (M_H^2)_{\rm bare} + O(\lambda, g^2, h^2)\Lambda^2, \tag{1.1}$$

where $\Lambda$ is the next higher scale in the theory. If there were no higher scale one would simply interpret $\Lambda$ as an ultraviolet cutoff and take the view that $M_H$ is a measured parameter and that $(M_H)_{\rm bare}$ is not an observable. However, the theory is presumably embedded in some larger theory that cuts off the integral at the finite scale of the new physics. For example, if the next scale is gravity then $\Lambda$ is the Planck scale $M_P = G_N^{-1/2} \sim 10^{19}$ GeV. If there is a simple grand unified theory, one would expect $\Lambda$ to be of order the unification scale $M_X \sim 10^{14}$ GeV. Hence, the natural scale for $M_H$ is $O(\Lambda)$, which is much larger than the expected value. There must be a fine-tuned and apparently highly contrived cancelation between the bare value and the correction, to more than 30 decimal places in the case of gravity. If the cutoff is provided by a grand unified theory there is a separate hierarchy problem at tree-level. The tree-level couplings between the Higgs field and the superheavy fields lead to the expectation that $M_H$ is equal to the unification scale unless unnatural fine-tunings are employed.

One solution to this Higgs/hierarchy problem is the possibility that the $W$ and $Z$ bosons are composite. However, in this case one would apparently eliminate the successes of the $SU(2) \times U(1)$ gauge theory. Another approach is to eliminate elementary Higgs fields in favor of a dynamical mechanism in which they are replaced by bound states of fermions. Technicolor and composite Higgs models are in this category. The third possibility is supersymmetry, which prevents large renormalizations by enforcing cancellations between the various diagram contributions. However, most grand unified versions do not explain why $(M_W/M_X)^2$ is so small in the first place.

## 1.4 Strong $CP$ Problem

Another fine-tuning problem is the strong $CP$ problem. One can add an additional term $\frac{\Theta}{32\pi^2} g_s^2 F\tilde{F}$ to the QCD lagrangian which breaks $P$, $T$ and $CP$ symmetry. $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}/2$ is the dual field. This term, if present, would induce an electric dipole moment $d_N$ for the neutron. The rather stringent limits on the dipole moment lead to the upper bound $\Theta < 10^{-10}$. The question is, therefore, why is $\Theta$ so small? It is not sufficient to just say that it is zero because $CP$ violation in the weak interactions leads to a radiative correction or renormalization of $\Theta$ by $\mathcal{O}(10^{-3})$. Therefore, an apparently contrived fine-tuning is needed to cancel this correction against the bare value. Solutions include the possibility that $CP$ violation is not induced directly by phases in the Yukawa couplings, as is usually assumed in the standard model, but is somehow violated spontaneously. $\Theta$ then would be a calculable parameter induced at loop level, and it is possible to make $\Theta$ sufficiently small. However, such models lead to difficult phenomenological and cosmological problems. Alternately, $\Theta$ becomes unobservable if there is a massless $u$ quark. However, most phenomenological estimates are not consistent with $m_u = 0$. Another possibility is the Peccei-Quinn mechanism, in which an extra global $U(1)$ symmetry is imposed on the theory in such a way that $\Theta$ becomes a dynamical variable which is zero at the minimum of the potential. Such models imply the existence of very light pseudoscalar particles called axions. Laboratory, astrophysical, and cosmological constraints allow only the range $10^8 - 10^{12}$ GeV for the scale at which the above $U(1)$ symmetry is broken.

## 1.5 Graviton Problem

Gravity is not fundamentally unified with the other interactions in the standard model, although it is possible to graft on classical general relativity by hand. However, this is not a quantum theory, and there is no obvious way to generate one within the standard model context. In addition to the fact that gravity is not unified and not quantized there is another difficulty, namely the cosmological constant. The cosmological constant can be thought of as energy of the vacuum. The energy density induced by spontaneous symmetry breaking is some 50 orders of magnitude larger than the observational upper limit. This implies the necessity of severe fine-tuning between the generated and bare pieces, which do not have any a priori reason to be related. Possible solutions include Kaluza-Klein and supergravity theories. These unify gravity but do not solve the problem of quantum gravity or yield renormalizable theories of quantum gravity, nor do they provide any obvious solution to the cosmological constant problem. Superstring theories unify gravity and may yield finite theories of quantum gravity and all the other interactions. It is not clear whether or not they solve the cosmological constant problem.

## 1.6 Dark sector problem

On the other hand Cosmology, the science of the Universe, has its own Standard Model. It is the so-called Hot Big-Bang model. The expansion of the Universe, the Bing-Bang Nucleosynthesis and the Cosmic Microwave Background Radiation have established the theoretical framework of Cosmology (which is based essentially on the Theory of General Relativity and the Cosmological Principle, namely that the Universe is homogeneous and isotropic on large scales) into the Standard Model of modern cosmology. In the last decade or so we have entered an era of precision cosmology. The basic quantities have been measured and now what remains is to try to understand them. What is surprising, is the fact that the Universe today seems to expand with an accelerating rate, while one would expect that because of the attractive nature of gravity the universe should be decelerating. This means that the dominant component in the universe is some strange "material" with negative pressure, called Dark Energy. The rest of the universe consists of photons, neutrinos (only a tiny fraction of the energy budget) and non-relativistic matter, most of which is in some unknown non-baryonic form, called Dark Matter. So we see that in its majority the Universe consists of something that we do not know what it is. The nature and detailed characteristics of dark matter and dark energy are the major theoretical challenges for modern cosmology.

## 1.7 The brane-world idea

Recently it has been suggested that there might exist some extra spatial dimensions. Of course this is not a new proposal. Instead it is essentially a revival of the old Kaluza-Klein idea. In the traditional Kaluza-Klein sense the extra dimensions are compactified on a small enough radius to evade detection in the form of Kaluza-Klein modes. However now there is a different setting where the extra dimensions could be large, under the assumption that ordinary matter is confined onto a three-dimensional subspace, called brane (more precisely "3-brane", referring to the three spatial dimensions) embedded in a higher dimensional space, called bulk. In fact the idea that we might be living inside a defect embedded in a higher dimensional space has already a long history. In the context of an ordinary higher dimensional gauge field theory it was proposed that we might live on a codimension one solitonic object. However, it was soon realized that in contrast to scalar and spin-1/2 fields, it would be difficult to confine gauge fields on such an object.

The situation is drastically different in the context of type-I string theory. A few developments led to an exciting possibility and renewed interest in the whole idea. First, with the discovery of the D-branes as an essential part of the spectrum in type-I string theory, one could conjecture that we inhabit such a D-brane embedded in a ten-dimensional bulk. The usual solitonic defect of field theory was thus replaced by an appropriate collection of D-branes, which by construction confine the gauge fields, together with all the ingredients of the standard model. All known matter and forces lie on our brane-world, with the exception of gravity which acts in the bulk space as well. It was however pointed out that the gravitational force on the brane was consistent with all laboratory and astrophysical data as long as the extra dimensions were smaller than a characteristic scale. This led to the exciting possibility of two extra dimensions in the sub-millimeter range. Furthermore, it was demonstrated in the context of an appropriate effective five-dimensional theory of gravity, that once we take into account the back reaction of the brane energy-momentum onto the geometry of spacetime, the graviton is effectively confined on the brane and Newton's law is reproduced to an excellent accuracy at large distances, even with a non-compact extra dimension.

The present work comprises my research in the field of brane cosmology. Brane-worlds open-up new ways to attack the Dark Matter, Dark Energy problems and these were exploited in what follows. Our work is organized as follows: In Part I, which includes chapters 2 and 3, we present the Theoretical framework of our discussion. We introduce, briefly and for completeness, the fundamentals of the Brane-World scenario, the Standard Models of particle physics and of Cosmology, as well as some basic knowledge about Supersymmetry. In Part II, the rest of the present work, we present our research in the field of brane cosmology. In chapter 4 we discuss axino dark matter in the brane-world scenario. We present sneutrino inflation and supersymmetric hybrid inflation in chapters 5 and 6 respectively. The role of brane-bulk energy exchange and a concrete model with our universe as a global late-time attractor is discussed in the seventh chapter. We summarize our results and finish with some conclusions in the last chapter.

# PART I

## The Theoretical Framework

# Chapter 2

# The brane-world scenario

In the present section we shall describe various realizations of the brane-world idea [1].

## 2.1 Randall-Sundrum localization

We will consider the case of a five-dimensional bulk with coordinates $x^M = (x^\mu, y)$, $\mu = 0, 1, 2, 3$. We also consider a three-brane located at $y = 0$. Apart from the five-dimensional Einstein term we also have a constant energy density (brane tension) on the brane, and a non-vanishing cosmological constant in the bulk. We can summarize the effective action as

$$S = \int dy\, d^4x \sqrt{g}(M^3 R - \Lambda) - \int d^4\xi dy \delta(y) \sqrt{\hat{g}} V_b \tag{2.1}$$

where $\hat{g}_{ab}$ is the induced metric on the brane $\hat{g}_{ab} = g_{MN} \frac{\partial x^M}{\partial \xi^a} \frac{\partial x^N}{\partial \xi^b}$. We will pick a static gauge for the brane coordinates $\xi^a = x^a$. To simplify matters, we will also consider an orbifold structure under $y \to -y$. Thus, the two parts of space-time separated by the brane are mirror symmetric around the position of the brane.

We would like to solve the equations of motion stemming from action (2.1). Let us first seek solutions invariant under the orbifold action which are flat along the brane, and depend only on the fifth coordinate $y$.

The ansatz for the five-dimensional metric is

$$ds^2 = e^{2A(y)} dx^\mu dx_\mu + dy^2 \tag{2.2}$$

In order for the flat-brane ansatz to provide a solution, the two vacuum energies must be related

$$V_b^2 = -12\Lambda M^3 \tag{2.3}$$

This implies that the vaccum energy $\Lambda$ must be negative. We will also define the RS (AdS) energy scale

$$K = -\frac{\Lambda}{V_b} \tag{2.4}$$

The gravitational equations have the solution

$$A(y) = -K|y| \tag{2.5}$$

The space on the one side of the brane is a slice of $AdS_5$ patched-up with its mirror image at $y = 0$. Indeed defining $r = e^{Ky}$ for $y > 0$ and scaling $x^\mu \to x^\mu/K$ we obtain

$$ds^2 = \frac{1}{K^2 r^2}(dr^2 + dx^\mu dx_\mu), \quad r \geq 1 \tag{2.6}$$

which is the $r \geq 1$ slice of $AdS_5$ in Poincare coordinates with AdS energy scale $K$. Note that the orbifold has removed the boundary of $AdS_5$ at $r = 0$.

An interesting further question concerns the effective interactions mediated by gravity in this background. To find them we must study the small fluctuations around this solution. Direct variation of the equations along the brane longitudinal directions and gauge fixing gives a scalar equation for the static graviton propagator

$$M^3(-e^{-2A}\nabla_x^2 - \partial_y^2 - 4A'\partial_y)G(x;y) = \delta(y)\delta^{(3)}(x) \tag{2.7}$$

This can be Fourier transformed along the $x^i$ coordinates obtaining

$$M^3(-e^{-2A}\vec{p}^2 - \partial_y^2 - 4A'\partial_y)G(\vec{p};y) = \delta(y) \tag{2.8}$$

Imposing the symmetry $G(\vec{p},y) = G(\vec{p},-y)$ we obtain the solution

$$G(y,\vec{p}) = Bw^2 K_2(\frac{wp}{K}) \tag{2.9}$$

where $p = |\vec{p}|$, $w = e^{K|y|}$, $K_2$ is the standard Bessel function and

$$B = \frac{1}{2M^3 p K_1(\frac{p}{K})} \tag{2.10}$$

We can now investigate the force mediated by the graviton fluctuations on the brane by evaluating

$$G(\vec{p},0) = \frac{1}{2M^3 p}\frac{K_2(\frac{p}{K})}{K_1(\frac{p}{K})} \tag{2.11}$$

The static gravitational potential between two unit sources on the brane (upon transforming back to configuration space) becomes

$$V(r) = \frac{1}{2\pi^2 r} \int_0^\infty dppsin(pr)G(\vec{p}, 0) = \frac{K}{4\pi M^3 r} + \delta V(r) \tag{2.12}$$

with

$$\delta V(r) = \frac{K}{4\pi M^3 r} \int_0^\infty dq\, sin(qr)\, \frac{K_0(q)}{K_1(q)} \tag{2.13}$$

where here $r^2 = \vec{x}^2$ is the spatial distance on the brane. We can now compute the correction to the gravitational potential for two extreme cases.

For $Kr \gg 1$ the main contribution to the integral (2.13) comes from small $q$ and we obtain

$$\delta V(r) \simeq \frac{1}{8\pi K M^3 r^2} \tag{2.14}$$

Thus, at long distances gravity is four-dimensional with sub-leading corrections. The effective four-dimensional Planck scale is $M_p^2 = M^3/K$.

For $Kr \ll 1$ the main contribution to the integral comes from large $q$ and we obtain

$$\delta V(r) \simeq \frac{1}{4\pi^2 M^3 r^2} \tag{2.15}$$

Thus, at short distances gravity is five-dimensional. This is completely analogous to compactification with radius $1/K$. The RS setup is thus an alternative mechanism to compactification for transforming five-dimensional gravity into four-dimensional at large distances.

## 2.2 Brane-Induced gravity

We now describe an alternative realization of four-dimensional gravity that comes under the name of brane-induced gravity. First we consider the simplest case of a five-dimensional bulk with coordinates $x^M = (x^\mu, y), \mu = 0, 1, 2, 3$ . We also consider a three brane located at $y = 0$. Apart from the five-dimensional Einstein term we would like to study the effects of a four-dimensional Einstein term localized on the brane. The relevant action is

$$M^3 \int dy\, d^4x \sqrt{g} R + r_c \int d^4\xi dy \delta(y) \sqrt{\hat{g}} \hat{R} \tag{2.16}$$

where $\hat{g_{ab}}$ is the induced metric on the brane and $\hat{R}$ the induced Ricci scalar. We also parameterized the coefficient of the four-dimensional term in terms of a new length scale $r_c$.

We are interested in the gravitational interaction, generated by the action (2.16), as perceived on the brane. We will evaluate first the static propagator of (2.16). Although there is interesting physics in the tensor structure, we will neglect it for the moment and consider instead the scalar propagator. Placing the source on the brane (at the origin) we must solve

$$M^3(\nabla_3^2 + \partial_y^2 + r_c\delta(y)\nabla_3^2)G(\vec{x}, y) = -\delta(y)\delta^{(3)}(x) \tag{2.17}$$

Fourier transforming in the 3-spatial coordinates $\vec{x}$ we obtain

$$M^3(\vec{p}^2 - \partial_y^2 + r_c\delta(y)\vec{p}^2)G(\vec{p}, y) = \delta(y) \tag{2.18}$$

The solution can be found by first solving the equations away from the position of the brane, and then matching along $y = 0$. The result is

$$G(\vec{p}, y) = \frac{e^{-|\vec{p}||y|}}{M^3(2|\vec{p}| + r_c\vec{p}^2)} \tag{2.19}$$

For the source and the probe being on the brane $y = 0$ the static propagator becomes

$$G(\vec{p}, 0) = \frac{1}{M^3(2|\vec{p}| + r_c\vec{p}^2)} \tag{2.20}$$

By Fourier transforming back we obtain the static gravitational potential

$$V(r) = \frac{1}{2\pi^2 r}\int_0^\infty dppsin(pr)G(\vec{p}, 0) \tag{2.21}$$

We are now ready to study the behavior of the gravitational force in various regimes.

For long distances $pr_c \ll 1$ the potential can be approximated as

$$V(r) \sim \frac{1}{M^3 r^2} \tag{2.22}$$

This is the behavior of five-dimensional gravity with Planck scale $M$.

For short distances $pr_c \gg 1$ we obtain

$$V(r) \sim \frac{1}{M^3 r_c r} \tag{2.23}$$

This is the behavior of four-dimensional gravity with effective Planck scale $M_p^2 = M^3 r_c$. Thus, we have a situation which is inverted with respect to normal compactification: four-dimensional gravity at short distances and five-dimensional gravity at long distances.

## 2.3 Randall-Sundrum plus brane–induced gravity

In this subsection we will investigate what happens when both mechanisms are at work simultaneously. The relevant effective action now is

$$\int dy\, d^4x\sqrt{g}(M^3R-\Lambda)+\int d^4\xi dy\delta(y)\sqrt{\hat{g}}(M^3r_c\hat{R}-V_b) \tag{2.24}$$

The crucial observation here is that since the RS solution is flat on the brane , it is not affected by the presence of the localized Einstein term. Thus with the RS fine-tuning $V_b^2=-12\Lambda M^3$ the solution (2.2), (2.5) is still valid.

Now the equation for the static (scalar) graviton propagator is modified to

$$M^3(-e^{-2A}\nabla_x^2-\partial_y^2-4A'\partial_y-r_c\delta(y)\nabla_x^2)G(x;y)=\delta(y)\delta^{(3)}(x) \tag{2.25}$$

This can be Fourier transformed along the $x^i$ coordinated obtaining

$$M^3(-e^{-2A}p^2-\partial_y^2-4A'\partial_y+r_c\delta(y)p^2)G(\vec{p};y)=\delta(y) \tag{2.26}$$

Imposing the symmetry $G(\vec{p},-y)=G(\vec{p},y)$ we obtain the solution

$$G(p,y)=Bw^2K_2(\frac{wp}{K}) \tag{2.27}$$

where the constant $B$ is given by

$$B=\frac{1}{M^3p(2K_1(\frac{p}{K})+r_cpK_2(\frac{p}{K}))} \tag{2.28}$$

We investigate the force mediated by the graviton fluctuations on the brane by evaluating

$$G(\vec{p},0)=\frac{1}{M^3p}\,\frac{K_2(\frac{p}{K})}{2K_1(\frac{p}{K})+r_cpK_2(\frac{p}{K})} \tag{2.29}$$

We distinguish two separated cases.

For $Kr_c\gg 1$ the potential exhibits four-dimensional behavior $\sim 1/r$ for all distances, with an effective Planck scale $M_{pl}^2\simeq M^3r_c$.

For $Kr_c\ll 1$ we find five-dimensional behavior $\sim 1/r^2$ for the potential for distances $r_c\ll r\ll 1/K$, while for $r\gg 1/K$ or $r\ll r_c$ the potential displays four-dimensional behavior. For short distances the Planck scale is $M_p^2\simeq M^3r_c$, while for long distances the effective Planck scale is $\tilde{M}_p^2\simeq M^3/K$.

# Chapter 3

# Preliminaries

## 3.1 The Standard Model of particle physics and supersymmetry

### 3.1.1 The SM of particle physics

#### The Standard Model Lagrangian

The Standard Model [2] is a gauge theory of the microscopic interactions. The strong interaction part is a gauge theory, based on the gauge group $SU(3)$ and is described by the Lagrangian

$$L_{SU_3} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \sum_r \bar{q}_{r\alpha} i \not{D}^\alpha_\beta \, q^\beta_r \tag{3.1}$$

where

$$F^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_s f_{ijk} \, G^j_\mu \, G^k_\nu \tag{3.2}$$

is the field strength tensor for the gluon fields $G^i_\mu$, $i = 1, \cdots, 8$, with $g_s$ the QCD gauge coupling constant, and the structure constants $f_{ijk}$ $(i, j, k = 1, \cdots, 8)$ are defined by

$$[\lambda^i, \lambda^j] = 2i f_{ijk} \lambda^k \tag{3.3}$$

where the $SU(3)$ $\lambda$ matrices are defined in Table 3.1. The $F^2$ term leads to three and four-point gluon self-interactions. The second term in $L_{SU_3}$ is the gauge covariant derivative for the quarks: $q_r$ is the $r^{\text{th}}$ quark flavor, $\alpha, \beta = 1, 2, 3$ are color indices, and

$$D^\alpha_{\mu\beta} = (D_\mu)_{\alpha\beta} = \partial_\mu \delta_{\alpha\beta} + i g_s \, G^i_\mu \, L^i_{\alpha\beta} \tag{3.4}$$

| | |
|---|---|
| $\lambda^i = \begin{pmatrix} \tau^i & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3$ | |
| $\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ | $\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$ |
| $\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ | $\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$ |
| $\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ | |

Table 3.1: The $SU(3)$ matrices.

where the quarks transform according to the triplet representation matrices $L^i = \lambda^i/2$. The color interactions are diagonal in the flavor indices, but in general change the quark colors. They are purely vector (parity conserving). There are no bare mass terms for the quarks in (3.1). These would be allowed by QCD alone, but are forbidden by the chiral symmetry of the electroweak part of the theory. The quark masses will be generated later by spontaneous symmetry breaking. There are in addition effective ghost and gauge-fixing terms which enter into the quantization of both the $SU(3)$ and electroweak lagrangians, and there is the possibility of adding an (unwanted) term which violates $CP$ invariance.

The electroweak theory is based on the $SU(2) \times U(1)$ gauge group. Its Lagrangian is

$$L_{SU_2 \times U_1} = L_{\text{gauge}} + L_{\varphi} + L_f + L_{\text{Yukawa}} \tag{3.5}$$

The gauge part is

$$L_{\text{gauge}} = -\frac{1}{4} F^i_{\mu\nu} F^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{3.6}$$

where $W^i_\mu$, $i = 1,\ 2,\ 3$ and $B_\mu$ are respectively the $SU(2)$ and $U(1)$ gauge fields, with field strength tensors

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ F_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g \epsilon_{ijk} W^j_\mu W^k_\nu \end{aligned} \tag{3.7}$$

where $g(g')$ is the $SU(2)$ $(U(1))$ gauge coupling and $\epsilon_{ijk}$ is the totally antisymmetric symbol. The $SU(2)$ fields have three and four-point self-interactions. $B$ is a $U(1)$ field associated with the weak hypercharge $Y = Q - T_3$, where

$Q$ and $T_3$ are respectively the electric charge operator and the third component of weak $SU(2)$. It has no self-interactions. The $B$ and $W_3$ fields will eventually mix to form the photon and $Z$ boson.

The scalar part of the lagrangian is

$$L_\varphi = (D^\mu\varphi)^\dagger D_\mu\varphi - V(\varphi) \tag{3.8}$$

where $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ is a complex Higgs scalar, which is a doublet under $SU(2)$ with $U(1)$ charge $Y_\varphi = +\frac{1}{2}$. The gauge covariant derivative is

$$D_\mu\varphi = \left(\partial_\mu + ig\frac{\tau^i}{2}W_\mu^i + \frac{ig'}{2}B_\mu\right)\varphi \tag{3.9}$$

where the $\tau^i$ are the Pauli matrices. The square of the covariant derivative leads to three and four-point interactions between the gauge and scalar fields.

$V(\varphi)$ is the Higgs potential. The combination of $SU(2)\times U(1)$ invariance and renormalizability restricts $V$ to the form

$$V(\varphi) = +\mu^2\varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2 \tag{3.10}$$

For $\mu^2 < 0$ there will be spontaneous symmetry breaking. The $\lambda$ term describes a quartic self-interaction between the scalar fields. Vacuum stability requires $\lambda > 0$.

The fermion term is

$$L_F = \sum_{m=1}^{F}\left(\bar{q}^0_{mL} i \not{D} q^0_{mL} + \bar{l}^0_{mL} i \not{D} l^0_{mL} + \bar{u}^0_{mR} i \not{D} u^0_{mR} + \bar{d}^0_{mR} i \not{D} d^0_{mR} + \bar{e}^0_{mR} i \not{D} e^0_{mR}\right) \tag{3.11}$$

In (3.11) $m$ is the family index, $F \geq 3$ is the number of families, and $L(R)$ refer to the left (right) chiral projections $\psi_{L(R)} \equiv (1 \mp \gamma_5)\psi/2$. The left-handed quarks and leptons

$$q^0_{mL} = \begin{pmatrix} u^0_m \\ d^0_m \end{pmatrix}_L \qquad l^0_{mL} = \begin{pmatrix} \nu^0_m \\ e^{-0}_m \end{pmatrix}_L \tag{3.12}$$

transform as $SU(2)$ doublets, while the right-handed fields $u^0_{mR}$, $d^0_{mR}$, and $e^{-0}_{mR}$ are singlets. Their $U(1)$ charges are $Y_{q_L} = \frac{1}{6}$, $Y_{l_L} = -\frac{1}{2}$, $Y_{\psi_R} = q_\psi$. The superscript 0 refers to the weak eigenstates, i.e. fields transforming according to definite $SU(2)$ representations. They may be mixtures of mass eigenstates (flavors). The quark color indices $\alpha = r,\ g,\ b$ have been suppressed. The gauge covariant derivatives are

$$\begin{aligned}
D_\mu q^0_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W^i_\mu + i\frac{g'}{6}B_\mu\right) q^0_{mL} \\
D_\mu l^0_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W^i_\mu - i\frac{g'}{2}B_\mu\right) l^0_{mL} \\
D_\mu u^0_{mR} &= \left(\partial_\mu + i\frac{2}{3}g' B_\mu\right) u^0_{mR} \\
D_\mu d^0_{mR} &= \left(\partial_\mu - i\frac{g'}{3}B_\mu\right) d^0_{mR} \\
D_\mu e^0_{mR} &= \left(\partial_\mu - ig' B_\mu\right) e^0_{mR} \qquad (3.13)
\end{aligned}$$

from which one can read off the gauge interactions between the $W$ and $B$ and the fermion fields. The different transformations of the $L$ and $R$ fields (i.e. the symmetry is chiral) is the origin of parity violation in the electroweak sector. The chiral symmetry also forbids any bare mass terms for the fermions.

The last term in (3.5) is

$$-L_{\text{Yukawa}} = \sum_{m,n=1}^{F} \left[\Gamma^u_{mn}\bar{q}^0_{mL}\tilde{\varphi}u^0_{mR} + \Gamma^d_{mn}\bar{q}^0_{mL}\varphi d^0_{nR} + \Gamma^e_{mn}\bar{l}^0_{mn}\varphi e^0_{nR}\right] + \text{H.C.} \qquad (3.14)$$

where the matrices $\Gamma_{mn}$ describe the Yukawa couplings between the single Higgs doublet, $\varphi$, and the various flavors $m$ and $n$ of quarks and leptons. One needs representations of Higgs fields with $Y = \frac{1}{2}$ and $-\frac{1}{2}$ to give masses to the down quarks, the electrons, and the up quarks. The representation $\varphi^\dagger$ has $Y = -\frac{1}{2}$, but transforms as the $2^*$ rather than the 2. However, in $SU(2)$ the $2^*$ representation is related to the 2 by a similarity transformation, and $\tilde{\varphi} \equiv i\tau^2\varphi^\dagger = \begin{pmatrix} \varphi^{0^\dagger} \\ -\varphi^- \end{pmatrix}$ transforms as a 2 with $Y_{\tilde{\varphi}} = -\frac{1}{2}$. All of the masses can therefore be generated with a single Higgs doublet if one makes use of both $\varphi$ and $\tilde{\varphi}$. The fact that the fundamental and its conjugate are equivalent does not generalize to higher unitary groups. Furthermore, in supersymmetric extensions of the standard model supersymmetry forbids the use of a single Higgs doublet in both ways in the lagrangian, and one must add a second Higgs doublet. Similar statements apply to most theories with an additional $U(1)$ gauge factor, i.e. a heavy $Z'$ boson.

### Spontaneous Symmetry Breaking

Gauge invariance (and therefore renormalizability) does not allow mass terms in the lagrangian for the gauge bosons or for chiral fermions. Massless gauge bosons are not acceptable for the weak interactions, which are known to be short-ranged. Hence, the gauge invariance must be broken spontaneously, which preserves the renormalizability. The idea is simply that the lowest energy (vacuum) state does not respect the gauge symmetry and induces effective masses for particles propagating through it.

Let us introduce the complex vector

$$v = \langle 0|\varphi|0\rangle = \text{constant} \tag{3.15}$$

which has components that are the vacuum expectation values of the various complex scalar fields. $v$ is determined by rewriting the Higgs potential as a function of $v$, $V(\varphi) \to V(v)$, and choosing $v$ such that $V$ is minimized. That is, we interpret $v$ as the lowest energy solution of the classical equation of motion. The quantum theory is obtained by considering fluctuations around this classical minimum, $\varphi = v + \varphi'$.

The single complex Higgs doublet in the standard model can be rewritten in a Hermitian basis as

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2) \\ \frac{1}{\sqrt{2}}(\varphi_3 - i\varphi_4 \end{pmatrix} \tag{3.16}$$

where $\varphi_i = \varphi_i^\dagger$ represent four hermitian fields. In this new basis the Higgs potential becomes

$$V(\varphi) = \frac{1}{2}\mu^2 \left(\sum_{i=1}^{4} \varphi_i^2\right) + \frac{1}{4}\lambda \left(\sum_{i=1}^{4} \varphi_i^2\right)^2 \tag{3.17}$$

which is clearly $O_4$ invariant. Without loss of generality we can choose the axis in this four-dimensional space so that $\langle 0|\varphi_i|0\rangle = 0, \quad i = 1, 2, 4$ and $\langle 0|\varphi_3|0\rangle = \nu$. Thus,

$$V(\varphi) \to V(v) = \frac{1}{2}\mu^2\nu^2 + \frac{1}{4}\lambda\nu^4 \tag{3.18}$$

which must be minimized with respect to $\nu$. For $\mu^2 > 0$ the minimum occurs at $\nu = 0$. That is, the vacuum is empty space and $SU(2) \times U(1)$ is unbroken at the minimum. On the other hand, for $\mu^2 < 0$ the $\nu = 0$ symmetric point is unstable, and the minimum occurs at some nonzero value of $\nu$ which breaks the $SU(2) \times U(1)$ symmetry. The point is foun d by requiring

$$V'(\nu) = \nu(\mu^2 + \lambda\nu^2) = 0 \tag{3.19}$$

which has the solution $\nu = (-\mu^2/\lambda)^{1/2}$ at the minimum. (The solution for $-\nu$ can also be transformed into this standard form by an appropriate $O_4$ transformation.) The dividing point $\mu^2 = 0$ cannot be treated classically. It is necessary to consider the one loop corrections to the potential, in which case it is found that the symmetry is again spontaneously broken.

We are interested in the case $\mu^2 < 0$, for which the Higgs doublet is replaced, in first approximation, by its classical value $\varphi \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix} \equiv v$. The generators $L^1$, $L^2$, and $L^3 - Y$ are spontaneously broken (e.g. $L^1 v \neq 0$). On the other hand, the vacuum carries no electric charge ($Qv = (L^3 + Y)v = 0$), so the $U_{1Q}$ of electromagnetism is not broken. Thus, the electroweak $SU(2) \times U(1)$ group is spontaneously broken down, $SU(2) \times U_{1Y} \rightarrow U_{1Q}$.

To quantize around the classical vacuum, write $\varphi = v + \varphi'$, where $\varphi'$ are quantum fields with zero vacuum expectation value. To display the physical particle content it is useful to rewrite the four hermitian components of $\varphi'$ in terms of a new set of variables using the Kibble transformation:

$$\varphi = \frac{1}{\sqrt{2}} e^{i\sum \xi^i L^i} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix} \tag{3.20}$$

$H$ is a hermitian field which will turn out to be the physical Higgs scalar. If we had been dealing with a spontaneously broken global symmetry the three hermitian fields $\xi^i$ would be the massless pseudoscalar Goldstone bosons that are necessarily associated with broken symmetry generators. However, in a gauge theory they disappear from the physical spectrum. To see this it is useful to go to the unitary gauge

$$\varphi \rightarrow \varphi' = e^{-i\sum \xi^i L^i} \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix} \tag{3.21}$$

in which the Goldstone bosons disappear. In this gauge, the scalar covariant kinetic energy term takes the simple form

$$\begin{aligned} (D_\mu\varphi)^\dagger D^\mu\varphi &= \frac{1}{2}(0\ \nu) \left[\frac{g}{2}\tau^i W_\mu^i + \frac{g'}{2} B_\mu\right]^2 \begin{pmatrix} 0 \\ \nu \end{pmatrix} + H \text{ terms} \\ &\rightarrow M_W^2 W^{+\mu} W_\mu^- + \frac{M_Z^2}{2} Z^\mu Z_\mu + H \text{ terms} \end{aligned} \tag{3.22}$$

where the kinetic energy and gauge interaction terms of the physical $H$ particle have been omitted. Thus, spontaneous symmetry breaking generates mass terms for the $W$ and $Z$ gauge bosons

$$\begin{aligned} W^{\pm} &= \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \\ Z &= -\sin\theta_W B + \cos\theta_W W^3 \end{aligned} \tag{3.23}$$

The photon field

$$A = \cos\theta_W B + \sin\theta_W W^3 \tag{3.24}$$

remains massless. The masses are

$$M_W = \frac{g\nu}{2} \tag{3.25}$$

and

$$M_Z = \sqrt{g^2 + g'^2}\frac{\nu}{2} = \frac{M_W}{\cos\theta_W} \tag{3.26}$$

where the weak angle is defined by $\tan\theta_W \equiv g'/g$. One can think of the generation of masses as due to the fact that the $W$ and $Z$ interact constantly with the condensate of scalar fields and therefore acquire masses, in analogy with a photon propagating through a plasma. The Goldstone boson has disappeared from the theory but has reemerged as the longitudinal degree of freedom of a massive vector particle.

It can be shown that $G_F/\sqrt{2} \sim g^2/8M_W^2$, where $G_F = 1.16639(2) \times 10^{-5}\ GeV^{-2}$ is the Fermi constant determined by the muon lifetime. The weak scale $\nu$ is therefore

$$\nu = 2M_W/g \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246\ GeV \tag{3.27}$$

Similarly, $g = e/\sin\theta_W$, where $e$ is the electric charge of the positron. Hence, to lowest order

$$M_W = M_Z\cos\theta_W \sim \frac{(\pi\alpha/\sqrt{2}G_F)^{1/2}}{\sin\theta_W} \tag{3.28}$$

where $\alpha \sim 1/137.036$ is the fine structure constant. Using $\sin^2\theta_W \sim 0.23$ from neutral current scattering, one expects $M_W \sim 78\ GeV$, and $M_Z \sim 89\ GeV$. (These predictions are increased by $\sim (2-3)\ GeV$ by loop corrections.) The $W$ and $Z$ were discovered at CERN by two groups (UA1 and UA2) in 1983. Subsequent measurements of their masses and other properties have been in perfect agreement with the standard model expectations (including the higher-order corrections), as is described in the articles of by Schaile and Einsweiler.

After symmetry breaking the Higgs potential becomes

$$V(\varphi) = -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda\nu H^3 + \frac{\lambda}{4}H^4 \tag{3.29}$$

The third and fourth terms represent the cubic and quartic interactions of the Higgs scalar. The second term represents a (tree-level) mass

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\nu \tag{3.30}$$

The weak scale is given in (3.27), but the quartic Higgs coupling $\lambda$ is unknown, so $M_H$ is not predicted. A priori, $\lambda$ could be anywhere in the range $0 \leq \lambda < \infty$. There is now an experimental lower limit $M_H \geq 60$ GeV from LEP. Otherwise, the decay $Z \rightarrow Z^* H$ would have been observed (There are also theoretical lower limits on $M_H$ in the $\sim 10\ GeV$ range, depending on $m_t$, when higher-order corrections are included).

### 3.1.2 Basics of Supersymmetry

#### The motivation for supersymmetry

It is widely accepted that the Standard Model of gauge interactions describing the laws of physics at the weak scale is extraordinarily successful. The agreement between theory and experimental data is very good. Yet, we believe that the present structure is incomplete. Only to remind a few drawbacks, the theory has too many parameters, it does not describe the fermion masses and why the number of generations is three. It contains fundamental scalars, something difficult to reconcile with our current understanding of non-supersymmetric field theory. Finally, it does not incorporate gravity.

It is tempting to speculate that a new (but yet undiscovered) symmetry, supersymmetry [3], may provide answers to these fundamental questions. Supersymmetry is the almost universally accepted framework for constructing extensions of the Standard Model. Supersymmetry can be formulated either as a global or a local symmetry. In the latter case it includes gravity, and is therefore called supergravity. Supersymmetry is the only framework in which we seem to be able to understand light fundamental scalars. It addresses the question of parameters: first, unification of gauge couplings works much better with than without supersymmetry; second, it is easier to attack questions such as fermion masses in supersymmetric theories, in part simply due to the presence of fundamental scalars. Supersymmetry seems to be intimately connected with gravity. So there are a number of theoretical arguments that suggest that nature might be supersymmetric, and that supersymmetry might manifest itself at energies of order the weak interaction scale.

### The supersymmetry algebra and supermultiplets

We begin with some basics, that apply to both global supersymmetry and supergravity.

In the low-energy regime, phenomenology requires the type of supersymmetry known as $N = 1$ (one generator). In this section, we present some features of $N = 1$ supersymmetric theories, that are likely to be relevant for inflation.

The basic supersymmetry algebra is given by

$$\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \tag{3.31}$$

where $Q_\alpha$ and $\overline{Q}_{\dot{\beta}}$ are the supersymmetry generators (bars stand for conjugate), $\alpha$ and $\beta$ run from 1 to 2 and denote the two-component Weyl spinors (quantities with dotted indices transform under the $(0, \frac{1}{2})$ representation of the Lorentz group, while those with undotted indices transform under the $(\frac{1}{2}, 0)$ conjugate representation). $\sigma^\mu$ is a matrix four vector, $\sigma^\mu = (-\mathbf{1}, \vec{\sigma})$ and $P_\mu$ is the generator of spacetime displacements (four-momentum).

The chiral and vector superfields are two irreducible representations of the supersymmetry algebra containing fields of spin less than or equal to one. Chiral fields contain a Weyl spinor and a complex scalar; vector fields contain a Weyl spinor and a (massless) vector. In superspace a chiral superfield may be expanded in terms of the Grassmann variable $\theta$ (the fermionic coordinates)

$$\phi(x, \theta) = \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) \tag{3.32}$$

where $\theta^2 \equiv \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta$. Here $x$ denotes a point in spacetime, $\phi(x)$ is the complex scalar, $\psi$ the fermion, and $F$ is an auxiliary field. As in this expression, we shall generally use the same symbol to represent a superfield and its scalar component. Under a supersymmetry transformation with anticommuting parameter $\zeta$, the component fields transform as

$$\delta\phi = \sqrt{2}\zeta\psi \tag{3.33}$$

$$\delta\psi = \sqrt{2}\zeta F + \sqrt{2} i \sigma^\mu \bar{\zeta} \partial_\mu \phi \tag{3.34}$$

$$\delta F = -\sqrt{2} i \partial_\mu \psi \sigma^\mu \bar{\zeta} \tag{3.35}$$

Here and in the following, for any generic two-component Weyl spinor $\lambda$, $\bar{\lambda}$ indicates the complex conjugate of $\lambda$. For a gauge theory one has to introduce vector superfields and the physical content is most transparent in the Wess-Zumino gauge. In this gauge and for the simplest case of an abelian group $U(1)$, the vector superfield may be written as

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \tfrac{1}{2}\theta^2\bar{\theta}^2 D \tag{3.36}$$

Here $A_\mu$ is the gauge field, $\lambda_\alpha$ is the gaugino, and $D$ is an auxiliary field. The analog of the gauge invariant field strength is a chiral field:

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \tfrac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu} + \theta^2\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\bar{\lambda}^{\dot{\beta}} \tag{3.37}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and where $\bar{\sigma}^\mu = (-\mathbf{1}, -\vec{\sigma})$. Regarding the supersymmetry transformations, let us just note that

$$\delta\lambda = i\zeta D + \zeta\sigma^\mu\bar{\sigma}^\nu F_{\mu\nu} \tag{3.38}$$

Global supersymmetry is defined as invariance under these transformations with $\zeta$ independent of spacetime position, and local supersymmetry (supergravity) as invariance with $\zeta$ depending on spacetime position. In the latter case one has to introduce another supermultiplet containing the graviton and the gravitino.

Global supersymmetry may be regarded as a limit of supergravity, in which roughly speaking gravity is made negligible by taking $M_{\rm Pl}$ to infinity. *For most purposes* it is a good approximation if the vevs of all relevant scalar fields and auxiliary fields are much less than $M_{\rm Pl}$.

### The Lagrangian of global supersymmetry

We focus first on global supersymmetry, with the usual restriction that it be renormalizable.

To write down the action for a set of chiral superfields, $\phi_i$, transforming in some representation of a gauge group $G$, one introduces, for each gauge generator, a vector superfield, $V^a$. Defining the matrix $V = T^a V_a$, where $T^a$ are the hermitian generators of the gauge group $G$ in the representation defined by the scalar fields and excluding the possible Fayet-Iliopoulos term to be discussed later, the most general renormalizable lagrangian, written in superspace, is then

$$\mathcal{L} = \sum_n \int d^2\theta d^2\bar{\theta}\phi_n^\dagger e^V\phi_n + \frac{1}{4k}\int d^2\theta W_\alpha^2 + \int d^2\theta W(\phi_n) + \text{h.c.} \tag{3.39}$$

where in the adjoint representation $\mathrm{Tr}(T^aT^b) = k\delta^{ab}$ and $W(\phi_n(x,\theta))$ is a fundamental object known as superpotential. The corresponding function of the scalar components $\phi_n(x)$, denoted by the same name and symbol, is a holomorphic function of the $\phi_n$. For simplicity, we shall pretend that there is a single gauge $U(1)$ interaction, with coupling constant $g$. This is adequate since such an interaction is the only one that we consider in detail. (To be precise, we consider a $U(1)$ with a Fayet-Iliopoulos term.) In the case of several $U(1)$'s, there are no cross-terms in the potential from the $D$-terms, i.e. $V_D$ is simply expressed as $\sum_n (V_D)_n$.

To write this down in terms of component fields, we need the covariant derivative

$$D_\mu = \partial_\mu - \frac{i}{2} g A_\mu \tag{3.40}$$

In terms of the component fields, the lagrangian takes the form:

$$\begin{aligned}\mathcal{L} &= \sum_n \left( D_\mu\phi_n^* D^\mu\phi_n + iD_\mu\bar{\psi}_n\bar{\sigma}^\mu\psi_n + |F_n|^2 \right) \\ &- \frac{1}{4}F_{\mu\nu}^2 - i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + \frac{1}{2}D^2 + \frac{g}{2}D\sum_n q_n\phi_n^*\phi_n \\ &- \left[ i\sum_n \frac{g}{\sqrt{2}}\bar{\psi}_n\bar{\lambda}\phi_n - \sum_{nm}\frac{1}{2}\frac{\partial^2 W}{\partial\phi_n\partial\phi_m}\psi_n\psi_m \right. \\ &+ \left. \sum_n F_n\left(\frac{\partial W}{\partial\phi_n}\right) \right] + \text{c.c.}\end{aligned} \tag{3.41}$$

At the end of the second line, $q_n$ are the $U(1)$-charges of the fields $\phi_n$. The equations of motion for the auxiliary fields $F_n$ and $D$ are the constraints:

$$F_n = -\left(\frac{\partial W}{\partial\phi_n}\right)^* \tag{3.42}$$

$$D = -\frac{g}{2}\sum_n q_n|\phi_n|^2 \tag{3.43}$$

Eq. (3.41) contains the gauge invariant kinetic terms for the various fields, which specify their gauge interactions. It also contains, after having made use of Eqs. (3.42) and (3.43), the scalar field potential,

$$V = V_F + V_D \tag{3.44}$$

$$V_F \equiv \sum_n |F_n|^2 \tag{3.45}$$

$$V_D \equiv \frac{1}{2}D^2 \tag{3.46}$$

This separation of the potential into an $F$ term and a $D$ term is crucial for inflation model-building, especially when it is generalized to the case of supergravity.

The potential specifies the masses of the scalar fields, and their interactions with each other. The first term in the third line specifies the interactions of gaugino and scalar fields, while the second specifies the masses of the chiral fermions and their interactions with the scalars. All of these non-gauge interactions are called Yukawa couplings.

To have a renormalizable theory, $W$ is at most cubic in the fields, corresponding to a potential which is at most quartic.

From the above expressions, in particular Eq. (3.45), one sees that the overall phase of $W$ is not physically significant. An internal symmetry can either leave $W$ invariant, or alter its phase. The latter case corresponds to what is called an R-symmetry. Because $W$ is holomorphic, the internal symmetries restrict its form much more than is the case for the actual potential $V$. In particular, terms in $W$ of the form $\frac{1}{2}m\phi_1^2$ or $m\phi_1\phi_2$, which would generate a mass term $m^2|\phi_1|^2$ in the potential, are usually forbidden.[1] As a result, scalar particles usually acquire masses only from the vevs of scalar fields (i.e. from the spontaneous breaking of an internal symmetry) and from supersymmetry breaking. The same applies to the spin-half partners of scalar fields, with the former contribution the same in both cases.

In the case of a $U(1)$ gauge symmetry, one can add to the above lagrangian what is called a Fayet-Iliopoulos term

$$-2\xi \int d^2\theta d^2\bar{\theta}\ V \tag{3.47}$$

This corresponds to adding a contribution $-\xi$ to the $D$ field, so that (3.43) becomes

$$D = -\frac{g}{2}\sum_n q_n|\phi_n|^2 - \xi \tag{3.48}$$

The $D$ term of the potential therefore becomes

$$V_D = \frac{1}{2}\left(\frac{g}{2}\sum_n q_n|\phi_n|^2 + \xi\right)^2 \tag{3.49}$$

From now on, we shall use a more common notation, where $\xi$ and the

[1] An exception is the $\mu$ term of the MSSM, $\mu H_U H_D$, which gives mass to the Higgs fields.

charges are redefined so that

$$V_D = \frac{1}{2}g^2 \left( \sum_n q_n |\phi_n|^2 + \xi \right)^2 \tag{3.50}$$

This is equivalent to

$$D = -g \left( \sum_n q_n |\phi_n|^2 + \xi \right) \tag{3.51}$$

A Fayet-Iliopoulos term may be present in the underlying theory from the very beginning, or appears in the effective theory after some heavy degrees of freedom have been integrated out.

### Spontaneously broken global susy

Global supersymmetry breaking may be either spontaneous or explicit. However here we shall discuss only the first case. For spontaneous breaking, the lagrangian is supersymmetric as given in the last subsection. But the generators $Q_\alpha$ fail to annihilate the vacuum. Instead, they produce a spin-half field, which may be either a chiral field $\psi_\alpha$ or a gauge field $\lambda_\alpha$. The condition for spontaneous susy breaking is therefore to have a nonzero vacuum expectation value for $\{Q_\alpha, \psi_\beta\}$ or $\{Q_\alpha, \lambda_\beta\}$.

The former quantity is defined by Eq. (3.34), and the latter by Eq. (3.38). The quantities $\partial_\mu \phi$ and $F_{\mu\nu}$ contain derivatives of fields, and are supposed to vanish in the vacuum. It follows that susy is spontaneously broken if, and only if, at least one of the auxiliary fields $F_n$ or $D$ has a non-vanishing vev.

In the true vacuum, one defines the scale $M_\mathrm{S}$ of global supersymmetry breaking by

$$M_\mathrm{S}^4 = \sum_n |F_n|^2 + \frac{1}{2}D^2 \tag{3.52}$$

or equivalently

$$M_\mathrm{S}^4 = V \tag{3.53}$$

(In the simplest case $D$ vanishes and there is just one $F_n$.)

When we go to supergravity, part of $V$ is still generated by the supersymmetry breaking terms, but there is also a contribution $-3|W|^2/M_\mathrm{Pl}^2$. This allows $V$ to vanish in the true vacuum as is (practically) demanded by observation.

During inflation, $V$ is positive so the negative term is smaller than the susy-breaking terms. In most models of inflation it is negligible. In any case, $V$ is at least as big as the susy breaking term, so the search for a model of inflation is also a search for a susy-breaking mechanism in the early Universe.

Spontaneous symmetry breaking can be either tree-level (already present in the lagrangian) or dynamical (generated only by quantum effects like condensation). The spontaneous breaking in general breaks the equality between the scalar and spin-$\frac{1}{2}$ masses, in each chiral supermultiplet. But at tree level the breaking satisfies a simple relation, which can easily be derived from the lagrangian (3.41). Ignoring mass mixing for simplicity, one finds in the case of symmetry breaking by an $F$-term,

$$\sum_n \left(m_{n1}^2 + m_{n2}^2 - 2m_{n\mathrm{f}}^2\right) = 0 \tag{3.54}$$

Here $n$ labels the chiral supermultiplets, $m_{n\mathrm{f}}$ is the fermion mass while $m_{n1}$ and $m_{n2}$ are the scalar masses. In the case of symmetry breaking by a $D$ term, coming from a $U(1)$, the right hand side of Eq. (3.54) becomes $D\mathrm{Tr}\mathbf{Q}$. But in order to cancel gauge anomalies, it is strongly desirable that $\mathrm{Tr}\mathbf{Q} = 0$ which recovers Eq. (3.54).

## 3.2 The Standard Model of Cosmology, the early Universe and inflation

### 3.2.1 The SM of cosmology

#### The Robertson-Walker Metric

Cosmology [4] as the application of general relativity (GR) to the entire universe would seem a hopeless endeavor were it not for a remarkable fact – the universe is spatially homogeneous and isotropic on the largest scales.

"Isotropy" is the claim that the universe looks the same in all directions. Direct evidence comes from the smoothness of the temperature of the cosmic microwave background. "Homogeneity" is the claim that the universe looks the same at every point. It is harder to test directly, although some evidence comes from number counts of galaxies. More traditionally, we may invoke the "Copernican principle," that we do not live in a special place in the universe. Then it follows that, since the universe appears isotropic around us, it should be isotropic around every point; and a basic theorem of geometry states that isotropy around every point implies homogeneity.

We may therefore approximate the universe as a spatially homogeneous and isotropic three-dimensional space which may expand (or, in principle, contract) as a function of time. The metric on such a spacetime is necessarily of the Robertson-Walker (RW) form.

Therefore, the most general spacetime metric consistent with homogeneity and isotropy is

$$ds^2 = -dt^2 + a^2(t)\left[d\rho^2 + f^2(\rho)\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{3.55}$$

where the three possibilities for $f(\rho)$ are

$$f(\rho) = \{\sin(\rho), \quad \rho, \quad \sinh(\rho)\} \tag{3.56}$$

This is a purely geometric fact, independent of the details of general relativity. We have used spherical polar coordinates $(\rho, \theta, \phi)$, since spatial isotropy implies spherical symmetry about every point. The time coordinate $t$, which is the proper time as measured by a comoving observer (one at constant spatial coordinates), is referred to as cosmic time, and the function $a(t)$ is called the scale factor.

There are two other useful forms for the RW metric. First, a simple change of variables in the radial coordinate yields

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{3.57}$$

where

$$k = \begin{cases} +1 & \text{if } f(\rho) = \sin(\rho) \\ 0 & \text{if } f(\rho) = \rho \\ -1 & \text{if } f(\rho) = \sinh(\rho) \end{cases} \tag{3.58}$$

Geometrically, $k$ describes the curvature of the three-dimensional space. $k = +1$ corresponds to positively curved spatial sections (locally isometric to 3-spheres); $k = 0$ corresponds to local flatness, and $k = -1$ corresponds to negatively curved (locally hyperbolic) spatial sections.

Note that we have not chosen a normalization such that $a_0 = 1$. We are not free to do this and to simultaneously normalize $|k| = 1$, without including explicit factors of the current scale factor in the metric. In the flat case, where $k = 0$, we can safely choose $a_0 = 1$.

A second change of variables, which may be applied to either (3.55) or (3.57), is to transform to *conformal time*, $\tau$, via

$$\tau(t) \equiv \int^t \frac{dt'}{a(t')} \tag{3.59}$$

Applying this to (3.57) yields

$$ds^2 = a^2(\tau)\left[-d\tau^2 + \frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{3.60}$$

where we have written $a(\tau) \equiv a[t(\tau)]$ as is conventional. The conformal time does not measure the proper time for any particular observer, but it does simplify some calculations.

A particularly useful quantity to define from the scale factor is the *Hubble parameter* (sometimes called the Hubble constant), given by

$$H \equiv \frac{\dot{a}}{a} \tag{3.61}$$

The Hubble parameter relates how fast the most distant galaxies are receding from us to their distance from us via Hubble's law,

$$v \simeq Hd. \tag{3.62}$$

This is the relationship that was discovered observationally by Edwin Hubble, and has been verified to high accuracy by modern observational methods.

### The Friedmann Equations

As mentioned, the RW metric is a purely kinematic consequence of requiring homogeneity and isotropy of our spatial sections. We next turn to dynamics, in the form of differential equations governing the evolution of the scale factor $a(t)$. These will come from applying Einstein's equation,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{3.63}$$

to the RW metric.

Before diving right in, it is useful to consider the types of energy-momentum tensors $T_{\mu\nu}$ we will typically encounter in cosmology. For simplicity, and because it is consistent with much we have observed about the universe, it is often useful to adopt the perfect fluid form for the energy-momentum tensor of cosmological matter. This form is

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu} \tag{3.64}$$

where $U^\mu$ is the fluid four-velocity, $\rho$ is the energy density in the rest frame of the fluid and $p$ is the pressure in that same frame. The pressure is necessarily isotropic, for consistency with the RW metric. Similarly, fluid elements will be comoving in the cosmological rest frame, so that the normalized four-velocity in the coordinates of (3.57) will be

$$U^{\mu} = (1, 0, 0, 0) \tag{3.65}$$

The energy-momentum tensor thus takes the form

$$T_{\mu\nu} = \begin{pmatrix} \rho & \\ & p g_{ij} \end{pmatrix} \tag{3.66}$$

where $g_{ij}$ represents the spatial metric (including the factor of $a^2$).

Armed with this simplified description for matter, we are now ready to apply Einstein's equation (3.63) to cosmology. Using (3.57) and (3.64), one obtains two equations. The first is known as the Friedmann equation,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{k}{a^2} \tag{3.67}$$

where an overdot denotes a derivative with respect to cosmic time $t$ and $i$ indexes all different possible types of energy in the universe. This equation is a constraint equation, in the sense that we are not allowed to freely specify the time derivative $\dot{a}$; it is determined in terms of the energy density and curvature. The second equation, which is an evolution equation, is

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi G \sum_i p_i - \frac{k}{2a^2} \tag{3.68}$$

It is often useful to combine (3.67) and (3.68) to obtain the *acceleration equation*

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i) \tag{3.69}$$

In fact, if we know the magnitudes and evolutions of the different energy density components $\rho_i$, the Friedmann equation (3.67) is sufficient to solve for the evolution uniquely. The acceleration equation is conceptually useful, but rarely invoked in calculations.

The Friedmann equation relates the rate of increase of the scale factor, as encoded by the Hubble parameter, to the total energy density of all matter in the universe. We may use the Friedmann equation to define, at any given time, a critical energy density,

$$\rho_c \equiv \frac{3H^2}{8\pi G} \tag{3.70}$$

for which the spatial sections must be precisely flat ($k = 0$). We then define the density parameter

$$\Omega_{\rm total} \equiv \frac{\rho}{\rho_c} \,, \tag{3.71}$$

which allows us to relate the total energy density in the universe to its local geometry via

$$\begin{aligned} \Omega_{\rm total} > 1 &\quad \Leftrightarrow \quad k = +1 \\ \Omega_{\rm total} = 1 &\quad \Leftrightarrow \quad k = 0 \\ \Omega_{\rm total} < 1 &\quad \Leftrightarrow \quad k = -1 \end{aligned} \tag{3.72}$$

It is often convenient to define the fractions of the critical energy density in each different component by

$$\Omega_i = \frac{\rho_i}{\rho_c} \tag{3.73}$$

Energy conservation is expressed in GR by the vanishing of the covariant divergence of the energy-momentum tensor,

$$\nabla_\mu T^{\mu\nu} = 0 \tag{3.74}$$

Applying this to our assumptions – the RW metric (3.57) and perfect-fluid energy-momentum tensor (3.64) – yields a single energy-conservation equation,

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{3.75}$$

This equation is actually not independent of the Friedmann and acceleration equations, but is required for consistency. It implies that the expansion of the universe (as specified by $H$) can lead to local changes in the energy density. Note that there is no notion of conservation of "total energy," as energy can be interchanged between matter and the spacetime geometry.

One final piece of information is required before we can think about solving our cosmological equations: how the pressure and energy density are related to each other. Within the fluid approximation used here, we may assume that the pressure is a single-valued function of the energy density $p = p(\rho)$. It is often convenient to define an equation of state parameter, $w$, by

$$p = w\rho \tag{3.76}$$

This should be thought of as the instantaneous definition of the parameter $w$; it need represent the full equation of state, which would be required to calculate the behavior of fluctuations. Nevertheless, many useful cosmological matter sources do obey this relation with a constant value of $w$. For example,

$w = 0$ corresponds to pressureless matter, or dust – any collection of massive non-relativistic particles would qualify. Similarly, $w = 1/3$ corresponds to a gas of radiation, whether it be actual photons or other highly relativistic species.

A constant $w$ leads to a great simplification in solving our equations. In particular, using (3.75), we see that the energy density evolves with the scale factor according to

$$\rho(a) \propto \frac{1}{a(t)^{3(1+w)}} \tag{3.77}$$

Note that the behaviors of dust ($w = 0$) and radiation ($w = 1/3$) are consistent with what we would have obtained by more heuristic reasoning. Consider a fixed *comoving* volume of the universe - i.e. a volume specified by fixed values of the coordinates, from which one may obtain the physical volume at a given time $t$ by multiplying by $a(t)^3$. Given a fixed number of dust particles (of mass $m$) within this comoving volume, the energy density will then scale just as the physical volume, i.e. as $a(t)^{-3}$, in agreement with (3.77), with $w = 0$.

To make a similar argument for radiation, first note that the expansion of the universe (the increase of $a(t)$ with time) results in a shift to longer wavelength $\lambda$, or a *redshift*, of photons propagating in this background. A photon emitted with wavelength $\lambda_e$ at a time $t_e$, at which the scale factor is $a_e \equiv a(t_e)$ is observed today ($t = t_0$, with scale factor $a_0 \equiv a(t_0)$) at wavelength $\lambda_o$, obeying

$$\frac{\lambda_o}{\lambda_e} = \frac{a_0}{a_e} \equiv 1 + z \tag{3.78}$$

The redshift $z$ is often used in place of the scale factor. Because of the redshift, the energy density in a fixed number of photons in a fixed comoving volume drops with the physical volume (as for dust) and by an extra factor of the scale factor as the expansion of the universe stretches the wavelengths of light. Thus, the energy density of radiation will scale as $a(t)^{-4}$, once again in agreement with (3.77), with $w = 1/3$.

Thus far, we have not included a cosmological constant $\Lambda$ in the gravitational equations. This is because it is equivalent to treat any cosmological constant as a component of the energy density in the universe. In fact, adding a cosmological constant $\Lambda$ to Einstein's equation is equivalent to including an energy-momentum tensor of the form

$$T_{\mu\nu} = -\frac{\Lambda}{8\pi G} g_{\mu\nu} \tag{3.79}$$

This is simply a perfect fluid with energy-momentum tensor (3.64) with

$$\begin{aligned} \rho_\Lambda &= \frac{\Lambda}{8\pi G} \\ p_\Lambda &= -\rho_\Lambda \end{aligned} \tag{3.80}$$

so that the equation-of-state parameter is

$$w_\Lambda = -1 \tag{3.81}$$

This implies that the energy density is constant,

$$\rho_\Lambda = \text{constant} \tag{3.82}$$

Thus, this energy is constant throughout spacetime; we say that the cosmological constant is equivalent to *vacuum energy.*

Similarly, it is sometimes useful to think of any nonzero spatial curvature as yet another component of the cosmological energy budget, obeying

$$\begin{aligned} \rho_{\text{curv}} &= -\frac{3k}{8\pi Ga^2} \\ p_{\text{curv}} &= \frac{k}{8\pi Ga^2} \end{aligned} \tag{3.83}$$

so that

$$w_{\text{curv}} = -1/3 \tag{3.84}$$

It is not an energy density, of course; $\rho_{\text{curv}}$ is simply a convenient way to keep track of how much energy density is lacking, in comparison to a flat universe.

### Flat Universes

It is much easier to find exact solutions to cosmological equations of motion when $k = 0$. Fortunately for us, nowadays we are able to appeal to more than mathematical simplicity to make this choice. Indeed, modern cosmological observations, in particular precision measurements of the cosmic microwave background, show the universe today to be extremely spatially flat.

In the case of flat spatial sections and a constant equation of state parameter $w$, we may exactly solve the Friedmann equation (3.77) to obtain

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{2/3(1+w)} \tag{3.85}$$

| Type of Energy | $\rho(a)$ | $a(t)$ |
|---|---|---|
| Dust | $a^{-3}$ | $t^{2/3}$ |
| Radiation | $a^{-4}$ | $t^{1/2}$ |
| Cosmological Constant | constant | $e^{Ht}$ |

Table 3.2: A summary of the behaviors of the most important sources of energy density in cosmology. The behavior of the scale factor applies to the case of a flat universe; the behavior of the energy densities is perfectly general.

where $a_0$ is the scale factor today, unless $w = -1$, in which case one obtains $a(t) \propto e^{Ht}$. Applying this result to some of our favorite energy density sources yields Table 3.2.

Note that the matter- and radiation-dominated flat universes begin with $a = 0$; this is a singularity, known as the Big Bang. We can easily calculate the age of such a universe:

$$t_0 = \int_0^1 \frac{da}{aH(a)} = \frac{2}{3(1+w)H_0} \tag{3.86}$$

Unless $w$ is close to $-1$, it is often useful to approximate this answer by

$$t_0 \sim H_0^{-1} \tag{3.87}$$

It is for this reason that the quantity $H_0^{-1}$ is known as the *Hubble time*, and provides a useful estimate of the time scale for which the universe has been around.

**Horizons**

One of the most crucial concepts to master about FRW models is the existence of *horizons*. This concept will prove useful in understanding the shortcomings of what we are terming the standard cosmology.

Suppose an emitter, $e$, sends a light signal to an observer, $o$, who is at $r = 0$. Setting $\theta =$ constant and $\phi =$ constant and working in conformal time, for such radial null rays we have $\tau_o - \tau = r$. In particular this means that

$$\tau_o - \tau_e = r_e \tag{3.88}$$

Now suppose $\tau_e$ is bounded below by $\bar{\tau}_e$; for example, $\bar{\tau}_e$ might represent the Big Bang singularity. Then there exists a maximum distance to which the observer can see, known as the *particle horizon distance*, given by

$$r_{\rm ph}(\tau_o) = \tau_o - \bar{\tau}_e \tag{3.89}$$

Similarly, suppose $\tau_o$ is bounded above by $\bar{\tau}_o$. Then there exists a limit to spacetime events which can be influenced by the emitter. This limit is known as the *event horizon distance*, given by

$$r_{\rm eh}(\tau_o) = \bar{\tau}_o - \tau_e \tag{3.90}$$

These horizon distances may be converted to *proper horizon distances* at cosmic time $t$, for example

$$d_H \equiv a(\tau) r_{\rm ph} = a(\tau)(\tau - \bar{\tau}_e) = a(t) \int_{t_e}^{t} \frac{dt'}{a(t')} \tag{3.91}$$

Just as the Hubble time $H_0^{-1}$ provides a rough guide for the age of the universe, the Hubble distance $cH_0^{-1}$ provides a rough estimate of the horizon distance in a matter- or radiation-dominated universe.

### 3.2.2 The early Universe

In this subsection we use what we know of the laws of physics and the universe today to infer conditions in the early universe. Early times were characterized by very high temperatures and densities, with many particle species kept in (approximate) thermal equilibrium by rapid interactions. We will therefore have to move beyond a simple description of non-interacting "matter" and "radiation," and discuss how thermodynamics works in an expanding universe.

#### Describing Matter

We have discussed how to describe matter as a perfect fluid, described by an energy-momentum tensor

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu} \tag{3.92}$$

where $U^\mu$ is the fluid four-velocity, $\rho$ is the energy density in the rest frame of the fluid and $p$ is the pressure in that same frame. The energy-momentum tensor is covariantly conserved,

$$\nabla_\mu T^{\mu\nu} = 0 \tag{3.93}$$

In a more complete description, a fluid will be characterized by quantities in addition to the energy density and pressure. Many fluids have a conserved quantity associated with them and so we will also introduce a *number flux density* $N^\mu$, which is also conserved

$$\nabla_\mu N^\mu = 0 \tag{3.94}$$

For non-tachyonic matter $N^\mu$ is a timelike 4-vector and therefore we may decompose it as

$$N^\mu = nU^\mu \tag{3.95}$$

We can also introduce an *entropy flux density* $S^\mu$. This quantity is not conserved, but rather obeys a covariant version of the second law of thermodynamics

$$\nabla_\mu S^\mu \geq 0 \tag{3.96}$$

Not all phenomena are successfully described in terms of such a local entropy vector (e.g. black holes); fortunately, it suffices for a wide variety of fluids relevant to cosmology.

The conservation law for the energy-momentum tensor yields, most importantly, equation (3.75), which can be thought of as the first law of thermodynamics

$$dU = TdS - pdV \tag{3.97}$$

with $dS = 0$.

It is useful to resolve $S^\mu$ into components parallel and perpendicular to the fluid 4-velocity

$$S^\mu = sU^\mu + s^\mu \tag{3.98}$$

where $s_\mu U^\mu = 0$. The scalar $s$ is the rest-frame entropy density which, up to an additive constant (that we can consistently set to zero), can be written as

$$s = \frac{\rho + p}{T} \tag{3.99}$$

In addition to all these quantities, we must specify an equation of state, and we typically do this in such a way as to treat $n$ and $s$ as independent variables.

### Particles in Equilibrium

The various particles inhabiting the early universe can be usefully characterized according to three criteria: in equilibrium vs. out of equilibrium (decoupled), bosonic vs. fermionic, and relativistic (velocities near $c$) vs. non-relativistic. In this section we consider species which are in equilibrium with the surrounding thermal bath.

Let us begin by discussing the conditions under which a particle species will be in equilibrium with the surrounding thermal plasma. A given species remains in thermal equilibrium as long as its interaction rate is larger than the expansion rate of the universe. Roughly speaking, equilibrium requires it to be possible for the products of a given reaction to have the opportunity to recombine in the reverse reaction and if the expansion of the universe is rapid enough this won't happen. A particle species for which the interaction rates have fallen below the expansion rate of the universe is said to have *frozen out* or *decoupled*. If the interaction rate of some particle with the background plasma is $\Gamma$, it will be decoupled whenever

$$\Gamma \ll H \tag{3.100}$$

where the Hubble constant $H$ sets the cosmological timescale.

As a good rule of thumb, the expansion rate in the early universe is "slow," and particles tend to be in thermal equilibrium (unless they are very weakly coupled). This can be seen from the Friedmann equation when the energy density is dominated by a plasma with $\rho \sim T^4$; we then have

$$H \sim \left(\frac{T}{M_{\rm Pl}}\right) T \tag{3.101}$$

Thus, the Hubble parameter is suppressed with respect to the temperature by a factor of $T/M_{\rm Pl}$. At extremely early times (near the Planck era, for example), the universe may be expanding so quickly that no species are in equilibrium; as the expansion rate slows, equilibrium becomes possible. However, the interaction rate $\Gamma$ for a particle with cross-section $\sigma$ is typically of the form

$$\Gamma = n\langle\sigma v\rangle \ , \tag{3.102}$$

where $n$ is the number density and $v$ a typical particle velocity. Since $n \propto a^{-3}$, the density of particles will eventually dip so low that equilibrium can once again no longer be maintained. In our current universe, no species are in equilibrium with the background plasma (represented by the CMB photons).

Now let us focus on particles in equilibrium. For a gas of weakly-interacting particles, we can describe the state in terms of a *distribution function* $f(\mathbf{p})$, where the three-momentum $\mathbf{p}$ satisfies

$$E^2(\mathbf{p}) = m^2 + |\mathbf{p}|^2 \tag{3.103}$$

| | Relativistic Bosons | Relativistic Fermions | Non-relativistic (Either) |
|---|---|---|---|
| $n_i$ | $\frac{\zeta(3)}{\pi^2} g_i T^3$ | $\left(\frac{3}{4}\right) \frac{\zeta(3)}{\pi^2} g_i T^3$ | $g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T}$ |
| $\rho_i$ | $\frac{\pi^2}{30} g_i T^4$ | $\left(\frac{7}{8}\right) \frac{\pi^2}{30} g_i T^4$ | $m_i n_i$ |
| $p_i$ | $\frac{1}{3}\rho_i$ | $\frac{1}{3}\rho_i$ | $n_i T \ll \rho_i$ |

Table 3.3: Number density, energy density, and pressure, for species in thermal equilibrium.

The distribution function characterizes the density of particles in a given momentum bin. (In general it will also be a function of the spatial position $\mathbf{x}$, but we suppress that here.) The number density, energy density, and pressure of some species labeled $i$ are given by

$$\begin{aligned} n_i &= \frac{g_i}{(2\pi)^3} \int f_i(\mathbf{p}) d^3p \\ \rho_i &= \frac{g_i}{(2\pi)^3} \int E(\mathbf{p}) f_i(\mathbf{p}) d^3p \\ p_i &= \frac{g_i}{(2\pi)^3} \int \frac{|\mathbf{p}|^2}{3E(\mathbf{p})} f_i(\mathbf{p}) d^3p \end{aligned} \tag{3.104}$$

where $g_i$ is the number of spin states of the particles. For massless photons we have $g_\gamma = 2$, while for a massive vector boson such as the $Z$ we have $g_Z = 3$. In the usual accounting, particles and antiparticles are treated as separate species; thus, for spin-1/2 electrons and positrons we have $g_{e^-} = g_{e^+} = 2$. In thermal equilibrium at a temperature $T$ the particles will be in either Fermi-Dirac or Bose-Einstein distributions,

$$f(\mathbf{p}) = \frac{1}{e^{E(\mathbf{p})/T} \pm 1} \tag{3.105}$$

where the plus sign is for fermions and the minus sign for bosons.

We can do the integrals over the distribution functions in two opposite limits: particles which are highly relativistic ($T \gg m$) or highly non-relativistic ($T \ll m$). The results are shown in Table 3.3, in which $\zeta$ is the Riemann zeta function, and $\zeta(3) \approx 1.202$.

From Table 3.3 we can extract several pieces of relevant information. Relativistic particles, whether bosons or fermions, remain in approximately equal abundances in equilibrium. Once they become non-relativistic, however, their abundance plummets, and becomes exponentially suppressed with respect to the relativistic species. This is simply because it becomes progressively harder for massive particle-antiparticle pairs to be produced in a plasma with $T \ll m$.

It is interesting to note that, although matter is much more dominant than radiation in the universe today, since their energy densities scale differently the early universe was radiation-dominated. We can write the ratio of the density parameters in matter and radiation as

$$\frac{\Omega_{\rm M}}{\Omega_{\rm R}} = \frac{\Omega_{\rm M0}}{\Omega_{\rm R0}} \left(\frac{a}{a_0}\right) = \frac{\Omega_{\rm M0}}{\Omega_{\rm R0}} (1+z)^{-1} \tag{3.106}$$

The redshift of matter-radiation equality is thus

$$1 + z_{\rm eq} = \frac{\Omega_{\rm M0}}{\Omega_{\rm R0}} \approx 3 \times 10^3 \tag{3.107}$$

This expression assumes that the particles that are non-relativistic today were also non-relativistic at $z_{\rm eq}$; this should be a safe assumption, with the possible exception of massive neutrinos, which make a minority contribution to the total density.

At this point we should stress that even decoupled photons maintain a thermal distribution; this is not because they are in equilibrium, but simply because the distribution function redshifts into a similar distribution with a lower temperature proportional to $1/a$. We can therefore speak of the "effective temperature" of a relativistic species that freezes out at a temperature $T_f$ and scale factor $a_f$:

$$T_i^{\rm rel}(a) = T_f \left(\frac{a_f}{a}\right) \tag{3.108}$$

For example, neutrinos decouple at a temperature around $1 \sim MeV$; shortly thereafter, electrons and positrons annihilate into photons, dumping energy (and entropy) into the plasma but leaving the neutrinos unaffected. Consequently, we expect a neutrino background in the current universe with a temperature of approximately 2K, while the photon temperature is 3K.

A similar effect occurs for particles which are non-relativistic at decoupling, with one important difference. For non-relativistic particles the temperature is proportional to the kinetic energy $\frac{1}{2}mv^2$, which redshifts as $1/a^2$. We therefore have

$$T_i^{\rm non-rel}(a) = T_f \left(\frac{a_f}{a}\right)^2 \tag{3.109}$$

In either case we are imagining that the species freezes out while relativistic /non-relativistic and stays that way afterward; if it freezes out while relativistic and subsequently becomes non-relativistic, the distribution function will be distorted away from a thermal spectrum.

The notion of an effective temperature allows us to define a corresponding notion of an effective number of relativistic degrees of freedom, which in turn permits a compact expression for the total relativistic energy density. The effective number of relativistic degrees of freedom (as far as energy is concerned) can be defined as

$$g_* = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4 \tag{3.110}$$

(The temperature $T$ is the actual temperature of the background plasma, assumed to be in equilibrium.) Then the total energy density in all relativistic species comes from adding the contributions of each species, to obtain the simple formula

$$\rho = \frac{\pi^2}{30} g_* T^4 \tag{3.111}$$

We can do the same thing for the entropy density. From (3.99), the entropy density in relativistic particles goes as $T^3$ rather than $T^4$, so we define the effective number of relativistic degrees of freedom for entropy as

$$g_{*S} = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3 \tag{3.112}$$

The entropy density in relativistic species is then

$$s = \frac{2\pi}{45} g_{*S} T^3 \tag{3.113}$$

Numerically, $g_*$ and $g_{*S}$ will typically be very close to each other. In the Standard Model, we have

$$g_* \approx g_{*S} \sim \begin{cases} 100, & T > 300\ MeV \\ 10, & 1\ MeV < T < 300\ MeV \\ 3, & T < 1\ MeV \end{cases} \tag{3.114}$$

The events that change the effective number of relativistic degrees of freedom are the QCD phase transition at $300 \sim MeV$, and the annihilation of electron/positron pairs at $1 \sim MeV$.

Because of the release of energy into the background plasma when species annihilate, it is only an approximation to say that the temperature goes as

$T \propto 1/a$. A better approximation is to say that the comoving entropy density is conserved,

$$s \propto a^{-3} \tag{3.115}$$

This will hold under all forms of adiabatic evolution; entropy will only be produced at a process like a first-order phase transition or an out-of-equilibrium decay. (In fact, we expect that the entropy production from such processes is very small compared to the total entropy, and adiabatic evolution is an excellent approximation for almost the entire early universe.) Combining entropy conservation with the expression (3.113) for the entropy density in relativistic species, we obtain a better expression for the evolution of the temperature,

$$T \propto g_{*S}^{-1/3} a^{-1} \tag{3.116}$$

The temperature will consistently decrease under adiabatic evolution in an expanding universe, but it decreases more slowly when the effective number of relativistic degrees of freedom is diminished.

### Thermal Relics

As we have mentioned, particles typically do not stay in equilibrium forever; eventually the density becomes so low that interactions become infrequent, and the particles freeze out. Since essentially all of the particles in our current universe fall into this category, it is important to study the relic abundance of decoupled species. (Of course it is also possible to obtain a significant relic abundance for particles which were never in thermal equilibrium; examples might include baryons produced by GUT baryogenesis, or axions produced by vacuum misalignment.) In this subsection we will typically neglect factors of order unity.

We have seen that relativistic, or *hot*, particles have a number density that is proportional to $T^3$ in equilibrium. Thus, a species $X$ that freezes out while still relativistic will have a number density at freeze-out $T_f$ given by

$$n_X(T_f) \sim T_f^3 \tag{3.117}$$

Since this is comparable to the number density of photons at that time, and after freeze-out both photons and our species $X$ just have their number densities dilute by a factor $a(t)^{-3}$ as the universe expands, it is simple to see that the abundance of $X$ particles today should be comparable to the abundance of CMB photons,

$$n_{X0} \sim n_{\gamma 0} \sim 10^2 \text{ cm}^{-3} \tag{3.118}$$

We express this number as $10^2$ rather than 411 since the roughness of our estimate does not warrant such misleading precision. The leading correction to this value is typically due to the production of additional photons subsequent to the decoupling of $X$; in the Standard Model, the number density of photons increases by a factor of approximately 100 between the electroweak phase transition and today, and a species which decouples during this period will be diluted by a factor of between 1 and 100 depending on precisely when it freezes out. So, for example, neutrinos which are light ($m_\nu <$ MeV) have a number density today of $n_\nu = 115\ \mathrm{cm}^{-3}$ per species, and a corresponding contribution to the density parameter (if they are nevertheless heavy enough to be nonrelativistic today) of

$$\Omega_{0,\nu} = \left(\frac{m_\nu}{92\ \mathrm{eV}}\right) h^{-2} \tag{3.119}$$

(In this final expression we have secretly taken account of the missing numerical factors, so this is a reliable answer.) Thus, a neutrino with $m_\nu \sim 10^{-2}$ eV would contribute $\Omega_\nu \sim 2 \times 10^{-4}$. This is large enough to be interesting without being large enough to make neutrinos be the dark matter. That's good news, since the large velocities of neutrinos make them free-stream out of overdense regions, diminishing primordial perturbations and leaving us with a universe which has much less structure on small scales than we actually observe.

Now consider instead a species $X$ which is nonrelativistic or *cold* at the time of decoupling. It is much harder to accurately calculate the relic abundance of a cold relic than a hot one, simply because the equilibrium abundance of a nonrelativistic species is changing rapidly with respect to the background plasma, and we have to be quite precise following the freeze-out process to obtain a reliable answer. The accurate calculation typically involves numerical integration of the Boltzmann equation for a network of interacting particle species; here, we cut to the chase and simply provide a reasonable approximate expression. If $\sigma_0$ is the annihilation cross-section of the species $X$ at a temperature $T = m_X$, the final number density in terms of the photon density works out to be

$$n_X(T < T_f) \sim \frac{1}{\sigma_0 m_X M_P} n_\gamma \tag{3.120}$$

Since the particles are nonrelativistic when they decouple, they will certainly be nonrelativistic today, and their energy density is

$$\rho_X = m_X n_X \tag{3.121}$$

We can plug in numbers for the Hubble parameter and photon density to obtain the density parameter,

$$\Omega_X = \frac{\rho_X}{\rho_{\rm cr}} \sim \frac{n_\gamma}{\sigma_0 M_{\rm Pl}^3 H_0^2} \tag{3.122}$$

Numerically, when $\hbar = c = 1$ we have 1 GeV$\sim 2 \times 10^{-14}$ cm, so the photon density today is $n_\gamma \sim 100$ cm$^{-3}$ $\sim 10^{-39}$ GeV$^{-3}$. The Hubble constant is $H_0 \sim 10^{-42}$ GeV, and the Planck mass is $M_{\rm Pl} \sim 10^{18}$ GeV, so we obtain

$$\Omega_X \sim \frac{1}{\sigma_0 (10^9 \text{ GeV}^2)} \tag{3.123}$$

It is interesting to note that this final expression is independent of the mass $m_X$ of our relic, and only depends on the annihilation cross-section; that's because more massive particles will have a lower relic abundance. Of course, this depends on how we choose to characterize our theory; we may use variables in which $\sigma_0$ is a function of $m_X$, in which case it is reasonable to say that the density parameter does depend on the mass.

One candidate for Cold Dark Matter (CDM) is a Weakly Interacting Massive Particle (WIMP). The annihilation cross-section of these particles, since they are weakly interacting, should be $\sigma_0 \sim \alpha_W^2 G_F$, where $\alpha_W$ is the weak coupling constant and $G_F$ is the the Fermi constant. Using $G_F \sim$ (300GeV)$^{-2}$ and $\alpha_W \sim 10^{-2}$, we get

$$\sigma_0 \sim \alpha_W^2 G_F \sim 10^{-9} \text{ GeV}^{-2} \tag{3.124}$$

Thus, the density parameter in such particles would be

$$\Omega_X \sim 1 \tag{3.125}$$

In other words, a stable particle with a weak interaction cross section naturally produces a relic density of order the critical density today, and so provides a perfect candidate for cold dark matter. A paradigmatic example is provided by the lightest supersymmetric partner (LSP), if it is stable and supersymmetry is broken at the weak scale. Such a possibility is of great interest to both particle physicists and cosmologists, since it may be possible to produce and detect such particles in colliders and to directly detect a WIMP background in cryogenic detectors in underground laboratories; this will be a major experimental effort over the next few years.

### Baryogenesis

The symmetry between particles and antiparticles, firmly established in collider physics, naturally leads to the question of why the observed universe is composed almost entirely of matter and no primordial antimatter.

If large domains of matter and antimatter exist, then annihilations would take place at the interface between them. If the typical size of such a domain was small enough, then the energy released by these annihilations would result in a diffuse $\gamma$-ray background and a distortion of the cosmic microwave radiation, neither of which is observed.

While the above considerations put an experimental upper bound on the amount of antimatter in the universe, strict quantitative estimates of the relative abundances of baryonic matter and antimatter may also be obtained from the standard cosmology. The baryon number density does not remain constant during the evolution of the universe, instead scaling like $a^{-3}$, where $a$ is the cosmological scale factor. It is therefore convenient to define the baryon asymmetry of the universe in terms of the quantity

$$\eta = \frac{n_B}{s} \tag{3.126}$$

where $s$ is the entropy density and $n_B$ is the difference between the baryon number density and the anti-baryon number density. The range of $\eta$ consistent with the observational data is

$$2.6 \times 10^{-10} < \eta < 6.2 \times 10^{-10} \tag{3.127}$$

Thus the natural question arises: As the universe cooled from early times to today, what processes, both particle physics and cosmological, were responsible for the generation of this very specific baryon asymmetry?

As pointed out by Sakharov, a small baryon asymmetry $\eta$ may have been produced in the early universe if three necessary conditions are satisfied

- Baryon number (B) violation
- Violation of C (charge conjugation symmetry) and CP (the combination of C and parity)
- departure from thermal equilibrium

The first condition should be clear since, starting from a baryon symmetric universe with $\eta = 0$, baryon number violation must take place in order to evolve into a universe in which $\eta$ does not vanish. The second Sakharov criterion is required because, if $C$ and $CP$ are exact symmetries, one can prove that the total rate for any process which produces an excess of baryons is equal to the rate of the complementary process which produces an excess of antibaryons and so no net baryon number can be created. That is to say that the thermal average of the baryon number operator $B$, which is odd under both $C$ and $CP$, is zero unless those discrete symmetries are violated. $CP$ violation is present either if there are complex phases in the Lagrangian which cannot be reabsorbed by field redefinitions (explicit breaking) or if some Higgs scalar field acquires a VEV which is not real (spontaneous breaking). We will discuss this in detail shortly.

Finally, to explain the third criterion, one can calculate the equilibrium average of $B$ at a temperature $T = 1/\beta$:

$$\begin{aligned} \langle B \rangle_T &= \mathrm{Tr}\,(e^{-\beta H} B) = \mathrm{Tr}\,[(CPT)(CPT)^{-1} e^{-\beta H} B)] \\ &= \mathrm{Tr}\,(e^{-\beta H} (CPT)^{-1} B (CPT)] = -\mathrm{Tr}\,(e^{-\beta H} B) \end{aligned} \tag{3.128}$$

where we have used that the Hamiltonian $H$ commutes with $CPT$. Thus $\langle B \rangle_T = 0$ in equilibrium and there is no generation of net baryon number.

Of the three Sakharov conditions, baryon number violation and $C$ and $CP$ violation may be investigated only within a given particle physics model, while the third condition – the departure from thermal equilibrium – may be discussed in a more general way, as we shall see. Let us discuss the Sakharov criteria in more detail.

**Baryon Number Violation**

Grand Unified Theories (GUTs) [5] describe the fundamental interactions by means of a unique gauge group $G$ which contains the Standard Model (SM) gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The fundamental idea of GUTs is that at energies higher than a certain energy threshold $M_{\mathrm{GUT}}$ the group symmetry is $G$ and that, at lower energies, the symmetry is broken down to the SM gauge symmetry, possibly through a chain of symmetry breakings. The main motivation for this scenario is that, at least in supersymmetric models, the (running) gauge couplings of the SM unify at the scale $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV, hinting at the presence of a GUT involving a higher symmetry with a single gauge coupling.

Baryon number violation seems very natural in GUTs. Indeed, a general property of these theories is that the same representation of $G$ may contain both quarks and leptons, and therefore it is possible for scalar and gauge bosons to mediate gauge interactions among fermions having different baryon number.

It is well-known that the most general renormalizable Lagrangian invariant under the SM gauge group and containing only color singlet Higgs fields is automatically invariant under global abelian symmetries which may be identified with the baryonic and leptonic symmetries. These, therefore, are accidental symmetries and as a result it is not possible to violate $B$ and $L$ at tree-level or at any order of perturbation theory. Nevertheless, in many cases the perturbative expansion does not describe all the dynamics of the theory and, indeed, in 1976 't Hooft realized that nonperturbative effects (instantons) may give rise to processes which violate the combination $B + L$, but not the orthogonal combination $B - L$. The probability of these processes occurring today is exponentially suppressed and probably irrelevant. However, in more extreme situations – like the primordial universe at very high temperatures – baryon and lepton number violating processes may be fast enough to play a significant role in baryogenesis.

### CP violation

$CP$ violation in GUTs arises in loop-diagram corrections to baryon number violating bosonic decays. Since it is necessary that the particles in the loop also undergo $B$-violating decays, the relevant particles are the $X$, $Y$, and $H_3$ bosons in the case of $SU(5)$.

In the electroweak theory things are somewhat different. Since only the left-handed fermions are $SU(2)_L$ gauge coupled, $C$ is maximally broken in the SM. Moreover, $CP$ is known not to be an exact symmetry of the weak interactions. This is seen experimentally in the neutral kaon system through $K_0$, $\bar{K}_0$ mixing. Thus, $CP$ violation is a natural feature of the standard electroweak model.

While this is encouraging for baryogenesis, it turns out that this particular source of $CP$ violation is not strong enough. The relevant effects are parameterized by a dimensionless constant which is no larger than $10^{-20}$. This appears to be much too small to account for the observed BAU and, thus far, attempts to utilize this source of CP violation for electroweak baryogenesis have been unsuccessful. In light of this, it is usual to extend the SM in some fashion that increases the amount of $CP$ violation in the theory while not leading to results that conflict with current experimental data. One concrete example of a well-motivated extension in the minimal supersymmetric standard model (MSSM).

In some scenarios, such as GUT baryogenesis, the third Sakharov condition is satisfied due to the presence of superheavy decaying particles in a rapidly expanding universe. These generically fall under the name of out-of-equilibrium decay mechanisms.

The underlying idea is fairly simple. If the decay rate $\Gamma_X$ of the superheavy particles $X$ at the time they become nonrelativistic (i.e. at the temperature $T \sim M_X$) is much smaller than the expansion rate of the universe, then the $X$ particles cannot decay on the time scale of the expansion and so they remain as abundant as photons for $T \lesssim M_X$. In other words, at some temperature $T > M_X$, the superheavy particles $X$ are so weakly interacting that they cannot catch up with the expansion of the universe and they decouple from the thermal bath while they are still relativistic, so that $n_X \sim n_\gamma \sim T^3$ at the time of decoupling.

Therefore, at temperature $T \simeq M_X$, they populate the universe with an abundance which is much larger than the equilibrium one. This overabundance is precisely the departure from thermal equilibrium needed to produce a final nonvanishing baryon asymmetry when the heavy states $X$ undergo $B$ and $CP$ violating decays.

The out-of-equilibrium condition requires very heavy states: $M_X \gtrsim (10^{15} - 10^{16})$ GeV and $M_X \gtrsim (10^{10} - 10^{16})$ GeV, for gauge and scalar bosons, respectively, if these heavy particles decay through renormalizable operators.

Since the linear combination $B - L$ is left unchanged by sphaleron transitions, the baryon asymmetry may be generated from a lepton asymmetry. Indeed, sphaleron transition will reprocess any lepton asymmetry and convert (a fraction of) it into baryon number. This is because $B + L$ must be vanishing and the final baryon asymmetry results to be $B \simeq -L$.

In the SM as well as in its unified extension based on the group $SU(5)$, $B - L$ is conserved and no asymmetry in $B - L$ can be generated. However, adding right-handed Majorana neutrinos to the SM breaks $B - L$ and the primordial lepton asymmetry may be generated by the out-of-equilibrium decay of heavy right-handed Majorana neutrinos $N_L^c$ (in the supersymmetric version, heavy scalar neutrino decays are also relevant for leptogenesis). This simple extension of the SM can be embedded into GUTs with gauge groups containing $SO(10)$. Heavy right-handed Majorana neutrinos can also explain the smallness of the light neutrino masses via the see-saw mechanism.

### 3.2.3 Inflation

So far we have described what is known as *the standard cosmology.* This framework is a towering achievement, describing to great accuracy the physical processes leading to the present day universe. However, there remain outstanding issues in cosmology. Many of these come under the heading of initial condition problems and require a more complete description of the sources of energy density in the universe. The most severe of these problems

| Fixed Point | $1 + 3w > 0$ | $1 + 3w < 0$ |
|---|---|---|
| $\Omega = 0$ | attractor | repeller |
| $\Omega = 1$ | repeller | attractor |
| $\Omega = \infty$ | attractor | repeller |

Table 3.4: Behavior of the density parameter near fixed points.

eventually led to a radical new picture of the physics of the early universe - *cosmological inflation*, which is the subject of this subsection.

We will begin by describing some of the problems of the standard cosmology.

### The Flatness Problem

The Friedmann equation may be written as

$$\Omega - 1 = \frac{k}{H^2 a^2} \tag{3.129}$$

where for brevity we are now writing $\Omega$ instead of $\Omega_{\text{total}}$. Differentiating this with respect to the scale factor, this implies

$$\frac{d\Omega}{da} = (1 + 3w)\frac{\Omega(\Omega - 1)}{a} \tag{3.130}$$

This equation is easily solved, but its most general properties are all that we shall need and they are qualitatively different depending on the sign of $1 + 3w$. There are three fixed points of this differential equation, as given in Table 3.4.

Observationally we know that $\Omega \simeq 1$ today – i.e. we are very close to the repeller of this differential equation for a universe dominated by ordinary matter and radiation ($w > -1/3$). Even if we only took account of the luminous matter in the universe, we would clearly live in a universe that was far from the attractor points of the equation. It is already quite puzzling that the universe has not reached one of its attractor points, given that the universe has evolved for such a long time. However, we may be more quantitative about this. If the only matter in the universe is radiation and dust, then in order to have $\Omega$ in the range observed today requires (conservatively)

$$0 \leq 1 - \Omega \leq 10^{-60} \tag{3.131}$$

This remarkable degree of fine tuning is the flatness problem. Within the context of the standard cosmology there is no known explanation of this fine-tuning.

### The Horizon Problem

The *horizon problem* stems from the existence of particle horizons in FRW cosmologies, as discussed in a previous subsection. Horizons exist because there is only a finite amount of time since the Big Bang singularity, and thus only a finite distance that photons can travel within the age of the universe. Consider a photon moving along a radial trajectory in a flat universe (the generalization to non-flat universes is straightforward). In a flat universe, we can normalize the scale factor to

$$a_0 = 1 \tag{3.132}$$

without loss of generality. A radial null path obeys

$$0 = ds^2 = -dt^2 + a^2 dr^2 \tag{3.133}$$

so the comoving (coordinate) distance traveled by such a photon between times $t_1$ and $t_2$ is

$$\Delta r = \int_{t_1}^{t_2} \frac{dt}{a(t)} \tag{3.134}$$

To get the physical distance as it would be measured by an observer at any time $t$, simply multiply by $a(t)$. For simplicity let's imagine we are in a matter-dominated universe, for which

$$a = \left(\frac{t}{t_0}\right)^{2/3} \tag{3.135}$$

The Hubble parameter is therefore given by

$$\begin{aligned} H &= \frac{2}{3} t^{-1} \\ &= a^{-3/2} H_0 \end{aligned} \tag{3.136}$$

Then the photon travels a comoving distance

$$\Delta r = 2H_0^{-1}\left(\sqrt{a_2} - \sqrt{a_1}\right) \tag{3.137}$$

The comoving horizon size when $a = a_*$ is the distance a photon travels since the Big Bang,

$$r_{\rm hor}(a_*) = 2H_0^{-1}\sqrt{a_*} \tag{3.138}$$

The physical horizon size, as measured on the spatial hypersurface at $a_*$, is therefore simply

$$d_{\rm hor}(a_*) = a_* r_{\rm hor}(a_*) = 2H_*^{-1} \tag{3.139}$$

Indeed, for any nearly-flat universe containing a mixture of matter and radiation, at any one epoch we will have

$$d_{\rm hor}(a_*) \sim H_*^{-1} \tag{3.140}$$

where $H_*^{-1}$ is the Hubble distance at that particular epoch. This approximate equality leads to a strong temptation to use the terms "horizon distance" and "Hubble distance" interchangeably; this temptation should be resisted, since inflation can render the former much larger than the latter, as we will soon demonstrate.

The horizon problem is simply the fact that the CMB is isotropic to a high degree of precision, even though widely separated points on the last scattering surface are completely outside each others' horizons. When we look at the CMB we were observing the universe at a scale factor $a_{\rm CMB} \approx 1/1200$; meanwhile, the comoving distance between a point on the CMB and an observer on Earth is

$$\begin{aligned} \Delta r &= 2H_0^{-1}(1-\sqrt{a_{\rm CMB}}) \\ &\approx 2H_0^{-1} \end{aligned} \tag{3.141}$$

However, the comoving horizon distance for such a point is

$$\begin{aligned} r_{\rm hor}(a_{\rm CMB}) &= 2H_0^{-1}\sqrt{a_{\rm CMB}} \\ &\approx 6\times 10^{-2}H_0^{-1} \end{aligned} \tag{3.142}$$

Hence, if we observe two widely-separated parts of the CMB, they will have non-overlapping horizons; distinct patches of the CMB sky were causally disconnected at recombination. Nevertheless, they are observed to be at the same temperature to high precision. The question then is, how did they know ahead of time to coordinate their evolution in the right way, even though they were never in causal contact? We must somehow modify the causal structure of the conventional FRW cosmology.

### Unwanted Relics

We have already talked about grand unified theories (GUTs). If grand unification occurs with a simple gauge group $G$, any spontaneous breaking of $G$ satisfies $\pi_2(G/H) = \pi_1(H)$ for any simple subgroup $H$. In particular, breaking down to the standard model will lead to magnetic monopoles [6], since

$$\pi_2(G/H) = \pi_1([SU(3)\times SU(2)\times U(1)]/\mathcal{Z}_6) = \mathcal{Z} \tag{3.143}$$

(The gauge group of the standard model is, strictly speaking, $[SU(3) \times SU(2) \times U(1)]/\mathcal{Z}_6$. The $\mathcal{Z}_6$ factor only affects the global structure of the group and not the Lie algebra, and thus is usually ignored by particle physicists.)

Using the Kibble mechanism, the expected relic abundance of monopoles works out to be

$$\Omega_{0,\text{mono}} \sim 10^{11} \left(\frac{T_{\text{GUT}}}{10^{14}\ \text{GeV}}\right)^3 \left(\frac{m_{\text{mono}}}{10^{16}\ \text{GeV}}\right) \tag{3.144}$$

This is far too big; the monopole abundance in GUTs is a serious problem for cosmology if GUTs have anything to do with reality.

In addition to monopoles, there may be other model-dependent relics predicted by our favorite theory. If these are incompatible with current limits, it is necessary to find some way to dilute their density in the early universe.

### The General Idea of Inflation

The horizon problem especially is an extremely serious problem for the standard cosmology because at its heart is simply causality. Any solution to this problem is therefore almost certain to require an important modification to how information can propagate in the early universe. Cosmological inflation is such a mechanism.

Before getting into the details of inflation we will just sketch the general idea here. The fundamental idea is that the universe undergoes a period of accelerated expansion, defined as a period when $\ddot{a} > 0$, at early times. The effect of this acceleration is to quickly expand a small region of space to a huge size, diminishing spatial curvature in the process, making the universe extremely close to flat. In addition, the horizon size is greatly increased, so that distant points on the CMB actually are in causal contact and unwanted relics are tremendously diluted, solving the monopole problem. As an unexpected bonus, quantum fluctuations make it impossible for inflation to smooth out the universe with perfect precision, so there is a spectrum of remnant density perturbations; this spectrum turns out to be approximately scale-free, in good agreement with observations of our current universe.

### Slowly-Rolling Scalar Fields

If inflation is to solve the problems of the standard cosmology, then it must be active at extremely early times. Thus, we would like to address the earliest times in the universe amenable to a classical description. We expect this to be at or around the Planck time $t_P$ and since Planckian quantities arise often in inflation we will retain values of the Planck mass in the equations of this section. There are *many* models of inflation, but because of time constraints we will concentrate almost exclusively on the *chaotic inflation* model of Linde. We have borrowed heavily in places here from the excellent text of Liddle and Lyth.

Consider modeling matter in the early universe by a real scalar field $\phi$, with potential $V(\phi)$. The energy-momentum tensor for $\phi$ is

$$T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}(\nabla_\alpha \phi)(\nabla_\beta \phi) + V(\phi)\right] \tag{3.145}$$

For simplicity we will specialize to the homogeneous case, in which all quantities depend only on cosmological time $t$ and set $k = 0$. A homogeneous real scalar field behaves as a perfect fluid with

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{3.146}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{3.147}$$

The equation of motion for the scalar field is given by

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{3.148}$$

which can be thought of as the usual equation of motion for a scalar field in Minkowski space, but with a friction term due to the expansion of the universe. The Friedmann equation with such a field as the sole energy source is

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \tag{3.149}$$

A very specific way in which accelerated expansion can occur is if the universe is dominated by an energy component that approximates a cosmological constant. In that case the associated expansion rate will be exponential, as we have already seen. Scalar fields can accomplish this in an interesting way. From (3.146) it is clear that if $\dot{\phi}^2 \ll V(\phi)$ then the potential energy of the scalar field is the dominant contribution to both the energy density and the pressure, and the resulting equation of state is $p \simeq -\rho$, approximately that of a cosmological constant. the resulting expansion is certainly accelerating. In a loose sense, this negligible kinetic energy is equivalent to the fields slowly rolling down its potential; an approximation which we will now make more formal.

Technically, the *slow-roll approximation* for inflation involves neglecting the $\ddot{\phi}$ term in (3.148) and neglecting the kinetic energy of $\phi$ compared to the potential energy. The scalar field equation of motion and the Friedmann equation then become

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H} \tag{3.150}$$

$$H^2 \simeq \frac{8\pi G}{3} V(\phi) \tag{3.151}$$

where a prime denotes a derivative with respect to $\phi$.

These conditions will hold if the two *slow-roll conditions* are satisfied. These are

$$\begin{aligned} |\epsilon| &\ll 1 \\ |\eta| &\ll 1 \end{aligned} \tag{3.152}$$

where the *slow-roll parameters* are given by

$$\epsilon \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \tag{3.153}$$

and

$$\eta \equiv M_{\rm Pl}^2 \frac{V''}{V} \tag{3.154}$$

It is easy to see that the slow roll conditions yield inflation. Recall that inflation is defined by $\ddot{a}/a > 0$. We can write

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 \tag{3.155}$$

so that inflation occurs if

$$\frac{\dot{H}}{H^2} > -1 \tag{3.156}$$

But in slow-roll

$$\frac{\dot{H}}{H^2} \simeq -\epsilon \tag{3.157}$$

which will be small. Smallness of the other parameter $\eta$ helps to ensure that inflation will continue for a sufficient period.

It is useful to have a general expression to describe how much inflation occurs, once it has begun. This is typically quantified by the number of *e-folds*, defined by

$$N(t) \equiv \ln\left(\frac{a(t_{\rm end})}{a(t)}\right) \tag{3.158}$$

Usually we are interested in how many efolds occur between a given field value $\phi$ and the field value at the end of inflation $\phi_{\rm end}$, defined by $\epsilon(\phi_{\rm end}) = 1$. We also would like to express $N$ in terms of the potential. Fortunately this is simple to do via

$$N(t) \equiv \ln\left(\frac{a(t_{\rm end})}{a(t)}\right) = \int_t^{t_{\rm end}} H\ dt \simeq \frac{1}{M_{\rm Pl}^2}\int_\phi^{\phi_{\rm end}} \frac{V}{V'}\ d\phi \tag{3.159}$$

The issue of initial conditions for inflation is one that is quite subtle and we will not get into a discussion of that here. Instead we will remain focused on chaotic inflation, in which we assume that the early universe emerges from the Planck epoch with the scalar field taking different values in different parts of the universe, with typically Planckian energies. There will then be some probability for inflation to begin in some places, and we shall focus on those.

### Vacuum Fluctuations and Perturbations

Recall that the structures - clusters and superclusters of galaxies - we see on the largest scales in the universe today, and hence the observed fluctuations in the CMB, form from the gravitational instability of initial perturbations in the matter density. The origin of these initial fluctuations is an important question of modern cosmology.

Inflation provides us with a fascinating solution to this problem - in a nutshell, quantum fluctuations in the inflaton field during the inflationary epoch are stretched by inflation and ultimately become classical fluctuations. Let's sketch how this works.

Since inflation dilutes away all matter fields, soon after its onset the universe is in a pure vacuum state. If we simplify to the case of exponential inflation, this vacuum state is described by the Gibbons-Hawking temperature

$$T_{\rm GH} = \frac{H}{2\pi} \simeq \frac{\sqrt{V}}{M_{\rm Pl}} \tag{3.160}$$

where we have used the Friedmann equation. Because of this temperature, the inflaton experiences fluctuations that are the same for each wavelength

$\delta\phi_k = T_{\rm GH}$. Now, these fluctuations can be related to those in the density by

$$\delta\rho = \frac{dV}{d\phi}\delta\phi \tag{3.161}$$

Inflation therefore produces density perturbations on every scale. The amplitude of the perturbations is nearly equal at each wavenumber, but there will be slight deviations due to the gradual change in $V$ as the inflaton rolls. We can characterize the fluctuations in terms of their spectrum $A_{\rm S}(k)$, related to the potential via

$$A_{\rm S}^2(k) \sim \left.\frac{V^3}{M_{\rm Pl}^6 (V')^2}\right|_{k=aH} \tag{3.162}$$

where $k = aH$ indicates that the quantity $V^3/(V')^2$ is to be evaluated at the moment when the physical scale of the perturbation $\lambda = a/k$ is equal to the Hubble radius $H^{-1}$. Note that the actual normalization of (3.162) is convention-dependent, and should drop out of any physical answer.

The spectrum is given the subscript "S" because it describes scalar fluctuations in the metric. These are tied to the energy-momentum distribution, and the density fluctuations produced by inflation are adiabatic — fluctuations in the density of all species are correlated. The fluctuations are also Gaussian, in the sense that the phases of the Fourier modes describing fluctuations at different scales are uncorrelated. These aspects of inflationary perturbations — a nearly scale-free spectrum of adiabatic density fluctuations with a Gaussian distribution — are all consistent with current observations of the CMB and large-scale structure, and have been confirmed to new precision by WMAP and other CMB measurements.

It is not only the nearly-massless inflaton that is excited during inflation, but any nearly-massless particle. The other important example is the graviton, which corresponds to tensor perturbations in the metric (propagating excitations of the gravitational field). Tensor fluctuations have a spectrum

$$A_{\rm T}^2(k) \sim \left.\frac{V}{M_{\rm Pl}^4}\right|_{k=aH} \tag{3.163}$$

The existence of tensor perturbations is a crucial prediction of inflation which may in principle be verifiable through observations of the polarization of the CMB. Although CMB polarization has already been detected, this is only the $E$-mode polarization induced by density perturbations; the $B$-mode polarization induced by gravitational waves is expected to be at a much lower level, and represents a significant observational challenge for the years to come.

For purposes of understanding observations, it is useful to parameterize the perturbation spectra in terms of observable quantities. We therefore write

$$A_{\mathrm{S}}^2(k) \propto k^{n_{\mathrm{S}}-1} \tag{3.164}$$

and

$$A_{\mathrm{T}}^2(k) \propto k^{n_{\mathrm{T}}} \tag{3.165}$$

where $n_{\mathrm{S}}$ and $n_{\mathrm{T}}$ are the "spectral indices". They are related to the slow-roll parameters of the potential by

$$n_{\mathrm{S}} = 1 - 6\epsilon + 2\eta \tag{3.166}$$

and

$$n_{\mathrm{T}} = -2\epsilon \tag{3.167}$$

Since the spectral indices are in principle observable, we can hope through relations such as these to glean some information about the inflaton potential itself.

Our current knowledge of the amplitude of the perturbations already gives us important information about the energy scale of inflation. Note that the tensor perturbations depend on $V$ alone (not its derivatives), so observations of tensor modes yields direct knowledge of the energy scale. If large-scale CMB anisotropies have an appreciable tensor component (possible, although unlikely), we can instantly derive $V_{\mathrm{inflation}} \sim (10^{16}\ \mathrm{GeV})^4$. (Here, the value of $V$ being constrained is that which was responsible for creating the observed fluctuations; namely, 60 $e$-folds before the end of inflation.) This is remarkably reminiscent of the grand unification scale, which is very encouraging. Even in the more likely case that the perturbations observed in the CMB are scalar in nature, we can still write

$$V_{\mathrm{inflation}}^{1/4} \sim \epsilon^{1/4} 10^{16}\ \mathrm{GeV} \tag{3.168}$$

where $\epsilon$ is the slow-roll parameter defined in (3.153). Although we expect $\epsilon$ to be small, the 1/4 in the exponent means that the dependence on $\epsilon$ is quite weak; unless this parameter is extraordinarily tiny, it is very likely that $V_{\mathrm{inflation}}^{1/4} \sim 10^{15}$-$10^{16}$ GeV.

We should note that this discussion has been phrased in terms of the simplest models of inflation, featuring a single canonical, slowly-rolling scalar field. A number of more complex models have been suggested, allowing for departures from the relations between the slow-roll parameters and observable quantities; some of these include hybrid inflation, inflation with novel kinetic terms, the curvaton model, low-scale models, brane inflation and models where perturbations arise from modulated coupling constants. This list is necessarily incomplete, and continued exploration of the varieties of inflationary cosmology will be a major theme of theoretical cosmology into the foreseeable future.

### Reheating and Preheating

Clearly, one of the great strengths of inflation is its ability to redshift away all unwanted relics, such as topological defects. However, inflation is not discerning, and in doing so any trace of radiation or dust-like matter is similarly redshifted away to nothing. Thus, at the end of inflation the universe contains nothing but the inflationary scalar field condensate. How then does that matter of which we are made arise? How does the hot big bang phase of the universe commence? How is the universe *reheated*?

Inflation ends when the slow-roll conditions are violated and, in most models, the field begins to fall towards the minimum of its potential. Initially, all energy density is in the inflaton, but this is now damped by two possible terms. First, the expansion of the universe naturally damps the energy density. More importantly, the inflaton may decay into other particles, such as radiation or massive particles, both fermionic and bosonic. To take account of this one introduces a phenomenological decay term $\Gamma_\phi$ into the scalar field equation. The inflaton undergoes damped oscillations and decays into radiation which equilibrates rapidly at a temperature known as the *reheat temperature* $T_{\rm RH}$.

# PART II

## Cosmological evolution, inflation and dark matter in brane cosmology

# Chapter 4

# Axino dark matter

We discuss dark matter in the brane world scenario. We work in the Randall-Sundrum type II brane world and assume that the lightest supersymmetric particle is the axino. We find that the axinos can play the role of cold dark matter in the universe, provided that the five-dimensional Planck mass is bounded both from below and from above. This is possible for higher reheating temperatures compared to the conventional four-dimensional cosmology due to a novel expansion law for the universe.

## 4.1 Introduction

There are good theoretical reasons for which particle physics proposes that new exotic particles must exist. The most compelling solution to the strong CP-problem of quantum chromodynamics (QCD), which can be stated as "why is the $\Theta$ parameter in QCD so small?", is the one proposed by Peccei and Quinn [7]. An additional global, chiral symmetry is introduced, now known as Peccei-Quinn (PQ) symmetry, which is spontaneously broken at the PQ scale $f_\alpha \geq 10^8\, GeV$ [8]. Since $U(1)_{PQ}$ is a spontaneously broken global symmetry, there must be a Nambu-Goldstone boson associated with this symmetry. However, because $U(1)_{PQ}$ suffers from a chiral anomaly, this boson is not massless but acquires a small mass. The pseudo Nambu-Goldstone boson associated with this spontaneous symmetry breaking is the axion [9], which has not yet been detected. On the other hand, supersymmetry (SUSY) is an ingredient that appears in many theories for physics beyond the standard model. SUSY solves the hierarchy problem and predicts that every particle we know should be escorted by its superpartner. The axino is the superpartner of the axion. In order for the supersymmetric solution of the hierarchy problem to work, it is necessary that the SUSY becomes manifest at relatively low energies, less than a few $TeV$, and therefore the required superpartners must have masses below this scale (for supersymmetry and supergravity see e.g. [10]).

One of the theoretical problems in modern cosmology is to understand the nature of cold dark matter in the universe. There are good reasons, both observational and theoretical, to suspect that a fraction of 0.22 of the energy density in the universe is in some unknown “dark” form [11]. Many lines of reasoning suggest that the dark matter consists of some new, as yet undiscovered, massive particle which experiences neither electromagnetic nor color interactions. In SUSY models which are realized with R-parity conservation the lightest supersymmetric particle (LSP) is stable. A popular cold dark matter candidate is the LSP, provided that it is electrically and color neutral. Certainly the most theoretically developed LSP is the lightest neutralino. However, there are other dark matter candidates as well, for example the gravitino and the axino. In this work we assume that the axino is the LSP. Axinos are special because they have unique properties: They are very weekly interacting and their mass can span a wide range, from very small ($\sim eV$) to large ($\sim GeV$) values. What is worth stressing is that, in contrast to those of the neutralino and the gravitino, axino mass does not have to be of the order of the SUSY breaking scale in the visible sector, $M_{SUSY} \sim 100GeV - 1TeV$. The first paper to show that the axinos can be CDM was [12]. There are however some early works on axino cosmology (see e.g. [13], [14]).

We believe that some time in its early history, the universe experienced an inflationary phase [15]. According to the inflationary paradigm, during the slow-roll phase of inflation the universe undergoes a rapid expansion, and consequently any initial population of axinos is diluted away. After slow-roll a reheating phase follows and leads the universe to the radiation era of the standard hot Big-Bang cosmology of temperature $T_R$. As the PQ symmetry is restored at $f_\alpha$, we consider only values of $T_R$ up to the PQ scale, which we take to be $f_\alpha = 10^{11}\, GeV$. Another important scale is the temperature $T_D$ at which axinos decouple from the thermal bath. For $T_R > T_D$, there has been an early phase in which axinos were in thermal equilibrium with the thermal bath. The axino density parameter is then given by the equilibrium number density [16]

$$\Omega_{\tilde{\alpha}} h^2 \sim \frac{m_{\tilde{\alpha}}}{2\, keV} \tag{4.1}$$

If we require that $\Omega_{\tilde{\alpha}} h^2 \sim 0.1$ then the axino mass $m_{\tilde{\alpha}} \sim 0.2\ keV$. For an axino mass in the range $m_{\tilde{\alpha}} \leq 1\ keV$, $1\ keV \leq m_{\tilde{\alpha}} \leq 100\ keV$ and

$m_{\tilde{a}} \geq 100keV$, we refer to hot, warm and cold axino dark matter respectively. So we see that for $T_R > T_D$, axinos can only be hot dark matter. For $T_R < T_D$, the axinos are out of thermal equilibrium so that the production mechanisms have to be considered in detail.

In order to generate a large enough abundance of axinos, one needs to repopulate the universe (after inflation) with them. There are two generic ways of achieving this. First, they can be generated through thermal production (TP), namely via scattering and decay processes of ordinary particles and sparticles still in the thermal bath. Second, axinos may also be produced via non-thermal production mechanisms (NTP) possibly present during the reheating phase. In [17] the authors considered both NTP and TP and they found that $T_R$ had to be relatively low, below some $10^6\,GeV$. However NTP mechanisms are strongly model dependent and we shall not consider them here. In [16] the authors using specific techniques (the hard thermal loop resummation technique [18] together with the Braaten-Yuan prescription [19]) computed the thermal production rate of axinos in supersymmetric QCD and evaluated the relic axino abundance. They found that axinos provide the density of cold dark matter observed by WMAP for relatively small reheating temperature after inflation $T_R \leq 10^6\,GeV$, essentially in agreement with [17]. Such a low reheating temperature excludes some models for inflation and the baryon asymmetry in the universe has to be explained by a mechanism that works efficiently at relatively small temperatures, excluding thermal leptogenesis [20] (for thermal leptogenesis in brane world cosmology see [21]).

The purpose of this paper is to show that this fact can be dealt with in the context of the brane world scenario. Our brane world model is the Randall-Sundrum type II model (RSII) [22], and in fact its supersymmetric extended model [23]. However, the cosmological solution of this extended model is the same as that in the non-supersymmetric model, since Einstein's equations belong to the bosonic part. The RSII model offers a novel expansion law for the observable four-dimensional universe. We find that the axino abundance today is proportional to the transition temperature, at which the modified expansion law in the brane world cosmology connects to the standard one, rather than the reheating temperature after inflation as in the standard cosmology. This means that even though the reheating temperature can be very high, the axinos can play the dominant part of the cold dark matter in the universe. Other works that discuss dark matter in brane cosmology are [24].

Let us see in more detail the thermal production of axinos (in standard cosmology). We assume that after inflation axinos are far from thermal equilibrium. With the axino number density $n_{\tilde{a}}$ being much smaller than the photon number density $n_\gamma$, the evolution of $n_{\tilde{a}}$ with cosmic time $t$ can be described by the Boltzmann equation

$$\frac{dn_{\tilde{a}}}{dt} + 3Hn_{\tilde{a}} = C_{\tilde{a}} \tag{4.2}$$

where $C_{\tilde{a}}$ is the collision term, while the second term on the left-hand side accounts for the dilution of the axinos due to the expansion of the universe described by the Hubble parameter $H$. It is convenient to define the dimensionless quantity

$$Y_{\tilde{a}} = \frac{n_{\tilde{a}}}{s} \tag{4.3}$$

where $s$ is the entropy density for the relativistic degrees of freedom in the primordial plasma

$$s(T) = h_{eff}(T)\frac{2\pi^2}{45}T^3 \tag{4.4}$$

with $h_{eff}(T) \simeq g_{eff}(T)$ in the radiation dominated epoch and $g_{eff}$ counts the total number of effectively massless degrees of freedom (those species with mass $m_i \ll T$). When all the degrees of freedom are relativistic $g_{eff} = 915/4 = 228.75$. Replacing the cosmic time $t$ with the temperature $T$, the number density $n_{\tilde{a}}$ with the number-to-entropy ratio $Y_{\tilde{a}}$ and using conservation of the entropy per comoving volume (see for example the first reference in [15]), the Boltzmann equation can be cast into the form

$$\frac{dY_{\tilde{a}}}{dT} = \frac{C_{\tilde{a}}(T)}{Ts(T)H(T)} \tag{4.5}$$

where $H(T)$ is the Hubble parameter as a function of the temperature $T$ for the radiation dominated era

$$H(T) = \sqrt{\frac{\pi^2 g_{eff}}{90}}\frac{T^2}{M_{pl}} \tag{4.6}$$

where $M_{pl} = 2.4 \times 10^{18}\, GeV$ is the reduced Planck mass. In terms of the number-to-entropy ratio $Y_{\tilde{a}}$, the axino density parameter is given by

$$\Omega h^2 = \frac{\rho_{\tilde{a}} h^2}{\rho_{cr}} = \frac{m_{\tilde{a}} n_{\tilde{a}} h^2}{\rho_{cr}} = \frac{m_{\tilde{a}} Y_{\tilde{a}} s(T_0) h^2}{\rho_{cr}} \tag{4.7}$$

Here we make use of the following values

$$T_0 = 2.73K = 2.35 \times 10^{-13}\, GeV \tag{4.8}$$

$$h_{eff}(T_0) = 3.91 \tag{4.9}$$

$$\rho_{cr}/h^2 = 8.1 \times 10^{-47}\, GeV^4 \tag{4.10}$$

The collision term $C_{\tilde{\alpha}}$ has been computed in supersymmetric QCD by the authors of [16]

$$C_{\tilde{\alpha}}(T) = \frac{3\zeta(3)(N_c^2-1)g^6T^6}{4096\pi^7 f_\alpha^2}\left(0.4336n_f + (N_c+n_f)\,ln\left(\frac{1.38T^2}{m_g}\right)\right) \quad (4.11)$$

where $N_c = 3$, $n_f = 6$ and $g$ is the QCD coupling constant

$$g(T) = \left(\frac{1}{4\times\pi\times 0.118} + \frac{3}{8\pi^2}\,ln\left(\frac{T}{M_Z}\right)\right)^{-1/2} \quad (4.12)$$

The collision term $C_{\tilde{\alpha}}$ assumes that gluons and gluinos are in thermal equilibrium and hence the expression that gives $C_{\tilde{\alpha}}$ is only valid for $T > m_{\tilde{g}}$. The thermal axino production proceeds basically during the hot radiation dominated epoch, that is at temperatures above that at matter-radiation equality $T_{eq}$. Integrating the Boltzmann equation the axino yield at the present temperature of the universe $T_0$ is given by

$$Y_{\tilde{\alpha}}(T_0) = \int_{T_{eq}}^{T_R} dT\,\frac{C_{\tilde{\alpha}}}{Ts(T)H(T)} \simeq \frac{C_{\tilde{\alpha}}(T_R)}{s(T_R)H(T_R)} \quad (4.13)$$

or

$$Y_{\tilde{\alpha}}(T_0) = \frac{3\zeta(3)45\sqrt{90}(N_c^2-1)g^6\left(0.4336n_f + (N_c+n_f)\,ln\left(\frac{1.38T_R^2}{m_g}\right)\right)M_{pl}}{2h_{eff}\sqrt{g_{eff}}\,4096\pi^{10}f_\alpha^2}\,T_R \quad (4.14)$$

and finally the axino density parameter is obtained

$$\Omega h^2 = 5.5g^6 ln\left(\frac{1.108}{g}\right)\left(\frac{m_{\tilde{\alpha}}}{0.1GeV}\right)\left(\frac{T_R}{10^4 GeV}\right)\left(\frac{10^{11}\,GeV}{f_\alpha}\right)^2 \quad (4.15)$$

Considering $f_\alpha = 10^{11}\,GeV$, axinos can be cold dark matter for masses $m_{\tilde{\alpha}} \geq 100\,keV$ and reheating temperatures $T_R \leq 10^6\,GeV$ [16].

Recently the brane world models have been attracting a lot of attention as a novel higher dimensional theory. In these models, it is assumed that the standard model particles are confined on a 3-brane while gravity resides in the whole higher dimensional spacetime. The model first proposed by Randall and Sundrum (RSII) [22], is a simple and interesting one, and its cosmological evolutions have been intensively investigated [25, 26, 27]. According to that model, our four-dimensional universe is realized on the 3-brane with a positive tension located at the UV boundary of five-dimensional AdS spacetime. In the bulk there is just a cosmological constant $\Lambda_5$, whereas on the brane there is matter with energy-momentum tensor $\tau_{\mu\nu}$. Also, the five-dimensional Planck mass is denoted by $M_5$ and the brane tension is denoted by $T$. If Einstein's equations hold for the five-dimensional bulk, then it has been shown in [28] that the effective four-dimensional Einstein's equations induced on the brane can be written as

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \frac{8\pi}{m_p^2}\tau_{\mu\nu} + (\frac{1}{M_5^3})^2 \pi_{\mu\nu} - E_{\mu\nu} \tag{4.16}$$

where $g_{\mu\nu}$ is the induced metric on the brane, $\pi_{\mu\nu} = \frac{1}{12}\,\tau\,\tau_{\mu\nu} + \frac{1}{8}\,g_{\mu\nu}\,\tau_{\alpha\beta}\,\tau^{\alpha\beta} - \frac{1}{4}\,\tau_{\mu\alpha}\,\tau_{\nu}^{\alpha} - \frac{1}{24}\,\tau^2\,g_{\mu\nu}$, $\Lambda_4$ is the effective four-dimensional cosmological constant, $m_p$ is the usual four-dimensional Planck mass and $E_{\mu\nu} \equiv C^{\alpha}_{\beta\rho\sigma}\,n_\alpha\,n^\rho\,g^\beta_\mu\,g^\sigma_\nu$ is a projection of the five-dimensional Weyl tensor $C_{\alpha\beta\rho\sigma}$, where $n^\alpha$ is the unit vector normal to the brane. The tensors $\pi_{\mu\nu}$ and $E_{\mu\nu}$ describe the influence of the bulk in brane dynamics. The five-dimensional quantities are related to the corresponding four-dimensional ones through the relations

$$m_p = 4\sqrt{\frac{3\pi}{T}}\,M_5^3 \tag{4.17}$$

and

$$\Lambda_4 = \frac{1}{2M_5^3}\left(\Lambda_5 + \frac{T^2}{6M_5^3}\right) \tag{4.18}$$

In a cosmological model in which the induced metric on the brane $g_{\mu\nu}$ has the form of a spatially flat Friedmann-Robertson-Walker model, with scale factor $a(t)$, the Friedmann-like equation on the brane has the generalized form (see e.g. the second reference in [25])

$$H^2 = \frac{\Lambda_4}{3} + \frac{8\pi}{3m_p^2}\rho + \frac{1}{36M_5^6}\rho^2 + \frac{C}{a^4} \tag{4.19}$$

where $C$ is an integration constant arising from $E_{\mu\nu}$. The cosmological constant term and the term linear in $\rho$ are familiar from the four-dimensional conventional cosmology. The extra terms, i.e the "dark radiation" term and the term quadratic in $\rho$, are there because of the presence of the extra dimension. Adopting the Randall-Sundrum fine-tuning

$$\Lambda_5 = -\frac{T^2}{6M_5^3} \tag{4.20}$$

the four-dimensional cosmological constant vanishes. So the generalized Friedmann equation takes the final form

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{\rho_0}\right) + \frac{C}{a^4} \tag{4.21}$$

where

$$\rho_0 = 96\pi G M_5^6 \tag{4.22}$$

with $G$ the Newton's constant. The second term proportional to $\rho^2$ and the dark radiation are new ingredients in the brane world cosmology and lead to a non-standard expansion law. The dark radiation term is severely constrained by the success of the Big Bang Nucleosynthesis (BBN), since the term behaves like an additional radiation at the BBN era [29]. So, for simplicity, we neglect the term in the following analysis. The five-dimensional Planck mass is also constrained by the BBN, which is roughly estimated as $M_5 \geq 10 TeV$ [30]. A more stringent constraint may be obtained by requiring that relative corrections to the Newtonian law of gravity should be small on scales $r \geq 1\, mm$. This gives $M_5 > 10^8\, GeV$ [31].

One can see that the evolution of the early universe can be divided into two eras. In the low energy regime $\rho \ll \rho_0$ the first term dominates and we recover the usual Friedmann equation of the conventional four-dimensional cosmology. In the high density regime $\rho_0 \ll \rho$ the second term dominates and we get an unconventional expansion law for the universe. In between there is a transition temperature $T_t$ for which $\rho(T_t) = \rho_0$. The transition temperature $T_t$ is determined as

$$T_t = 1.6 \times 10^7 \left(\frac{100}{g_{eff}}\right)^{1/4} \left(\frac{M_5}{10^{11}\, GeV}\right)^{3/2} GeV \tag{4.23}$$

once $M_5$ is given. Using the transition temperature the generalized Friedmann-like equation (for the radiation era) can be rewritten in the form

$$H = H_{st} \sqrt{1 + \frac{T^4}{T_t^4}} \tag{4.24}$$

with $H_{st}$ the Hubble parameter in standard four-dimensional Big-Bang cosmology. Assuming a transition temperature $T_R \gg T_t$ and $T_t \gg T_{eq}$, the following integral can be computed to a very good approximation

$$\int_{T_{eq}}^{T_R} dT \, \frac{1}{\sqrt{1 + \frac{T^4}{T_t^4}}} = \int_{T_{eq}}^{T_t} dT \, \frac{1}{\sqrt{1 + \frac{T^4}{T_t^4}}} + \int_{T_t}^{T_R} dT \, \frac{1}{\sqrt{1 + \frac{T^4}{T_t^4}}} \simeq 2T_t \tag{4.25}$$

Therefore, the axino yield resulting on integrating the Boltzmann equation in brane cosmology is

$$Y_{\tilde{a}}(T_0) = \int_{T_{eq}}^{T_R} dT \, \frac{C_{\tilde{a}}(T)}{T s(T) H_{st}(T)} \frac{1}{\sqrt{1 + \frac{T^4}{T_t^4}}} \simeq \frac{C_{\tilde{a}}(T_R)}{T_R s(T_R) H_{st}(T_R)} 2T_t \tag{4.26}$$

The expression is only valid for $T_t > m_{\tilde{g}}$. So we see that essentially the reheating temperature $T_R$ in the axino parameter density $\Omega_{\tilde{\alpha}}$ is replaced by the transition temperature $T_t$. Therefore, axinos can play the role of cold dark matter in the universe for

$$m_{\tilde{\alpha}} \geq 100\, keV, \;\; T_t \leq 2 \times 10^6\, GeV \tag{4.27}$$

independently of the reheating temperature. This is the main result in this work. At this point we would like to stress the fact that the axino abundance does depend on the reheating temperature, but only through the coupling constant $g(T_R)$. The function $g(T)$ is a very slow-varying function of the temperature (for example, $g(T = 10^6\, GeV) = 0.986$ and $g(T = 10^{10}\, GeV) = 0.852$), so practically we can consider reheating temperatures of the order of $\sim 10^{10}\, GeV$ and neglect the dependence on it. Note that according to the analysis of [32], with a transition temperature $T_t \leq 10^6\, GeV$ the gravitino problem can be avoided. It is interesting to note that in order that the axinos can play the role of cold dark matter in the universe, the five-dimensional Planck mass $M_5$ can only take values in a range between an upper limit and a lower limit. If $M_5$ becomes too high, the transition temperature $T_t$ will be higher than $\sim 10^6\, GeV$ and this sets an upper bound for $M_5$: $M_5 \leq 2.9 \times 10^{10}\, GeV$. On the other hand, if $M_5$ becomes too low, the reheating temperature will be smaller than the gluino mass, $m_{\tilde{g}} \sim 1\, TeV$. Thus, we get a lower bound for $M_5$: $M_5 > 1.8 \times 10^8\, GeV$. So we find a window for the five-dimensional Planck mass

$$1.8 \times 10^8\, GeV < M_5 \leq 2.9 \times 10^{10}\, GeV \tag{4.28}$$

It is interesting to note that this range for the five-dimensional Planck mass is compatible with the bounds mentioned before coming from the BBN and modifications to Newton's law, namely that $M_5 > 10^8\, GeV$.

To illustrate the above ideas let us present a specific example. We consider the case in which the five-dimensional Planck mass is $M_5 = 10^{10}\, GeV$. Then the transition temperature $T_t$ is found to be $T_t = 4 \times 10^5\, GeV$. We also assume that the reheating temperature is $T_R = 10^{10}\, GeV$. If the axinos are to be the cold dark matter in the universe, their parameter density has to be $\Omega_{\tilde{\alpha}} h^2 = 0.113$. This happens for $m_{\tilde{\alpha}} \simeq 511\, keV$. Our treatment is valid as long as axinos are never in thermal equilibrium after inflation. One can easily check that this is the case. For that we need to compare the Hubble parameter $H(T)$ to the rate $\Gamma(T)$ of the reaction that maintain the axinos in thermal equilibrium. For $T < f_\alpha$ the reaction rate is [14]

$$\Gamma \sim \frac{\alpha_s^3}{16\pi f_\alpha^2}\, T^3 \tag{4.29}$$

We can see that after inflation $H(T) \gg \Gamma(T)$ for all the values of $M_5$ in the allowed range.

Summarizing, we have discussed dark matter in Randall-Sundrum type II brane world assuming that the axino is the LSP. We have seen that axinos can be the dominant part of the cold dark matter in the universe if their mass $m_{\tilde{\alpha}} \geq 100\ keV$ and the transition temperature $T_t \leq 2 \times 10^6\ GeV$ independently of the reheating temperature $T_R$ after inflation (provided that $T_R \gg T_t$, which is true for $T_R \sim 10^{10}\ GeV$). Therefore, in contrast to the case for the conventional four-dimensional cosmology, high values for $T_R$ such as $10^{10}\ GeV$ are allowed, in accord with most inflationary models and baryogenesis through leptogenesis.

# Chapter 5

# Sneutrino inflation

We discuss sneutrino inflation in the brane-world scenario. We work in the Randall-Sundrum type II brane-world, generalized with the introduction of the Gauss-Bonnet (GB) term, a correction to the effective action in string theories. We find that a viable inflationary model is obtained with a reheating temperature appropriate to lead to the right baryon asymmetry and render the gravitino safe for cosmology. In specific realizations we satisfy all the observational constaints without the unnaturally small Yukawa couplings required in other related approaches.

## 5.1 Introduction

Inflation [15] has become the standard paradigm for the early Universe, because it solves some outstanding problems present in the standard Hot Big-Bang cosmology, like the flatness and horizon problems, the problem of unwanted relics, such as magnetic monopoles, and produces the cosmological fluctuations for the formation of the structure that we observe today. The recent spectacular CMB data from the WMAP satellite [11, 33] have strengthen the inflationary idea, since the observations indicate an *almost* scale-free spectrum of Gaussian adiabatic density fluctuations, just as predicted by simple models of inflation. According to chaotic inflation with a potential for the inflaton field $\phi$ of the form $V = (1/2)m^2\phi^2$, the WMAP normalization condition requires for the inflaton mass $m$ that $m = 1.8 \times 10^{13}\, GeV$ [34]. However, a yet unsolved problem about inflation is that we do not know how to integrate it with ideas in particle physics. For example, we would like to identify the inflaton, the scalar field that drives inflation, with one of the known fields of particle physics.

One of the most exciting experimental results in the last years has been the discovery of neutrino oscillations [35]. These results are nicely explained if neutrinos have a small but finite mass [36]. The simplest models of neutrino masses invoke heavy gauge-singlet neutrinos that give masses to the light neutrinos via the seesaw mechanism [37]. If we require that light neutrino masses $\sim 10^{-1}$ to $10^{-3}\, eV$, as indicated by the neutrino oscillations data, we find that the heavy singlet neutrinos weight $\sim 10^{10}$ to $10^{15}\, GeV$ [38], a range that includes the value of the inflaton mass compatible with WMAP. On the other hand, the hierarchy problem of particle physics is elegantly solved by supersymmetry (see e.g. [10]), according to which every known particle comes with its superpartner, the sparticle. In supersymmetric models the heavy singlet neutrinos have scalar partners with similar masses, the sneutrinos, whose properties are ideal for playing the role of the inflaton [39, 34].

Superstring theory includes, apart from the fundamental string, other extended objects called p-branes. A special class of p-branes are D(irichlet)p-branes, where open strings can end. D-brane physics has motivated the brane-world idea, which has attracted a lot of interest over the last years. In a brane-world scenario our universe is modeled by a 3-brane embedded in a five-dimensional bulk spacetime. In the simpest cases, all the standard model fields (open string sector) are confined on the brane, while gravity (closed string sector) propagates in the bulk. The brane is a hypersurface that splits the five-dimensional manifold into two parts and plays the role of a boundary of spacetime. Usually the brane is considered to be infinitely thin and the matching conditions can be used to relate the bulk dynamics to what we observe on the brane. The model first proposed by Randall and Sundrum (RS II) [22] offers a viable alternative to the standard Kaluza-Klein treatment of the extra dimensions and together with various extensions has been intensively investigated for its cosmological consequences (see e.g. [25] and for reviews [40]).

In four dimensions, the Einstein tensor is the only second-rank tensor that (i) is symmetric, (ii) is divergence free, (iii) it depends only on the metric and its first derivatives, and (iv) is linear in second derivatives of the metric. However, in $D > 4$ dimensions more complicated tensors with the above properties exist. For example, in five dimensions the second order Lovelock tensor reads

$$H_{ab} = RR_{ab} - 2R_{ac}R^c_b - 2R^{cd}R_{acbd} + R^{cde}_a R_{bcde} - \frac{1}{4}g_{ab}(R^2 - 4R_{cd}R^{cd} + R^{cdes}R_{cdes}) \quad (5.1)$$

and can be obtained from an action containing the GB term [41]

$$\mathcal{L}_{GB} = R^2 - 4R_{ab}R^{ab} + R^{abcd}R_{abcd} \tag{5.2}$$

Higher order curvature terms appear also in the low-energy effective field equations arising in string theory. Brane-worlds are string-inspired and so it is natural to include such terms in the five-dimensional field equations.

It is important to note that in the context of extra dimensions and the brane-world idea one obtains on the brane a generalized Friedmann equation, which is different from the usual one of conventional four-dimensional cosmology. This means that the rate of expansion of the universe in this novel cosmology is altered and accordingly the description of the physics in the early universe can be different from the standard treatment. So it would be very interesting to study the cosmological implications of these new ideas about extra dimensions and braneworlds. The Friedmann-like equation for a GB brane-world has been derived in [42, 43, 41].

Sneutrino inflation in the context of Randall-Sundrum type II model has been analyzed in [44]. However, it would be interesting to study the effect of the GB term. After all, this term is a high energy modification to general relativity and as such it is expected to be important in the early universe. Furthermore, as it has been shown in [45], the quadratic potential $V \sim \phi^2$ for the inflaton is observationally more favoured when the GB term is present. The purpose of the present work is to discuss sneutrino inflation in the context of a GB braneworld.

The present chapter is organized as follows. There are five sections of which this introduction is the first. In section 2 we describe sneutrino inflation in a GB brane-world. Section 3 contains the discussion of reheating, gravitino production and baryogenesis through leptogenesis. Our results are summarized in section 4 and we conclude with a discussion section 5.

## 5.2 Sneutrino inflation in a GB brane-world

### 5.2.1 GB brane-world

Here we review GB brane-world, following essentially [45]. The five-dimensional bulk action for the GB braneworld scenario is given by

$$\begin{aligned} S &= \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-^{(5)}g}\,[-2\Lambda_5 + R \\ &\quad + \alpha\left(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}\right)] \\ &\quad - \int_{\text{brane}} d^4x\,\sqrt{-g}\,\lambda + S_{\text{mat}} \end{aligned} \tag{5.3}$$

where $\alpha > 0$ is the GB coupling, which has dimensions of $length^2$, $\lambda > 0$ is the brane tension, $\Lambda_5 < 0$ is the bulk cosmological constant and $S_{\rm mat}$ denotes the matter action. The fundamental energy scale of gravity is the five-dimensional scale $M_5$ with $\kappa_5^2 = 8\pi/M_5^3$. For the discussion to follow we define a new mass scale through the relation $\alpha = 1/M_*^2$.

The GB term may be viewed as the lowest-order stringy correction to the five-dimensional Einstein-Hilbert action with $\alpha \ll 1/\mu^2$, where $1/\mu$ is the bulk curvature scale, $|R| \sim \mu^2$. The Randall-Sundrum type models are recovered for $\alpha = 0$. Moreover, for an anti-de Sitter bulk, it follows that $\Lambda_5 = -3\mu^2(2-\xi)$, where

$$\xi \equiv 4\alpha\mu^2 \ll 1 \tag{5.4}$$

Imposing a $Z_2$ reflection symmetry across the brane in an anti-de Sitter bulk and assuming that a perfect fluid matter source is confined on the brane, one obtains the modified Friedmann equation

$$\kappa_5^2(\rho+\lambda) = 2\mu\sqrt{1+\frac{H^2}{\mu^2}}\left[3-\xi+2\xi\frac{H^2}{\mu^2}\right] \tag{5.5}$$

This can be rewritten in the useful form

$$H^2 = \frac{\mu^2}{\xi}\left[(1-\xi)\cosh\left(\frac{2\chi}{3}\right)-1\right] \tag{5.6}$$

where $\chi$ is a dimensionless measure of the energy density $\rho$ on the brane defined by

$$\rho+\lambda = m_\alpha^4\sinh\chi \tag{5.7}$$

with

$$m_\alpha = \left[\frac{8\mu^2(1-\xi)^3}{\xi\kappa_5^4}\right]^{1/8} \tag{5.8}$$

the characteristic GB energy scale.

The requirement that one should recover general relativity at low energies leads to the relation

$$\kappa_4^2 = \frac{\mu}{1+\xi}\,\kappa_5^2 \tag{5.9}$$

where $\kappa_4^2 = 8\pi/M_{pl}^2$ and $M_{pl}$ is the four-dimensional Planck scale. Since $\xi \ll 1$, we have $\mu \approx M_5^3/M_{pl}^2$. Furthermore, the brane tension is fine-tuned to zero effective cosmological constant on the brane

$$\kappa_5^2\lambda = 2\mu(3-\xi) \tag{5.10}$$

The GB energy scale $m_\alpha$ is larger than the RS energy scale $\lambda^{1/4}$, since we consider that the GB term is a correction to RS gravity. Using (5.10) this implies [46] $\xi \lesssim 0.15$, which is consistent with Eq. (5.4).

Expanding Eq. (5.6) in $\chi$, we find three regimes for the dynamical history of the brane universe

$$\rho \gg m_\alpha^4 \quad \Rightarrow \quad H^2 \approx \left[\tfrac{\mu^2\kappa_5^2}{4\xi}\,\rho\right]^{2/3} \quad \text{(GB)} \tag{5.11}$$

$$m_\alpha^4 \gg \rho \gg \lambda \quad \Rightarrow \quad H^2 \approx \tfrac{\kappa_4^2}{6\lambda}\,\rho^2 \quad \text{(RS)} \tag{5.12}$$

$$\rho \ll \lambda \quad \Rightarrow \quad H^2 \approx \tfrac{\kappa_4^2}{3}\,\rho \quad \text{(GR)} \tag{5.13}$$

Eqs. (5.11)-(5.13) are considerably simpler than the full Friedmann equation and for inflation we shall assume the first one (GB).

### 5.2.2 Chaotic inflation in a GB brane-world

We will consider the case in which the energy momentum on the brane is dominated by the sneutrino inflaton field $\phi$ confined on the brane with a self-interaction potential $V(\phi) = (1/2)M^2\,\phi^2$, where $M$ is the mass of the sneutrino field. The field $\phi$ is a function of time only, as dictated by the isotropy and homogeneity of the observed four-dimensional universe. A homogeneous scalar field behaves like a perfect fluid with pressure $p = (1/2)\dot\phi^2 - V$ and energy density $\rho = (1/2)\dot\phi^2 + V$. We shall assume that there is no energy exchange between the brane and the bulk, so the energy-momentum tensor $T_{\mu\nu}$ of the scalar field is conserved, that is $\nabla^\nu T_{\mu\nu} = 0$. In terms of the pressure $p$ and the energy density $\rho$ the continuity equation takes the form

$$\dot\rho + 3H(p+\rho) = 0 \tag{5.14}$$

where $H$ is the Hubble parameter $H = \dot a/a$. This is equivalent to the equation of motion for the scalar field $\phi$

$$\ddot\phi + 3H\dot\phi + V'(\phi) = 0 \tag{5.15}$$

the Klein-Gordon equation for $\phi$ in a Robertson-Walker background. The equation that governs the dynamics of the expansion of the universe is the Friedmann-like equation of the previous subsection. Inflation takes place in the early stages of the evolution of the universe, so we suppose that inflation takes place in the GB high energy regime

$$H^2 = \left(\frac{\mu^2\kappa_5^2}{4\xi}\,\rho\right)^{2/3} \tag{5.16}$$

In the slow-roll approximation the slope and the curvature of the potential must satisfy the two constraints $\epsilon \ll 1$ and $|\eta| \ll 1$, where $\epsilon$ and $\eta$ are the two slow-roll parameters which are defined by

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \tag{5.17}$$

$$\eta \equiv \frac{V''}{3H^2} \tag{5.18}$$

In this approximation the equation of motion for the scalar field takes the form

$$\dot{\phi} \simeq -\frac{V'}{3H} \tag{5.19}$$

while the generalized Friedmann equation becomes ($V \gg \dot{\phi}^2$)

$$H^2 \simeq \left(\frac{\mu^2 \kappa_5^2}{4\xi} V\right)^{2/3} \tag{5.20}$$

The number of e-folds during inflation is given by

$$N \equiv ln \frac{a_f}{a_i} = \int_{t_i}^{t_f} H dt \tag{5.21}$$

Before presenting all the formulae, it would perhaps be useful at this point to describe what follows. Any model of inflation should i) solve the flatness and horizon problems, ii) reproduce the amplitude for density perturbations (COBE normalization), iii) predict a nearly scale-invariant spectrum, and iv) predict very small tensor perturbations. For a strong enough inflation we take $N = 70$, which is enough to solve the horizon and flatness problems. Using the equations of motion we shall compute the spectral index, as well as the scalar and tensor perturbations. We will then fix the remaining parameters by requiring that the amplitude of scalar perturbations is reproduced. This will lead to a prediction of the spectral index and the tensor-to-scalar ratio.

According to a recent analysis [47], at $1 - \sigma$

$$A_s \simeq 2 \times 10^{-5} \tag{5.22}$$

$$-0.048 < n_s - 1 < 0.016 \tag{5.23}$$

with $A_s$ the amplitude of the density perturbations and $n_s$ the spectral index. On large cosmological scales, data [47] give for the tensor perturbations

$$r < 0.47 \qquad\qquad 95\% \text{ c.l.} \tag{5.24}$$

with $r$ the tensor-to-scalar ratio defined as $r = 16\, A_t^2/A_s^2$ (consistent with the normalization of Ref. [33] in the low energy limit), where $A_t$ is the amplitude of the tensor perturbations.

In the slow-roll approximation the number of e-folds and the slow-roll parameters are given by the formulae

$$\epsilon \simeq \frac{V'^2}{9V^{5/3}} \left(\frac{4\xi}{\mu^2\kappa_5^2}\right)^{2/3} \tag{5.25}$$

$$\eta \simeq \frac{V''}{3V^{2/3}} \left(\frac{4\xi}{\mu^2\kappa_5^2}\right)^{2/3} \tag{5.26}$$

$$N \simeq -3 \left(\frac{\mu^2\kappa_5^2}{4\xi}\right)^{2/3} \int_{\phi_*}^{\phi_{end}} \frac{V^{2/3}}{V'}\, d\phi \tag{5.27}$$

where $\phi_{end}$ is the value of the inflaton at the end of inflation, which is determined from the condition that the maximum of $\epsilon, |\eta|$ equals unity, and the $*$ denotes the point at which observable quantities are computed. The main cosmological constraint (normalization condition) comes from the amplitude of the scalar perturbations [48]

$$A_s = \frac{4}{5}\,\frac{H^2}{M_{pl}^2 |H'(\phi)|} \tag{5.28}$$

where the right-hand side is evaluated at the horizon-crossing when the comoving scale equals the Hubble radius during inflation and $M_{pl} = 1.22 \times 10^{19}\ GeV$ is the four-dimensional Planck mass. In the present context the amplitude of the scalar perturbations is given by

$$A_s^2 = \frac{144 V^{8/3}}{25 M_{pl}^4 V'^2} \left(\frac{\mu^2\kappa_5^2}{4\xi}\right)^{2/3} \tag{5.29}$$

The spectral index for the scalar perturbations $n_s$ is given in terms of the slow-roll parameters

$$n_s - 1 \equiv \frac{d\, lnA_s^2}{d\, lnk} = 2\eta - 6\epsilon \tag{5.30}$$

and is found to be

$$n_s = \frac{2N-3}{2N} = 0.98 \tag{5.31}$$

while the tensor-to-scalar ratio $r$ is given by

$$r = \frac{3M_*^2 M_{pl}^2}{2N^{3/2} M M_5^3} \tag{5.32}$$

with $A_t$ the amplitude of the tensor perturbations [48]

$$A_t = \frac{2}{5\sqrt{\pi}} \frac{H}{M_{pl}} \tag{5.33}$$

where again the right-hand side is evaluated at the horizon-crossing. Taking the normalization condition into account we obtain for $M$

$$M = 3.4 \times 10^{-5} \frac{M_*^{2/3} M_{pl}^{4/3}}{M_5} \tag{5.34}$$

and for the tensor-to-scalar ratio

$$r = 75.3 \frac{M_*^{4/3} M_{pl}^{2/3}}{M_5^2} \tag{5.35}$$

## 5.3 Reheating, gravitino production and leptogenesis

### 5.3.1 Reheating

We start by introducing three heavy right-handed neutrinos $N_i$ which only interact with leptons and Higgs. The superpotential that describes their interactions is [49]

$$W = f_{ia} N_i L_a H_u \tag{5.36}$$

where $f_{ia}$ is the matrix for the Yukawa couplings, $H_u$ is the superfield of the Higgs doublet that couples to up-type quarks and $L_a$ $(a = e, \mu, \tau)$ is the superfield of the lepton doublets. We assume that the scalar partner of the lightest right-handed neutrino plays the role of the inflaton. After inflation the inflaton decays into normal particles which quickly thermalize. This is the way the universe reenters the radiation dominated era. The sneutrino inflaton decays into leptons and Higgs and their antiparticles according to the superpotential (5.36) and the decay rate is given by [49]

$$\Gamma_\phi = \frac{1}{4\pi} f^2 M \tag{5.37}$$

with $M$ the sneutrino mass and $f^2 \equiv \sum_a |f_{1a}|^2$. The reheating temperature after inflation is defined by assuming instantaneous conversion of the inflaton energy into radiation, when the decay rate of the inflaton $\Gamma_\phi$ equals the expansion rate $H$. In GB braneworld cosmology $H$ is given by

$$H = \left(\frac{\kappa_5^2}{16\alpha}\right)^{1/3} \rho^{1/3} \tag{5.38}$$

and in the radiation dominated era the energy density of the universe is given by

$$\rho = \rho_R = g_{eff}\, \frac{\pi^2}{30}\, T^4 \tag{5.39}$$

with $g_{eff} = 228.75$ the effective number of relativistic degrees of freedom in the MSSM for $T \gg 1\ TeV$. Thus we obtain

$$H = \left(\frac{\kappa_5^2}{16\alpha}\right)^{1/3} \left(g_{eff}\, \frac{\pi^2}{30}\, T^4\right)^{1/3} \tag{5.40}$$

The condition $H(T_R) = \Gamma_\phi$ gives for the reheating temperature

$$T_R = \left(\frac{15 M^3 M_5^3}{16\pi^6 g_{eff} M_*^2}\, f^6\right)^{1/4} \tag{5.41}$$

After inflation, the direct out-of-equilibrium decays of the sneutrino inflaton generate the lepton asymmetry which is partially converted into a baryon asymmetry via sphaleron effects. This requires that $T_R < M$ or that

$$f^2 < \left(\frac{16\pi^6 g_{eff} M M_*^2}{15 M_5^3}\right)^{1/3} \tag{5.42}$$

### 5.3.2 Gravitino production

Any viable inflationary model should avoid the gravitino problem [50]. This means that for unstable gravitinos that decay after Big-Bang Nucleosynthesis (BBN), their decay products should not alter the abundances of the light elements in the universe that BBN predicts. This requirement sets an upper bound for the gravitino abundance

$$\eta_{3/2} \equiv \frac{n_{3/2}}{n_\gamma} \leq \frac{\zeta_{max}}{m_{3/2}} \tag{5.43}$$

with $m_{3/2} \sim 100\ GeV - 1\ TeV$ the gravitino mass, $n_\gamma$ the photon number density and $\zeta_{max}$ a parameter related to the maximum gravitino abundance allowed by the BBN predictions. According to the analysis of the authors of [51], $\zeta_{max} = 5 \times 10^{-12}\ GeV$ for $m_{3/2} = 100\ GeV$. To find the gravitino abundance one has to integrate Boltzmann equation

$$\frac{dn_{3/2}}{dt} + 3Hn_{3/2} = C_{3/2}(T) \tag{5.44}$$

with $C_{3/2}(T)$ the collision term responsible for the thermal production of gravitinos as a function of the temperature $T < T_R$. The rate for the thermal production of gravitinos is dominated by QCD processes since the strong coupling is considerably larger than the electroweak couplings. Taking into account 10 two-body processes involving left-handed quarks, squarks, gluons and gluinos, the authors of [52] computed the collision term $C_{3/2}(T)$ in the framework of supersymmetric QCD. They obtained

$$C_{3/2}(T) = a(T) \left(1 + b(T)\,\frac{m_{\tilde{g}}^2}{m_{3/2}^2}\right) \frac{T^6}{M_{pl}^2} \tag{5.45}$$

where $m_{\tilde{g}} \sim 1\;TeV$ is the gluino mass and $a(T), b(T)$ are two slowly-varying functions of the temperature, estimated to be [44]

$$a(T_R) = 2.38,\;\; b(T_R) = 0.13 \tag{5.46}$$

If we assume that the quantity $sa^3$ is constant during the expansion of the universe, where $a$ is the scale factor and $s$ is the entropy density $s = h_{eff}\,(2\pi^2 T^3)/45$, then the integration of Boltzmann equation gives

$$\eta_{3/2}(T) = \frac{h_{eff}(T)}{h_{eff}(T_R)}\,\frac{C_{3/2}(T_R)}{H(T_R)n_\gamma(T_R)} \tag{5.47}$$

with $h_{eff}$ the effective number of relativistic degrees of freedom. For $T \gg 1\,TeV$ all particles are relativistic and for the MSSM $h_{eff}(T_R) \sim g_{eff}(T_R) = 915/4 = 228.75$, while $h_{eff}(T) = 43/11$ for $T < 1\;MeV$. Thus, using (5.43) with $m_{3/2} = 100\;GeV$ one is led to the following upper bound for the reheating temperature

$$T_R \leq 1.63 \times 10^{-8}\,\frac{M_{pl}^{6/5} M_*^{2/5}}{M_5^{3/5}} \equiv T_0 \tag{5.48}$$

At this point we should also check whether the contribution of the gravitinos to the energy density of the universe is compatible with the observed matter density of the universe, $\Omega_m h^2 < 0.143$ [11], where $h = (H/100)\,\frac{Mpc\,sec}{Km}$. From the gravitino abundance we can calculate their normalized density

$$\Omega_{3/2}h^2 = m_{3/2}\eta_{3/2}n_{\gamma 0}h^2\rho_{cr}^{-1} \tag{5.49}$$

with $n_{\gamma 0} = 3.15 \times 10^{-39}\;GeV^3$ the photon density today and $\rho_{cr} = 8.07 \times 10^{-47}h^2\;GeV^4$ the critical density. For $m_{3/2} = 100\;GeV$ we obtain

$$\Omega_{3/2}h^2 = 1.86 \times 10^9\,\frac{M_5 T_R^{5/3}}{M_{pl}^2 M_*^{2/3}} \tag{5.50}$$

Using the WMAP bound on the matter density of the universe, $\Omega_m h^2 < 0.143$ we get the following relation between $T_R$, $M_5$ and $M_*$

$$T_R < 8.54 \times 10^{-7} \frac{M_{pl}^{6/5} M_*^{2/5}}{M_5^{3/5}} \tag{5.51}$$

which is less stringent than the constraint (5.48) coming from BBN.

### 5.3.3 Direct leptogenesis from sneutrino decay

Any lepton asymmetry $Y_L \equiv n_L/s$ produced before the electroweak phase transition is partially converted into a baryon asymmetry $Y_B \equiv n_B/s$ via sphaleron effects [53]. The resulting $Y_B$ is

$$Y_B = C\, Y_L \tag{5.52}$$

with the fraction $C$ computed to be $C = -8/15$ in the MSSM [54]. The lepton asymmetry, in turn, is generated by the direct out-of-equilibrium decays of the sneutrino inflaton after inflation and is given by [49]

$$Y_L = \frac{3}{4} \frac{T_R}{M}\, \epsilon \tag{5.53}$$

with $\epsilon$ the $CP$ asymmetry in the sneutrino decays. For convenience we parametrize the $CP$ asymmetry in the form

$$\epsilon = \epsilon^{max}\, sin\delta_L \tag{5.54}$$

where $\delta_L$ is an effective leptogenesis phase and $\epsilon^{max}$ is the maximum asymmetry which is given by [55]

$$\epsilon^{max} = \frac{3}{8\pi} \frac{M\sqrt{\Delta m_{atm}^2}}{v^2 sin^2\beta} \tag{5.55}$$

with $v = 174\,GeV$ the electroweak scale, $tan\beta$ the ratio of the vevs of the two Higgs doublets of the MSSM and $\Delta m_{atm}^2 = 2.6 \times 10^{-3}\, eV^2$ the mass squared difference measured in atmospheric neutrino oscillation experiments. For simplicity we shall take $sin\beta \sim 1$ (large $tan\beta$ regime), in which case the maximum $CP$ asymmetry is given by

$$\epsilon^{max} = 2 \times 10^{-10} \left( \frac{M}{10^6\, GeV} \right) \tag{5.56}$$

Combining the above formulae we obtain

$$Y_B = 8 \times 10^{-11} |sin\delta_L| \left( \frac{T_R}{10^6\, GeV} \right) \tag{5.57}$$

From the WMAP data [11] we know that

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 6.1 \times 10^{-10} \tag{5.58}$$

If we recall that the entropy density for relativistic degrees of freedom is $s = h_{eff} \frac{2\pi^2}{45} T^3$ and that the number density for photons is $n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3$, one easily obtains for today that $s = 7.04 n_\gamma$. Thus, using (5.57) we have

$$T_R = \frac{1.08 \times 10^6}{|sin\delta_L|}\, GeV \tag{5.59}$$

from which we get a lower bound for the rehetaing temperature

$$T_R \geq 1.08 \times 10^6\, GeV \tag{5.60}$$

## 5.4 Results

Let us summarize the results obtained above. We take $M_5$ and $M_*$ to be two independent mass scales, in principle anywhere between the four-dimensional Planck mass $M_{pl}$ and the electroweak scale, $v \sim 200\; GeV$. First we present all the constraints that have to be satisfied. We have mentioned that $\xi \ll 1$ and that in the GB regime $\rho \gg m_\alpha^4$. These lead to the constraints

$$M_* \gg \frac{2M_5^3}{M_{pl}^2} \tag{5.61}$$

and

$$M_* \ll M \tag{5.62}$$

respectively. On the other hand, the sneutrino drives inflation and simultaneously produces the lepton asymmetry through its direct out-of-equilibrium decay after the inflationary era. This requires the reheating temperature to be smaller than the sneutrino inflaton mass, namely $T_R < M$. Furthermore, the gravitino abundance constraint requires $T_R \leq T_0$. So we see that the reheating temperature has to be lower than both $M$ and $T_0$. Now the question arises, whether $M$ is larger than $T_0$ or vise versa. We have checked that for $M_5$ and $M_*$ in their allowed range, $M$ is always larger than $T_0$. Thus, the requirement that $T_R \leq T_0$ also guarantees that $T_R < M$. Hence, for given $M_5$ and $M_*$, the reheating temperature is bounded both from below and from above as follows

$$1.08 \times 10^6 \; GeV \leq T_R \leq T_0 \tag{5.63}$$

Of course, $T_0$ should not be lower than the minimum of the reheating temperature

$$T_0 \geq 1.08 \times 10^6 \; GeV \tag{5.64}$$

Combining all the constraints mentioned above we find an upper bound for $M_*$

$$M_* \leq 3 \times 10^{11} \; GeV \tag{5.65}$$

Then, for a given value for $M_*$, $M_5$ has to range between a maximum and a minimum value. If $M_5$ gets too small, the tensor-to-scalar ratio gets larger than the observed value, while if $M_5$ gets too large, then the constraint (5.61) or (5.62) is not satisfied. For example, for the extremum values of $M_*$

- For $M_* = 3 \times 10^{11} \; GeV$

$$1.31 \times 10^{15} \; GeV \leq M_5 \leq 2.42 \times 10^{15} \; GeV \tag{5.66}$$

- while for $M_* = 200 \; GeV$

$$9.97 \times 10^{8} \; GeV \leq M_5 \leq 5.3 \times 10^{12} \; GeV \tag{5.67}$$

We see that $M_5$ can be very close to the unification scale $M_{GUT} \sim 10^{16} \; GeV$ (but remains lower than that) and not lower than $10^8 \; GeV$. Interestingly, our findings are compatible with experiments to probe deviations from Newton's law, which currently imply that $M_5 \geq 10^8 \; GeV$ [46]. Finally, for all the allowed values of $M_5$ and $M_*$, we find that the constraint (5.64) is always satisfied and that the tensor perturbations are always negligible.

So far we have treated $M_*$ as a phenomenological parameter of the model. However, the GB coupling $\alpha$ is related to the string mass scale $M_{string}$ and it is defined to be $\alpha = 1/(8M_{string}^2)$ [56]. Thus, $M_*^2 = 8M_{string}^2$. M-theory seems to allow arbitrary values for the string scale. Experimental limits imply that is is not lower than $\mathcal{O}(TeV)$. If the string scale is around a few TeV [57, 58, 59], observation of novel effects in forthcoming experiments becomes a realistic possibility (see e.g. [60]). For the special case $M_{string} = 7 \; TeV$ or $M_* = 19.81 \; TeV$ we obtain

$$2.13 \times 10^{10} \; GeV \leq M_5 \leq 2.45 \times 10^{13} \; GeV \tag{5.68}$$

For the minimum value $M_5 = 2.13 \times 10^{10}\ GeV$ we obtain for $M$, tensor-to-scalar ratio and reheating temperature the following

$$r = marginal \quad (5.69)$$
$$M = 3.28 \times 10^{13}\ GeV \quad (5.70)$$

and

$$1.08 \times 10^{6}\ GeV \leq T_R \leq 4.34 \times 10^{10}\ GeV \quad (5.71)$$

while for the maximum value $M_5 = 2.45 \times 10^{13}\ GeV$ we obtain

$$r = 3.56 \times 10^{-7} \quad (5.72)$$
$$M = 2.85 \times 10^{10}\ GeV \quad (5.73)$$

and

$$1.08 \times 10^{6}\ GeV \leq T_R \leq 6.33 \times 10^{8}\ GeV \quad (5.74)$$

Finally, for the Yukawa coupling $f^2$ we find

- for $M_5 = 2.13 \times 10^{10}\ GeV$, $f^2 < 6.79 \times 10^{-2}$,
- while for $M_5 = 2.45 \times 10^{13}\ GeV$, $f^2 < 5.63 \times 10^{-6}$

However, phenomenological issues such as neutrino masses and axion scale, seem more natural if $M_{string}$ is in the range of $10^{10} - 10^{14}\ GeV$ [61] centered around $10^{12}\ GeV$. For the case $M_{string} \sim 10^{11}\ GeV$ or $M_* = 3 \times 10^{11}\ GeV$ as mentioned already we obtain

$$1.31 \times 10^{15}\ GeV \leq M_5 \leq 1.42 \times 10^{15}\ GeV \quad (5.75)$$

For the minimum value $M_5 = 1.31 \times 10^{15}\ GeV$ we obtain for $M$, tensor-to-scalar ratio and reheating temperature the following

$$r = marginal \quad (5.76)$$
$$M = 3.27 \times 10^{13}\ GeV \quad (5.77)$$

and

$$1.08 \times 10^{6}\ GeV \leq T_R \leq 4.33 \times 10^{10}\ GeV \quad (5.78)$$

while for the maximum value $M_5 = 1.42 \times 10^{15}\ GeV$ we obtain

$$r = 0.4 \quad (5.79)$$
$$M = 3.01 \times 10^{13}\ GeV \quad (5.80)$$

and

$$1.08 \times 10^{6}\ GeV \leq T_R \leq 4.12 \times 10^{10}\ GeV \quad (5.81)$$

Finally, for the Yukawa coupling $f^2$ we find

- for $M_5 = 1.31 \times 10^{15}\ GeV$, $f^2 < 0.07$,
- while for $M_5 = 1.42 \times 10^{15}\ GeV$, $f^2 < 0.06$

Note that in contrast to the standard four-dimensional [34] or to the Randall-Sundrum sneutrino inflation [44] scenarios, in all cases treated above, the Yukawa coupling $f^2$ in the presence of the GB term need not be unnaturally small.

## 5.5 Conclusions

In the present work we have examined sneutrino inflation in the GB braneworld. The GB term appears in the low-energy effective field equations of string theories and it is the lowest order stringy correction to the five-dimensional Einstein gravity. Inflation is driven by the sneutrino inflaton, which is the scalar superpartner of the lightest of the heavy singlet neutrinos, that might explain in a natural way the tiny neutrino masses via the seesaw mechanism. The sneutrino inflaton, apart from driving inflation, also produces the lepton asymmetry that partially is converted to the baryon asymmetry via sphaleron effects. We find that we can get a viable inflationary model that reproduces the correct amplitude for density perturbations and predicts a nearly scale-invariant spectrum and negligible tensor perturbations. Furthermore, the reheating temperature after inflation is such that the gravitino does not upset the BBN results and the required lepton asymmetry is generated. Our analysis shows that all these are simultaneously achieved for a wide range of values of the five-dimensional Planck mass $M_5$ and the mass scale $M_*$ set by the GB coupling.

# Chapter 6

# D-term inflation

We consider hybrid inflation in the braneworld scenario. In particular, we consider inflation in global supersymmetry with the D-terms in the scalar potential for the inflaton field to be the dominant ones (D-term inflation). We find that D-term dominated inflation can naturally accomodate all requirements of the successful hybrid inflationary model also in the framework of D-brane cosmology with global supersymmetry. The reheating temperature after inflation can be high enough ($\sim 10^{10}\, GeV$ or higher) for successful thermal leptogenesis.

## 6.1 Introduction

Recently there has been considerable interest in higher dimensional cosmological models. In those models our four-dimensional world lives on a three-dimensional extended object (brane) which is embedded in a higher dimensional space (bulk). The models of this kind are string-inspired ones, as it is known that in Type I string theory [62] there are two sectors, the open and the closed ones, and that the theory contains extended objects, called D-branes, where open strings can end. The fields in the closed sector (including gravity) can propagate in the bulk, whereas the fields in the open sector are confined to the brane. In such string-inspired scenarios the extra dimensions need not be small [57, 63] and in fact they can even be noncompact [22]. It is important to note that in the context of extra dimensions and the braneworld idea one discovers a generalized Friedmann equation, which is different from the usual Friedmann equation in conventional cosmology. This means that the rate of expansion of the universe in this novel cosmology is altered and accordingly the physics in the early universe can be different from what we know already. So it would be very interesting to study the cosmological implications of these new ideas about extra dimensions and braneworlds. Perhaps the best laboratory for such a study is inflation [15], which has become the standard paradigm in the Big-Bang cosmology and which is in favour after the recent discovery from WMAP satellite (see e.g [11]) that the universe is almost flat. It is known that there is not a theory for inflation yet. All we have is a big collection of inflationary models. The single-field models for inflation, such as 'new' [64] or 'chaotic' [65], are characterized by the disadvantage that they require 'tiny' coupling constants in order to reproduce the observational data. This difficulty was overcome by Linde who proposed, in the context of non-supersymmetric GUTS, the hybrid inflationary scenario [66]. We remark that before that, the authors in [67] worked out a string-inspired version of hybrid inflation. It turns out that one can consider hybrid inflation in supersymmetric theories (for a review on supersymmetry and supergravity see [10]) too. In fact, inflation looks more natural in supersymmetric theories rather in non-supersymmetric ones [68]. In a supersymmetric theory, the tree-level potential is the sum of an F-term and a D-term. These two terms have rather different properties and in all inflationary models only one of them dominates [69]. The case of F-term inflation (where F-terms dominate) was considered for the first time in [70], while the case of D-term inflation (where D-terms dominate) was considered in [71]. In fact, if one considers supergravity then D-term inflation looks more promising, since it avoids the problem associated with the inflaton mass [71]. F-term inflation in braneworld was studied in [72]. In the present note we discuss the implications of D-term inflation. Before proceeding our discussion, let us specify our setup. The braneworld model that we shall consider is the supersymmetric version of the RS II model (see e.g. [23]). However, the cosmological solution of this extended model is the same as that in the non-supersymmetric model, since Einstein's equations belong to the bosonic part. The only sourse in the bulk is a five-dimensional cosmological constant. There is matter confined to the brane and during inflation, which is the cosmological era we shall be interested in, this matter is dominated by a scalar field, called the inflaton field $\phi$.

The present chapter consists of six sections of which this introduction is the first. We present D-term inflation in the second section and brane cosmology in the third. Our results for the inflationary dynamics on the brane are discussed in the fourth section. We discuss reheating after inflation in the fifth section and finally we conclude in the sixth section.

## 6.2 D-term inflation

In this section we explain what D-term inflation is, following essentially [68]. Inflation, by definition, breaks global supersymmetry since it requires a non-zero cosmological constant $V$ (false vacuum energy of the inflaton). For a D-term spontaneous breaking of supersymmetry a term linear in the auxiliary field $D$ is needed (Fayet-Iliopoulos mechanism [73]). If the theory contains an abelian $U(1)$ gauge symmetry (anomalous or not), the Fayet-Iliopoulos D-term

$$\xi \int d^4\theta V = \xi D \tag{6.1}$$

where $V$ is the vector superfield, is supersymmetric and gauge invariant and therefore allowed by the symmetries. We remark that an anomalous $U(1)$ symmetry is usually present in string theories and the anomaly is cancelled by the Green-Schwarz mechanism. However, here we will consider a non-anomalous $U(1)$ gauge symmetry. In the context of global supersymmetry, D-term inflation is derived from the superpotential

$$W = \lambda \Phi \Phi_+ \Phi_- \tag{6.2}$$

where $\Phi, \Phi_-, \Phi_+$ are three chiral superfields and $\lambda$ is the superpotential coupling. Under the $U(1)$ gauge symmetry the three chiral superfields have sharges $Q_\Phi = 0, Q_{\Phi_+} = +1$ and $Q_{\Phi_-} = -1$, respectively. The superpotential given above leads to the following expression for the scalar potential

$$V(\phi_+, \phi_-, |\phi|) = \lambda^2 (|\phi|^2 (|\phi_+|^2 + |\phi_-|^2) + |\phi_+ \phi_-|^2) + \frac{g^2}{2}(|\phi_+|^2 - |\phi_-|^2 + \xi)^2 \tag{6.3}$$

where $\phi$ is the scalar component of the superfield $\Phi$, $\phi_\pm$ are the scalar components of the superfields $\Phi_\pm$, $g$ is the gauge coupling of the $U(1)$ symmetry and $\xi$ is a Fayet-Iliopoulos term, chosen to be positive. The global minimum is supersymmetry conserving, but the gauge group $U(1)$ is spontaneously broken

$$\langle \phi \rangle = \langle \phi_+ \rangle = 0, \;\; \langle \phi_- \rangle = \sqrt{\xi} \tag{6.4}$$

However, if we minimize the potential, for fixed values of $\phi$, with respect to other fields, we find that for $\phi > \phi_c = \frac{g}{\lambda}\sqrt{\xi}$, the minimum is at $\phi_+ = \phi_- = 0$. Thus, for $\phi > \phi_c$ and $\phi_+ = \phi_- = 0$ the tree-level potential has a vanishing curvature in the $\phi$ direction and large positive curvature in the remaining two directions $m_\pm^2 = \lambda^2 |\phi|^2 \pm g^2 \xi$.

For arbitrary large $\phi$ the tree-level value of the potential remains constant and equal to $V_0 = (g^2/2)\xi^2$, thus $\phi$ plays naturally the role of an inflaton field. Along the inflationary trajectory the F-term vanishes and the universe is dominated by the D-term, which splits the masses in the $\Phi_+$ and $\Phi_-$ superfields, resulting to the one-loop effective potential for the inflaton field. The radiative corrections are given by the Coleman-Weinberg formula [74]

$$\Delta V_{1-loop} = \frac{1}{64\pi} \sum_i (-1)^{F_i} m_i^4 ln \frac{m_i^2}{\Lambda^2} \tag{6.5}$$

where $\Lambda$ stands for a renormalization scale which does not affect physical quantities and the sum extends over all helicity states $i$, with fermion number $F_i$ and mass squared $m_i^2$. The radiative corrections given by the above formula lead to the following effective potential for D-term inflation

$$V(\phi) = \frac{g^2\xi^2}{2} \left(1 + \frac{g^2}{16\pi^2} ln \frac{|\phi|^2\lambda^2}{\Lambda^2}\right) \tag{6.6}$$

The end of inflation is determined either by the failure of the slow-roll conditions or when $\phi$ approaches $\phi_c$.

## 6.3 Effective gravitational equations on the brane

Here we review the basic equations of brane cosmology. We work essentially in the context of Randall-Sundrum II model [22]. In the bulk there is just a cosmological constant $\Lambda_5$, whereas on the brane there is matter with energy-momentum tensor $\tau_{\mu\nu}$. Also, the brane has a tension $T$. The five-dimensional Planck mass is denoted by $M_5$. If Einstein's equations hold in the five-dimensional bulk, then it has been shown in [28] that the effective four-dimensional Einstein's equations induced on the brane can be written as

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \frac{8\pi}{M_p^2} \tau_{\mu\nu} + (\frac{8\pi}{M_5^3})^2 \pi_{\mu\nu} - E_{\mu\nu} \tag{6.7}$$

where $g_{\mu\nu}$ is the induced metric on the brane, $\pi_{\mu\nu} = \frac{1}{12}\,\tau\,\tau_{\mu\nu} + \frac{1}{8}\,g_{\mu\nu}\,\tau_{\alpha\beta}\,\tau^{\alpha\beta} - \frac{1}{4}\,\tau_{\mu\alpha}\,\tau_\nu^\alpha - \frac{1}{24}\,\tau^2\,g_{\mu\nu}$, $\Lambda_4$ is the effective four-dimensional cosmological constant, $M_p$ is the usual four-dimensional Planck mass and $E_{\mu\nu} \equiv C^\alpha_{\beta\rho\sigma}\,n_\alpha\,n^\rho\,g^\beta_\mu\,g^\sigma_\nu$ is a projection of the five-dimensional Weyl tensor $C_{\alpha\beta\rho\sigma}$, where $n^\alpha$ is the unit vector normal to the brane. The tensors $\pi_{\mu\nu}$ and $E_{\mu\nu}$ describe the influence of the bulk in brane dynamics. The five-dimensional quantities are related to the corresponding four-dimensional ones through the relations

$$M_p = \sqrt{\frac{3}{4\pi}} \frac{M_5^3}{\sqrt{T}} \tag{6.8}$$

and

$$\Lambda_4 = \frac{4\pi}{M_5^3}\left(\Lambda_5 + \frac{4\pi T^2}{3M_5^3}\right) \tag{6.9}$$

In a cosmological model in which the induced metric on the brane $g_{\mu\nu}$ has the form of a spatially flat Friedmann-Robertson-Walker model, with scale factor $a(t)$, the Friedmann-like equation on the brane has the generalized form (see e.g. the second reference in [25])

$$H^2 = \frac{\Lambda_4}{3} + \frac{8\pi}{3M_p^2}\rho + (\frac{4\pi}{3M_5^3})^2\rho^2 + \frac{C}{a^4} \tag{6.10}$$

where $C$ is an integration constant arising from $E_{\mu\nu}$. The cosmological constant term and the term linear in $\rho$ are familiar from the four-dimensional conventional cosmology. The extra terms, i.e. the "dark radiation" term and the term quadratic in $\rho$, are there because of the presence of the extra dimension. Adopting the Randall-Sundrum fine-tuning

$$\Lambda_5 = -\frac{4\pi T^2}{3M_5^3} \tag{6.11}$$

the four-dimensional cosmological constant vanishes. Furthermore, the term with the integration constant $C$ will be rapidly diluted during inflation and can be ignored. So the generalized Friedmann equation takes the final form

$$H^2 = \frac{8\pi}{3M_p^2}\rho\left(1 + \frac{\rho}{2T}\right) \tag{6.12}$$

We notice that in the low density regime $\rho \ll T$ we recover the usual Friedmann equation. However, in the high energy regime $\rho \gg T$ the unity can be neglected and then the Friedmann-like equation becomes

$$H^2 = \frac{4\pi\rho^2}{3TM_p^2} \tag{6.13}$$

Note that in this regime the Hubble parameter is linear in $\rho$, while in conventional cosmology it goes with the square root of $\rho$.

## 6.4 Inflationary dynamics on the brane

As already mentioned, we will consider the case in which the energy-momentum on the brane is dominated by a scalar field $\phi$ confined on the brane with a self-interaction potential $V(\phi)$ given in (6.6). The field $\phi$ is a function of time only, as dictated by the isotropy and homogeneity of the observed four-dimensional universe. A homogeneous scalar field behaves like a perfect fluid with pressure $p = (1/2)\dot{\phi}^2 - V$ and energy density $\rho = (1/2)\dot{\phi}^2 + V$. There is no energy exchange between the brane and the bulk, so the energy-momentum tensor $T_{\mu\nu}$ of the scalar field is conserved, that is $\nabla^\nu T_{\mu\nu} = 0$. This is equivalent to the continuity equation for the pressure $p$ and the energy density $\rho$

$$\dot{\rho} + 3H(p + \rho) = 0 \tag{6.14}$$

where $H$ is the Hubble parameter $H = \dot{a}/a$. Therefore we get the equation of motion for the scalar field $\phi$, which is the following

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{6.15}$$

This is of course the Klein-Gordon equation for a scalar field in a Robertson-Walker background. The equation that governs the dynamics of the expansion of the universe is the Friedmann-like equation of the previous section. Inflation takes place in the early stages of the evolution of the universe, so in the Friedmann equation the extra term dominates and therefore the equation for the scale factor is

$$H^2 = \frac{4\pi\rho^2}{3TM_p^2} \tag{6.16}$$

In the slow-roll approximation the slope and the curvature of the potential must satisfy the two constraints $\epsilon \ll 1$ and $|\eta| \ll 1$, where $\epsilon$ and $\eta$ are the two slow-roll parameters which are defined by

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \tag{6.17}$$

$$\eta \equiv \frac{V''}{3H^2} \tag{6.18}$$

In this approximation the equation of motion for the scalar field takes the form

$$\dot{\phi} \simeq -\frac{V'}{3H} \tag{6.19}$$

while the generalized Friedmann equation becomes ($V \gg \dot{\phi}^2$)

$$H^2 \simeq \frac{4\pi V^2}{3TM_p^2} \tag{6.20}$$

The number of e-folds during inflation is given by

$$N \equiv ln\frac{a_f}{a_i} = \int_{t_i}^{t_f} H dt \tag{6.21}$$

For a strong enough inflation we take $N = 60$. In the slow-roll approximation the number of e-folds and the slow-roll parameters are given by the formulae [31]

$$\epsilon \simeq \frac{M_p^2}{16\pi}\left(\frac{V'}{V}\right)^2 \frac{4T}{V} \tag{6.22}$$

$$\eta \simeq \frac{M_p^2}{8\pi}\left(\frac{V''}{V}\right) \frac{2T}{V} \tag{6.23}$$

$$N \simeq -\frac{8\pi}{M_p^2}\int_{\phi_i}^{\phi_f} \frac{V}{V'}\frac{V}{2T}\, d\phi \tag{6.24}$$

The main cosmological constraint comes from the amplitude of the scalar perturbations which is given in this new context by [31]

$$A_s^2 = \frac{512\pi}{75M_p^6}\frac{V^3}{V'^2}\left(\frac{V}{2T}\right)^3 \tag{6.25}$$

where the right-hand side is evaluated at the horizon-crossing when the comoving scale equals the Hubble radius during inflation. Finally, the spectral index for the scalar perturbations is given in terms of the slow-roll parameters

$$n_s - 1 \equiv \frac{d\, lnA_s^2}{d\, lnk} = 2\eta - 6\epsilon \tag{6.26}$$

and the tensor-to-scalar ratio is given by

$$\frac{A_t^2}{A_s^2} = \epsilon\,\frac{T}{V} \tag{6.27}$$

In what follows we will assume that $g \sim 0.5$ and that inflation ends at $\phi_c = (g/\lambda)\sqrt{\xi}$. To make sure that the slow-roll conditions are satisfied we impose the constraint

$$\frac{TM_p^2\lambda^2}{16\pi^3 g^2\xi^3} \ll 1 \tag{6.28}$$

Also, we have assumed that the potential $V$ is much larger than the brane tension $T$. Therefore another constraint to be satisfied is

$$\frac{g^2\xi^2}{4T} \gg 1 \tag{6.29}$$

Now that we have written all the necessary formulae, we can proceed to the presentation of our results. For arbitrary $\lambda$ it is not possible to satisfy both the datum from COBE that $A_s = 2\times 10^{-5}$ and the slow-roll conditions. For this to happen the superpotential coupling $\lambda$ has to be smaller or equal to 0.0245 (approximately). Then, for a given value for $\lambda$, the brane tension cannot become arbitrarily large because in that case the constraint that the potential should be much larger than the brane tension is not satisfied. We find the following upper bound for the brane tension $T$

$$T \leq 10^{55} - 10^{56}\, GeV^4 \tag{6.30}$$

Now that we have set upper bounds for $T$ and $\lambda$ so that our constraints and the data from COBE are satisfied, we can compute the spectral index $n_s$ and the tensor-to-scalar ratio $r$. We find for the spectral index $n_s = 0.983 - 0.998$ and for the ratio $r \sim 10^{-4}$ or lower.

A detailed analysis shows that for a particular value for $\lambda$ (below the upper bound of course) the spectral index does not depend on $T$ and is always very close to 1. As $\lambda$ becomes smaller and smaller the spectral index slightly increases and gets even closer to 1. Also, in all cases the tensor perturbations are negligible. Finally, we find that for the maximum value for the brane tension, $M_5^{(max)} \sim 10^{15} GeV$ and $\left(\sqrt{\xi}\right)_{max} = (3.99 - 6.74) \times 10^{14} GeV$, whereas $\sqrt{\xi}$ becomes smaller as $T$ decreases. We note that according to our analysis $\lambda$ a priori can take arbitrarily small values. However, this would be unnatural and for that reason we do not consider values for $\lambda$ smaller than $5 \times 10^{-4}$. In that case we find that the values of the inflaton remain safely below Planck mass and therefore global supersymmetry is a good approximation. Our result differs from a similar study [75], in which the discussion leads to the conclusion that supergravity corrections are important.

## 6.5 Reheating

Finally, let us turn to the discussion of reheating after inflation and to the computation of the reheating temperature $T_R$. After slow-roll the inflaton decays with a decay rate $\Gamma$ and the decay products quickly thermalize. This is the way the universe reenters the radiation era of standard Big-Bang cosmology. The reheating temperature $T_R$ is related to two more cosmological topics, namely the gravitino problem [50] and the baryogenesis through leptogenesis. In gravity mediated SUSY breaking models and for an interesting range of the gravitino mass, $m_{3/2} \sim 0.1 - 1 TeV$, if the gravitino is unstable it has a long lifetime and decays after the BBN. The decay products destroy light elements produced by the BBN and hence the primordial abundance of the gravitino is constrained from above to keep the success of the BBN.

This leads to an upper bound on the reheating temperature $T_R$ after inflation, since the abundance of the gravitino is proportional to $T_R$. A detailed analysis derived a stringent upper bound $T_R \leqslant 10^6 - 10^7\, GeV$ when gravitino has hadronic modes [51, 76]. On the other hand, primordial lepton asymmetry is converted to baryon asymmetry [77] in the early universe through the “sphaleron” effects of the electroweak gauge theory [53]. This baryogenesis through leptogenesis requires a lower bound on the reheating temperature. Leptogenesis can be thermal or non-thermal. For a thermal leptogenesis $T_R \geqslant 2 \times 10^9\, GeV$ [78], whereas for non-thermal leptogenesis $T_R \geqslant 10^6\, GeV$ [79]. It seems that it is impossible to satisfy both constaints for the reheating temperature coming from leptogenesis and the gravitino problem. However, the authors of [32] have showed that in the brane world scenario, that we discuss here, it is possible to solve the gravitino problem allowing for the reheating temperature to be as high as $10^{10}\, GeV$. According to reference [32] the gravitino abundance is proportional not to the reheating temperature, as is the case in conventional cosmology, but to a transition temperature $T_t$ between high temperatures ($T_R$) and low ones (today's temperature $T_0$). That way the requirement for not over-production of gravitino leads to an upper bound for this transition temperature and not for the reheating temperature, which can be as high as a satisfactory leptogenesis requires.

The reheating temperature is given by the formula

$$T_R = \left( \sqrt{\frac{3T}{\pi}}\, \frac{15 \Gamma M_p}{\pi^2 g_{eff}} \right)^{1/4} \tag{6.31}$$

where $g_{eff}$ is the effective number of degrees of freedom at the reheating temperature and for the MSSM is $g_{eff} = \frac{915}{4}$. Assuming that the inflaton $\phi$ decays to the lighest of the three heavy right handed neutrinos $\psi$

$$\phi \;\rightarrow\; \psi + \psi \tag{6.32}$$

the decay rate of the inflaton is [69]

$$\Gamma = \frac{m_{infl}}{8\pi} \left( \frac{M_1}{\sqrt{\xi}} \right)^2 \tag{6.33}$$

where $m_{infl}$ is the inflaton mass, $M_1$ is the smallest of the three neutrino mass eigenvalues and $m_{infl} > 2M_1$. The mass of the inflaton is given in terms of the coupling constant $g$ and the Fayet-Iliopoulos parameter $\xi$ by

$$m_{infl} = \sqrt{2}\, g\, \sqrt{\xi} \tag{6.34}$$

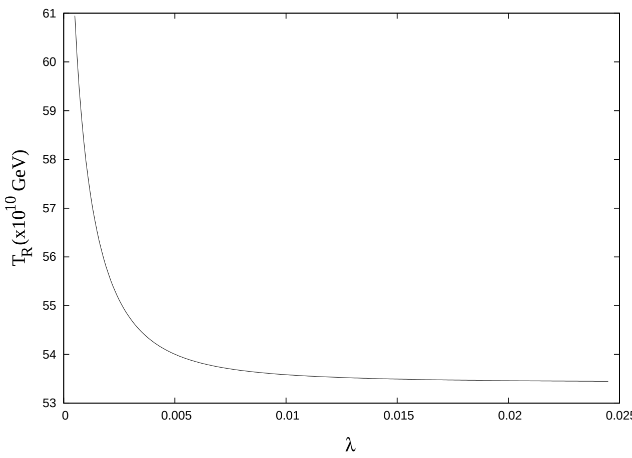


Figure 6.1: Reheating temperature $T_R$ versus the superpotential coupling $\lambda$ for $M_1 = 10^{10}\ GeV$ and $\sqrt{\xi} = 10^{14}\ GeV$.

If the value of the mass of the lightest right handed neutrino is $M_1 = 10^{10}\ GeV$, which is a representative value, then the reheating temperature $T_R$ can be one to two orders of magnitude larger than the right handed neutrino mass, depending on the values of the superpotential coupling $\lambda$ and the Fayet-Iliopoulos term $\xi$ (see Figures 6.1 and 6.2). So we see that the reheating temperature is of the right order of magnitude for thermal leptogenesis. When the right handed neutrino mass increases (remaining though smaller than $m_{infl}/2$), the reheating temperature increases too and in fact it goes like $\sim \sqrt{M_1}$. For example, if $\lambda = 0.01$ and $\sqrt{\xi} = 10^{14}\ GeV$, then $T_R = 4.15 \times 10^{11}\ GeV$ for $M_1 = 6 \times 10^9\ GeV$ and $T_R = 3.79 \times 10^{12}\ GeV$ for $M_1 = 5 \times 10^{11}\ GeV$. Finally, for a given $M_1$ and a given $\xi$, when $\lambda$ increases then $T_R$ decreases, but only slighty so as to remain of the right order of magnitude for a successful leptogenesis (see Figure 6.1). Also, for a given value of $M_1$ and $\lambda$, when $\xi$ increases, $T_R$ increases also (see Figure 6.2).

## 6.6 Conclusions

To summarize, we have reexamined supersymmetric D-term dominated hybrid inflation in brane cosmology. We have found that we can reproduce the observational data provided that each of the brane tension, five-dimensional Planck mass and the superpotential coupling does not exceed a particular value. For a given value for the superpotential coupling, when the brane tension takes the maximum allowed value then the scale of inflation $\sqrt{\xi}$ is of

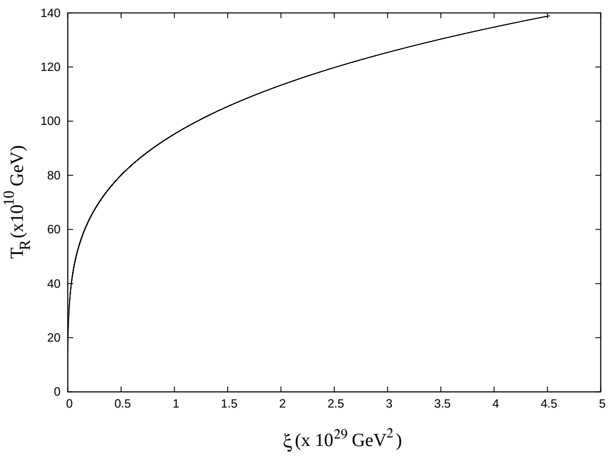


Figure 6.2: Reheating temperature $T_R$ versus the Fayet-Iliopoulos term $\xi$ for superpotential coupling $\lambda = 0.01$ and $M_1 = 10^{10}\ GeV$.

the order of $\sim 10^{14}\ GeV$. This value of the inflationary scale is lower than the (supersymmetric) GUT scale, but close to it. Also, we have found that for natural values of the superpotential coupling $\lambda$ the inflaton field cannot take large values and stays well below the four-dimensional Planck mass, consistent with the global supersymmetry approximation adopted here. Furthermore, we have seen that our results are compatible with the corresponding results in the standard four-dimensional cosmology. This means that the advantages of the hybrid model are naturally preserved in the framework of brane cosmology. Finally, our study shows that the reheating temperature after inflation can naturally be of order $10^{10}\ GeV$ (or larger) allowing for a successful thermal leptogenesis.

# Chapter 7

# Brane-Bulk energy exchange

The role of brane-bulk energy exchange and of an induced gravity term on a single braneworld of negative tension and vanishing effective cosmological constant is studied. It is shown that for the physically interesting cases of dust and radiation a unique global attractor which can realize our present universe (accelerating and $0 < \Omega_{m0} < 1$) exists for a wide range of the parameters of the model. For $\Omega_{m0} = 0.3$, independently of the other parameters, the model predicts that the equation of state for the dark energy today is $w_{DE,0} = -1.4$, while $\Omega_{m0} = 0.03$ leads to $w_{DE,0} = -1.03$. In addition, during its evolution, $w_{DE}$ crosses the $w_{DE} = -1$ line to smaller values.

## 7.1 Introduction

In cosmologies where the present universe is realized as a finite point during the cosmic evolution, the answer to the coincidence question "why it is that today $\Omega_{m0}$ and $\Omega_{DE,0}$ are of the same order of magnitude", relies on appropriate choice of *initial conditions*. By contrast, in a scenario in which the present universe is in its asymptotic era (close to a fixed point) the answer to the above question reduces to an appropriate choice of the *parameters* of the model. However, this latter situation is not easily realized if today's universe is accelerating, because:

If the energy density of a perfect fluid with equation of state $w > -1/3$ of any cosmological system is conserved, all fixed points of the system with $\Omega_m \neq 0$ are decelerating.

Indeed, with $\rho$ the energy density of the perfect fluid with conservation equation $\dot{\rho} + 3(1+w)H\rho = 0$, the Hubble equation of an arbitrary cosmology can be written in the form

$$H^2 = 2\gamma(\rho + \rho_{DE}) \tag{7.1}$$

where $\gamma = 4\pi G_N/3$. Then, the equation governing $\rho_{DE}$ can always be brought into the form $\dot{\rho}_{DE} + 3(1 + w_{DE})H\rho_{DE} = 0$, where $w_{DE}$ is time-dependent and distiguishes one model from the other. It can be easily seen that $d(\Omega_m/\Omega_{DE})/d\ln a = 3(\Omega_m/\Omega_{DE})(w_{DE} - w)$ and $2q = 1 + 3(w\Omega_m + w_{DE}\Omega_{DE})$, where $\Omega_m = 2\gamma\rho/H^2$, $\Omega_{DE} = 2\gamma\rho_{DE}/H^2$ and $q = -\ddot{a}/aH^2$. At the fixed point (denoted by $*$) $d(\Omega_m/\Omega_{DE})/d\ln a = 0$. For $\Omega_{m*} \neq 0$ one obtains $w_{DE*} = w$, and $2q_* = 1 + 3w > 0$.

Thus, independently of the cosmological model, the only way our accelerating universe with $\Omega_{m*} \neq 0$ can be close to a late time fixed point is by violating the standard conservation equation of matter. In 4-dimensional theories, an accelerating late time cosmological phase characterized by a frozen ratio of dark matter/dark energy appears in coupled dark energy scenarios [80] as a result of the interaction of the dark matter with other energy-momentum components, such as scalar fields. In higher dimensional theories, where the universe is represented as a 3-brane, this violation could be the result of energy exchange between the brane and the bulk. In particular in five dimensions, a universe with fixed points characterized by $\Omega_{m*} \neq 0$, $q_* < 0$ was realized in [27] in the context of the Randall-Sundrum braneworld scenario with energy influx from the bulk. However, these fixed points cannot represent the present universe, since they have $\Omega_{m*} > 2$. In this paper we present a brane-bulk energy exchange model with induced gravity whose global attractor can represent today's universe.

Let us consider an arbitrary cosmology in the form (7.1). Instances of such cosmologies arise in braneworld models or in theories with modified 4-dimensional actions leading to $H^2 = f(\rho)$, or in cosmologies where $\rho_{DE}$ is due to additional fields. Assuming that as a result of some interaction $\rho$ is not conserved, it will satisfy an equation of the form

$$\dot{\rho} + 3(1 + w)H\rho = -T \tag{7.2}$$

Then, the equation governing $\rho_{DE}$ can always be brought into the form

$$\dot{\rho}_{DE} + 3(1 + w_{DE})H\rho_{DE} = T \tag{7.3}$$

where $w_{DE}$ is time and model dependent. Whenever a fixed point of the system satisfies

$$H_*T_* \neq 0 \quad , \quad \dot{\rho} = \dot{\rho}_{DE} = 0 \tag{7.4}$$

one obtains

$$w_{DE*} = -1 - \frac{1+w}{\Omega_{m*}^{-1} - 1} \tag{7.5}$$

Equation (7.5) is model-independent, in the sense that it does not depend on the form of $T$ or the function $w_{DE}(t)$. For $\Omega_{m*} < 1$ equation (7.5) gives $w_{DE*} < -1$. Specifically, for $w = 0$ and $\Omega_{m*} = \Omega_{CDM} = 0.3$ one obtains $w_{DE*} = -1.4$, while for $\Omega_{m*} = \Omega_{bar} = 0.03$, $w_{DE*} = -1.03$.

The cosmology discussed in the present paper has a global attractor of the form (7.4), (7.5) [81]. Moreover, the universe during its evolution crosses the $w_{DE} = -1$ barrier from higher values. This behavior is favored by several recent model-independent [82] as well as model-dependent [83, 84, 85, 86] analyses of the astronomical data.

## 7.2 The model

We consider the model described by the gravitational brane-bulk action [87]

$$S = \int d^5x\sqrt{-g}\,(M^3R - \Lambda) + \int d^4x\sqrt{-h}\,(m^2\hat{R} - V) \tag{7.6}$$

where $R, \hat{R}$ are the Ricci scalars of the bulk metric $g_{AB}$ and the induced metric $h_{AB} = g_{AB} - n_A n_B$ respectively ($n^A$ is the unit vector normal to the brane and $A, B = 0, 1, 2, 3, 5$). The bulk cosmological constant is $\Lambda/2M^3 < 0$, the brane tension is $V$, and the induced-gravity crossover scale is $r_c = m^2/M^3$.

We assume the cosmological bulk ansatz

$$ds^2 = -n(t,y)^2dt^2 + a(t,y)^2\gamma_{ij}dx^idx^j + b(t,y)^2dy^2 \tag{7.7}$$

where $\gamma_{ij}$ is a maximally symmetric 3-dimensional metric, parametrized by the spatial curvature $k = -1, 0, 1$. The non-zero components of the five-dimensional Einstein tensor are

$$G_{00} = 3\Big\{\frac{\dot{a}}{a}\Big(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\Big) - \frac{n^2}{b^2}\Big[\frac{a''}{a} + \frac{a'}{a}\Big(\frac{a'}{a} - \frac{b'}{b}\Big)\Big] + \frac{kn^2}{a^2}\Big\} \tag{7.8}$$

$$\begin{aligned} G_{ij} = {} & \frac{a^2}{b^2}\gamma_{ij}\Big\{\frac{a'}{a}\Big(\frac{a'}{a} + \frac{2n'}{n}\Big) - \frac{b'}{b}\Big(\frac{n'}{n} + \frac{2a'}{a}\Big) + \frac{2a''}{a} + \frac{n''}{n}\Big\} \\ & + \frac{a^2}{n^2}\gamma_{ij}\Big\{\frac{\dot{a}}{a}\Big(\frac{2\dot{n}}{n} - \frac{\dot{a}}{a}\Big) - \frac{2\ddot{a}}{a} + \frac{\dot{b}}{b}\Big(\frac{\dot{n}}{n} - \frac{2\dot{a}}{a}\Big) - \frac{\ddot{b}}{b}\Big\} - k\gamma_{ij} \end{aligned} \tag{7.9}$$

$$G_{05} = 3\Big(\frac{n'}{n}\frac{\dot{a}}{a} + \frac{a'}{a}\frac{\dot{b}}{b} - \frac{\dot{a}'}{a}\Big) \tag{7.10}$$

$$G_{55} = 3\Big\{\frac{a'}{a}\Big(\frac{a'}{a} + \frac{n'}{n}\Big) - \frac{b^2}{n^2}\Big[\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\Big(\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\Big)\Big] - \frac{kb^2}{a^2}\Big\} \tag{7.11}$$

where primes indicate derivatives with respect to $y$, while dots derivatives with respect to $t$. The five-dimensional Einstein equations take the usual form

$$G_{AC} = \frac{1}{2M^3} T_{AC}|_{tot} \tag{7.12}$$

where

$$T^A_C|_{tot} = T^A_C|_{v,B} + T^A_C|_{m,B} + T^A_C|_{v,b} + T^A_C|_{m,b} + T^A_C|_{ind} \tag{7.13}$$

is the total energy-momentum tensor,

$$T^A_C|_{v,B} = \mathrm{diag}(-\Lambda, -\Lambda, -\Lambda, -\Lambda, -\Lambda) \tag{7.14}$$

$$T^A_C|_{v,b} = \mathrm{diag}(-V, -V, -V, -V, 0)\frac{\delta(y)}{b} \tag{7.15}$$

$$T^A_C|_{m,b} = \mathrm{diag}(-\rho, p, p, p, 0)\frac{\delta(y)}{b} \tag{7.16}$$

$T^A_C|_{m,B}$ is any possible additional energy-momentum in the bulk, the brane matter content $T^A_C|_{m,b}$ consists of a perfect fluid with energy density $\rho$ and pressure $p$, while the contributions arising from the scalar curvature of the brane are given by

$$T^0_0|_{ind} = \frac{6m^2}{n^2}\Big(\frac{\dot{a}^2}{a^2} + \frac{kn^2}{a^2}\Big)\frac{\delta(y)}{b} \tag{7.17}$$

$$T^i_j|_{ind} = \frac{2m^2}{n^2}\Big(\frac{\dot{a}^2}{a^2} - \frac{2\dot{a}\dot{n}}{an} + \frac{2\ddot{a}}{a} + \frac{kn^2}{a^2}\Big)\delta^i_j\frac{\delta(y)}{b} \tag{7.18}$$

Assuming a $\mathbb{Z}_2$ symmetry around the brane, the singular part of equations (7.12) gives the matching conditions

$$\frac{a'_{o^+}}{a_o b_o} = -\frac{\rho + V}{12M^3} + \frac{r_c}{2n_o^2}\Big(\frac{\dot{a}_o^2}{a_o^2} + \frac{kn_o^2}{a_o^2}\Big) \tag{7.19}$$

$$\frac{n'_{o^+}}{n_o b_o} = \frac{2\rho + 3p - V}{12M^3} + \frac{r_c}{2n_o^2}\Big(\frac{2\ddot{a}_o}{a_o} - \frac{\dot{a}_o^2}{a_o^2} - \frac{2\dot{a}_o\dot{n}_o}{a_o n_o} - \frac{kn_o^2}{a_o^2}\Big) \tag{7.20}$$

(the subscript o denotes the value on the brane), while from the 05, 55 components of equations (7.12) we obtain

$$\frac{n'_o}{n_o}\frac{\dot{a}_o}{a_o} + \frac{a'_o}{a_o}\frac{\dot{b}_o}{b_o} - \frac{\dot{a}'_o}{a_o} = \frac{T_{05}}{6M^3} \tag{7.21}$$

$$\frac{a'_o}{a_o}\Big(\frac{a'_o}{a_o} + \frac{n'_o}{n_o}\Big) - \frac{b_o^2}{n_o^2}\Big[\frac{\ddot{a}_o}{a_o} + \frac{\dot{a}_o}{a_o}\Big(\frac{\dot{a}_o}{a_o} - \frac{\dot{n}_o}{n_o}\Big)\Big] - \frac{kb_o^2}{a_o^2} = \frac{T_{55} - \Lambda b_o^2}{6M^3} \tag{7.22}$$

where $T_{05}, T_{55}$ are the 05 and 55 components of $T_{AC}|_{m,B}$ evaluated on the brane. Substituting the expressions (7.19), (7.20) in equations (7.21), (7.22), we obtain the semi-conservation law and the Raychaudhuri equation

$$\dot{\rho} + 3\frac{\dot{a}_o}{a_o}(\rho + p) = -\frac{2n_o^2}{b_o}T_5^0 \tag{7.23}$$

$$\begin{aligned} \Big(H_o^2 + \frac{k}{a_o^2}\Big)\Big[1 - \frac{r_c^2(\rho + 3p - 2V)}{24m^2}\Big] + \frac{r_c^2(\rho + 3p - 2V)(\rho + V)}{144m^4} \\ + \Big(\frac{\dot{H}_o}{n_o} + H_o^2\Big)\Big[1 - \frac{r_c^2}{2}\Big(H_o^2 + \frac{k}{a_o^2}\Big) + \frac{r_c^2(\rho + V)}{12m^2}\Big] = \frac{\Lambda - T_5^5}{6M^3} \end{aligned} \tag{7.24}$$

where $H_o = \dot{a}_o/a_o n_o$ is the Hubble parameter of the brane. One can easily check that in the limit $m \to 0$, equation (7.24) reduces to the corresponding second order equation of the model without $\hat{R}$ [27]. Energy exchange between the brane and the bulk has also been investigated in [88, 89, 90].

Since only the 55 component of $T_{AC}|_{m,B}$ enters equation (7.24), one can derive a cosmological system that is largely independent of the bulk dynamics, if at the position of the brane the contribution of this component relative to the bulk vacuum energy is much less important than the brane matter relative to the brane vacuum energy, or schematically

$$\Big|\frac{T_5^5}{\Lambda}\Big| \ll \Big|\frac{\rho}{V}\Big| \tag{7.25}$$

Then, for $|\Lambda|$ not much larger than the Randall-Sundrum value $V^2/12M^3$, the term $T_5^5$ in equation (7.24) can be ignored. Alternatively, the term $T_5^5$ can be ignored in equation (7.24) if simply

$$\Big|\frac{T_5^5}{\Lambda}\Big| \ll 1 \tag{7.26}$$

Note that relations (7.25) and (7.26) are only boundary conditions for $T_5^5$, which in a realistic description in terms of bulk fields will be translated into boundary conditions on these fields. In the special case where (7.25), (7.26) are valid throughout the bulk, the latter remains unperturbed by the exchange of energy with the brane.

One can now check that a first integral of equation (7.24) is

$$\begin{aligned} H_o^4 - \frac{2H_o^2}{3}\Big(\frac{\rho + V}{2m^2} + \frac{6}{r_c^2} - \frac{3k}{a_o^2}\Big) + \Big(\frac{\rho + V}{6m^2} - \frac{k}{a_o^2}\Big)^2 + \\ + \frac{4}{r_c^2}\Big(\frac{\Lambda}{12M^3} - \frac{k}{a_o^2}\Big) - \frac{\chi}{3r_c^2} = 0 \end{aligned} \tag{7.27}$$

with $\chi$ satisfying

$$\dot{\chi} + 4n_oH_o\chi = \frac{r_c^2 n_o^2 T}{m^2 b_o}\Big(H_o^2 - \frac{\rho + V}{6m^2} + \frac{k}{a_o^2}\Big) \tag{7.28}$$

and $T = 2T_5^0$ is the discontinuity across the brane of the 05 component of the bulk energy-momentum tensor. The solution of (7.27) for $H_o$ is

$$H_o^2 = \frac{\rho + V}{6m^2} + \frac{2}{r_c^2} - \frac{k}{a_o^2} \pm \frac{1}{\sqrt{3}r_c}\Big[\frac{2(\rho + V)}{m^2} + \frac{12}{r_c^2} - \frac{\Lambda}{M^3} + \chi\Big]^{\frac{1}{2}} \tag{7.29}$$

and equation (7.28) becomes

$$\dot{\chi} + 4n_oH_o\chi = \frac{2n_o^2 T}{m^2 b_o}\Big\{1 \pm \frac{r_c}{2\sqrt{3}}\Big[\frac{2(\rho + V)}{m^2} + \frac{12}{r_c^2} - \frac{\Lambda}{M^3} + \chi\Big]^{\frac{1}{2}}\Big\} \tag{7.30}$$

At this point we find it convenient to employ a coordinate frame in which $b_o = n_o = 1$ in the above equations. This can be achieved by using Gauss normal coordinates with $b(t, z) = 1$, and by going to the temporal gauge on the brane with $n_o = 1$. It is also convenient to define the parameters

$$\lambda = \frac{2V}{m^2} + \frac{12}{r_c^2} - \frac{\Lambda}{M^3} \tag{7.31}$$

$$\mu = \frac{V}{6m^2} + \frac{2}{r_c^2} \tag{7.32}$$

$$\gamma = \frac{1}{12m^2} \tag{7.33}$$

$$\beta = \frac{1}{\sqrt{3}r_c} \tag{7.34}$$

For a perfect fluid on the brane with equation of state $p = w\rho$ our system is described by equations (7.23), (7.29), (7.30), which simplify to (we omit the subscript o in the following)

$$\dot{\rho} + 3(1 + w)H\rho = -T \tag{7.35}$$

$$H^2 = \mu + 2\gamma\rho \pm \beta\sqrt{\lambda + 24\gamma\rho + \chi} - \frac{k}{a^2} \tag{7.36}$$

$$\dot{\chi} + 4H\chi = 24\gamma T\Big(1 \pm \frac{1}{6\beta}\sqrt{\lambda + 24\gamma\rho + \chi}\Big) \tag{7.37}$$

while the second order equation (7.24) for the scale factor becomes

$$\frac{\ddot{a}}{a} = \mu - (1 + 3w)\gamma\rho \pm \beta\frac{\lambda + 6(1 - 3w)\gamma\rho}{\sqrt{\lambda + 24\gamma\rho + \chi}} \tag{7.38}$$

Finally, setting $\psi \equiv \sqrt{\lambda + 24\gamma\rho + \chi}$, equations (7.36), (7.37), (7.38) take the form

$$H^2 = \mu + 2\gamma\rho \pm \beta\psi - \frac{k}{a^2} \tag{7.39}$$

$$\dot{\psi} + 2H\Big(\psi - \frac{\lambda + 6(1-3w)\gamma\rho}{\psi}\Big) = \pm\frac{2\gamma T}{\beta} \tag{7.40}$$

$$\frac{\ddot{a}}{a} = \mu - (1+3w)\gamma\rho \pm \beta\frac{\lambda + 6(1-3w)\gamma\rho}{\psi} \tag{7.41}$$

Throughout, we will assume $T(\rho) = A\rho^{\nu}$, with $\nu > 0$, $A$ constant parameters [27, 26]. Notice that the system of equations (7.35)-(7.37) has the influx-outflow symmetry $T \to -T$, $H \to -H$, $t \to -t$. For $T = 0$ the system reduces to the cosmology studied in [91].

We will be referring to the upper (lower) $\pm$ solution as Branch A (Branch B). We shall be interested in a model that reduces to the Randall-Sundrum vacuum in the absence of matter, i.e. it has vanishing effective cosmological constant. This is achieved for $\mu = \mp\beta\sqrt{\lambda}$, which, given that $m^2V + 12M^6$ is negative (positive) for branches A (B), is equivalent to the fine-tuning $\Lambda = -V^2/12M^3$. Notice that for Branch A, $V$ is necessarily negative. Cosmologies with negative brane tension in the induced gravity scenario have also been discussed in [92].

Consider the case $k = 0$. The system possesses the obvious fixed point $(\rho_*, H_*, \psi_*) = (0, 0, \sqrt{\lambda})$. However, for $sgn(H)T < 0$ there are non-trivial fixed points, which are found by setting $\dot{\rho} = \dot{\psi} = 0$ in equations (7.35), (7.40). For $w \leq 1/3$ these are:

$$\begin{aligned}\frac{2T(\rho_*)^2}{9(1+w)^2\rho_*^2} &= 2\mu + (1-3w)\gamma\rho_* \\ &\pm \sqrt{9(1+w)^2\gamma^2\rho_*^2 + 4\beta^2[\lambda + 6(1-3w)\gamma\rho_*]}\end{aligned} \tag{7.42}$$

$$H_* = -\frac{T(\rho_*)}{3(1+w)\rho_*} \tag{7.43}$$

$$\psi_*^2 \pm \frac{3(1+w)}{\beta}\gamma\rho_*\psi_* - [\lambda + 6(1-3w)\gamma\rho_*] = 0 \tag{7.44}$$

Equation (7.41) gives

$$\Big(\frac{\ddot{a}}{a}\Big)_* = \frac{T(\rho_*)^2}{9(1+w)^2\rho_*^2} \tag{7.45}$$

which is positive, and also, it has the same form (as a function of $\rho_*$) as in the absence of $\hat{R}$. The deceleration parameter is found to have the value

$$q_* = -1 \tag{7.46}$$

which means $\dot{H}_* = 0$. Furthermore, at this fixed point we find

$$\Omega_{m*} \equiv \frac{2\gamma\rho_*}{H_*^2} = \frac{18(1+w)^2}{A^2}\gamma\rho_*^{3-2\nu} \tag{7.47}$$

Equation (7.42), when expressed in terms of $\Omega_{m*}$, has only one root for each branch

$$\rho_* = \frac{\beta}{2\gamma}\frac{6(1-3w)\beta \pm \sqrt{\lambda}(1-3w-4\Omega_{m*}^{-1})}{(2\Omega_{m*}^{-1}+1+3w)(\Omega_{m*}^{-1}-1)} \tag{7.48}$$

However, it can be seen from (7.48) that for $-1 \leq w \leq 1/3$ and $\Omega_{m*} < 1$ the Branch B is inconsistent with equation (7.42). On the contrary, Branch A with $-1 \leq w \leq 1/3$ and $\Omega_{m*} < 1$ is consistent for $0 < 6(1-3w)\beta + \sqrt{\lambda}(1-3w-4\Omega_{m*}^{-1}) < 3\sqrt{4(1-3w)^2\beta^2-(1+w)^2\lambda}$. Thus, since we are interested in realizing the present universe as a fixed point, Branch B should be rejected, and from now on we will only consider Branch A. So, we have seen until now that *for negative brane tension, we can have a fixed point of our model with acceleration and* $0 < \Omega_{m*} < 1$. This behavior is qualitatively different from the one obtained in the context of the model presented in [27] (for $-1/3 \leq w \leq 1/3$), where for positive brane tension we have $\Omega_{m*} > 2$, while for negative brane tension the universe necessarily exhibited deceleration; therefore, in that model the idea that the present universe is close to a fixed point could not be realized.

Concerning the negative brane tension the following remarks are in order: (a) In the conventional, non-supersymmetric setting, it is well known that a negative tension brane with or without induced gravity is accompanied by tachyonic bulk gravitational modes [93]; however, including the GB corrections relevant at high-energies, the tachyonic modes can be completely removed for a suitable range of the parameters [94]. (b) As shown in [95], in supersymmetric theories, spacetimes with two branes of opposite tension are stable; in particular, there is no instability due to expanding "balooning" modes on the negative brane. It is, however, unclear what happens in models with supersymmetry unbroken in the bulk but softly broken on the brane. (c) Finally, it has been shown [96] that with appropriate choice of boundary conditions, both at the linearized level as well as in the full theory, the gravitational potential of a mass on a negative tension brane has the correct $1/r$ attractive behaviour.

## 7.3 Critical point analysis

We shall restrict ourselves to the flat case $k = 0$. In order to study the dynamics of the system, it is convenient to use (dimensionless) flatness parameters such that the state space is compact [97]. Defining

$$\omega_m = \frac{2\gamma\rho}{D^2} \quad , \quad \omega_\psi = \frac{\beta\psi}{D^2} \quad , \quad Z = \frac{H}{D} \tag{7.49}$$

where $D = \sqrt{H^2 - \mu}$, we obtain the equations

$$\omega_m + \omega_\psi = 1 \tag{7.50}$$

$$\begin{aligned} \omega_m' = \omega_m \Big[ & (1+3w)(\omega_m - 1)Z - \frac{A}{\sqrt{|\mu|}}\Big(\frac{|\mu|\omega_m}{2\gamma}\Big)^{\nu-1}(1-Z^2)^{\frac{3}{2}-\nu} \\ & - 2Z(1-Z^2)\frac{1 - Z^2 - 3(1-3w)\beta^2\mu^{-1}\omega_m}{1-\omega_m}\Big] \end{aligned} \tag{7.51}$$

$$Z' = (1-Z^2)\Big[(1-Z^2)\frac{1 - Z^2 - 3(1-3w)\beta^2\mu^{-1}\omega_m}{1-\omega_m} - 1 - \frac{1+3w}{2}\omega_m\Big] \tag{7.52}$$

with $' = d/d\tau = D^{-1}d/dt$. Note that $-1 \le Z \le 1$, while both $\omega$'s satisfy $0 \le \omega \le 1$. The deceleration parameter is given by

$$q = \frac{1}{Z^2}\Big[\frac{1+3w}{2}\omega_m - (1-Z^2)\frac{\omega_m - Z^2 - 3(1-3w)\beta^2\mu^{-1}\omega_m}{1-\omega_m}\Big] \tag{7.53}$$

and $H' = -HZ(q+1)$. The system of equations (7.51)-(7.52) inherits from equations (7.35)-(7.37) the symmetry $A \to -A$, $Z \to -Z$, $\tau \to -\tau$. The system written in the new variables contains only three parameters. However, going back to the physical quantities $H$, $\rho$ one will need specific values of two more parameters.

It is obvious that the points with $|Z| = 1$ have $H = \infty$. Therefore, from (7.39) it arises that the infinite density $\rho = \infty$ big bang (big crunch) singularity, when it appears, is represented by one of the points with $Z = 1$ ($Z = -1$). The points with $\omega_m = 1$, $|Z| \neq 1, 0$ have $\omega_m' = \infty$, $Z' = \infty$ and finite $\rho$, $H$; for $w \le 1/3$, one has in addition $\ddot{a}/a = +\infty$, i.e. divergent 4D curvature scalar on the brane.

The system possesses, generically, the fixed point (a) $(\omega_{m*}, \omega_{\psi*}, Z_*) = (0, 1, 0)$, which corresponds to the fixed point $(\rho_*, H_*, \psi_*) = (0, 0, \sqrt{\lambda})$ discussed above. For $\nu \le 3/2$ there are in addition the fixed points (b) $(\omega_{m*}, \omega_{\psi*}, Z_*) = (0, 1, 1)$ and (c) $(\omega_{m*}, \omega_{\psi*}, Z_*) = (0, 1, -1)$. All these critical points

| | $\nu < 3/2$ | $\nu = 3/2$ | $\nu > 3/2$ |
|---|---|---|---|
| No. of F.P. | 1 | 0 or 1 | 1 |
| Nature | A | A | S |

Table 7.1: The fixed points for $w = 0$, influx.

are either non-hyperbolic, or their characteristic matrix is not defined at all; thus, their stability cannot be studied by first order perturbation analysis. In cases like these, one can find non-conventional behaviors (such as saddle-nodes and cusps [98]) of the flow-chart near the critical points. There are two more candidate fixed points (d) $(\omega_{m*}, \omega_{\psi*}, Z_*) = (1, 0, 1)$ and (e) $(\omega_{m*}, \omega_{\psi*}, Z_*) = (1, 0, -1)$, whose existence cannot be confirmed directly from the dynamical system, since they make equations (7.51), (7.52) undetermined. Apart from the above, there are other critical points given by

$$\frac{A}{\sqrt{|\mu|}}\Big(\frac{|\mu|\omega_{m*}}{2\gamma}\Big)^{\nu-1} = -\frac{3(1+w)\,Z_*}{(1-Z_*^2)^{\frac{3}{2}-\nu}} \tag{7.54}$$

$$(1+3w)\omega_{m*}^2 + (1-3w)\Big[1 - \frac{6\beta^2}{\mu}(1-Z_*^2)\Big]\omega_{m*} - 2[1-(1-Z_*^2)^2] = 0 \tag{7.55}$$

They exist only for $AZ_* < 0$ and correspond to the ones given by equations (7.42)-(7.44). For the physically interesting case $w = 0$ with influx we scanned the parameter space and were convinced that for $\nu \neq 3/2$ there is always only one fixed point; for $\nu < 3/2$ this is an attractor (A), while for $\nu > 3/2$ this is a saddle (S). For $w = 0$, $\nu = 3/2$ there is either one fixed point (attractor) or no fixed points, depending on the parameters. For the other characteristic value $w = 1/3$, we concluded that for $\nu < 3/2$ there is only one fixed point (attractor), for $\nu > 2$ there is only one fixed point (saddle), while for $3/2 < \nu < 2$ there are either two fixed points (one attractor and one saddle) or no fixed points at all, depending on the parameters. For $w = 1/3$, $\nu = 3/2$ there is either one fixed point (attractor) or no fixed points. Finally, for $w = 1/3$, $\nu = 2$ there is either one fixed point (saddle) or no fixed points. These results were obtained numerically for a wide range of parameters and are summarized in Tables 7.1 and 7.2.

The approach to an attractor described by the linear approximation of (7.51)-(7.52) is exponential in $\tau$ and takes infinite time $\tau$ for the universe to reach it. Given that near this fixed point the relation between the cosmic time $t$ and the time $\tau$ is linear, we conclude that it also takes infinite cosmic time to reach the attractor.

| | $\nu < 3/2$ | $\nu = 3/2$ | $3/2 < \nu < 2$ | $\nu = 2$ | $\nu > 2$ |
|---|---|---|---|---|---|
| No. of F.P. | 1 | 0 or 1 | 0 or 2 | 0 or 1 | 1 |
| Nature | A | A | A,S | S | S |

Table 7.2: The fixed points for $w = 1/3$, influx.

Defining $\epsilon = sgn(H)$, we see from (7.51)-(7.52) that the lines $Z = \epsilon$ ($\nu \leq 3/2$), $\omega_m = 0$ are orbits of the system. Furthermore, the family of solutions with $Z \approx \epsilon$ and $dZ/d\omega_m = Z'/\omega'_m \approx 0$ is approximately described for $\nu < 3/2$ by $\omega'_m = \epsilon(1 + 3w)\omega_m(\omega_m - 1)$, and thus, they move away from the point $(\omega_{m*}, Z_*) = (1, 1)$, while they approach the point $(\omega_{m*}, Z_*) = (1, -1)$. In addition, the solution of this equation is $\omega_m = [1+ce^{\epsilon(1+3w)\tau}]^{-1}$, with $c > 0$ an integration constant. Using this solution in equation $H'/H = -Z(q + 1)$ we find that for $w = 1/3$, $H/H_o = \sqrt{\omega_m}/(1 - \omega_m)$, where $H_o$ is another integration constant. Then, the equation for $\omega_m(t)$ becomes $d\omega_m/dt = -2\epsilon\omega_m\sqrt{H_o^2\omega_m - \mu(1 - \omega_m)^2}$, and can be integrated giving $t$ as a function of $\omega_m$ or $H$. Therefore, in the region of the big bang/big crunch singularity one obtains $a(t) \sim \sqrt{\epsilon t}$, $\rho(t) \sim t^{-2}$, as in the standard radiation dominated big-bang scenario. This means that for $\nu < 3/2$ the energy exchange has no observable effects close to the big bang/big crunch singularity.

Since our proposal relies on the existence of an attractor, we shall restrict ourselves to the case $\nu < 3/2$. It is convenient to discuss the four possible cases separately:

(i) $w = 0$ with influx. The generic behavior of the solutions of equations (7.51)-(7.52) is shown in Figure 7.1. We see that all the expanding solutions approach the global attractor. Furthermore, there is a class of collapsing solutions which bounce to expanding ones. Finally, there are solutions which collapse all during their lifetime to a state with finite $\rho$ and $H$. The physically interesting solutions are those in the upper part of the diagram emanating from the big bang $(\omega, Z) \approx (1, 1)$. These solutions start with a period of deceleration. The subsequent evolution depends on the value of $3\beta^2/|\mu|$, which determines the relative position of the dashed and dotted lines. Specifically, for $3\beta^2/|\mu| > 1$ (the case of Figure 7.1) one distinguishes two possible classes of universe evolution. In the first, the universe crosses the dashed line entering the acceleration era still with $w_{DE} > -1$, and finally it crosses the dotted line to $w_{DE} < -1$ approaching the attractor. In the second, while in the deceleration era, it first crosses the dotted line to $w_{DE} < -1$, and then the dashed line entering the eternally accelerating era. For $3\beta^2/|\mu| \leq 1$,

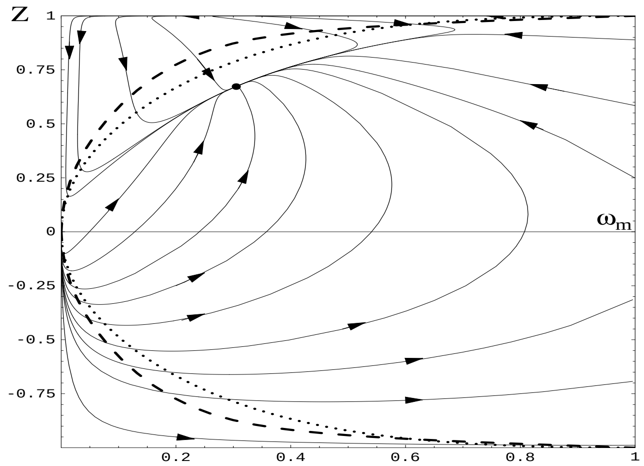


Figure 7.1: Influx, $w = 0$, $\nu < 3/2$. The arrows show the direction of increasing cosmic time. The dotted line corresponds to $w_{DE} = -1$. The region inside (outside) the dashed line corresponds to acceleration (deceleration). The region with $Z > 0$ represents expansion, while $Z < 0$ represents collapse. The present universe is supposed to be close to the global attractor.

the dotted line lies above the dashed line, and, consequently, only the second class of trajectories exists. To connect with the discussion in the introduction, notice that the Friedmann equation (7.39) can be written in the form (7.1) with dark energy $\rho_{DE} = (\beta\psi + \mu)/2\gamma$. Using (7.40), the equation for $\rho_{DE}$ takes the form (7.3) with

$$w_{DE} = \frac{-1}{3(1-\omega_m)}\left[2Z^2 - \omega_m - 1 - 6(1-3w)\frac{\beta^2}{\mu}\frac{\omega_m(1-Z^2)}{Z^2-\omega_m}\right] \quad (7.56)$$

The global attractor (7.42)-(7.44) satisfies relations (7.4) and consequently, $w_{DE}$ evolves to the value $w_{DE*}$ given by (7.5). As for the bouncing solutions, they approach the attractor after they cross the line $Z^2 = \omega_m$, where $w_{DE}$ jumps from $+\infty$ to $-\infty$; however, the evolution of the observable quantities is regular.

(ii) $w = 0$ with outflow. The generic behavior in this case is obtained from Figure 7.1 by the substitution $Z \to -Z$ and $\tau \to -\tau$, which reflects the diagram with respect to the $\omega_m$ axis and converts attractors to repelers.

(iii) $w = 1/3$ with outflow. Figure 7.2 depicts the flow diagram of this case. Even though in the case of radiation in general $w_{DE} > -1/3$ from equation (7.56), there are both acceleration and deceleration regions. Furthermore, from equation (7.5) it is $\Omega_{m*} > 1$.

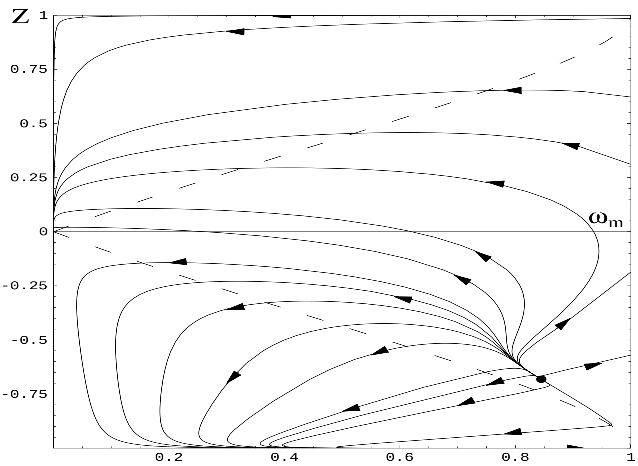


Figure 7.2: Outflow, $w = 1/3$, $\nu < 3/2$. The arrows show the direction of increasing cosmic time. The region inside (outside) the dashed line corresponds to acceleration (deceleration). The region with $Z > 0$ represents expansion, while $Z < 0$ represents collapse.

(iv) $w = 1/3$ with influx. This arises like in (ii) by reflection of Figure 7.2 and resembles Figure 7.1.

## 7.4 Conclusions

In this work, we studied the role of brane-bulk energy exchange on the cosmological evolution of a brane with negative tension, zero effective cosmological constant, and in the presence of the induced curvature scalar term in the action. Adopting the physically motivated $\rho^\nu$ power-law form for the energy transfer and assuming a cosmological constant in the bulk, an autonomous system of equations was isolated. In this scenario, the "dark energy" is a result of the geometry and the brane-bulk energy exchange. The negative tension of the brane is necessary in order to realize the present universe (accelerating with $0 < \Omega_{m0} < 1$) as being close to a future fixed point of the evolution equations. We studied the possible cosmologies using bounded normalized variables and the corresponding global phase portraits were obtained. By studying the number and nature of the fixed points we demonstrated numericaly that our present universe can be easily realized as a late-time fixed point of the evolution. This provides an alternative answer to the coincidence problem in cosmology, which does not require specific fine-tuning of the initial data. Furthermore, the equation of state for the dark energy at the attractor is uniquely specified by the value $\Omega_{m0}$. Remarkably, for $\Omega_{m0} = 0.3$, one obtains $w_{DE,0} = -1.4$, independently of the other parameters, while for the other suggestive value $\Omega_{m0} = 0.03$, $w_{DE,0} = -1.03$. In the past, the function $w_{DE}$ crosses the line $w_{DE} = -1$ to larger values.

It would be interesting to investigate if the above partial success of the present scenario persists after one tries to fit the supernova data and the detailed CMB spectum [99]. Of course, the nature of the content of the bulk and of the mechanism of energy exchange with the brane is another crucial open question, which we hope to deal with in a future publication.

# Chapter 8

# Conclusions

We have studied dark matter, inflation and dark energy in the brane-world scenario. These are three topics of fundamental importance for modern cosmology. According to recent observations (Cosmic Microwave Background, Supernovae, galaxy serveys), most of the energy content of our universe is in the form of dark matter and dark energy. In addition, inflation is responsible for producing the cosmological fluctuations for the formation of the structure that we observe today. The brane-world idea is inspired from M/string theory and although brane models are not yet derived from the fundamental theory, at least they contain the key features of string theory, such as extra dimensions, higher-dimensional objects (branes), higher order corrections to gravity (Gauss-Bonnet) etc. Inflation and the dark sector have been addressed in the framework of standard four-dimensional cosmology. However, it is challenging to try to study them using alternative gravitational theories such as braneworlds. Furthermore, since string theory claims to give us a fundamental description of nature, it is important to study what kind of cosmology they predict. The essence of the brane-world scenario is that the standard model, with its matter and gauge interactions, is localized on a three-dimensional hypersurface, called brane, embedded in a higher-dimensional spacetime, called the bulk. Gravitons, the mediators of the gravitational interaction, are free to propagate into the whole bulk and thus connect the standard model sector with the dynamics of the extra dimensions. Brane models are capable of giving non-conventional cosmologies. In standard four-dimensional cosmology the Hubble parameter goes like the square of the energy density. However, in brane models this is not true any more. The relation between the Hubble parameter and the energy density is more complicated and the specific form depends on the model at hand. These novel cosmologies can be used in a two-fold way: On one hand they help us in attacking in a different context longstanding problems of standard cosmology. On the other hand they offer us an ideal laboratory for testing ideas coming from M/string theory. We believe that brane cosmology is an exciting field and that it deserves further study.

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