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Orientifolds, Anomalies and the Standard Model

A Doctoral Dissertation

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2005

Preprint typeset in JHEP style - HYPER VERSION hep-th/0503055

Thesis: Orientifolds, Anomalies and the Standard Model

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Abstract: In this thesis, we study aspects of D-brane realizations of the Standard Model. Specifically, we study orientifold models with rotation and translation elements that break supersymmetry, provide the general consistency conditions and derive the massless spectrum for these type of orientifolds. These models contain in general anomalous $U(1)$ gauge fields. The Green-Schwarz mechanism cancels the anomaly and provides a mass term for the anomalous gauge fields. We calculate the bare mass for supersymmetric and non-supersymmetric vacua and we show that higher dimensional anomalies can affect the masses of the anomalous $U(1)$s. Phenomenological aspects are also discussed. We evaluate the contribution of the extra $U(1)$ fields to the anomalous moments and it is shown that this imposes constraints on the magnitude of the string scale.

Keywords: string theory, orientifolds, non-supersymmetric string theories, model building, Anomalous U(1), string phenomenology.

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1. Introduction

The Standard Model (SM) of physics has been confirmed to a great accuracy in many experiments. Despite the fact that the Higgs particle remains experimentally elusive, few doubt that there will be major surprises in this direction.

In a related direction, however there is concrete experimental evidence that neutrinos have (tiny) masses and mixings and the SM should be extended to accommodate this. Many ideas exist on how this can be achieved and we are awaiting experimental evidence to delineate any particular direction. On the other hand there are some theoretical issues that make physicists believe that the SM is not the final story. Some of these are:

- (Quantum) gravity is not incorporated. It is not a renormalizable theory.
- The SM suffers from the *hierarchy problem*. It is believed that SM particles are coming from a Ground Unified Theory that spontaneously broke to $SU(3)\times SU(2)\times U(1)$. The breaking scale of this unified theory turns out to be $M_U \sim 10^{16}$GeV, which is very far from the electroweak breaking scale. In order to evaluate the potential for the Higgs, we have to fine-tune the parameters that receive contributions from all orders in perturbation theory.

Several ideas have been put forward to deal with the large hierarchy of scales. One is the so called theory of *technicolor* which considers all scalar fields in the SM as bound states of fermions by a new set of interactions [9].

Another idea is a new symmetry, the so-called *supersymmetry* that relates fermions and bosons. Supersymmetry (if it exists) is obviously broken at low energy. However, it solves the hierarchy problem, since above the supersymmetry breaking scale there are no radiative corrections to the masses of the fields.

String theory, after an initial short life as a theory of hadrons, regained popularity because it was found to be the only consistent framework known that provided a workable unified theory of quantum gravity [1, 2, 3, 4].

Initially, it was the heterotic string theory [10] that first provided a picture of grand unification, supersymmetry and quantum gravity. There were several models which at low energies came close to the SM [11]. It predicts that the unification scale is close to the four-dimensional Planck scale, $M_P \sim 10^{19}$GeV, giving an answer to the hierarchy problem.

Recently, other ten-dimensional supersymmetric string theories (type-IIA/B closed and Type I closed and open strings) have come into focus. Moreover, It has been shown that all superstring theories are vacuum states of a larger eleven-dimensional theory so-called *M-theory* [12, 13], with non-perturbative dualities relating the strong coupling behavior of one theory to the weak coupling behavior of another. The resurgence of interest in these theories is also partly due to the discovery of solitonic objects (D-branes and NS5-branes) that are contained in these theories. In particular, D-branes have provided a new geometrical interpretation of gauge theories [14, 15].

In this search for new string vacua, a new possibility also emerged, namely that the string scale could be much lower than the four-dimensional Planck scale and in particular it could be as low as a few TeV [16], opening new avenues for experimental confirmation of all such theories that had until now been considered more as mathematical structures than as physical models. Such ground states are the so called $orientifolds$ that are generalized compactifications of type I string theory (compactifications of superstring theories are expected since their critical dimension is $D = 10$). Crucially, these models contain D-branes whose (localized) fluctuations should describe the SM particles while gravity is naturally included in the closed sector.

In such orientifold models the non-abelian couplings of the D-brane gauge fields and the relation between the four-dimensional Planck scale and the string scale is given by:

$$\frac{1}{g_{YM}^2} = \frac{V_{||}}{g_s} \qquad , \qquad \frac{M_P^2}{M_s^2} = \frac{V_6}{g_s V_{||}} \ , \tag{1.1}$$

where V_6 and $V_{||}$ are the volumes of the 6-dimensional compact manifold and the longitudinal sub-manifold of the D-branes respectively. Therefore, if $V_6/V_{||} \gg 1$ and if the theory is kept perturbative $g_s < 1$, the string scale can be anywhere between the Planck scale and a few TeV.

Supersymmetry breaking in these models is achieved by various geometric mechanisms, such as:

- Intersecting branes [17, 18].
- Non-freely acting supersymmetry-breaking orbifolds that generically induce breaking in the open sector.
- Freely acting supersymmetry-breaking orbifolds, such as the Scherk-Schwarz mechanism that we will explore in detail later on. [19, 20]

In these models there is no hierarchy problem, since above the string scale there is no field theoretic running of couplings. However, a low string scale requires some of the internal dimensions to be larger than the string scale. Therefore, the "old" problem changes form and maybe rephrased as the "new" hierarchy problem: why the minimum of the potential of the moduli is at $R \gg 1$? This question still remains an open problem in string theory.

In this thesis we study some aspects of D-brane realizations of the SM. We start with an introductory chapter to string theory and superstring theory and also discuss some issues of compactifications and orbifold constructions. The following chapter explains the foundations of unoriented open and closed string theories, the orientifolds. The presence of extended dynamical objects (D-branes) is necessary to make the theory consistent [23, 27, 29, 28, 24].

During my thesis, I worked on this field of research in collaboration with A.B. Hammou and N. Irges and we provided general consistency conditions for supersymmetric and non-supersymmetric orientifolds (Scherk-Schwarz deformation breaking [20, 21, 30, 31, 32, 33, 34]) and we also gave the general structure of the massless spectrum of these models.

As we mentioned above, in the orientifold models gauge interactions are described by open strings whose ends are confined on the D-branes, while gravity is mediated by closed strings in the bulk [35, 37, 38, 39, 40, 41]. Ordinary matter is preferably generated by the fluctuations of the open strings and is thus also localized on the appropriate D-branes. Consistency conditions and Wilson lines can provide a D-brane configuration that will localize the Standard Model gauge group and massless spectrum on a stack of 3 plus 2 plus 1 at least D-branes. The rest of the D-branes being further away will not affect the local properties of the model.

These D-brane configurations naturally provide some extra $U(1)$ gauge fields. Such $U(1)$ fields have generically four-dimensional anomalies which are cancelled via the Green-Schwarz mechanism [42, 43, 28, 45, 54]. A scalar axionic field (zero-form, or its dual two-form) is responsible for the cancellation of the anomalies of each anomalous boson. This mechanism gives a mass to the anomalous $U(1)$ fields and breaks the associated gauge symmetry.

If the string scale is around a few TeV, observation of such anomalous $U(1)$ gauge bosons becomes a realistic possibility [55, 56].

As was shown in [47], it is possible to compute the bare masses of the anomalous $U(1)$s by evaluating the ultraviolet tadpole of the one-loop open string diagram with the insertion of two gauge bosons on different boundaries. In this limit, the diagrams of the annulus with both gauge bosons in the same boundary and the Möbius strip do not contribute when vacua have cancelled tadpoles.

It turns out that $U(1)$ gauge fields that are free of four-dimensional anomalies can still be massive [36, 46, 47]. Herein we show that this is due to the presence of mass-generating six-dimensional anomalies. If there are decompactification limits in the theory, then six-dimensional anomalies can affect four-dimensional masses. This work was result of my research: In six dimensions, two types of field are necessary to cancel the anomalies, a scalar axion and a two-form. There is also a four-form field but it is dual to the scalar. Via the Green-Schwarz mechanism, the pseudoscalar axions give mass to the anomalous $U(1)$ fields. However, the two-forms are not involved in mass generation. It is shown that four-dimensional non-anomalous $U(1)$s can have masses if their decompactification limits suffer from six-dimensional anomalies. We calculate the masses of the anomalous $U(1)$s of various six-dimensional orientifolds and we compare our results with decompactification limits of the four-dimensional orientifolds Z_6' and Z_6 [48].

Chapter 5 is result of my research where we are interested in the masses of the anomalous $U(1)$s in non-supersymmetric models since such models are of the type that will eventually represent the low energy physics of the Standard Model. In particular, intersecting-brane realizations of the Standard Model are generically non-supersymmetric. We calculate the mass formulae using the "background field method" [50] and find that they are the same as the supersymmetric ones when we have cancellation of all tadpoles [51]. In cases where NSNS tadpoles do not vanish, there are extra contributions proportional to the non-vanishing tadpole terms. The mass formulae derived earlier in this section are valid even if we add Wilson lines that move the branes away from the fixed points. The Wilson lines generically break the gauge group and they will affect the masses of the anomalous $U(1)$s through the traces of the model dependent γ matrices. The formulae, are applied to a Z_2 non-supersymmetric orientifold model, with RR and NSNS tadpoles to be cancelled, where supersymmetry is broken by a Scherk-Schwarz deformation [34].

The Green-Schwarz mechanism is not the only source for the masses of anomalous $U(1)$s. In Standard Model realizations, the Higgs is necessarily charged under one of the anomalous $U(1)$s. As it was described in [49], the Higgs contribution to the mass of these $U(1)s$ is $g_A\sqrt{M^2+e_H^2\langle H\rangle^2}$, where g_A is the gauge coupling of the anomalous $U(1)$ and e_H is the $U(1)$ charge of the Higgs. The Higgs contribution to the $U(1)$ mass can be obtained from the effective field theory unlike the ultraviolet mass which can only be calculated in string theory.

In the last chapter, based on a D-brane realization of the Standard Model [41], we make some phenomenological predictions and we evaluate the contribution of the massive anomalous $U(1)$s to the anomalous magnetic moment (AMM) of muon $\alpha_{muon} = (g-2)/2$. These contributions are currently in the range allowed by experiment. Finally, we use the precise measurement of $\alpha_{muon} = (g-2)/2$ from the Brookhaven AGS experiment [57] to provide precise constrains for the masses of the anomalous $U(1)$s in the TeV range. This work has been done during my thesis, in collaboration with E. Kiritsis [55].

2. String Theory

2.1 Bosonic String

String theory is a quantum theory where the fundamental object is a 1-dimensional element: *the string*. The lagrangian that describes such an object in flat space is the so-called "Nambu-Goto" action:

$$S_{NG} = -T \int dA \tag{2.1}$$

This action is the direct generalization of the relativistic point particle lagrangian where the mass of the particle has been replaced by the tension of the string T and the world-line ds by the the world-volume dA.

Suppose that ξ_i with $i = 1, 2$ are coordinates of the world-sheet and $G_{\mu\nu}$ is the metric of a D-dimensional spacetime where the string propagates. If spacetime is flat $G_{\mu\nu} = \eta_{\mu\nu}$, the Nambu-Goto action takes the form:

$$S_{NG} = -T \int \sqrt{-\det G_{ij}} d^2\xi = -T \int \sqrt{(\dot{X}X')^2 - \dot{X}^2 X'^2} d^2\xi \tag{2.2}$$

where $G_{ij} = G_{\mu\nu}\partial_i X^\mu \partial_j X^\nu$ the induced metric.

The square root in the Nambu-Goto action (2.2) makes the treatment of the quantum theory quite complicated. To overcome this difficulty, Polyakov introduced an intrinsic fluctuating metric $g_{\alpha\beta}$ on the worldsheet. For flat spacetime, his action takes the form:

$$S_P = -\frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{-\det g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu + \frac{\langle\varphi\rangle}{4\pi} \int d^2\xi \sqrt{-\det g} R \tag{2.3}$$

where φ the dilaton field. This action describes 2-dimensional gravity coupled to D worldsheet scalars. The last term in (2.3) is a topological invariant, the Euler character of the $2D$ surface.

The stress tensor of the scalars is defined as the variation of the matter-action with respect to the metric:

$$T_{\alpha\beta} = -\frac{2}{T} \frac{1}{\sqrt{-\det g}} \frac{\delta S_P}{\delta g^{\alpha\beta}} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2} g_{\alpha\beta} g^{\gamma\delta} \partial_\gamma X^\mu \partial_\delta X_\mu \tag{2.4}$$

The $2D$ Einstein equations give the classical solution for the metric $g_{\alpha\beta}$:

$$T_{\alpha\beta} = 0 \quad \Rightarrow \quad g_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu \tag{2.5}$$

Notice that the zero in the right part of (2.5) is due to the fact that $2D$ gravity is Ricci flat. Substituting back the classical solution to the Polyakov action we find the Nambu-Goto action, where $T = (2\pi\alpha')^{-1}$. Therefore, the two actions are equivalent at least classically.

From now on we will take the Polyakov action as the starting point of our study. The symmetries of this action are:

- Poincaré invariance
- Local two dimensional reparametrization invariance
- Conformal invariance

Using the above symmetries we can give to the Polyakov action a convenient form. This is the so-called *conformal gauge* where the worldsheet metric becomes flat:

$$g_{\alpha\beta} = \eta_{\alpha\beta} \ . \tag{2.6}$$

It is convenient to work in Euclidean signature by performing a Wick rotation $\tau \to -i\tau$. We also make a conformal transformation that maps a cylinder to a complex plane:

$$z = e^{\tau - i\sigma} \ , \quad \bar{z} = e^{\tau + i\sigma} \ . \tag{2.7}$$

In the z plane, equal times contours are concentric circles. The $\tau \to -\infty$ gets mapped to $z = 0$. The Polyakov action takes the form:

$$S_P \sim \int d^2z \ \partial X^\mu \bar{\partial} X_\mu \ . \tag{2.8}$$

The classical equations of motion (EOM) can be evaluated by varying the action with respect to the fields. In the conformal gauge, the EOM for the bosons are:

$$\partial \bar{\partial} X^\mu = 0 \ . \tag{2.9}$$

Even if we have fixed the gauge, we have to impose the equations which where found by the variation of the metric $g_{\alpha\beta}$ (2.5):

$$T_{\alpha\beta} = 0 \ . \tag{2.10}$$

The later are known as the *Virasoro constraints.*

2.1.1 Solving the string equations of motion

In general, there are two kinds of string with different boundary conditions: *closed* and *open* strings:

- Closed Strings: $X^\mu(\tau, \sigma + 2\pi) = X^\mu(\tau, \sigma)$. The solution is:

$$\begin{aligned} X^\mu(z, \bar{z}) &= X^\mu_L(z) + X^\mu_R(\bar{z}) \qquad \text{where:} \\ X^\mu_L(z) &= \frac{x^\mu}{2} - i\frac{\alpha'}{2} p^\mu_L \log z + i\sqrt{\frac{\alpha'}{2}} \sum_{k \neq 0} \frac{\tilde{\alpha}^\mu_k}{k} z^{-k} \ , \\ X^\mu_R(\bar{z}) &= \frac{x^\mu}{2} - i\frac{\alpha'}{2} p^\mu_R \log \bar{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{k \neq 0} \frac{\alpha^\mu_k}{k} \bar{z}^{-k} \ , \end{aligned} \tag{2.11}$$

where for non-compact dimensions $p_L^\mu = p_R^\mu = p^\mu$. The $\alpha_k^\mu, \tilde{\alpha}_k^\mu$ are Fourier modes where k runs over all integers. Reality conditions give relations between opposite sign Fourier modes: $(\alpha_k^\mu)^* = \alpha_{-k}^\mu$ and $(\tilde{\alpha}_k^\mu)^* = \tilde{\alpha}_{-k}^\mu$.

- Open Strings: There are two different boundary conditions that can be imposed to the ends of an open string:

  - Neumann boundary conditions (N): $\partial_\sigma X^\mu|_{end} = 0$,
  - Dirichlet boundary conditions (D): $\partial_\tau X^\mu|_{end} = 0$.

  These two choices, eliminate an extra boundary term that appears from the variation of the Polyakov action. Therefore, open strings can have different boundary conditions on their endpoints. All the possible combinations are: NN, DD, ND with different solutions:

$$
\begin{aligned}
X_{NN}^\mu(z,\bar{z}) &= x^\mu - i\alpha' p^\mu \log z\bar{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{k\neq 0} \frac{\alpha_k^\mu}{k}(z^{-k} + \bar{z}^{-k}) \ , \\
X_{DD}^\mu(z,\bar{z}) &= -\frac{c^\mu}{2\pi} \log(z/\bar{z}) + i\sqrt{\frac{\alpha'}{2}} \sum_{k\neq 0} \frac{\alpha_k^\mu}{k}(z^{-k} - \bar{z}^{-k}) \ , \\
X_{ND,DN}^\mu(z,\bar{z}) &= \sqrt{\frac{\alpha'}{2}} \sum_{k\in\mathbb{Z}+1/2} \frac{\alpha_k^\mu}{k}(z^{-k} \mp \bar{z}^{-k}) \ .
\end{aligned}
\tag{2.12}
$$

  The open strings have been parametrized as $\sigma \in [0,\pi]$. The x^μ and p^μ are the position and momentum of the center of mass of the open string. Notice also that we have imposed two different conditions in the DD case: $X^\mu|_{\sigma=0} = 0$ and $X^\mu|_{\sigma=\pi} = c^\mu$. This will be very important later on.

The physical states obey also the Virasoro constrains. In the conformal gauge, these constrains take the form: $T_{zz} = 0$, $T_{\bar{z}\bar{z}} = 0$. Defining the Fourier modes of these elements of the stress-tensor, we have:

- Closed strings

$$
\begin{aligned}
L_m &= \frac{1}{2\pi i} \oint dz z^{n+1} T_{zz}(z) = \frac{1}{2} \sum_n \alpha_{m-n}^\mu \alpha_{\mu,n} \ , \\
\bar{L}_m &= \frac{1}{2\pi i} \oint d\bar{z} \bar{z}^{n+1} T_{\bar{z}\bar{z}}(\bar{z}) = \frac{1}{2} \sum_n \tilde{\alpha}_{m-n}^\mu \tilde{\alpha}_{\mu,n} \ .
\end{aligned}
\tag{2.13}
$$

- Open Strings

$$
L_m = \frac{1}{2\pi i} \int_C \left[dz z^{m+1} T_{zz} + d\bar{z} \bar{z}^{m+1} T_{\bar{z}\bar{z}} \right] = \frac{1}{2} \sum_n \alpha_{m-n}^\mu \alpha_{\mu,n} \ , \tag{2.14}
$$

where we have used (2.11, 2.12).

In the Hamiltonian picture we have the equal-τ Poisson brackets for the dynamical variables and their conjugate momenta:

$$\{X^\mu(\sigma,\tau),\partial_\tau X^\nu(\sigma',\tau)\}_{PB}=2\pi\alpha'\eta^{\mu\nu}\delta(\sigma-\sigma') \tag{2.15}$$

and $\{X^\mu,X^\nu\}_{PB}=\{\partial_\tau X^\mu,\partial_\tau X^\nu\}_{PB}=0$. Inserting (2.11, 2.12) in the above we find relations for the oscillator modes:

$$\begin{aligned}&\{\alpha^\mu_m,\alpha^\nu_n\}_{PB}=\{\tilde\alpha^\mu_m,\tilde\alpha^\nu_n\}_{PB}=-im\eta^{\mu\nu}\delta_{m+n,0}\ ,\\ &\{\alpha^\mu_m,\tilde\alpha^\nu_n\}_{PB}=0\ ,\qquad \{x^\mu,p^\nu\}_{PB}=\eta^{\mu\nu}\ .\end{aligned} \tag{2.16}$$

In the open string case there are no $\tilde\alpha^\mu_m$. Using these relations we find that the Virasoro constraints form the *classical Virasoro algebra*:

$$\begin{aligned}&\{L_m,L_n\}_{PB}=-i(m-n)L_{m+n}\ ,\\ &\{\bar L_m,\bar L_n\}_{PB}=-i(m-n)\bar L_{m+n}\ ,\qquad \{L_m,\bar L_n\}_{PB}=0\ .\end{aligned} \tag{2.17}$$

2.1.2 Quantization

There are various different ways to quantize the classical bosonic string. All these ways agree whenever they can be applied. We will describe the *light-cone* method that is based on first solving the Virasoro constraints and then replacing the fields with operators and the Poisson brackets with commutators.

However, even after we have fixed the conformal gauge, there is some invariance leftover. Defining $X^\pm=\frac{1}{2}(X^0\pm X^1)$ and using this symmetry we can eliminate all the oscillators from the "+" direction. After imposing the Virasoro constraints (2.10), we can express all the α^-_m oscillators as functions of α^i_m. Since we have solved the Virasoro constraints, we can now quantize the string by the usual field $\to$ operators and $\{\ ,\ \}_{PB}\to -i[\ ,\ ]$ replacements in (2.16). The index $i=2,\cdots,D-1$.

The choice of the light-cone gauge, however, obscures the Lorentz-invariance of the theory. Finding the operators $M^{\mu\nu}$ that generate Lorentz transformations, and varying that they will have the correct algebra with p^μ, one finds that this is true only in $D=26$ spacetime dimensions.

2.1.3 Spectrum

From all the α^μ_m, $\tilde\alpha^\mu_m$, we define as *raising* and *lowering* operators, modes with $m<0$ and $m>0$ respectively. By the commutation relations we realize that x^μ and p^μ commute with all α^μ_n and therefore we can diagonalize one of them. Choosing the momenta, the ground state is labeled by $|p^\mu\rangle$:

$$\alpha^\nu_m|p^\mu\rangle=0\quad\text{for}\ \ m>0\ . \tag{2.18}$$

The zero modes of the Virasoro operators define the mass-shell condition for the physical states:

$$
\begin{aligned}
M^2 &= \frac{2}{\alpha'}\left(L_0+\bar{L}_0-2\right) \qquad \text{for closed strings} ,\\
M^2 &= \frac{1}{\alpha'}\left(L_0-1\right) \qquad\qquad \text{for open strings} .
\end{aligned} \tag{2.19}
$$

There is an extra constraint $L_0-\bar{L}_0=0$ for the closed strings, from the fact that there is not any special initial point on the string.

From (2.19) is clear that the ground state is a tachyon for both, closed and open strings. The massless states for the closed strings can be decomposed into:

$$
\alpha^i_{-1}\tilde{\alpha}^j_{-1}|p\rangle \rightarrow G^{ij}+B^{ij}+\Phi . \tag{2.20}
$$

which are a spin-2 particle G^{ij} (graviton), an antisymmetric tensor B^{ij} and a scalar Φ, respectively, the massless state of the open string:

$$
\alpha^i_{-1}|p\rangle , \tag{2.21}
$$

is a vector of $SO(24)$. The tachyon and the absence of space-time fermions make obvious the need of another enlarged theory, this theory is the *supersting*.

2.2 Chan-Paton factors

We can add a non-dynamical degree of freedom to the endpoints of an open string. Considering that the endpoints can take values $i=1,2,\ldots,N$, the ground-state is labelled, in addition to the momentum, by the charge on the endpoints: $|p^\mu;ij\rangle$. These labels are called *Chan-Paton* factors. In general, we can have N^2 different labels (for oriented strings) that give rise to a $U(n)$ gauge group. Originally, the motivation of this was to introduce $SU(3)$ flavor quantum numbers: the endpoints are like quarks and antiquarks, connected by a color-electric flux tube.

Since each open string state has N^2 copies, we can introduce Hermitian matrices λ^α_{ij}, normalized such that:

$$
Tr[\lambda^\alpha\lambda^\beta]=\delta^{\alpha\beta} . \tag{2.22}
$$

The λs form a complete set of states for the two endpoints. Therefore, they form a representation of $U(N)$. Massless vectors will be associated with this gauge symmetry.

Interactions of open strings imply that the endpoint of one string will be the end point of the other. Therefore, a tree-level amplitude of k external open strings will contain an extra term:

$$
Tr[\lambda^{\alpha_1}\lambda^{\alpha_2}\cdots\lambda^{\alpha_k}] . \tag{2.23}
$$

String amplitudes have an obvious $U(N)$ global symmetry $\lambda^\alpha\rightarrow U\lambda^\alpha U^\dagger$.

Later on we will see that these labels are associated with same extended dynamical objects, the D-branes.

2.3 Superstrings

As we mentioned before, the bosonic string spectrum contains a tachyon and no spacetime fermions. A generalization of the Polyakov action has been shown to solve these problems, which contains fermions on the worldsheet (we leave aside the Euler number)[1]:

$$S_{SP} = -\frac{1}{4\pi a'}\int d^2\xi\sqrt{g}\Big(g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu + \frac{i}{2}\bar{\psi}^\mu_M \partial\!\!\!/ \psi_{M\mu} \\ + \frac{i}{2}\bar{\chi}_\alpha\gamma^\beta\gamma^\alpha\psi_{M\mu}\left(\partial_b X^\mu - \frac{i}{4}\bar{\chi}_b\psi^\mu_M\right)\Big) \ , \qquad (2.24)$$

where $\psi^\mu_M = \begin{pmatrix}\psi^\mu \\ \tilde{\psi}^\mu\end{pmatrix}$ are two-dimensional Majorana spinors, the superpartners of X^μ couple to two-dimensional supergravity fields: the zweibein e^a_α [2] and the Majorana gravitino χ_α. The last term is inserted to manifest local worldsheet supersymmetry. This action has the following symmetries:

- Local worldsheet supersymmetry .
- Local super-Weyl invariance .
- Worldsheet Lorentz invariance .
- Worldsheet reparametrization invariance .

Choosing a gauge (analog of the conformal gauge) we can eliminate the gravitational fields. Finally, the action takes the form:

$$S_{SP} = -\frac{1}{4\pi a'}\int d^2z\left(\partial X^\mu\bar{\partial}X^\nu + \psi^\mu\bar{\partial}\psi^\nu + \tilde{\psi}^\mu\partial\tilde{\psi}^\nu\right)\eta_{\mu\nu} \ . \qquad (2.25)$$

The EOM for the fermions denote that ψ and $\tilde{\psi}$ are holomorphic and antiholomorphic functions of $z, \bar{z}$.

2.3.1 Solving the equations of motion

As we mention in the previous section, there are two sectors living on the worldsheet: the bosonic and the fermionic sector. The boundary conditions for the bosonic sector are identical to the ones in the bosonic string (2.11) giving the same results as above. For the fermionic sector we can make two inequivalent choices of boundary conditions:

$$\text{Ramond (R):} \qquad \psi^\mu(\sigma + 2\pi) = \psi^\mu(\sigma) \qquad (2.26)$$

$$\text{Neveu-Schwarz (NS):} \qquad \psi^\mu(\sigma + 2\pi) = -\psi^\mu(\sigma) \qquad (2.27)$$

[1]First, we will explore the closed superstring theory.

[2]We remind that $e^\alpha_a e^\beta_b g_{\alpha\beta} = \eta_{ab}$, for $a, b, \alpha, \beta = 0, 1$ and the Dirac matrices are $\gamma^\alpha = e^\alpha_a\gamma^a$.

Expanding the fermions we find (in the $z, \bar{z}$ basis):

$$\psi^\mu(z) = \sum_{r\in\mathbb{Z}+\nu} \frac{\psi_r^\mu}{z^{r+1/2}} \quad , \qquad \tilde{\psi}^\mu(z) = \sum_{r\in\mathbb{Z}+\tilde{\nu}} \frac{\tilde{\psi}_r^\mu}{\bar{z}^{r+1/2}} \tag{2.28}$$

where $\nu = 0, 1/2$ in the R and NS sector respectively. Since the left and right movers of the closed string do not interact, we can make four inequivalent choices for the periodicity conditions of the fermions that are called RR, RNS, NSR, NSNS.

In addition to the Virasoro operators L_m, which come from the Fourier expansions of the (bosonic) energy-momentum tensor, there are also the G_r operators which come from the Fourier expansions of the fermionic energy-momentum tensor:

$$\begin{aligned} L_m &= \frac{1}{2}\sum_m : \alpha^\mu_{m-n}\alpha_{\mu,n} : + \frac{1}{2}\sum_r (r - \frac{m}{2}) : \psi^\mu_{m-r}\psi_{\mu,r} : + \delta_{m,0}\Delta \ , \\ G_r &= \sum_n \alpha^\mu_n \psi_{\mu,r-n} \ , \end{aligned} \tag{2.29}$$

where r is half-odd integer for the NS sector and integer in the R sector. For each fermionic coordinate, the corresponding normal ordering shift Δ is $-1/48$ and $1/24$ in the NS and R respectively. Each periodic bosonic coordinate contributes $-1/24$. As a result, in D dimensions in the light-cone basis, we have a total $-\frac{1}{16}(D-2)$ from the NS sector and 0 from the R.

As in the bosonic case, we can go to the light-cone gauge and solve the “super” Virasoro constrains:

$$G_r|physical\rangle = 0 \ , \quad r > 0 \ ; \qquad (L_n - \delta_{n,0})|physical\rangle = 0 \ , \quad n \geq 0 \ . \tag{2.30}$$

Finally, we eliminate the “+” and we express the “−” coordinates as a function of the “i”s for both bosonic and fermionic states.

The critical dimension for the supersymmetric version of the bosonic string is $D = 10$.

Next, we quantize the theory. Canonical quantization requires, in addition to the quantum version of (2.16) for the bosonic modes, also anticommutation relations for the fermionic modes:

$$\{\psi^\mu_r, \psi^\nu_s\} = \{\tilde{\psi}^\mu_r, \tilde{\psi}^\nu_s\} = \eta^{\mu\nu}\delta_{r+s} \ . \tag{2.31}$$

(Since we are in the light-cone gauge two coordinates have been expressed as functions of the other coordinates. Therefore, $\mu \to i = 2, \cdots, 9$).

2.3.2 Spectrum

There are three independent left moving sectors living on the worldsheet of the closed superstring[3]: For the bosonic sector α^i_n, the annihilation, creation operators and the vacuum state are identical to those introduced for the pure bosonic closed string. The NS and the R fermionic modes are new sectors and we will study them separately:

[3]Same study can take place also for the right moving modes.

- The NS sector: The anticommutation relations for $\nu = 1/2$ show that we can define the ground state to be annihilated by all $r > 0$ modes:

$$\psi^i_r|0\rangle_{NS} = 0 \qquad \text{for } r > 0 \ . \tag{2.32}$$

Obviously, all modes with $r < 0$ are raising operators.

- The R sector: In the R sector there are zero modes. For the non-zero modes we define again:

$$\psi^i_r|vacuum\rangle_R = 0 \qquad \text{for } r > 0 \ . \tag{2.33}$$

The ψ^i_0 satisfy an $O(8)$ Clifford algebra: $\{\psi^i_0, \psi^j_0\} = \delta^{ij}$. The R vacuum is degenerate and the fermionic zero modes change ground state. We can choose a basis:

$$\psi^\pm_i = \frac{1}{\sqrt{2}}(\psi^{2i+2}_0 \pm \psi^{2i+3}_0) \ , \tag{2.34}$$

where $\{\psi^+_i, \psi^-_j\} = \delta^{ij}$. The ψ^-_i will be the annihilation operators. Thus, the R vacuum is

$$|vacuum\rangle_R = |s_0, s_1, s_2, s_3\rangle_R \qquad s_i = \pm 1/2 \ , \tag{2.35}$$

and it is constructed by $2^{8/2} = 16$ ground states. These ground states can be decomposed into the $\mathbf{8}_s$ with an even number of $-1/2$s and the $\mathbf{8}_c$ with odd number of $-1/2$s (even or odd is clearly a convention).

The mass formula for the superstring is again provided by the L_0, $\bar{L}_0$ constraint and it is:

$$M^2 = \frac{2}{\alpha'}\left(L_0 + \bar{L}_0\right) \ . \tag{2.36}$$

The NS vacuum is clearly tachyonic due to the non-vanishing of Δ and $\bar{\Delta}$ (2.29).

In order to achieve spacetime supersymmetry and eliminate the tachyon, the spectrum is projected onto states with an odd number of fermions. This is called the *GSO projection*[4]. The GSO operators are defined as

$$GSO_{NS} = (-1)^F \ , \qquad GSO_R = (-1)^{\sum_i s_i} \ , \tag{2.37}$$

where F is the worldsheet fermion number. To eliminate the tachyon we keep the NS states that have an odd number of fermions. However, things are not so clear in the R sector. Which states should we project out? The $\mathbf{8}_s$ or the $\mathbf{8}_c$? This question

[4]There is another reason for projection out the odd or the even fermionic modes and this is modular invariance. We will come back in this when we will discuss the 1-loop amplitudes.

has a relative answer since spinor or conjugate-spinor is just a matter of definition. The question is: "What should be the GSO projection to the left compared to the right movers?" Since left and right movers are disconnected, we can make the same or different choice. There are two inequivalent theories that are called *Type IIA* and *Type IIB*, where in the A we choose different and in the B the same GSO projections for the two sectors. The massless spectrum of the two theories are (in $SO(8)$ content):

$$\begin{aligned}
&\text{Type IIA:} \quad (\mathbf{8}_v \oplus \mathbf{8}_s)_L \otimes (\mathbf{8}_v \oplus \mathbf{8}_c)_R \\
&\text{Type IIB:} \quad (\mathbf{8}_v \oplus \mathbf{8}_s)_L \otimes (\mathbf{8}_v \oplus \mathbf{8}_s)_R
\end{aligned} \tag{2.38}$$

The spectrum is provided below. The G^{ij} is the graviton. The ψ^i are gravitino with different and same chirality in A and B theories respectively.

	NSNS	RR	NSR	RNS
Type IIA	$\Phi \oplus B^{ij} \oplus G^{ij}$	$A^i \oplus C^{ijk}$	$\psi^i_{\dot\alpha}$	$\tilde\psi^j_{\beta}$
Type IIB	$\Phi \oplus B^{ij} \oplus G^{ij}$	$\Phi' \oplus B'^{ij} \oplus D^{ijkl}$	$\psi^i_{\dot\alpha}$	$\tilde\psi^j_{\dot\beta}$

The above massless spectra are described by $10D$ supergravity theories, the so called: *Type IIA* and *Type IIB*.

2.4 Open Superstrings

Before we describe the open strings we have to mention that a pure open string theory cannot be consistent. Open strings can always interact by themselves giving open and closed strings. The complete theory is one that describes both open+closed strings and is called *Type I*. We will explore this theory later on when we will discuss the orientifold models.

The open superstring action is again (2.25) where $\sigma \in [0, \pi]$ and $\tau \in (-\infty, +\infty)$. The bosonic sector has the same solutions as in the pure bosonic case (2.12). We have again two choices for the fermionic boundary conditions:

$$\psi^\mu(0,\tau) = \tilde\psi^\mu(0,\tau) \quad , \qquad \psi^\mu(\pi,\tau) = \tilde\psi^\mu(\pi,\tau) \ , \tag{2.39}$$

$$\psi^\mu(0,\tau) = -\tilde\psi^\mu(0,\tau) \quad , \qquad \psi^\mu(\pi,\tau) = \tilde\psi^\mu(\pi,\tau) \ . \tag{2.40}$$

Traditionally, we want the, so called, R sector to have the same moding as the bosonic part. Therefore, in the NN and DD open strings the R sector is the one with boundary conditions (2.39) and NS with (2.40). To visualize the connection of the open and closed R and NS sectors, we can combine ψ^m, $\tilde\psi^m$ in a single field Ψ^μ with the extended range $\sigma \in [0, 2\pi]$. Defining $\Psi^\mu(\sigma) \equiv \psi^\mu(\sigma)$ and $\Psi^\mu(2\pi - \sigma) \equiv \tilde\psi^\mu(\sigma)$. These left moving fields are periodic in R and antiperiodic in NS. This is called the *doubling trick* and allows us to treat the open sector as the left moving sector of the closed string.

Having express the open string as the left moving sector of a closed string, we can use the quantization procedure and GSO projection introduced in the previous chapter (2.3.2). The open string spectrum is the same to the left moving spectrum of the closed string.

For the ND open strings the choice of the R and NS sector is the opposite to the one of NN and DD ones. This interchanges the properties between the R and NS giving the spinorial vacuum to the NS sector. We will describe all these cases in more detail later on.

2.5 Compactification

As we mentioned before, string theory lives in $10D$. Therefore, if we want to discuss interesting phenomenological aspects we have to somehow reduce the visible dimensions to our familiar $4D$ spacetime. One of the most straight-forward ideas is to compactify the extra six dimensions to a compact manifold:

$$\mathbb{R}^{(1,3)} \times \mathcal{M}_6 \ , \tag{2.41}$$

where the $1+3$ real dimensions form the Minkowski space.

The effects of compact dimensions in a theory are many. Kaluza and Klein had shown in the beginning of the 20^{th} century that in a theory in $D = 5$ dimensions with one compact $x^4 = x^4 + 2\pi\mathcal{R}$, the momentum in the compact dimension is quantized such that $p_n^4 = n/\mathcal{R}$. Massless scalars in $5D$ can be expanded $\phi(x^N) = \sum \phi_m(x^\mu)e^{imx^4/\mathcal{R}}$ (M, N run in all and μ, ν run in the non-compact dimensions) giving a family of scalars of mass $m_m = m/\mathcal{R}$ in $4D$:

$$\partial^M \partial_M \phi(x^N) = 0 \qquad \rightarrow \qquad \left(\partial^\mu \partial_\mu - \frac{m^2}{\mathcal{R}^2}\right) \phi_m(x^\nu) = 0 \ . \tag{2.42}$$

This family is a tower of states characterized by m which are called the *Kaluza-Klein modes*.

$5D$ gravity with one compact dimension also has interesting effects. Decompose G^{MN} into $G^{\mu\nu}$, $G^{\mu 4}$, G^{44}. As is known, $5D$ local coordinate transformations are a symmetry where:

$$x^M \rightarrow x^M + \epsilon^M(x) \ , \qquad G^{MN} \rightarrow G^{MN} - \partial^M \epsilon^N - \partial^N \epsilon^M \ . \tag{2.43}$$

Local transformations of the type $\epsilon^4(x^\mu), \epsilon^\mu = 0$ (rotations of the circle) can be interpreted as gauge transformation of a "vector" field $G_{\mu 4} = A_\mu$: $A^\mu \rightarrow A^\mu - \partial^\mu \epsilon^4$.

The effective action for the massless theory in a curved background contains the graviton G_{MN}, an antisymmetric tensor B_{MN}, and a dilaton Φ. Considering that none of these fields is x^5 dependant the graviton-dilaton action becomes:

$$\begin{aligned} S_{symmetric} &= \frac{1}{16\pi G^N_{(5)}} \int d^5x \sqrt{-G_{(5)}} e^{-2\Phi} \left(R^{(5)} + 4\partial_M \Phi \partial^M \Phi\right) \\ &= \frac{1}{16\pi G^N_{(4)}} \int d^4x \left(\sqrt{G_{(4)}}\right) e^{-2\Phi_4} \times \\ &\qquad \left(R^{(5)} - \partial_\mu \phi \partial^\mu \phi + 4\partial_\mu \Phi_4 \partial^\mu \Phi_4 - \frac{1}{4} e^{2\phi} F^{\mu\nu} F_{\mu\nu}\right) \end{aligned} \tag{2.44}$$

where $G_{44} = e^{2\phi}$, $\Phi_4 = \Phi - \phi/2$, the Newton constants in $5D$ and $4D$ are related through $G^N_{(5)} = 2\pi \mathcal{R} G^N_{(4)}$. Therefore, $5D$ gravity plus the dilaton in a spacetime with one compact dimension can be interpreted as $4D$ gravity coupled to electromagnetism and two scalar fields ϕ, Φ_4.

Closed strings that live in spaces with compact dimensions have another very interesting effect that does not appear in particle physics. They can rap around the compact dimension. This gives a topological charge the *winding number* n. Solving again the EOM for the bosonic string living in compact dimensions, we find that the momenta in compact dimensions are not equal any more. The solution for the closed string is given again by (2.11) with:

$$p_L = \frac{m}{\mathcal{R}} + \frac{n\mathcal{R}}{\alpha'} \quad , \quad p_R = \frac{m}{\mathcal{R}} - \frac{n\mathcal{R}}{\alpha'} \, , \tag{2.45}$$

for the compact dimensions.

Notice that if we exchange $m \leftrightarrow n$ and $\mathcal{R} \to 1/\mathcal{R}$ we end up with a theory where $p_L \to p_L$ and $p_R \to -p_R$. It is important to mention that the spectrum and the currents also respect this property. This property is called *T-duality* and implies that conformal field theory cannot distinguish a circle of radius $\mathcal{R}$ from another of radius $1/\mathcal{R}$. It states that two a priori different theories are in fact equivalent. In the next sections, we will find some more interesting properties of this duality.

2.6 Orbifolds

There is a class of exactly soluble compactifications on spaces known as *orbifolds* [22]. The notion of orbifold arises when we consider a manifold M that has a discrete symmetry group G. We may consider a new manifold $\tilde{M} = M/G$, which is obtained from the old one by moding out the symmetry group G. If G is freely acting, the manifold $\tilde{M}$ is smooth. If the manifold $\tilde{M}$ has fixed-points, it has conical singularities at the fixed points.

Orbifolds are interesting in the context of CFT and string theory, since they provide spaces for string compactifications that are richer than tori, but admit an exact CFT description. Moreover, although their classical geometry can be singular, strings propagate smoothly on them. In other words, the correlation functions of the associated CFT are finite.

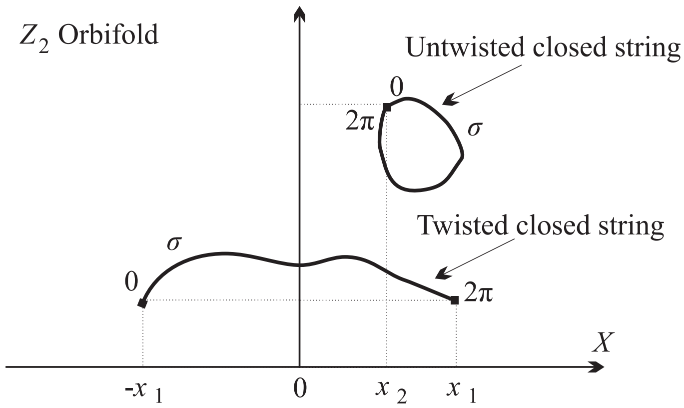


Figure 1: The closed strings of the Z_2 orientifold.

To be more precise, we will explore a specific example. Consider a circle S^1 parametrized by $X = X + 2\pi\mathcal{R}$. The orbifold action will be a Z_2 discrete symmetry where: $G = \{1, R\}$ and $R: \ X \to -X$. This identification gives rise to two sectors: the so-called untwisted and twisted sectors. The fact that the string wavefunction must be invariant under the element R gives rise to the *untwisted sector*. Imposing the reflection condition on the mode expansion (2.11) we find:

$$m = n = 0 \ , \quad \alpha_k \to -\alpha_k \ , \quad \tilde{\alpha}_k \to -\tilde{\alpha}_k \ . \tag{2.46}$$

In addition, we also have a new sector in the closed string spectrum, in which, the boundary conditions for the bosonic part of the string are twisted:

$$X(\sigma + 2\pi) = -X(\sigma) \ . \tag{2.47}$$

These strings are called *twisted sector* strings and they are closed only under the identification (2.47) (Fig.1). Solving the EOM using the twisted boundary condition we find:

$$X = x_{fixed\ points} + i\sqrt{\frac{\alpha'}{2}} \sum_{r\in\mathbb{Z}+v} \left(\frac{\alpha_r}{r} z^{-r} + \frac{\tilde{\alpha}_r}{r} \bar{z}^{-r} \right) \ . \tag{2.48}$$

where for the Z_2 case $v_R \equiv g = 1/2$[5]. Notice the differences between the mode expansions (2.11) and (2.48) in the absence of momenta and in the modding of the oscillators. The center of mass is localized on $x_{fixed\ points} = 0, \ \pi\mathcal{R}$, the fixed points of the manifold. Therefore, we have one ground state on each fixed poind $|H^{0,\pi\mathcal{R}}\rangle$ that is annihilated by the positive moding α_r. The action of R on the oscillator modes of the twisted sector is again given by (2.46).

We have also to impose boundary conditions on the fermionic twisted sectors. Since in the untwisted sector the R and the bosonic sector have the same moding,

[5] We denote by g the rotation angle of a Z_2 element R.

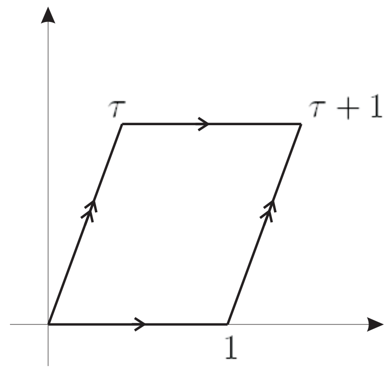


Figure 2: The torus as a periodic lattice.

we define as R twisted sector the half moded one. Therefore, the NS sector has the zero modes.

In general, we could project with an element $\alpha = e^{2\pi i v}$ (as we mention above, the reflection element R is a special case with $v_R = 1/2$). The untwisted wavefunction should be invariant under the action of the new element and the twisted states will be modes of the general kind: α_{k+v}. Notice that right movers will be moded as: $\tilde{\alpha}_{k-v}$. The fermions on the other hand will be also twisted with $\alpha = e^{2\pi i(v+\nu)}$ where $\nu = 0, 1/2$ for R and NS respectively. The field expansion will be similar to (2.28) where the moding will run to: $r \in \mathbb{Z} + \nu + v$. Notice also that different sectors will be localized on different fixed points.

The existence of the two sectors has its origins in a deeper reason that is *modular invariance* of the 1-loop diagram.

2.6.1 Partition function and modular invariance

Consider the 1-loop vacuum to vacuum amplitude of an oriented closed string which is obviously a torus diagram. To evaluate the path-integral we have to sum over all possible tori. The torus is a two dimensional surface that can be seen as two independent one-cycles, parametrized as $\sigma^1, \sigma^2 \in [0,1]$. It is completely specified by giving a flat metric and a *complex structure* τ with $Im(\tau) \geq 0$ that cannot be changed by any infinitesimal diffeomorphisms or Weyl rescaling. Defining complex coordinates $w = \sigma^1 + \tau\sigma^2$ and $\bar{w} = \sigma^1 + \bar{\tau}\sigma^2$, the periodicity conditions become:

$$w \to w + 1 \ , \qquad w \to w + \tau \ . \tag{2.49}$$

The torus can be thought as a point of the complex plane w identified under two translation vectors corresponding to the complex numbers 1 and τ .

Not all τ describe different tori. The periodicity conditions show that transformations of the type:

$$\tau' = \frac{a\tau + b}{c\tau + d} \ , \qquad \text{with:} \ \ ad - bc = 1 \ , \tag{2.50}$$

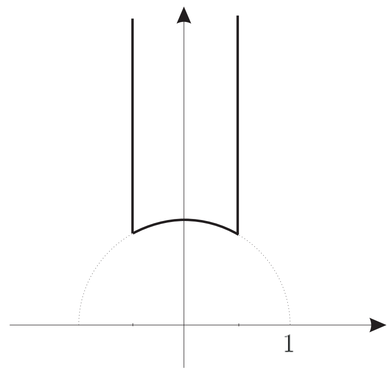


Figure 3: Fundamental domain of the torus.

keep the torus invariant. This is the group SL(2,$\mathbb{Z}$). The generators of this group are:

$$T : \tau \to \tau + 1 \ , \qquad S : \tau \to -1/\tau \ . \tag{2.51}$$

It can be shown that the fundamental domain $\mathcal{F}$ of the modular group of the torus is $|\tau^1| \leq 1/2$ and $||\tau|| \geq 1$ (Fig.3).

The path-integral of a conformal field theory on a torus is the 1-loop vacuum energy. As we mentioned above, the Hamiltonian is $H = L_0 + \bar{L}_0$, which is the generator of translations in worldsheet time τ. The generator of rotations around σ is $P = L_0 - \bar{L}_0$. Putting everything together we have:

$$\mathcal{T} = \int e^{-S} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Tr[e^{-2\pi\tau H} e^{2\pi i \sigma P}] = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Tr[q^{L_0-1} \bar{q}^{\bar{L}_0-1}] \ , \tag{2.52}$$

where $q = e^{2\pi i \tau}$. This is the so-called *torus partition function*, since expanding in powers of $q, \bar{q}$, the powers in the expansion refer to the mass squared level of excitations. Notice that (2.52) does not contain divergencies since the integration area, $\mathcal{F}$, does not touch the origin.

One very important property of (2.52) is that it is *modular invariant* (invariant under (2.51)). This property is crucial and it means that we correctly integrate over all inequivalent tori.

The partition function of an orbifold has to be modified since we want to project onto states that in the Z_2 case have $R = +1$. The untwisted contribution is

$$\mathcal{T}_U = \frac{1}{2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Tr_U[(1 + R) q^{L_0-1} \bar{q}^{\bar{L}_0-1}] \ . \tag{2.53}$$

The trace part of the 1 term is modular invariant like (2.52). However, the trace part with the insertion of R is not. We have to add some extra terms/sectors if we want the total partition function to be modular invariant. The twisted sector recovers this problem giving the full modular invariant partition function:

$$Z_{U+T} = \frac{1}{2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Tr_{U+T}[(1+R) q^{L_0} \bar{q}^{\bar{L}_0}] \ . \qquad (2.54)$$

We will describe more precisely the orbifold construction in the next section where we will discuss a generalized version of it: the orientifold.

3. Orientifolds

Orientifolds are generalized orbifolds, where except from the orbifold discrete symmetry we include orientation reversal on the worldsheet [23, 27, 29, 28, 24]. This expansion generates a theory of unoriented closed strings (plus open strings as we will see in a while).

Orientation reversal, Ω, means interchanging of left and right movers. Here, we shall consider the Type IIB closed theory since it contains a symmetry on left and right modes[6]. Ω interchanges $\sigma \to -\sigma$ (or $z \leftrightarrow \bar{z}$). Looking at (2.11, 2.48) we realize that the action of Ω on the oscillators is to interchange $p_L \leftrightarrow p_R$ and $\alpha \leftrightarrow \tilde{\alpha}$. The action of Ω on the bosonic zero modes of a compact dimension is:

$$\Omega|m,n\rangle = |m,-n\rangle \ . \tag{3.1}$$

The action and the quantization procedure preserve the worldsheet parity. For all the oscillator modes we have:

$$\Omega\alpha_k^\mu\Omega^{-1} = \tilde{\alpha}_k^\mu \ , \quad \Omega\tilde{\alpha}_k^\mu\Omega^{-1} = \alpha_k^\mu \ , \quad \Omega\psi_r^\mu\Omega^{-1} = \tilde{\psi}_r^\mu \ , \quad \Omega\tilde{\psi}_r^\mu\Omega^{-1} = -\psi_r^\mu \ , \tag{3.2}$$

for integer and half-integer r. The minus in the last equation is included to give $\Omega\psi\tilde{\psi}\Omega^{-1} = \psi\tilde{\psi}$, so that the graviton is invariant under the Ω projection.

The total orientifold group contains elements of two kinds: internal symmetries of the worldsheet theory, forming a group G, and elements of the form $\Omega \cdot g$, where g is some symmetry element that is taken from a group G'. Closure implies that $\Omega \cdot g \cdot \Omega \cdot g' \in G$ for $g, g' \in G'$. The full orientifold group is $G + \Omega G'$.

In our study we will concentrate on groups where $G = G'$. For simplicity, the compact manifold will be formed by three tori where our $10D$ space will be parametrized as:

$$\mathbb{R}^4 \times T_1^2 \times T_2^2 \times T_3^2 \ . \tag{3.3}$$

We define complex coordinates for each torus: $z^i = X^{2i+2} + iX^{2i+3}$ and similarly for the complex fermionic states $\psi^i = \psi^{2i+2} + i\psi^{2i+3}$, for $i = 1, 2, 3$. In general, G contains two kinds of elements: rotations and translations:

- *Rotation* elements are a subgroup of the Poincaré group and they are defined as:

$$\alpha = \exp\left(2\pi i(v_\alpha^1 J_{45} + v_\alpha^2 J_{67} + v_\alpha^3 J_{89})\right) \ , \tag{3.4}$$

  where J_{mn} are $SO(6)$ Cartan generators. The resulting manifold has fixed points. To preserve some of the supersymmetry, the v_α^is should satisfy the

[6] We could also take the Type IIA for a Ω' element that changes also the chirality of the left and right moving fermions.

condition $\sum_i v_\alpha^i = 0$. This ensures that there are gravitini in both the NSR and RNS untwisted sectors.

Notice that a Z_2 element R_i (to preserve some of the supersymmetries)[7] leaves unaffected one torus T_i^2. We denote the components of the rotation vector as $u_R^i \equiv g^i$. For example, an R_1 element has a shift vector: $g_1 = \{0, 1/2, -1/2\}$. Such elements will play a key role in the rest of our studies. In the next table we show the general rotation elements of G and we denote the tori in which they act:

Elements of G	T_1^2	T_2^2	T_3^2
$\alpha:\ v_\alpha^3 = 0$	X	X	
$\alpha:\ v_\alpha^3 \neq 0$	X	X	X
R_1		X	X
R_2	X		X
R_3	X	X	

The direct action of such an element on the bosonic zero modes of a compact dimension is:

$$\alpha\ |m, n\rangle = |e^{2\pi i v_\alpha} m, e^{2\pi i v_\alpha} n\rangle\ , \tag{3.5}$$

where m, n complex momentum and winding numbers coming from the complex parametrization of the coordinates. On the oscillation states the action is: for the bosonic and NS sector:

$$\alpha\ z^i = e^{2\pi i v_\alpha^i} z^i\quad , \qquad \alpha\ \psi^i = e^{2\pi i v_\alpha^i} \psi^i\ . \tag{3.6}$$

The conjugate fields z^{-i}, ψ^{-i} transform with the phase $e^{-2\pi i v_\alpha^i}$. On the R sector, the action on the non-zero modes is similar to the one on the NS, however, the R vacuum transform as:

$$\alpha\ |s_0 s_1 s_2 s_3\rangle = e^{2\pi i v_\alpha \cdot s} |s_0 s_1 s_2 s_3\rangle\ , \tag{3.7}$$

where we extended $v_\alpha = \{0, v_\alpha^1, v_\alpha^2, v_\alpha^3\}$. The action on the right movers is the same to the one on the left movers.

- *Translation* elements h are freely acting elements which are also a subgroup of the Poincaré group. The generic symmetry of a d-dimensional toroidal CFT contains the $\mathrm{U(1)_L^d} \times \mathrm{U(1)_R^d}$ chiral symmetry. The transformations associated

[7] Z_2 reflecting elements are also denoted in the literature as I. In particular, the reflection element that does not break supersymmetry acts on the coordinates of two tori and it is denoted by I_4.

with it are arbitrary lattice translations. They act on a state with momenta m_i and windings n_i as

$$h_{\rm translation} = \exp\left[2\pi i \sum_{i=1}^{d}(m_i\theta_i + n_i\phi_i)\right] \ , \tag{3.8}$$

where θ_i, ϕ_i are rational in order to obtain a discrete group. There are also symmetries that are subgroups of the $O(d,d)$ group not broken by the moduli G_{ij} and B_{ij}. These depend on the point of the moduli space. For the rest of our study we will concentrate on translation elements that act on one only coordinate as momentum shifts of order N ($\theta = 1/N, \ \phi_i = 0$). Generalization to more dimensions and to winding shifts is straight forward.

Clearly, the action h_N affects only the bosonic zero modes of the states where by acting in direct it gives an eigenvalue, and by twisting it shifts the winding number:

$$\begin{array}{lll} \text{Direct action} & h_N : |m,n\rangle \rightarrow e^{2\pi i m/N}|m,n\rangle \ . \\ \text{Twist} & h_N : |m,n\rangle \rightarrow |m,n+1/N\rangle \ . \end{array} \tag{3.9}$$

Translation elements that are accompanied by elements that treat bosons and fermions in a different way break supersymmetry. These type of actions are called *Sherck-Schwarz* deformations (SS) [20, 21]. For this work we will consider only Z_2 SS deformations:

$$h = (-1)^{\mathcal{F}} \ h_2 \ , \tag{3.10}$$

where $\mathcal{F}$ is the space-time fermion number. The geometric action of this element is to halve the radius of the corresponding dimension that it acts onto: $X \rightarrow X + \pi R$. Notice that:

- The $\tilde{\alpha}^{\mu}_{-1}|S^I_{\alpha}\rangle \otimes |m,n\rangle$ has the space-time quantum numbers of the gravitino. It transforms with $(-1)^{m+1}$ sign under the h action. Therefore, the massless state $|m,n\rangle = |0,0\rangle$ is projected out (massless gravitino) but not the $|m,n\rangle = |1,0\rangle$ state that has mass $m^2_{3/2} = 1/4R^2$ ($\rightarrow$ massive gravitino).
- Supersymmetry is broken spontaneously and it restores in the large radius limit $R \rightarrow \infty$.

Rotation α and translation h elements belong in G only if they commute $[\alpha, h] = 0$. Therefore, in the direction where a Z_2 Scherk-Schwarz deformation acts, we can only consider rotations by Z_2 elements R.

To summarize, in the supersymmetric case the most general rotation element α has rotation vector $v_\alpha = (v^1_\alpha, v^2_\alpha, v^3_\alpha)$ with $v^3_\alpha = 0$ or $v^3_\alpha \neq 0$. On the other hand, in non-supersymmetric models where a SS deformation acts onto one coordinate of the third torus T_3^2, the most general rotation element α will act as $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ or $v_\alpha = (v_\alpha^1, v_\alpha^2, 1/2)$[8].

3.1 The closed string spectrum

The closed string spectrum is constructed combining left and right states with the same chirality, invariant under the orbifold action. We will concentrate on the massless states. Untwisted states:

- From the NSNS states we have a graviton $((\psi^{\{\mu}_{-1/2}\tilde{\psi}^{\nu\}}_{-1/2} - \delta^{\mu\nu}\psi^{\rho}_{-1/2}\tilde{\psi}_{-1/2,\rho})|0,0\rangle)$, an axion $(\psi^{[\mu}_{-1/2}\tilde{\psi}^{\nu]}_{-1/2}|0,0\rangle)$ and a dilaton $(\psi^{\rho}_{-1/2}\tilde{\psi}_{-1/2,\rho}|0,0\rangle)$. Since parity projects onto symmetric states the axion is eliminated.

  The matter states depend on the orbifold action (v_1, v_2, v_3). Consider a state:

  $$\alpha\ \psi^{\pm i}_{-1/2}\tilde{\psi}^{\pm j}_{-1/2}|0;0\rangle \rightarrow e^{\pm 2\pi i(v_i+\tilde{v}_j)k}\psi^{\pm i}_{-1/2}\tilde{\psi}^{\pm j}_{-1/2}|0;0\rangle\ . \tag{3.11}$$

  It will not be excluded only in the case $(v_i + \tilde{v}_j)k \in \mathbb{Z}$. Ω projection will keep only left-right invariant states.

- Similar for the RR sector:

  $$\alpha\ |s_i;\tilde{s}_j\rangle \rightarrow e^{\pm 2\pi i(s_i\cdot v_i+\tilde{s}_j\cdot\tilde{v}_j)k}|s_i;\tilde{s}_j\rangle\ , \tag{3.12}$$

  where $i,j = 0,1,2,3$. The invariant states are those where $(s_i\cdot v_i + \tilde{s}_j\cdot\tilde{v}_j)k \in \mathbb{Z}$. Ω projects onto antisymmetric combinations of left-right.

- Finally, for the RNS and NSR the procedure is similar to the above. Invariant states under Ω are taken by the symmetric combination NSR+RNS.

For the twisted sector the procedure is similar. However, only Z_2 twisted elements are invariant under the Ω projection as it was mentioned in section 2.6.

3.2 Klein Bottle

Consider the 1-loop vacuum amplitude of a theory with orientation reversal. Consider also $\alpha \in G$, an element of a N dimensional group G. The 1-loop partition function for this generalized orbifold will be:

$$\frac{1}{2\times N}\sum_{\alpha} Tr[(1+\Omega)\alpha q^{L_0}\bar{q}^{\bar{L}_0}] \tag{3.13}$$

Notice that we project onto even states under Ω (+1 eigenvalue) since odd states do not form a closed set under interactions. The amplitudes that do not contain Ω

[8]The former can be written in the form $v_\beta + g_{i=1,2}$ with $v_\beta^3 = 0$ a rotation in the $T_1^2 \times T_2^2$ torus and $g_{i=1,2}$ a Z_2 element. Therefore, without loss of generality we can take α such that $v_\alpha^3 = 0$.

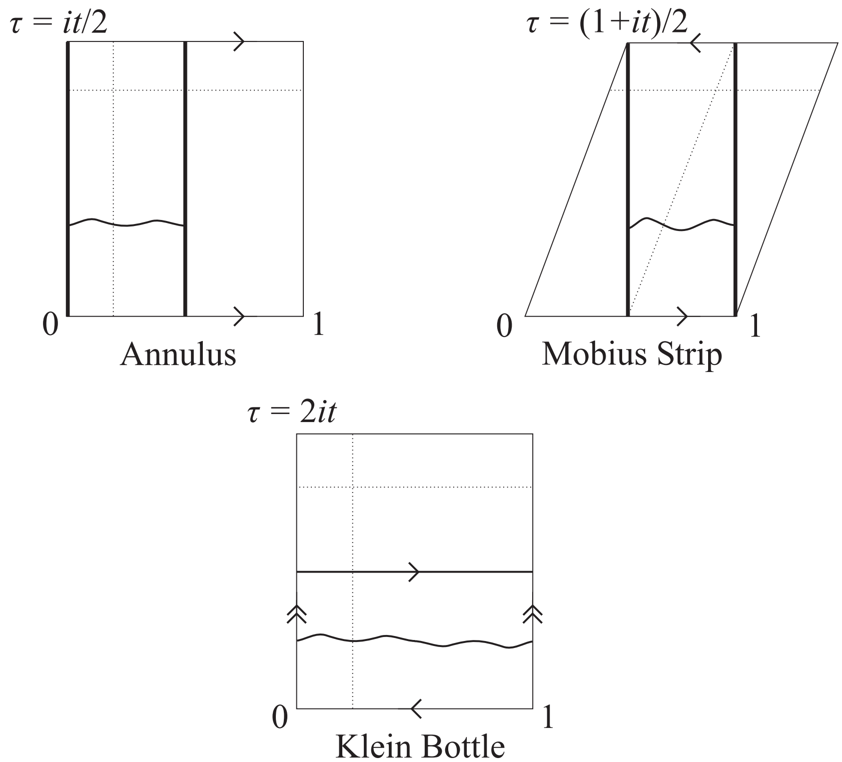


Figure 4: Covering tori and fundamental cells for the three one loop surfaces. The cycles are represented by dashed lines. The point M' is the image of M under the appropriate involutions.

describe the propagation of oriented closed strings. It is the usual torus amplitudes $\mathcal{T}$. The amplitude that contains Ω describes strings that propagate and flip orientation. Geometrically, this is described by an unoriented two-dimensional surface with Euler number zero (equal to the Euler number of the torus): the Klein Bottle amplitude $\mathcal{K}$.

Topologically, the Klein Bottle can be obtained from its covering torus with complex structure $\tau = 2i\tau_2$, if the lattice translations are supplemented by the anticonformal involution $w \to 1 - \bar{w} + i\tau_2$. This representation will be denoted as $\mathcal{K}$. There is a second choice of polygon that defines an inequivalent horizontal time. It is obtained by halving the horizontal side while doubling the vertical one and thus leaving the area unchanged. The end result has the virtue of displaying an equivalent representation of this surface as a tube terminating on two crosscaps, and the horizontal side is the proper time elapsed as a closed oriented string propagates between the crosscaps. The change of orientation is taking place on the crosscaps. This will be denoted as $\tilde{\mathcal{K}}$ (Fig.4).

To evaluate the path-integral we have to integrate over all possible $\mathcal{K}$, that means integration on $\tau_2 \in [0, \infty)$. We will see that this integration gives in general ultraviolent (UV) divergences (due to tadpoles) [26].

3.2.1 Supersymmetric Orientifolds

In this section we will evaluate the UV limit of (3.13). First, we will concentrate on supersymmetric cases and after we will generalize to include the Scherk-Schwarz deformation that spontaneously breaks supersymmetry.

We can work out the contribution of an element $\alpha \in G$ to the Klein Bottle amplitude by using the trace formula:

$$\mathcal{K}_\alpha = Tr_{U+T}\left[\Omega\alpha \quad q^{L_0}\bar{q}^{\bar{L}_0}\right] \ ,$$

where the subscripts U and T refer to the untwisted and twisted closed string states of the type IIB orbifold model considered. General twisted states have different moding between left and right movers (we recall (2.48) and comments below) which coincide only for the Z_2 case. Therefore, only Z_2 twisted sectors will survive the Ω reflection. The contribution to the Klein Bottle amplitude of an element $\alpha \in G$ can be written in the form

$$\mathcal{K}_\alpha \sim T[{}^{0}_{2v_\alpha}] + T[{}^{g}_{2v_\alpha}] \ , \tag{3.14}$$

where the second term exists only in case where there are Z_2 factors denoted by g (section 2.6). The form of $T[{}^{u}_{v}]$ is given in the appendix (B.1, B.2). In the transverse channel, the contribution of an element α corresponds to a propagation of a closed string state projected by $(\Omega\alpha)^2 = \alpha^2$ which explains the $2v_\alpha$ factor in (3.14).

As we mentioned above, the 1-loop diagram gives in general UV divergencies, since $\tau_2 \in [0, \infty)$. The way to compute the divergent contribution is to evaluate $\tilde{\mathcal{K}}$. In this picture, the horizontal side is the proper time elapsed as a closed oriented string propagates between two crosscaps. Technically, to go from the one representation of the $\mathcal{K}$ to the other $\tilde{\mathcal{K}}$ we need to perform a modular transformation, $l = 1/4t$, where t is the loop modulus and l the cylinder length [27]. To extract the divergencies we evaluate the UV limit by taking $l \to \infty$.

If the orbifold group G contains Z_2 factors denoted by R_i [9], then there is an extra contribution since $(\Omega R_i \alpha)^2 = \alpha^2$:

$$T[{}^{0}_{2g_i v_\alpha}] + T[{}^{g_j}_{2g_i v_a}] \ , \tag{3.15}$$

where i, j denote the different Z_2 elements in G. In general, elements $\alpha \in G$ can leave one torus unaffected or act on all tori. Without loss of generality, we consider as the unaffected torus the T_3^2 (3.3). Therefore, the various orientifolds can be classified by $v_\alpha^3 = 0$ or $v_\alpha^3 \neq 0$.

[9] We remind that R_i is a Z_2 rotation element which leaves unaffected the T_i^2 torus and acts on the other two tori.

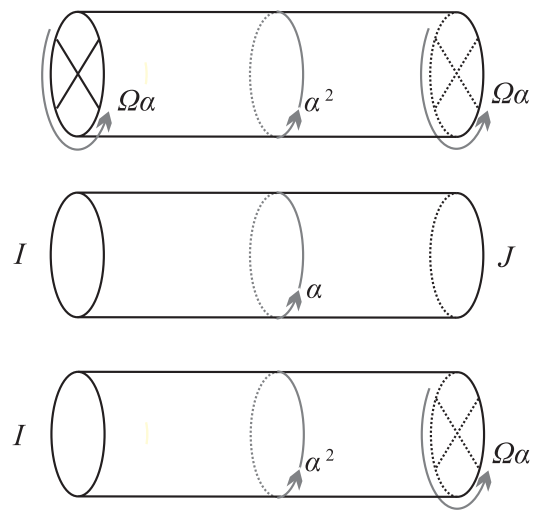


Figure 5: Klein-bottle, Annulus and Möbius strip. The one-loop amplitudes become tree-level in the transverse picture where an α^2-twisted closed string propagates between crosscaps and boundaries.

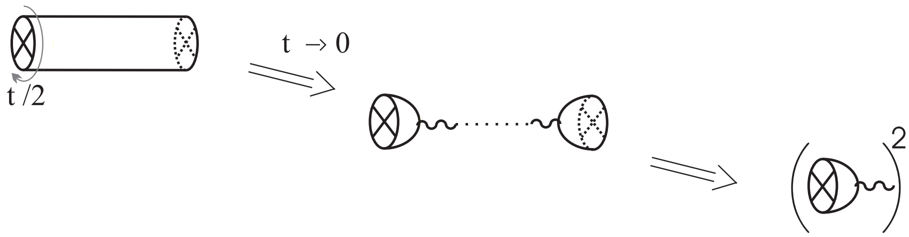


Figure 6: We can factorize and compute the contributions to the divergences of each of the two crosscups between which closed strings propagate.

Taking the UV limit $l \to \infty$ of $\tilde{\mathcal{K}}$, we can factorize and compute the contributions to the divergences of each of the two crosscups, between which closed strings propagate (Fig.6). The results are provided in the appendix where we use representative pictures for the tadpoles. The type of the twist of the emitted closed string (α^2) is marked on the right of the tadpole. Using this notation, we can classify all cases in a compact way:

- The contribution to the Klein Bottle from an element α with $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ will be:

    - If the orbifold group G does not include R factors, the only contribution of α to the massless tadpoles will come from the untwisted sector states (the first term in (3.14)):

$$(1_{NS}-1_R)\Big(\Omega\alpha\ \alpha^2\Big)^2 . \tag{3.16}$$

- In case the group G contains Z_2 factors, $R \in G$, that commute with α, we have extra contributions from the twisted states. We classify the contributions to the tadpoles by the different Z_2 elements that are included in G:

  i. If $R_3 \in G$ we have two sources of divergences:

$$(1_{NS}-1_R)\Big(\Omega\alpha\ \alpha^2+\Omega R_3\alpha\ \alpha^2\Big)^2 , \tag{3.17}$$

  ii. If $R_i \in G$ for a given $i = 1$ or 2:

$$(1_{NS}-1_R)\Big(\Omega\alpha\ \alpha^2+\Omega R_i\alpha\ \alpha^2\Big)^2 , \tag{3.18}$$

  iii. If $R_l \in G$ with $l = 1, 2, 3$:

$$(1_{NS}-1_R)\Big(\Omega\alpha\ \alpha^2+\sum_{l=1}^{3}\Omega R_l\alpha\ \alpha^2\Big)^2 . \tag{3.19}$$

All the amplitudes above are proportional to $(1_{NS} - 1_R)$ and their multiplicatives appear as perfect squares [23, 24]. We should mention that for this kind of orbifold action all the amplitudes are volume depended ($\mathcal{V}_3$ is the volume of the third torus which is not affected by α). They are of the general form:

$$(1_{NS}-1_R)\left[K_1\sqrt{\mathcal{V}_3}+\frac{K_2}{\sqrt{\mathcal{V}_3}}\right]^2 , \tag{3.20}$$

where K_1 and K_2 are constants.

- Next, we consider the case where α acts on all tori ($v_\alpha = (v^1_\alpha, v^2_\alpha, v^3_\alpha)$ with $v_\alpha^{l=1,2,3} \neq 0$). We can classify again:

  - If G contains no R factors.

$$(1_{NS}-1_R)\Big(\Omega\alpha\ \alpha^2\Big)^2 . \tag{3.21}$$

  - In case G contains R factors:

    i. If $R_i \in G$ for a given i.

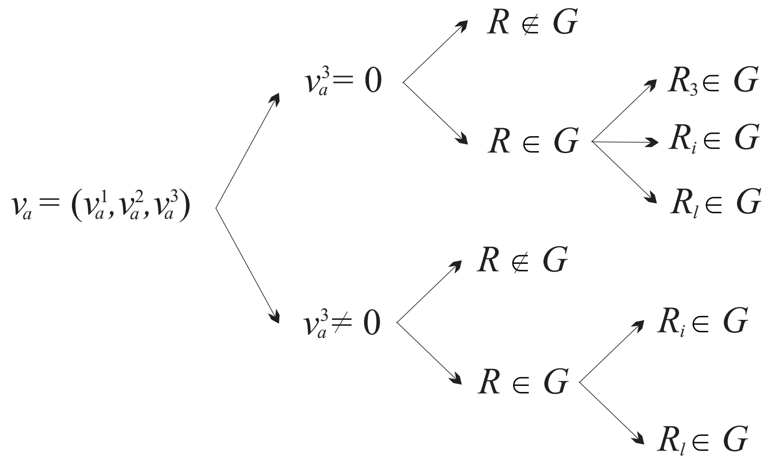
$$(1_{NS}-1_R)\Big(\Omega\alpha\ \alpha^2+\Omega R_i\alpha\ \alpha^2\Big)^2 . \tag{3.22}$$

ii. If $R_l \in G$ for $l = 1, 2, 3$.

$$(1_{NS} - 1_R)\left(\Omega\alpha \otimes \!\!\sim\!\sim \alpha^2 + \sum_{l=1}^{3} \Omega R_l \alpha \otimes \!\!\sim\!\sim \alpha^2 \right)^2 . \qquad (3.23)$$

All the amplitudes are again perfect squares as in (3.20) without the volume dependence.

Schematically, the above classification of tadpoles can be visualized as:



3.2.2 O-planes

Tadpoles in general are amplitudes for creation of a single particle from the vacuum. They are artifacts of the perturbation theory and they appear in higher loops.

The tadpoles that we found in the Klein-Bottle amplitude (3.2), can be interpreted as sources of massless closed fields in space-time introduced by the orientifold (Ω) projection. They couple to the massless IIB fields, in particular the metric (so they have energy or tension), the dilaton and the RR-forms (under which they are minimally coupled). Such sources are localized in sub-manifolds of spacetime, typically $p+1$ dimensional hyperplanes that are known as orientifold planes, O_p.

Depending on the tension and charge, we define the following notation: O_+ an O-plane with negative tension and charge, O_- an O-plane with positive tension and charge. The $\bar{O}_+$ and $\bar{O}_-$ have same tension and opposite charge to the O_+ and O_- respectively.

The NSNS tadpoles can be seen by an analogous phenomenon in field theory. Consider for example the action:

$$\int d^dx \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + Q\phi \right) . \qquad (3.24)$$

The equation of motion is: $\partial_\mu \partial^\mu \phi = Q$. If we expand around $\phi(x) = 0$ we will encounter Feynman diagrams like:

$$Q \otimes\!\sim\!\sim\!\sim\!\otimes Q \quad \sim \quad \frac{1}{k^2} = \int_0^\infty dl \exp(-k^2 l) , \qquad (3.25)$$

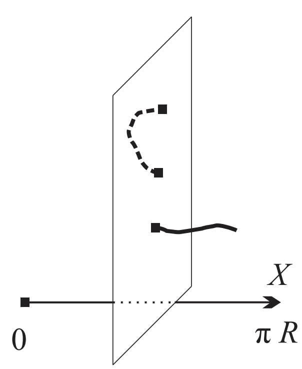
that have divergences at vanishing momentum. From (3.25) we realize that the divergence originates as $l \to \infty$. We could avoid this divergence if we had expanded around the correct vacuum. The NSNS divergence in $\tilde{\mathcal{K}}$ has the same origin. If we had expanded around the correct non-constant metric and dilaton, the amplitude would converge.

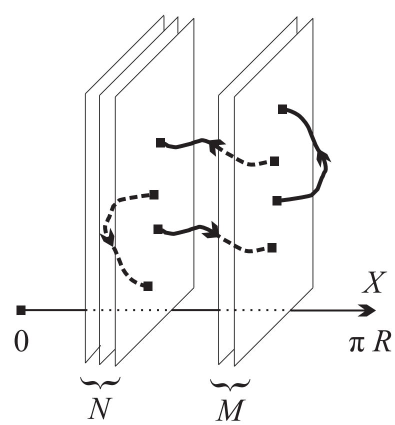
The RR tadpoles have different origin and they need to be cancelled since they refer to not vanishing charges. In noncompact spaces this may be acceptable since the Faraday lines can end at infinity. However, this is not possible in compact spaces where a non vanishing of the total charge violates the Gauss law.

3.3 Open strings and D-branes

Stability of the above unoriented closed string theory requires a new "twisted" sector under Ω (analogous to the twisted sector of the orbifold construction). This is the *open string* sector.

Open strings have endpoints. We can always define a $p+1$ dimensional hyperplane, that is called the *Dp-brane*, where the ends of the open strings attach. Open strings that are attached to the brane can freely move in the p longitudinal directions (they obey Neumann boundary conditions) and they are fixed in the remaining $9-p$ transverse directions (where they obey Dirichlet boundary conditions).





As we mentioned before, we can introduce charges on the endpoints of the open strings, the Chan-Paton factors (2.2). In this picture, strings with different charges can be interpreted as strings ending on different stacks of branes.

There is an interesting property of T-duality on D-branes. When we T-dualize a dimension, Dirichlet boundary conditions become Neumann and vice versa. This implies that if we T-dualize a longitudinal or a transverse direction of a D_p-brane, it will become D_{p-1} or D_{p+1} brane, respectively.

D-branes are dynamical objects that couple to the NSNS and RR states. The lagrangian of a D-brane is given by:

$$S_{D-brane} = -\tau_p \int d^{p+1}\xi \; e^{-\phi}\sqrt{\det(G_{ab}+B_{ab}+2\pi\alpha' F_{ab})}$$
$$+\mu_p \int_{\mathcal{M}_{p+1}} C_{p+1}\wedge Tr[e^{B+2\pi\alpha' F}] \; . \tag{3.26}$$

The first term is called the "Dirac-Born-Infeld action" and it contains the induced metric on the brane G_{ab} (that is connected with the spacetime metric via $G_{ab} = G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$), an antisymmetric tensor B_{ab}, [10] coming from the closed string sector and the field strength of a gauge field, F_{ab}, that lives in the brane. τ_p is the tension of the brane. The second term is the "Wess-Zumino" action that describes the coupling of the D-brane to the RR sector. C_{p+1} are the RR forms and μ_p the RR charge.

3.3.1 Orientifold action on open strings

In general, an open string is denoted as $|\Psi, ab\rangle$, where Ψ refers to the worldsheet fields and ab to the Chan-Paton indices that are associated to the string endpoints on Dp-branes and Dq-branes. The Chan-Paton labels are contracted with a hermitian matrix λ_{ab}. The action of a group element α of the orientifold group G is given by:

$$\alpha \; : \; |\Psi, ab\rangle \to (\gamma_{\alpha,p})_{aa'}|\alpha\Psi, a'b'\rangle(\gamma_{\alpha,q})^{-1}_{b'b} \; , \tag{3.27}$$

where γ_α unitary matrices associated to α. The action of the same element accompanied with Ω gives:

$$\Omega\alpha \; : \; |\Psi, ab\rangle \to (\gamma_{\Omega\alpha,p})_{aa'}|\alpha\Psi, b'a'\rangle(\gamma_{\Omega\alpha,q})^{-1}_{b'b} \; . \tag{3.28}$$

Since $1 \in G$, acting twice with Ω we find:

$$\gamma_\Omega^T = \pm\gamma_\Omega \; . \tag{3.29}$$

A nontrivial argument of Gimon and Polchinski shows that for D9-branes the γ_Ω is symmetric and for D5-branes antisymmetric.

The worldsheet parity Ω acts by interchanging the string $\sigma \to \pi - \sigma$ (or $z \to -\bar{z}$ in the complex plane basis): $X(\sigma) \to X(\pi-\sigma)$ and $\psi(\sigma) \to \pm\tilde{\psi}(\pi-\sigma)$. Applying this on (2.12) and (2.28) we find that:

$$\Omega\alpha^\mu_m\Omega^{-1} = \pm e^{i\pi m}\alpha^\mu_m \; , \qquad \Omega\psi^\mu_m\Omega^{-1} = \epsilon e^{i\pi m}\psi^\mu_m \; , \tag{3.30}$$

where "+" is for NN and "−" is for DD strings (2.12). These transformations are valid for integer and half-integer m. The $\epsilon = \pm 1$ leaving an irrelevant sign freedom. There is no corresponding result for the ND sector since Ω takes it to a different DN sector.

The action of the rotating elements α on the open strings is the same as the action on the closed ones (3.6, 3.7).

[10] B_{ab} is introduced due to the T-duality that connects D(p+1) and D(p-1) branes.

Rotation elements on Chan-Paton factors

The action of the orientifold elements on the Chan-Paton (CP) factors is defined in such a way that the total open string wavefunction $|\Psi, ab\rangle\lambda_{ab}$ will remain invariant under the orientifold action. As an example, we will provide the transformation rules for the massless open spectrum of a generic orientifold model.

- For Dp-Dp states, where $p = 9, 5_i$:
  - The massless NS sector is $\psi^M_{-1/2}|0, ab\rangle\lambda_{ab}$. This includes gauge bosons for $M = \mu$ and matter scalars for $M = \pm i$, with $i = 1, 2, 3$. For the gauge fields, the $\psi^\mu_{-1/2}$ do not transform under α. However, for the scalars, the $\psi^i_{-1/2}$ acquire a phase $e^{\pm 2\pi i v^i_\alpha}$. Therefore, to construct totally invariant states, the λ_{ab}s should transform in the opposite way:

$$\psi^\mu_{-1/2}|0, ab\rangle\lambda^{(0)}_{ab} : \lambda^{(0)} = \gamma_{\alpha,p}\lambda^{(0)}\gamma^{-1}_{\alpha,p} , \qquad \lambda^{(0)} = -\gamma_{\Omega,p}\lambda^{(0)T}\gamma^{-1}_{\Omega,p} , \tag{3.31}$$

$$\psi^i_{-1/2}|0, ab\rangle\lambda^{(i)}_{ab} : \lambda^{(i)} = e^{2\pi i v^i_\alpha}\gamma_{\alpha,p}\lambda^{(i)}\gamma^{-1}_{\alpha,p} , \qquad \lambda^{(i)} = -\gamma_{\Omega,p}\lambda^{(i)T}\gamma^{-1}_{\Omega,p} , \tag{3.32}$$

    on the fixed points. Scalar fields $\psi^j_{-1/2}|0, ab\rangle$ on D5_i-branes with $i \neq j$ transform with a minus sign in the Ω projection due to the DD boundary conditions on the j directions transverse to the brane:

$$\psi^j_{-1/2}|0, ab\rangle\lambda^{(j)}_{ab} : \lambda^{(j)} = e^{2\pi i v^j_\alpha}\gamma_{\alpha,5_i}\lambda^{(j)}\gamma^{-1}_{\alpha,5_i} , \qquad \lambda^{(j)} = \gamma_{\Omega,5_i}\lambda^{(j)T}\gamma^{-1}_{\Omega,5_i} \tag{3.33}$$

    In case we can move some D5_i-branes away from the fixed points, rotation elements do not act on the fields and we should omit the first equation in (3.31, 3.32, 3.33).
  - The massless R sector is the vacuum: $|s_0s_1s_2s_3, ij\rangle\lambda_{ij}$. GSO requires an even number of "$-1/2$"s. Using the $\sum_i v^i_\alpha = 0$ we find that: states with $s_0 = s_1 = s_2 = s_3$ do not transform and their relative CP matrix transforms as $\lambda^{(0)}$. However, states that have: $s_i = s_0 \neq s_j = s_k$ transform with a phase $e^{\pm 2\pi i v^i_\alpha}$ and their relative CP matrices transform as the $\lambda^{(i)}$s.

- For D9-D5_i states:
  - The massless NS sector is $|s_js_k, ab\rangle\lambda_{ab}$. GSO projection requires $s_j = s_k$. These fields transform under α acquiring a phase $e^{2\pi i(v^j_\alpha s_j + v^k_\alpha s_k)}$. The λ_{ab} transform as:

$$|s_js_k, ab\rangle\lambda_{ab} : \lambda_{59} = e^{2\pi i(v^j_\alpha s_j + v^k_\alpha s_k)}\gamma_{\alpha,5_i}\lambda_{5_i9}\gamma^{-1}_{\alpha,9} , \tag{3.34}$$

    on the fixed points. Obviously, there is no constraint due to Ω, since it relates different states ND$\leftrightarrow$DN.

– The massless R sector is the vacuum: $|s_0 s_i, ab\rangle\lambda_{ab}$. GSO projection requires $s_0 = s_i$. These fields transform under α acquiring a phase $e^{2\pi i v_\alpha^i s_i}$. The λ_{ab} transform as:

$$|s_0 s_i, ab\rangle\lambda_{ab} : \lambda_{59} = e^{2\pi i v_\alpha^i s_i}\gamma_{\alpha,5_i}\lambda_{5_i 9}\gamma_{\alpha,9}^{-1} \, . \tag{3.35}$$

Notice that the condition $\sum_i v_\alpha^i = 0$ relates bosonic and fermionic states as it was stated above.

Scherk-Schwarz deformation on Chan-Paton

Similarly to the above we can define the action of translation elements on the Chan-Paton factors. We will concentrate onto the Z_2 Scherk-Schwarz deformation element h (3.10).

The action of this element is defined in such a way that supersymmetry will be restored if we take the decompactification limit of the torus where it acts.

- Consider the Dp-Dp string states with $p = 9, 5_i$. Bosonic states do not transform under h. To ensure totally invariant states, their relative λ should also not transform:

$$\psi^M_{-1/2}|0, ab\rangle\lambda_{ab} \quad : \quad \lambda = \gamma_{h,p}\lambda\gamma_{h,p}^{-1} \, . \tag{3.36}$$

  Space-time fermionic states acquire a minus sign and their λ should also transform in this way:

$$|s_0 s_1 s_2 s_3, ab\rangle\lambda_{ab} \quad : \quad \lambda = -\gamma_{h,p}\lambda\gamma_{h,p}^{-1} \, . \tag{3.37}$$

- For the mixed 95_i states, space-time bosons do not transform, giving:

$$|s_j s_k, ab\rangle\lambda_{ab} \quad : \quad \lambda = \gamma_{h,9}\lambda\gamma_{h,5_i}^{-1} \, , \tag{3.38}$$

  where $j \neq i \neq k$ and GSO projection demands $s_j = s_k$. Space-time fermions acquire a minus sign that must be eliminated by the transformation of their relative λ:

$$|s_0 s_i, ab\rangle\lambda_{ab} \quad : \quad \lambda = -\gamma_{h,9}\lambda\gamma_{h,5_i}^{-1} \, , \tag{3.39}$$

  where GSO projection demands $s_0 = s_i$.

Having the transformation conditions of the Chan-Paton matrices, we need the analytic expressions of the γs to find the representations of the massless fields. Later, we will see that γ matrices obey some consistency conditions.

3.4 Annulus

To evaluate the 1-loop partition function, we have to include also the contribution of the new sector of the theory (the open strings). The 1-loop diagram of an open string is the *Annulus*. The annulus can be taken from the torus with the involution $z \to -\bar{z}$ and $z \to 1 - \bar{z}$ (Fig.4). The τ is purely imaginary and the τ_2 is the proper time of an open string that sweeps the annulus $\mathcal{A}$. However, there is a distinct horizontal choice that defines the proper time elapsed while a closed string propagates between the two boundaries $\tilde{\mathcal{A}}$. These boundaries are the D-branes that the open string ends on.

The Annulus amplitudes can be computed for all kinds of D-branes existing in the theory and the contribution of an element α is given by the trace formula:

$$\mathcal{A}_{IJ,\alpha} = Tr_{IJ}\left[\alpha\, q^{L_0}\right] \ ,$$

where now the trace is over all open string states attached between I and J D-branes. When there are no reflecting elements R in the theory, only D9-branes are necessary to cancel the RR and NSNS charges. However, when there are R_i-factors we need in addition D5$_i$-branes extended along the $R^4 \times T_i^2$ and sitting on the R_i-fixed points of the other tori. The contribution of an element α can be written in the form:

$$\mathcal{A}_\alpha = \left(Tr[\gamma_{\alpha,9}]^2 + Tr[\gamma_{\alpha,5_i}]^2\right) T\left[{}^{0}_{v_\alpha}\right] + 2Tr[\gamma_{v_\alpha,9}]Tr[\gamma_{\alpha,5_i}]T\left[{}^{g_i}_{v_\alpha}\right] \ . \tag{3.40}$$

To extract the tadpole contributions we need to perform a modular transformation to the transverse channel, $l = 1/2t$, and then take the limit $l \to \infty$ [27]. We can perform a similar factorization to the one that we already did for the Klein-Bottle (Fig.6) and evaluate the tadpoles for the different D-branes:

- for an element α such that: $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$:
  - If G contains no Z_2-factors, then the only contribution to the tadpoles in the annulus amplitude is coming from the 99 strings

$$(1_{NS} - 1_R)\Big(\mathrm{D9} \ \ \alpha\Big)^2 \ . \tag{3.41}$$

  - In the case where the group G contains R-factors, then we have also contributions from the corresponding D5-branes. As in the Klein Bottle case, we have the following cases:

    i. if $R_3 \in G$,

$$(1_{NS} - 1_R)\Big(\mathrm{D9} \ \ \alpha + \mathrm{D5}_3 \ \ \alpha\Big)^2 \ , \tag{3.42}$$

ii. if $R_i \in G$, for a given $i = 1$ or 2,

$$(1_{NS} - 1_R)\left(\mathrm{D9}\ \alpha + \mathrm{D5}_i\ \alpha\right)^2 , \tag{3.43}$$

iii. if $R_l \in G$, with $l = 1, 2, 3$

$$(1_{NS} - 1_R)\left(\mathrm{D9}\ \alpha + \sum_{l=1}^{3} \mathrm{D5}_l\ \alpha\right)^2 . \tag{3.44}$$

In all the above cases the general structure is again proportional to zero ($1_{NS} - 1_R$) and the multiplicative is a function of the volume of the unaffected torus:

$$(1_{NS} - 1_R)\left[A_1\sqrt{\mathcal{V}_3} + \frac{A_2}{\sqrt{\mathcal{V}_3}}\right]^2 . \tag{3.45}$$

The A_1 and A_2 are functions of the traces of the Chan-Paton factors, $Tr[\gamma_{\alpha,I}]$. The A_1 is the contribution of strings that are longitudinal to the torus which is unaffected by v_α (they have Neumann boundary conditions in this torus). Therefore, it is proportional to $Tr[\gamma_{\alpha,9}]$ and $Tr[\gamma_{\alpha,5_3}]$. The A_2 is the contribution of the strings that are transverse to $\mathcal{V}_3$ and it is a function of $Tr[\gamma_{\alpha,5_i}]$ for $i = 1, 2$.

- If now $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$, then the classification is similar to the Klein Bottle one:

  - If the orbifold group G has no Z_2 factors, we have just the contribution of the 99 strings.

$$(1_{NS} - 1_R)\left(\mathrm{D9}\ \alpha\right)^2 . \tag{3.46}$$

  - If the group G contains R factors, then:

    i. if $R_i \in G$ for a given i, we should include its corresponding $D5_i$-branes as well

$$(1_{NS} - 1_R)\left(\mathrm{D9}\ \alpha + \mathrm{D5}_i\ \alpha\right)^2 , \tag{3.47}$$

    ii. if $R_l \in G$ with $l = 1, 2, 3$, we should include its corresponding $D5_l$-branes

$$(1_{NS} - 1_R)\left(\mathrm{D9}\ \alpha + \sum_{l=1}^{3} \mathrm{D5}_l\ \alpha\right)^2 . \tag{3.48}$$

The structure of these amplitudes is similar to (3.45) without the volume dependance.

3.5 Möbius Strip

In general, we can expect that there are closed strings which propagate between the two sources of closed strings: the O-planes and the D-branes. The amplitude that describes such transmission is topologically a 1-loop amplitude and contributes at the same level in string perturbation theory as the Klein-bottle and the Annulus. It is the so called *Möbius Strip*.

The Möbius Strip can be taken from a double covered torus by the involution: $z \to 1-\bar{\tau}_2+i\tau_2$ (Fig.4). The parameter τ_2 describes the proper time elapsed while an open string sweeps the Möbius Strip $\mathcal{M}$. There is again an alternative choice where the horizontal parameter describes a closed string propagating between a boundary and a crosscap $\tilde{\mathcal{M}}$.

The contribution of an element α accompanied by Ω is:

$$\mathcal{M}_{I,\alpha} = Tr_I \left[\Omega\alpha \; q^{L_0}\right] \;, \tag{3.49}$$

where the trace is over open strings attached to a DI-brane. Finally, this contribution has the form:

$$\begin{aligned}\mathcal{M}_\alpha = - &\Big(Tr[\gamma^T_{\Omega\alpha,9}\gamma^{-1}_{\Omega\alpha,9}]T[{}^{\,0}_{v_\alpha}] + Tr[\gamma^T_{\Omega R_i\alpha,9}\gamma^{-1}_{\Omega R_i\alpha,9}]T[{}^{\;\,0}_{g_i v_\alpha}] \\ &+ Tr[\gamma^T_{\Omega\alpha,5_i}\gamma^{-1}_{\Omega\alpha,5_i}]T[{}^{\;\,0}_{g_i v_\alpha}] + Tr[\gamma^T_{\Omega g_i v,5_i}\gamma^{-1}_{\Omega g_i v,5_i}]T[{}^{\,0}_{v_\alpha}]\Big) \;,\end{aligned}$$

the overall minus sign is conventional. However, we should make the same choice of sign as for the identity element of G. To extract the tadpole conditions we must perform a modular transformation to the transverse channel of the form $P = TST^2ST$ where, $T : \tau \to \tau + 1$ and $S : \tau \to -1/\tau$, where in this case $l = 1/8t$. Finally, we take the UV limit $l \to \infty$ (Fig.4).

The Möbius strip transverse channel amplitude is the mean value between the transverse channel Klein Bottle and Annulus amplitudes [23, 24]. Therefore, comparing the UV limit of the Möbius strip amplitude (3.49) with the mean value of the UV limits of the Klein Bottle and Annulus amplitudes, we obtain the following constraints on the matrices $\gamma_{\alpha,I}$ and $\gamma_{\Omega.\alpha,I}$:

$$\begin{aligned}&Tr[\gamma^T_{\Omega\alpha,9}\gamma^{-1}_{\Omega\alpha,9}] = Tr[\gamma_{\alpha^2,9}] \;, \qquad Tr[\gamma^T_{\Omega R_i\alpha,9}\gamma^{-1}_{\Omega R_i\alpha,9}] = -Tr[\gamma_{\alpha^2,9}] \;, \\ &Tr[\gamma^T_{\Omega\alpha,5_i}\gamma^{-1}_{\Omega\alpha,5_i}] = -Tr[\gamma_{\alpha^2,5_i}] \;, \quad Tr[\gamma^T_{\Omega R_i\alpha,5_i}\gamma^{-1}_{\Omega R_i\alpha,5_i}] = Tr[\gamma_{\alpha^2,5_i}] \;, \\ &Tr[\gamma^T_{\Omega R_j\alpha,5_i}\gamma^{-1}_{\Omega R_j\alpha,5_i}] = -Tr[\gamma_{\alpha^2,5_i}] \;,\end{aligned} \tag{3.50}$$

where in the last equation $i \neq j$ and $i,j = 1,2,3$. These constraints appear for either $v_\alpha = (v^1_\alpha, v^2_\alpha, 0)$ or $v_\alpha = (v^1_\alpha, v^2_\alpha, v^3_\alpha)$.

3.6 Tadpole conditions

The massless part of the transverse channel amplitudes $\tilde{\mathcal{K}}_\alpha + \tilde{\mathcal{A}}_\alpha + \tilde{\mathcal{M}}_\alpha$ provide the tadpole conditions. Let us examine all the different cases for an element α^2 where:

- α is such that $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$:
  - If G contains no Z_2 factors:
$$Tr[\gamma_{\alpha^2,9}] = 32 \prod_l \cos \pi v_\alpha^l \ . \tag{3.51}$$
  - If G contains Z_2 factors, then we have the following cases:
    i. if $R_3 \in G$,
$$Tr[\gamma_{\alpha^2,9}] + 4\prod_l \sin 2\pi v_\alpha^l \ Tr[\gamma_{\alpha^2,5_3}] = 32 \left(\prod_l \cos \pi v_\alpha^l + \prod_l \sin \pi v_\alpha^l \right) , \tag{3.52}$$
    ii. if $R_i \in G$ for a given $i = 1$ or 2,
$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] &= 32 \ \prod_l \cos \pi v_\alpha^l \ , \\ 2 \sin 2\pi v_\alpha^j \ Tr[\gamma_{\alpha^2,5_i}] &= 32 \ \cos \pi v_\alpha^i \sin \pi v_\alpha^j \ . \end{aligned} \tag{3.53}$$
    iii. if $R_l \in G$ with $l = 1, 2, 3$,
$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] + 4\prod_l \sin 2\pi v_\alpha^l Tr[\gamma_{\alpha^2,5_3}] &= 32 \left(\prod_l \cos \pi v_\alpha^l + \prod_l \sin \pi v_\alpha^l \right) , \\ \sum_{i \neq j = 1,2} 2 \sin 2\pi v_\alpha^j Tr[\gamma_{\alpha^2,5_i}] &= 32 \sum_{i\neq j=1,2} \epsilon_{ij} \cos \pi v_\alpha^i \sin \pi v_\alpha^j \ . \end{aligned} \tag{3.54}$$
- α is such that $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$:
  - If G does not contain any Z_2 factors:
$$Tr[\gamma_{\alpha^2,9}] = 32 \ \prod_l \cos \pi v_\alpha^l \ . \tag{3.55}$$
  - If G does contain Z_2 factors, then:
    i. if $R_i \in G$ for a given i:
$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] \ + \ 4\prod_{l\neq i} \sin 2\pi v_\alpha^l \ Tr[\gamma_{\alpha^2,5_i}] \\ = 32 \left(\prod_l \cos \pi v_\alpha^l + \prod_l \sin \pi v_\alpha^l \right) , \end{aligned} \tag{3.56}$$
    ii. if $R_l \in G$ with $l = 1, 2, 3$:
$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] \ + \ 4\sum_{i=1}^{3} \prod_{l\neq i} \sin 2\pi v_\alpha^l Tr[\gamma_{\alpha^2,5_i}] \\ = 32 \left(\prod_l \cos \pi v_\alpha^l + \sum_i \cos \pi v_\alpha^i \prod_{l \neq i} \sin \pi v_\alpha^l \right) . \end{aligned} \tag{3.57}$$

Notice that in all these cases, the tadpole conditions hold for both NS and R sectors due to supersymmetry.

The tadpole condition for an element $\alpha \in G$ that is not the square of any other element of G (there is no element $\beta \in G$ such that $\alpha = \beta^2$), will receive contribution only from the Annulus amplitude. If this element is such that $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ or $g_3 v_\alpha$, tadpole conditions will be the same as before with zeros in the right hand side of (3.51-3.54). It is not difficult to work out the tadpole conditions for elements $g_i v_\alpha$:

$$\begin{aligned} Tr[\gamma_{R_i\alpha,9}] &+ 4\sin\pi v_\alpha^i \cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,5_3}] \\ &+ 2\cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,5_i}] + 2\sin\pi v_\alpha^i Tr[\gamma_{R_i\alpha,5_j}] = 0 \ , \end{aligned} \tag{3.58}$$

where it is understood that $i \neq j = 1,2$ and the different terms exist only if the corresponding Rs do. If $v_\alpha = (v_\alpha^1, v_\alpha^2, v_\alpha^3)$, the tadpole conditions are the same as (3.55-3.57) without the right hand side (*i.e.* the right hand side is zero).

In the next chapters, we will give some applications of the formulae we have obtained in this section and compare with the supersymmetric orientifolds already studied in the literature.

3.7 Breaking Supersymmetry with Scherk-Schwarz deformation

In this section we include a Z_2 Scherk-Schwarz (SS) deformation in order to break supersymmetry. Without loss of generality we consider that the translation h_2 of (3.10) acts on a direction of the third torus T_3^2. This deformation is compatible only with an orbifold action that commutes with it, therefore, we will restrict ourselves to elements a with rotation angles of the form $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ or $v_\alpha g_i$ where g_i is the rotation angle of R_i, a Z_2 element which leaves the coordinates of the T_i^2 torus invariant and gives a minus sign to the others, $i = 1, 2$.

3.7.1 Klein Bottle

The trace in $\mathcal{K}$, is taken over all states, and gives rise to a term coming from the zero modes:

$$\sum_{m,n} q^{\alpha' p_L^2/4} \bar{q}^{\alpha' p_R^2/4} \langle m, n|\Omega\alpha|m, n\rangle \ . \tag{3.59}$$

By (3.1), we realize that the h-twisted sector does not survive the Ω projection since it has shifted windings and Ω keeps only $n = 0$ states. This sector can survive iff there is R, a Z_2 element in G where, acting with Ω keeps the $n \neq 0$ states: $\Omega R\ |m, n\rangle \rightarrow\ |-m, n\rangle$. In this case, the invariant states are those with vanishing momenta, $m = 0$. Therefore, the h-twisted sector will survive this projection if h and R act in the same direction. It is easy to realize that R and Rh twisted fields generate O_5 and $\bar{O}_5$-planes sitting on the corresponding fixed points [21, 31].

To extract the massless tadpole contribution we need to perform a modular transformation, $l = 1/4t$, and then take $l \rightarrow \infty$, as in the previous section. In addition to (3.15), we will have extra contributions from the h and hR_i twisted sector:

$$T\big[{}^{h}_{2g_i v_\alpha}\big], \quad T\big[{}^{h}_{2g_i h v_\alpha}\big], \quad T\big[{}^{hg_j}_{2g_i v_\alpha}\big], \quad T\big[{}^{hg_j}_{2g_i h v_\alpha}\big] \ , \tag{3.60}$$

where $i \neq j = 1, 2$ ($T[{}^{a}_{b}]$ are provided in the appendix). These sectors contribute as $(1_{NS} + 1_R)$ to the tadpoles.

- If the orbifold group G does not contain a Z_2 element, the contribution to the Klein Bottle will come only from the untwisted sector as for the case without SS (3.16). There is no contribution from $\mathcal{K}_{h\alpha}$ due to the shift (it gives rise only to massive states).

- If the group G contains an R factor, we have also contributions from the twisted states.

  i. If $R_3 \in G$, the contribution is exactly as before (without SS (3.17)) because the Scherk-Schwarz deformation is acting transverse to the R_3 factor and so the twisted states by $R_3 h$ do not contribute.

  ii. if $R_i \in G$ for a given $i = 1$ or 2

$$\begin{aligned} &1_{NS}\left(\Omega\alpha \bigotimes\!\rightsquigarrow \alpha^2 + \Omega R_i\alpha \bigotimes\!\rightsquigarrow \alpha^2 + \Omega R_i h\alpha \bigotimes\!\rightsquigarrow \alpha^2\right)^2 \\ &- 1_R\left(\Omega\alpha \bigotimes\!\rightsquigarrow \alpha^2 + \Omega R_i\alpha \bigotimes\!\rightsquigarrow \alpha^2 - \Omega R_i h\alpha \bigotimes\!\rightsquigarrow \alpha^2\right)^2 \end{aligned} \tag{3.61}$$

  iii. if $R_l \in G$ with $l = 1, 2, 3$.

$$\begin{aligned} 1_{NS}\Bigg(&\Omega\alpha \bigotimes\!\rightsquigarrow \alpha^2 + \Omega R_3\alpha \bigotimes\!\rightsquigarrow \alpha^2 \\ &+ \sum_{i=1}^{2}\left(\Omega R_i\alpha \bigotimes\!\rightsquigarrow \alpha^2 + \Omega R_i h\alpha \bigotimes\!\rightsquigarrow \alpha^2\right)\Bigg)^2 \\ -1_R\Bigg(&\Omega\alpha \bigotimes\!\rightsquigarrow \alpha^2 + \Omega R_3\alpha \bigotimes\!\rightsquigarrow \alpha^2 \\ &+ \sum_{i=1}^{2}\left(\Omega R_i\alpha \bigotimes\!\rightsquigarrow \alpha^2 - \Omega R_i h\alpha \bigotimes\!\rightsquigarrow \alpha^2\right)\Bigg)^2 \end{aligned} \tag{3.62}$$

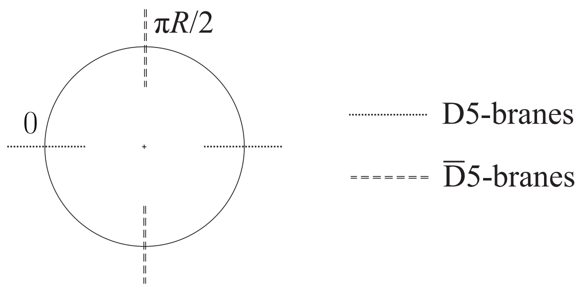
All these amplitudes are perfect squares as they should be. However, the cases (ii.) and (iii.) do not appear as $(1_{NS} - 1_R)$ any more. This dissimilarity of the coefficients of the NS and R oscillators is due to the effect of SS deformation and the breaking of supersymmetry via the term $\Omega R_i \alpha h$. All the amplitudes have the general form:

$$1_{NS}\left[K_{NS,1}\sqrt{\mathcal{V}_3} + \frac{K_{NS,2}}{\sqrt{\mathcal{V}_3}}\right]^2 - 1_R\left[K_{R,1}\sqrt{\mathcal{V}_3} + \frac{K_{R,2}}{\sqrt{\mathcal{V}_3}}\right]^2 \ , \tag{3.63}$$

where $K_{NS,2} \sim (1+1)f(v_\alpha)$, $K_{R,2} \sim (1-1)f(v_\alpha) = 0$ and $f(v_\alpha)$ is a function of the vector v_α. This explains the appearance of the factor of 2 in the NS sector in (3.61) and (3.62) and the absence of the factor proportional to $1/\sqrt{\mathcal{V}_3}$ in the R sector.

3.7.2 Annulus

To cancel these tadpoles one needs to add D9, $\mathrm{D5}_3$ and $\mathrm{D5}_i$-branes as well as $\mathrm{D5}_i$-antibranes in the case $R_i \in G$, with $i = 1, 2$, where the Scherk-Schwarz element h acts in the T_3^2 torus. The anti $\mathrm{D5}_i$-branes sit on the $R_i h$ fixed points [31].



The contribution from the annulus amplitudes to the tadpole conditions are the same as for the case without SS deformation, with in addition the anti-$\mathrm{D5}_{i\neq 3}$-brane sector when $R_i \in G$. Note that the annulus amplitudes between the same type of branes contribute as $(1_{NS} - 1_R)$ whereas, the ones between a D-brane and an anti-D-brane give $(1_{NS} + 1_R)$. The contribution of the element α in the Annulus amplitudes is:

- When G does not contain Z_2 factors[11], the contribution is the same as (3.41).

- When G contains Z_2 factors, then:

  i. if $R_3 \in G$, the contribution is the same as (3.42).

  ii. if $R_i \in G$ for a given $i = 1$ or 2:

$$1_{NS}\left(\mathrm{D9}\;\alpha + \mathrm{D5}_i\;\alpha + \overline{\mathrm{D5}}_i\;\alpha\right)^2$$
$$- \;1_R\left(\mathrm{D9}\;\alpha + \mathrm{D5}_i\;\alpha - \overline{\mathrm{D5}}_i\;\alpha\right)^2 \qquad (3.64)$$

[11]what we mean by Z_2 factors are those other than the Scherk-Schwarz deformation h.

iii. if $R_l \in G$ with $l = 1, 2, 3$:

$$\begin{aligned} & 1_{NS}\Bigg(\mathrm{D9}\;\alpha + \mathrm{D5}_3\;\alpha \\ & \qquad\qquad + \sum_{i=1}^{2}\left(\mathrm{D5}_i\;\alpha + \overline{\mathrm{D5}}_i\;\alpha\right)\Bigg)^2 \\ & -1_R\Bigg(\mathrm{D9}\;\alpha + \mathrm{D5}_3\;\alpha \\ & \qquad\qquad + \sum_{i=1}^{2}\left(\mathrm{D5}_i\;\alpha - \overline{\mathrm{D5}}_i\;\alpha\right)\Bigg)^2 \end{aligned} \tag{3.65}$$

The massless contribution from the above amplitudes is not proportional to $(1_{NS} - 1_R)$ as in the supersymmetric case. The general form is a function of the volume of the unaffected torus:

$$1_{NS}\left[A_{NS,1}\sqrt{\mathcal{V}_3} + \frac{A_{NS,2}}{\sqrt{\mathcal{V}_3}}\right]^2 - 1_R\left[A_{R,1}\sqrt{\mathcal{V}_3} + \frac{A_{R,2}}{\sqrt{\mathcal{V}_3}}\right]^2 . \tag{3.66}$$

The general A_is are again functions of the traces of the Chan-Paton factors, $Tr[\gamma_{\alpha,I}]$. The $A_{NS,1}$ and $A_{R,1}$ are proportional to $Tr[\gamma_{\alpha,9}]$ and $Tr[\gamma_{\alpha,5_3}]$. The $A_{NS,2}$ and $A_{R,2}$ are proportional to $Tr[\gamma_{\alpha,5_i}]$ and $Tr[\gamma_{\alpha,\bar{5}_i}]$, for $i = 1, 2$. This is precisely the effect of the anti $\overline{\mathrm{D}}$-branes.

3.7.3 Möbius Strip

Finally, the Möbius strip amplitude that is derived in two inequivalent ways (as a direct amplitude and as the mean value of the Klein Bottle and the Annulus amplitudes) leads to the same constraints as before (3.50), where in addition:

- if $R_i \notin G$, then:

  i. if $R_3 \notin G$:

$$Tr[\gamma^T_{\Omega h\alpha,9}\gamma^{-1}_{\Omega h\alpha,9}] = \pm Tr[\gamma_{\alpha^2,9}] , \tag{3.67}$$

  ii. if $R_3 \in G$:

$$Tr[\gamma^T_{\Omega h\alpha,5_3}\gamma^{-1}_{\Omega h\alpha,5_3}] = \pm Tr[\gamma_{\alpha^2,5_3}] . \tag{3.68}$$

  The signs are the same for the $D9$ and $D5_3$ sectors due to T-duality. Examples of this cases have been discussed in [34].

- If $R_i \in G$, for a given $i = 1$ or 2:

$$Tr[\gamma^T_{\Omega h\alpha,I}\gamma^{-1}_{\Omega h\alpha,I}] = Tr[\gamma_{\alpha^2,I}], \quad I = 9, 5_3, 5_i, \bar{5}_i . \tag{3.69}$$

  In all these cases $\gamma^2_{R,I} = -1$, with $I = 9, 5_l, \bar{5}_i$ for all Rs.

3.7.4 Tadpole conditions

The tadpole conditions for an element α such that $v_\alpha = (v_\alpha^1, v_\alpha^2, 0)$ are classified as:

- G contains no Z_2 factors:

$$Tr[\gamma_{\alpha^2,9}] = 32 \prod_l \cos \pi v_\alpha^l \ , \tag{3.70}$$

  the tadpole condition is the same as in the case without SS deformation (3.51).

- G contains Z_2 factors:

  i. if $R_3 \in G$:

$$\begin{aligned} Tr[\gamma_{\alpha^2,9}] \, + \, 4 \prod_l \sin 2\pi v_\alpha^l \ Tr[\gamma_{\alpha^2,5_3}] & \\ = 32 \left(\prod_l \cos \pi v_\alpha^l + \prod_l \sin \pi v_\alpha^l \right) \ , \end{aligned} \tag{3.71}$$

  ii. if $R_i \in G$ for a given $i = 1$ or 2:

$$\begin{aligned} & Tr[\gamma_{\alpha^2,9}] = 32 \ \prod_l \cos \pi v_\alpha^l \\ 1_{NS}: \quad & 2 \sin 2\pi v_\alpha^j \ \left(Tr[\gamma_{\alpha^2,5_i}] + Tr[\gamma_{\alpha^2,\bar{5}_i}] \right) = 32 \ \cos \pi v_\alpha^i \ \sin \pi v_\alpha^j \ , \\ 1_R: \quad & 2 \sin 2\pi v_\alpha^j \ \left(Tr[\gamma_{\alpha^2,5_i}] - Tr[\gamma_{\alpha^2,\bar{5}_i}] \right) = 0 \ , \end{aligned} \tag{3.72}$$

  iii. if $R_l \in G$ with $l = 1, 2, 3$:

$$\begin{aligned} & Tr[\gamma_{\alpha^2,9}] + 4 \prod_l \sin 2\pi v_\alpha^l Tr[\gamma_{\alpha^2,5_3}] = 32 \left(\prod_l \cos \pi v_\alpha^l + \prod_l \sin \pi v_\alpha^l \right) \\ 1_{NS}: \quad & 2 \sum_{i \neq j = 1,2} \sin 2\pi v_\alpha^j \left(Tr[\gamma_{\alpha^2,5_i}] + Tr[\gamma_{\alpha^2,\bar{5}_i}] \right) \\ & \qquad = 32 \sum_{i \neq j = 1,2} \epsilon_{ij} \ \cos \pi v_\alpha^i \sin \pi v_\alpha^j \ , \\ 1_R: \quad & 2 \sum_{i \neq j = 1,2} \sin 2\pi v_\alpha^j \left(Tr[\gamma_{\alpha^2,5_i}] - Tr[\gamma_{\alpha^2,\bar{5}_i}] \right) = 0 \ . \end{aligned} \tag{3.73}$$

Finally, there could be elements that cannot be expressed as the square of any other element in G, these elements will not receive contribution from the Klein Bottle amplitude. For such elements the tadpole conditions are the same as in (3.70-3.73) with zero on the right hand side. For the elements $h\alpha$ and $R_3 h\alpha$ the tadpole conditions are as in (3.70) and (3.71) and because $D5_i$ and $D\bar{5}_i$ are transverse to the direction where h acts, there are no conditions on $Tr[\gamma_{h\alpha,5_i}]$ and $Tr[\gamma_{h\alpha,\bar{5}_i}]$. For the element $R_i\alpha$ the tadpole condition is:

$$\begin{aligned} Tr[\gamma_{R_i\alpha,9}] &+ 4\sin\pi v_\alpha^i \cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,5_3}] \\ &+ 2\cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,5_i}] + 2\sin\pi v_\alpha^i Tr[\gamma_{R_i\alpha,5_j}] = 0 \ , \end{aligned} \tag{3.74}$$

where the D5_i-antibranes do not contribute because they do not sit on the fixed points of this element. This condition is valid for both NS and R sectors (the contribution is proportional to $(1_{NS} - 1_R)$). For the element $R_ih\alpha$ we find:

$$\begin{aligned} 1_{NS}: \quad Tr[\gamma_{R_ih\alpha,9}] &+ 4\sin\pi v_\alpha^i \cos\pi v_\alpha^j Tr[\gamma_{R_ih\alpha,5_3}] \\ &+ 2\cos\pi v_\alpha^j Tr[\gamma_{R_i\alpha,\bar{5}_i}] + 2\sin\pi v_\alpha^i Tr[\gamma_{R_ih\alpha,\bar{5}_j}] = 0 \ , \\ 1_R: \quad Tr[\gamma_{R_ih\alpha,9}] &+ 4\sin\pi v_\alpha^i \cos\pi v_\alpha^j Tr[\gamma_{R_ih\alpha,5_3}] \\ &- 2\cos\pi v_\alpha^j Tr[\gamma_{R_ih\alpha,\bar{5}_i}] - 2\sin\pi v_\alpha^i Tr[\gamma_{R_ih\alpha,\bar{5}_j}] = 0 \ , \end{aligned} \tag{3.75}$$

where the $D5_i$-branes do not contribute because they do not sit on the fixed points of $R_ih\alpha$.

3.8 Solving the tadpole conditions

A simple way to impose the tadpole conditions on the Chan-Paton matrices λ is to recast them in a Cartan-Weyl basis. In this case, constraints on the λs will emerge as restrictions on weight vectors [28].

Suppose that we imply some constraints on the λs and we find that they are constrained to be generators of a specific Lie-algebra. Therefore, they can be organized into charged generators: $\lambda_a = E_a$ and Cartan generators, $\lambda_I = H_I$ such that:

$$[H_I, E_a] = \rho_I^a E_a \ , \tag{3.76}$$

where ρ_I^a the roots associated to the generators E_a of the Lie-algebra. The matrix γ_α and its powers represent the action of the orientifold on the Chan-Paton factors, and they correspond to elements of a discrete subgroup of the abelian group spanned by the Cartans. Hence, we can write:

$$\gamma_\alpha = e^{-2\pi i V_\alpha \cdot H} \ , \tag{3.77}$$

where the *shift* vector V_α has the dimension of the number of the Cartan matrices of the initial Lie-group. Different elements have different shift vectors that are defined by the relevant tadpole conditions.

Recalling the formula $e^{-B}Ae^{B} = \sum_{n=0}^{\infty}[A,B]_n$, with $[A,B]_{n+1} = [[A,B]_n, B]$ and $[A,B]_0 = A$, and using (3.76), it is easy to show that $\gamma_\alpha E_a \gamma_\alpha^{-1} = e^{-2\pi i \rho_a \cdot V_\alpha} E_a$. All the equations that provide the massless spectrum can be expressed in the following way:

$$\rho_a \cdot V_{pq}^\alpha = f_{pq} \ , \tag{3.78}$$

where "f_{pq}" is a number associated with the transformation of the various strings that are stretched between Dp-Dq branes. Notice the difference between 99, $5_i 5_i$ and 95_i strings:

$$f_{99,5_i5_i} = \begin{cases} 0+k & \text{for vectors} \\ v^i_\alpha + k & \text{for scalars } \psi_i \\ s\cdot v_\alpha + k & \text{for fermions} \end{cases} \quad , \; f_{95_i} = \begin{cases} s_j v_j + s_l v_l + k & \text{for scalars} \\ s_i v_i + k & \text{for fermions} \end{cases} \tag{3.79}$$

where $k \in \mathbb{Z}$. In the next section, we will be more precise by exploring the: $SO(N)$, $U(N)$ and $USp(N)$ algebras which always appear in orientifold constructions.

Orientifolds with commuting γs

Consider the action of Ω on 99 states. The parity transformation is represented by a symmetric $\gamma_{\Omega,9}$ matrix. The constraint, $\lambda = -\gamma_{\Omega,9}\lambda\gamma^{-1}_{\Omega,9}$, restricts the original 32×32 matrices λ to be generators of the $SO(32)$ algebra. Therefore, the Cartan will be $H^I_{ij} = \delta_{i,2I}\delta_{j,2I} - \delta_{i,2I+1}\delta_{j,2I+1}$[12]. The roots ρ_α have the form $(\underline{\pm1,\pm1,0,\ldots,0})$, where the underlining indicates that all possible permutations must be considered.

Every rotation element that commutes with $\gamma_{\Omega,9}$ can be written in the form (3.77). We have mentioned already that $\gamma^N_\alpha = \pm1$ [28]. For even elements γ_α only the minus sign is allowed, and the shift vector can have the general form:

$$V_\alpha = \frac{1}{2N}\Big(1,\ldots,1,3,\ldots,3,\ldots\ldots,N-1,\ldots,N-1\Big)\ . \tag{3.80}$$

The number of the entries is determined by the tadpole conditions. In the case where there are two commuting rotation elements that commute also with $\gamma_{\Omega,9}$, they can both be expressed in the form of (3.80). However, we should be careful that the mixed tadpole conditions between the commuting elements are satisfied.

Consider now the action of Ω on 55 states. The parity transformation is represented by an antisymmetric $\gamma_{\Omega,5}$ matrix. The constraint on λ restricts the original 32×32 matrices λ to be generators of the $USp(32)$ algebra. The roots ρ_α have the same form as the ones above: $(\underline{\pm1,\pm1,0,\ldots,0})$, however, we have to add some extra long ones: $(\underline{\pm2,0,\ldots,0})$. Whenever, the D5-branes are on top of the fixed points, the long roots are projected out. If all D5-branes sit at the same fixed point, we can take $V_{55} = V_{99}$, giving the same spectrum for both cases.

The 95 sector is handled using an auxiliary $SO(64) \supset SO(32)_{(99)} \otimes SO(32)_{(55)}$ algebra. Since we have generators acting simultaneously on both D9 and D5 branes, only roots of the form:

$$\rho_{(95)} = \rho_{(9)} \otimes \rho_{(5)} = (\underline{\pm1,0,\ldots,0};\underline{\pm1,0,\ldots,0})\ , \tag{3.81}$$

must be considered. The shift vector is defined as $W_{(95)} = V_{(9)} \otimes V_{(5)}$.

[12] We have normalize the Cartans as $Tr[H_I H_J] = 2\delta_{IJ}$.

Each commuting element gives an extra contribution to the spectrum. Consider an orientifold of the type $Z'_{2N} = Z_2 \times Z_N$, where Z_N commutes with the Z_2. We can consider the direct shift vector V_{2N} and evaluate the massless spectrum. However, we will separate and study the action of the Z_N on the spectrum created by Z_2:

- We will first evaluate the spectrum of a single Z_2 element that is acting as $v_2 = (1/2, -1/2, 0)$ and $V = \frac{1}{4}(1, 1, \ldots, 1)$. Using the technique that we describe above, we find that the Z_2 orientifold has gauge group $U(16)$. The Cartans of $U(16)$ are the same as the ones of the $SO(32)$, however, the roots of the $SO(32)$ that give the adjoint of the $U(16)$ are only the: $(\underline{+1, -1, 0, \ldots})$.

  There are scalar fields, $\psi^1_{1/2}|0\rangle$, $\psi^2_{1/2}|0\rangle$ in the $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ and $\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$, associated to the $(\underline{+1, +1, 0, \ldots})$ and $(\underline{-1, -1, 0, \ldots})$ roots of the initial $SO(32)$, respectively. There are also scalars, $\psi^3_{1/2}|0\rangle$ in the adjoint.

- On top of that, we have to act with an extra Z_N element that acts as $v_N = (0, -1/N, 1/N)$ (without loss of generality). The new gauge group will be given by the condition $\rho V_N = 0 \mathrm{mod} \mathbb{Z}$ where $\rho = (\underline{+1, -1, 0, \ldots})$. Similarly, scalars $\psi^1_{1/2}|0\rangle$, $\psi^2_{1/2}|0\rangle$ will have $\rho V_N = 0 \bmod \mathbb{Z}$, $\rho V_N = -1/N \bmod \mathbb{Z}$, where the roots are $(\underline{+1, +1, 0, \ldots})$ and $(\underline{-1, -1, 0, \ldots})$ that gave the antisymmetric reps in the Z_2 case. Finally, the $\psi^3_{1/2}|0\rangle$, will have $\rho V_N = 1/N \bmod \mathbb{Z}$ where again $\rho = (\underline{+1, -1, 0, \ldots})$. Similarly for the 55 and 59 sectors.

Rotation elements that commute with γ_Ω and have $\gamma_\alpha^N = +1$, have shift vector in the general form:

$$V_\alpha = \frac{1}{N}\Big(0, \ldots, 0, 1, \ldots, 1, \ldots\ldots, (N-1)/2, \ldots, (N-1)/2\Big) \,. \tag{3.82}$$

The number of the entries is again determined by the tadpole conditions.

Orientifolds with non-commuting γs

In the previous section, we studied the action of various commuting elements on the λs. We showed that we can use the shift vectors to evaluate the spectrum. Any extra condition breaks the representations further.

When we have non commuting γs we cannot apply directly the above method since we cannot diagonalize all γ matrices together. Models with non-commuting elements contain $Z_2 \times Z_2$ as a subgroup. The $Z_2 \times Z_2$ orientifold contains three R_i reflecting elements (where $i = 1, 2, 3$) that each generate different D5_i-branes. The γ_{R_i}s do not commute since all of them should have $\gamma^2_{R_i} = -1$. After some tedious calculations we find that the gauge group is $USp(16)$ for all branes. There are also scalars in the antisymmetric rep of $USp(16)$ [29]. 95_i and 5_i5_j states transform in bifundamental representations.

Having the spectrum of $Z_2 \times Z_2$, we can apply extra shift elements on it. We will use as a basis the Cartan and the roots of the $USp(16)$. As an example, we will consider the $Z_2 \times Z_6$ (which is equivalent to $Z_2 \times Z_2 \times Z_3$) orientifold. We will act just with the shift vector of Z_3 and we will use the proper roots for each field. The shift vector in this case will be:

$$V_\alpha = \frac{1}{N}\Big(0, \ldots, 1, \ldots\ldots, (N-1)/2, \ldots, (N-1)\ldots\Big)\ , \tag{3.83}$$

where $N = 3$ [13]. To find the gauge group of $Z_2 \times Z_6$, we will use the roots of $USp(16)$ and for the scalars we will remove the long roots.

Scherk-Schwarz deformation

The action of Scherk-Schwarz deformations on open strings is similar to the action of rotation elements.

The γ_h can in general be $\gamma_h^2 = \pm 1$. A generic choice for these two cases is:

$$V_h = \frac{1}{4}\begin{cases} (1_a, -1_b) & \text{for } \gamma_h^2 = -1, \\ \\ (2_a, 0_b) & \text{for } \gamma_h^2 = +1, \end{cases} \tag{3.84}$$

where the index refers to the number of the same components in the vector. In the case where $\gamma_h^2 = -1$, we have $a = b$, however, there is no constraint for $\gamma_h^2 = +1$. The related f_h for the Scherk-Schwarz deformation is just:

$$f_h = \begin{cases} 0 + k & \text{for spacetime bosons,} \\ 1/2 + k & \text{for spacetime fermions,} \end{cases} \tag{3.85}$$

where again $k \in \mathbb{Z}$.

3.9 Applications

As we mention above, we can simplify the initial problem of finding the reps of the orientifold group by using the proper shift vector $V = \{V_i\}$ with number of identical entries n_i (3.80,3.82).

Even elements

Consider a shift vector of an even element where $\gamma^N = -1$. By the definition (3.80), we have: $V_i = (2i-1)/2N$. Therefore, the massless spectrum will be in general:

- Vectors in: $\prod_i U(n_i)$.

[13]Notice that this shift vector is the same with the one provided in (3.82) upon rotation. For example the "1"s and "N-1"s can be identified upon rotation. The reason for this choice is that tadpole conditions between different elements cannot be satisfied with all elements in the form (3.80, 3.82).

- Scalars $\psi^I_{-1/2}|0\rangle$ in: $(n_i, n_{1-i+f_{99}N})$, $(n_i, \overline{n}_{i-f_{99}N})$, $(\overline{n}_i, \overline{n}_{1-i-f_{99}N})$.

Notice that there will be antisymmetric reps iff $2i-1 = f_{99}N$. Similarly for the fermions.

According to (3.79), 55 states states form similar reps. The 95_i states have:

- Scalars $|s_j, s_k\rangle$ in: $(n_i, \tilde{n}_{1-i+f_{95}N})$, $(n_i, \overline{\tilde{n}}_{i-f_{95}N})$, $(\overline{n}_i, \tilde{n}_{i+f_{95}N})$, $(\overline{n}_i, \overline{\tilde{n}}_{1-i-f_{95}N})$.

- Fermions $|s_0, s_i\rangle$ in: $(n_i, \tilde{n}_{1-i+s_iv_iN})$, $(n_i, \overline{\tilde{n}}_{i-s_iv_iN})$, $(\overline{n}_i, \tilde{n}_{i+s_iv_iN})$, $(\overline{n}_i, \overline{\tilde{n}}_{1-i-s_iv_iN})$.

Odd elements

Consider a shift vector of an odd element where $\gamma^N = 1$. By the definition (3.82), we have: $V_i = (i-1)/N$. Therefore, the massless spectrum will be in general:

- Vectors in: $A \times \prod_{i\neq 1}^{(N+1)/2} U(n_i)$. Where $A = \{SO(n_1), USp(n_1)\}$ depending on the existence of commuting or non-commuting Z_2 elements.

- Scalars $\psi^I_{-1/2}|0\rangle$ in: $(n_i, n_{2-i+f_{99}N})$, $(n_i, \overline{n}_{i-f_{99}N})$, $(\overline{n}_i, \overline{n}_{2-i-f_{99}N})$.

Representations of $SO(n_1)$ or $USp(n_1)$ appear as $n_1 + \bar{n}_1$ that represent the vector $n_{1,v}$. There will be antisymmetric reps in the $U(n_i)$ iff $2i-2 = v_I N$. Similarly for the fermions.

In case there are D5-branes, 55 states form similar reps to the above. The 95_i states have:

- Scalars $|s_j, s_k\rangle$ in: $(n_i, \tilde{n}_{2-i+f_{95}N})$, $(n_i, \overline{\tilde{n}}_{i-f_{95}N})$, $(\overline{n}_i, \tilde{n}_{i+f_{95}N})$, $(\overline{n}_i, \overline{\tilde{n}}_{2-i-f_{95}N})$.

- Fermions $|s_0, s_i\rangle$ in: $(n_i, \tilde{n}_{2-i+s_iv_iN})$, $(n_i, \overline{\tilde{n}}_{i-s_iv_iN})$, $(\overline{n}_i, \tilde{n}_{-i+s_iv_iN})$, $(\overline{n}_i, \overline{\tilde{n}}_{2-i-s_iv_iN})$.

Scherk-Schwarz deformation

Scherk-Schwarz deformation commutes with each rotation element. Therefore, we can represent the γ_hs with a shift vector (3.85). In general, each component of the rotating shift vector can have different components of the SS deforming vector. For example, consider V_h where $\gamma_h^2 = -1$. Components V_i will split $V_i \to V_i^1 + V_i^2$ with $n_i = n_i^1 + n_i^2$. The components of the SS deformation will be: $V_i^h \to V_i^{h,1} + V_i^{h,2}$ where $V_i^{h,1} = -V_i^{h,2} = 1$. Following the same spirit, we realize that the action of the SS deformation breaks the representations. We can summarize by considering a representation:

$$(m,n)_+ \to \begin{cases} (m_1, n_1) + (m_2, n_2) & \text{bosons,} \\ (m_1, n_2) + (m_2, n_1) & \text{fermions,} \end{cases} \tag{3.86}$$

$$(n,m)_- \to \begin{cases} (m_1, n_2) + (m_2, n_1) & \text{bosons,} \\ (m_1, n_1) + (m_2, n_2) & \text{fermions,} \end{cases} \tag{3.87}$$

where the index $+,-$ denote the $\gamma_h^2 = \pm 1$ and m, n are both in fundamental or antifundamental reps. The bifundamental reps split for both $\gamma_h^2 = \pm 1$, as follows:

$$(m, \overline{n})_{\pm} \rightarrow \begin{cases} (m_1, \overline{n}_1) + (m_2, \overline{n}_2) & \text{bosons,} \\ (m_1, \overline{n}_2) + (m_2, \overline{n}_1) & \text{fermions,} \end{cases} \tag{3.88}$$

Therefore, the effect of the SS deformation on the open strings in a given supersymmetric model is to break the gauge group for $\gamma_h^2 = -1$ as

$$U(N) \rightarrow U(n) \times U(N-n) \ , \qquad SO(2N) \rightarrow U(N) \ , \tag{3.89}$$

whereas for $\gamma_h^2 = +1$ as

$$U(N) \rightarrow U(n) \times U(N-n) \ , \qquad SO(N) \rightarrow SO(n) \times SO(N-n) \ . \tag{3.90}$$

3.9.1 Some specific examples

Supersymmetric $T^2 \times K3$

The first example of groups are supersymmetric models with $G = Z_N$ for $N = 2, 3, 4, 6$ acting on T^4 [27]. The tadpole conditions are given by (3.70-3.71) with $v_\alpha^1 = -v_\alpha^2 = k/N$, $v_\alpha^3 = 0$ leading for odd N:

$$Tr[\gamma_{2k,9}] = 32 \cos^2 \frac{\pi k}{N} \ ,$$

whereas for even N:

$$\begin{aligned} Tr[\gamma_{2k,9}] - 4 \ \sin^2 \frac{2k\pi}{N} \ Tr[\gamma_{2k,5_3}] &= 32 \ \cos \frac{2k\pi}{N} \ , \\ Tr[\gamma_{2k-1,9}] - 4 \ \sin^2 \frac{(2k-1)\pi}{N} \ Tr[\gamma_{2k-1,5_3}] &= 0 \ . \end{aligned}$$

Solving these equations, we find γs and by the (3.3.1) we find the gauge group and massless spectrum of these models which are provided in the appendix I.

Non-supersymmetric $T^2 \times K3$

Next, consider an orientifold of the type $G = Z_N \times Z_2'$. The extra Z_2' is a freely acting SS deformation h which acts in a transverse circle of T^4/Z_N and breaks supersymmetry spontaneously.

Upon projecting this orbifold by the world sheet parity Ω, the massless limit of the tree channel Klein Bottle amplitude has non-vanishing RR tadpoles and thus reveals the presence of orientifold planes in the background. Besides the $O9$-plane that extends in the non-compact directions, wraps the $T^2 \times T^4$ and it is present for any N, for even N the model contains also $O5$-planes that extend along the non-compact directions, wrap around the T^2 and sit at the α^k-fixed points of the transverse T^4. In order to cancel the associated to the orientifold planes massless tadpoles one has to introduce D9 and D5-branes. The contribution of the D-branes to the tadpoles is encoded in the massless limit of the transverse channel Annulus and Möbius strip amplitudes.

The matrices $\gamma_{1,9}$ and $\gamma_{1,5}$ that correspond to the identity element of $Z_N \times Z_2'$ can be chosen to be the 32×32 identity matrices, so that $Tr[\gamma_{1,9}] = Tr[\gamma_{1,5}] = 32$. This is a constraint on the number of D-branes that originates from tadpole cancellation in the untwisted sector. The twisted tadpole conditions on the other hand in the α^k twisted sector, for N even are given by [27]

$$Tr[\gamma_{\alpha^{2k-1},9}] - 4\sin^2\frac{(2k-1)\pi}{N}\; Tr[\gamma_{\alpha^{2k-1},5}] = 0\ , \tag{3.91}$$

$$Tr[\gamma_{\alpha^{2k},9}] - 4\sin^2\frac{2\pi k}{N}\; Tr[\gamma_{\alpha^{2k},5}] - 32\cos\frac{2\pi k}{N} = 0\ , \tag{3.92}$$

whereas for N odd they read

$$Tr[\gamma_{\alpha^{2k},9}] - 32\cos^2\frac{\pi k}{N} = 0\ . \tag{3.93}$$

From the $\alpha^k h$ and h twisted sectors we do not get further constraints on $Tr[\gamma_{\alpha^k h,9}]$, $Tr[\gamma_{\alpha^k h,5}]$, $Tr[\gamma_{h,9}]$ and $Tr[\gamma_{h,5}]$. Notice that for N even, the tadpole conditions are consistent with T-duality transformations along the T^4 torus that exchanges the $D9$ and $D5$-branes. On the other hand, for the circle along which the shift is performed, we have a freedom in taking $\gamma_{h,9}^2 = \pm 1$ and also $\gamma_{h,5}^2 = \pm 1$, however T-duality constrains them to have the same sign. In summary, we will obtain two open string spectra for each N, related by Wilson lines.

Let us describe the massless spectrum starting from the closed string sector. The closed string spectra of the supersymmetric T^4/Z_N orientifolds have been computed in [27]. Sectors twisted by h do not contribute to the massless part of the Torus and the Klein-Bottle since they correspond to half integer winding [24]. Every other massless sector in the Torus is the same as in the corresponding supersymmetric model [14] plus an identical sector where the sign of the fermions is reversed. This simply means that h projects out the fermions altogether from the closed string sector. The bosons remain multiplied by a factor of two which is cancelled by the 1/2 of the h-projector $(1 + h)/2$ in the trace. The Klein-Bottle on the other hand remains the same as in the corresponding supersymmetric model. The extra 1/2 from the h-projector is now cancelled by a factor of two coming from the doubling of the surviving the Ω projection states, since any sector and its projected by h counterpart give the same contribution to the Klein-Bottle. The closed string spectrum therefore for any N is just the bosonic part of the corresponding supersymmetric model compactified on a T^2 torus. The full open string spectrum will be presented in the appendix (I) for each

[14] By corresponding supersymmetric model we simply mean the model obtained by eliminating the SS part, which is supersymmetric for all values of N discussed here.

value of N considered here. It is easy to check that the spectrum do not suffer from irreducible gauge anomalies. This is due to the fact that all fermions are in vector like representations. Alternatively, the models we have considered are effectively five dimensional and therefore do not have anomalies.

3.10 Chapter Summary

In this chapter we give an introduction to the orientifolds and we explore the breaking of supersymmetry by the Scherk-Schwarz deformation. We give general formulae for the tadpole conditions and we provide the general form of the massless spectrum.

4. D-brane realization of the Standard Model and anomalies

One of the important motivations in favor of string theory is the fact that it seems to include in principle all the ingredients required to embed the Standard Model (SM) inside a full unified theory with gravity. A standard approach that tries to embed the SM into string theory is the so called *top-down approach*. One starts by a string theory and tries to reduce the number of dimensions, supersymmetries and the gauge group by an appropriate orientifold compactification leading to a massless spectrum as similar as possible to the SM.

Lately, the string theories that are analyzed are open string theories (orientifolds) where the SM gauge group can be obtained from the D-branes. A low string scale compatible with the known value of the Planck scale can be easily accommodated in ground states of unoriented open and closed strings. Solvable vacua of this type are orientifolds of closed strings. Such vacua include various type of D-branes stretching their worldvolumes in the four non-compact dimensions while wrapping additional worldvolume dimensions around cycles of the compact six torus. Moreover, they include non-dynamical orientifold planes that cancel the charges of the D-branes, implementing the (un)orientability condition and stabilizing the vacuum (cancellation of tadpoles).

Since masses of open strings are proportional to their lengths, it is obvious that the branes that give rise to the SM fields must be very close together in the internal space. Thus, we can talk about the local group of SM D-branes and we may focus our discussion on this. The presence of other branes further away may affect global rather than local properties of the model (but can be important for the overall stability of the configuration).

As we mention before, the standard relation between the string scale and the Planck scale, namely $M_P^2 = \frac{V_6}{g_s^2} M_s^2$ implies that the internal volume must be very large in string units. The hierarchy problem in this context is the question of what stabilizes the value of $V_6 \gg 1$. No compelling answer exists to this question so we will bypass it and move on. However, the possibility of low string scale M_s [16] in these theories and supersymmetry breaking at that scale without suffering directly from the ordinary hierarchy problem of the scalar masses makes these theories particularly interesting. If the string scale is around a few TeV, observation of novel effects at the near future experiments becomes a realistic possibility.

The minimal D-brane configuration that can successfully accommodate the SM gauge group consists of three sets of branes[15] with gauge symmetry $U(3) \times U(2) \times U(1)$[16]. The SM particles are considered as open string states attached on different

[15] As we mentioned above, N coincident D-branes typically generate a Unitary group $U(N)$.

[16] Bottom to top model building shows that we have to introduce another single D-brane which provides an extra $U(1)'$ gauge boson [41]. However, we can omit this extra brane for the rest of our studies since it does not participate to the hypercharge

stacks of D-branes. Therefore, in these models the SM fields are open strings that are stretched onto a stack of 3, a stack of 2 and one brane (at least):

$$U(3) \times U(2) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1)_3 \times U(1)_2 \times U(1) \ . \tag{4.1}$$

Notice that every stack of branes supplies the model with extra abelian gauge fields. Such $U(1)$ fields have generically four-dimensional anomalies. In the rest of this chapter we will discuss about anomalies and the Green-Schwarz mechanism that cancels them.

4.1 Anomalies

Anomalies are generated when classical symmetries are broken at the quantum level [6, 7, 8]. There are *Global* and *Local* (Gauge) anomalies. Global anomalies contribute finitely to physical processes. As an example, the decay rate $\pi^0 \rightarrow \gamma\gamma$ that receive contribution from the anomalies providing the correct experimental number for three colored quarks.

Gauge anomalies afflict symmetries necessary to normalize the theory and they must be avoided. The longitudinal polarization of a gauge field related to them does not decouple. The axial Ward-identities contain an anomaly (axial current is not conserved) leading to inconsistences. Anomalies arise in Parity violating (chiral) theories. This means that left and right handed fermions do not transform in the same way under the gauge symmetry.

Consider an effective action of Dirac fermions coupled to gauge fields $\Gamma(A_\mu, \psi)$, that

$$e^{i\Gamma[A_\mu]} \sim \int [D\bar{\psi}][D\psi] \ \exp\left\{ - \int d^d x \bar{\psi} \left(\frac{1+\gamma^{d+1}}{2} \right) \not{D}\psi \right\} \ . \tag{4.2}$$

In general, we can evaluate the Anomalies by:

- Functional integration (which is the so-called "Fujikawa's method"):

  In this case, the anomalies appear as a phase factor due to the variation of the fermion measure $[D\bar{\psi}][D\psi]$. Therefore, the variation of the lagrangian of (4.2) does not vanish:

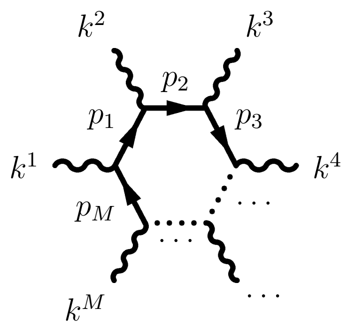
  $$\delta\mathcal{L} = \frac{\mathcal{A}_{a\ldots b\ldots}}{32\pi^2} \epsilon_{\mu\nu\ldots\rho\sigma\ldots} F_a^{\mu\nu} \ldots F_b^{\rho\sigma} \ldots \quad , \tag{4.3}$$

  where $\mathcal{A}_{a\ldots b\ldots}$ the anomaly.

- Directly from the Feynman-diagrams:

  In gauge theories, the longitudinal components of the associated external gauge field in physical processes should decouple to ensure unitarity. Therefore, one can take a diagram with on-shell external gauge fields and check whether the matrix elements with one polarization vector longitudinal and the rest transverse and physical vanishes or not.

Anomalies arise when some of these diagrams do not vanish. It has been shown that anomalies arise from parity-violation amplitudes since they contain an $\epsilon^{\mu\cdots\rho}$ tensor, which is coming from the trace of chiral fermions in the loops.



$$\sim \mathcal{A}\int d^dx\; Tr\left[\frac{i}{\not p_1}\not\zeta_1\frac{i}{\not p_2}\not\zeta_2\dots\frac{i}{\not p_M}\not\zeta_M\left(\frac{1+\gamma^{d+1}}{2}\right)\right]\;,$$

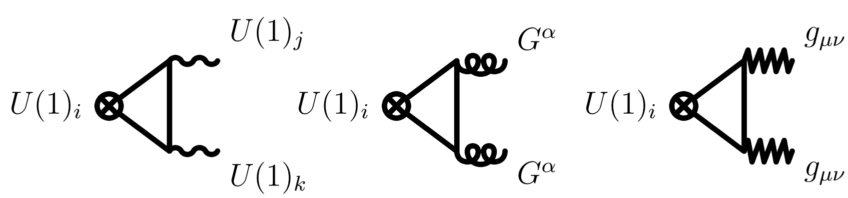
where p_i the momenta of the internal fermionic propagators, $\zeta(k_i)$ the polarization vectors of each external gauge boson. The parity matrix is projecting out all the right-fermions. $\mathcal{A} = Tr[t_1^\alpha t_2^\alpha \dots t_M^\alpha]$ is the group theory factor, where t_i^α the generators of the gauge group in the representation of the internal fermions. The emitted bosons are physical and on-shell ($k \cdot \zeta = k \cdot k = 0$). An $\epsilon^{\mu\cdots\rho}$ is arising from the trace with γ^{d+1}. The number of the external bosons in the anomalous amplitudes is $1 + d/2$.

This diagram is divergent and has to be regulated. Pauli-Villars method for example supply with masses m the internal fermions and at the end of the computation we take the limit $m \to \infty$. Careful evaluation shows that taking one of the polarization vectors longitudinal, the matrix element does not vanish. The form of the anomaly is proportional to (4.3).

In $4D$, the anomalous diagram is a triangle with three external bosons. In a theory of gauge group $U(N)$, the group theory factor implies that the possible anomalous diagrams can be:

$$SU(N)^3\;,\quad U(1)\times SU(N)^2\;,\quad U(1)^3\;. \tag{4.4}$$

In general, there can also be gravitational anomalies. However, we will not discuss them in the present study. The two last diagrams introduce the concept of the *anomalous* $U(1)$s. The Feynman diagrams which contribute to the $U(1)$ anomalies are:



(4.5)

Consider for simplicity only one anomalous $U(1)$. In terms of a gauge transformation $A^{(0)}_\mu \to A^{(0)}_\mu + \partial_\mu \epsilon$ of the effective action, the anomalies are:

$$\delta_\epsilon S = \int d^4x \Big\{ \epsilon \ \left(A_1 \ F \wedge F + A_2 \ Tr[G_a \wedge G_a] + A_3 \ R \wedge R \right) \Big\} \ , \tag{4.6}$$

where $A_1 = Tr[Q^3]$, $A_2 = Tr[QT^aT^a]$ and $A_3 = Tr[Q]$ the group theory factors. We suppress the indexes for simplicity. We will concentrate our study in the mixed (second) anomalous diagram and we will describe the Green-Schwarz mechanism that cancels the anomaly [42, 43, 28]. Generalization of this mechanism is strait forward for the rest of the anomalies.

4.2 Green-Schwarz mechanism

In this section we will explore the Green-Schwarz mechanism in $4D$. The fields that contribute to the anomaly cancellation are antisymmetric tensors $B^k_{\mu\nu}$ and they are coming from the kth twisted closed string spectrum (they are RR fields). We will consider one anomalous $U(1)$ and one antisymmetric $B_{\mu\nu}$. The generalization is straightforward. The lagrangian in terms of the RR 2-form is

$$\begin{aligned} \mathcal{L}_B = & -\frac{1}{4g_0^2} F^{(0)} F^{(0)} - \frac{1}{4g_a^2} Tr[F^a F^a] \\ & + \frac{1}{2} c_1 \tilde{H} \tilde{H} + c_3 \tilde{F}^{(0)}_{\mu\nu} B^{\mu\nu} + 2 c_3 c_2 \tilde{\Omega}^{(\alpha)\mu} A^{(0)}_\mu \ , \end{aligned} \tag{4.7}$$

where $F^{(0)}$, F^a the field strengths of the anomalous $U(1)$ ($A^{(0)}$) and the non-abelian $SU(N)$ (G^μ_a) gauge fields. The field strength of the RR field $H_{\mu\nu\rho}$ is modified by a Chern-Simons term $\Omega^{(\alpha)} = Tr\left[\gamma_k \left(G^\alpha dG^\alpha - \frac{2i}{3} G^\alpha \wedge G^\alpha \wedge G^\alpha \right)\right]$:

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + c \sum_\alpha \Omega^{(\alpha)}_{\mu\nu\rho} \ , \tag{4.8}$$

Notice the twist γ_k matrix that represents the action of the orbifold group α_k. All c_is are constants. c is of order of $[mass]^2$. The third term in (4.7) is provided by the way that the RR-forms couple to gauge field strength [25, 52, 54], :

$$Tr[\gamma e^{iF}] \wedge C \quad \to \quad c_3 Tr[\gamma_k \lambda] \tilde{F}^{(0)} \wedge B \ , \tag{4.9}$$

where C is a sum over RR forms of various degrees (terms of the correct degree of total form are kept).

It is more convenient to express the lagrangian (4.7) using the Poincaré dual of $B_{\mu\nu}$ scalar field α (axion):

$$\begin{aligned} \mathcal{L}_\alpha = & -\frac{1}{4g_0^2} F^{(0)} F^{(0)} - \frac{1}{4g_a^2} Tr[F^a F^a] \\ & - \frac{1}{2} \left(d\alpha - 2c_1 A^{(0)} \right)^2 - \frac{1}{2} c_2 \alpha Tr[F^a \tilde{F}^a] \ . \end{aligned} \tag{4.10}$$

Notice that the third term in the lagrangian is not invariant under a $U(1)$ gauge transformation unless the axion α also transforms like:

$$A_\mu^{(0)} \rightarrow A_\mu^{(0)} + \partial_\mu \epsilon \ , \qquad \alpha \rightarrow \alpha + 2c_1\epsilon \ . \tag{4.11}$$

However, this transformation of the axion generates a non-invariance coming from the fourth term in (4.10). This term will annihilate the anomalous term that is generated by the fermionic transformation, giving an anomaly free gauge theory. The total variation of the lagrangian under the above gauge transformation is:

$$\delta_\epsilon \mathcal{L}_{total} = -\left(c_1c_2 - \frac{\mathcal{A}}{32\pi^2}\right) \ \epsilon \ Tr[F^a\tilde{F}^a] \ , \tag{4.12}$$

where the first term is coming from the variation of $\mathcal{L}_\alpha$ and the second are the mixed anomalies from the variation of the measure of the chiral fermions. The anomaly is cancelled for: $\mathcal{A} = 32\pi^2 c_1c_2$.

The NSNS-twisted moduli m (SUSY partner of the α that they form together a complex scalar field $\phi = m+ia$) couple to the vector fields generating Fayet-Iliopoulos D-terms:

$$S_{FI} = \int d^4x \frac{1}{g_0^2}\left(m + \sum_i q_i|\Phi_i|^2\right)^2 \ . \tag{4.13}$$

where Φ_i denote various open strings with charge q_i under the anomalous $U(1)$s. More details are provided in appendix (F).

On the fixed points we have: $\langle m\rangle = 0$. The global $U(1)_0$ remains unbroken despite the fact that the gauge boson became massive [53]. Away from the fixed points we have: $\langle m\rangle \neq 0$. Restoration of SUSY (that is more economical state for the system) implies that the charged scalars will acquire a non-vanishing VEV. This breaks the global $U(1)_0$ symmetry.

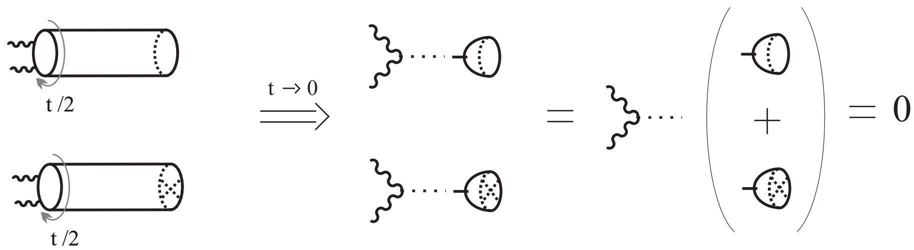
4.3 Calculation of the bare mass of the anomalous $U(1)$s

In this section we will evaluate the contribution to the anomalous $U(1)$ mass for supersymmetric orientifolds.

Closer look to (4.10) shows that these terms are coming from different orders in string perturbation theory. The $(\partial\alpha^i)^2$ is a tree-level (sphere) term, the $A^i\partial\alpha^i$ comes in the disk and the quadratic term in the gauge fields is a one-loop contribution. To clarify this, we mention that g_i^2 is proportional to $g_s = e^\phi$ and every power of the axion absorbs a dilaton factor $e^{-\phi}$ because it is a RR filed. The string perturbation series are weighted by $g_s^{-\chi}$ where $\chi = 2 - 2h - c - b$ is the Euler character and h, c and b denote the handle, the cross-cups and the boundaries of a closed orientable Riemann surface respectively.

The diagrams at one-loop that contribute to terms quadratic in the gauge bosons (anomalous $U(1)$s) are the genus-one surfaces with boundaries: the annulus and the

Möbius strip. In the infrared (IR) region they diverge logarithmically and give the logarithmic running of the couplings. In the ultraviolet (UV) region the tadpoles of the annulus with both gauge bosons inserted in the same boundary and the Möbius strip vanish due to the tadpole cancellation.



However, in this UV limit the annulus amplitude with the gauge bosons inserted in opposite boundaries provides the mass-term of the anomalous $U(1)$ [47]. Since we are interested in the anomalous gauge boson mass, we concentrate on the latter diagram. The gauge boson vertex operator is

$$\tilde{V}^a = \lambda^a \epsilon_\mu (\partial X^\mu + i(p \cdot \psi)\psi^\mu) e^{ip\cdot X} \ , \tag{4.14}$$

where λ is the Chan-Paton matrix and ϵ^μ is the polarization vector. The 2-point annulus amplitude is given by

$$\mathcal{A}^{ab} = -\frac{1}{4G} \int [d\tau][dz] \int \frac{d^d p}{(2\pi)^d} \sum_k \langle \tilde{V}^a(\epsilon_1, p_1, z) \tilde{V}^b(\epsilon_2, p_2, z_0) \rangle_k \ , \tag{4.15}$$

where we keep undetermined the number of non-compact dimensions d and G denotes the order of the orientifold group. The fundamental polygon of the annulus is $[0, t/2] \otimes [0, 1/2]$ (Fig.4). The index k denotes the various orbifold sectors that we may have. Using the translation symmetry of the annulus, we fix the position of one VO to $z_0 = 1/2$. The other VO is placed on the imaginary axis with $z \in [0, t/2]$.

The leading term of (4.15) is

$$\mathcal{A}^{ab} = \int \frac{d^d p}{(2\pi)^d} \Big(\zeta_1 \cdot \zeta_2 \ p_1 \cdot p_2 - \zeta_1 \cdot p_2 \ p_1 \cdot \zeta_2 \Big) \sum_k Tr[\gamma_k \lambda^\alpha] Tr[\gamma_k \lambda^\beta] \mathcal{A}_k^{ab} \ , \tag{4.16}$$

where

$$\mathcal{A}_k^{ab} = -\frac{1}{2G} \int [d\tau][dz] e^{-p_1\cdot p_2 \langle X(z) X(1/2) \rangle} \Big(\langle \psi(z)\psi(1/2)\rangle^2 - \langle X(z) \partial X(1/2) \rangle^2 \Big) Z_k^{ab} \ , \tag{4.17}$$

since the p-independent terms vanish due to supersymmetry. The bosonic and fermionic correlation functions are given in the appendix (D.4, D.5).

It appears that the amplitude (4.15) has a kinimatical multiplicative factor that is $\mathcal{O}(p^2)$, thus would seem to provide a leading correction only to the anomalous gauge boson coupling. We will see however, that after integration over the position

z and the annulus modulus τ_2, a term proportional to $1/p_1 \cdot p_2$ appears from the ultraviolet (UV) region (as a result of the quadratic UV divergence in the presence of anomalous $U(1)$s) that will provide the mass-term.

Strictly speaking, the amplitude above is zero on-shell if we enforce the physical state conditions $\zeta \cdot p = p^2 = 0$ and momentum conservation $p_1 + p_2 = 0$. There is however a consistent off-shell extension, without imposing momentum conservation, that has given consistent results in other cases (see [3] for a discussion) and we adopt it here. We will thus impose momentum conservation only at the end of the calculation.

Spin structure summation of the partition function Z_k^{ab}, gives zero due to space-time supersymmetry[17]. Therefore, terms in the correlation functions which are spin-structure independent vanish. The only spin-dependant term lies in the fermionic correlation function:

$$\langle\psi(z-1/2)\psi(0)\rangle^2[^\alpha_\beta]_{annulus} = -2\pi i\partial_\tau \log\vartheta[^\alpha_\beta](0|\tau) \ . \tag{4.18}$$

Equ. (4.18) is independent of z, the position of the second VO. Thus, we can easily integrate on dz. Using the modular transformations of the theta functions and keeping the leading order of δ, we have:

$$\int_0^{\tau_2} dz\ e^{-\delta\langle X(z)X(0)\rangle} = \int_0^{\tau_2} dz\ \tau_2^{\delta/2}\frac{(2\pi\eta^3(\tau))^\delta}{\vartheta[^0_1](z/\tau| - 1/\tau)} = \tau_2^{1+\delta/2}[2\pi\eta^3(\tau)]^\delta + ... \tag{4.19}$$

Following the procedure of [47] we rewrite (4.17) as:

$$\mathcal{A}_k^{ab} = -\frac{1}{2G}\int [d\tau]\tau_2^{1+\delta/2}[2\pi\eta^3(\tau)]^\delta F_k^{ab} \ . \tag{4.20}$$

defining F_k^{ab} as a term which contains all the spin-structure and the orbifold information:

$$F_k^{ab} = \sum_{\alpha\beta}\eta^{\alpha\beta}\left[-2\pi i\partial\log\vartheta[^\alpha_\beta]\right]\left[\frac{1}{(2\pi\tau)^3}\frac{\vartheta[^\alpha_\beta]}{\eta^3}\right] Z^{ab}_{int,k}[^\alpha_\beta] \ , \tag{4.21}$$

where $\eta^{\alpha\beta} = \frac{1}{2}(-1)^{\alpha+\beta+\alpha\beta}$. The first bracket is denoting the VO insertion in the annulus diagram. The second is the six-dimensional partition function.

The integral on t has a logarithmic divergence in δ in the IR and a pole in the UV.

$$\mathcal{A}_k^{ab} = \frac{2C_k^{ab}}{\pi\delta|G|} + \mathcal{O}(\log\delta) \ . \tag{4.22}$$

The on-shell limit $[(\zeta_1\cdot\zeta_2)(p_1\cdot p_2) - (\zeta_1\cdot p_2)(p_1\cdot\zeta_2)]/p_1\cdot p_2 \to \zeta_1\cdot\zeta_2 = \zeta^2$ provides the un-normalized mass matrix.

[17]Later we will evaluate the bare mass of the anomalous $U(1)$s also for the non-supersymmetric case.

4.3.1 $\mathcal{N}$=1 supersymmetric sectors

In orientifold models, any element $\alpha \in G$ which acts onto all tori ($v^i_\alpha \neq 0$ for any $i = 1, 2, 3$) provides an $\mathcal{N} = 1$ supersymmetric sector. In that case, it is straightforward to evaluate the various $C_k^{pq,UV}$ and the mass formulae:

$$\frac{1}{2} M^2_{pp,ab}\Big|_{\mathcal{N}=1} = \sum_{\substack{k \\ \mathcal{N}=1 \text{ sectors}}} -\frac{1}{\pi^3|G|}\prod_{i=1}^{3} |\sin[\pi k v_j]|\ Tr[\gamma_k \lambda^a_p] Tr[\gamma_k \lambda^b_p]\ , \tag{4.23}$$

$$\frac{1}{2} M^2_{95,ab}\Big|_{\mathcal{N}=1} = \sum_{\substack{k \\ \mathcal{N}=1 \text{ sectors}}} \frac{\sin(\pi k v_1)}{2\pi^3|G|}\ \frac{1}{2}\prod_{j=1}^{3} \frac{\sin[\pi k v_j]}{|\sin[\pi k v_j]|}\ Tr[\gamma_k \lambda^a_9] Tr[\gamma_k \lambda^b_5]\ , \tag{4.24}$$

where $p = 9, 5$ and we have divided the 59 contribution by two, to avoid overcounting.

4.3.2 $\mathcal{N}$=2 supersymmetric sectors

$\mathcal{N} = 2$ supersymmetric sectors are present when a two-torus remains invariant under the action of the appropriate orientifold element. Only massless states and their KK descendants survive the (4.21). In this case, the function $F_k^{ab}(t) = C_k^{ab} P_2(t)$ where C_k^{ab} is still given by (4.22). $P_2(t)$ is either the appropriate momentum lattice when these directions are NN (Neumann boundary conditions), or the winding lattice when these directions are DD (Dirichlet boundary conditions) [28]. No lattice sum can appear along ND directions. The open string momentum sum relevant in the NN case:

$$P_2(t) = \sum_{m\in\mathbb{Z}} e^{-4\pi\tau_2 \frac{\alpha'}{4}\left(\frac{m}{R}\right)^2} = \frac{R}{\sqrt{\alpha'\tau_2}} \sum_{w\in\mathbb{Z}} e^{-\frac{4\pi}{\tau_2}\frac{\alpha'}{4}\left(\frac{wR}{\alpha'}\right)^2}\ , \tag{4.25}$$

while the open string (DD) winding sum is:

$$W_2(t) = \sum_{w\in\mathbb{Z}} e^{-4\pi\tau_2 \frac{\alpha'}{4}\left(\frac{wR}{\alpha'}\right)^2} = \frac{1}{R}\sqrt{\frac{\alpha'}{\tau_2}} \sum_{m\in\mathbb{Z}} e^{-\frac{4\pi}{\tau_2}\frac{\alpha'}{4}\left(\frac{m}{R}\right)^2}\ . \tag{4.26}$$

The pole contribution of (4.20) has been evaluated in [47]:

$$\frac{4\mathcal{V}_2\ C_k^{ab,IR}}{\pi\delta} + \mathcal{O}(\log\delta)\ . \tag{4.27}$$

We now proceed to evaluate the contributions to the mass coming from $\mathcal{N} = 2$ sectors of abelian orientifolds. For such sectors, one of the kv_i is integer. We will choose without loss of generality $kv_3 =$ integer. We compute:

$$\frac{1}{2} M^2_{ab,NN}\Big|_{\mathcal{N}=2} = \sum_{\substack{k \\ \mathcal{N}=2 \text{ sectors}}} -\frac{2\mathcal{V}_2}{\pi^3|G|}\prod_{j=1}^{2} |\sin[\pi k v_j]| Tr[\gamma_k \lambda^a] Tr[\gamma_k \lambda^b]\ , \tag{4.28}$$

$$\frac{1}{2} M^2_{ab,DD}\Big|_{\mathcal{N}=2} = \sum_{\substack{k \\ \mathcal{N}=2 \text{ sectors}}} -\frac{1}{2\mathcal{V}_2\pi^3|G|}\prod_{j=1}^{2} |\sin[\pi k v_j]| Tr[\gamma_k \lambda^a] Tr[\gamma_k \lambda^b]\ . \tag{4.29}$$

Finally, for the 59 case, the relevant $\mathcal{N}=2$ sector is when the longitudinal torus is untwisted. In this case, we evaluate:

$$\frac{1}{2}M^2_{ab,DN}\bigg|_{\mathcal{N}=2} = \sum_{\substack{k \\ \mathcal{N}=2 \text{ sectors}}} (-1)^{kv_1}\frac{\mathcal{V}_2}{2\pi^3|G|}Tr[\gamma_k\lambda^a]Tr[\gamma_k\lambda^b] \ . \tag{4.30}$$

We have divided the 59 contribution by an additional factor of two. In the case where the two-torus corresponds to DD boundary conditions (in a D7-D3 configuration for instance), one should replace $\mathcal{V}_2 \to 1/4\mathcal{V}_2$.

We should mention, that the above masses are unormalized. To obtain the normalized mass matrix, we must also take into account the kinetic terms of the $U(1)$ gauge bosons which are

$$S_{\text{kinetic}} = -\frac{1}{4g_s}\left[\mathcal{V}_1\mathcal{V}_2\mathcal{V}_3\sum_i F_i^2 + \mathcal{V}_3\sum_j \tilde{F}_j^2\right] \ . \tag{4.31}$$

where i and j denote the gauge groups that are coming from different stacks of D9 and D5-branes. This implies $M^2_{99} \to M^2_{99}/(\mathcal{V}_1\mathcal{V}_2\mathcal{V}_3)$, $M^2_{55} \to M^2_{55}/\mathcal{V}_3$ and $M^2_{95} \to M^2_{95}/(\mathcal{V}_3\sqrt{\mathcal{V}_1\mathcal{V}_2})$.

4.4 Applications on $\mathcal{N}$=1 orientifolds

We are going to apply our formulae on various orientifolds. First, we will compute how many anomalous $U(1)$s appear in the various orientifold models by evaluating the mixed-anomaly traces that give the anomalous $U(1)$s. Our normalization of the square casimir, cubic casimir and the $U(1)$ charge of the $SU(N)$ representations are given in the following table:

$SU(N)$ Reps	Square Casimir	Cubic Casimir	$U(1)$ Charge
$\square$	$T(\square) = 1$	$A(\square) = \ \ 1$	$Q(\square) = \ \ 1$
$\overline{\square}$	$T(\overline{\square}) = 1$	$A(\overline{\square}) = -1$	$Q(\overline{\square}) = -1$
$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	$T(\begin{smallmatrix}\square\\ \square\end{smallmatrix}) = N-2$	$A(\begin{smallmatrix}\square\\ \square\end{smallmatrix}) = \ \ N-4$	$Q(\begin{smallmatrix}\square\\ \square\end{smallmatrix}) = \ \ 2$
$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$	$T(\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}) = N-2$	$A(\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}) = -N+4$	$Q(\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}) = -2$

For the evaluation of the mass matrix of the anomalous $U(1)$s, the normalized generators of the anomalous $U(1)_i$ are defined as:

$$\lambda_i^\alpha = \frac{1}{2\sqrt{n_i}}\sum Q_i^\alpha \cdot H \ , \tag{4.32}$$

where α denotes the type of brane. The $Q_i^\alpha = (0,\ldots,0,1,\ldots,1,0,\ldots,0)$ is a 16-dimensional vector with n_i entries of 1s where the $SU(n_i-1)$ lives. We normalize the λ matrices with $Tr[\lambda^2] = 1/2$. Thus, the relevant trace is:

$$Tr[\gamma_k^\alpha\lambda_i^\alpha] = Tr[e^{-2\pi ikV^\alpha\cdot H}Q_i^\alpha\cdot H] = -\frac{i}{\sqrt{n_i}}\sin[2\pi kV_i^\alpha] \ , \tag{4.33}$$

where V_i^α are the overlapping components of V^α and Q^α [28].

4.4.1 The four-dimensional Z_6' orientifold

The orbifold rotation vector is $(v_1, v_2, v_3) = (1, -3, 2)/6$. Since there is an order two twist ($k = 3$), we have one set of D5-branes. Tadpole cancellation implies the existence of 32 D9-branes and 32 D5-branes that we put together at one of the fixed points of the Z_2 action (namely the origin). The Chan-Paton 'shift' vectors are

$$V_{5,9} = \frac{1}{12}(1,1,1,1,5,5,5,5,3,3,3,3,3,3,3,3)\ . \tag{4.34}$$

The gauge group has a factor of $U(4) \times U(4) \times U(8)$ coming from the D9-branes and an isomorphic factor coming from the D5-branes. The massless spectrum is provided in Table.1. Different sectors preserve different supersymmetries. The $\mathcal{N} = 1$ sectors correspond to $k = 1, 5$, while for $k = 2, 3, 4$ we have $\mathcal{N} = 2$ sectors.

The four-dimensional anomalies of the $U(1)$s have been computed in [28] and the anomaly matrix is

$$A_{QTT}[Z_6'] \sim \begin{pmatrix} 2 & 2 & 4\sqrt{2} & -2 & 0 & -2\sqrt{2} \\ -2 & -2 & -4\sqrt{2} & 0 & 2 & 2\sqrt{2} \\ 0 & 0 & 0 & 2 & -2 & 0 \\ -2 & 0 & -2\sqrt{2} & 2 & 2 & 4\sqrt{2} \\ 0 & 2 & 2\sqrt{2} & -2 & -2 & -4\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 \end{pmatrix}\ , \tag{4.35}$$

there are two linear combinations that are free of four-dimensional anomalies: $\sqrt{2}(A_1 + A_2) - A_3$ and $\sqrt{2}(\tilde{A}_1 + \tilde{A}_2) - \tilde{A}_3$.

The contribution to the mass matrix [47] is

$$\begin{aligned} \frac{1}{2}M^2_{aa,ij} &= -\frac{\sqrt{3}}{24\pi^3}\left(Tr[\gamma_1\lambda_i^a]Tr[\gamma_1\lambda_j^a] + Tr[\gamma_5\lambda_i^a]Tr[\gamma_5\lambda_j^a]\right) \\ &- \frac{1}{4\pi^3}\left(\mathcal{V}_2\delta_{a,9} + \frac{1}{4\mathcal{V}_2}\delta_{a,5}\right)\left(Tr[\gamma_2\lambda_i^a]Tr[\gamma_2\lambda_j^a] + Tr[\gamma_4\lambda_i^a]Tr[\gamma_4\lambda_j^a]\right) \\ &- \frac{\mathcal{V}_3}{3\pi^3}Tr[\gamma_3\lambda_i^a]Tr[\gamma_3\lambda_j^a]\ , \end{aligned} \tag{4.36}$$

for $a = 5, 9$ where $\delta_{a,b}$ is the Kronecker delta. The mixed 59 annulus diagrams give a contribution to the mass

$$\begin{aligned} \frac{1}{2}M^2_{95,ij} = &- \frac{\sqrt{3}}{48\pi^3}\left(Tr[\gamma_1\lambda_i^9]Tr[\gamma_1\lambda_j^5] + Tr[\gamma_5\lambda_i^9]Tr[\gamma_5\lambda_j^5]\right. \\ &+ \left. Tr[\gamma_2\lambda_i^9]Tr[\gamma_2\lambda_j^5] - Tr[\gamma_4\lambda_i^9]Tr[\gamma_4\lambda_j^5]\right) \\ &- \frac{\mathcal{V}_3}{12\pi^3}Tr[\gamma_3\lambda_i^9]Tr[\gamma_3\lambda_j^5]\ . \end{aligned} \tag{4.37}$$

The unormalized mass matrix [47] has eigenvalues and eigenvectors:

$$m_1^2 = 6\mathcal{V}_2 \quad , \quad -A_1 + A_2 \tag{4.38}$$

$$m_2^2 = \frac{3}{2\mathcal{V}_2} \quad , \quad -\tilde{A}_1 + \tilde{A}_2 \tag{4.39}$$

$$m_{3,4}^2 = \frac{5\sqrt{3} + 48\mathcal{V}_3 \pm \sqrt{3}\alpha}{12} \quad , \quad \frac{-3 \pm \alpha}{4\sqrt{2}(4\sqrt{3}\mathcal{V}_3 - 1)}(A_1 + A_2 - \tilde{A}_1 - \tilde{A}_2) - A_3 + \tilde{A}_3; \tag{4.40}$$

$$m_{5,6}^2 = \frac{15\sqrt{3} + 80\mathcal{V}_3 \pm \beta}{12} \quad , \quad \frac{9\sqrt{3} \mp \beta}{4\sqrt{2}(20\mathcal{V}_3 - 3\sqrt{3})}(A_1 + A_2 + \tilde{A}_1 + \tilde{A}_2) + A_3 + \tilde{A}_3; \tag{4.41}$$

with $\alpha = \sqrt{25 - 128\sqrt{3}\mathcal{V}_3 + 768\mathcal{V}_3^2}$ and $\beta = \sqrt{5(135 - 384\sqrt{3}\mathcal{V}_3 + 1280\mathcal{V}_3^2)}$. Note that the eigenvalues are invariant under the T-duality symmetry of the theory $\mathcal{V}_2 \to 1/4\mathcal{V}_2$. Thus, all $U(1)$s become massive, including the two anomaly free combinations. This result is unexpected since there is no obvious mechanism that provides a mass to non-anomalous $U(1)$s.

4.4.2 The four-dimensional Z_6 orientifold

The orbifold rotation vector is $(v_1, v_2, v_3) = (1, 1, -2)/6$. Since there is an order two twist ($k = 3$), we have one set of $D5$-branes that are stretched in the 4D Minkowski and wrap the third torus T_3^2. Tadpole cancellation implies the existence of 32 $D9$-branes and 32 $D5$-branes that we put together at one of the fixed points of the Z_2 action (namely the origin). The Chan-Paton 'shift' vectors are

$$V_{5,9} = \frac{1}{12}(1,1,1,1,1,1,5,5,5,5,5,5,3,3,3,3) \,. \tag{4.42}$$

The gauge group has a factor of $U(6) \times U(6) \times U(4)$ coming from the $D9$-branes and an isomorphic factor coming from the $D5$-branes. The massless spectrum is provided in the appendix Table.1. This orientifold has different supersymmetries in different sectors. The $\mathcal{N} = 1$ sectors correspond to $k = 1, 2, 4, 5$, while for $k = 3$ we have $\mathcal{N} = 2$ sectors.

The four-dimensional anomalies of the $U(1)$s have been computed in [28] and the anomaly matrix is

$$A_{QTT}[Z_6] \sim \begin{pmatrix} 6 & -3 & \sqrt{6} & 3 & 0 & \sqrt{6} \\ 3 & -6 & -\sqrt{6} & 0 & -3 & -\sqrt{6} \\ -9 & 9 & 0 & -3 & 3 & 0 \\ 3 & 0 & \sqrt{6} & 6 & -3 & \sqrt{6} \\ 0 & -3 & -\sqrt{6} & 3 & -6 & -\sqrt{6} \\ -3 & 3 & 0 & -9 & 9 & 0 \end{pmatrix} , \tag{4.43}$$

there are three linear combinations that are free of anomalies: $A_1 + A_2 - \sqrt{\frac{3}{2}} A_3$, $\tilde{A}_1 + \tilde{A}_2 - \sqrt{\frac{3}{2}} \tilde{A}_3$ and $A_3 - \tilde{A}_3$.

The contributions to the mass matrix [47] are:

$$\begin{aligned}\frac{1}{2} M^2_{aa,ij} = & -\frac{\sqrt{3}}{48\pi^3} \Big(Tr[\gamma_1 \lambda^a_i] Tr[\gamma_1 \lambda^a_j] + Tr[\gamma_5 \lambda^a_i] Tr[\gamma_5 \lambda^a_j] \\ & +3 (Tr[\gamma_2 \lambda^a_i] Tr[\gamma_2 \lambda^a_j] + Tr[\gamma_4 \lambda^a_i] Tr[\gamma_4 \lambda^a_j]) \Big) \\ & -\frac{\mathcal{V}_3}{3\pi^3} Tr[\gamma_3 \lambda^a_i] Tr[\gamma_3 \lambda^a_j] \ , \end{aligned} \tag{4.44}$$

for $a = 5, 9$, while

$$\begin{aligned}\frac{1}{2} M^2_{59,ij} = & -\frac{\sqrt{3}}{48\pi^3} \Big(Tr[\gamma_1 \lambda^5_i] Tr[\gamma_1 \lambda^9_j] + Tr[\gamma_5 \lambda^5_i] Tr[\gamma_5 \lambda^9_j] \\ & +3 (Tr[\gamma_2 \lambda^5_i] Tr[\gamma_2 \lambda^9_j] + Tr[\gamma_4 \lambda^5_i] Tr[\gamma_4 \lambda^9_j]) \Big) \\ & -\frac{\mathcal{V}_3}{12\pi^3} Tr[\gamma_3 \lambda^5_i] Tr[\gamma_3 \lambda^9_j] \ . \end{aligned} \tag{4.45}$$

Notice that the $\mathcal{N} = 2$ sector contributes with a term proportional to $\mathcal{V}_3$. The mass matrix of the anomalous $U(1)$s has the following eigenvalues and eigenstates [47]:

$$m_1^2 = 0 \quad , \quad A_1 + A_2 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{6}(A_3 - \tilde{A}_3); \tag{4.46}$$

$$m_2^2 = \frac{3\sqrt{3}}{2} \quad , \quad A_1 - A_2 - \tilde{A}_1 + \tilde{A}_2; \tag{4.47}$$

$$m_3^2 = 3\sqrt{3} \quad , \quad A_1 - A_2 + \tilde{A}_1 - \tilde{A}_2; \tag{4.48}$$

$$m_4^2 = 8\mathcal{V}_3 \quad , \quad -\sqrt{\frac{3}{2}}(A_1 + A_2 - \tilde{A}_1 - \tilde{A}_2) - A_3 + \tilde{A}_3; \tag{4.49}$$

$$m_{5,6}^2 = \frac{7\sqrt{3} + 80\mathcal{V}_3 \pm \beta}{12} \quad , \quad \frac{40\mathcal{V}_3 - \sqrt{3} \pm \beta}{12\sqrt{2} - 40\sqrt{6}\mathcal{V}_3}(A_1 + A_2 + \tilde{A}_1 + \tilde{A}_2) + A_3 + \tilde{A}_3; \tag{4.50}$$

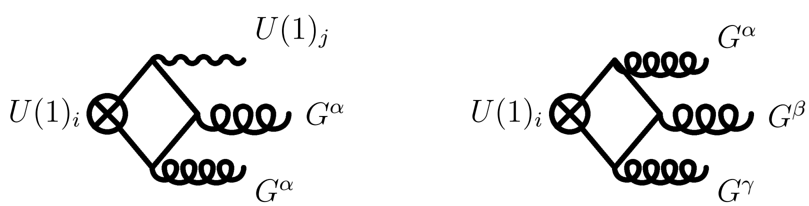
where $\beta = \sqrt{147 - 1040\sqrt{3}\mathcal{V}_3 + 6400\mathcal{V}_3^2}$. Again, two non-anomalous $U(1)$s acquire masses.

As we have seen in the two last examples of Z'_6 and Z_6 orientifold models, $U(1)$ gauge fields that are free of four-dimensional anomalies can still be massive. This is unexpected and we should study the contribution of higher anomalies in the mass-generation of the $U(1)$s. We will especially study the six-dimensional anomalies since we cannot have eight-dimensional anomalies in supersymmetric orientifold models (which obey the condition: $\sum_i v_i = 0$). We will show that if there are decompactification limits in the theory, six-dimensional anomalies affect four-dimensional masses.

4.5 The structure of six-dimensional mixed gauge anomalies

In the previous section we computed the bare masses of the anomalous $U(1)$s by evaluating the ultraviolet tadpole of the one-loop open string diagram with the insertion of two gauge bosons on different boundaries. It turns out that $U(1)$ gauge fields that are free of four-dimensional anomalies can still be massive. This is unexpected and we should study the contribution of higher anomalies in the mass-generation of the $U(1)$s. We will especially study the six-dimensional anomalies since we cannot have eight-dimensional anomalies in orientifold models that obey the condition $\sum_i v_i = 0$. We will show that if there are decompactification limits in the theory, six-dimensional anomalies affect four-dimensional masses.

In six dimensions, the leading diagram that can give a contribution to anomalies is the square diagram [43]. The mixed group theory factors that do not identically vanish are these with two or three external non-abelian gauge bosons. The Feynman diagrams that eventually contain anomalies are:



Therefore, in the presence of an anomalous $U(1)$ field, the effective action is not invariant under a transformation $\delta A^i = d\epsilon^i$:

$$\delta_{\epsilon^i} S|_{gauge} = \int d^6x \left\{ \epsilon^i \ \left(A_{QQTT}\ F^j \wedge Tr[G^2] + A_{QTTT}\ Tr[G^3] \right) \right\} , \qquad (4.51)$$

where $A_{QQTT} = Tr[Q_i Q_j T^\alpha T^\alpha]$, $A_{QTTT} = Tr[Q_i T^\alpha \{T^\beta T^\gamma\}]$ the group theory factors. Powers of forms are understood as wedge products. We denote by $G_{\mu\nu}$ the field strength of a non-abelian gauge field W_μ.

Gauge invariance is preserved by the six-dimensional Green-Schwarz mechanism. However, two inequivalent fields should contribute to this cancellation. The cancellation of the first anomalous term is arranged by a 2-form B^i (RR twisted field) which transform under the $U(1)$ transformation like $\delta B^i = -\epsilon^i F^i$. The lagrangian of this field is:

$$S_{QQTT} = \int d^6x \left[-\frac{1}{4g_i^2} F^{i2}_{\mu\nu} - \frac{1}{12} \left[dB^i + \Omega_{A^i} \right]^2 + A_{QQTT}\ B^i \wedge Tr[G^2] \right] , \qquad (4.52)$$

where the last term is proportional to the anomaly of the first diagram. The 3-form $\Omega_{A^i} = A^i dA^i$ is the Chern-Simons term of the abelian gauge field A^i_μ. This part of the action does not generate a mass for the gauge boson.

By the (4.52), we can evaluate the action in terms of the dual 2-form λ of B [56]. Using $Tr[G_i \tilde{G}_i] = d\Omega_{W_i}$, where $\Omega_{W_i} = Tr[W_i dW_i + \frac{2}{3} W_i^3]$ is the Chern-Simons term for the non-abelian gauge field W^i, we finally find:

$$\tilde{S}_{QQTT} = \int d^6x \left[-\frac{1}{4g_i^2} F^{i2}_{\mu\nu} - \frac{1}{12} \left[d\lambda^i - 6A_{QQTT}\Omega_{W^i} \right]^2 - \frac{1}{6} \Omega_{A^i} \wedge (d\lambda^i - 6A_{QQTT}\Omega_{W^i}) \right] . \tag{4.53}$$

The λ^i are invariant under $U(1)$ gauge transformations and transform like $\delta\lambda^i = 6C_1\ Tr[G\epsilon^i]$ under a non-abelian gauge transformation $\delta W_i^\mu = D^\mu \epsilon_i$ so that the action is gauge invariant. Thus, under a $U(1)$ gauge transformation the variation of $\Omega_{A^i} \wedge d\lambda^i$ (since $\delta\Omega_{A^i} = d\epsilon F$) vanishes due to integration by parts and the term $\Omega_{A^i} \wedge \Omega_{W^i}$ cancels the first anomaly in (4.51).

The second anomaly is cancelled by a pseudoscalar axion that transforms under the $U(1)$ transformation as $\delta\alpha^i = -\epsilon^i$:

$$S_{QTTT} = \int d^6x \left[-\frac{1}{4g_i^2} F^{i2}_{\mu\nu} + \frac{M^2}{2} (A^i + d\alpha^i)^2 + A_{QTTT}\alpha^i\ Tr[G^3] \right] . \tag{4.54}$$

This action supplies a mass term for the $U(1)$ gauge field and breaks the gauge symmetry in six dimensions.

4.6 Six-dimensional mass formulae

The general mass formulae for the anomalous $U(1)$ gauge fields in six-dimensional orientifolds can be easily evaluated in the same way that we did for the four-dimensional cases. N=1 six-dimensional orientifolds are created as T^4/Z_N where $N = 2, 3, 4, 6$. The results for strings attached on the same kind of branes (untwisted states) are (E.3)

$$\frac{1}{2} M^2_{aa} = -\frac{4}{\pi^2 N} \sum_k \sin^2 \frac{\pi k}{N}\ Tr[\gamma_k \lambda^a] Tr[\gamma_k \lambda^a] , \tag{4.55}$$

where $a = 5, 9$ denotes the kind of D-branes on which the open string is attached. In the case where strings have one end on a $D5$ and the other on a $D9$-brane (twisted states) we have:

$$\frac{1}{2} M^2_{59} = -\frac{1}{\pi^2 N} \sum_k Tr[\gamma_k \lambda^5] Tr[\gamma_k \lambda^9] . \tag{4.56}$$

We should mention, that the above masses are again unormalized. To obtain the normalized mass matrix, we must also take into account the kinetic terms of the $U(1)$ gauge bosons which are again (4.31), however, the volume of the torus that the D5-branes is longitudinal to, should be normalized to identity. This implies $M^2_{99} \to M^2_{99}/(\mathcal{V}_1\mathcal{V}_2)$, $M^2_{55} \to M^2_{55}$ and $M^2_{95} \to M^2_{95}/(\sqrt{\mathcal{V}_1\mathcal{V}_2})$.

4.7 Six-dimensional $\mathcal{N}$=1 orientifolds examples

Usual six-dimensional decompactification limits of four-dimensional supersymmetric orientifolds are the $\mathcal{N}$=1 orientifolds of Type IIB string theory, $\mathbb{R}^6 \times K3/Z_N$ where the only possible choices are $N = 2, 3, 4, 6$. Thus, we will apply the above general formulae on these orientifolds. Tadpole cancellation guaranties that the models are free of irreducible non-Abelian anomalies [28, 43].

4.7.1 Z_2 orientifold

For the Z_2, the tadpole condition gives 32 $D9$ and 32 $D5$-branes [23, 27]. The characteristic vectors are:

$$V_{5,9} = \frac{1}{4}(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1) \, . \tag{4.57}$$

The gauge group is $U(16)_9 \times U(16)_5$. The massless states are given in Table.2. We are interested in anomalous diagrams with one abelian and three non-abelian gauge bosons $U(1) \times SU(N)^3$ since their cancellation provides the six-dimensional mass-term. We find:

$$A_{QTTT} = 32 \cdot \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix} \, , \tag{4.58}$$

where the columns label the U(1)s, while the rows label the non-abelian factors. The matrix has two non-zero eigenvalues and both anomalous $U(1)$s are expected to become massive [44]. The unormalized mass matrix for the anomalous $U(1)$s is calculated by the use of (4.55), (4.56) and (4.33):

$$\frac{1}{2}M^2 = -\frac{1}{2\pi^2}\begin{pmatrix} 4\, Tr[\gamma_1\lambda^9]Tr[\gamma_1\lambda^9] & Tr[\gamma_1\lambda^9]Tr[\gamma_1\lambda^5] \\ Tr[\gamma_1\lambda^5]Tr[\gamma_1\lambda^9] & 4\, Tr[\gamma_1\lambda^5]Tr[\gamma_1\lambda^5] \end{pmatrix} = \frac{8}{\pi^2}\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} . \tag{4.59}$$

As it was expected from the effective field theory computation of the anomalies, there are two massive eigenstates: $\pm A + \tilde{A}$ with masses $24/\pi^2$, $40/\pi^2$ (we denote with A the gauge boson that is coming from the D9-branes and with $\tilde{A}$ the one that is coming from the D5).

4.7.2 Z_3 orientifold

The Z_3 orientifold does not contain a Z_2 reflection element. Thus, there are no D5-branes. The characteristic vector is:

$$V_9 = \frac{1}{3}(1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0) \, , \tag{4.60}$$

and the gauge group $U(8) \times SO(16)$. From the massless spectrum which is provided in Table.2 we find that the single gauge boson suffers from mixed non-abelian anomalies [44].

$$A_{QTTT} = 48. \tag{4.61}$$

Using (4.33) we find the mass of this gauge boson:

$$\frac{1}{2}M^2 = \frac{32}{3\pi^2}\sum_{k=1}^{2}\sin^2\frac{\pi k}{3}\sin^2\frac{2\pi k}{3} = \frac{12}{\pi^2} \, . \tag{4.62}$$

4.7.3 Z_4 orientifold

The Z_4 orientifold contains 32 D9-branes and 32 D5-branes. The characteristic vectors are:

$$V_{5,9} = \frac{1}{8}(1,1,1,1,1,1,1,1,3,3,3,3,3,3,3,3) \ , \tag{4.63}$$

and the gauge group is $U(8)_9 \times U(8)_9 \times U(8)_5 \times U(8)_5$. The massless spectrum is provided in Table.2. The $U(1) \times SU(N)^3$ anomalies are:

$$A_{QTTT} = 16 \cdot \begin{pmatrix} 3 & -1 & -1 & 0 \\ -1 & 3 & 0 & -1 \\ -1 & 0 & 3 & -1 \\ 0 & -1 & -1 & 3 \end{pmatrix} \ . \tag{4.64}$$

where again the columns label the U(1)s and the rows the non-abelian factors $SU(8)_9^2 \times SU(8)_5^2$. Notice that we have two equal matrices in the diagonal blocks and two other ones equal in the off-diagonal blocks. This is a consequence of the fact that the D9 and D5 branes are related by T-duality and split in isomorphic groups. All those models are T-selfdual . The anomaly matrix has four non-zero eigenvalues [44].

The mass matrix of the anomalous U(1) masses is

$$\frac{1}{2}M^2 = \frac{4}{\pi^2} \begin{pmatrix} 3 & -1 & 1 & 0 \\ -1 & 3 & 0 & 1 \\ 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{pmatrix} \ . \tag{4.65}$$

Diagonalizing this matrix, we find four massive $U(1)$ fields that are in accordance with the anomalies. The massive $U(1)$ fields are $-A_1 - A_2 + \tilde{A}_1 + \tilde{A}_2$, $A_1 + \tilde{A}_2$, $A_2 + \tilde{A}_1$, $-A_1 + A_2 - \tilde{A}_1 + \tilde{A}_2$ with masses $4/\pi^2$, $12/\pi^2$, $12/\pi^2$, $20/\pi^2$ respectively.

4.7.4 Z_6 orientifold

The Z_6 orientifold contains 32 D9-branes and 32 D5-branes. The characteristic vectors are:

$$V_{5,9} = \frac{1}{12}(1,1,1,1,5,5,5,5,3,3,3,3,3,3,3,3) \tag{4.66}$$

and the gauge group $U(4)_9 \times U(4)_9 \times U(8)_9 \times U(4)_5 \times U(4)_5 \times U(8)_5$. The massless spectrum is provided in Table.2. The $U(1) \times SU(N)^3$ anomalies are:

$$A_{QTTT} = 8 \cdot \begin{pmatrix} 3 & 0 & -2 & -1 & 0 & 0 \\ 0 & 3 & -2 & 0 & -1 & 0 \\ -1 & -1 & 4 & 0 & 0 & -2 \\ -1 & 0 & 0 & 3 & 0 & -2 \\ 0 & -1 & 0 & 0 & 3 & -2 \\ 0 & 0 & -2 & -1 & -1 & 4 \end{pmatrix} \ . \tag{4.67}$$

The columns are the $U(1)$s and the rows the non-abelian factors, always in the ordered form of Table.2. The (4.67) has five non-zero and one zero eigenvalue which corresponds to $A_1 + A_2 + A_3 + \tilde{A}_1 + \tilde{A}_2 + \tilde{A}_3$. Our result is in accordance with [44] where it had been shown that one of the six $U(1)$ factor remains unbroken. The independent axions that participate in the cancellation of the anomaly and the mass generation are only five.

The mass matrix for the anomalous $U(1)$s is

$$\frac{1}{2}M^2 = \frac{2}{\pi^2}\begin{pmatrix} 3 & 0 & -\sqrt{2} & 1 & 0 & 0 \\ 0 & 3 & -\sqrt{2} & 0 & 1 & 0 \\ -\sqrt{2} & -\sqrt{2} & 4 & 0 & 0 & 2 \\ 1 & 0 & 0 & 3 & 0 & -\sqrt{2} \\ 0 & 1 & 0 & 0 & 3 & -\sqrt{2} \\ 0 & 0 & 2 & -\sqrt{2} & -\sqrt{2} & 4 \end{pmatrix}. \tag{4.68}$$

Diagonalizing the mass matrix, we find that five $U(1)$ fields become massive and one remains massless. The effective field theory computation agrees with the result above.

4.8 4D Anomalous $U(1)$s and the relation to 6D anomalies upon decompactifications

4.8.1 Decompactification of the Z_6' orientifold

The axions that cancel the anomalies, being twisted RR fields, are localized on the fixed points of the internal dimensions. Since there are various orbifold sectors k, there are also various axions α_k^i localized on the fixed points of the internal tori where the k-th orbifold element acts [54]. Thus, in the Z_6' orientifold, the α_1^i, α_5^i axions are living in the 4D Minkowski space, the α_2^i, α_4^i in 4D Minkowski space plus the second torus T_2^2 and the α_3^i in 4D Minkowski space plus the third torus T_3^2.

The decompactification limit of the first torus ($\mathcal{V}_1 \to \infty$) does not have any special interest since none of the fields become six-dimensional.

Decompactification of the second torus ($\mathcal{V}_2 \to \infty$)

If we decompactify the second torus ($\mathcal{V}_2 \to \infty$) the 99 states that are coming from the $k = 2, 4$ sectors and the α_2^i, α_4^i axions become 6 dimensional fields. The gauge group is enhanced and can be found by the action of γ_2, γ_4 on the Chan-Paton factors. The fields of the other sectors remain four-dimensional and do not contribute to six-dimensional anomalies. The 'shift' vector will be $2V_9$, where V_9 is given in (4.34). Following the known procedure we find that the four-dimensional $U(4)\times U(4)\times U(8)$ gauge group is enhanced in $U(8) \times SO(16)$. The generators of the $U(4)_1 \times U(4)_2$ are enhanced in the generators of the $U(8)$ as $T_{U(8)} \sim T_{U(4)_1} \oplus \overline{T}_{U(4)_2}$ and the generators of the $U(8)$ in the generators of the $SO(16)$.

The rest of the matter fields are combined with some Kaluza-Klein states, that now become massless, to give the representations of the greater gauge group. The $(4,4,1)$, $(\bar{4},\bar{4},1)$ are now contained in the adjoint of the $U(8)$ as the $(1,1,28)$, $(1,1,\overline{28})$ are contained in the adjoint of the $SO(16)$. The $(6,1,1)$, $(1,\bar{6},1)$ form the antisymmetric $(28,1)$. The $(\bar{4},4,1)$ form the $(\overline{28},1)$. Finally, the $(4,1,8)$, $(1,\bar{4},\bar{8})$, $(4,1,\bar{8})$ and $(1,\bar{4},8)$ form the bi-fundamental $(8,16)$. Thus, the effective gauge group is the one that it was taken from the Z_3 six-dimensional orientifold (Table.2).

The spectrum of the Z_3 six-dimensional orientifold contains an anomalous gauge boson (chapter 4.7.2). By the way that the $U(4)\times U(4)\times U(8)$ gauge group is enhanced in $U(8)\times SO(16)$, we find that the anomalous gauge boson is $A_1 - A_2$ and becomes massive due to the six-dimensional Green-Schwarz mechanism. This mass can be evaluated by the six dimensional formulae and it is given in (4.62). The A_1+A_2 and A_3 are enhanced in the non-Abelian factors and they have no anomalies.

The contribution of the six-dimensional masses to the four-dimensional ones can be found by taking the $\mathcal{V}_2\to\infty$ limit of (4.36):

$$\frac{1}{2}M^2_{99,ij} = -\ \frac{1}{4\pi^3}\left(Tr[\gamma_2\lambda^9_i]Tr[\gamma_2\lambda^9_j] + Tr[\gamma_4\lambda^9_i]Tr[\gamma_4\lambda^9_j]\right)\ , \tag{4.69}$$

which is the same as the formula of the masses in the six-dimensional Z_3 orientifold (4.62) upon normalization. The sectors $k=2,4$ of the four-dimensional Z_6' orientifold in this limit are the $k=1,2$ sectors of the six-dimensional Z_3 orientifold. Using (4.32) and (4.34), we evaluate the mass-matrix of the anomalous $U(1)$s. The mass-matrix has two zero eigenvalues, with eigenvectors: A_3, A_1+A_2 and a massive state with eigenvalue:

$$-A_1 + A_2\ , \qquad m^2 = \frac{3}{\pi^3}\ , \tag{4.70}$$

as it was expected by the way that the initial $U(4)\times U(4)\times U(8)$ gauge group is enhanced in $U(8)\times SO(16)$. This six-dimensional contribution affects the four-dimensional mass (4.38).

The results confirm that anomalous gauge bosons in six-dimensions that become massive through the six-dimensional Green-Schwarz mechanism, contribute to the four-dimensional mass generation by a normalized term.

Decompactification of the third torus ($\mathcal{V}_3\to\infty$)

If we decompactify the third torus ($\mathcal{V}_3\to\infty$), all the string states from the $k=3$ sector and the a^i_3 axions become six-dimensional. The new gauge group can be found by the action of the γ_3 on the Chan-Paton. The orbifold rotation $3(v_1,v_2)=(1,-1)/2$ shows that D5-branes survive in this limit. The 'shift' vector is now $3V_a$ where V_a is given in(4.34). The four-dimensional $U(4)_\alpha\times U(4)_\alpha\times U(8)_\alpha$ gauge group (where $\alpha=5,9$) is enhanced to $U(16)_\alpha$ that is the gauge group of the Z_2 six-dimensional orientifold. The generators are $T_{U(16)}\sim T_{U(4)_1}\oplus T_{U(4)_2}\oplus \overline{T}_{U(8)}$. Therefore, $(1,\bar{4},\bar{8})_a$,

$(4,1,8)_a$, $(\bar{4},4,1)_a$ are enhanced in the adjoint of the $U(16)_a$. The $(6,1,1)_a$, $(1,4,\bar{8})_a$, $(1,1,\overline{28})_a$, $(4,4,1)_a$ form the antisymmetric 120_a. The $(\bar{4},1,8)_a$, $(\bar{4},\bar{4},1)_a$, $(1,1,28)_a$, $(1,\bar{6},1)_a$ are enhanced in the $\overline{120}_a$.

From the way that the generators are formed we can expect that the abelian factor of $U(16)_9$, $A \sim A_1 + A_2 - \sqrt{2}A_3$ where the coefficients are coming from the normalization of the generators of different rank. Similarly for the abelian factor of $U(16)_5$, $\tilde{A} \sim \tilde{A}_1 + \tilde{A}_2 - \sqrt{2}\tilde{A}_3$. As we have seen in section 4.7.1, the new gauge group contains two anomalous bosons in six dimensions which are linear combinations of the A and $\tilde{A}$. The other mass eigenstates are embedded in the non-abelian factors. The masses of the six-dimensional gauge bosons have been found in (4.59). The contribution of the six-dimensional mass-terms to the four-dimensional mass generation can be found by taking the $\mathcal{V}_3 \to \infty$ limit in (4.36), (4.37) and these are ($a = 5, 9$):

$$\frac{1}{2}M^2_{aa,ij} = -\frac{1}{3\pi^3}Tr[\gamma_3\lambda^a_i]Tr[\gamma_3\lambda^a_j] \ , \tag{4.71}$$

and for 59 states:

$$\frac{1}{2}M^2_{59,ij} = -\frac{1}{12\pi^3}Tr[\gamma_3\lambda^5_i]Tr[\gamma_3\lambda^9_j] \ , \tag{4.72}$$

which are the same (upon normalization) with the contributions of the six-dimensional generation of the Z_2 orientifold (section 4.7.1). In this limit, the $k = 3$ sector of the six-dimensional Z'_6 orientifold is the $k = 1$ sector of the six-dimensional Z_2 one. The mass-matrix has four zero eigenvalues, with eigenvectors: $\sqrt{2}\tilde{A}_1 + \tilde{A}_3$, $-\tilde{A}_1 + \tilde{A}_2$, $\sqrt{2}A_1 + A_3$, $-A_1 + A_2$ and two massive states with eigenvalues:

$$\begin{aligned} A_1 + A_2 - \sqrt{2}A_3 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{2}\tilde{A}_3 \quad &, \qquad m_3^2 = \frac{4}{\pi^3} \ , \\ -A_1 - A_2 + \sqrt{2}A_3 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{2}\tilde{A}_3 \quad &, \qquad m_5^2 = \frac{20}{3\pi^3} \ . \end{aligned} \tag{4.73}$$

The two massive states are the anomalous $U(1)$ which have been found in the spectrum of the original six-dimensional Z_2 orientifold. The indices are taken from the four-dimensional counting and denote which masses are affected by six-dimensional anomalies. Notice that the linear combinations agree with our expectations.

Another interesting limit of the Z'_6 orientifold is $\mathcal{V}_3 \to 0$. In this limit, the two linear combinations that are free of four-dimensional anomalies become massless. This is consistent with the fact that the six-dimensional anomalies which are responsible for their masses cancel locally in this limit.

4.8.2 Decompactification of the Z_6 orientifold

In the Z_6 orientifold, the $\alpha^i_1, a^i_2, a^i_4, \alpha^i_5$ axions are living in the 4D Minkowski space, and the α^i_3 in 4D Minkowski space plus the third torus T_3.

The decompactification limits of the first and second tori ($\mathcal{V}_1, \mathcal{V}_2 \to \infty$) do not have any special interest since none of the fields become six-dimensional and there are no six-dimensional anomalies.

Decompactification of the third torus ($\mathcal{V}_3 \to \infty$)

If we decompactify the third torus ($\mathcal{V}_3 \to \infty$), all the string states from the $k = 3$ sector and the a_3^i axions become six-dimensional. The rest of the sectors and axions remain four-dimensional and do not contribute to six-dimensional anomalies. The new gauge group can be found by the action of the γ_3 on the Chan-Paton. The orbifold rotation $3(v_1, v_2) = (1, -1)/2$ shows that D5-branes survive in this limit. The 'shift' vector is now $3V_a$ where V_a is given in(4.42). The old $U(6) \times U(6) \times U(4)$ gauge group is enhanced to $U(16)$, which is the gauge group of the Z_2 six-dimensional orientifold (Table 1). The generators are combined as $T_{U(16)} \sim T_{U(6)_1} \oplus T_{U(6)_2} \oplus \overline{T}_{U(4)}$. Therefore, we can determine how the old spectrum is enhanced to the new one. The $(\bar{6}, 1, \bar{4})$, $(1, 6, 4)$ and $(6, \bar{6}, 1)$ combine in the adjoint of $U(16)$. The $(15, 1, 1)$, $(1, 6, \bar{4})$ are in the antisymmetric 120 and $(1, \overline{15}, 1)$, $(\bar{6}, 1, 4)$ in the $\overline{120}$.

By the way that the generators of the $U(6)^2 \times U(4)$ are enhanced to the $U(16)$ we can expect that the six-dimensional $U(1)$ gauge boson of the $U(16)$ will be a linear combination $A_1 + A_2 - \sqrt{\frac{2}{3}} A_3$ where the normalization coefficient in front of A_3 takes into account the difference of the rank. Similarly for the tilde.

The contributions of the six-dimensional anomalies to the four-dimensional mass generation are given by the $\mathcal{V}_3 \to \infty$ limit in (4.44), (4.45). We find (for $a = 5, 9$):

$$\frac{1}{2} M^2_{aa,ij} = -\frac{1}{3\pi^3} Tr[\gamma_3 \lambda_i^a] Tr[\gamma_3 \lambda_j^a] \ , \tag{4.74}$$

while, for twisted open strings:

$$\frac{1}{2} M^2_{59,ij} = -\frac{1}{12\pi^3} Tr[\gamma_3 \lambda_i^5] Tr[\gamma_3 \lambda_j^9] \ , \tag{4.75}$$

which are the same (upon normalization) as the contributions of the six-dimensional generation of the Z_2 orientifold (section 4.7.1). The mass-matrix has four zero eigenvalues, with eigenvectors: $\sqrt{\frac{2}{3}} \tilde{A}_1 + \tilde{A}_3$, $-\tilde{A}_1 + \tilde{A}_2$, $\sqrt{\frac{2}{3}} A_1 + A_3$, $-A_1 + A_2$ and two massive states with eigenvalue:

$$\begin{aligned} & A_1 + A_2 - \sqrt{\frac{2}{3}} A_3 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{\frac{2}{3}} \tilde{A}_3 \ , \qquad m_4^2 = \frac{4}{\pi^3} \ , \\ & A_1 + A_2 - \sqrt{\frac{2}{3}} A_3 + \tilde{A}_1 + \tilde{A}_2 - \sqrt{\frac{2}{3}} \tilde{A}_3 \ , \qquad m_5^2 = \frac{20}{3\pi^3} \ . \end{aligned} \tag{4.76}$$

The two massive states are the anomalous $U(1)$s which have been found in the spectrum of the original six-dimensional Z_2 orientifold. It is easy to verify that the four-dimensional massless state $A_1 + A_2 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{6}(A_3 - \tilde{A}_3)$ (4.50) is still massless in six dimensions.

4.9 Chapter Conclusions

In this chapter we have shown that four-dimensional non-anomalous $U(1)$s can become massive if in decompactification limits they suffer from six-dimensional anomalies.

We have studied several four-dimensional orientifolds. In the decompactification limit, there are sectors in such orientifolds that become six dimensional. The original four-dimensional massless spectrum, combined with Kaluza-Klein states that become massless in this limit, enhanced to the massless spectrum of six-dimensional orientifolds. Some RR axions also become six-dimensional fields.

In the $6D$ orientifolds, we have calculated the stringy anomalous $U(1)$ masses that are in accordance with six-dimensional anomalies. The six-dimensional RR axions contribute to the mass-generation of the anomalous $U(1)$s through the Green-Schwarz mechanism.

We verified that the six-dimensional mass-matrix is the same as the volume dependant contribution to the four-dimensional matrix. Thus, six-dimensional anomalies play indirectly a role in four-dimensional masses and explain why some non-anomalous $U(1)$ gauge bosons have a non-zero mass.

5. Anomalous $U(1)$s masses in non-supersymmetric open string vacua

In this section, we are interested in the masses of the anomalous $U(1)$s in non-supersymmetric models since such are the models that will eventually represent the low energy physics of the Standard Model. In particular, intersecting-brane realizations of the Standard Model are generically non-supersymmetric. We calculate the mass formulae using the "background field method" and find that they are the same as the supersymmetric ones when we have cancellation of all tadpoles. In cases where NSNS tadpoles do not vanish, there are extra contributions proportional to the non-vanishing tadpole terms.

The formulae are valid even if we add Wilson lines that move the branes away from the fixed points. The Wilson lines generically break the gauge group and they will affect the masses of the anomalous $U(1)$s through the traces of the model dependent γ matrices.

5.1 Computing with the background-field method

Our purpose is to evaluate the bare masses of the anomalous $U(1)$ which appear in the one-loop amplitudes with boundaries where two gauge fields are inserted [47]. Here we will use another technique which is based on turning on a magnetic field on the D-branes and pick out the second order terms to this magnetic field. This method is called "the background-field method" [50]. We turn on different magnetic fields B_a in every stack of branes, longitudinal to x^1, a non-compact dimension,

$$F^a_{23} = B_a Q_a \ , \tag{5.1}$$

where Q_a are the $U(1)_a$ generators from every stack of branes. The effect of the magnetic field on the open-string spectrum is to shift the oscillator frequencies of the string non-compact $x^2 + ix^3$ coordinate by an amount ϵ_a:

$$\epsilon_a = \frac{1}{\pi}[\arctan(\pi q_i^a B_a) + \arctan(\pi q_j^a B_a)] \ , \tag{5.2}$$

where q_i^a, q_j^a are the $U(1)_a$ charges of the i, j endpoints. The Chan-Paton states λ_{ij} that describe the endpoint i, j of the open string, are the generators of gauge group that remains after the orientifold construction. Diagonalizing these matrixes, we can replace the Q_i with λ_{ii}.

The expansion of the one-loop vacuum energy is:

$$\Lambda(B) = \frac{1}{2}\left(\mathcal{T} + \mathcal{K} + \mathcal{A}(B) + \mathcal{M}(B)\right) = \Lambda_0 + \frac{1}{2}\left(\frac{B}{2\pi}\right)^2 \Lambda_2 + \dots \ , \tag{5.3}$$

where B one of the different magnetic fields. Generically, it appears a linear to B term that is a pour tadpole and it is coming from the RR sector. This term vanishes when we have tadpole cancellation. The quadratic term in the background field contains a lot of information. In the IR limit, we have a logarithmic divergence whose coefficient is the β-function. The UV limit provides the mass-term of the anomalous gauge bosons. The finite part of this term is the threshold correction in the gauge couplings [50]. The annulus amplitude in the Z_N type I orientifolds (without the magnetic field) can be written as:

$$\mathcal{A}^{ab} = -\frac{1}{2N}\sum_{k=1}^{N-1}\int_0^\infty \frac{dt}{t}\mathcal{A}_k^{ab}(q) \ , \tag{5.4}$$

where a, b the different kind of D-branes at the ends of the open strings. The $\mathcal{A}_k^{ab}$ is the contribution of the kth sector:

$$\mathcal{A}_k^{ab} = \frac{1}{4\pi^4 t^2}\mathrm{Tr}[\gamma_a^k]\mathrm{Tr}[\gamma_b^k]\sum_{\alpha,\beta=0,1}\eta^{\alpha\beta}\frac{\vartheta\left[\substack{\alpha\\ \beta}\right]}{\eta^3}Z_{int,k}^{ab}\left[\substack{\alpha\\ \beta}\right]|_{\mathcal{A}} \ . \tag{5.5}$$

Similarly, we can exchange $\mathcal{A}$ with $\mathcal{M}$ in (5.4) to have an analogous expression for the Möbius strip. The $\mathcal{M}_k^a$ is given by:

$$\mathcal{M}_k^{a} = -\frac{1}{4\pi^4 t^2}\mathrm{Tr}[\gamma_a^{2k}]\sum_{\alpha,\beta=0,1}\eta^{\alpha\beta}\frac{\vartheta\left[\substack{\alpha\\ \beta}\right]}{\eta^3}Z_{int,k}^{a}\left[\substack{\alpha\\ \beta}\right]|_{\mathcal{M}} \ . \tag{5.6}$$

In the presence of the background magnetic field B_a, the above amplitudes become:

$$\begin{aligned}
\mathcal{A}_k^{ab}(B) &= \frac{i}{4\pi^3 t}\mathrm{Tr}\left[(B_a\lambda_a\gamma_a^k\otimes\gamma_b^k+\gamma_a^k\otimes B_b\lambda_b\gamma_b^k)\sum_{\alpha\beta}\eta^{\alpha\beta}\frac{\vartheta\left[\substack{\alpha\\ \beta}\right](\frac{i\epsilon t}{2})}{\vartheta\left[\substack{1\\ 1}\right](\frac{i\epsilon t}{2})}\right]Z_{int,k}^{ab}\left[\substack{\alpha\\ \beta}\right]|_{\mathcal{A}} \ , \\
\mathcal{M}_k^{a}(B) &= -\frac{i}{2\pi^3 t}\mathrm{Tr}\left[B_a\lambda_a\gamma_a^{2k}\sum_{\alpha\beta}\eta^{\alpha\beta}\frac{\vartheta\left[\substack{\alpha\\ \beta}\right](\frac{i\epsilon t}{2})}{\vartheta\left[\substack{1\\ 1}\right](\frac{i\epsilon t}{2})}\right]Z_{int,k}^{a}\left[\substack{\alpha\\ \beta}\right]|_{\mathcal{M}} \ .
\end{aligned} \tag{5.7}$$

Notice that the only differences from (5.5, 5.6) are in the contribution of the non-compact part of the partition functions. This is expected since the presence of the magnetic fields affect only the x^2, x^3 coordinates. Therefore, the expressions (5.7) are valid for all kinds of orientifold models.

Since we are interested in the quadratic B^2 terms of the above amplitudes, we expand the above formulae to quadratic order in the background field, using the following Taylor expansions:

$$\epsilon \simeq \begin{cases} B_a\lambda_a\otimes 1 + 1\otimes B_b\lambda_b & \text{in} \quad \mathcal{A}^{ab}, \\ 2B_a\lambda_a & \text{in} \quad \mathcal{M}^a. \end{cases} \tag{5.8}$$

The zero-order B terms are the amplitudes in the absence of the magnetic field (5.5, 5.6). These expressions give the tadpole cancellation conditions in virtue of the UV divergences. The linear to B terms appear from the $a = b = 1$ sector in (5.7). This is a pour tadpole and vanishes when we have tadpole cancellation. Therefore, it does not affect higher order in B amplitudes. The second order-terms on B are:

$$\mathcal{A}^{ab}_{2,k} = \pi^2 i \left[\mathrm{Tr}[\lambda_a^2\gamma_a^k]\mathrm{Tr}[\gamma_b^k] + \mathrm{Tr}[\gamma_a^k]\mathrm{Tr}[\lambda_b^2\gamma_b^k] + 2\mathrm{Tr}[\lambda_a\gamma_a^k]\mathrm{Tr}[\lambda_b\gamma_b^k]\right] F_k^{ab}|_{\mathcal{A}} \ , \quad (5.9)$$

$$\mathcal{M}^{a}_{2,k} = -4\pi^2 i \ \mathrm{Tr}[\lambda_a^2\gamma_a^{2k}] \ F_k^{aa}|_{\mathcal{M}} \ , \quad (5.10)$$

defining F_k^{ab} as a term which contains all the spin-structure and the orbifold information:

$$F_k^{ab}|_\sigma = \frac{1}{4\pi^4}\sum_{\alpha\beta}\eta_{\alpha\beta}\ \pi i\partial_\tau \left[\log\frac{\vartheta[^\alpha_\beta](0|\tau)}{\eta(\tau)}\right]\frac{\vartheta[^\alpha_\beta](0|\tau)}{\eta^3(\tau)} Z^{ab}_{int,k}[^a_b]|_\sigma \ , \quad (5.11)$$

for both surfaces (the choice of τ define the surface σ). Note that the $a = b = 1$ sector is not contained in the (5.11). This term can be formally written as the supertrace over states from the open ab k-orbifold sector:

$$F_k^{ab}|_\sigma = \frac{|G|}{(2\pi)^2} Str^{ab}_{k,\mathrm{open}}\left[\frac{1}{12} - s^2\right] e^{-tM^2/2}\Big|_\sigma \ , \quad (5.12)$$

where the s is the $4D$ helicity. Thus, for:

- *large* τ_2 we have:
$$\lim_{\tau_2\to\infty} F_k^{ab} = C^{ab}_{k,IR} + \mathcal{O}[e^{-2\pi\tau_2}] \ , \quad (5.13)$$
with $C^{ab}_{k,IR} = \frac{|G|}{(2\pi)^2} Str_k\left[\frac{1}{12} - s^2\right]_{open}$.

- *small* τ_2 we have
$$\lim_{\tau_2\to 0} F_k^{ab} = \frac{1}{\tau_2}\left[C^{ab}_{k,UV} + \mathcal{O}[e^{-\frac{\pi}{2\tau_2}}]\right] \ , \quad (5.14)$$
where $C^{ab}_{k,IR} = \frac{|G|}{(2\pi)^2} Str_k\left[\frac{1}{12} - s^2\right]_{closed}$. The helicity supertrace is now in the closed-string k-sector mapped from the open k-sector dy a modular transformation.

Notice that in the annulus amplitude (5.9), the two first terms are proportional to the square of the B field. This cases are proportional to annulus amplitudes $\mathcal{A}_2$, where two vertex-operators (VOs) are on the same boundary. In the last component of (5.9), the B fields are coming from the opposite D-branes and is proportional to $\mathcal{A}_{11}$, with the VOs on different boundaries. The (5.10) is proportional to a Möbius strip amplitude with the insertion of two VOs.

The IR limit $t \to \infty$ can be found easily using the (5.13). We regularize the integral by $\mu \to 1/t^2$ and we find the β-function:

$$
\begin{aligned}
b &= -\frac{2}{N}\sum_{k=1}^{N-1} \lim_{t\to\infty} \left(\mathcal{A}^{ab}_{2,k}(t) + \mathcal{M}^{a}_{2,k}(t)\right) \\
&= -\frac{2\pi^2 i}{N}\sum_{k=1}^{N-1}\Bigg[\Big(\mathrm{Tr}[\lambda_a^2\gamma_a^k]\mathrm{Tr}[\gamma_b^k] + \mathrm{Tr}[\gamma_a^k]\mathrm{Tr}[\lambda_b^2\gamma_b^k] \\
&\quad +2\mathrm{Tr}[\lambda_a\gamma_a^k]\mathrm{Tr}[\lambda_b\gamma_b^k]\Big) C^{ab}_{k,IR}|_{\mathcal{A}} - 4\mathrm{Tr}[\lambda_a^2\gamma_a^{2k}]C^{a}_{k,IR}|_{\mathcal{M}}\Bigg] . \qquad (5.15)
\end{aligned}
$$

For the UV limit $t \to 0$, we use the (5.14) and we regularize the integral by $\mu \le t$. The A_2 and M together are giving terms proportional to the tadpole cancellation conditionsTherefore, when we have vanishing of RR and NSNS tadpoles, the masses of the anomalous gauge bosons are given by $\mathcal{A}_{11}$:

$$
\frac{1}{2}M^2_{aa} = \frac{\pi^2 i}{N}\sum_{k=1}^{N-1}\mathrm{Tr}[\lambda_a\gamma_a^k]^2 C^{ab}_{k,UV}|_{\mathcal{A}} , \qquad (5.16)
$$

$$
\frac{1}{2}M^2_{59} = \frac{\pi^2 i}{2N}\sum_{k=1}^{N-1}\mathrm{Tr}[\lambda_5\gamma_5^k]\mathrm{Tr}[\lambda_9\gamma_9^k] C^{59}_{k,UV}|_{\mathcal{A}} , \qquad (5.17)
$$

where $\alpha = 5, 9$. When we have non-vanishing NSNS tadpoles there is an extra contribution to the mass formulas, proportional to the non-vanishing tadpole.

The formulae (5.16, 5.17) still hold even if we add Wilson lines. Generically, adding a Wilson line we shift the windings or the momenta in a coordinate with Neumann or Dirichlet boundary conditions respectively. This breaks the gauge group. In the transverse (closed) channel the shifts appears as phases $e^{2\pi i n\theta}$ where θ the shift and n the momenta or windings respectively to the above. Since only the massless states contribute in the UV limit, the effect of the Wilson line will appear only in the traces of the γ matrices.

The threshold correction [3] is the finite part of (5.9) and (5.10). Generically we have:

$$
\frac{16\pi^2}{g^2} = \frac{16\pi^2}{g_0^2} - \frac{1}{2N}\sum_{k=1}^{N-1}\int_{\mu}^{1/\mu^2}\frac{dt}{t}\left(\mathcal{A}^{ab}_2 + \mathcal{M}^{a}_2\right) - b\log\frac{\mu^2}{M^2} - \frac{1}{2}M^2_{ab}\frac{1}{\mu} , \qquad (5.18)
$$

where we separate the divergencies from the quadratic terms to B. The above formulae for the β-function, the corrections to the gauge couplings and the masses of the anomalous $U(1)$s are the same to the supersymmetric ones found in [47, 50]. Next, we will apply the above formulae to a non-supersymmetric model that has been constructed by Scherk-Schwarz deformation [34].

5.2 A four-dimensional non-supersymmetric orientifold example

In this section we will evaluate the masses of the anomalous $U(1)$s in a Z_2 orientifold model where supersymmetry is broken by a Scherk-Schwarz deformation. The spectrum is provided in Table.3 [34]. We remind that the tadpole cancellation provides two different solutions that depend on the inequivalent choices of $\gamma_h^2 = \pm 1$ where γ_h the action of h on the Chan-Paton matrixes. The 16-dimensional 'shift' vector of the Z_2 orientifold is (4.57) The 'shift' vector of the SS deformation is as it was defined in (3.84). In both cases $a + b = 16$, however we implement for simplicity $a = b = 8$. The massless spectrums are provided in Table 1. The gauge group in both cases is the same. The only difference appears in the exchange of the antisymmetric reps with the bi-fundamental $(8, 8) + (\overline{8}, \overline{8})$ in the $(99)/(55)$ matter sector. The spectrum is anomaly-free in $4D$ since it is non-chiral.

The internal annulus partition functions for 99, 55 and 59 strings are:

$$Z^{99,55}_{int,k}[{}^{\alpha}_{\beta}] = -\sum_{s,r=0}^{1}(-1)^{\alpha s+\beta r}\left[(-1)^{s\cdot m_4}P_{m_4}P_{m_5}\right]\frac{\vartheta[{}^{\alpha}_{\beta}](0|\tau)}{\eta(\tau)}(2\sin\frac{\pi k}{2})^2\prod_{j=1}^{2}\frac{\vartheta[{}^{\alpha}_{\beta+2v_jk}](0|\tau)}{\vartheta[{}^{1}_{1+2v_jk}](0|\tau)}\ ,$$

$$Z^{59}_{int,k}[{}^{\alpha}_{\beta}] = 2\sum_{s,r=0}^{1}(-1)^{\alpha s+\beta r}\left[(-1)^{s\cdot m_4}P_{m_4}P_{m_5}\right]\frac{\vartheta[{}^{\alpha}_{\beta}](0|\tau)}{\eta(\tau)}\prod_{j=1}^{2}\frac{\vartheta[{}^{\alpha+1}_{\beta+2v_jk}](0|\tau)}{\vartheta[{}^{0}_{1+2v_jk}](0|\tau)}\ . \tag{5.19}$$

For $s = r = 0$, we have the internal partition function of a $T^2 \times K^3/Z_2$ orientifold. s denotes the direct action of the SS deformation and r the twisted sector. The $(-1)^{s\cdot m_4}P_{m_4}P_{m_5}$ is the lattice sum over momenta along the first torus T^2:

$$(-1)^{s\cdot m_i}P_{m_i}(i\tau_2/2) = \frac{1}{\eta(i\tau_2/2)}\sum_{m_i}(-1)^{s\cdot m_i}q^{\frac{\alpha'}{4}\left(\frac{m_i}{R_i}\right)^2}\ , \tag{5.20}$$

For $s = 1$ we have the SS deformation that shifts the m_4 momenta. As we mention before, $r = 0, 1$ denotes the h untwisted and twisted sectors respectively. However we will neglect the twisted sector since it requires the insertion of anti-D-branes [34].

To evaluate the masses of the anomalous bosons, we insert (5.19) and (5.11) in the mass formulae. After some 'thetacology' we find $F^{\alpha\beta}_{k=1}$ for $\alpha, \beta = 5, 9$. In the UV region, only the first terms in both formulae contribute to the mass of the anomalous $U(1)$s. Terms (that contains the SS action h) after the Poisson re-summation become proportional to $W_{\nu_4+1/2}$ and do not contribute to the $C^{99,55,59}_{UV}$. Since SS deformation does not contribute to the mass terms of the anomalous $U(1)$s, we can directly evaluate their masses for both two inequivalent solutions ($\gamma_h^2 = \pm 1$):

$$\begin{aligned}\frac{1}{2}M^2_{\alpha\alpha,ij} &= -\frac{4\pi^2}{4}\mathrm{Tr}[\lambda^a_i\gamma_g]\mathrm{Tr}[\lambda^a_j\gamma_g]\frac{\mathcal{V}_1}{\pi^2\alpha'}\\ &= -\frac{\mathcal{V}_1}{\alpha'}\left(-\frac{i}{\sqrt{8}}\sin[2\pi V^a_i]\right)\left(-\frac{i}{\sqrt{8}}\sin[2\pi V^a_j]\right) = \frac{\mathcal{V}_1}{8\alpha'}\ ,\end{aligned} \tag{5.21}$$

$$\frac{1}{2}M^2_{59,ij} = \frac{4\pi^2}{2\times 4}\mathrm{Tr}[\lambda^5_i\gamma_g]\mathrm{Tr}[\lambda^9_j\gamma_g]\frac{\mathcal{V}_1}{2\pi^2\alpha'} = -\frac{\mathcal{V}_1}{32\alpha'}\ , \tag{5.22}$$

where $\alpha = 5, 9$. The mass-matrix has two massless gauge bosons $-\tilde{A}_1 + \tilde{A}_2$, $-A_1 + A_2$ and two massive $A_1 + A_2 + \tilde{A}_1 + \tilde{A}_2$, $-A_1 - A_2 + \tilde{A}_1 + \tilde{A}_2$ with masses $3\mathcal{V}_1/32\alpha'$, $5\mathcal{V}_1/32\alpha'$ respectively.

There are no anomalous $U(1)$s in these models since the spectrum is non-chiral. However, the existence of the two massive gauge bosons are the consequence of $6D$ anomalies [36, 46, 47, 48]. The decompactification limit of the first torus (where the SS deformation acts) leads to the N=1 $6D$ Z_2 orientifolds that contains two anomalous $U(1)$s that become massive via the Green-Schwarz mechanism. Therefore, axions that participate in the anomaly cancellation in the $6D$ model, contribute to the $4D$ masses of the anomalous $U(1)$s by volume dependant terms. The ratio of the masses found in [48] for the Z_2 supersymmetric orientifold are the same to the above.

5.3 Chapter Conclusions

In this section we evaluated the general mass formula for the anomalous $U(1)$s in non-supersymmetric orientifolds. We have shown that the supersymmetric formulae of [47] are also valid in non-supersymmetric orientifolds provided that the tadpoles cancel.

6. Anomalous $U(1)$s and spontaneous symmetry breaking

In D-brane realizations of the Standard Model we must have at least two Higgs in order to be able to give masses to all quarks and leptons [47]. Generically, each Higgs is charged under the anomalous $U(1)$s.

We will analyze here the case of a single anomalous $U(1)$ coupled to a Higgs field in order to discuss the relevant effects. Consider a toy model with an anomalous $U(1)$ gauge field A'_μ, chiral charged fermions and a complex Higgs. We also have an axion a to cancel the anomalies. The relevant part of the low-energy effective Lagrangian can be written as:

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4g_{A'}^2}F_{A'}^2 + M_s^2(\partial a + A')^2 + D_\mu H D^\mu H^* + V(|H|^2) \\ & + Q_L\bar{\psi}_L \not{A}'\psi_L + Q_R\bar{\psi}_R\not{A}'\psi_R + h\bar{\psi}_L\psi_R H + c.c.\end{aligned} \tag{6.1}$$

This Lagrangian (6.1) is invariant under the "anomalous" $U(1)$ transformations.

$$\begin{aligned}&A'_\mu \to A'_\mu + \partial_\mu\epsilon \ , \qquad && a \to a - \epsilon \\ &\psi_L \to e^{iQ_L\epsilon}\psi_L \ , && \psi_R \to e^{iQ_R\epsilon}\psi_R \\ &H \to e^{i(Q_R-Q_L)\epsilon}H\end{aligned} \tag{6.2}$$

There are two sources of gauge symmetry breaking. One is the stringy mass term and the other is the non-zero expectation value of the Higgs. Writing $H = re^{i\phi}$, the Higgs potential fixes the vacuum expectation value $\langle r\rangle = v$. The kinetic term of the Higgs field gives an extra contribution to the A' mass term:

$$v^2(\partial\phi + \Delta Q A')^2 \ . \tag{6.3}$$

To proceed with the one-loop calculation, it is necessary to add a gauge fixing term

$$\mathcal{L}_{gaugefixing} = \lambda\Big(\partial A' + \frac{cM_s^2\alpha}{\lambda} - \frac{\Delta Q v^2\phi}{\lambda}\Big)^2 \ , \tag{6.4}$$

which keeps A'_μ orthogonal to a and ϕ. Redefining $\tilde{a} = Ma$ and $\tilde{\phi} = v\phi$ we can diagonalize the axions by an $SO(2)$ rotation

$$\begin{pmatrix} a' \\ \phi' \end{pmatrix} = \begin{pmatrix} \cos\theta' & \sin\theta' \\ -\sin\theta' & \cos\theta' \end{pmatrix}\begin{pmatrix} \tilde{a} \\ \tilde{\phi} \end{pmatrix} \ , \tag{6.5}$$

where $\cos\theta' = \frac{cM_s}{\sqrt{c^2M_s^2+v^2\Delta Q^2}}$ and $\sin\theta' = \frac{\Delta Q v}{\sqrt{c^2M_s^2+v^2\Delta Q^2}}$. Now, the effective Lagrangian has the form:

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4g_{A'}^2}F_{A'}^2 + m_{A'}^2A'^2 + (\partial b')^2 + m_{b'}^2 b'^2 + (\partial\phi')^2 \\ & + Q_L\bar{\psi}_L\not{A}'\psi_L + Q_R\bar{\psi}_R\not{A}'\psi_R + hv\bar{\psi}_L\psi_R e^{i(\sin\theta' b' + \cos\theta'\phi')/v} + c.c.\end{aligned} \tag{6.6}$$

The masses are:

$$m_\psi = hv \quad , \qquad m_{A'} = \sqrt{c^2 M_s^2 + v^2 \Delta Q^2} \ ,$$
$$m_{\phi'} = 0 \quad , \qquad m_{a'} = \sqrt{c^2 M_s^2 + v^2 \Delta Q^2}/\sqrt{\lambda}. \tag{6.7}$$

We define $m_B = \mu$ for simplicity. The propagators are:

$$D^{\mu\nu}_{A'}(k) = \frac{-ig^{\mu\nu}}{k^2 - \mu^2} + (1 - \lambda^{-1})\frac{ik^\mu k^\nu}{(k^2 - \mu^2)(k^2 - \mu^2/\lambda)} \quad ,$$
$$G_{\phi'}(k) = \frac{i}{k^2} \quad , \qquad G_{a'}(k) = \frac{i}{k^2 - \mu^2/\lambda} \tag{6.8}$$

We will gauge fix $\phi' = 0$ (physical gauge) and the Yukawa couplings between the physical axion and the fermions is:

$$h_{eff} = h\frac{cM_s}{\sqrt{c^2 M_s^2 + v^2 \Delta Q^2}} \ . \tag{6.9}$$

In order to suppress this interaction we must have $cM_s \ll v\Delta Q$.

7. A D-brane realization of the Standard Model

Bottom to top model building shows that the SM can be embedded in a product of unitary groups appearing on D-brane stacks as a subgroup of $U(3) \times U(2) \times U(1) \times U(1)'$ [41]. However, for the rest of our study we will omit the last single brane that provides the $U(1)'$ gauge boson since it does not participate to the hypercharge. We will concentrate onto $U(3) \times U(2) \times U(1)$[18]. Each $U(n)$ factor arises from n coincident D-branes. As $U(3) = SU(3) \times U(1)$, a string with one end on this group of branes is a triplet under $SU(3)$ with $Q_3 = \pm 1$ abelian charge. Thus, Q_3 is identified with the gauged baryon number. Similarly, the second factor arises from two coincident D-branes ("weak" branes) and the gauged overall abelian charge Q_2 is identified with the weak-doublet number. Both collections have their own gauge couplings g_3, g_2 that are functions of the string coupling g_s and possible compactification volumes. The necessity for at least an extra $U(1)$ factor is due to the fact that we cannot express the hypercharge as a linear combination of baryon and weak-doublet numbers[19]. The U(1) brane can be in principle independent of the other branes and has in general a different gauge coupling g_1. In [41], the $U(1)$ brane has been put on top of either the color or the weak D-branes. Thus, g_1 is equal to either g_3 or g_2.

Let us denote by Q_3, Q_2 and Q_1 the three $U(1)$ charges of $U(3) \times U(2) \times U(1)$. These charges can be fixed so that they lead to the right hypercharge. In order that we can match the measured gauge couplings with the ones appropriate for the brane-configuration and also avoid hierarchy problems we find that we have to put the $U(1)$ brane on top of the color branes. Consequently we set $g_1 = g_3$. This fixes the string scale to be between 6 to 8 TeV [41]. There are two possibilities for charge assignments. Under $SU(3) \times SU(2) \times U(1)_3 \times U(1)_2 \times U(1)_1$ the members of a given quark and lepton family have the following quantum numbers:

$$\begin{array}{ll} Q(\mathbf{3},\mathbf{2};1,1+2z,0)_{1/6} & L(\mathbf{1},\mathbf{2};0,1,z)_{-1/2} \\ u^c(\bar{\mathbf{3}},\mathbf{1};-1,0,0)_{-2/3} & l^c(\mathbf{1},\mathbf{1};0,0,1)_{1} \\ d^c(\bar{\mathbf{3}},\mathbf{1};-1,0,1)_{1/3} & \end{array} \tag{7.1}$$

where $z = 0, -1$. From (7.1) and the requirement that the Higgs doublet has hypercharge 1/2, one finds two possible assignments for it:

$$H \quad (\mathbf{1},\mathbf{2};0,1+2z,1)_{1/2} \qquad H' \quad (\mathbf{1},\mathbf{2};0,-(1+2z),0)_{1/2} \tag{7.2}$$

[18] In fact the minimal embedding is in $U(3) \times U(2)$, however such an embedding has phenomenological problems: proton stability cannot be protected and some SM fields cannot get masses.

[19] It turns out that a complete collection of SM D-branes (one that can accommodate all the endpoints of SM strings) includes a fourth $\mathrm{U}(1)_b$ component that does not participate in the hypercharge. Such a D-brane wraps the large dimensions, and consequently its coupling is ultra weak. It is also anomalous and thus massive [41] . Due to its weak coupling its contributions to magnetic moments are negligible compared to the ones we consider. We will thus ignore it in this paper.

The trilinear Yukawa terms are

$$z = 0 \ : \qquad H'Qu^c \quad , \quad H^\dagger Ll^c \ , \quad H^\dagger Qd^c \tag{7.3}$$

$$z = -1 \ : \qquad H'Qu^c \quad , \quad H'^\dagger Ll^c \ , \quad H^\dagger Qd^c \tag{7.4}$$

In each case, two Higgs doublets are necessary to give masses to all quarks and leptons. The $U(3)$ and $U(1)$ branes are D3 branes. The U(2) branes are D7 branes whose four extra longitudinal directions are wrapped on a four-torus of volume 2.5 in string units [41]. The spectator $U(1)_b$ brane is stretching in the bulk but the fermions that end on it do not have KK excitations. Thus, the only SM field that has KK excitations is a linear combination of the hypercharge gauge boson and the two anomalous $U(1)$ gauge bosons. The masses of KK states, are shifted from the basic state by multiples of $0.8M_s$.

We will now describe the structure of the gauge sector for the D-brane configuration above. We denote by A^i_μ the $U(1)_i$ gauge fields and $F^i_{\mu\nu}$ their corresponding field strengths. Also we denote $G^\beta_{\mu\nu}$ the field strengths of the non-abelian gauge group where β runs over the two simple factors. There is also a set of two axion fields b^α with normalized kinetic terms. Starting from the kinetic terms of the gauge fields and requesting for the cancellation of the $QT^\alpha T^\alpha$ mixed anomalies, we can write down the most general low energy action

$$\begin{aligned}
\mathcal{L} = &-\frac{1}{4}\sum_i F^i_{\mu\nu}F^{i,\mu\nu} + \sum_i \bar\psi Q_i \not{A}^i \psi - \frac{1}{4}\sum_a Tr G^a_{\mu\nu}G^{a,\mu\nu} \\
&+ \sum_{\alpha,\beta} \Lambda_{\alpha,\beta}\frac{b^\alpha}{M_s}\epsilon^{\mu\nu\rho\sigma} Tr[G^\beta_{\mu\nu}G^\beta_{\rho\sigma}] + \sum_\alpha (\partial_\mu b^\alpha - M_s\lambda^{\alpha i}A^i_\mu)(\partial^\mu b^\alpha - M_s\lambda^{\alpha j}A^{j,\mu}) \\
&+ \sum_{\alpha,i,j}\frac{C_{\alpha ij}}{M_s}\epsilon^{\mu\nu\rho\sigma}\partial_\mu b^\alpha A^i_\nu F^j_{\rho\sigma} + \sum_{i,j,k}\frac{D_{ijk}}{M_s}\epsilon^{\mu\nu\rho\sigma}A^i_\mu A^j_\nu F^k_{\rho\sigma} \\
&+ \sum_\alpha Z_\alpha \frac{b^\alpha}{M_s}\epsilon^{\mu\nu\rho\sigma} Tr[R_{\mu\nu}R_{\rho\sigma}]
\end{aligned} \tag{7.5}$$

where charge operators Q_i contain all coupling dependence. The last term involves the curvature two-form $R_{\mu\nu}$ and is responsible for the cancellation of the gravitational anomalies. Under $U(1)$ gauge transformations (modified by the anomaly)

$$A^i_\mu \to A^i_\mu + \partial_\mu \epsilon^i \quad , \quad b^\alpha \to b^\alpha + \sum_i \lambda^{\alpha i}A^i_\mu \tag{7.6}$$

we have

$$\begin{aligned}
&D_{ijk} = -D_{jik} \quad , \quad \sum_a \Lambda_{\alpha,\beta}\lambda^{\alpha,\iota} = Tr[Q^i T_\beta T_\beta] \\
&D_{ijk} = -\sum_a C_{\alpha ij}\lambda^{\alpha k} = Tr[Q^i Q^j Q^k] \quad , \quad \sum_a Z_\alpha \lambda^{\alpha,i} = Tr[Q^i]
\end{aligned} \tag{7.7}$$

The only free parameters which are not fixed by the anomalies are $\lambda^{\alpha i}$. These define the mass matrix of gauge bosons $M^2_{ij} = M_s^2 \lambda^{\alpha i}\lambda^{bj}$. This matrix is symmetric and has a zero eigenvalue corresponding to the non-anomalous hypercharge. The $\lambda^{\alpha i}$ can be computed by a string calculation. The parameters remaining in the mass matrix is the 2×2 submatrix of the anomalous gauge bosons.

Now, we will describe the couplings of the gauge fields in more details. The two first terms of (7.5) are written as

$$\mathcal{L} = -\frac{1}{4}\sum_i F^i F^i + \sum_i \frac{g_i}{\sqrt{i}}\bar{\psi} Q_i \not{A}^i \psi \tag{7.8}$$

where g_i are the $SU(i)$ coupling constants and the charges have the standard integral normalization (7.1). We will set $x = \frac{g_3/\sqrt{3}}{g_2/\sqrt{2}} = \sqrt{5/3}$ as $g_2/g_3 \sim \sqrt{0.4}$ [41]. Doing a $O(3)$ rotation, we can go to a basis where the kinetic terms of the $U(1)$ gauge fields are still diagonal, while one of them corresponds to the hypercharge: $A_i = U_{ij}\widetilde{A}_j$ with $A_Y = \widetilde{A}_1$. This rotation is different in each theory.

For the $z = 0$ case we use

$$U = \begin{pmatrix} \frac{2\sqrt{3}}{\sqrt{28+9x^2}} & -\frac{\sqrt{16+9x^2}\sin\theta}{\sqrt{28+9x^2}} & \frac{\sqrt{16+9x^2}\sin\theta\sqrt{3}}{\sqrt{28+9x^2}} \\ -\frac{3x}{\sqrt{28+9x^2}} & -\frac{2(-2\sqrt{28+9x^2}\cos\theta+3\sqrt{3}x\sin\theta)}{\sqrt{28+9x^2}\sqrt{16+9x^2}} & \frac{2(2\sqrt{28+9x^2}\sin\theta+3\sqrt{3}x\cos\theta)}{\sqrt{28+9x^2}\sqrt{16+9x^2}} \\ \frac{4}{\sqrt{28+9x^2}} & \frac{3x\sqrt{28+9x^2}\cos\theta+8\sqrt{3}\sin\theta}{\sqrt{28+9x^2}\sqrt{16+9x^2}} & \frac{3x\sqrt{28+9x^2}\sin\theta-8\sqrt{3}\cos\theta}{\sqrt{28+9x^2}\sqrt{16+9x^2}} \end{pmatrix} \tag{7.9}$$

and the $U(1)$ charges:

$$Q_Y \sim Q_1 - \frac{Q_2}{2} + \frac{2Q_3}{3}$$

$$\begin{aligned} Q_\alpha \sim -\sqrt{3}x(16+9x^2)\sin\theta Q_1 + 2(2\sqrt{28+9x^2}\cos\theta - 3\sqrt{3}x\sin\theta)Q_2 \\ +(3x^2\sqrt{28+9x^2}\cos\theta + 8\sqrt{3}x\sin\theta)Q_3 \end{aligned}$$

$$\begin{aligned} Q_b \sim \sqrt{3}x(16+9x^2)\cos\theta Q_1 + 2(2\sqrt{28+9x^2}\sin\theta + 3\sqrt{3}x\cos\theta)Q_2 \\ +(3x^2\sqrt{28+9x^2}\sin\theta - 8\sqrt{3}x\cos\theta)Q_3 \end{aligned} \tag{7.10}$$

We can obtain the $z = -1$ case from the one above by $x \to -x$. The matrix U is now

$$U = \begin{pmatrix} \frac{2\sqrt{3}}{\sqrt{28+9x^2}} & -\frac{\sqrt{16+9x^2}\sin\theta}{\sqrt{28+9x^2}} & \frac{\sqrt{16+9x^2}\sin\theta\sqrt{3}}{\sqrt{28+9x^2}} \\ \frac{3x}{\sqrt{28+9x^2}} & \frac{2(2\sqrt{28+9x^2}\cos\theta+3\sqrt{3}x\sin\theta)}{\sqrt{28+9x^2}\sqrt{16+9x^2}} & -\frac{2(-2\sqrt{28+9x^2}\sin\theta+3\sqrt{3}x\cos\theta)}{\sqrt{28+9x^2}\sqrt{16+9x^2}} \\ \frac{4}{\sqrt{28+9x^2}} & \frac{-3x\sqrt{28+9x^2}\cos\theta+8\sqrt{3}\sin\theta}{\sqrt{28+9x^2}\sqrt{16+9x^2}} & -\frac{3x\sqrt{28+9x^2}\sin\theta+8\sqrt{3}\cos\theta}{\sqrt{28+9x^2}\sqrt{16+9x^2}} \end{pmatrix} \tag{7.11}$$

and the charges:

$$Q_Y \sim Q_1 + \frac{Q_2}{2} + \frac{2Q_3}{3}$$

$$Q_\alpha \sim -\sqrt{3}x(16+9x^2)\sin\theta Q_1 + 2(2\sqrt{28+9x^2}\cos\theta + 3\sqrt{3}x\sin\theta)Q_2$$
$$+(-3x^2\sqrt{28+9x^2}\cos\theta + 8\sqrt{3}x\sin\theta)Q_3$$

$$Q_b \sim \sqrt{3}x(16+9x^2)\cos\theta Q_1 + 2(2\sqrt{28+9x^2}\sin\theta - 3\sqrt{3}x\cos\theta)Q_2$$
$$-(3x^2\sqrt{28+9x^2}\sin\theta - 8\sqrt{3}x\cos\theta)Q_3 \qquad (7.12)$$

The parameter θ can be used to diagonalize the mass matrix of the two anomalous $U(1)$s A_α and A_b. The two eigenvalues μ_α^2, μ_b^2 and θ parametrize effectively the 2×2 mass matrix. The masses of the anomalous $U(1)$ gauge fields have also contributions from the Higgs effect since the Higgses are also charged under the anomalous $U(1)$s (appendix). However, such corrections are of order of m_Z/M_s and are thus subleading for our purposes. String theory calculations indicate that $\mu_{\alpha,b}$ are a factor of 5-10 below the string scale [41]. Thus they are expected to be in the TeV range.

7.1 Phenomenological aspects - Calculation of lepton anomalous magnetic moment in the presence of an anomalous $U(1)$

The recent precise measurement of the anomalous magnetic moment (AMM) of muon $\alpha_{muon} = (g-2)/2$ from the Brookhaven AGS experiment [57] gave

$$\alpha_{muon}^{exp} = 116592023(151)\times 10^{-11} \qquad (7.13)$$

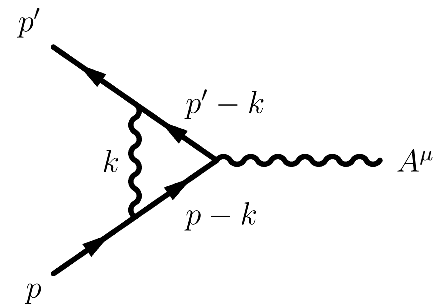
The difference between the experimental value (7.13) and the theoretical expectation, (for a review see [58]), due to standard model (SM) is

$$\delta\alpha_{muon} = \alpha_{muon}^{exp} - \alpha_{muon}^{SM} = (43\pm 16)\times 10^{-10}. \qquad (7.14)$$

The experimental precision is unprecented and it is going to reach $\pm 4\times 10^{-10}$ soon. It becomes thus important to examine the signals of physics beyond the SM. Various explanations for a discrepancy have been proposed building on earlier computations [60]. Many of those assume SUSY broken at a mass scale not far above the weak scale [63, 64, 65, 62, 66]. Other approaches include large or warped extra dimensional models, extended gauge structure and other alternatives [67, 68, 69, 70].

Here, we compute such $(g-2)_{\rm anom}$ contributions from the anomalous $U(1)$s and show that they are in the range implied by the experimental result. We use (7.13) to provide precise constrains for the masses of the anomalous U(1)'s in the TeV range.

To derive the AMM of a lepton, we consider the three-point function of two leptons and a photon where a gauge boson or the two scalars can be exchanged on the internal line:

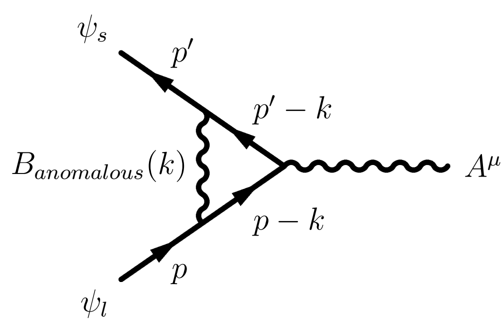


We sandwich the above diagram between two on-shell spinors, so we can use the Gordon decomposition and the mass-shell conditions. Our goal is to write the expression in the form:

$$\bar{u}(p')\Big\{\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2)\Big\}u(p) \tag{7.15}$$

where $q_\mu = p'_\mu - p_\mu$. The $F_2(q^2=0)$ will give us a correction of the AMM of the lepton which propagates. In the present calculation, we have to include diagrams which are coming from the non trivial couplings between the anomalous $U(1)$s and leptons. The external vector gauge abelian field is the photon, the internal propagating fields with momentum k can be the anomalous $U(1)$ gauge boson or the scalars (axions). We will outline here these calculations. More details can be found in appendix B.

As the anomalous $U(1)$ couples differently to left and right leptons, it is neccesary to consider diagrams where chirality is conserved (L-L, R-R diagrams) and others where chirality is different (L-R, R-L). The corresponding diagrams are

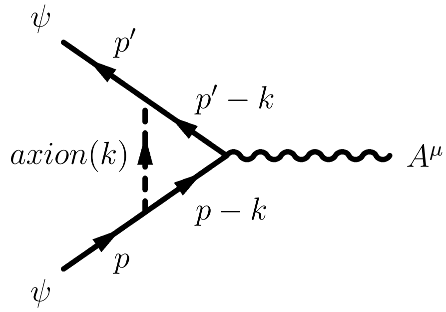


and in algebraic form:

$$\bar{u}(p')[\int \frac{d^4k}{(2\pi)^4}(iQ_s\gamma_\nu P_s)\frac{i}{\not{p}'-\not{k}-m}\gamma_\mu\frac{i}{\not{p}-\not{k}-m}(iQ_l\gamma_\rho P_l)D^{\nu\rho}(k)]u(p) \tag{7.16}$$

where $s,l = L,R$ label the chirality.

The propagator of $U(1)$ contains the arbitrary gauge fixing parameter λ. In a non-chiral theory λ disappears because of the mass-sell conditions of the two spinors which sandwich the diagrams (7.16). In a chiral theory, we need the contribution of b' with mass (6.7) in order to obtain a gauge invariant result. We also have to add the one-loop diagrams of ϕ'. These diagrams are:



where "axion" stands for b' or ϕ'. In algebraic form they are given by:

$$\frac{m^2 \Delta Q^2}{\mu^2} \bar{u}(p') \int \frac{d^4k}{(2\pi)^4} \gamma_5 \frac{i}{\not{p}' - \not{k} - m} \gamma_\mu \frac{i}{\not{p} - \not{k} - m} \gamma_5 G_{b'}(k) u(p) \tag{7.17}$$

for the b' axion and

$$\frac{(hc)^2 M_s^2}{\mu^2} \bar{u}(p') \int \frac{d^4k}{(2\pi)^4} \gamma_5 \frac{i}{\not{p}' - \not{k} - m} \gamma_\mu \frac{i}{\not{p} - \not{k} - m} \gamma_5 G_{\phi'}(k) u(p) \tag{7.18}$$

for ϕ'. We expect the sum of the three diagrams to be λ-independent. In the appendix we show it explicitly. In view of this, we can use any gauge for the evaluation. For simplicity, we choose the Feynman - t'Hooft gauge $\lambda = 1$

The steps of this calculation are as follow:

a. Express the denominator as a perfect square using the Feynman parameter trick and shifting the loop momentum.

b. Move all the $\not{p}'$ to the left, all the $\not{p}$ to the right and make use of the on-shell spinor conditions.

c. Perform the momentum integral of the loop after a Wick rotation to Euclidean space.

d. Distinguish terms proportional to p_μ and p'_μ.

e. Integrate the remaining variables that resulted from Feynman parameter trick.

Following the steps above, we find for the anomalous $U(1)$ exchanged diagram (details can be found in Appendix B): For L-L and R-R diagrams:

$$-\frac{Q_L^2 + Q_R^2}{16 m \pi^2} (p_\mu + p'_\mu) \int_0^1 dx \frac{x(x^2 - 3x + 2)}{x^2 + (1-x)\frac{\mu^2}{m^2}} \tag{7.19}$$

For mixed diagrams (L-R and R-L):

$$-\frac{Q_L Q_R}{16 m \pi^2} (p_\mu + p'_\mu) \int_0^1 dx \frac{2x(1-x)}{x^2 + (1-x)\frac{\mu^2}{m^2}} \tag{7.20}$$

The axion b' exchange diagram gives

$$\frac{\Delta Q^2}{16m\pi^2}\frac{m^2}{\mu^2}(p_\mu + p'_\mu)\int_0^1 dx\frac{x^3}{x^2+(1-x)\frac{\mu^2}{m^2}} \tag{7.21}$$

The diagram for the axion ϕ' has the same integral with (7.21) in the limit $\mu \to 0$. Since however the axion is expected to get a small mass from non-perturbative effects we will consider it with $m_{\phi'}$ small. In this case we obtain

$$\frac{(hc)^2}{16m\pi^2}\frac{M_s^2}{\mu^2}(p_\mu + p'_\mu)\int_0^1 dx\frac{x^3}{x^2+(1-x)\frac{m_{\phi'}^2}{m^2}} \tag{7.22}$$

As $M_s/\mu \sim 1$, the limit of (7.22) for $m_{\phi'} \to 0$ is:

$$\frac{h^2}{16m\pi^2}(p_\mu + p'_\mu)\frac{1}{2} \tag{7.23}$$

7.2 Anomalous magnetic moment of muon in the D-brane realization of the standard model

Using the results above we can now embark in the calculation of the AMM of the muon in the D-brane realization of the SM. To do this we have to include the contribution of (7.19) and (7.20) for both anomalous $U(1)$s as well as the (7.21) and (7.23) for the axion diagrams to the SM result[20].

$$\delta\alpha = \frac{h^2}{16\pi^2} + \frac{1}{8\pi^2}\times \tag{7.24}$$
$$\sum_{i=\alpha,b}\left(\frac{m}{\mu_i}\right)^2\int_0^1 dx\frac{x(m^2\Delta Q_i^2 x^2 + \mu_i^2(4Q_{iL}Q_{iR} - (2-x)(Q_{iL}^2+Q_{iR}^2))(1-x)}{m^2x^2+\mu_i^2(1-x^2)}$$

In our case $\mu_i \gg m$, therefore we expand the contributions and keep the terms up to second order in (μ_i/m). The final result is

$$\alpha_{muon}^{U(3)\times U(2)\times U(1)} = \alpha_{muon}^{SM} + \sum_{i=\alpha,b}\frac{Q_{iL}^2 - 3Q_{iL}Q_{iR} + Q_{iR}^2}{12\pi^2}\left(\frac{m}{\mu_i}\right)^2 + \frac{h^2}{16\pi^2} \tag{7.25}$$

where $Q_{\alpha L}, Q_{\alpha R}, Q_{bL}, Q_{bL}$ are the rotated by (7.10) or (7.12), charges of (7.1). We use as Q_{iL} and Q_{iR} the charges of the L and l^c in (7.1).

Using the measured difference (7.14) we can express one of the unknown variables as a function of the two others. Thus, for $z=0$ we can find the μ_α and μ_β dependence of $\tan\theta$. We have to solve a second order equation:

$$(12\pi^2\mu_\alpha^2\mu_b^2(\delta\alpha - \alpha_{\phi'}) + m^2(817\mu_\alpha^2 - 1220\mu_b^2))\tan^2\theta + 26\sqrt{215}m^2(\mu_\alpha^2 - \mu_b^2)\tan\theta$$
$$+12\pi^2\mu_\alpha^2\mu_b^2(\delta\alpha - \alpha_{\phi'}) - 1220m^2\mu_\alpha^2 + 817m^2\mu_b^2 = 0 \tag{7.26}$$

[20]We use for simplicity $m = m_{muon}$.

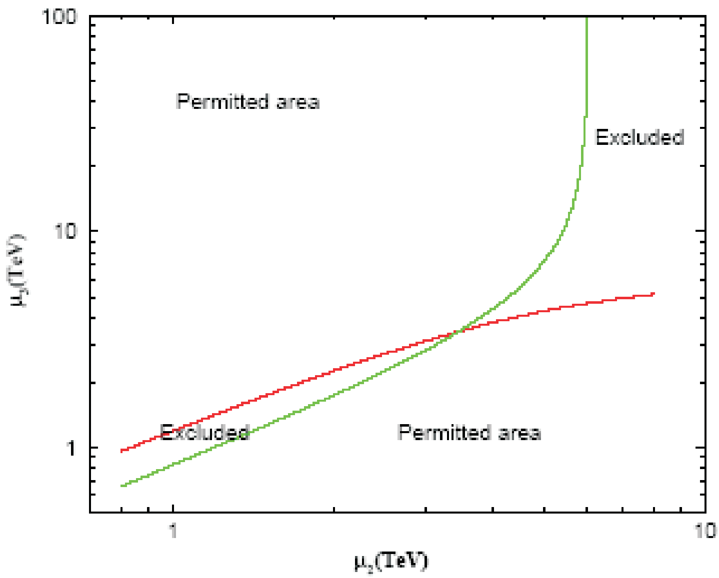


Figure 7: The $z = 0$ model. Between the two plots is the excluded area, where the determinant of the second order equation is negative.

where we denote as $\alpha_{\phi'}$ the contribution from the axion ϕ'. As $\tan\theta$ is real, the discriminant must be positive. We can easily find the excluded area in the μ_2,μ_3 plane where this discriminant is negative. In Fig.7 we plot this area for the z=0 model.

For the $z = -1$ model we obtain

$$(12\pi^2\mu_\alpha^2\mu_b^2(\delta\alpha - \alpha_{\phi'}) - m^2(10363\mu_\alpha^2 + 580\mu_b^2))\tan^2\theta - 362\sqrt{215}m^2(\mu_\alpha^2 - \mu_b^2)\tan\theta$$
$$+12\pi^2\mu_\alpha^2\mu_b^2(\delta\alpha - \alpha_{\phi'}) - m^2(580\mu_\alpha^2 + 10363m^2\mu_b^2) = 0 \tag{7.27}$$

and the allowed area is plotted in Fig.8. As mentioned before the anomalous $U(1)$ masses are expected to be in the TeV range. Thus, there is little allowed space in this case in order to reproduce the experimental result.

Until now we have evaluated diagrams of the lowest lying string states. The massive oscillator string states at level n have masses equal to $\sqrt{n}M_s$. The ratio of the contribution of such a state to that of a low lying state is expected to scale as the square of the ratio of the masses. Thus corrections due to the first massive level are in the 1-5% range and higher levels are further suppressed. There are also KK states that can contribute. However their masses as mentioned earlier are as large as the string scale and thus give suppressed contributions.

7.3 Chapter Conclusions

In this chapter we have analyzed contributions to the anomalous magnetic moment

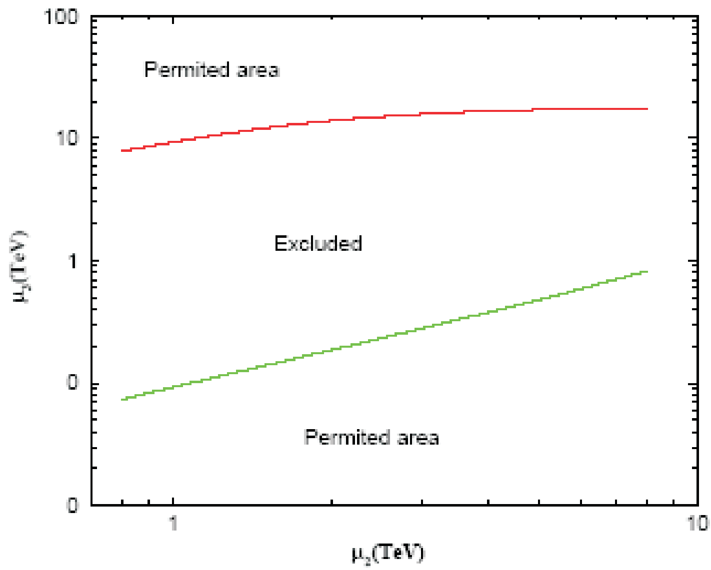


Figure 8: The $z = -1$ model. Between the two plots is the excluded area where the determinant of the second order equation is negative.

of leptons in the minimal D-brane realization of the Standard Model. We have shown that the two anomalous massive gauge bosons present with masses in the TeV range, provide contributions that have the correct order of magnitude to accommodate the recent experimental data [57]. Further contributions from string oscillators and KK states are expected to be sufficiently suppressed.

8. Conclusions

In this thesis we have studied specific kind of open string theories generated by orientifold models. We have provided the general consistency conditions and we have derived the general formulae for the corresponding massless spectrum of various open string theories. This classification is very important in model building procedures which target to embed the Standard Model in string theory.

Applying the same ideas and technics, we could presumably provide general formulae also in other string theory models with intersecting branes, fluxes or asymmetric orientifolds.

As we have mentioned, all open string models that approach the Standard Model contain anomalous $U(1)$ gauge fields. The anomaly is cancelled via the Green-Schwarz mechanism that generates a mass for the corresponding anomalous gauge boson. We have evaluated the bare masses of the anomalous $U(1)$s in four-dimensional supersymmetric orientifolds. However, we have found that there are cases where even non-anomalous $U(1)$s acquire a mass and we have showed that this is due to six-dimensional anomalies that upon decompactifications affect the four-dimensional theory.

We have also evaluated the general formulae for the bare mass of anomalous $U(1)$s in non-supersymmetric orientifolds. This is important since the proper D-brane realization of the Standard Model will have broken supersymmetry.

These results and formulae have direct implications for model building both in string theory and field theory orbifolds. They provide a necessary and sufficient condition for a non-anomalous $U(1)$ to remain massless (the hypercharge for example). One has just to check the associated higher dimensional anomalies in the various decompactification limits.

We have studied other mass sources for the anomalous $U(1)$s. The D-brane realizations of the Standard Model require that the Higgses (usually there are more than one to give masses to all quarks and leptons) are charged under the anomalous $U(1)$s. This generates an extra mass source for the anomalous bosons.

Usually, there are mixings between the axions that cancel the anomalies and the Higgses of the theory, therefore some of the axions acquire masses. The study of these massive axions can provide very interesting results that could eventually also be tested at LHC, if the string scale is of order of a few TeV.

Finally, we have evaluated the contribution of the extra $U(1)$ fields to the anomalous moments of the leptons and it has been shown that this imposes constraints on the magnitude of the string scale.

Acknowledgments

First of all I would like to thank Prof. Elias Kiritsis who offered me the possibility to do my Ph.D. under his supervision. I am also grateful to Prof. Theodore Tomaras for being a great teacher in my Diploma and in my Ph.D. time. I would like to thank Dr. Amine Bouziane Hammou for helping me very much and for very fruitful collaboration. Also I would like to thank Dr. Nikos Irges for help and collaboration.

I would like to thank Prof. Massimo Bianchi, Prof. Mariano Quiros and Dr. Marco Serone for useful discussions.

I would also like to thank Dr. Dominic Clancy, Dr. Amine Bouziane Hammou, Gregoris Panotopoulos and Anne-Lise Chagneau for proof-reading of my thesis.

I am also grateful to University of Crete and to Laboratoire de Physique Théorique Ecole Polytechnique for support and hospitality.

I would like to thank the Laboratoire de Physique Théorique de l' Ecole Normal Surériere for hospitality. I would also like to thank the Universitat Autònoma de Barcelona, the Universitá di Roma "Tor Vergata", the Scuola Internationale Superiore di Studi Advanzati (SISSA) for hospitality during the last stage of this thesis.

This thesis was financially supported from the "Herakleitos" program of the Greek Ministry of Education.

Finally, I would like to thank my parents for supporting me all these years of studies and all my friends.

Appendices

A. Definitions and identities

The Dedekind function is defined by the usual product formula (with $q = e^{2\pi i\tau}$)

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1-q^n) \ . \tag{A.1}$$

The Jacobi ϑ-functions with general characteristic and arguments are

$$\begin{aligned}
\vartheta[^\alpha_\beta](z|\tau) &= \sum_{n\in Z} e^{i\pi\tau(n-\alpha/2)^2} e^{2\pi i(z-\beta/2)(n-\alpha/2)} \\
\vartheta[^\alpha_\beta](z|\tau) &= \eta \ e^{i\pi\alpha(z+\beta/2)} q^{\frac{\alpha}{4}-\frac{1}{24}} \times \\
& \prod_{n=1}^{\infty}\left(1+e^{2\pi i(z+\beta/2)}q^{n+\frac{\alpha-1}{2}}\right)\left(1+e^{-2\pi i(z+\beta/2)}q^{n+\frac{\alpha-1}{2}}\right)
\end{aligned} \tag{A.2}$$

We define: $\vartheta_1(z|\tau) = \vartheta\left[^1_1\right](z|\tau)$, $\vartheta_2(z|\tau) = \vartheta\left[^1_0\right](z|\tau)$, $\vartheta_3(z|\tau) = \vartheta\left[^0_0\right](z|\tau)$, $\vartheta_4(z|\tau) = \vartheta\left[^0_1\right](z|\tau)$. The modular properties of these functions are:

$$\begin{aligned}
\eta(\tau+1) &= e^{i\pi/12}\eta(\tau) \ , \quad \vartheta[^\alpha_\beta](z|\tau+1) = e^{-\frac{i\pi}{4}\alpha(\alpha-2)}\vartheta[^{\ \ \alpha}_{\alpha+\beta-1}](z|\tau) \\
\eta(-1/\tau) &= \sqrt{-i\tau}\,\eta(\tau) \ , \quad \vartheta[^\alpha_\beta]\left(\frac{z}{\tau}\middle|\frac{-1}{\tau}\right) = \sqrt{-i\tau}\ e^{i\pi\left(\frac{\alpha\beta}{2}+\frac{z^2}{\tau}\right)}\ \vartheta[^{\ \beta}_{-\alpha}](z|\tau)
\end{aligned} \tag{A.3}$$

A very useful identity that is valid for $\sum h_i = \sum g_i = 0$ is

$$\sum_{\alpha,\beta=0,1} \eta_{\alpha\beta}\ \vartheta\left[^\alpha_\beta\right](v) \prod_{i=1}^{3} \vartheta\left[^{\alpha+h_i}_{\beta+g_i}\right](0) = \vartheta_1(-v/2) \prod_{i=1}^{3} \vartheta\left[^{1-h_i}_{1-g_i}\right](v/2) \ . \tag{A.4}$$

B. Partition functions and Lattices

Let us define some of the objects that we used in this paper. The oscillator dependant parts are:

$$T[^0_v] = \frac{1}{2}\sum_{a,b}(-1)^{a+b+ab}\frac{\vartheta[^a_b]}{\eta}\prod_i -2\sin\pi v_i \frac{\vartheta[^{\ \ a}_{b+2v_i}]}{\vartheta[^{\ \ 1}_{1+2v_i}]}. \tag{B.1}$$

$$T[^g_v] = \frac{1}{2}\sum_{a,b}(-1)^{a+b+ab}\frac{\vartheta[^a_b]}{\eta}\prod_i \frac{\vartheta[^{a+2g_i}_{b+2v_i}]}{\vartheta[^{1+2g_i}_{1+2v_i}]}. \tag{B.2}$$

The lattice parts are:

$$\Lambda_{m+a,n+b} = \frac{1}{\eta(q)\eta(\bar{q})}\sum_{m,n} q^{\frac{\alpha'}{4}\left(\frac{m+a}{R}+\frac{n+b}{\alpha'}R\right)^2} \bar{q}^{\frac{\alpha'}{4}\left(\frac{m+a}{R}+\frac{n+b}{\alpha'}R\right)^2} \tag{B.3}$$

and the momentum and winding parts:

$$P_m(i\tau_2/2) = \frac{1}{\eta(i\tau_2/2)} \sum_m q^{\frac{\alpha'}{4}\left(\frac{m}{R}\right)^2} \tag{B.4}$$

$$W_n(i\tau_2/2) = \frac{1}{\eta(i\tau_2/2)} \sum_n q^{\frac{\alpha'}{4}\left(\frac{nR}{\alpha'}\right)^2} \tag{B.5}$$

C. Twisted Tadpoles

Taking the UV limit of the transpose Klein Bottle and Annulus (between similar branes) amplitudes, we have the tadpoles square of an O-plane and a D-brane respectively. Therefore, we can factorize and compute the contributions of these hyperplanes. This can be a useful tool to evaluate the tadpole conditions for a specific model. The contributions have found[21]:

- Supersymmetric Models
  - $v_a^3 = 0$
    - O-Plane contributions (they all appear with opposite sign in the NS and R sectors.):

$$\Omega\alpha \quad \alpha^2 \sim \sqrt{\frac{\mathcal{V}_3}{\prod_{l=1}^2 2\sin 2\pi v_a^l}} \prod_{l=1}^2 2\cos\pi v_a^l$$

$$\Omega R_3\alpha \quad \alpha^2 \sim \sqrt{\frac{\mathcal{V}_3}{\prod_{l=1}^2 2\sin 2\pi v_a^l}} \prod_{l=1}^2 2\sin\pi v_a^l$$

$$\Omega R_i\alpha \quad \alpha^2 \sim \epsilon_{ij}\sqrt{\frac{1}{\mathcal{V}_3\prod_{l=1}^2 2\sin 2\pi v_a^l}} 2\cos\pi v_a^i 2\sin\pi v_a^j$$

where $i \neq j$

    - D-brane contribution (they also appear with opposite sign in the NS and R sectors):

$$\text{D9} \quad \alpha \sim \sqrt{\frac{\mathcal{V}_3}{\prod_{l=1}^2 2\sin\pi v_a^l}} Tr[\gamma_{\alpha,9}]$$

$$\text{D5}_3 \quad \alpha \sim \sqrt{\frac{\mathcal{V}_3}{\prod_{l=1}^2 2\sin\pi v_a^l}} \prod_{l=1}^2 2\sin\pi v_a^l Tr[\gamma_{\alpha,5_3}]$$

$$\text{D5}_i \quad \alpha \sim \sqrt{\frac{1}{\mathcal{V}_3\prod_{l=1}^2 2\sin\pi v_a^l}} 2\sin\pi v_a^j Tr[\gamma_{\alpha,5_i}]$$

where $i \neq j$

[21] We remind that $v_a = (v_a^1, v_a^2, v_a^3)$.

- $v_a^3 \neq 0$:
  - O-Plane contributions:

$$\Omega\alpha \quad \alpha^2 \sim \sqrt{\frac{1}{\prod_{l=1}^{3} 2\sin 2\pi v_a^l}} \prod_{l=1}^{3} 2\cos\pi v_a^l$$

$$\Omega R_i\alpha \quad \alpha^2 \sim \sqrt{\frac{1}{\prod_{l=1}^{3} 2\sin 2\pi v_a^l}} 2\cos\pi v_a^i \prod_{l\neq i} 2\sin\pi v_a^l$$

where $i = 1, 2, 3$

  - D-brane contributions:

$$\mathrm{D9} \quad \alpha \sim \sqrt{\frac{1}{\prod_{l=1}^{3} 2\sin\pi v_a^l}} Tr[\gamma_{\alpha,9}]$$

$$\mathrm{D5}_i \quad \alpha \sim \sqrt{\frac{1}{\prod_{l=1}^{3} 2\sin\pi v_a^l}} \prod_{l\neq i} 2\sin\pi v_a^l Tr[\gamma_{\alpha,5_i}]$$

where $i = 1, 2, 3$

- Non-Supersymmetric Models. Breaking SUSY by SS deformation, acting on the third torus where also $v_a^3 = 0$ some more tadpoles are added on the above:

$$\Omega R_i h\alpha \quad \alpha^2 \sim \epsilon_{ij} \sqrt{\frac{1}{\mathcal{V}_3 \prod_{l=1}^{2} 2\sin 2\pi v_a^l}} 2\cos\pi v_a^i 2\sin\pi v_a^j$$

where $i \neq j$

$$\overline{\mathrm{D5}}_i \quad \alpha \sim \epsilon_{ij} \sqrt{\frac{1}{\mathcal{V}_3 \prod_{l=1}^{2} 2\sin\pi v_a^l}} 2\sin\pi v_a^j Tr[\gamma_{\alpha,5_i}]$$

where $i \neq j$

These tadpoles have the same sign in both NS and R sectors.

D. Correlation functions on the annulus

We present here the derivation of the propagators that we will use for the calculation of the annulus $\mathcal{A}$. This surface can be defined as quotient of the torus $\mathcal{T}$ under the involution:

$$\mathcal{I}_{\mathcal{A}}(z) = 1 - \bar{z}. \tag{D.1}$$

Thus, the correlators can be expressed in terms of the propagators on the torus. For the bosonic case we have

$$\langle X(z)X(w)\rangle_{\mathcal{T}} = -\frac{1}{4}\log\left|\frac{\vartheta_1(z-w|\tau)}{\vartheta_1'(0|\tau)}\right|^2 + \frac{\pi(z_2-w_2)^2}{2\tau_2} \equiv P_B(z,w) \tag{D.2}$$

and symmetrizing under the involution:

$$\begin{aligned}\langle X(z)X(w)\rangle_{\mathcal{A}} &= \frac{1}{2}[P_B(z,w)+P_B(\mathcal{I}_{\mathcal{A}}(z),w)+P_B(z,\mathcal{I}_{\mathcal{A}}(w))+P_B(\mathcal{I}_{\mathcal{A}}(z),\mathcal{I}_{\mathcal{A}}(w)] \\ &= P_B(z,w)+P_B(z,1-\bar{w})\ . \end{aligned} \tag{D.3}$$

In the amplitude, the partial derivative of the above correlator (D.3) appears. Thus, we give the expression that we use for $w=1/2$:

$$\langle \partial_z X(z)X(1/2)\rangle_{\mathcal{A}} = -\frac{1}{2}\left[\partial_z \log\vartheta_1(z-1/2|\tau)+\frac{2\pi i z_2}{\tau_2}\right] \tag{D.4}$$

for $z=z_1+iz_2$. We remind also that $\partial_z=(\partial_{z_1}-i\partial_{z_2})/2$. For the fermionic correlators on the torus we have the identity:

$$\langle\psi(z)\psi(w)\rangle^2\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] = -\frac{1}{4}\mathcal{P}(z-w)-\pi i\partial_\tau\log\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)}{\eta(\tau)} \tag{D.5}$$

where $\mathcal{P}(z-w)$ is the Weierstrass function. Symmetrizing the torus propagator under the involution we find that (D.5) holds also for the annulus.

E. Computations in Type I orientifolds

In the appendix, we give some more details about the $6D$ computations of the mass term.

E.1 Open strings attached on the same kind of branes

The internal partition function of strings attached on the same kind of branes is:

$$Z^{aa}_{int,k}\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] = \prod_{j=1}^{2}(-2\sin\pi k v_j)\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+2kv_j\end{smallmatrix}\right](0|\tau)}{\vartheta\left[\begin{smallmatrix}1\\ 1+2kv_j\end{smallmatrix}\right](0|\tau)} \qquad \text{for a=5,9.} \tag{E.1}$$

After the use of (A.4) and the fact that $\vartheta\left[\begin{smallmatrix}1\\ 1\end{smallmatrix}\right](0|\tau)=0$, we find for the annulus amplitude:

$$\begin{aligned}\mathcal{A}^{aa}_k &= -\frac{1}{2N}\int[d\tau]\tau_2^{1+\delta/2}[2\pi\eta^3(\tau)]^\delta\left[\frac{1}{2\pi\tau^3}4\sin^2\frac{\pi k}{N}\right] \\ &= -\frac{(2\pi)^\delta}{\pi N}\sin^2\frac{\pi k}{N}\int_0^{i\infty}d\tau_2\tau_2^{-2+\delta/2}\eta^{3\delta}(\tau_2).\end{aligned} \tag{E.2}$$

We are interested in the UV limit of the above integral. The annulus moduli is $\tau_2=it/2$:

$$\begin{aligned}\mathcal{A}^{aa,UV}_k &= -\frac{(2\pi)^\delta}{\pi N}\sin^2\frac{\pi k}{N}2^{1-\delta/2}\int_0^1 dt\ t^{-2+\delta/2}\eta^{3\delta}(it/2) \\ &= -\frac{(2\pi)^\delta}{\pi N}\sin^2\frac{\pi k}{N}2^{1-\delta/2}\int_0^1 dt\ t^{-2+\delta/2}\left[\left(\frac{2}{t}\right)^{1/2}\eta\left(\frac{2}{t}\right)\right]^{3\delta} \\ &= -\frac{4}{\pi^2\delta N}\left(\frac{8}{\delta}\right)^\delta\sin^2\frac{\pi k}{N}\ \Gamma(1+\delta,\pi\delta/2).\end{aligned} \tag{E.3}$$

where $\Gamma(a,x)$ is the incomplete Γ-function and $\Gamma(1,0)=1$.

E.2 Open strings attached on different kind of branes

Strings attached on different kind of branes have coordinates X^a with mixed Dirichlet-Neumann boundary conditions. Those coordinates are half-integer moded and there are no windings or momenta. The fermionic sectors interchange modes between R and NS (since the R states should have same modes than the coordinates) keeping the total fermionic pact unchanged. Thus, the internal partition function for such strings is:

$$Z^{59}_{int,k}[^{\alpha}_{\beta}] = \prod_{j=1}^{2} \frac{\vartheta[^{\alpha+1}_{\beta+2kv_j}](0|\tau)}{\vartheta[^{0}_{1+2kv_j}](0|\tau)}. \tag{E.4}$$

Following the same procedure, like in the case of the strings with the same boundary conditions, we substitute (E.4) in (5.11) and after a bit of "thetacology" we find:

$$\mathcal{A}^{59}_k = -\frac{1}{2N}\int [d\tau]\tau_2^{1+\delta/2}[2\pi\eta^3(\tau)]^{\delta}\left[\frac{1}{2\pi\tau^3}\right] \tag{E.5}$$

The integral is the same as in the case of the strings having the same boundary conditions. Using the above result we find:

$$\mathcal{A}^{59,UV}_k = -\frac{1}{\pi^2\delta N}\left(\frac{8}{\delta}\right)^{\delta}\ \Gamma(1+\delta,\pi\delta/2). \tag{E.6}$$

F. D-terms and supersymmetry

Consider a generic Lagrangian that depends on chiral fields Φ_i, $\bar{\Phi}_i$ and an abelian vector field:

$$\begin{aligned}\mathcal{L}_{susy} = &\int d^2\theta d^2\bar{\theta}\ K(\Phi_i,\bar{\Phi}_i,V) + \frac{1}{4}\int d^2\theta\ f(\Phi_i)W^{\alpha}W_{\alpha} + \frac{1}{4}\int d^2\bar{\theta}\ f(\bar{\Phi}_i)\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} \\ &+\int d^2\theta\ W(\Phi_i) + \int d^2\bar{\theta}\ W(\bar{\Phi}_i)\end{aligned}$$

where as usual K the Kähler potential (arbitrary real function), f the gauge kinetic function and W the superpotential (holomorphic functions). The superfields have the component expansions:

$$\begin{aligned}\Phi_i &= \phi_i + \sqrt{2}\theta\psi_i + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\phi_i + \theta\theta F_i + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^{\mu}\partial_{\mu}\psi_i + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\phi_i \\ \bar{\Phi}_j &= \bar{\phi}_j + \sqrt{2}\bar{\theta}\bar{\psi}_j - i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\bar{\phi}_j + \bar{\theta}\bar{\theta}\bar{F}_j - \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\partial_{\mu}\bar{\psi}_j\bar{\sigma}^{\mu}\theta + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\bar{\phi}_j \\ V &= -\theta\sigma^{\mu}\bar{\theta}A_{\mu} + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D\end{aligned}$$

in the WZ gauge [5, 6]. Expanding the Lagrangian in component fields we have:

$$
\begin{aligned}
\mathcal{L}_{susy} = & -\frac{1}{4}A^\mu A_\mu \partial_V^2 K|_0 + \left(-\partial_\mu\phi_i\partial^\mu\bar{\phi}_j - \frac{i}{2}\psi_i\sigma^\mu\partial_\mu\bar{\psi}_j - \frac{i}{2}\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi_j\right) K_{i\bar{j}} \\
& + \left(\frac{i}{\sqrt{2}}\lambda\psi_i + \frac{i}{2}A^\mu\partial_\mu\phi_i\right)\partial_V K_i - \left(\frac{i}{\sqrt{2}}\bar{\lambda}\bar{\psi}_j + \frac{i}{2}A^\mu\partial_\mu\bar{\phi}_j\right)\partial_V K_{\bar{j}} + \frac{1}{2}\bar{\psi}_j\bar{\sigma}^\mu A_\mu\psi_i\partial_V K_{i\bar{j}} \\
& -\frac{1}{4}\Re(f_0)F^{\mu\nu}F_{\mu\nu} + i\Re(f_0)\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda - \frac{1}{4}\Im(f_0)F^{\mu\nu}\tilde{F}_{\mu\nu} \\
& -\frac{1}{2}\psi_i\psi_l\bar{F}_j K_{il\bar{j}} - \frac{i}{2}\bar{\psi}_j\bar{\sigma}_\mu\psi_i\partial^\mu\phi_l K_{il\bar{j}} + \frac{i}{2}\partial^\mu\bar{\phi}_j\bar{\psi}_m\bar{\sigma}_\mu\psi_l K_{i\bar{j}\bar{m}} + \frac{1}{4}\bar{\psi}_j\bar{\psi}_m\psi_i\psi_l K_{il\bar{j}\bar{m}} + \mathcal{V}[\phi_i,\bar{\phi}_j]
\end{aligned}
$$

The indexes $i, \bar{j}$ denote derivatives for $\phi_i, \bar{\phi}_j$ respectively. The $\Re, \Im$ denote Real and Imaginary parts. It is taken $V = 0 = \theta = \bar{\theta}$ and $f_0 \equiv f(\phi_0)$. The K are functions of only the lowest component of the chiral fields (the scalar fields). Notice that already, we can solve for the auxiliary fields: Where we define the real potential for the scalar fields:

$$
\mathcal{V}[\phi_i,\bar{\phi}_j] = W_i F_i + \frac{1}{4\Re f_0}(\partial_V K|_0)^2
$$

Green-Schwarz anomaly cancellation

Consider now a Kähler potential and a gauge function suitable for a model with anomalous $U(1)$s V_α. The form of these functions will be:

$$
K = K(\Phi_\alpha + \bar{\Phi}_\alpha + c^\alpha V_\alpha; \bar{\Phi}_i e^{2q_i^\alpha V_\alpha}\Phi_i) \quad , \quad f = f(\Phi_\alpha) \ .
$$

where i for various chiral fields and α for the axions. We do not include non anomalous $U(1)s$ for simplicity. We consider diagonalized axions, one to one with the anomalous $U(1)$s. Notice that the first combination gives:

$$
\begin{aligned}
\Phi_\alpha + \bar{\Phi}_\alpha + c^\alpha V_\alpha = & \ 2s_\alpha + \sqrt{2}\theta\psi_\alpha + \sqrt{2}\bar{\theta}\bar{\psi}_\alpha - 2\theta\sigma^\mu\bar{\theta}\left(\partial^\mu b_\alpha + \frac{c^\alpha}{2}A^\mu_\alpha\right) + \theta\theta F_\alpha + \bar{\theta}\bar{\theta}\bar{F}_\alpha \\
& + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}(\bar{\sigma}^\mu\partial_\mu\psi_\alpha + \sqrt{2}c^\alpha\bar{\lambda}_\alpha) - \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}(\partial_\mu\bar{\psi}_\alpha\bar{\sigma}^\mu + \sqrt{2}c^\alpha\lambda_\alpha)\theta \\
& + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(\Box s_\alpha + c^\alpha D_\alpha)
\end{aligned}
$$

We have separated the lowest component of the axionic superfield $\Phi_\alpha|_{\theta=\bar{\theta}=0} = \phi_\alpha = s_\alpha + ib_\alpha$. The axion appears always in the combination: $\partial^\mu b_\alpha + \frac{c^\alpha}{2}A^\mu_\alpha$ that is gauge invariant for $A^\mu_\alpha \to A^\mu_\alpha + \partial^\mu\epsilon_\alpha$ and $b_\alpha \to b_\alpha - \frac{c^\alpha}{2}\epsilon_\alpha$.

In heterotic string theory, there is at most one anomalous gauge boson and one axion is needed. In this case s_0 is the dilaton and b_0 the dual to the model independent antisymmetric tensor. In type I, there can be many anomalous $U(1)$s and the role of the axions $b_\alpha = \delta^k_\alpha b_k$ are played by the RR twisted fields (k denotes the sector).

The $s_\alpha = \delta^k_\alpha m_k$ are the NSNS twisted moduli corresponding to the blowup modes associated with the singularities of the orbifold.

We will focus in an effective field theory that is coming from an orientifold of Type IIB string theory. In this case the Kähler potential is quadratic to chiral fields

$$K = \frac{1}{2}\sum_\alpha (\Phi_\alpha + \bar{\Phi}_\alpha + c^\alpha V_\alpha)^2 + \sum_i \bar{\Phi}_i e^{2q_i^\alpha V_\alpha}\Phi_i \tag{F.1}$$

The lagrangian is simplified a lot:

$$\begin{aligned}
\mathcal{L}_{susy} = & -\frac{1}{4}\Re(f_\alpha)F_\alpha^{\mu\nu}F_{\alpha\mu\nu} + i\Re(f_\alpha)\bar{\lambda}_\alpha\bar{\sigma}^\mu\partial_\mu\lambda_\alpha - \frac{1}{4}\Im(f_\alpha)F_\alpha^{\mu\nu}\tilde{F}_{\alpha\mu\nu} \\
& -\left(\partial_\mu s_\alpha\partial^\mu s_\alpha + \left(\partial_\mu b_\alpha + \frac{c^\alpha}{2}A_{\alpha\mu}\right)^2 + \frac{i}{2}\psi_\alpha\sigma^\mu\partial_\mu\bar{\psi}_\alpha + \frac{i}{2}\bar{\psi}_\alpha\bar{\sigma}^\mu\partial_\mu\psi_\alpha\right) \\
& +\frac{ic^\alpha}{\sqrt{2}}(\lambda_\alpha\psi_\alpha - \bar{\lambda}_\alpha\bar{\psi}_\alpha) \\
& -(\partial_\mu + iq_i^\alpha A_{\alpha\mu})\phi_i(\partial^\mu - iq_i^\alpha A_\alpha^\mu)\bar{\phi}_i \\
& -\frac{i}{2}\bar{\psi}_i\bar{\sigma}^\mu\left(\partial_\mu + iq_i^\alpha A_{\alpha\mu}\right)\psi_i - \frac{i}{2}\psi_i\sigma^\mu\left(\partial_\mu - iq_i^\alpha A_{\alpha\mu}\right)\bar{\psi}_i \\
& +\sqrt{2}iq_i^\alpha\left(\lambda_\alpha\psi_i\bar{\phi}_i - \bar{\lambda}_\alpha\bar{\psi}_i\phi_i\right) + \mathcal{V}[\phi_\alpha,\bar{\phi}_\alpha,\phi_i,\bar{\phi}_j]
\end{aligned} \tag{F.2}$$

We should transform everything from the Weyl to the usual Dirac basis. We have two Weyl spinors $\lambda_\alpha^D = \begin{pmatrix} i\lambda_\alpha \\ -i\bar{\lambda}_\alpha \end{pmatrix}$, $\psi_\alpha^D = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}_\alpha \end{pmatrix}$ (obviously the indices are not spinor) and we construct Dirac spinors by $\psi_i^D = \begin{pmatrix} \psi_{2i-1} \\ \bar{\psi}_{2i} \end{pmatrix}$ for $i = 1\ldots N$. In case $q_{2i-1} \neq q_{2i}$ the model is chiral. If in addition $\sum_i q_i \neq 0$ the gauge boson A_α^μ is anomalous. Putting all together:

$$\begin{aligned}
\mathcal{L}_{susy} = & -\frac{1}{4}\Re(f_\alpha)F_\alpha^{\mu\nu}F_{\alpha\mu\nu} + i\Re(f_\alpha)\bar{\lambda}_\alpha\bar{\sigma}^\mu\partial_\mu\lambda_\alpha - \frac{1}{4}\Im(f_\alpha)F_\alpha^{\mu\nu}\tilde{F}_{\alpha\mu\nu} \\
& -\left(\partial_\mu s_\alpha\partial^\mu s_\alpha + \left(\partial_\mu b_\alpha + \frac{c^\alpha}{2}A_\alpha^\mu\right)^2 + \frac{i}{2}\bar{\psi}_\alpha^D\partial\!\!\!/\psi_\alpha^D\right) \\
& +\frac{c^\alpha}{2\sqrt{2}}\left(\bar{\lambda}_\alpha^D\psi_\alpha^D + \bar{\psi}_\alpha^D\lambda_\alpha^D\right) \\
& -(\partial_\mu + iq_i^\alpha A_{\alpha\mu})\phi_i(\partial^\mu - iq_i^\alpha A_\alpha^\mu)\bar{\phi}_i - i\bar{\psi}_i^D\gamma^\mu\left(\partial_\mu + i(q_{2i-1}^\alpha P_L + q_{2i}^\alpha P_R)A_{\alpha\mu}\right)\psi_i^D \\
& +\sqrt{2}\left[\bar{\lambda}_\alpha^D\left(q_{2i-1}^\alpha\bar{\phi}_{2i-1}P_L + q_{2i}^\alpha\phi_{2i}P_R\right)\psi_i^D + \bar{\psi}_i^{\;D}\left(q_{2i-1}^\alpha\phi_{2i-1}P_R + q_{2i}^\alpha\bar{\phi}_{2i}P_L\right)\lambda_\alpha^D\right] \\
& +\mathcal{V}[\phi_\alpha,\bar{\phi}_\alpha,\phi_i,\bar{\phi}_j]
\end{aligned}$$

Notice the 1/2 in front the Weyl spinor λ_α^D, ψ_α^D. The mass-matrix of the fermions is not diagonal.

The potential $\mathcal{V}[\phi_\alpha, \bar{\phi}_\alpha, \phi_i, \bar{\phi}_j]$ provides the D-term:

$$\frac{1}{4\Re f_0}(\partial_V K|_0)^2 = \frac{1}{2\Re f_0}\left(c^\alpha s_\alpha + \sum_i q_i^\alpha \bar{\phi}_i \phi_i\right)^2$$

G. The extended Standard Model fields

In this appendix we provide some more details about the masses of the fields and the gauge couplings. Based on (7.2) the Higgs expectation values have the form:

$$h = \frac{v}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}, \bar{h} = \frac{v}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}. \tag{G.1}$$

Thus, the covariant derivative of the Higgs (in the $z = 0$ model) is

$$D^\mu H = \frac{v}{\sqrt{2}}(\partial^\mu - i\frac{g_3\mathbf{1}}{\sqrt{\mathbf{2}}}A_1^\mu - i\frac{g_2\mathbf{1}}{\mathbf{2}}A_2^\mu - i\frac{g_2}{2}\tau_\alpha W_\alpha^\mu)\begin{pmatrix}1\\0\end{pmatrix}e^{i\phi} \tag{G.2}$$

$$D^\mu H' = \frac{v}{\sqrt{2}}(\partial^\mu + i\frac{g_2\mathbf{1}}{\mathbf{2}}A_2^\mu - i\frac{g_2}{2}\tau_\alpha W_\alpha^2)\begin{pmatrix}1\\0\end{pmatrix}e^{i\phi'} \tag{G.3}$$

where W_α, $\alpha = 1, 2, 3$ the $SU(2)$ gauge bosons. We normalize all $U(N)$ generators according to $TrT^\alpha T^b = \delta^{\alpha b}/2$ and measure the corresponding $U(1)_N$ charges with respect to the coupling $g_N/\sqrt{2N}$, with g_N the $SU(N)$ coupling constant as in [41]. We have also $g_1 = g_3$.

The mass matrix for the gauge bosons is

$$M = V^T m V \tag{G.4}$$

where $V^T = (A_1, A_2, A_3, W_3, W_1, W_2)$ and

$$m = \frac{v^2}{4}\begin{pmatrix} g_3^2 & \frac{g_2 g_3}{\sqrt{2}} & 0 & \frac{g_2 g_3}{\sqrt{2}} & 0 & 0 \\ \frac{g_2 g_3}{\sqrt{2}} & g_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{g_2 g_3}{\sqrt{2}} & 0 & 0 & g_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_2^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_2^2 \end{pmatrix}. \tag{G.5}$$

Doing a rotation with the matrix (7.9), we can go to a basis where $\tilde{A}_1$ is the hypercharge. The other two $U(1)$ bosons $\tilde{A}_2, \tilde{A}_3$ are anomalous and we expect two axions α_2, α_3 to cancel the anomalies. Inserting

$$\mathcal{L}_{axionicterms} = \frac{1}{2}(\partial\alpha_2 - M_2\tilde{A}_2)^2 + \frac{1}{2}(\partial\alpha_3 - M_3\tilde{A}_3)^2, \tag{G.6}$$

two elements of the rotated mass matrix will be shifted. Since $v \ll M_2, M_3 \sim M_s$, we can perturbatively diagonalize this matrix and find the new masses of these new fields. Finally, there is a massless state (photon), a "light" Z boson with mass

$$m_Z^2 = \frac{v^2 g_2^2 r^2}{2t^2} - v^4 \frac{g_2^2 g_3^2 r^2 s^2 (M_2^2 + M_3^2 + (M_2^2 - M_3^2)\cos 2\theta)}{64 t^4 M_2^2 M_3^2} + O\Big[\frac{M_Z^6}{M_s^4}\Big] \qquad \text{(G.7)}$$

and two heavy ones with masses:

$$\begin{aligned} \mu_2^2 &= M_2^2 + v^2 \tfrac{8g_2^4 t^2 \cos^2\theta + g_3 \sin\theta(-4g_2^2 t^3 \cos\theta + g_3(130 g_2^4 + 66 g_2^2 g_3^2 + 9 g_3^4)\sin\theta)}{2s^2 t^2} + O\Big[\tfrac{M_Z^4}{M_s^2}\Big] \\ \mu_3^2 &= M_3^2 + v^2 \tfrac{g_3^2(130 g_2^4 + 66 g_2^2 g_3^2 + 9 g_3^4)\cos^2\theta + 4g_2^2 g_3 t^3 \cos\theta \sin\theta + 8 g_2^4 t^2 \sin^2\theta}{2s^2 t^2} + O\Big[\tfrac{M_Z^4}{M_s^2}\Big] \end{aligned} \qquad \text{(G.8)}$$

where $t = \sqrt{14g_2^2 + 3g_3^2}$, $s = \sqrt{16g_2^2 + 6g_3^2}$, $f = \sqrt{11g_2^2 + 3g_3^2}$, $r = \sqrt{7g_2^2 + 3g_3^2}$ and $\mu_i = m_{A'_i}$, the masses of the new anomalous U(1)s. The old fields as functions of the new rotated fields are:

$$\begin{aligned} A_1 &\approx \tfrac{2\sqrt{3} t g_2 A'_1 - \sqrt{2} r s \sin\theta A'_2 + \sqrt{2} r s \cos\theta A'_3 - 6 g_2 g_3 W'_3}{2tr} \\ A_2 &\approx \tfrac{-\sqrt{6} g_3 s t A'_1 + 4 g_2 r (2t\cos\theta - 3 g_3 \sin\theta) A'_2 + 4 g_2 r (3 g_3 \cos\theta + 2t \sin\theta) A'_3 + 3\sqrt{2} g_3^2 s W'_3}{2rst} \\ A_3 &\approx \tfrac{2 g_2 s t A'_1 + \sqrt{6} r (g_3 t \cos\theta + 4 g_2^2 \sin\theta) A'_2 + \sqrt{6} r (-4 g_2^2 \cos\theta + g_3 t \sin\theta) A'_3 - 2\sqrt{3} g_2 g_3 s W'_3}{rst} \\ W_3 &\approx -\tfrac{\sqrt{3} g_3 A'_1 + t W'_3}{\sqrt{2} r} \end{aligned} \qquad \text{(G.9)}$$

where A'_1 and W'_3 are the photon and the Z^0.

It is necessary to add a R_ξ gauge fixing term. This will cancel some mixing terms which are coming from the kinetic terms of the Higgses and it will maintain the manifest unitarity of the theory with spontaneously broken gauge symmetry.

$$\begin{aligned} \mathcal{L}_{gaugefixing} = {} & \lambda (\partial A'_1)^2 \\ & + \mu \Big(\partial A'_2 - v^2 \frac{2(\phi - \phi') g_2^2 t \cos\theta - g_3 (f^2 \phi - 3 g_2^2 \phi') \sin\theta}{2\mu t s} - \frac{M_2}{2\mu}\alpha_2\Big)^2 \\ & + \rho \Big(\partial A'_3 - v^2 \frac{g_3 (f^2 \phi - 3\phi' g_2^2) \cos\theta + 2(\phi - \phi') g_2^2 t \sin\theta}{2\rho t s} - \frac{M_3}{2\rho}\alpha_3\Big)^2 \\ & + \sigma \Big(\partial W'_3 + v^2 \frac{(\phi + \phi') g_2 r}{2\sqrt{2}\sigma t}\Big)^2 \end{aligned} \qquad \text{(G.10)}$$

The gauge fixing terms give masses to the axions and to the Higgs. We can diagonalize perturbatively the mass-matrix of these fields. Considering $\mu = \lambda = \rho = \sigma$ we find one massless and three massive fields:

$$\begin{aligned} m_{\tilde{a}_2}^2 &= \frac{M_2^2}{4\mu} + O[M_Z^2] \quad , \qquad m_{\tilde{a}_3}^2 = \frac{M_3^2}{4\mu} + O[M_s^2] \\ m_{\tilde{\phi}}^2 &= \frac{1}{4\mu}\frac{g_2^2 r^2 v^4}{t^2} + O\Big[\frac{M_z^2}{M_s^2}\Big] \quad , \qquad m_{\tilde{\Phi}}^2 = 0 \end{aligned} \qquad \text{(G.11)}$$

The old fields as a functions of the new ones are:

$$\begin{aligned}
\alpha_2 &\approx \tilde{a}_2 - \frac{v^4(4g_2^2 g_3 t^3 \cos 2\theta + (112 g_2^6 - 106 g_2^4 g_3^2 - 66 g_2^2 g_3^4 - 9 g_3^6)\sin 2\theta)}{2t^2 s^2 M_2 M_3}\tilde{a}_3 + \frac{v^2 g_3 s^2 \sin\theta}{\sqrt{2} t M_2}\tilde{\phi} + \frac{v^2(4g_2^2\cos\theta - g_3 t \sin\theta}{\sqrt{2} s M_2}\tilde{\Phi} \\
\alpha_3 &\approx \tilde{a}_3 - \frac{v^2 g_3 s^2 \cos\theta}{\sqrt{2}2tM_3}\tilde{\phi} + \frac{v^2(g_3 t\cos\theta + 4g_2^2 \sin\theta}{\sqrt{2}s^2 M_3}\tilde{\Phi} \\
\phi &\approx \frac{v^2(2g_2^2 t\cos\theta - g_3 f^2 \sin\theta)}{tsM_2}\tilde{a}_2 + \frac{v^2(g_3 f^2\cos\theta + 2g_2^2 t\sin\theta)}{tsM_3}\tilde{a}_3 + \frac{1}{\sqrt{2}}\tilde{\phi} - \frac{1}{\sqrt{2}}\tilde{\Phi} \\
\phi' &\approx \frac{v^2 g_2^2(-2t\cos\theta + 3g_3\sin\theta)}{tsM_2}\tilde{a}_2 - \frac{v^2 g_2^2(3g_3\cos\theta + 2t\sin\theta)}{tsM_3}\tilde{a}_3 + \frac{1}{\sqrt{2}}\tilde{\phi} + \frac{1}{\sqrt{2}}\tilde{\Phi}
\end{aligned} \tag{G.12}$$

From the trilinear Yukawa couplings we can find how leptons couple to the new Higgses and axions.

H. The evaluation of lepton vertex functions

Here we will give some details about the calculation of the lepton AMM. Our goal is to separate from the vertex functions, terms proportional to $\sigma^{\mu\nu}q_m$. As the vertex functions are sandwiched by two on-shell spinors we can use the Gordon decomposition and try to distinguish terms proportional to p^μ and p'^μ. We will begin with (7.16) for the anomalous $U(1)$ diagram. We rewrite it here:

$$\bar{u}(p')[\int \frac{d^4k}{(2\pi)^4}(iQ_s\gamma_\nu P_s)\frac{i}{\not{p}' - \not{k} - m}\gamma_\mu \frac{i}{\not{p} - \not{k} - m}(iQ_l\gamma_\rho P_l)D^{\nu\rho}(k)]u(p) \tag{H.1}$$

where $s, l = L, R$ denote the chiralities. The propagator of the $U(1)$ $D^{\mu\nu}$ contains the gauge fixing parameter λ. This parameter is expected to disappear from physical gauge invariant couplings. We will verify explicitly here that λ disappears from the sum of all the vertex functions. The $D^{\mu\nu}$ consist of two terms, one independent and one dependent on λ. First, we will calculate the correction from the λ-independent part. In this case we have a fraction with three factors in the denominator. Using the Feynman parameter trick we write the denominator as follows:

$$\frac{1}{((p'-k)^2 - m^2)((p-k)^2 - m^2)((k^2 - \mu^2)} = 2! \int_0^1 dx \int_0^{1-x} dy \frac{1}{D^3} \tag{H.2}$$

where

$$D = k^2 - 2k(px + p'y) + p^2 x + p'^2 y - m^2(x+y) - \mu^2(1-x-y) \tag{H.3}$$

In order to express the denominator as a function of the norm of the momentum, we shift k to $k + px + p'y$. We find $D = k^2 - \Delta$ where

$$\Delta = m^2(x+y) + \mu^2(1-x-y) \tag{H.4}$$

Next, we will express the numerator of (H.1) in terms of k^μ in order to integrate on the internal momenta. Because of the symmetry, two identities are useful here:

$$\int \frac{d^4k}{(2\pi)^4}\frac{k^\mu}{D^3} = 0 \tag{H.5}$$

$$\int \frac{d^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{D^3} = \int \frac{d^4k}{(2\pi)^4}\frac{\frac{1}{4}k^2 g^{\mu\nu}}{D^3} \tag{H.6}$$

We keep only terms proportional to even powers of k^μ. We will separate chiral and mixed diagrams:
(1) $L-L$, $R-R$ diagrams. The numerator of (H.1) with $s=l$ has the form

$$\gamma_\nu \frac{1\pm\gamma_5}{2}(\not{A}+m)\gamma_\mu(\not{C}+m)\gamma^\nu \frac{1\pm\gamma_5}{2} \tag{H.7}$$

which, after some algebra becomes

$$\frac{1}{2}\gamma_\nu \not{A}\gamma_\mu \not{C}\gamma^\nu + \frac{1}{2}m^2\gamma_\nu\gamma_\mu\gamma^\nu. \tag{H.8}$$

Terms which contain one γ_5 are orthogonal to $\gamma_{\mu\nu}$ and we can ignore them. Also the second term in (H.8) does not contribute since it is proportional to γ_μ. Thus, only the first term remains. Shifting k to $k+px+p'y$ we obtain

$$\gamma^\nu((1-y)\not{p}'-x\not{p}-\not{k})\gamma_\mu((1-x)\not{p}-y\not{p}'-\not{k})\gamma_\nu \tag{H.9}$$

Moving all $\not{p}'$ to the left, all $\not{p}$ to the right, using (H.5), (H.6) and on-shell conditions, we find

$$4m[(1-2x-y+xy+x^2)p_\mu + (1-x-2y+xy+y^2)p_\mu] \tag{H.10}$$

Here there is a symmetry under the reflection $x \leftrightarrow y$. Thus, we can make the coefficients of p_μ and p'_μ equal adding the "reflected" terms and divide the result by 2. Now, only the integrals on x and y remain. Integrating on x and making a change of variables, we find:

$$-\frac{Q_s^2}{16m\pi^2}(p_\mu+p'_\mu)\int_0^1 dx\frac{x(x^2-3x+2)}{x^2+(1-x)\frac{\mu^2}{m^2}} \tag{H.11}$$

Our main interest is for $\mu \gg m$. Expanding, we find:

$$\frac{Q_s^2}{16m\pi^2}(p_\mu+p'_\mu)\Big(-\frac{2}{3}\Big(\frac{m}{\mu}\Big)^2 + \Big(-\frac{19}{12}-2\log\Big(\frac{m}{\mu}\Big)\Big)\Big(\frac{m}{\mu}\Big)^4 + O\Big(\frac{m}{\mu}\Big)^5\Big) \tag{H.12}$$

(2) $L-R$ and $R-L$ diagrams. The only difference from the above lies in the numerator. Working similarly, for $s \neq l$ in (H.1) we find

$$4m[(1-2x)p_\mu + (1-2y)p'_\mu] \tag{H.13}$$

and finally

$$-\frac{Q_L Q_R}{16m\pi^2}(p_\mu+p'_\mu)\int_0^1 dx\frac{2x(1-x)}{x^2+(1-x)\frac{\mu^2}{m^2}} \tag{H.14}$$

The expansion for $\mu \gg m$ gives:

$$\frac{Q_L Q_R}{16m\pi^2}(p_\mu+p'_\mu)\Big(2\Big(\frac{m}{\mu}\Big)^2 - 2\Big(-\frac{11}{3}-4\log\Big(\frac{m}{\mu}\Big)\Big)\Big(\frac{m}{\mu}\Big)^4 + O\Big(\frac{m}{\mu}\Big)^5\Big) \tag{H.15}$$

We will now calculate the contribution of the second (λ-dependent) term of the massive gauge field's propagator (6.8) in (7.16). The denominator contains four factors. We will use again the Feynman parameter trick.

Due to the projection operators, there are terms with two, one and no γ_5. Terms with one γ_5 do not contribute to (7.15) being orthogonal to both γ_μ, $\sigma_{\mu\nu}$. Terms without γ_5 vanish using mass-shell conditions of the fermions that sandwich the diagram. Only terms with two γ_5s remain. After a lot of Diracology we obtain

$$-(1-\lambda^{-1})\frac{\Delta Q^2(p_\mu+p'_\mu)}{16\pi^2}\int_0^1 dx\int_0^x dy\int_0^y dz\,\times \\ \left(-\frac{m(-1+3z)}{m^2y^2+\mu^2\left(x-y+\frac{1-x}{\lambda}\right)}+\frac{m^3zy^2}{\left(m^2y^2+\mu^2\left(x-y+\frac{1-x}{\lambda}\right)\right)^2}\right) \tag{H.16}$$

Now, we will calculate the axion diagrams (7.21) and (7.23). The β' axion diagram is equal to

$$\frac{m^2\Delta Q^2}{\mu^2}\bar{u}(p')\int\frac{d^4k}{(2\pi)^2}\gamma_5\frac{i}{\not{p}'-\not{k}-m}\gamma_\mu\frac{i}{\not{p}-\not{k}-m}\gamma_5 G_{b'}(k)u(p) \tag{H.17}$$

The only difference with the $U(1)$ diagram (H.1) is in the numerator. So, we focus on it and the result is

$$2[(x^2+yx)p_\mu+(y^2+xy)p'_\mu]. \tag{H.18}$$

Thus, the (H.17) contribution is

$$\frac{\Delta Q^2}{16m\pi^2}\frac{\mu^2}{m^2}(p_\mu+p'_\mu)\int_0^1 dx\frac{x^3}{x^2+(1-x)\frac{\mu^2}{\lambda m^2}} \tag{H.19}$$

In the entire contribution only (H.16) and (H.19) are λ dependent. Adding these two terms and calculating the λ derivative using Mathematica we find zero. Thus, λ disappears as it should and we can use the Feynman - t'Hooft gauge for simplicity. As we are interested in $\mu \gg m$, we expand (H.19):

$$\frac{\Delta Q^2}{16m\pi^2}\frac{\mu^2}{m^2}(p_\mu+p'_\mu)\left(\left(-\frac{11}{6}-2\log\left(\frac{m}{\mu}\right)\right)\left(\frac{m}{\mu}\right)^4+O\left(\frac{m}{\mu}\right)^5\right). \tag{H.20}$$

Let us now turn to the ϕ' diagram. The corresponding integral is the $\mu\to 0$ limit of the the integral in (H.19). However we will consider a more general case where μ is small. Keeping the same coupling constant as the above we have

$$\frac{h^2}{16m\pi^2}(p_\mu+p'_\mu)\int_0^1 dx\frac{x^3}{x^2+(1-x)\frac{m_{\phi'}^2}{m^2}} \tag{H.21}$$

Considering $m_{\phi'}$ very small we can expand (H.21) and we find

$$\frac{h^2}{16m\pi^2}(p_\mu+p'_\mu)\left(\frac{1}{2}+\left(1+log\left(\frac{m_{\phi'}}{m}\right)\right)\left(\frac{m_{\phi'}}{m}\right)^2\right)+O\left(\frac{m_{\phi'}}{m}\right)^3 \tag{H.22}$$

In the last formula there is h which is computable from SM. From (6.7) is obvious that we need to estimate the expectation value of the Higgs v. Using the mass of Z^0 $M_{Z^0} = 91.19GeV$, the electron charge e and the value of $\sin^2\theta_W = 0.23$ from SM we find $v = 2M_{Z^0}\sin\theta_W\sqrt{1-\sin^2\theta_W}/e$ so

$$h = \frac{em_{muon}}{2M_{Z^0}\sin\theta_W\sqrt{1-\sin^2\theta_W}} \tag{H.23}$$

I. Massless spectrums of some orientifolds

Twist Group Gauge Group	(99)/(55) matter	(95) matter
Z_6' $U(4)_9^2\times U(8)_9\times$ $U(4)_5^2\times U(8)_5$	$(\overline{4},1,8)+(1,4,\overline{8})+(6,1,1)$ $+(1,\overline{6},1)+(4,1,8)+(1,\overline{4},\overline{8})$ $+(\overline{4},4,1)+(4,4,1)+(\overline{4},\overline{4},1)$ $+(1,1,28)+(1,1,\overline{28})$	$(\overline{4},1,1;\overline{4},1,1)+(1,4,1;1,4,1)$ $+(1,\overline{4},1;1,1,8)+(1,1,8;1,\overline{4},1)$ $+(4,1,1;1,1,\overline{8})+(1,1,\overline{8};4,1,1)$
Z_6 $U(6)_9^2\times U(4)_9\times$ $U(6)_5^2\times U(4)_5$	$2(15,1,1)+2(1,\overline{15},1)$ $+2(\overline{6},1,4)+2(1,6,\overline{4})$ $+(\overline{6},1,\overline{4})+(1,6,4)+(6,\overline{6},1)$	$(6,1,1;6,1,1)+(1,\overline{6},1;1,\overline{6},1)$ $+(1,6,1;1,1,\overline{4})+(1,1,\overline{4};1,6,1)$ $+(\overline{6},1,1;1,1,4)+(1,1,4;\overline{6},1,1)$

Table 1: The transformations of the massless fermionic states in two D=4 orientifolds.

Twist Group Gauge Group	(99)/(55) matter	(95) matter
Z_2 $U(16)_9\times U(16)_5$	$2\times 120+2\times\overline{120}$	$(16;\overline{16})+(\overline{16};16)$
Z_3 $U(8)\times SO(16)$	$(8,16_v)+(\overline{8},16_v)$ $+(28,1)+(\overline{28},1)$	-
Z_4 $U(8)_9\times U(8)_9\times$ $U(8)_5\times U(8)_5$	$(28,1)+(\overline{28},1)$ $+(1,28)+(1,\overline{28})$ $+(8,\overline{8})+(\overline{8},8)$	$(8,1;\overline{8},1)+(\overline{8},1;8,1)$ $+(1,8;1,\overline{8})+(1,\overline{8};1,8)$
Z_6 $(U(4)^2\times U(8))_9\times$ $+(U(4)^2\times U(8))_5$	$(\underline{6,1},1)+(\underline{\overline{6},1},1)$ $+(\underline{4,1},\overline{8})+(\underline{\overline{4},1},8)$	$(4,1,1;\overline{4},1,1)+(\overline{4},1,1;4,1,1)$ $+(1,4,1;1,\overline{4},1)+(1,\overline{4},1;1,4,1)$ $+(1,1,8;1,1,\overline{8})+(1,1,\overline{8};1,1,8)$

Table 2: The transformations of the massless fermionic states in all the D=6 orientifolds. The underlined numbers denote all the possible permutations.

	$\mathbf{Z}_2$	
$\gamma_h^2=-1$ $\{U(a)\times U(b)\}_{9,5}$	(99)/(55) matter	(59) matter
Scalars	adjoint $+\ (a,b)+c.c.$	$(a,1;\bar a,1)+(1,b;1,\bar b)+c.c.$
Fermions	$(\genfrac{}{}{0pt}{}{\Box}{\Box},1)+(1,\genfrac{}{}{0pt}{}{\Box}{\Box})+2(a,\bar b)+c.c.$	$(a,1;1,\bar b)+(1,b;\bar a,1)+c.c.$
$\gamma_h^2=+1$ $\{U(a)\times U(b)\}_{9,5}$	(99)/(55) matter	(59) matter
Scalars	adjoint $+\ (\genfrac{}{}{0pt}{}{\Box}{\Box},1)+(1,\genfrac{}{}{0pt}{}{\Box}{\Box})+c.c.$	$(a,1;\bar a,1)+(1,b;1,\bar b)+c.c.$
Fermions	$(a,b)+2(a,\bar b)+c.c.$	$(a,1;1,\bar b)+(1,b;\bar a,1)+c.c.$
	$\mathbf{Z}_3$	
$\gamma_h^2=-1$ $U(a)\times U(b)\times U(8)$	(99)/(55) matter	
Scalars	adjoint $+(a,b,1)+(\bar a,1,8)+(1,b,\bar 8)+c.c.$	
Fermions	$2\left((a,\bar b,1)+(1,1,\genfrac{}{}{0pt}{}{\Box}{\Box})\right)+(\genfrac{}{}{0pt}{}{\Box}{\Box},1,1)+$ $+(1,\genfrac{}{}{0pt}{}{\Box}{\Box},1)+(\bar a,1,\bar 8)+(1,\bar b,8)+c.c.$	
$\gamma_h^2=+1$ $U(a)\times U(b)\times$ $SO(c)\times SO(d)$	(99) matter	
Scalars	adjoint $+(\genfrac{}{}{0pt}{}{\Box}{\Box},1,1,1)+(\bar a,1,c,1)$ $+(1,\genfrac{}{}{0pt}{}{\Box}{\Box},1,1)+(1,\bar b,1,d)+c.c.$	
Fermions	$2\left((a,\bar b,1,1)+(1,1,c,d)\right)+(\bar a,\bar b,1,1)$ $+(a,1,1,d)+(1,b,c,1)+c.c.$	

Table 3: The h action on the Chan-Paton charges breaks the gauge group of the six-dimensional supersymmetric orientifolds compactified on $K3$. For Z_2 we have $a+b=16$ and for Z_3: $a+b=8$.

	$\mathbf{Z}_4$	
$\gamma_h^2=-1$ $\{U(a)\times U(b)\times$ $U(c)\times U(d)\}_{9,5}$	(99)/(55) matter	(59) matter
Scalars	adjoint $+(\bar a,b,1,1)+(a,1,\bar c,1)$ $+(1,b,1,\bar d)+(1,1,c,d)+c.c.$	$(a,1_3;\bar a,1_3)+(1,b,1_2;1,b,1_2)+$ $(1_2,c,1;1_2,\bar c,1)+(1_3,d;1_3,\bar d)+c.c.$
Fermions	$2\times\big((a,b,1,1)+(1,1,c,d)\big)$ $+(\boxminus,1,1,1)+(\bar a,1,1,\bar d)+(1,\boxminus,1,1)$ $(1,\bar b,c,1)+(1,1,\boxminus,1)+(1,1,1,\boxminus)+c.c.$	$(a,1_3;1,\bar b,1_2)+(1,b,1_2;\bar a,1_3)+$ $(1_2,c,1;1_3,\bar d)+(1_3,d;1_2,\bar c,1)+c.c.$
$\gamma_h^2=+1$ $\{U(a)\times U(b)\times$ $U(c)\times U(d)\}_{9,5}$	(99)/(55) matter	(59) matter
Scalars	adjoint $+(\boxminus,1_3)+(\bar a,1,c,1)+(1,\boxminus,1_2)$ $+(1,\bar b,1,d)+(1_2,\overline{\boxminus},1)+(1_3,\overline{\boxminus})+c.c.$	$(a,1_3;\bar a,1_3)+(1,b,1_2;1,b,1_2)+$ $(1_2,c,1;1_2,\bar c,1)+(1_3,d;1_3,\bar d)+c.c.$
Fermions	$2\,\big((a,\bar b,1,1)+(1,1,c,\bar d)\big)+(\bar a,\bar b,1,1)$ $+(a,1,1,\bar d)+(1,b,\bar c,1)+(1,1,c,d)+c.c.$	$(a,1_3;1,\bar b,1_2)+(1,b,1_2;\bar a,1_3)+$ $(1_2,c,1;1_3,\bar d)+(1_3,d;1_2,\bar c,1)+c.c.$
	$\mathbf{Z}_6$	
$\gamma_h^2=-1$ $\{U(a)\times U(b)\times$ $U(c)\times U(d)\times$ $U(e)\times U(f)\}_{9,5}$	(99)/(55) matter	(59) matter
Scalars	adjoint $+(\bar a,b,1_4)+(a,1,\bar c,1_3)+$ $(1,b,1,\bar d,1_2)+(1_2,c,1,\bar e,1)+$ $(1_3,d,1,\bar f)+(1_4,e,f)+c.c.$	$(a,1_5;\bar a,1_5)+(1,b,1_4;1,b,1_4)+$ $(1_2,c,1_3;1_2,\bar c,1_3)+(1_4,e,1;1_4,\bar e,1)$ $(1_3,d,1_2;1_3,\bar d,1_2)+(1_5,f;1_5,\bar f)+c.c.$
Fermions	$2\,\big((a,\bar b,1_4)+(1_2,c,\bar d,1_2)+(1_4,e,\bar f)\big)+$ $(\bar a,1_2,\bar d,1_2)+(1,\bar b,c,1_3)+(1_2,\bar c,1_2,f)$ $+(1,b,1_4;\bar a,1_5)+(1_3,\bar d,e,1)+(\boxminus,1_5)$ $(1,\boxminus,1_4)+(1_4,\overline{\boxminus},1)+(1_5,\overline{\boxminus})+c.c.$	$(a,1_5;1,\bar b,1_4)+(1,b,1_4;\bar a,1_5)+$ $(1_2,c,1_3;1_3,\bar d,1_2)+(1_4,e,1;1_5,\bar f)$ $(1_3,d,1_2;1_2,\bar c,1_3)+(1_5,f;1_4,\bar e,1)$ $+c.c.$
$\gamma_h^2=+1$ $\{U(a)\times U(b)\times$ $U(c)\times U(d)\times$ $U(e)\times U(f)\}_{9,5}$	(99)/(55) matter	(59) matter
Scalars	adjoint $+(\bar a,1,\bar c,1_3)+(1,b,1,d,1_2)$ $(1_2,\bar c,1,e,1)+(1_3,\bar d,1,f)+(\boxminus,1_5)$ $+(1,\boxminus,1_4)+(1_4,\overline{\boxminus},1)+(1_5,\overline{\boxminus})$	$(a,1_5;\bar a,1_5)+(1,b,1_4;1,b,1_4)$ $(1_2,c,1_3;1_2,\bar c,1_3)+(1_4,e,1;1_4,\bar e,1)$ $(1_3,d,1_2;1_3,\bar d,1_2)+(1_5,f;1_5,\bar f)$
Fermions	$2\times\big((a,\bar b,1_4),\ (1_2,c,\bar d,1_2),\ (1_4,e,\bar f)\big)$ $(\bar a,\bar b,1_4)+(a,1_2,\bar d,1_2)+(1,b,\bar c,1_3)$ $(1_2,c,1_2,\bar f)+(1_3,d,\bar e,1)+(1_4,e,f)$	$(a,1_5;1,\bar b,1_4)+(1,b,1_4;\bar a,1_5)$ $(1_2,c,1_3;1_3,\bar d,1_2)+(1_2,c,1;1_3,\bar d)$ $(1_3,d,1_2;1_2,\bar c,1_3)+(1_3,d;1_2,\bar c,1)$

Table 4: The h action on the Chan-Paton charges breaks the gauge group of the six-dimensional supersymmetric orientifolds compactified on $K3$. For Z_4 we have $a+b=c+d=8$ and for Z_6: $2a+2b=c+d=2e+2f=8$.

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