Value At Risk,Forecasting and Non-Linear Transformations

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0.1 Abstract

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The VaR measure is a statistical quantile, expressing the minimum loss of a security or a portfolio for a given period and confidence level. The accuracy of the assessment of this measure is important as it contributes to the optimal portfolio selection and generally speaking, defines the level of risk that should be faced by an investor or by an institution. At this thesis, I am going to use the semi-parametric CAViaR model in order to estimate VaR. The main aim of this study is the comparison of the existing models and forecasting methods of VaR with those that use non-linear transformations. The above comparison would focus on assessing the predictive power to estimate the VaR measure between the initial and transformed data. The empirical part of this paper, is based on the forecasting ability of the above process for three financial indexes, Cac40, Nikkei225 and SP500.

0.2 Introduction

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In order to have a general idea about the Value at Risk(VaR), it would be essential to mention what actually is risk, in the introduction of this thesis. At first we could say that risk is the danger that an organization faces when there are losses by the change in a market price or, generally speaking, is the possibility of an undesirable result. In other words, when an investment's actual return is different from the expected return then this move has losses and risk. In economic terms, operations carried out on the market often involves risk, which is the likelihood of side effects or, more specific, the possibility of defaults to a portfolio because of changes in the market value of assets (Siriopoulos, 2010).There are three essential steps that concern the calculation of risk level: a) identification and classification of risk, b) measurement and c) management of this risk. According to Saunders and Cornett (2008) there are many classes of risk where the main categories are the following:

1. **Interest rate risk.** The risk incurred by a financial institution when the maturities of its assets and liabilities are mismatched.

2. **Market risk.** The risk incurred from assets and liabilities in a financial institution's trading book due to changes in interest rates, exchange rates and other prices.

3. **Credit risk.** The risk that promised cash flows from loans and securities held by financial institutions may not be paid in full.

4. **Off-balance-sheet risk.** The risk incurred by a financial institution as the result of activities related to its contingent assets and liabilities held off the balance sheet.

5. **Foreign exchanged risk.** The risk that exchange rate changes can affect the value of a financial institution's assets and liabilities denominated in nondomestic currencies.

6. **Country or sovereign risk.** The risk that repayments to foreign lenders or investors may be interrupted because of restrictions, intervention, or interference from foreign governments.

7. **Technology risk.** The risk incurred by a financial institution when its technological investments do not produce anticipated cost savings.

8. **Operational risk.** The risk that existing technology, auditing, monitoring and other support systems may malfunction or break down.

9. **Liquidity risk.** The risk that a sudden surge in liability withdrawals may require a financial institution to liquidate assets in a very short period of time and at less than fair market prices.

10. **Insolvency risk.** The risk that a financial institution may not have enough capital to offset a sudden decline in the value of its assets.

As a result, we can understand that the correct measurement of each of these categories, could determine the failure or the success of an institution or of an investor. So, a main step is to measure and quantify the level of the risk in order to avoid losses or to increase our gains. The risk of a portfolio or of an investment is connected with the variance's and covariance's change level so there are many methods of calculating the possibility of a change in the level of variance and the basic result is that when the volatility increases then the risk is greater (Siriopoulos, 2010).

As we can conclude, the financial disaster of nowadays demands the correct and the most reliable measurement of financial risk. VaR is a number of great importance and the benchmark for risk measurement and subsequent capital allocation for financial institutions worldwide, as chosen by the Basel Committee on Banking Supervision(Gerlach et al, 2011). VaR measure is a statistical quantile expressing the minimum loss of a security or a portfolio of a given period for a certain confidence level. The accuracy of the assessment of this measure is much important as it contributes to optimal portfolio selection.
At this thesis I will examine, the conditions under which we have the best result of forecasting the VaR number and I will compare the classical methods with those that use non-linear transformations. The rest of the thesis is organized as follows: **Chapter 1** presents analytically what actually is VaR and also methods of estimating and testing VaR are mentioned. In this chapter I mention the CAViaR method which is the basic process that I use for my analysis.
Chapter 2 reports some of non-linear transformations and I state the process in order to calculate the transformation parameter. **Chapter 3**, involves the empirical data analysis of the CAViaR model comparison using transformed and non-transformed data. Finally, at **Chapter 4** I refer the conclusions of the above comparison and generally of whole document.

Chapter 1

Theoretical Basis and Value at Risk

1.1 Introduction to Value at Risk

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To begin with, it would be important to make a historical reference about VaR. In the late of 1970s and 1980s, a number of major financial institutions started work on internal models to measure and aggregate risk across the institution as a whole. They started work on these models in the first instance for their own internal risk management purposes. As firms became more complex, it was becoming increasingly difficult, but also increasingly important, to be able to aggregate their risk taking account of how they interact with each other, and the main problem was that firms lacked the methodology to do so. The best known of these systems is the RiskMetrics system developed by JP Morgan. This system is said to have originated when it was major to have a daily report indicating risk and potential losses. In order to meet this demand, the Morgan staff had to develop a system to measure risk across different trading positions, across the whole institution, and also aggregate this risk into a single risk measure. The measure used was VaR (Kevin Dowd, 2002). As I have already mentioned above, VaR is a method of calculating risk, but it is important at this part to give a further explanation of this. Value at Risk, is an attempt to provide a single number summarizing the total risk in a portfolio of financial institutions. More specific we can define VaR as follows: I am X percent certain there will not be a loss of more than V dollars in the next N days. The variable V is the VaR of the portfolio and it is a function of two parameters: the time horizon (N days) and the confidence level(X) . It is the loss level over N days that has a probability of only (100-X) percentage of being exceeded, and we can represent this by the following figure: (Hull, 2009).

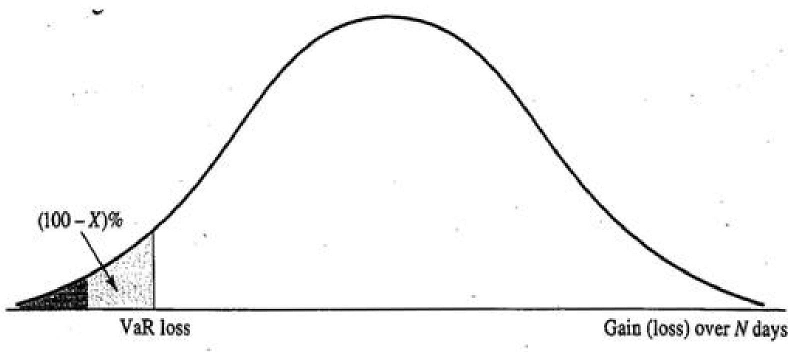

Figure 1.1: Value at Risk.

According to the book of Ruey S. Tsay (2005), VaR is used to ensure that the financial institution can still be in business after a catastrophic event. More specific, VaR can be defined as the maximal loss of a financial position during a given time period for a given probability (confidence level). In a mathematical way, suppose that at the time index t we are interested in the risk of financial position for the next l periods. We can define VaR with probability p as:

$$p = Pr[y_t < -VaR] = F_\ell(VaR)$$

where y_t is the change in value of the assets from time t to t=l, and $F_l(VaR)$ is the cumulative distribution function of y_t. This definition refers to a long position where we suffer a loss when $y_t < 0$. As for short position where we suffer a loss when the value of asset increases, $y_t > 0$, we have:

$$p = Pr[y_t > VaR] = 1 - Pr[y_t < VaR] = 1 - F(VaR)$$

Because the VaR represents a hypothetical loss, it is usually a positive number, and another general definition could be represented below:

$$Pr(y_t < -VaR) = \alpha$$

where $y_t = W(t) - W(t-1)$ is the change in the value of the portfolio. VaR number has many advantages, according to Kevin Dowd (2002) . The first is that it provides a common consistent measure of risk across different positions and risk factors. VaR provides us with a common risk yardstick, and this yardstick makes it possible for institutions to manage their risks in new ways that were not possible before. Secondly, VaR takes account of the correlations between different risk.

On the other hand, the disadvantages of VaR are the following: VaR estimates are too imprecise to be of much use, and empirical evidence suggests that different VaR models could give very different VaR estimates. The danger here is obvious: if VaR estimates are too inaccurate and users take them seriously, they could take on much bigger risks and lose more than they had bargained for. Another problem is that if VaR measures are used to control or renumerate risk-taking, traders will have an incentive to seek out positions where risk is over- or under-estimated and trade them. They will therefore take on more risk than suggested by VaR estimates so our VaR estimates will be biased downwards and their empirical evidence suggests that the magnitude of these underestimates can be very substantial. Furthermore, VaR players are dynamic hedgers, and need to revise their positions in the face of changes in market prices. If everyone uses VaR, there is then a danger that this hedging behavior will make uncorrelated risks become very correlated and firms will bear much greater risk than their VaR models might suggest.

To sum up this part of my thesis, I think that we can all understand the importance of this method. VaR information can be used in many ways. Some of them are: (1) Senior management can use it to set their overall risk target, and from that determine risk targets and position limits down the line. If they want the firm to increase its risks, they would increase the overall VaR target, and vice versa. (2) We can use it to determine capital allocation, where the riskier the activity, the higher the VaR and the greater the capital requirement. (3) VaR can be very useful for reporting and disclosing purposes, and firms increasingly make a point of reporting VaR information in their annual reports. (4) We can use VaR information to assess the risks of different investment opportunities before decisions are made. In short, VaR could help to provide a more consistent and integrated approach to the management of different risks, leading also to greater risk transparency and disclosure, and better strategic management.

1.2 Methods of Estimating VaR

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At this point, it would be appropriate to present the methods that we use in order to estimate VaR. We have three main methods and a number of models, which could be presented as examples to these methods, in order to estimate VaR:

1. **Non-parametric method.** According to this approach the VaR is estimated as the quantile of the empirical distribution of historical returns from a moving window of the most recent periods. The advantage of this is that it requires no distributional assumption and that it is easy to compute. A drawback of this process is that the VaR estimation can be poor and slow to converge to the actual VaR, especially for the extreme quantiles, and there is a difficulty in the choice of the number of observations to include in the moving window(Jeon, Taylor, 2013). An example of this method is the Historical Simulation:

   Historical Simulation. In this method, which is a simple one, we use elements of the past in order to estimate what may happen in the future. The first step in this process is to define the variables of the market that could affect the value of a portfolio. In substance, we calculate the change in the value of the portfolio between the present and the future and we determine a probable distribution (Hull, 2009). As for the simple historical simulation method (HS), VaR calculated at the time t+1, given the data from the experience level in a window of w observations at time t. This method has many advantages and disadvantages.
   As for the advantages we have: it is intuitive and conceptually simple, this approach is easy to implement on a spreadsheet, we use data that are readily available and this method provide results that are easy to report and easy to communicate to senior managers and interested outsiders and also it defines a distribution of the market variables. Opposite, we have the following drawbacks which characterize Historical Simulation: The biggest weakness is that the results are completely dependent on the data set and this could lead to many problems. More specific, if our data period was unusually quit, Historical Simulation will often produce VaR estimates that are low for the risk we are

actually facing and if our data period was unusually volatile we have the opposite result. Other disadvantage is that this approach could have, difficulty handling shifts that might take place during our sample period and is sometimes slow to reflect major events. It could be characterized as a simple and slow model and it cannot explain the upgrade of volatility (Kevin Dowd, 2002).

In this part, I am going to refer to time-varying methods of estimating VaR. As we know, volatility is the standard deviation of returns and so it could be considered as a measure of risk. Time-varying volatility implies that volatility is itself subject to extreme fluctuations, and in our case, stocks and other financial instruments exhibiting periods of high volatility and low volatility at various periods in time. So the following methods and models refer to time-varying methods.

2. **Parametric Method.** This method involves a parameterization of the behavior of prices, with conditional quantiles estimated using a conditional volatility forecast and an assumption for the shape of the distribution. An advantage of this process is the complete formation of the conditional returns distribution, and a major disadvantage is that the specification of the variance equation and the choice of distribution might be wrong. A classic example of parametric method is the GARCH model:

   GARH Model. These kind of models, Generalized Autoregressive Conditional Heteroscedasticity(GARCH), are econometric models used in cases where we need several parameters to explain adequately the volatility of assets' return (Tsay, 2005). These models are the most widely used statistical models for volatility and express the conditional variance as a linear function of lagged squared error terms and lagged conditional variance terms. To give a general definition, according to Tsay (2005), we can present mathematically a GARCH model as follows:

   $y_t = \sigma_t \varepsilon_t$

   $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i y_{t-i}^2 + \sum_{j=1}^{s} b_j \sigma_{t-j}^2$

where y_t is the shock of an asset return, and σ_t^2 is the variance, and ε_t is a sequence of iid random variables with mean zero and variance one. Generally in such a model, the distribution of tails is heavier compared with the normal distribution and this process provides a simple parametric procedure that could be used to describe the evolution of the variance. Keeping in mind that the variances cannot be negative, as a result the variables in such a model cannot be negative. The estimated variance which produced by a GARCH model, is the weighted average of three different provisions: firstly, the constant variance that indicates the long-term average, secondly the forecast that was made the previous year and finally the new information that it was not available at the previous forecast. In conclusion, these models express the volatility as linear function with the square of the time lag of the error and with the time lag of the variance. In my case, where I have to estimate VaR which is time-varying, I could define VaR as : $VaR_t = \mu_t + \sigma_t * Q(a)$, where μ_t is the mean number and Q(a) is the quantile under a distribution.

3. **Semi-parametric Method.** This third and last basic method, includes estimations of extreme value analysis and methods based on quantile regression. This method has the advantage of allowing the shape of the conditional returns distribution to be time-varying, and for the time-variation to be different for different quantiles of the distribution.

As I have already state, I am going to use the CAViaR models, which is a semi-parametric model, for classical methods and for those that use the non-linear transformations. The cause of creating this method is the need to focus on asymptotic form that the tails present instead of modeling the whole distribution. The fact that the market assets return changes during the time and coincides can show us that there is autocorrelation. So the VaR which is linked to the standard deviation of the distribution should exhibit similar behavior.

CAViaR specification. Mathematically, as it has already presented in the article of Engle and Manganelli (2004), we have:

$$f_t(b) = b_0 + \sum_{i=1}^{q} b_i f_{t-i}(b) + \sum_{j=1}^{r} b_j \ell(x_{t-j})$$

where x_t is a vector of time t observable variables and b_j is a p-vector of unknown parameters and p = q + r + 1 is the dimension of b and l is a function of a finite number of lagged values of observables. The role of $\ell(x_{t-j})$ is to link $f_t(b)$ to observable variables that belong to the information set.

In this research I am going to estimate three types of CAViaR models which are: (a) the Symmetric absolute value CAViar(S-CAViaR), (b) the Asymmetric Slope CAViaR(AS-CAViaR) and (c) the indirect AR- CAViaR but i am going to refer the Threshold CAViaR too(T-CAViaR). These models are as follow:

(a) Symmetric absolute value CAViaR(S-CAViaR) The mathematical type is as above:

$$f_t(b) = b_1 + b_2 f_{t-1}(b) + b_3 |y_{t-1}|$$

where b is the vector of betas:$b = (b_1, b_2 ...)$ and these kind of models rely on the magnitude of the error, rather than the squared error as in GARCH models. Also these models are symmetric to positive and negative observations, with linear responses and parameters. To account for financial market asymmetry, via the leverage effect the S-CAViaR model was extended in Engle and Manganelli (2004) to AS-CAViaR models (Gerlach et al, 2011).

(b) Asymmetric Slope CAViaR(AS-CAViaR) The mathematical type of this model as I have already mentioned above is:

$$f_t(b) = b_1 + b_2 f_{t-1}(b) + b_3 (y_{t-1}) + b_4 (y_{t-1})^-$$

This model was designed specifically to model the asymmetric leverage effect, which is the tendency for volatility to be greater following a negative return than a positive return of equal size (Jeon, Taylor, 2013).

(c) AR-Indirect CAViaR(AR-CAViaR). According to the article of Jeon and James, 2013, the mathematical type of the Indirect AR-Garch CAViaR is as follows:

$$f_t(\beta) = \beta_5 y_{t-1} + (\beta_1 + \beta_2((f_{t-1}(\beta) - \beta_3 y_{t-2})^2) + \beta_4((y_{t-1} - \beta_3 y_{t-2})^2))^{1/2}$$

The indirect AR(1)-GARCH(1,1) CAViaR model allows for the conditional mean to be time-varying.

(d) Threshold CAViaR(T-CAViaR). This model is not going to be used at this work but I think that it is important to present it because sometimes we make use of non-linear CAViaR models, in order to capture more flexible asymmetric and nonlinear responses via more general threshold nonlinear forms. This model is as follows:

$$f_t(b) = \begin{cases} b_1 + b_2 f_{t-1}(b) + b_3|y_{t-1}| & z_{t-1} < r \\ b_4 + b_5 f_{t-1}(b) + b_6|y_{t-1}| & z_{t-1} > r \end{cases}$$

where z is an observed threshold variable which could be exogenous, or self-exciting, that is, $z_t = y_t$ and r is the threshold value, typically set as r = 0, or estimated. Here each parameter in the dynamic quantile function can respond differently to positive and negative responses. We call this the T-CAViaR family and it includes the SAV$(r = \infty)$ and AS $(r = 0, b_4 = b_1, b_5 = b_2)$ CAViaR models as special cases(Gerlach et al. , 2011).

We focus our interest in CAViaR method because the data at this research concentrates on non-linear transformations, and we saw how it works. As it has concluded, in the article of Engle and Manganelli, 1999, all the existing models try to estimate the distribution of the returns and then recover its quantile in an indirect way but the model of CAViaR, estimate directly the quantile by estimating the evolution of the quantile over time using a special type of autoregressive process.

4. **Quantile Regression** Another important method of estimating VaR is the Quantile Regression. Many methods of quantile estimation in economics and finance are based on the assumption that financial returns have normal distributions. Under the assumption of a conditionally normal returns distribution, estimating conditional quantiles is equivalent to estimating the conditional volatility of returns. Financial time series and returns distributions are not well approximated by Gaussian models because market returns have

negative skewness and excess kurtosis.

Quantile regression is well suited to estimating conditional quantiles. Just as classical linear regression methods based on minimizing sums of squared residuals allow the estimation of models for conditional mean, quantile regression methods offer a mechanism for estimating models for the conditional quantiles (Xiao and Koenker, 2009). Quantile regression provides a technique to estimate conditional quantiles of y given one or more explanatory variables. It allows for a more precise description of the tails of the distribution of y and such a model is robust to heteroskedasticity, skewness and leptokurtosis which are common features of financial data (Baur, Dimpfl , Jung, 2007). If the conditional return distribution is not Gaussian but fat-tailed, then QR estimates will be more robust and efficient than the conditional estimates(Ma and Pohlman, 2008).

It would be more useful to represent this method mathematically, as it has represented in the literature. As Barnes and Hughes,2002, represent , a quantile regression model is as follows:

$$y_i = x_i^{'}\beta_\theta + u_{\theta i}$$

where β_θ is an unknown k*1 vector of regression parameters associated with the θ percentile,x_i is a k*1 vector of independent variables, y_i is the dependent variable of interest and $u_{\theta i}$ is an unknown error term. In our case, where we want to estimate VaR, y_i represent the returns,$x_i^{'}\beta_\theta$ is the $f_t(b)$ which represent VaR and the error term is equal with $u_{\theta i} = \bar{y}_i - f_t(b)$. The θ conditional quantile of y given x is

$$Quant_\theta(y_i|x_i) = x_i^{'}\beta_\theta$$

Many of the empirical quantile regression papers assume that the errors are independently and identically distributed (i.i.d.), the only necessary assumption concerning $u_{\theta i}$ is $Quant_\theta(u_{\theta i}|x_i) = 0$, that is, the conditional θ quantile of the error term is equal to zero. The quantile regression estimator can be found as the solution to the following minimization problem:

$$\hat{\beta_\theta} = argmin_\beta(\sum_{i=y_i>x_i'\beta} \theta|y_i - x'\beta| + \sum_{i=y_i<x'\beta} (1-\theta)|y_i - x_i'\beta|)$$

where as I stated above we can replace $x_i'\beta$ with $f_t(b)$. As I have already present quantile estimation provides a nonparametric approach to VaR calculation. It makes no specific distributional assumption on the return of a portfolio except that the distribution continues to hold within the prediction period and is a simple method (Tsay, 2005). There are some advantages of quantile regression over the classical methods. First of all, we have the whole distribution, so we would know not only the expected average return, but also the whole distribution of the return in the next period given the information known at the current moment. Secondly, the conditional mean result could be derived from the conditional quantile effect. In particular, if the distribution of effects is not too skewed, the conditional mean effect would be close to the median, so for the same sample, quantile regression reveals more information than other methods. (Ma and Pohlman, 2008).

On the other hand, this approach has some disadvantages. First, it assumes that the distribution of the return remains unchanged from the sample period to the prediction period. Given that VaR is concerned mainly with tail probability, this assumption implies that the predicted loss cannot be greater than that of the historical loss something that it is not valid in practice. Second, for extreme quantiles (i.e., when p is close to zero or unity), the empirical quantiles are not efficient estimates of the theoretical quantiles. Third, the direct quantile estimation fails to take into account the effect of explanatory variables that are relevant to the portfolio under study. In real application, VaR obtained by the empirical quantile can serve as a lower bound for the actual VaR (Tsay, 2005).

1.3 Methods of Testing VaR

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In this subsection I am going to refer methods that we can use in order to testing VaR number. Analytically, there are tests that we could follow so we can understand if the Value at Risk number that we found is valid or not. The most reliable of these methods are the backtesting, which includes Kupiec and Christoffersen test and finally the Violation Rate.

Backtesting

This is a statistical model which certifies that actual losses matching with the expected losses and this indicates that we can compare the historical VaR with the return of the portfolio. The comparison of these two, specifies the periods in which VaR underestimated or when the loss of the portfolio exceed the initial expected VaR. Analytically, using the definition of Kevin Dowd (2002), before we can use risk models with confidence, it is necessary to validate them, and the critical issue in model validation is backtesting, the application of quantitative, typically statistical methods to determine whether a models risk estimates are consistent with the assumptions on which the model is based.

According to the results of the method, if actual losses do not exceed the expected VaR, then this is the appropriate measure, while if actual losses exceed the expected VaR may not be accurate. This method should be used in a small time horizon so as to ensure sufficient observations and to prevent the effect of changes in the portfolio. Also the use of high levels of confidence should be avoided as they reduce the effectiveness and strength of the method. The backtesting model is an act of balance between two types of errors: rejecting an accurate model than by accepting an inaccurate model. A powerful statistic test should minimize the chance of these errors.

The main process of this model is as follows: the first requirement in backtesting is to obtain suitable data and our profit or losses data need cleaning to get rid of components that are not directly related to current or recent market risk taking. Moreover, we select a significance level, and then we estimate the probability associated with the null hypothesis being true (we would accept the null hypothesis if the estimated value of this probability exceeds the chosen significance level).

One problem with the standard statistical backtests is that they place a lot of weight on the estimated prob-value. We have only an estimate of this and the true probability , will always remain unknown. This raises a problem: we might get a poor prob-value estimate that leads us to make an incorrect estimation of the model and incorrectly reject a true model or incorrectly accept a false one. Finally, another process that we could follow in order to examine the adequacy of our initial model is to follow again a backtesting model with alternative data, position and confidence level (Kevin Dowd, 2002). The basic tests of backetesting process, are :

1. **Kupiec test**. This test, focuses on the property of unconditional coverage(which put restrictions about how often a violation of VaR happens (Campbell, 2005)). More specific, we test if the observed frequency of tail losses or the frequency of losses that exceed VaR, is consistent with the expected frequency. Under the null hypothesis that we have a good model, the number of tail losses have the following distribution:

   $$Pr(x|n,p) = \binom{n}{i} p^i (1-p)^{n-i}$$

   where n is the number of observations and p is the expected frequency of tail losses which equal to $1-\alpha$ confidence level. This test is simple, easily applicable and does not require a large number of observations. As Campbell (2005) refers, this test is concerned with whether or not the reported VaR is violated more or less than $a*100$ percent of the time. More specific, Kupiec proposed a proportion of failures test that examines how many times a financial institution's VaR is violated over a given time period. If the number of violations differs considerably from $a*100$ percent of the sample, then the accuracy of the underlying risk model maybe it will not be valid and it could understates or overstates the portfolio's underlying level of risk. Analytically, if we have a sample size of T observations and a number of violations N, we have to know whether or not $\hat{p} \equiv N/T$ is equal to α:

   $$H_0 : p = E(H_t) = \alpha$$

The propability of observing N violations over a sample size of T is driven by a binominal distribution and null hypothesis $H_0 : p = \alpha$ can be verified through a LR test:

$$LR = 2ln * (\frac{\hat{p}^N(1-\hat{p})^{T-N}}{\alpha^N(1-\alpha)^{T-N}}$$

which follows the chi-squared distribution with one degree of freedom. These kind of tests have two drawbacks: The first shortcoming is that these tests are known to have difficulty in detecting VaR measures that systematically under report risk. From a statistical view these tests exhibit low power in sample sizes consistent with the current regulatory framework, i.e. one year. A second disadvantage of these tests is that they focus exclusively on the unconditional coverage property of an adequate VaR measure and do not examine the extent to which the independence property is satisfied. Accordingly, these tests may fail to detect VaR measures that exhibit correct unconditional coverage but exhibit dependent VaR violations.

2. **Christoffersen test**. The idea of this test is to separate the initial hypothesis and then to test each case separately in order to avoid the problem of a possible independency. For example we could separate the null hypothesis in two sub-hypothesis: first the correct frequency of tail losses arises from the model and second tail losses are independent. In this way we can understand whether the backtesting has failed and more specific the reasons of this failure(Kevin Dowd, 2002). Generally speaking, Christoffersen test develops an independence test, employing a two state Markov process, and combines this with the Kupiec test to develop a joint likelihood ratio conditional coverage test that examines whether VaR estimates display correct conditional coverage at each point in time(Chen et al.,2012). This test examines whether or not the likelihood of a VaR violation depends on whether or not a VaR violation occurred on the previous day. If the VaR measure accurately reflects the underlying portfolio risk then the chance of violating today's VaR should be independent of whether or not yesterday's VaR was violated(Campbell,2005).

Christoffersen test has one main disadvantage: all independence tests start from the assertion that any accurate VaR measure will result in a series of independent hits. Accordingly, any test of the independence property must fully describe the way in which violations of the independence property may arise. There are many ways in which the independence property could be violated. It might be the case that the likelihood of violating tomorrow's VaR depends not on whether yesterday's VaR was violated but whether the VaR was violated one week ago.If this is the way in which the lack of the independence property manifests itself then the Christoffersen test will have no ability to detect this violation of the independence property.

Violation Rate

As Gerlach et al (2012) refer, a basic criterion in order to compare VaR models is the violation rate(VRate), defined as the proportion of observations for which the actual return is more extreme than the forecasted VaR level, over the forecast period. The mathematical function of VRate is as follows:

$$\hat{\alpha} = \frac{1}{m}\left(\sum_{t=n+1}^{n+m} I(y_t < f_t(\beta))\right)$$

where n is the learning sample size, m is the forecast sample size and $\hat{\alpha}$ is the VRate. A forecast model's VRate should be close to the confidence level a. Moreover, many times we estimate the ratio $\hat{\alpha}/a$ to compare the competing models, where models with $\hat{\alpha}/a \approx 1$ are most desirable. When $\hat{\alpha}/\alpha < 1$, risk and loss estimates are higher than actual , while alternatively, when $\hat{\alpha}/\alpha > 1$, risk estimates are lower than actual and financial institutions may not allocate sufficient capital to cover future losses. More specific , when we have that $\hat{\alpha} \approx \alpha$ it is the desirable and when we have that $\hat{\alpha}/\alpha < 1$ the risk is overestimated which is preferable than $\hat{\alpha}/\alpha > 1$ where the risk is underestimated because in the last case we have a short tail and less extreme values.

In the next chapter of my thesis, I will make a reference to non-linear transformations and parameter of transformation. I will mention four of them which are the Box-Cox, Yeo-Johnson, John -Drapper and Manly transformation, which is the transformation that I am going to use in my research.

Chapter 2

Non-linear transformations and CAViaR Models

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In this chapter I am going to see how to incorporate non-linear transformations on CAViaR models . More specific, as Manly (1976) stated , many authors have discussed that the use of such transformations could make the data more suitable for analysis, with the parameter λ , which will be mentioned below,being chosen to satisfy multiple criteria. So, at this point of the document, it is necessary to refer to some of these transformations theoretically. Generally speaking, the most important of these non-linear transformations that we use in economics are the following:

1. **Box Cox transformation.** According to the article of Sakia (1992) , Box and Cox proposed a parametric power transformation technique in order to reduce anomalies such as non-additivity, non-normality and heteroscedasticity. Tukey (1957) introduced a family of power transformations such that the transformed values are a monotonic function of the observations over some admissible range and indexed by

$$y_t^{\lambda} = \begin{cases} y_t^{\lambda} & \lambda \neq 0 \\ \log y_t & \lambda = 0 \end{cases}$$

for $y_t > 0$. However, this family has been modified by Box and Cox to take account of the discontinuity at $\lambda = 0$, such that

$$y_t^\lambda = \begin{cases} (y_t^\lambda - 1)/\lambda & \lambda \neq 0 \\ \log y_t & \lambda = 0 \end{cases}$$

This transformation is valid only for $y_t > 0$ and therefore, modifications have had to be made for negative observations. So they proposed a shifted power transformation which has the form below:

$$y_t^\lambda = \begin{cases} ((y_t + \lambda_2)^{\lambda_1} - 1)/\lambda_1 & \lambda_1 \neq 0 \\ \log(y_t + \lambda_2) & \lambda_1 = 0 \end{cases}$$

where λ_1 is the transformation parameter and λ_2 is chosen to be $y_t > -\lambda_2$. Box-Cox used the asymptotic distribution of the likelihood ratio to test some hypotheses about the parameter.

More specific as we can see at the article of Tsiotas (2008), the main characteristic of Box-Cox is its parameter λ which determines the specification. When $\lambda = 0$ it becomes linear expressing normal-like dependencies and when it takes values away from zero, becomes non-linear and gives rise to dependencies that deviate from normality.

2. **Yeo-Johnson transformation.** As we can read in the article of Yeo and Johnson (2000), the main purpose of this transformation was to reduce skewness and to approximate normality. As I have already mentioned, the basic Box-Cox transformation is valid for positive y_t. So it was important to introduce another parameter in order to handle situations where the response is negative. So, they first considered a modified modulus transformation which has different transformation parameters on the positive and negative line:

$$y_t^\lambda = \begin{cases} ((x+1)^{\lambda_+} - 1)/\lambda_+ & (x \geq 0, \lambda_+ \neq 0) \\ \log(x+1) & (x \geq 0, \lambda_+ = 0) \\ -((-x+1)^{\lambda_- - 1}/\lambda_- & (x < 0, \lambda_- \neq 0) \\ -\log(-x+1) & (x < 0, \lambda_- = 0) \end{cases}$$

Next , they imposed the condition that the second derivative $\partial^2 y(\lambda_+, \lambda_-, x)/\partial x^2$ be continuous at x=0. This forces the transformation to be smooth and implies that $\lambda_+ + \lambda_- = 2$. Then , they defined the power transformation , $y() : R * R \to R$, where:

$$y_t^\lambda = \begin{cases} ((x+1)^\lambda - 1)/\lambda & (x \geq 0, \lambda \neq 0) \\ \log(x+1) & (x \geq 0, \lambda = 0) \\ -((-x+1)^{2-\lambda} - 1)/(2-\lambda) & (x < 0, \lambda \neq 2)r \\ -log(-x+1) & (x < 0, \lambda = 2) \end{cases}$$

where y_t^λ is concave in x for $\lambda < 1$ and convex for $\lambda > 1$ and the number 1 in parentheses makes the transformed value have the same sign as the original value and it reduces y to the identity transformation for $\lambda = 1$.

3. **John-Draper transformation.** According to the relative article of John and Draper, 1980, it is possible to have a good transformation but maybe it is not satisfactory. For example, this transformation could reduce the skewness but there is a possibility that we do not need this correction for our data. So they introduced the following transformation:

$$y_t^\lambda = \begin{cases} sign((|y|+1)^\lambda - 1)/\lambda) & \lambda \neq 0 \\ sign(\log(|y|+1)) & \lambda = 0 \end{cases}$$

where the sign of y_t^λ is that associated with the observation y. This family of monotonic transformations, continuous at $\lambda = 0$, will be referred to as the modulus transformation and will be appropriate for dealing with a fairly symmetric but non-normal error distribution. The modulus transformation , is effective on a distribution which already possesses approximate symmetry about some central point and alters each half of the distribution through the same power transformation in an attempt to make the shape more normal.

4. **Manly transformation.**According to the article of Manly, 1976, this kind of transformation is an alternative one to Box-Cox, that I have already mention above. In essence the Manly transformation has the advantage of allowing negative data values, as Yeo-Johnson has it. The mathematical form of this transformation is:

$$y_t^\lambda = \begin{cases} \exp(\lambda y) - 1/\lambda & \lambda \neq 0 \\ x & \lambda = 0 \end{cases}$$

It is mentioned that we have to be careful at how we choose the number of λ and stressed that it is a transformation based on the Maximum Likelihood Estimation(M.L.E) of the parameter in order to make the transformed variable normally distributed.

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2.1 Estimation of transformation parameter

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It would be appropriate at this point, to refer en example of estimating the power transformation parameter λ, in order to ensure linearity.According to the article of Mu and He, 2007, the quantile regression model extends the notion of ordinary sample quantiles to a regression model in which the conditional quantiles have a linear form. Specifically, $(x_i, y_i), i = 1,,n$ is a random sample from a population, where x_i is a p*1 vector of regressors and y_i is a scalar dependent variable. The linear quantile regression model assumes that:

$$Q_\theta(Y_i|x_i) = f_t(\beta_\theta)$$

where $Q_\theta(Y_i|x_i)$ denotes the θ conditional quantile of Y, conditional on the regressor vector x_i. The linear coefficient, β_θ ,can be estimated by solving the following equation:

$$\hat{\beta_\theta} = argmin_{b\exists R_p} \sum_{i=1}^{n} \rho_\theta(y_i - f_t(\beta_\theta)$$

where $\rho_\theta(u) = |u| - (2\theta - 1)u$. A characteristic of quantile regression is "equivariance to monotone transformations".H() is a monotone function on R. Then, for any random variable Y, we have:

$$Q_\theta(h(Y)|x) = h(Q_\theta(Y|x))$$

That is, for any monotone transformation h, the conditional quantiles of the transformed variable. Transformations alter in a fundamental way what is being estimated in ordinary least squares regression. In particular, when the linearity fails, we extend the linear quantile regression model by incorporating a parametric power transformation on the dependent variable. The conditional quantile of the dependent variable Y given X=x is simply the inverse power transformation of the linear predictor $f_t(\beta_\theta)$. More analytically, on their article, they considered the power-transformed linear quantile regression model as :

$$Q_\theta(\Lambda_{\lambda_\theta}(Y_i)|x_i) = f_t(\beta_\theta)$$

where β_θ is a $p*1$ vector of unknown regression parameter,λ_θ is an unknown transformation parameter, and $\Lambda_\lambda()$ denotes the family of power transformations introduced by Box and Cox (1964), that I mentioned above.

A simple principle motivates our estimator: If a quantile regression model holds at the θ quantile, then, conditional on the covariates, the fraction of the observations that fall below the assumed θ quantile curve should be close to θ. That means that the proportion of negative residuals should be approximately θ. On the other hand, if a quantile regression model does not hold, we expect to see clear discrepancies between the proportions of negative residuals and θ somewhere in the covariate space. Our objective function simply adds up the squared differences between proportions of negative residuals and θ over many quadrants of the covariate space. Intuitively, a consistent estimator for λ_θ can be obtained by minimizing such a measure. Specifically, for a fixed λ, they fitted a linear quantile regression model based on the transformed data $(x_i, \Lambda_\lambda(y_i))_{i=1}^n$. An estimate for β_θ is obtained by solving:

$$\hat{\beta}(\lambda;\theta) = argmin_{b \exists R^p} \sum_{i=1}^{n} \rho_\theta(\Lambda_\lambda(y_i) - f_t(\beta)$$

They defined the following process of residuals indexed by $t = (t_1, t_2, , t_p) \exists R^p$:

$$R_n(t;\lambda;\theta) = (1/n) \sum_{j=1}^{n} I(X_j < t)[\theta - I(\Lambda_\lambda(Y_j) - f_t \hat{\beta}(\lambda;\theta) < 0)]$$

where I() is the indicator function. Here and for the rest of the article, $x < t$ for $x, t \exists R_p$ if and only if each component of x is less than or equal to the corresponding component of t. Thus, the size of $R_n(t, \lambda; \theta)$ can be used to distinguish the right transformation from a wrong alternative. Our objective function is then defined to be:

$$V_n(\lambda;\theta) = \int_{-\infty}^{\infty} [R_n(t,\lambda;\theta)]^2 d\hat{F_X}(t) = (1/n)\sum_{i=1}^{n}[R_n(X_i,\lambda;\theta)]^2$$

Where $\hat{F_X}(t) = n^{-1}\sum_{i=1}^{n} I(X_i < t)$ denotes the empirical distribution function of $X_1,,X_n$. Our proposed estimator for λ_θ is obtained as:

$$\hat{\lambda_n} = argmin_{\lambda \exists \Omega_{\lambda_\theta}} V_n(\lambda;\theta),$$

Where Ω_{λ_θ} denotes the parameter space for λ_θ. The corresponding estimate for β_θ is taken to be $\hat{\beta_n} = \hat{\beta}(\hat{\lambda_n};\theta)$. The objective function $V_n(\lambda;\theta)$ is not differentiable with respect to λ but it is a function of a single parameter.

The essential result that it is important to find out is whether or not, the use of non linear transformations give us a better result for the number of $\hat{\alpha}/\alpha$,comparing with the classic process, at the previous part of this thesis. In the next and last chapter of this research, I am going to present the whole statistical analysis. More analytically, the next chapter contains the tables, figures and the mathematical analysis of the initial data and also, the statistical analysis of the transformed data.

Chapter 3

Empirical Data Analysis

3.1 The Data

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To begin with, an important issue of this research is to mention some information that concern our data which will be used in order to calculate VaR. So, to implement our methodology on real data, I am going to use a sample of daily prices from Yahoo Finance that concern stock indexes. More specifically, I examine three main indexes: SP 500, CAC 40, Nikkei 225. The samples ranged from January 1, 2002 to September 30,2016 and I am interested in returns of these indexes, which defined from close prices,by taking the logarithmic difference of the daily price index as a percentage following the type bellow:

$$y_t = (\log(W_t) - \log(W_{t-1})) * 100$$

where W_t is the closing index value on day t. At Figure 3.1, at the end of the chapter p.40-43 below, I represent the distribution of daily returns and close prices respectively ,for each index from 2002 to 2016. We can observe a decline in every index from 2007 to 2009, which is a period that characterized by the global economic crisis.

By using R program I examined the basic statistical characteristics of my series as it shows below at Table 1. From these results we could derive that our data are not normal, they present autocorrelation and as for the other statistics we have leptokurtossis,because for every index the kurtosis is much greater than 3(kurtosis=3 for normal distribution) and negative skewness. In this case, that our data concern returns of financial indexes, these outcomes are logical because in real life financial time series are not normally distributed.

Table 1:Descriptive statistics

statistics	CAC 40	SP 500	NIKKEI 225
Mean	-0.00033	0.00737	0.00495
Median	0.01571	0.0266	0.02029
Sd	0.64697	0.53622	0.66935
Min	-4.11343	-4.11255	-5.25974
Max	4.60117	4.7586	5.74771
Kurtosis	5.18679	9.36206	6.74485
Skewness	-0.0003224	-0.225	-0.453

It is important to mention that, at this kind of data we usually have to separate the sample of each index in two subperiods the in sample and the out of sample. Empirical evidence based on out-of-sample forecast performance because is generally considered more trustworthy than evidence based on in-sample performance which can be more sensitive to external factors. Out-of-sample forecasts also better reflect the information available to the forecaster in real time and we could regard out-of-sample performance as the ultimate test of a forecasting model(Hansen, 2012).

More specific, in this work I am going to examine two forecasting periods. For the first one, I split the given data in the in sample period which is from 1/1/2002 to 12/31/2009, and it is used for initial parameter estimation and model selection, and an out-of-sample period, which is from 1/1/2010 to 09/30/2016 and it used to evaluate forecast performance and backtesting method. This period, as we can see at the Figure 3.2, is characterized by a rising in close prices of each index. For the second one, the in sample period is from 1/1/2002 to 12/31/2006 and the out of sample period is from 1/1/2007 to 12/31/2009. For each of these periods, I will estimate the VRate number and the Christoffersen numbers(which is more valid and preferable than Kupiec test which I mentioned above).

3.1.1 Forecasting period: 2010-2016

According to the tables that follow we have: as Table 2 represents, for $\alpha = 0.05$, we can conclude that the $\hat{\alpha}/\alpha$ number about the forecasting period, for S and AS-CAViaR and for all of our indexes is greater than 1 which means that the risk is underestimated. Opposite to this,in AR-CAViaR this number smaller than 1, fact that show us that the risk is overerestimated for Cac40 and Nikkei225, and the risk is underestimated for SP 500, because the $\hat{\alpha}/\alpha$ is greater than 1. As for $\alpha = 0.01$, we have that the number of $\hat{\alpha}/\alpha$ is greater than 1 for all of the indexes and for all of the models, which as I have already mention means that the risk is underestimated.

Table 2: $\hat{\alpha}/\alpha$ number

$\alpha = 0.05$	**CAC 40**	**NIKKEI225**	**SP500**
S-CAViaR	1.041	1.030	1.048
AS-CAViaR	1.12	1.152	1.26
AR-CAviaR	0.98	0.94	1.06
$\alpha = 0.01$	**CAC40**	**NIKKEI225**	**SP500**
S-CAViaR	1.50	1.79	1.59
AS-CAViaR	2.002	1.70	2.10
AR-CAViaR	1.90	1.55	1.41

As for the Christoffersen numbers , if the p-values of these are greater than $\alpha = 0.05$ or $\alpha = 0.01$ we accept our null hypothesis, which is that we have a right model and the calculated VaR is estimated correctly, and vice versa, we reject the null hypothesis when the p-values are smaller than $\alpha = 0.05$ or $\alpha = 0.01$. These tests show us the number of violations of VaR. So, according to the Table 3, and for $\alpha = 0.05$, regarding the number of Christoffersen test, we accept the null hypothesis for every model and index because the p-values are greater than $\alpha = 0.05$. For $\alpha = 0.01$, we accept the null hypothesis in every model and index apart from the case of AS and AR -CAviaR for Cac40.

Table 3: Christoffersen test and p-values

$\alpha = 0.05$	**CAC 40**	**NIKKEI225**	**SP500**
S-CAViaR	1.11(0.29)	0.55(0.45)	0.11(0.73)
AS-CAViaR	0.41(0.51)	0.006(0.93)	0.002(0.96)
AR-CAviaR	0.16(0.68)	0.42(0.51)	0.011(0.91)
$\alpha = 0.01$	**CAC40**	**NIKKEI225**	**SP500**
S-CAViaR	0.79(0.37)	2.46(0.11)	3.20(0.073)
AS-CAViaR	1.63(2.00e-01)	0.23(0.62)	1.03e-02(9.18e-01)
AR-CAviaR	1.28(2.56e-01)	0.64(0.42)	4.01(0.045)

Finally, as for the parameters's values of our regressions of each model , it is essential to refer that are statistically significant only if the p-value of the beta is smaller than the level of significance. As it represented at Table 4 and Table 4.1, for $\alpha = 0.05$, the parameter values are statistically significant in every of our cases except from b_3 in AS-CAViaR for CAC. As for $\alpha = 0.01$, we can conclude that we accept as statistically significant every b except from b_1, b_2, b_3 in S-CAViaR for Nikkei225, b_5 for Cac40 in AR-CAViaR, b_1, b_2, b_3, b_4, b_5 in AR-CAViaR for Nikkei225 and finally b_2, b_3, b_4, b_5 in AR-CAViaR for SP500.

Table 4: Parameters's values for $a = 0.05$.

Models	**CAC 40**	**NIKKEI225**	**SP500**
S-CAViaR			
b1	-0.014(0.010)	-0.012(0.004)	-0.003(0.001)
b2	0.88(0.022)	0.90(0.0028)	0.93(0.0016)
b3	-0.21(0.03)	-0.18(0.0027)	-0.12(0.002)
AS-CAviaR			
b1	-0.010(0.009)	-0.021(0.006)	-0.009(0.0009)
b2	0.94(0.047)	0.90(0.009)	0.96(0.0017)
b3	0.010(0.10)	-0.067(0.007)	0.031(0.0027)
b4	0.20(0.021)	0.22(0.015)	0.11(0.004)
AR-CAViaR			
b1	0.012(0.002)	0.013(0.01)	0.008(0.002)
b2	0.91(0.002)	0.91(0.004)	0.92(0.009)
b3	-0.11(0.006)	0.07(0.02)	-0.12(0.008)
b4	0.21(0.008)	0.19(0.014)	0.16(0.044)
b5	-0.009(0.002)	0.16(0.02)	-0.02(0.048)

Note: Numbers within parenthesis represent the p-value of each number.

Table 4.1: Parameters' values for $\alpha = 0.01$

Models	**CAC40**	**NIKKEI225**	**SP500**
S-CAViaR			
b1	-0.02(3.92e-05)	-0.08(0.23)	-0.017(0.003)
b2	0.93(4.14e-03)	0.76(0.19)	0.92(0.005)
b3	-0.15(1.39e-04)	-0.58(0.16)	-0.20(0.008)
AS-CAViaR			
b1	-0.03(0.001)	-0.24(0.0025)	-0.46(0.0004)
b2	0.91(0.0003)	0.71(0.018)	0.50(0.0001)
b3	-0.77(0.011)	-0.13(0.002)	-0.28(0.0002)
b4	0.23(0.0004)	0.63(0.006)	0.41(0.0009)
AR-CAViaR			
b1	0.04(0.003)	0.09(0.03)	0.018(0.002)
b2	0.92(0.001)	0.88(0.01)	0.93(0.01)
b3	-0.12(0.009)	0.25(0.05)	-0.20(0.16)
b4	0.24(0.008)	0.414(0.02)	0.26(0.18)
b5	-0.05(0.01)	0.417(0.04)	-0.14(0.17)

Note: Numbers within parenthesis represent the p-value of each number.

At Figure 3.2, at the end of this chapter p.43-51, I present graphically the relationship between each index and VaR about the period of forecast. In the next subsection I present the analysis about the second forecasting period, but I am going to refer only the results that concern the Christoffersen and $\hat{\alpha}/\alpha$ numbers.

3.1.2 Forecasting Period: 2007-2009

For the second forecasting period I am going to refer the results that concern the $\hat{\alpha}/\alpha$ number and the Chistoffersen test result. As Table 5 represents, for $\alpha = 0.05$, we can conclude that the $\hat{\alpha}/\alpha$ number about this forecasting period, for AS and AR-CAViaR and for all of our indexes is greater than 1 which means that the risk is underestimated. Opposite to this, for S-CAViaR this number is smaller than 1, so the risk is overererestimated for all of our indexes. As for $\alpha = 0.01$, we have that the number of $\hat{\alpha}/\alpha$ is greater than 1 for all of the indexes and for all of the models, which indicates that the risk is underestimated.

Table 5: $\hat{\alpha}/\alpha$ number

$\alpha = 0.05$	**CAC 40**	**NIKKEI225**	**SP500**
S-CAViaR	0.8616	0.9016	0.8741
AS-CAViaR	1.1724	1.26	1.26
AR-CAviaR	1.38	1.53	1.66
$\alpha = 0.01$	**CAC40**	**NIKKEI225**	**SP500**
S-CAViaR	1.044	1.092	1.059
AS-CAViaR	5.86	6.30	6.30
AR-CAViaR	2.34	2.32	3.31

According to the Table 6, and for $\alpha = 0.05$, regarding the number of Christoffersen test, we accept the null hypothesis for every model and index because the p-values are greater than $\alpha = 0.05$, except from the case of Nikkei and SP500 in AR CAViaR. For $\alpha = 0.01$, we accept the null hypothesis in every model and index.

Table 6: Christoffersen(C) test and p-values

$\alpha = 0.05$	**CAC 40**	**NIKKEI225**	**SP500**
S-CAViaR	0.2287(0.6324)	0.174(0.67)	0.21(0.64)
AS-CAViaR	3.25(0.07)	0.0025(0.96)	0.0025(0.96)
AR-CAviaR	1.029(0.31)	4.09(0.04)	5.84(0.015)
$\alpha = 0.01$	**CAC40**	**NIKKEI225**	**SP500**
S-CAViaR	0.169(0.68)	0.177(0.67)	0.171(0.67)
AS-CAViaR	3.25(0.07)	0.00025(0.096)	0.00026(0.096)
AR-CAviaR	0.86(0.35)	0.80(0.36)	1.71(0.019)

At Figure 3.3, at the end of this chapter p.52-60, I present graphically the relationship between each index and VaR about the period of forecast. In the next subsection I present the analysis about the second forecasting period, but I am going to refer only the results that concern the Christoffersen and $\hat{\alpha}/\alpha$ numbers.

In the next section, I will present the results by using the non-linear transformations. Given that we are interesting in VRate number, I am going to refer to tables and figures that concern the VRate and Christoffersen numbers, and I will not present the parameters values of my models.

3.2 Results of Non-linear transformations

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In this section I will present the statistical results that concern the transformed data.In order to make use of such transformations, I had to make some changes relating to the data. As for Manly transformations, the procedure that followed was: I transformed the initial data for each index by using the suitable equation of the transformation in R for every model and for every confidence level. Secondly, for every λ number which I found, I had to find the respective $\hat{\alpha}/\alpha$ number resulting from the above and take the appropriate one. As I did before , in order to take the $\hat{\alpha}/\alpha$, I used the backtesting process in R. At this point I will turn my attention to the results which concern the λ number and the equivalent $\hat{\alpha}/\alpha$ number.

3.2.1 Forecasting period: 2010-2014

At Table 7 , for $\alpha = 0.05$, for all of our indexes and models we have that the number of $\hat{\alpha}/\alpha$ approximate 1, which is the preferable outcome. Analytically, for all of our indexes and models, apart from Cac40 and SP in AS-CAviaR, $\hat{\alpha}/\alpha$ number is smaller than 1 so we overestimate risk. As for $\alpha = 0.01$, at Table 8, we may notice that for all of our indexes and models we have that the number of $\hat{\alpha}/\alpha$ is smaller but approximate 1, apart from all of our indexes in S -CAViaR where the $\hat{\alpha}/\alpha = 1$,and it is a little greater than 1, which is the preferable outcome.

Table 7: Optimal λ estimates and $\hat{\alpha}/\alpha$ number for $\alpha = 0.05$.

number of λ	$\hat{\alpha}/\alpha$
S-CAViaR	
Cac40,$\lambda = -0.05$	0.96
Nikkei225, $\lambda = -0.15$	0.9346
SP500, $\lambda = -0.05$	0.9658
AS-CAviaR	
Cac40,$\lambda = -0.20$	1.094
Nikkei225, $\lambda = -0.20$	0.9574
SP500, $\lambda = -0.20$	1.0845
AR-CAViaR	
Cac40,$\lambda = -0.05$	0.9947
Nikkei225, $\lambda = -0.20$	0.9826
SP500, $\lambda = 0.40$	0.9658

Table 8: Optimal λ and $\hat{\alpha}/\alpha$ number for $\alpha = 0.01$.

number of λ	$\hat{\alpha}/\alpha$
S-CAViaR	
Cac40,$\lambda = -0.15$	1.041
Nikkei225, $\lambda = -0.20$	1.018
SP500, $\lambda = -0.20$	1.06
AS-CAviaR	
Cac40,$\lambda = -0.40$	0.9281
Nikkei225, $\lambda = -0.25$	0.8793
SP500, $\lambda = -0.40$	0.9770
AR-CAViaR	
Cac40,$\lambda = -0.25$	0.9253
Nikkei225, $\lambda = -0.25$	0.8987
SP500, $\lambda = 0.00$	0.8833

Finally, it would be essential to represent the above results graphically. So, in Figure

3.4,at the end of the chapter p.61-69, the relationship of λ and $\hat{\alpha}/\alpha$ is illustrated diagrammatically. In these figures I take account not only the suitable λ number which give us the appropriate $\hat{\alpha}/\alpha$,as I stated in the above tables, but also we can see the outcome of VRate for more than 1 λs.

Comparison of the above results

Here, I will present analytically what we can conclude from the comparison between data that used non-linear transformations with those that did not. For every index, model and significance level we have:

a)S-CAViaR,$\alpha = 0.05$. For this model and for the no-transformed data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***1.041, 1.030 and 1.048*** respectively. For the same case and the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***0.96, 0.9346 and 0.9658***. So here, we prefer the transformed data results because they approximate 1 but they are smaller than this and as I have noticed we prefer to overestimate risk.

b)S-CAViaR, $\alpha = 0.01$. For this model and for our initial data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***1.50, 1.79 and 1.59*** respectively. For the same case and the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***1.041, 1.018 and 1.06***. So here, we prefer the transformed data results because they approximate 1 in contrast to the initial data.

c)AS-CAViaR, $\alpha = 0.05$.In this case for no-transformed data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***1.12, 1.152 and 1.26*** respectively. As for the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***1.094, 0.9574 and 1.0845***. So here, we prefer the transformed data results in any case because they approximate 1 ,but are greater than this in cases of CAC and SP, and are smaller than the results that we get in case of no-transformed data.

d)AS-CAViaR, $\alpha = 0.01$.Here we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***2.002, 1.70 and 2.10*** respectively. For the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***0.92, 0.87 and 0.97***. We prefer the transformed data results again, in all of the cases.

e)AR-CAViaR,$\alpha = 0.05$.For this model and for our initial data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***0.9832, 0.94 and 1.06*** respectively. For the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***0.9947, 0.9826 and 0.9658***. So here, we prefer the transformed data results again for all of our indexes, and we can see that these numbers approximate one and are smaller than this.

f)AR-CAViaR,$\alpha = 0.01$.For this model and for our initial data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***1.90, 1.55 and 1.41*** respectively. For the same case and the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***0.9253, 0.8987 and 0.8833***. We prefer the transformed data results again, in all of the cases.

3.2.2 Forecasting Period: 2007-2009

Analytically, at Table 9 , for $\alpha = 0.05$, for all of our indexes and models we have that the $\hat{\alpha}/\alpha$ is smaller than 1 except from the case of SP in AS-CAViaR and for CAC and SP in AR-CAViaR where is greater than 1 . As for $\alpha = 0.01$, at Table 10, we may notice that for all of our indexes and models we have that the number of $\hat{\alpha}/\alpha$ is smaller but approximate 1, apart from SP500 in AS-CAViaR and Nikkei in AR-CAViaR where the $\hat{\alpha}/\alpha$ is greater than 1. Finally, as I did before, at Figure 3.5, p.70-78, I represent the VRate numbers for more than 1 λs.

Table 9: Optimal λ and $\hat{\alpha}/\alpha$ number for $\alpha = 0.05$.

number of λ	$\hat{\alpha}/\alpha$
S-CAViaR	
Cac40,$\lambda = -0.25$	0.99
Nikkei225, $\lambda = -0.25$	0.98
SP500, $\lambda = -0.35$	0.95
AS-CAviaR	
Cac40,$\lambda = -0.50$	0.83
Nikkei225, $\lambda = -0.15$	1.006
SP500, $\lambda = -0.45$	1.45
AR-CAViaR	
Cac40,$\lambda = -0.20$	1.25
Nikkei225, $\lambda = -0.35$	0.98
SP500, $\lambda = -0.50$	1.13

Table 10: Optimal λ and $\hat{\alpha}/\alpha$ number for $\alpha = 0.01$.

number of λ	$\hat{\alpha}/\alpha$
S-CAViaR	
Cac40,$\lambda = -0.15$	0.52
Nikkei225, $\lambda = -0.35$	0.81
SP500, $\lambda = -0.20$	0.92
AS-CAviaR	
Cac40,$\lambda = -0.15$	0.92
Nikkei225, $\lambda = -0.20$	0.97
SP500, $\lambda = -0.50$	1.46
AR-CAViaR	
Cac40,$\lambda = -0.20$	1.04
Nikkei225, $\lambda = -0.35$	1.63
SP500, $\lambda = -0.25$	0.79

.

Comparison of the above results

.

As I did at previous section, for every model and significance level we have:

a)S-CAViaR,$\alpha = 0.05$. For no-transformed data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***0.86,0.90 and 0.87*** respectively. For the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***0.99,0.98 and 0.95***. So here, we prefer the transformed data results because they approximate 1 but are smaller than this and as I have noticed we prefer to overestimate risk.

b)S-CAViaR,$\alpha = 0.01$. As for no-transformed data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***1.044,1.092 and 1.059*** respectively. As for the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***0.52,0.81 and 0.92***. We prefer the transformed data results because they approximate 1 but are smaller than this, except from the case of CAC where we prefer the no-transformed data result which approximate 1 and is better than the outcome of transformed data.

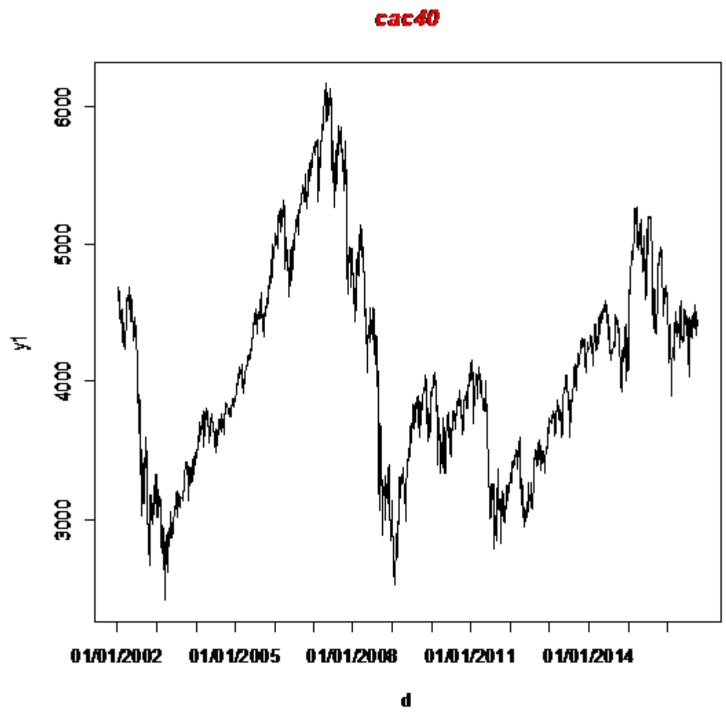
c)AS-CAViaR,$\alpha = 0.05$. As for the initial data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***1.17,1.26 and 1.26*** respectively. For the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***0.83, 1.006 and 1.45***. So here, we prefer the transformed data results in all of the cases except from SP500 where the $\hat{\alpha}/\alpha$ is greater by using the transformations.

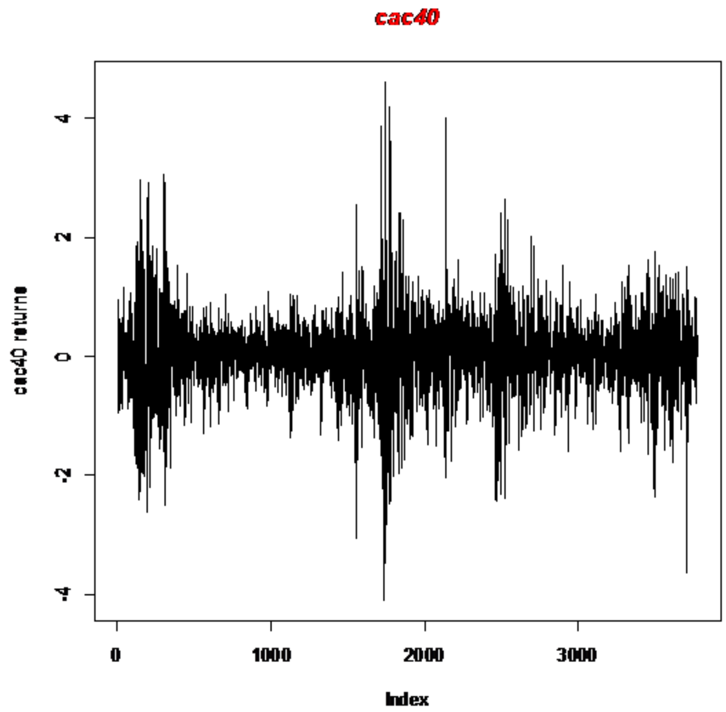
d)AS-CAViaR, $\alpha = 0.01$. In this case for our initial data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***5.86,6.30 and 6.30*** respectively. The transformed data results are equal with ***0.92,0.97 and 1.46***. So here, we prefer the transformed data results in all of the cases and the risk is overerestimated.

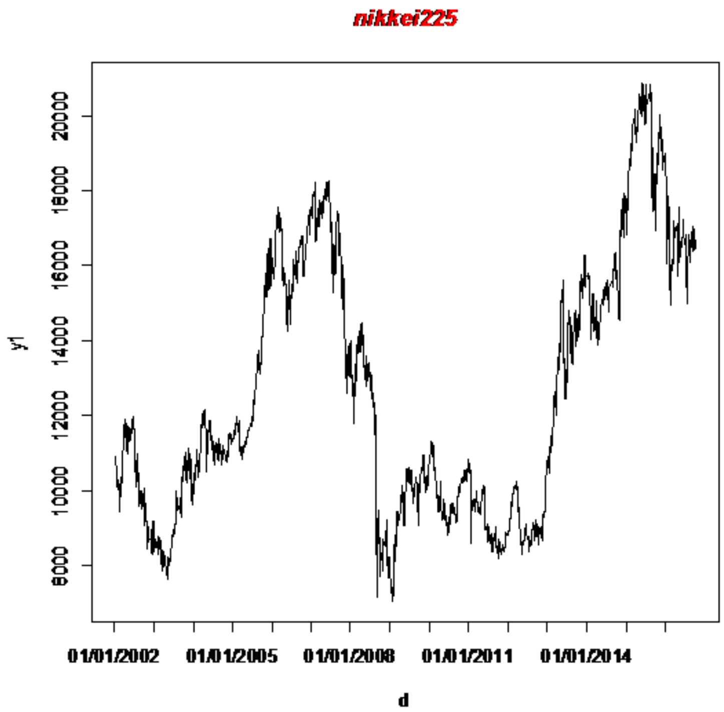
e)AR-CAViaR, $\alpha = 0.05$.For the initial data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ number is equal with ***1.38, 1.53 and 1.66*** respectively. For the transformed data we found that $\hat{\alpha}/\alpha$ is equal with ***1.25, 0.98 and 1.13***. So here, we prefer the transformed data results in all cases. The transformed data results are greater than 1 but smaller than the outcomes that concern no-transformed data.

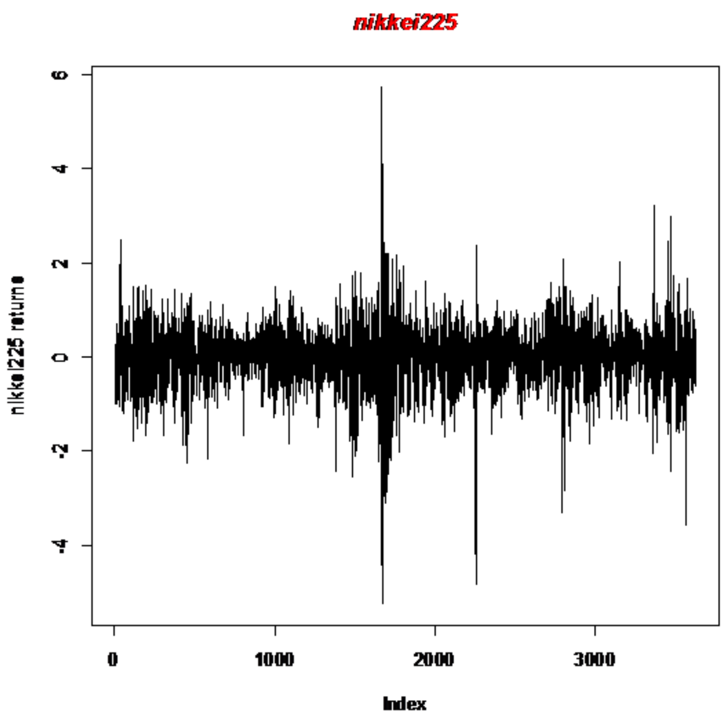
f)AR-CAViaR,$\alpha = 0.01$. Finally, for our initial data we have that for Cac40,Nikkei225 and SP500 the $\hat{\alpha}/\alpha$ is equal with ***2.34,2.32 and 3.31*** respectively. For the transformed data the $\hat{\alpha}/\alpha$ is equal with ***1.04, 1.63 and 0.79***. So, we prefer the transformed data results in all of the cases again.

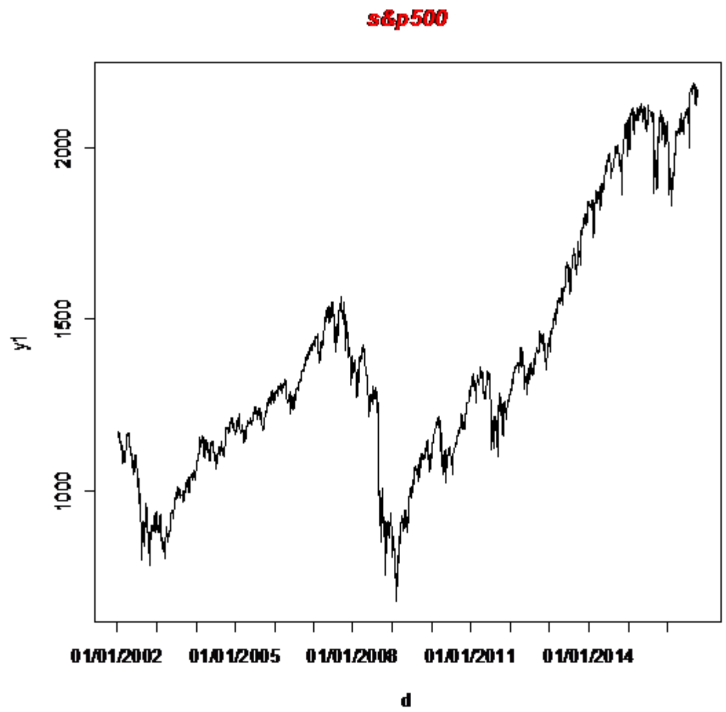
Figure 3.1: Close prices and daily returns of each index






nikkei225
y1
8000
10000
12000
14000
16000
18000
20000
01/01/2002
01/01/2005
01/01/2008
01/01/2011
01/01/2014
d


nikkei225
nikkei225 returns
-4
-2
0
2
4
6
0
1000
2000
3000
Index


s&p500
y1
2000
1500
1000
01/01/2002
01/01/2005
01/01/2008
01/01/2011
01/01/2014
d

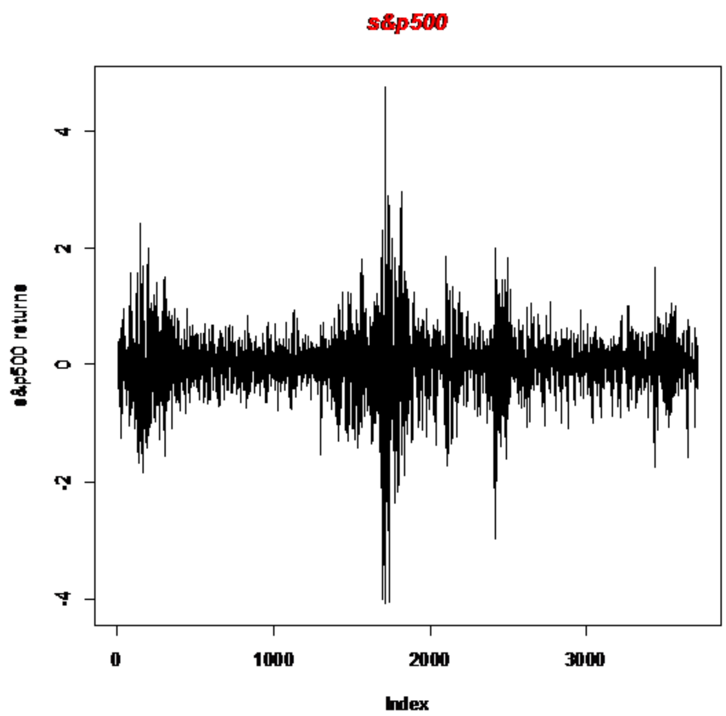
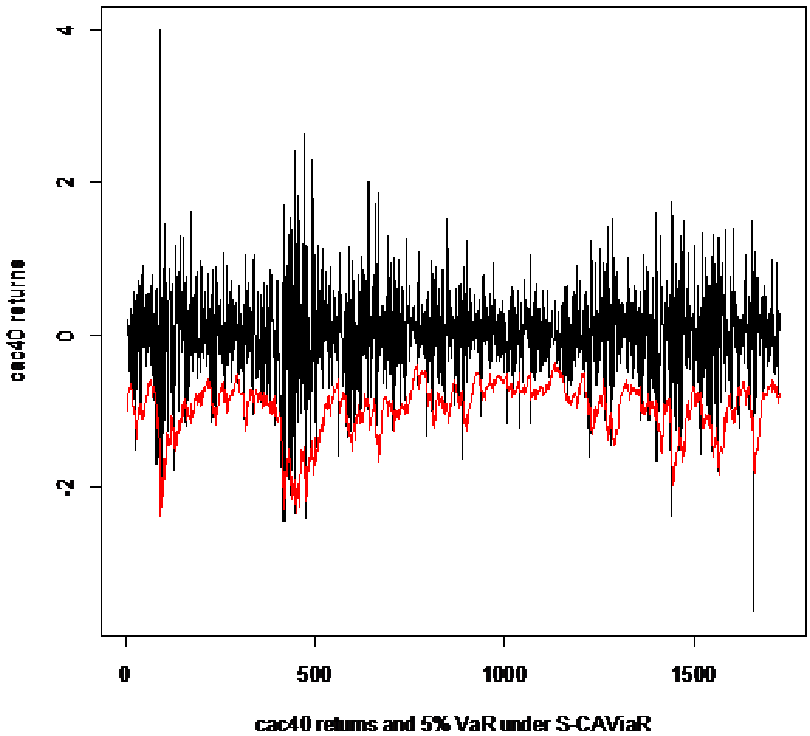
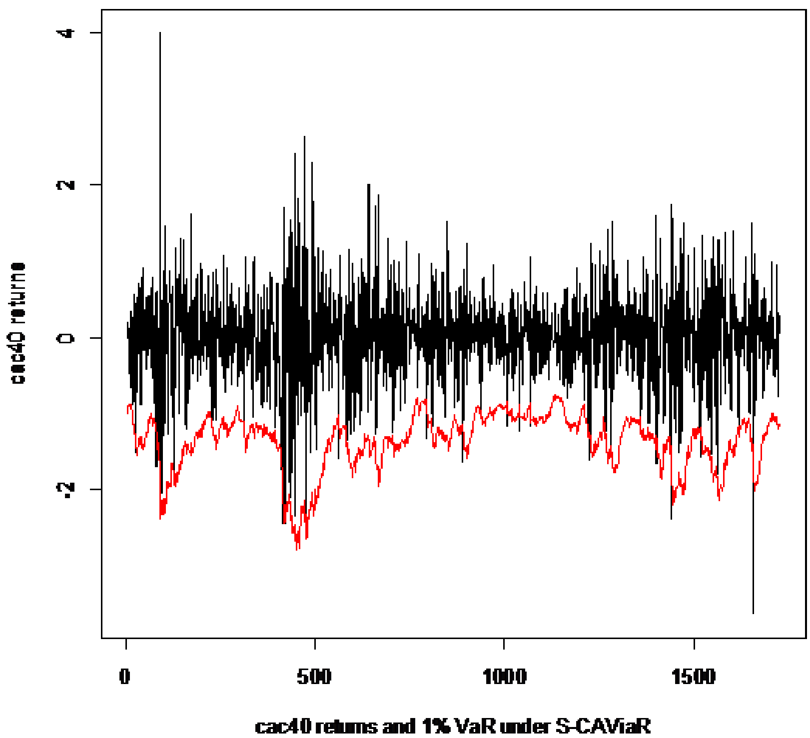
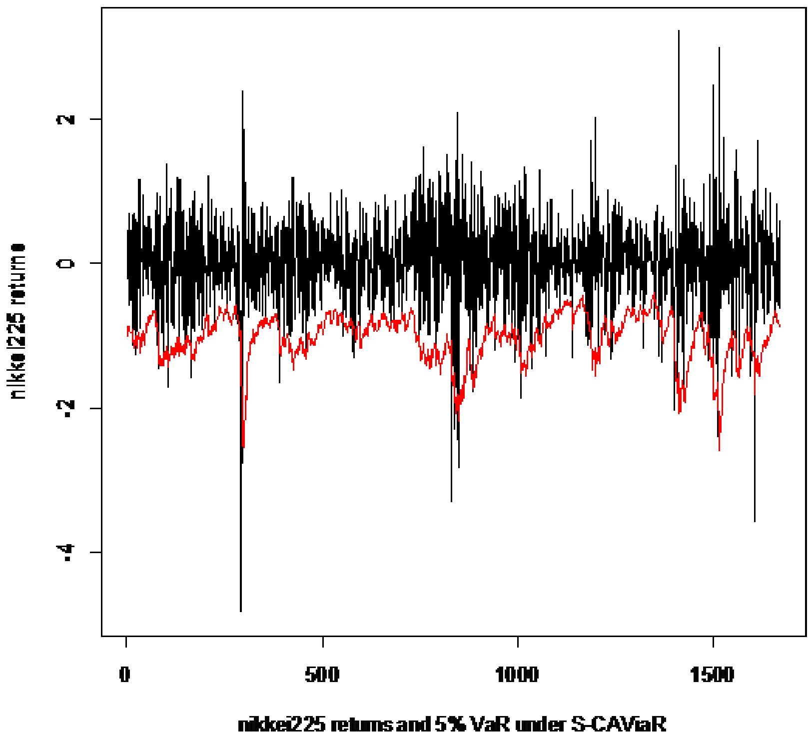

s&p500
s&p500 returns
4
2
0
-2
-4
0
1000
2000
3000
Index

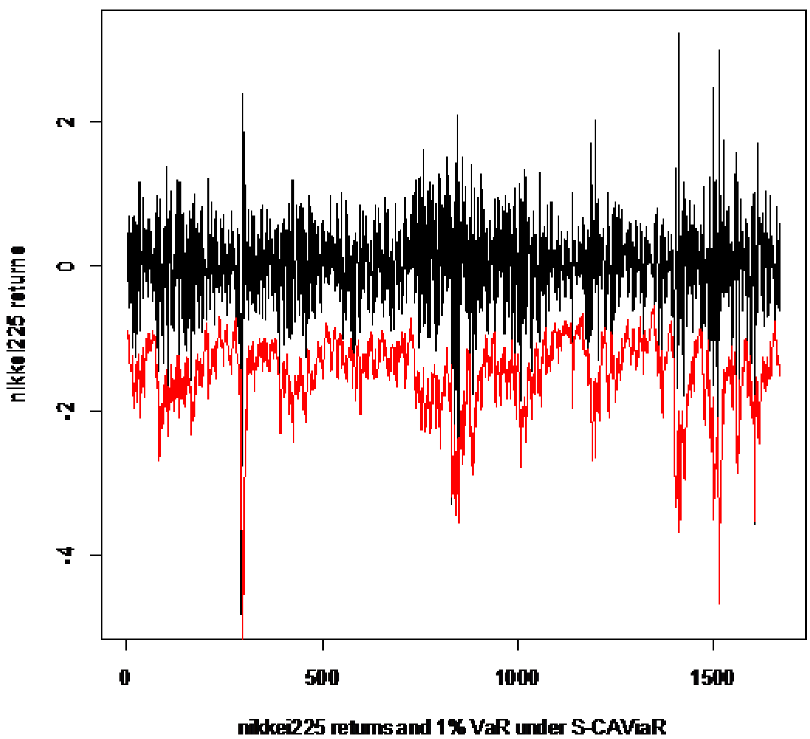
Figure 3.2: Returns and VaR for forecasting period 2010-2016



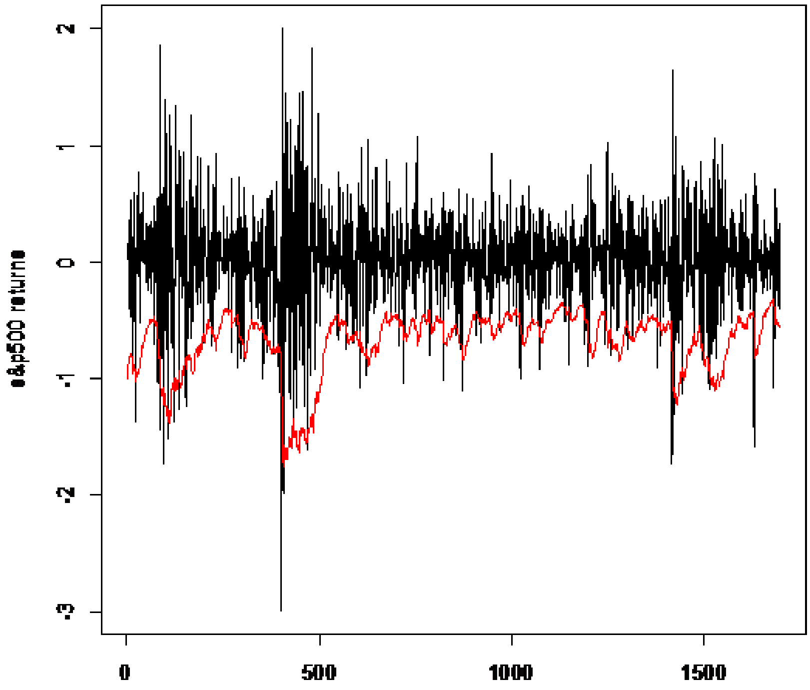




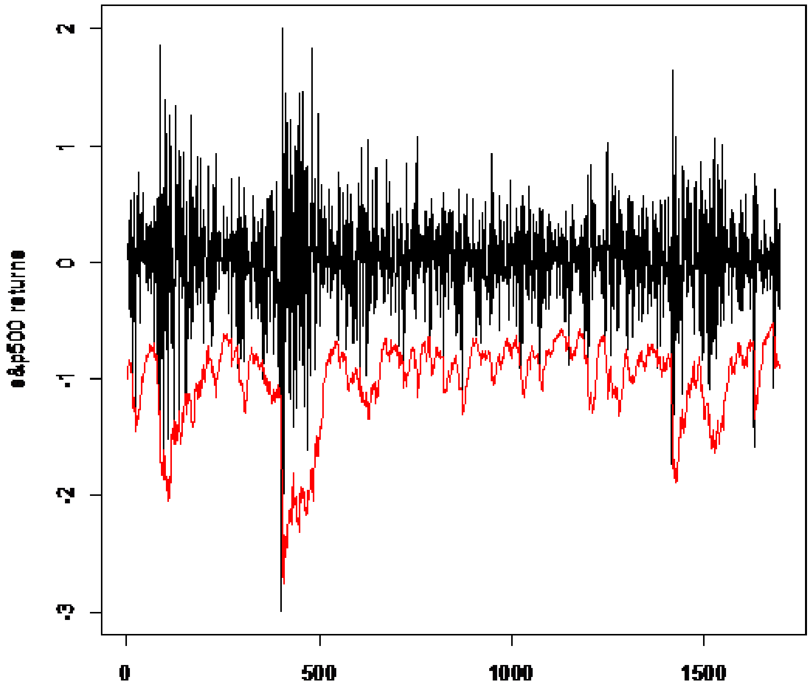
nikkei225 returns
2
0
-2
-4
0
500
1000
1500
nikkei225 returns and 5% VaR under S-CAViaR



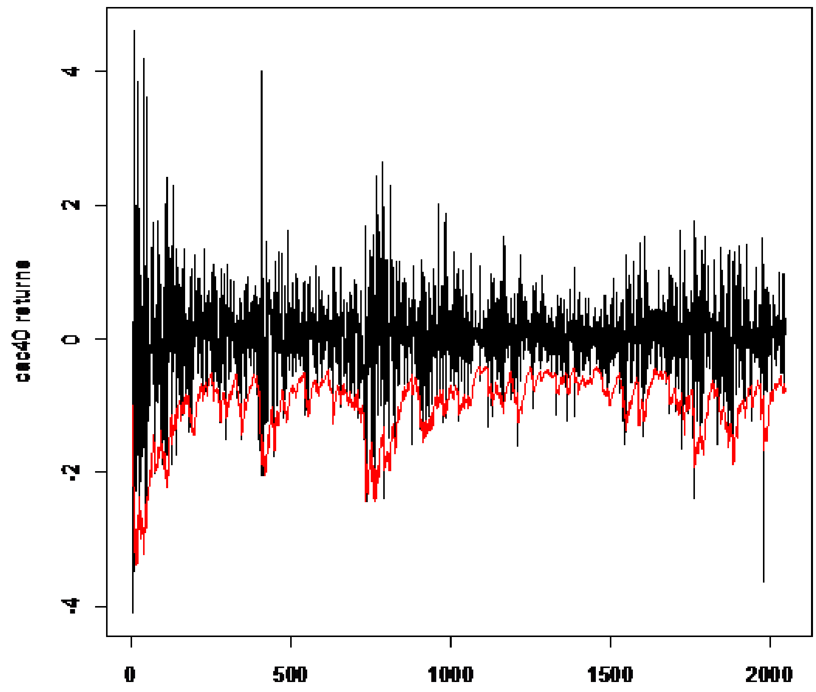
nikkei225 returns
2
0
-2
-4
0
500
1000
1500
nikkei225 returns and 1% VaR under S-CAViaR



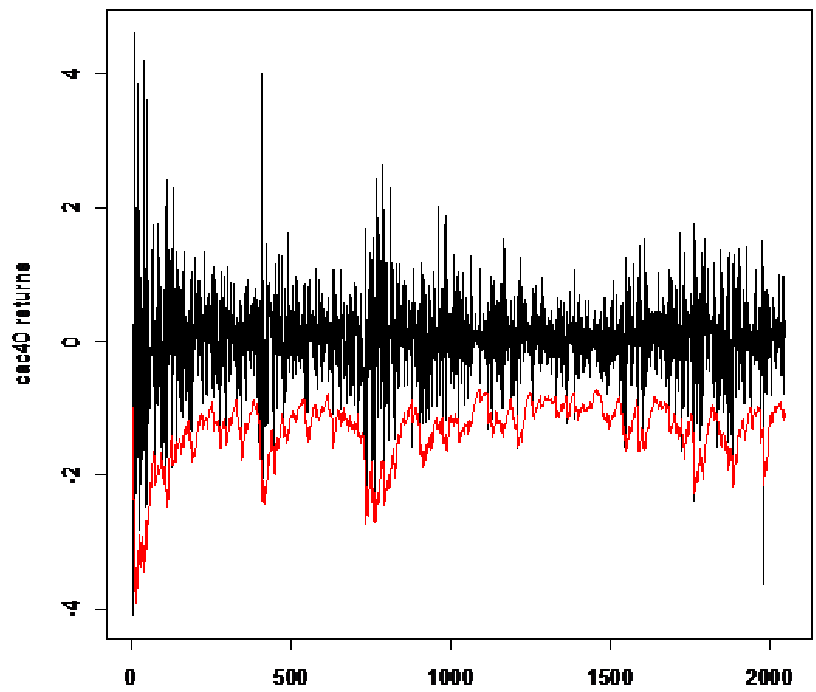
s&p500 returns and 5% VaR under S-CAViaR



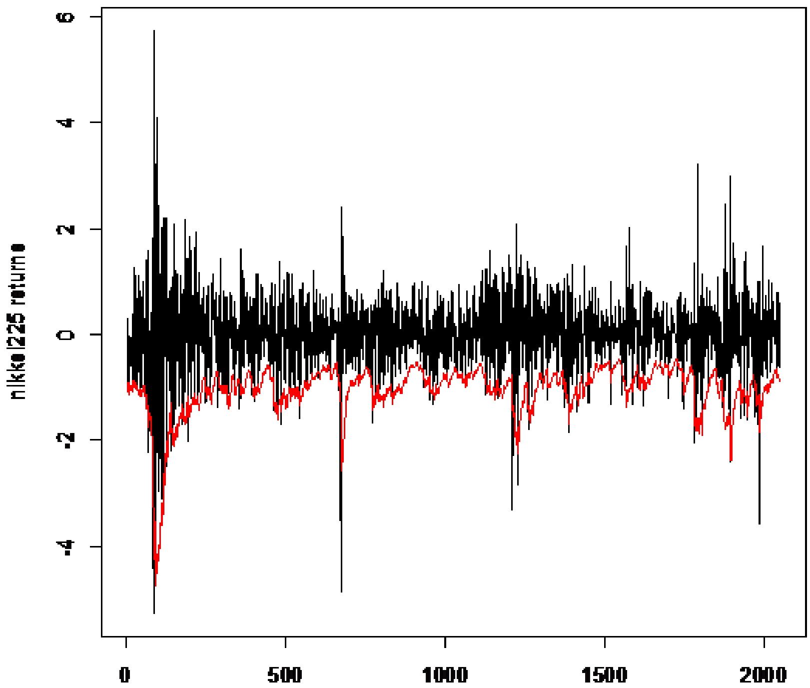
s&p500 returns and 1% VaR under S-CAViaR



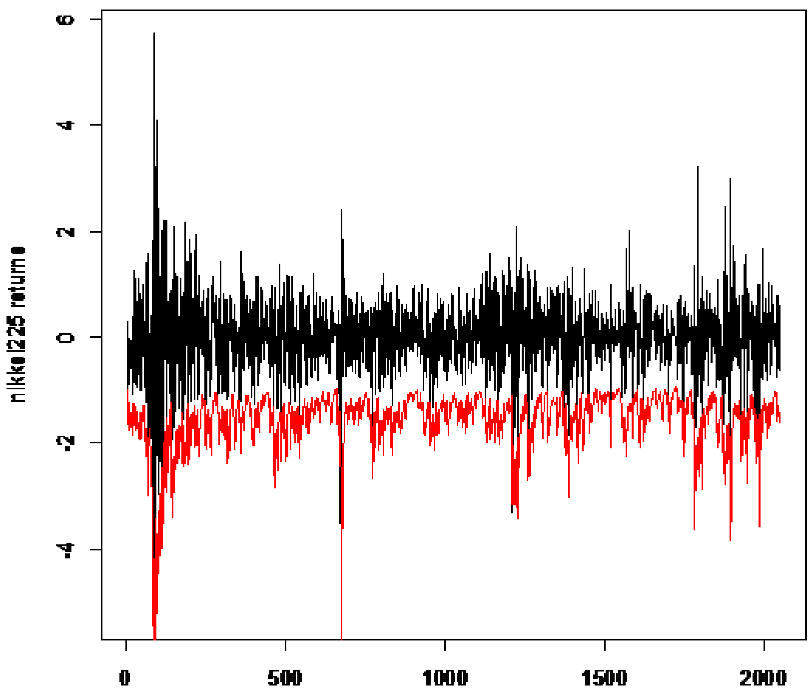
cac40 returns and 5% VaR under AS-CAViaR



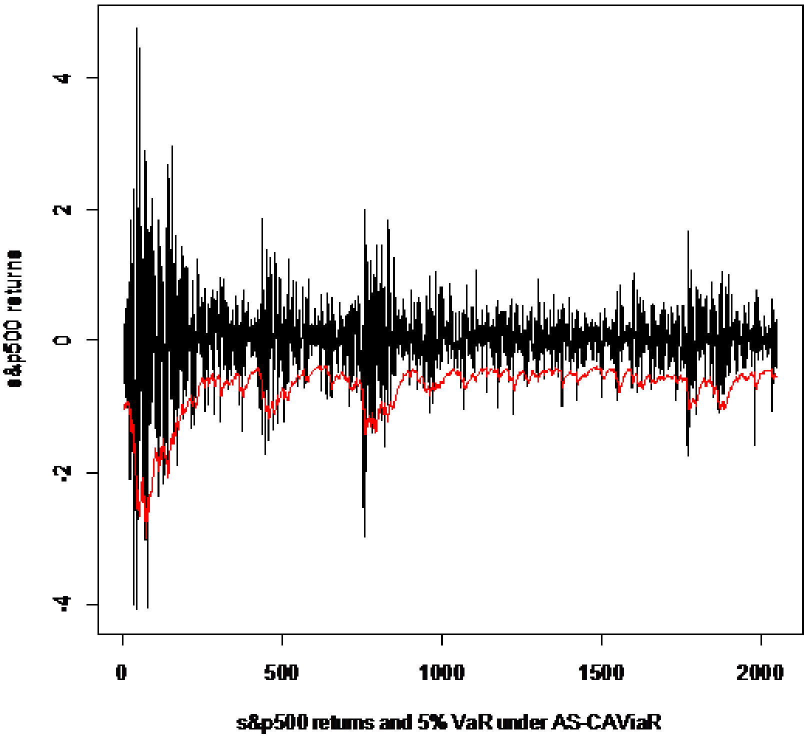
cac40 returns and 1% VaR under AS-CAViaR

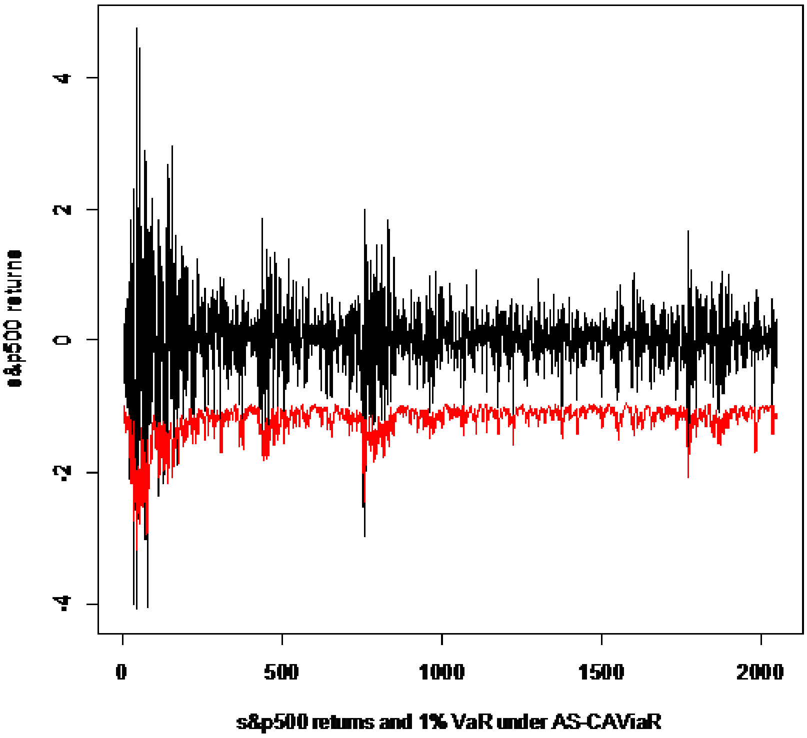


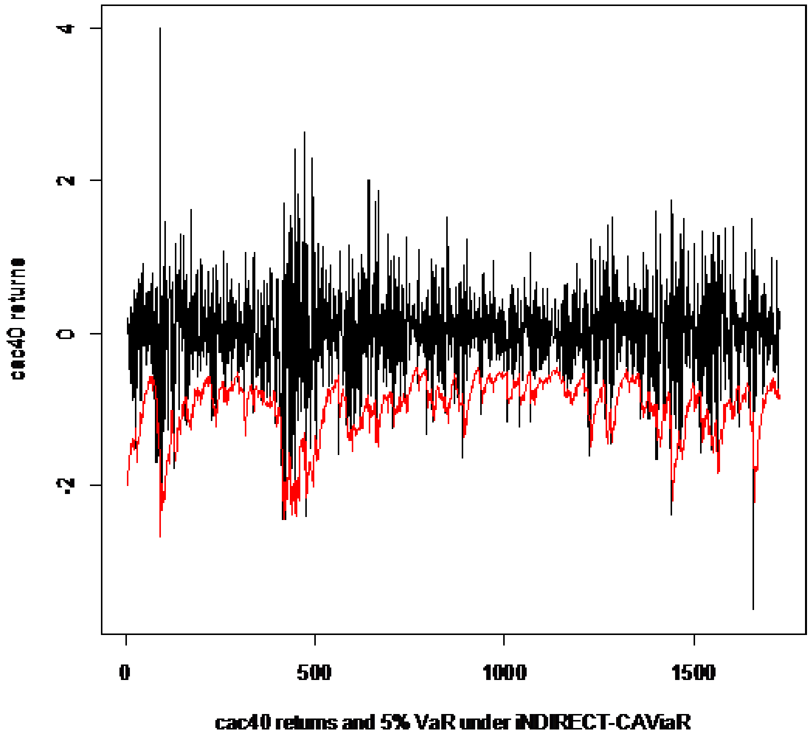
nikkei225 returns and 5% VaR under AS-CAViaR

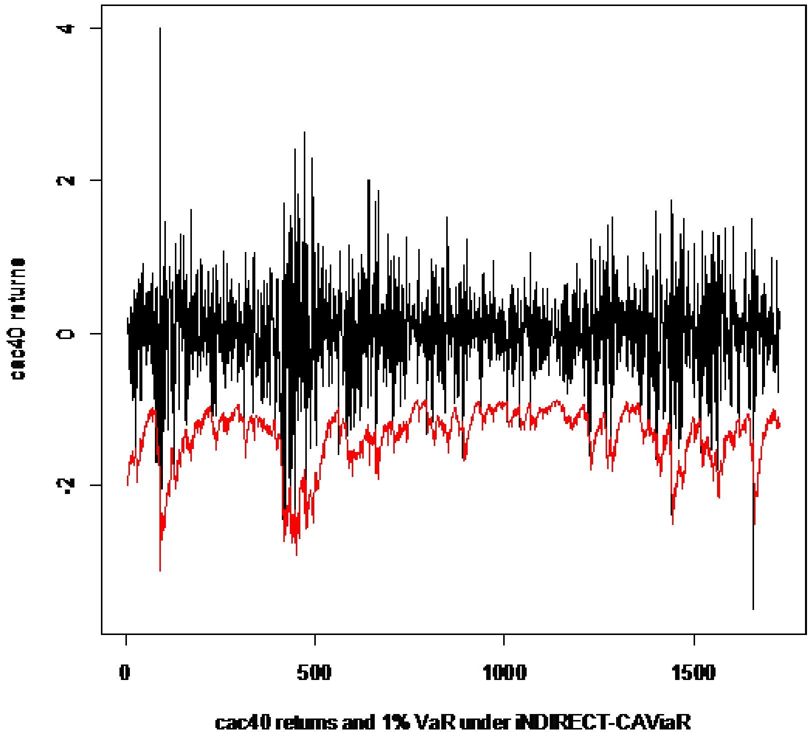


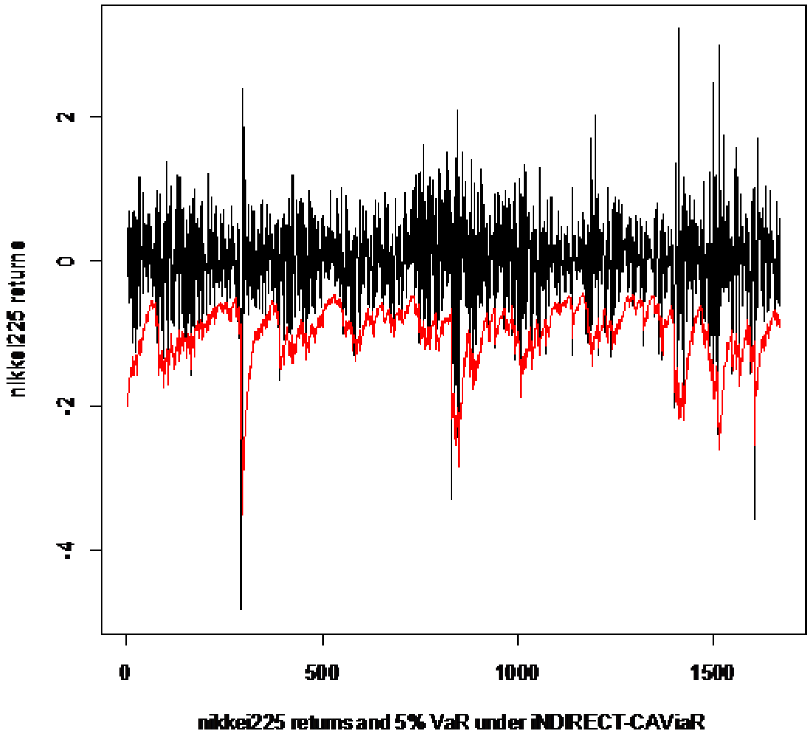
nikkei225 returns and 1% VaR under AS-CAViaR

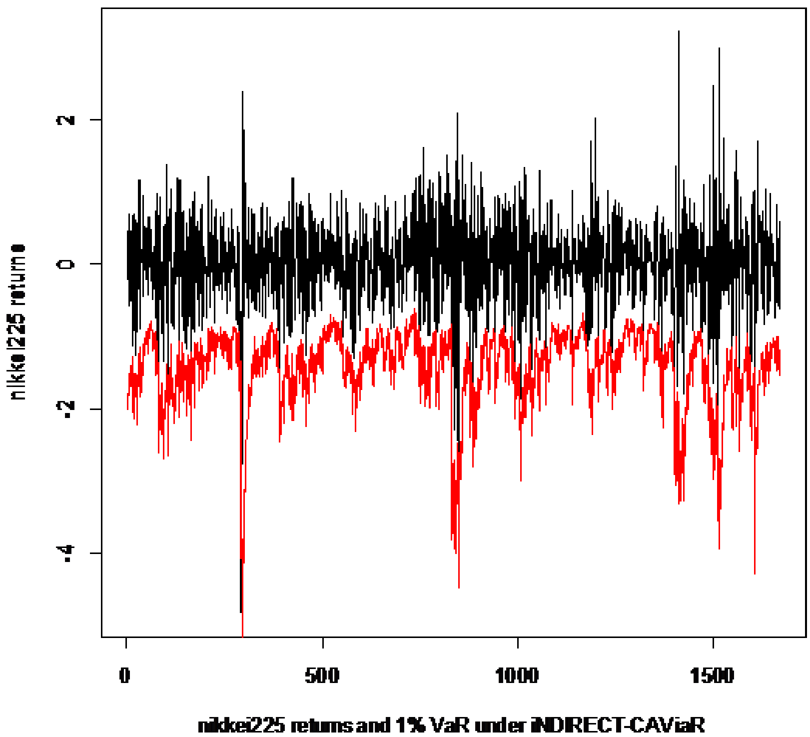

s&p500 returns
4
2
0
-2
-4
0
500
1000
1500
2000
s&p500 returns and 5% VaR under AS-CAViaR

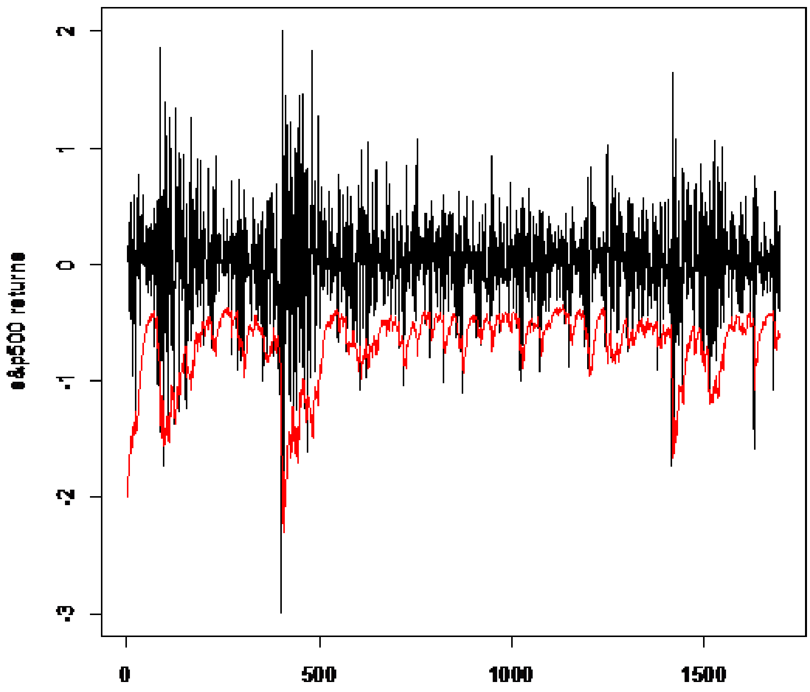

s&p500 returns
4
2
0
-2
-4
0
500
1000
1500
2000
s&p500 returns and 1% VaR under AS-CAViaR


cac40 returns
4
2
0
-2
0
500
1000
1500
cac40 returns and 5% VaR under iNDIRECT-CAViaR

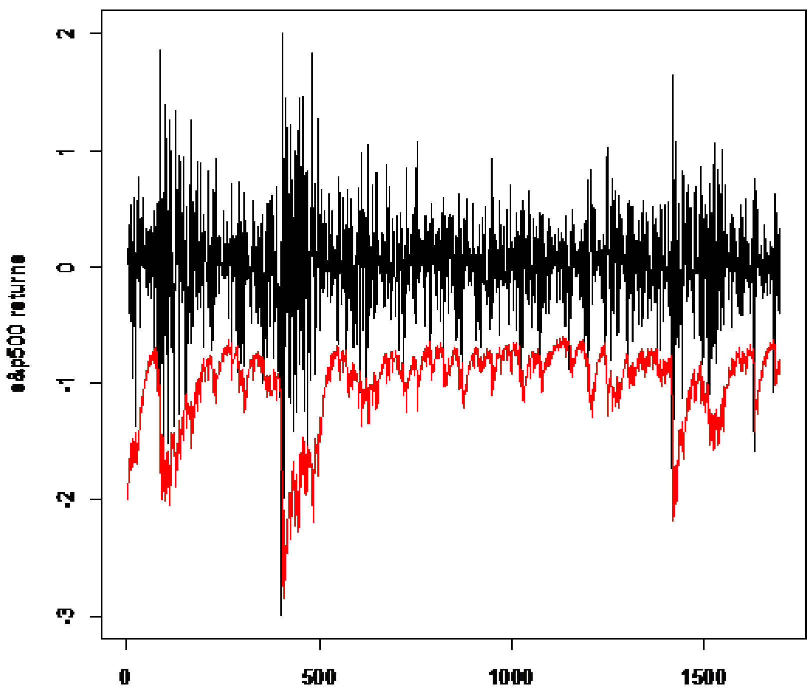

cac40 returns
4
2
0
-2
0
500
1000
1500
cac40 returns and 1% VaR under iNDIRECT-CAViaR


nikkei225 returns
2
0
-2
-4
0
500
1000
1500
nikkei225 returns and 5% VaR under INDIRECT-CAViaR


nikkei225 returns
2
0
-2
-4
0
500
1000
1500
nikkei225 returns and 1% VaR under INDIRECT-CAViaR

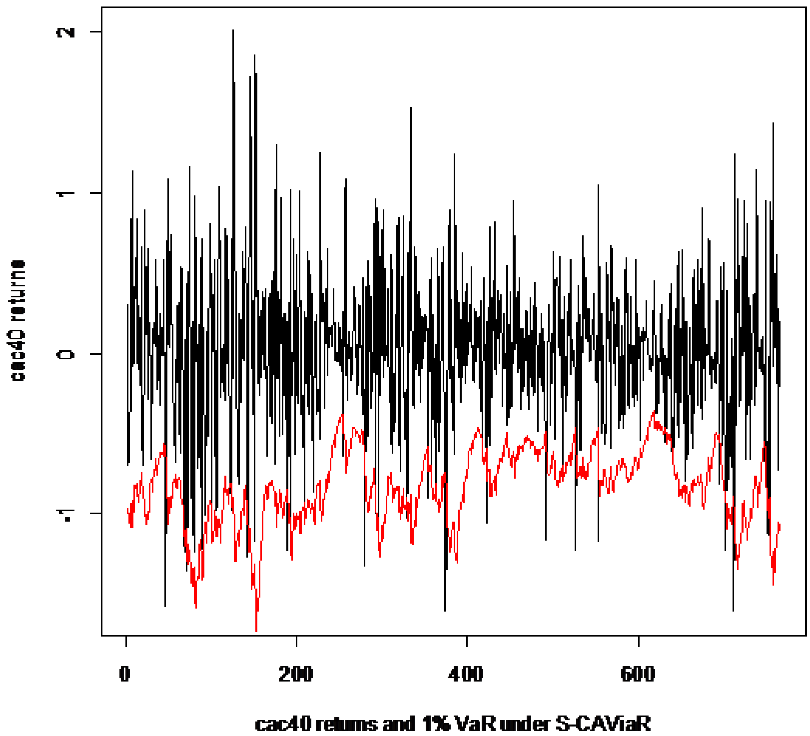


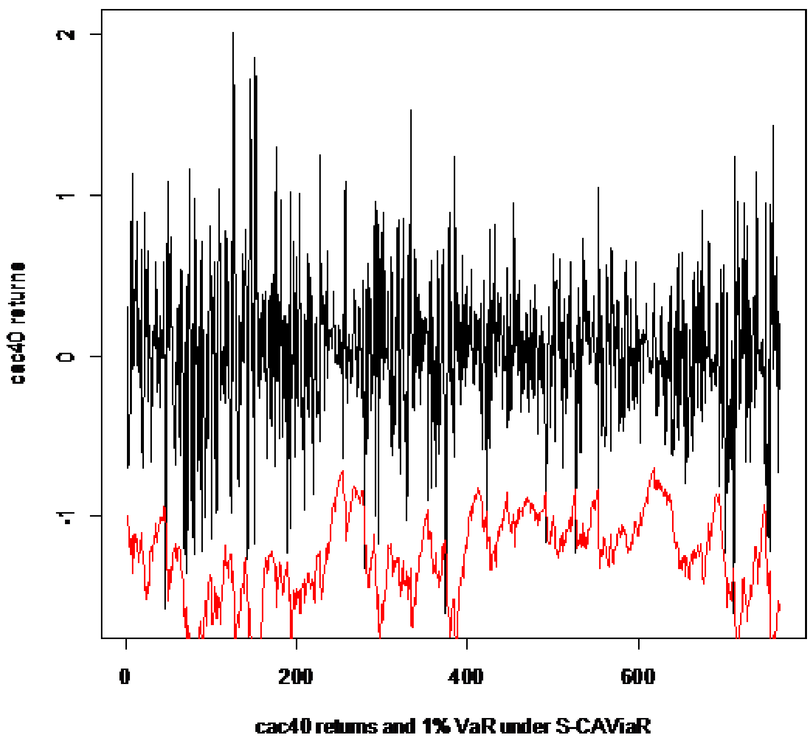
s7p500 returns and 5% VaR under iNDIRECT-CAViaR

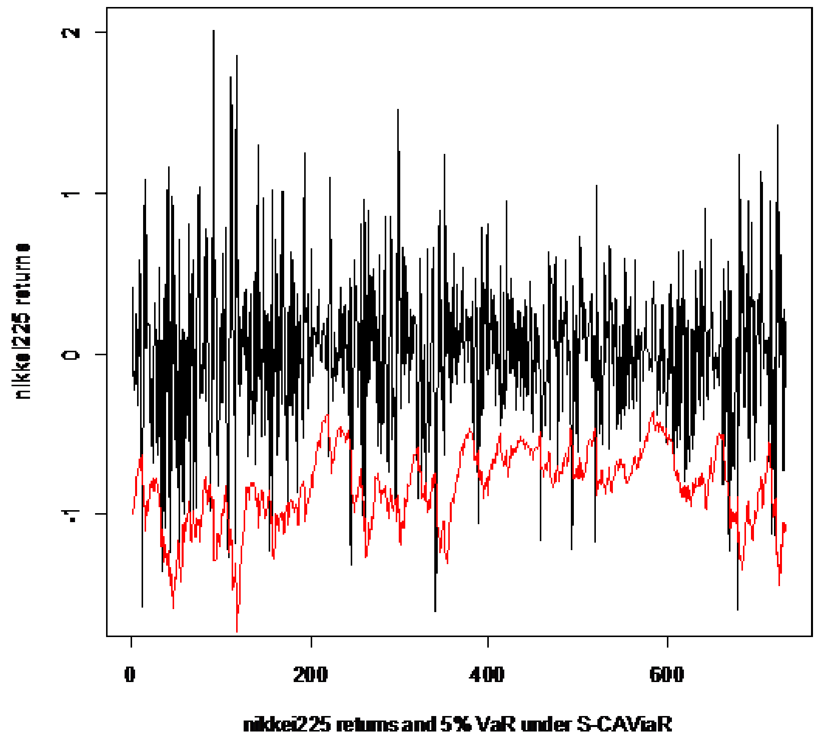


s7p500 returns and 1% VaR under iNDIRECT-CAViaR

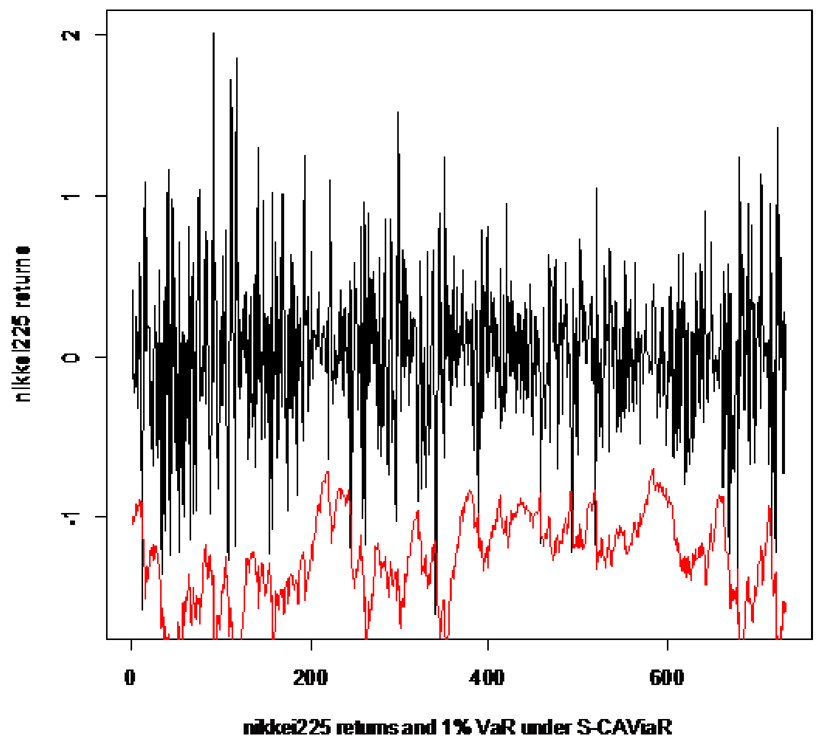
Figure 3.3: Returns and VaR for forecasting period 2007-2009



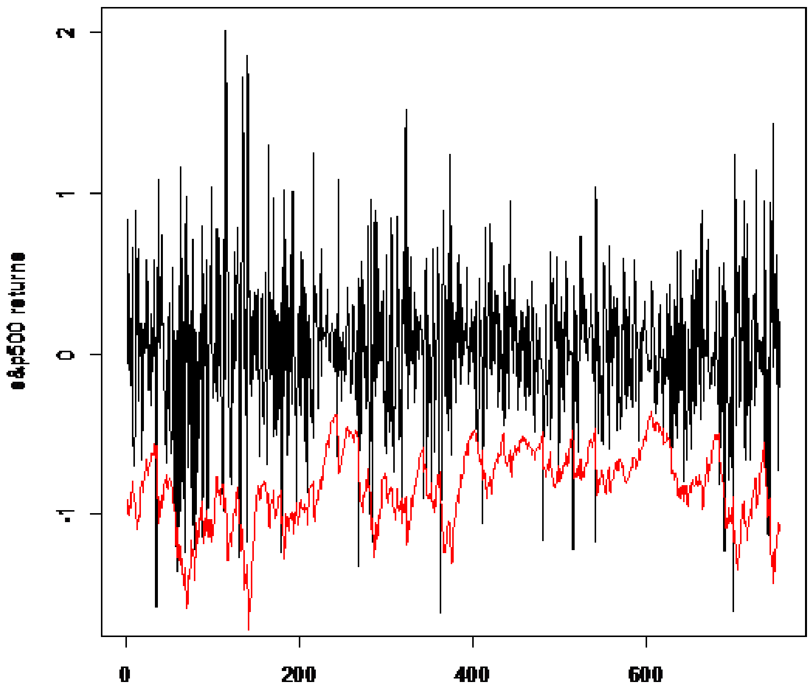




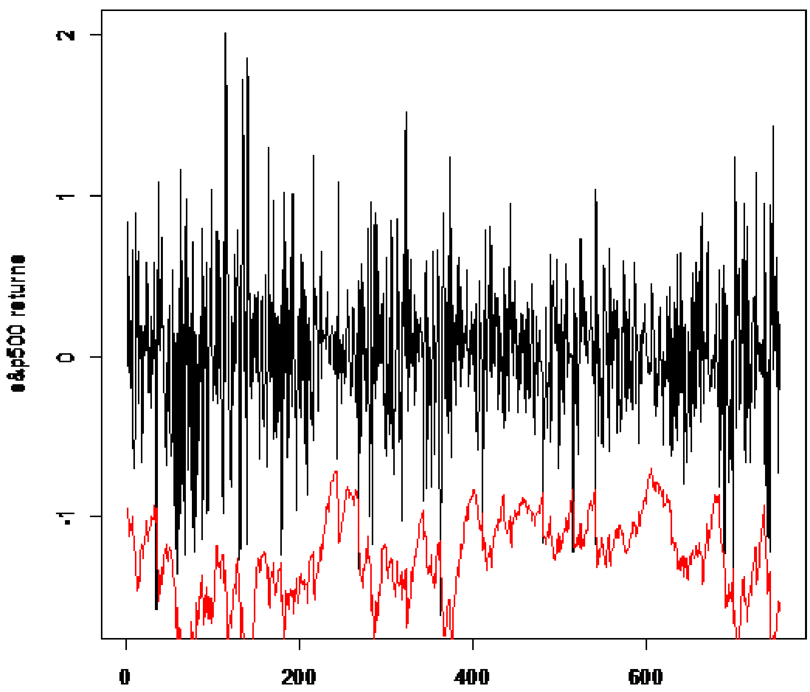
nikkei225 returns
2
1
0
-1
0
200
400
600
nikkei225 returns and 5% VaR under S-CAViaR



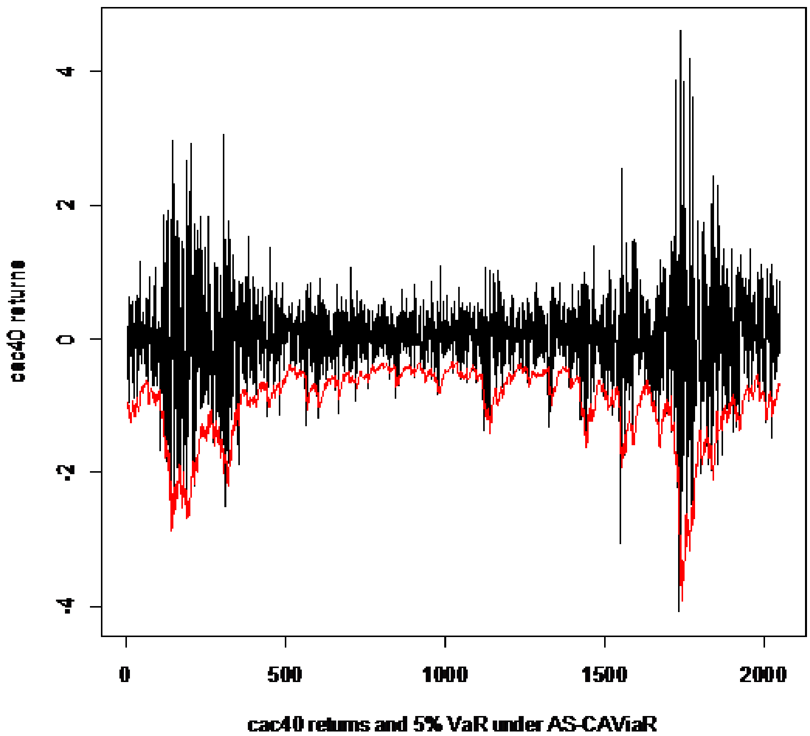
nikkei225 returns
2
1
0
-1
0
200
400
600
nikkei225 returns and 1% VaR under S-CAViaR

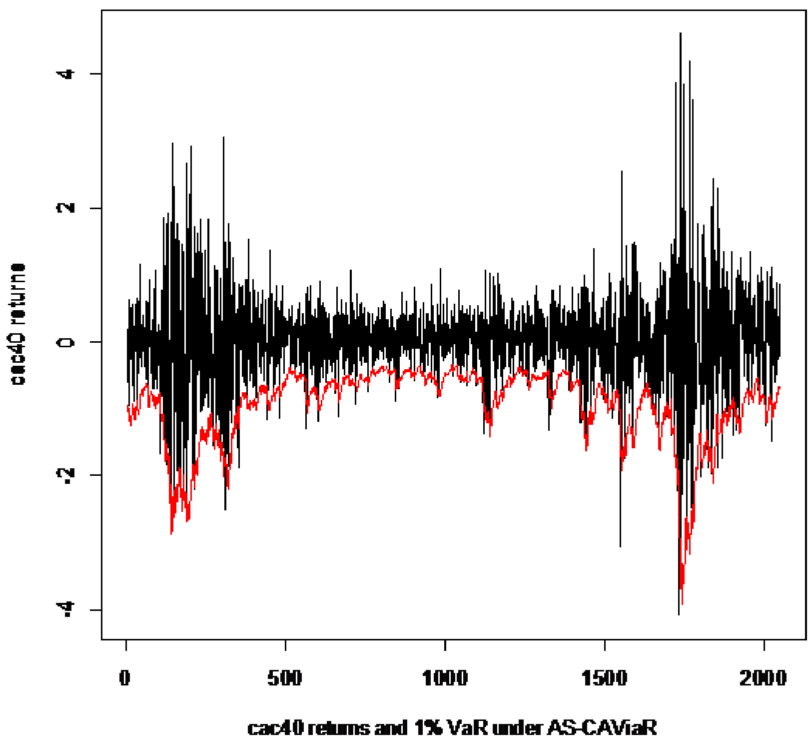


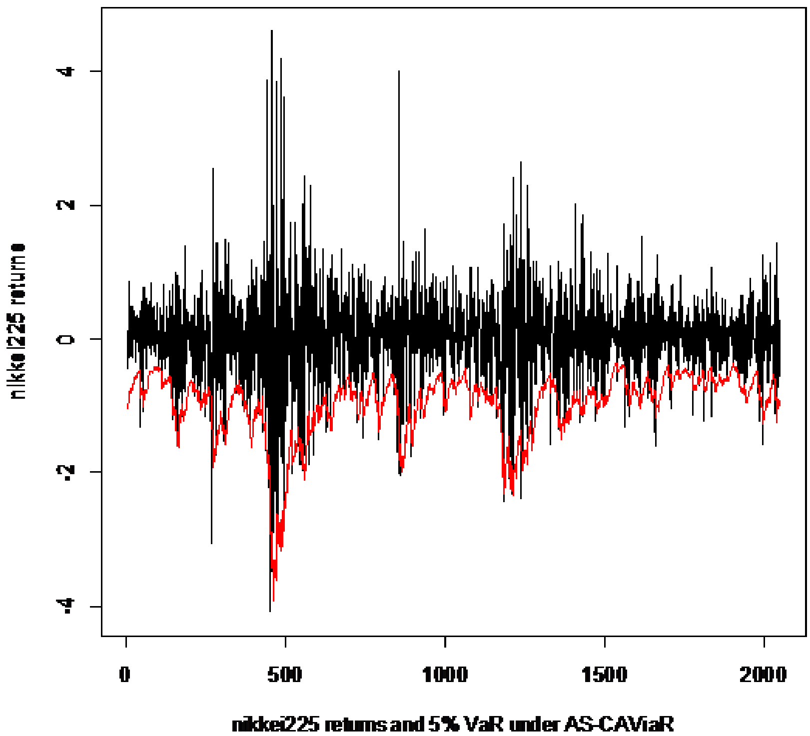
s&p500 returns and 5% VaR under S-CAViaR

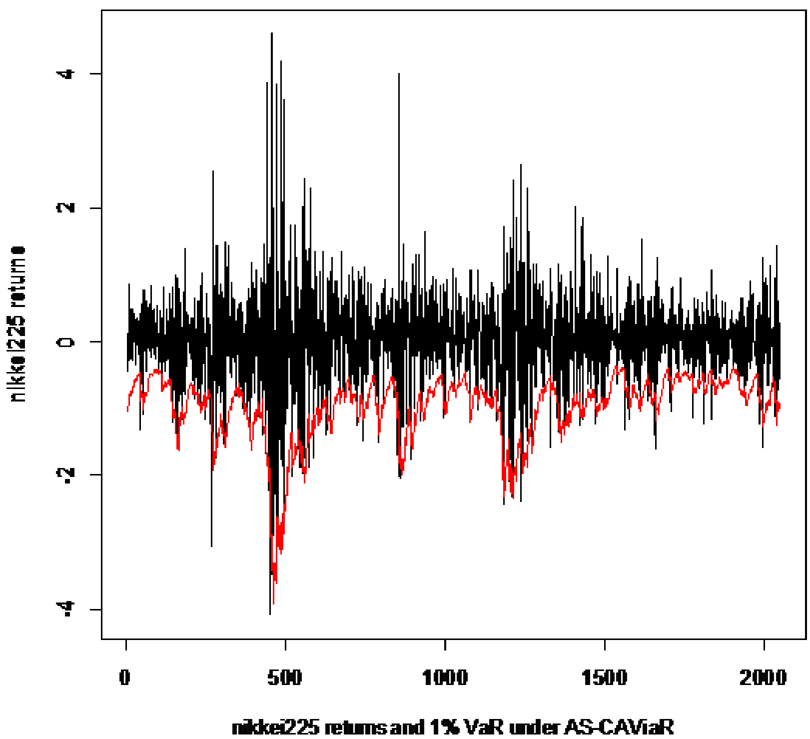


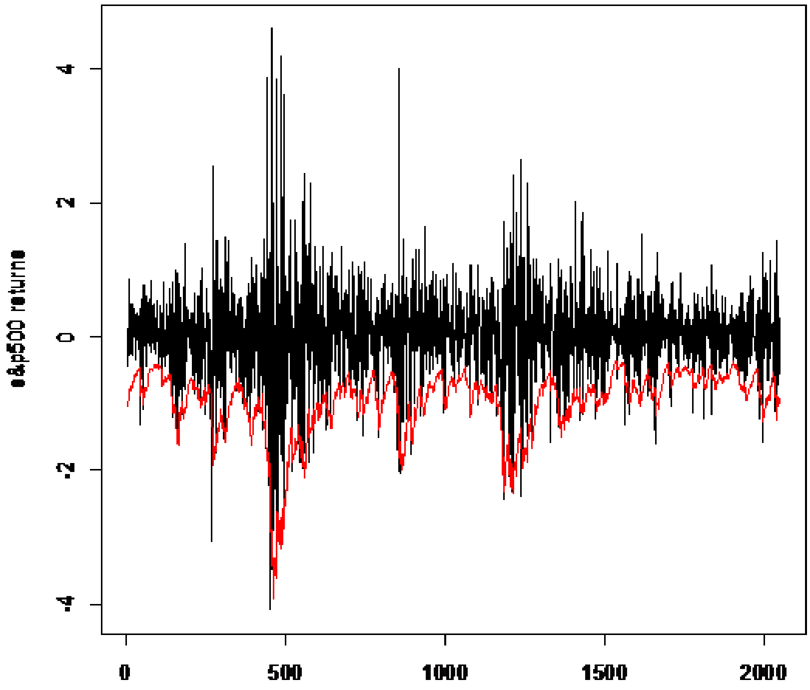
s&p500 returns and 1% VaR under S-CAViaR


cac40 returns
4
2
0
-2
-4
0
500
1000
1500
2000
cac40 returns and 5% VaR under AS-CAViaR

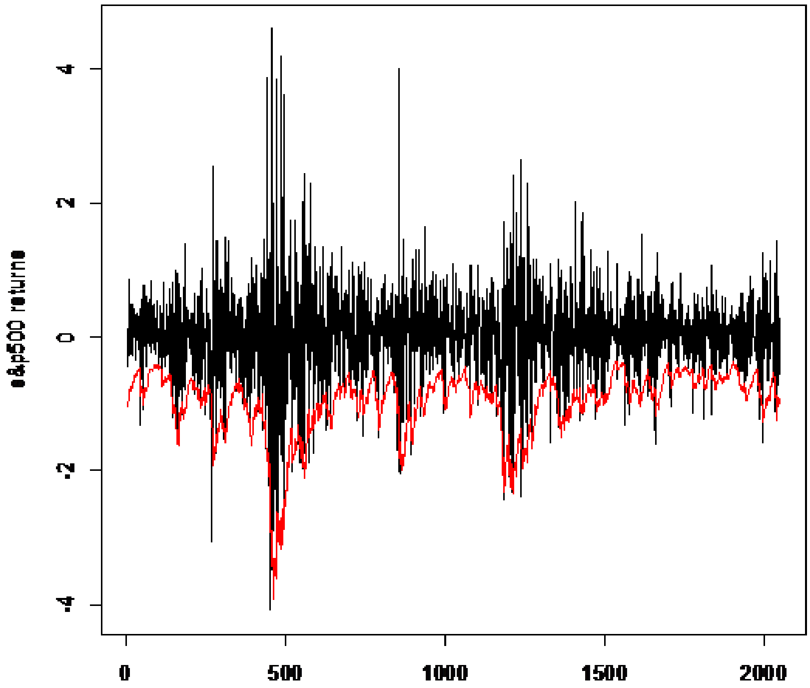

cac40 returns
4
2
0
-2
-4
0
500
1000
1500
2000
cac40 returns and 1% VaR under AS-CAViaR


nikkei225 returns
4
2
0
-2
-4
0
500
1000
1500
2000
nikkei225 returns and 5% VaR under AS-CAViaR

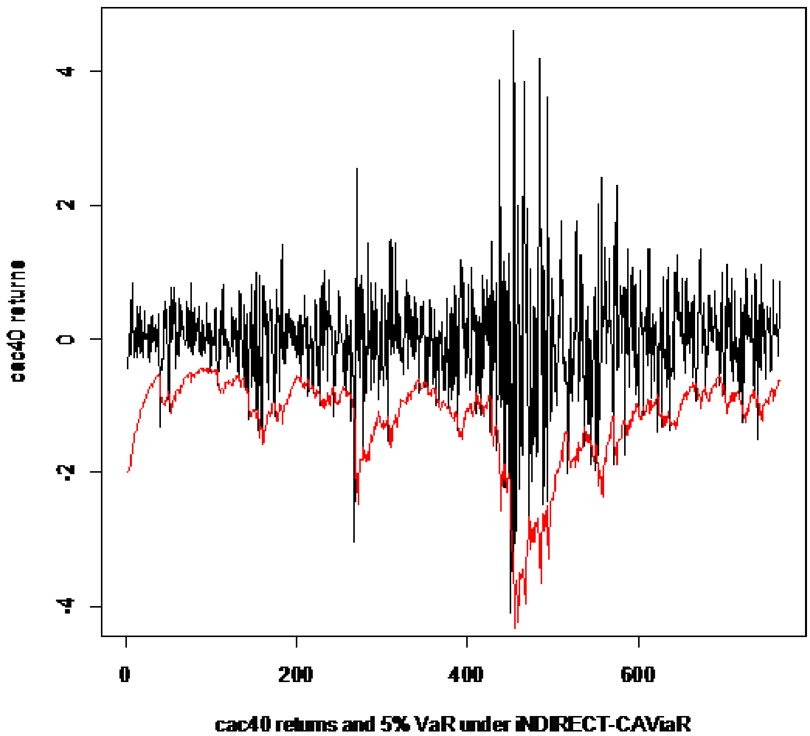

nikkei225 returns
4
2
0
-2
-4
0
500
1000
1500
2000
nikkei225 returns and 1% VaR under AS-CAViaR

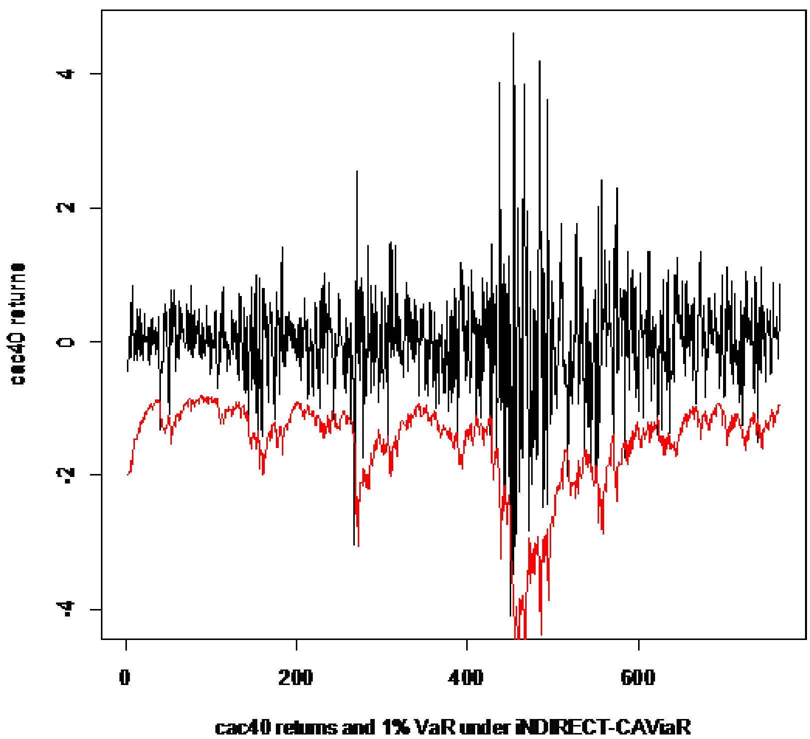


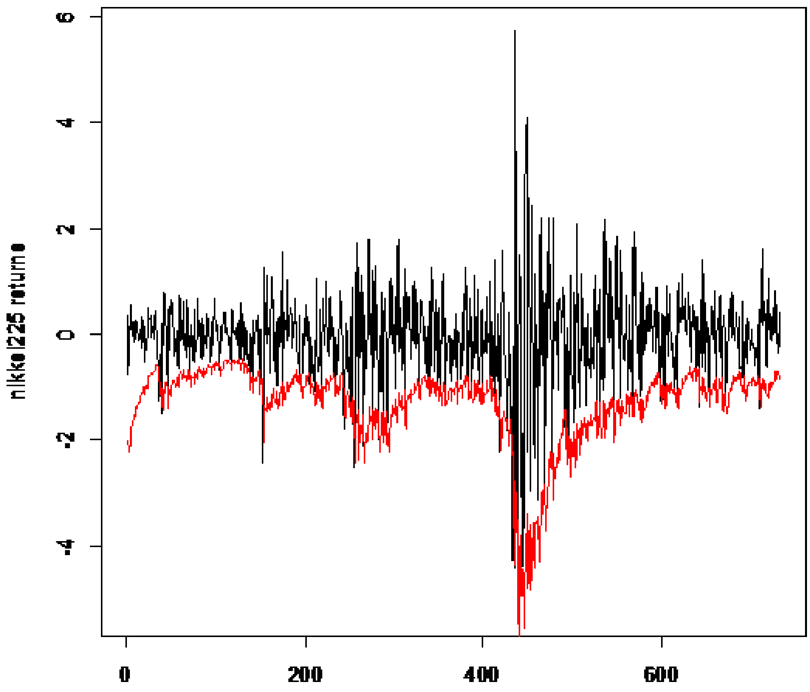
s&p500 returns and 5% VaR under AS-CAViaR



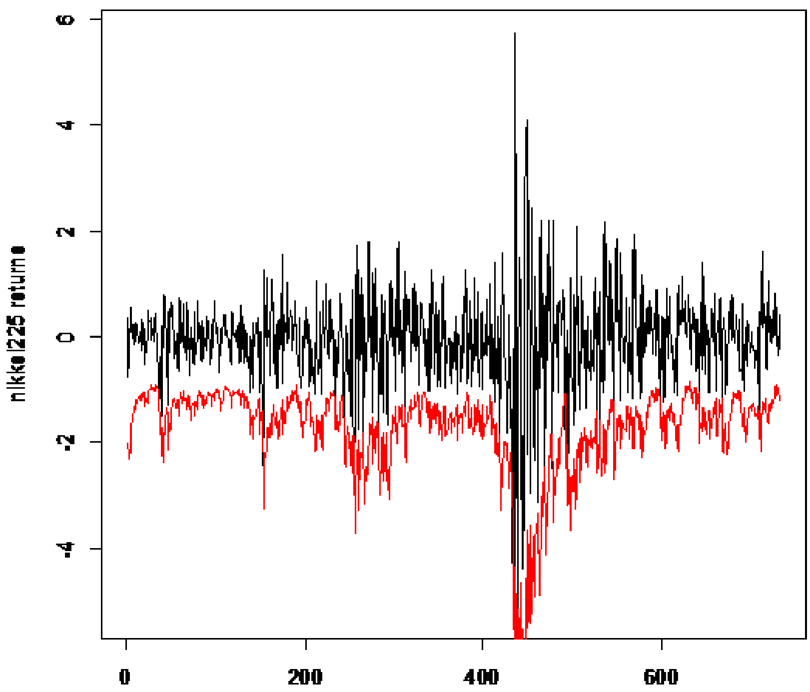
s&p500 returns and 1% VaR under AS-CAViaR


cac40 returns
4
2
0
-2
-4
0
200
400
600
cac40 returns and 5% VaR under iNDIRECT-CAViaR

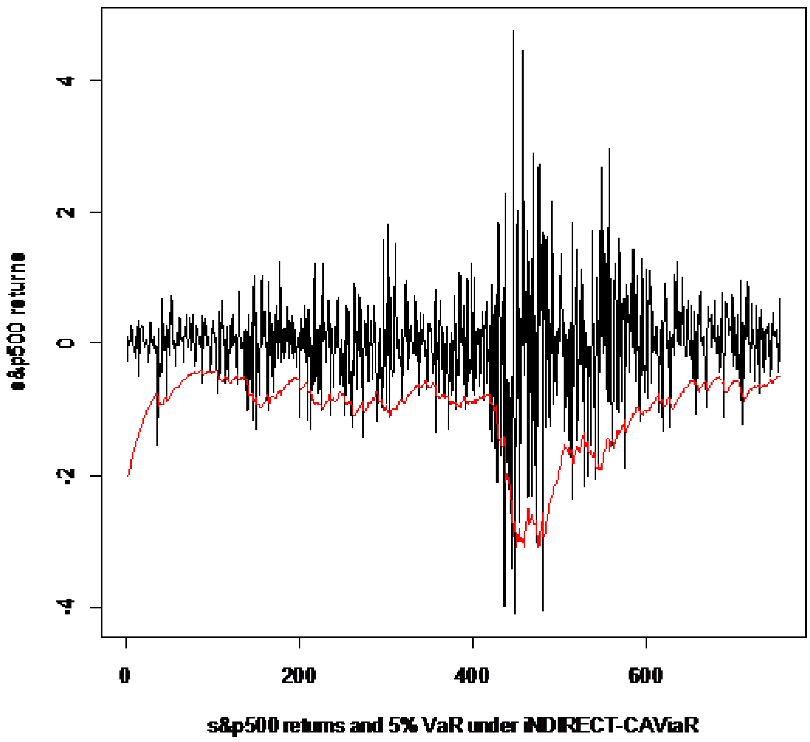

cac40 returns
4
2
0
-2
-4
0
200
400
600
cac40 returns and 1% VaR under iNDIRECT-CAViaR



nikkei225 returns and 5% VaR under INDIRECT-CAViaR



nikkei225 returns and 1% VaR under INDIRECT-CAViaR


s&p500 returns
-4
-2
0
2
4
0
200
400
600
s&p500 returns and 5% VaR under INDIRECT-CAViaR

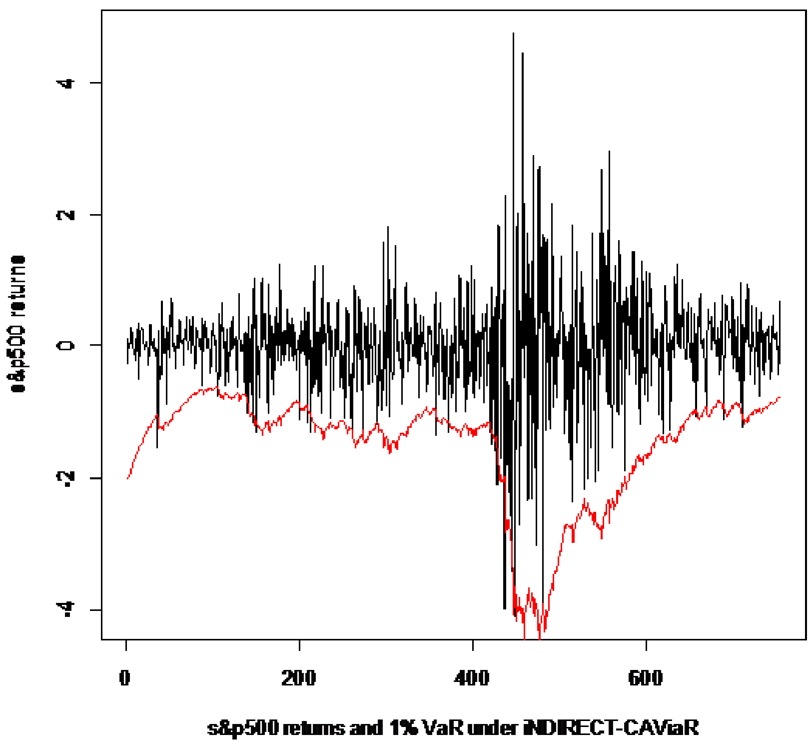
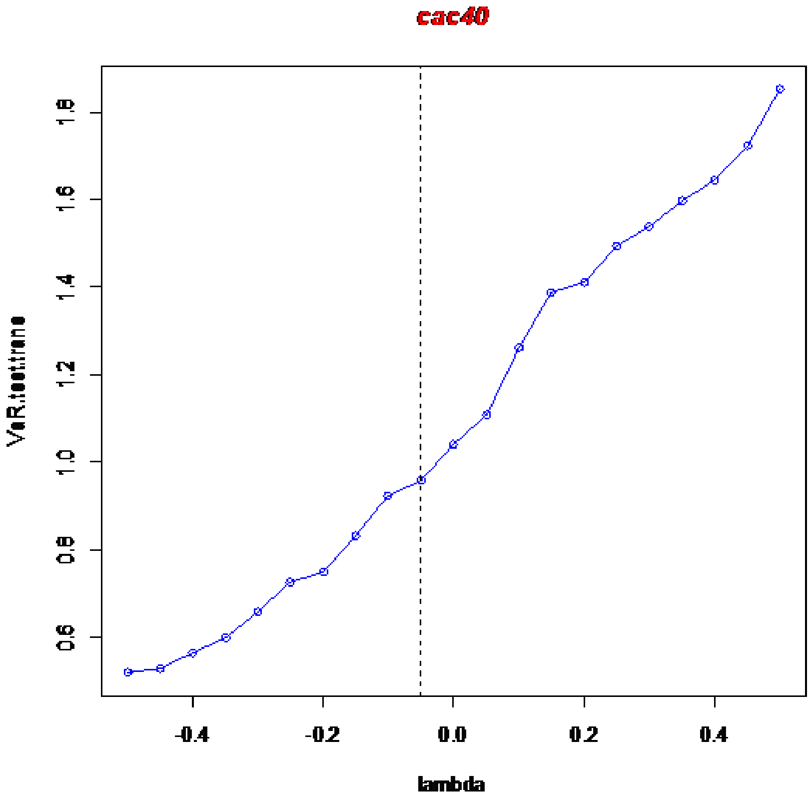
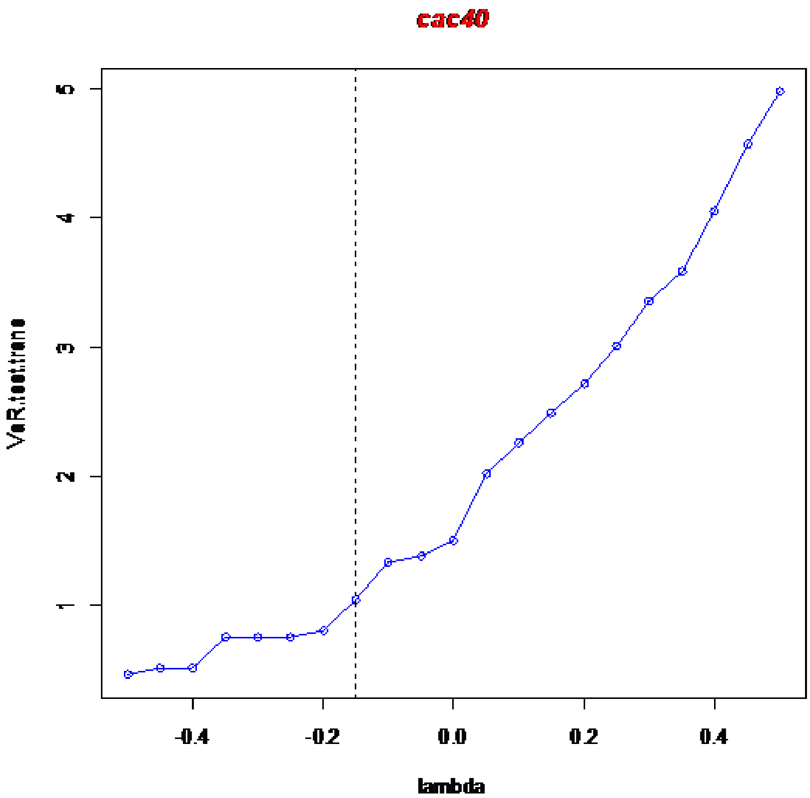
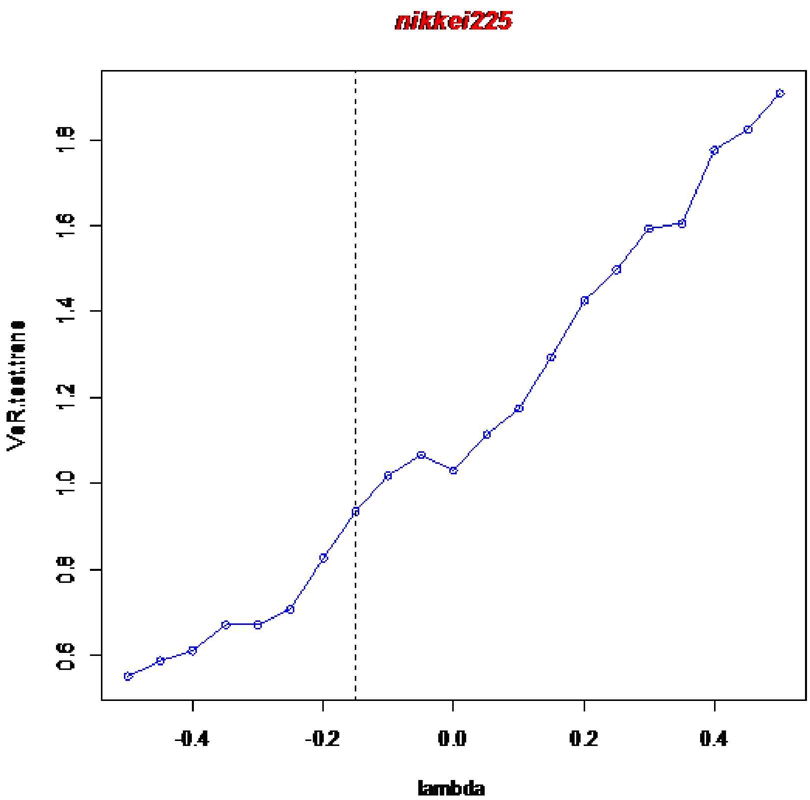

s&p500 returns
-4
-2
0
2
4
0
200
400
600
s&p500 returns and 1% VaR under INDIRECT-CAViaR

Figure 3.4: Number of λ and $\hat{\alpha}/\alpha$ for S-CAViaR, a=0.05 and a=0.01,AS-CAViaR, a=0.05 and a=0.01,AR-CAViaR, a=0.05 and a=0.01 respectively



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1.2
1.4
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1.8
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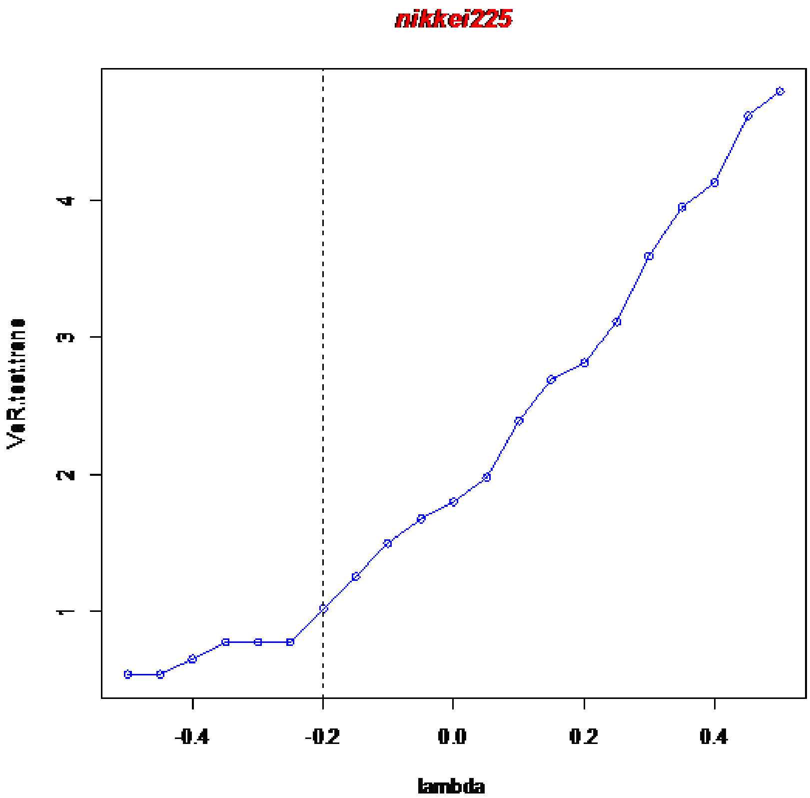
0.0
0.2
0.4
lambda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4
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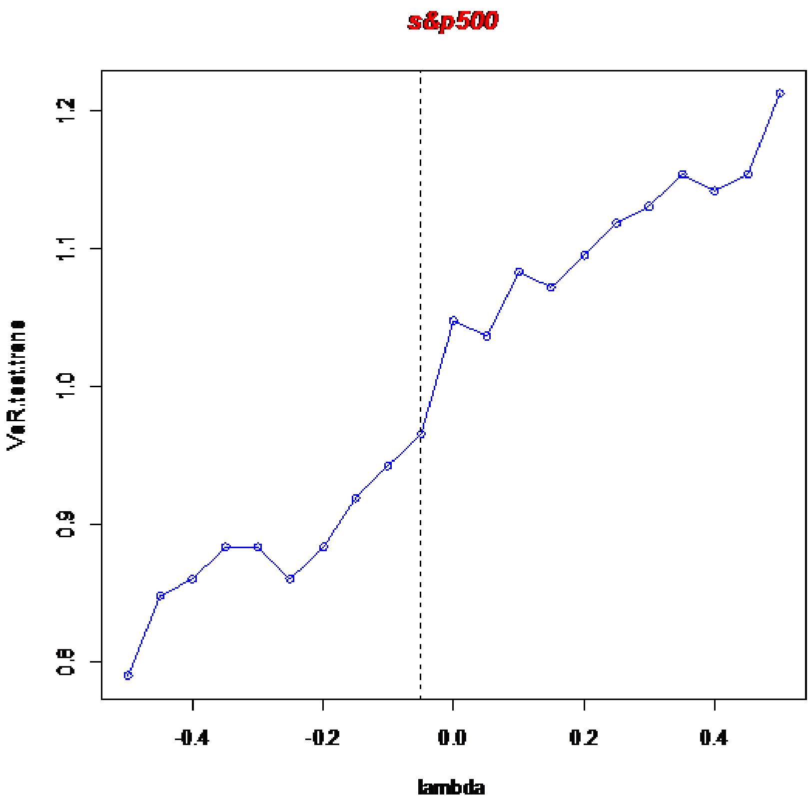
0.4
lambda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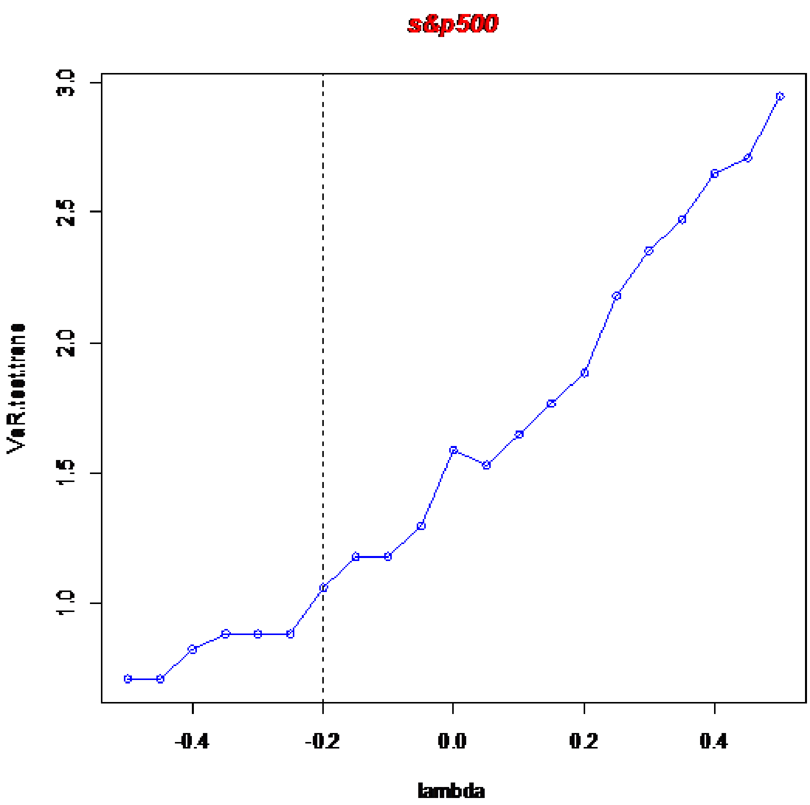
1.1
1.2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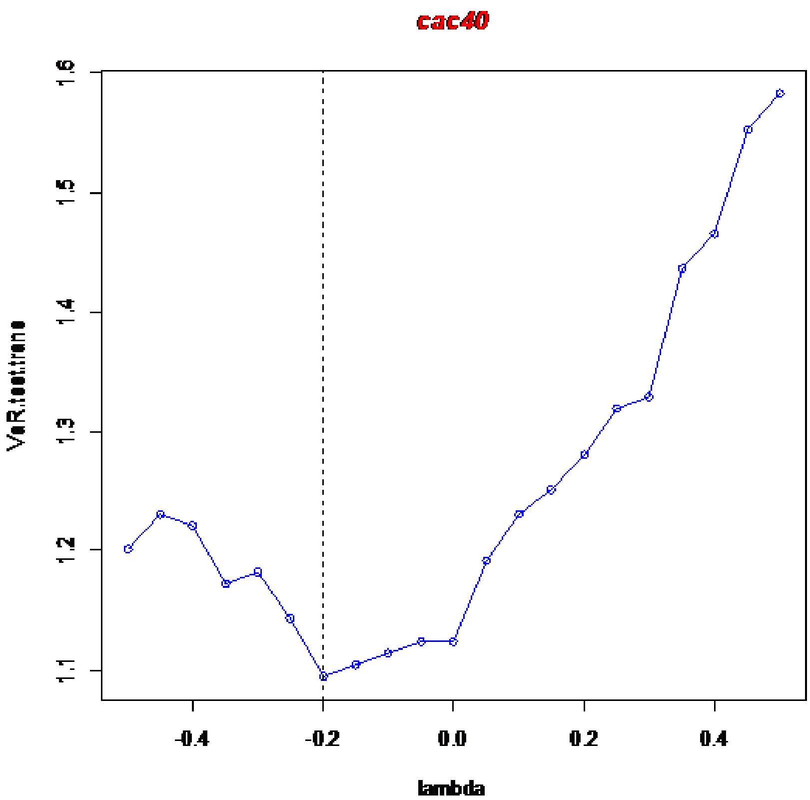
2.5
3.0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lambda
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1.3
1.4
1.5
1.6
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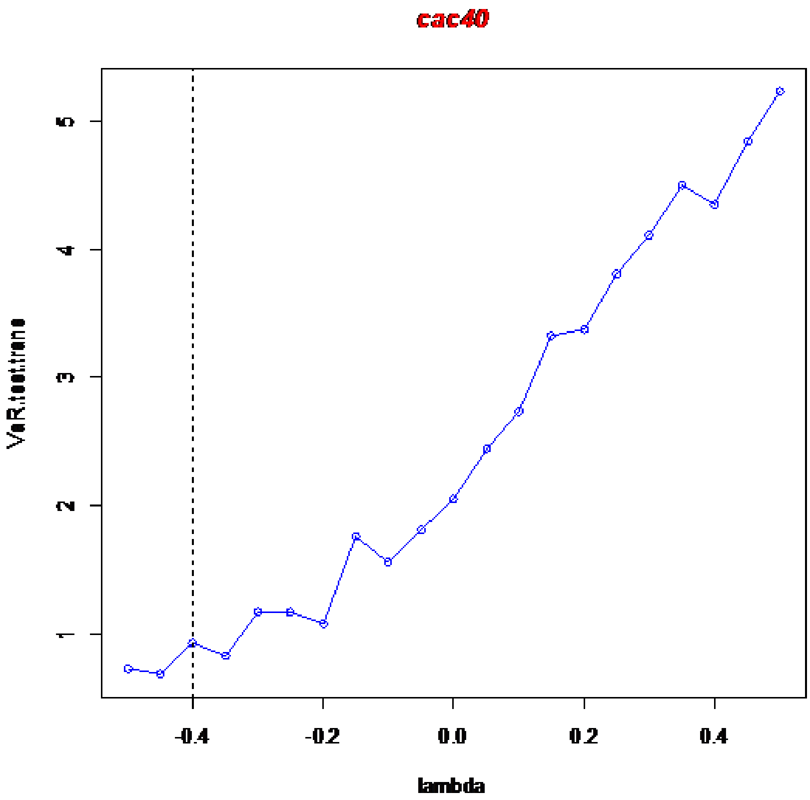
-0.2
0.0
0.2
0.4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lambda
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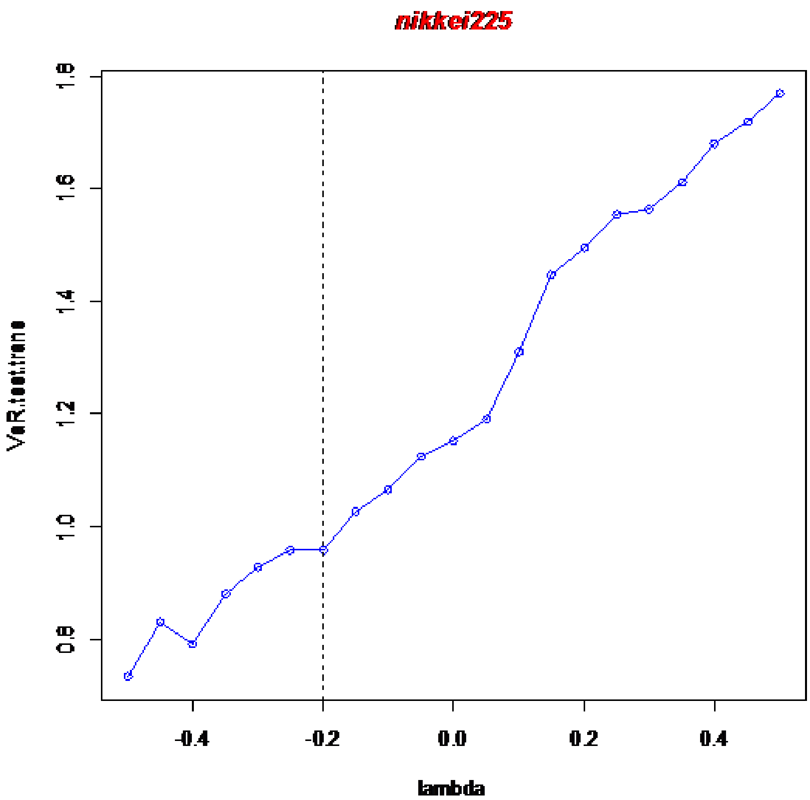
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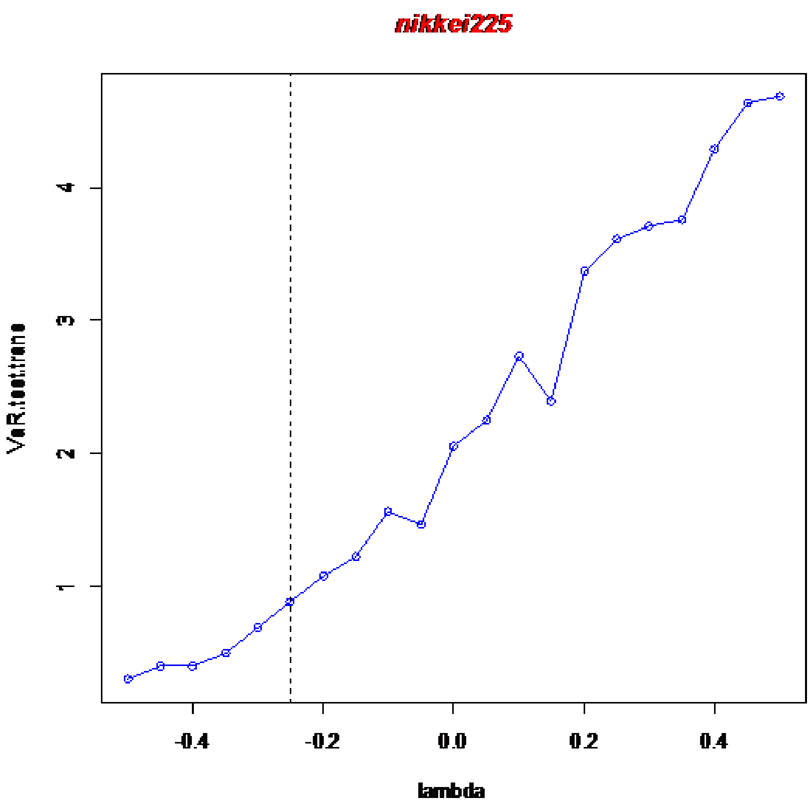
0.2
0.4
lambda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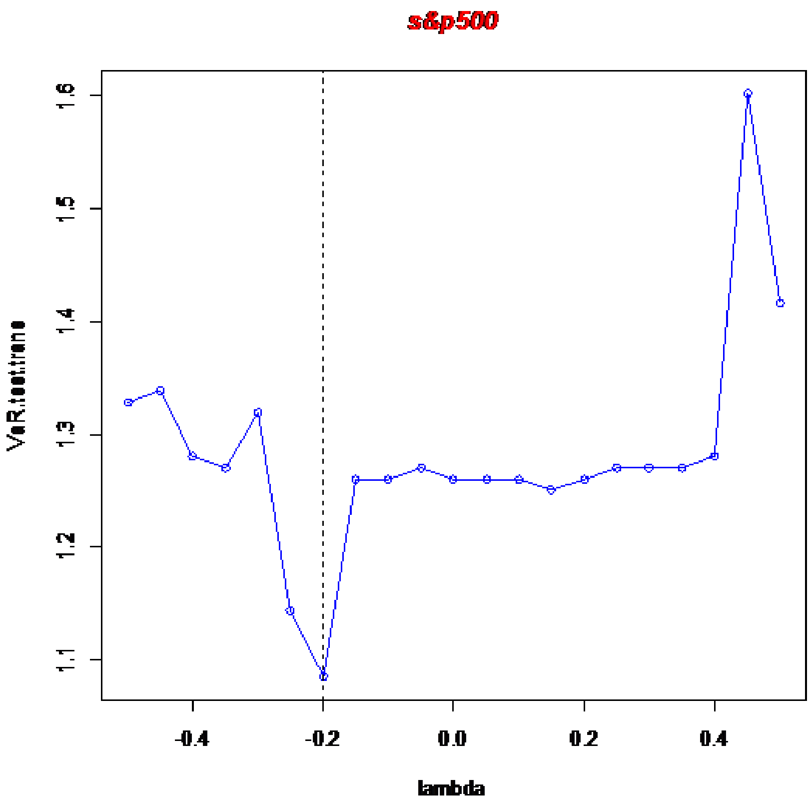
0.4
lambda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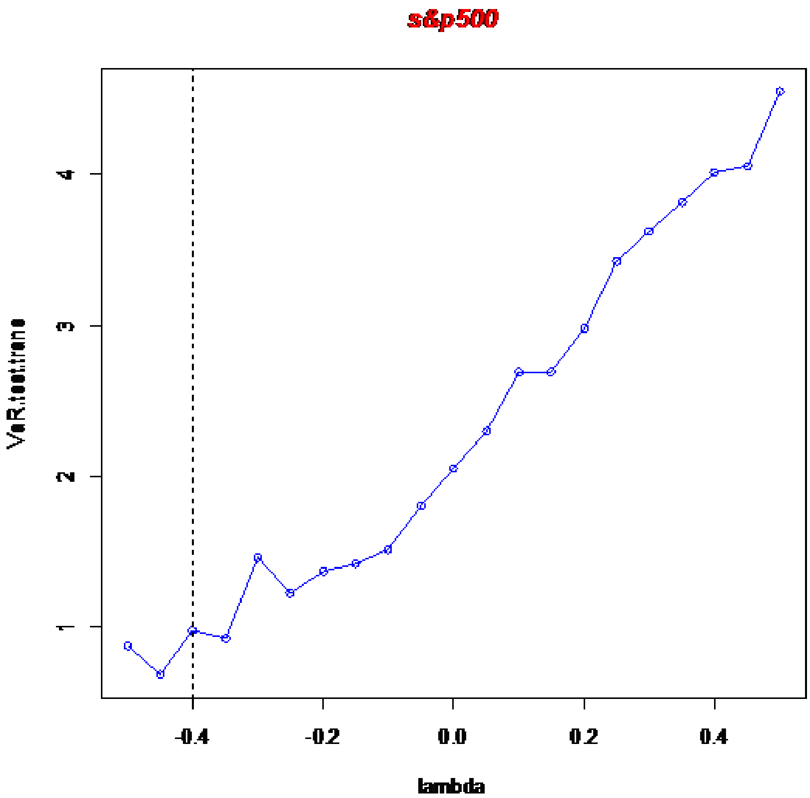
0.0
0.2
0.4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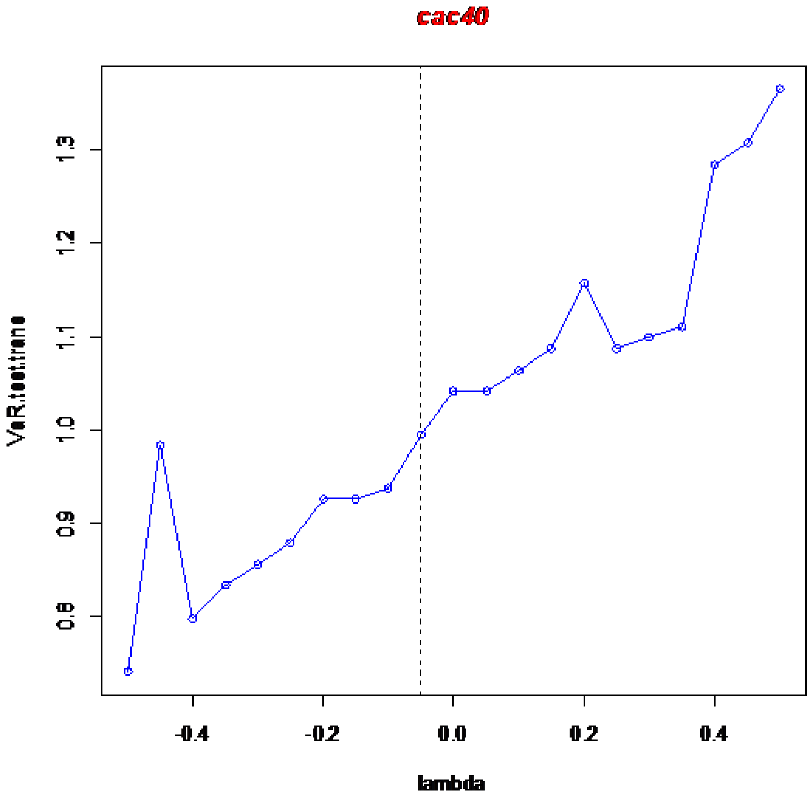
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lambda
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0.9
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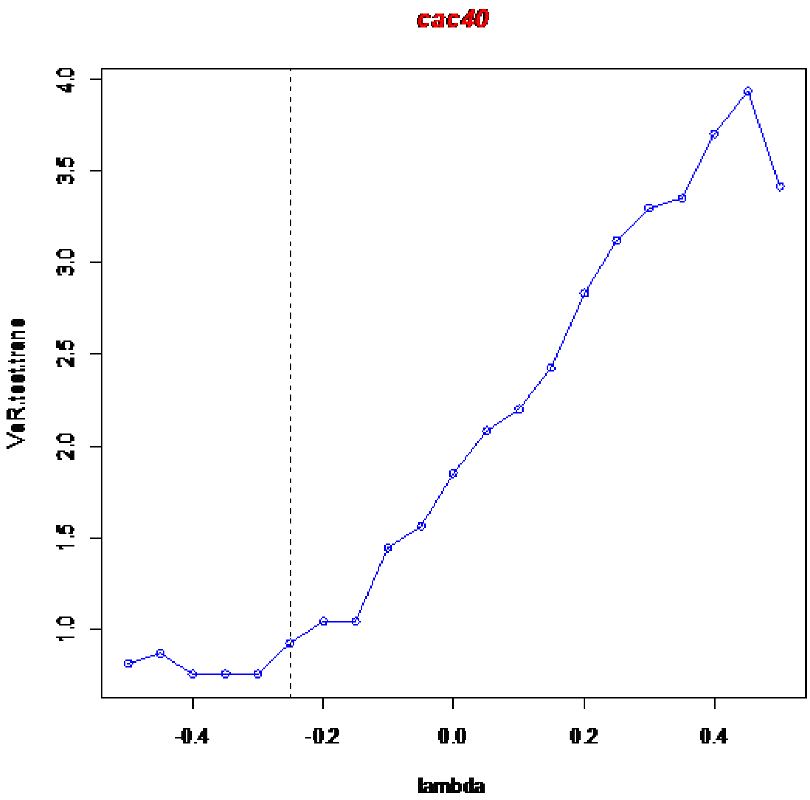
0.0
0.2
0.4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lambda
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2.5
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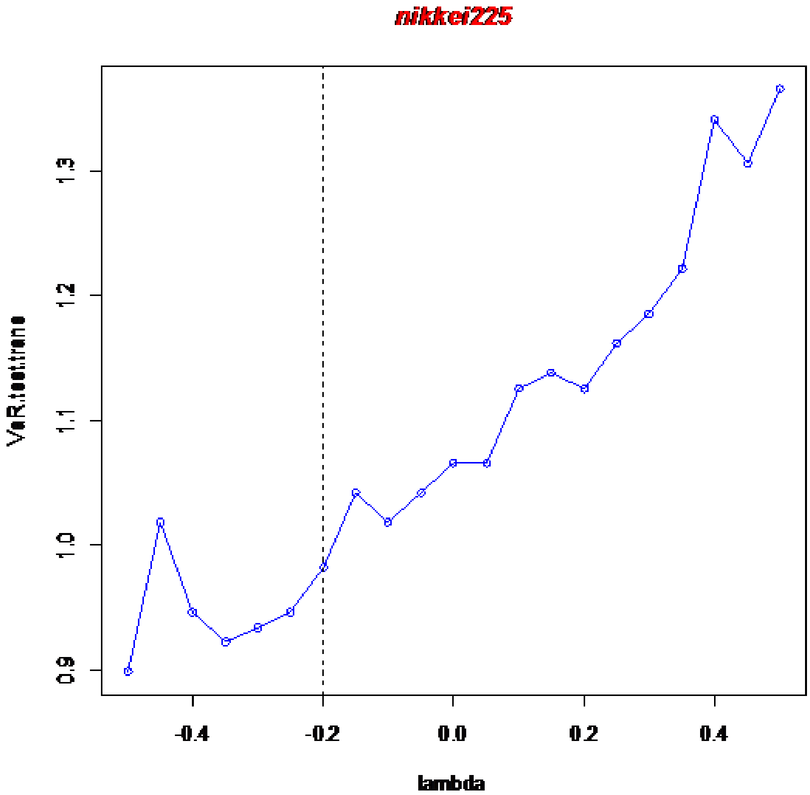
0.0
0.2
0.4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lambda
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0.2
0.4
0.9
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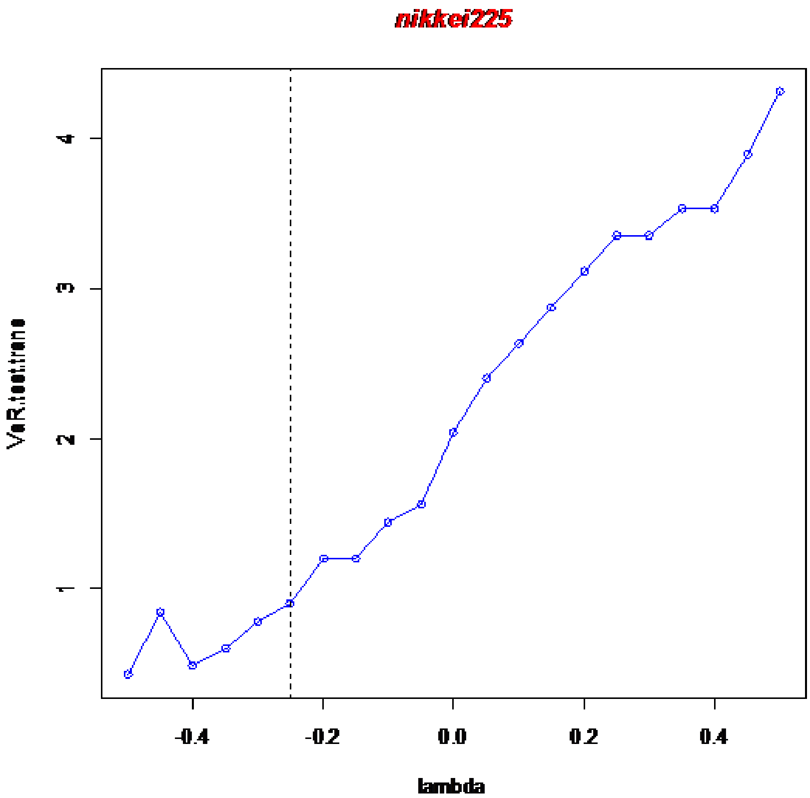
1.2
1.3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lambda
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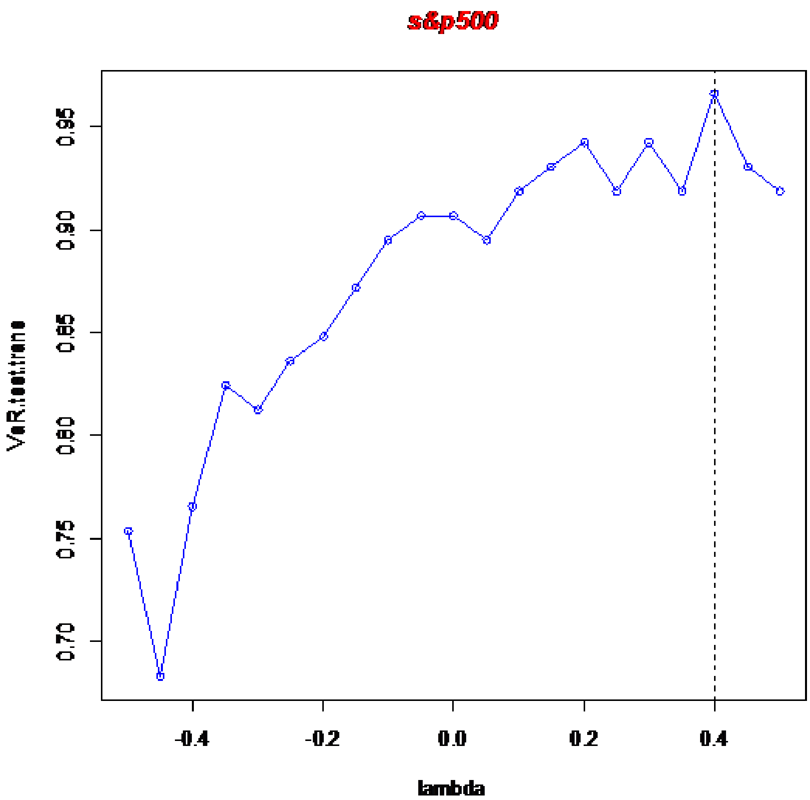
-0.2
0.0
0.2
0.4
1
2
3
4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VaR.test.trans
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0.80
0.85
0.90
0.95
-0.4
-0.2
0.0
0.2
0.4
lambda

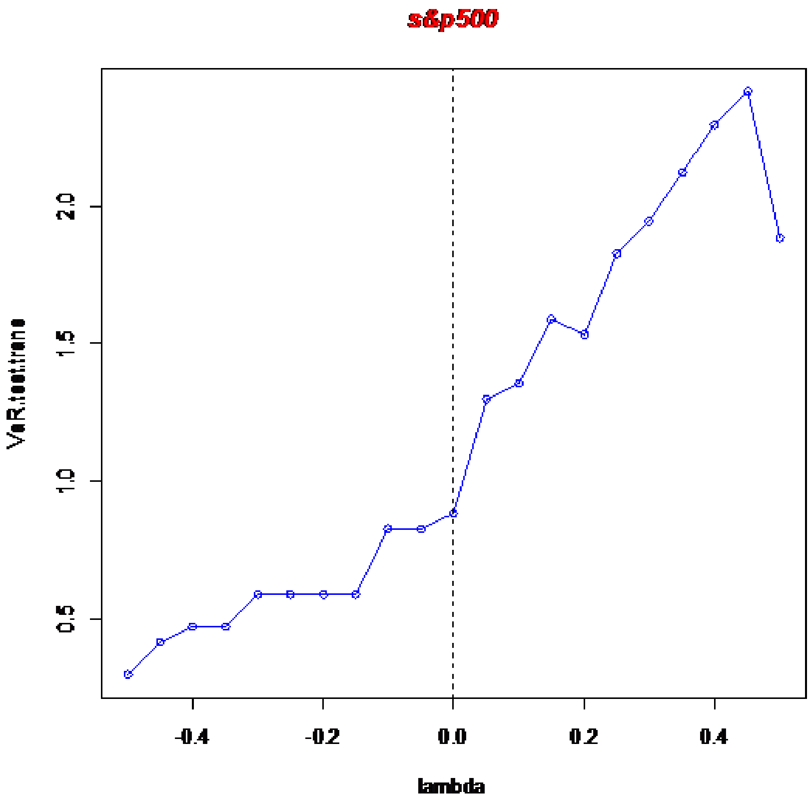
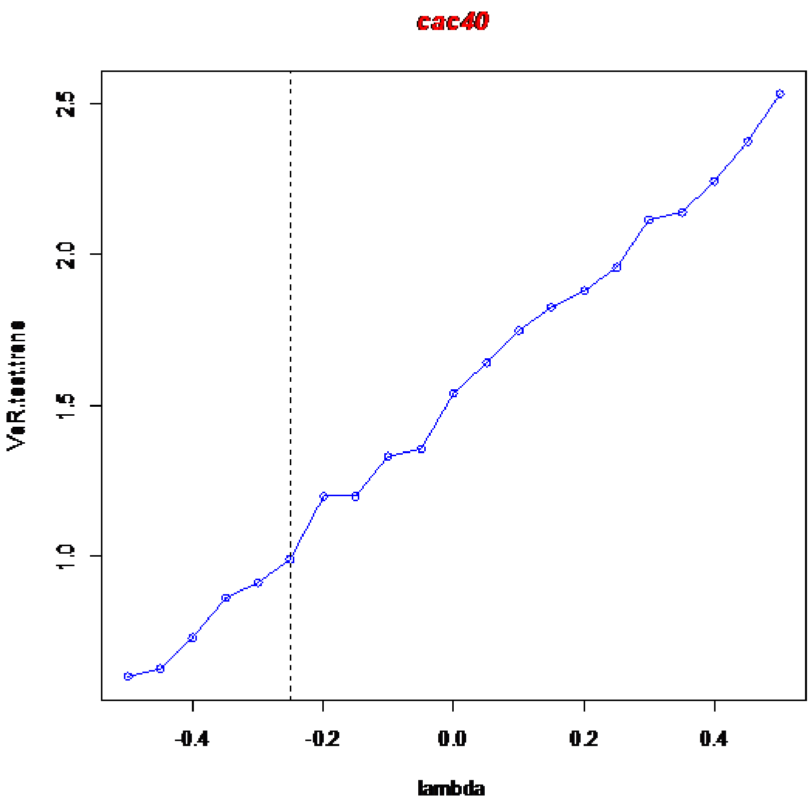
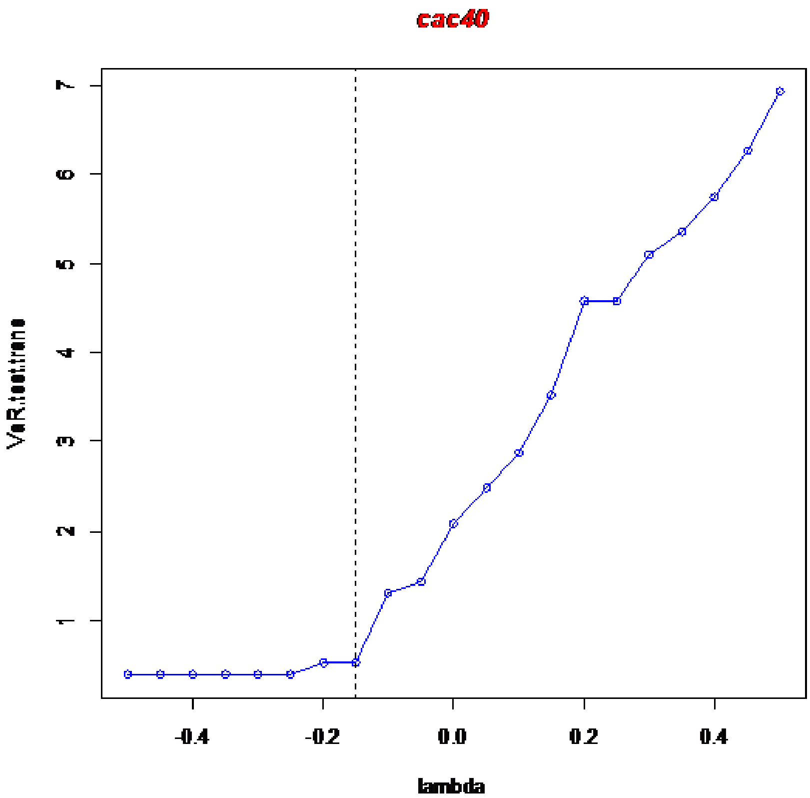
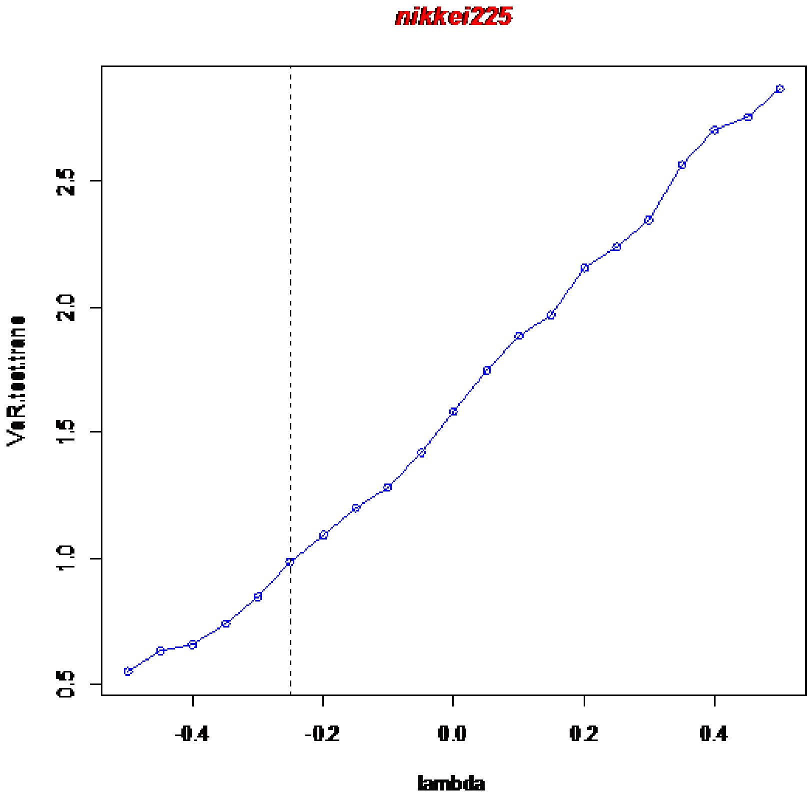

s&p500
VaR.test.trans
0.5
1.0
1.5
2.0
-0.4
-0.2
0.0
0.2
0.4
lambda

Figure 3.5: Number of λ and $\hat{\alpha}/\alpha$ for S-CAViaR, a=0.05 and a=0.01,AS-CAViaR, a=0.05 and a=0.01,AR-CAViaR, a=0.05 and a=0.01 respectively



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VaR.test.trans
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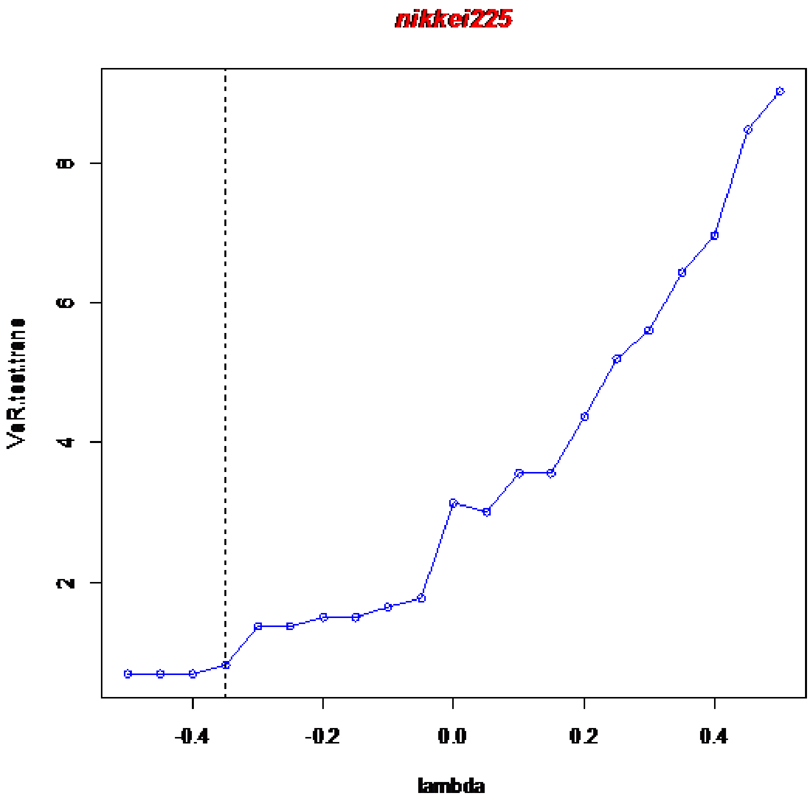
1.0
1.5
2.0
2.5
-0.4
-0.2
0.0
0.2
0.4
lambda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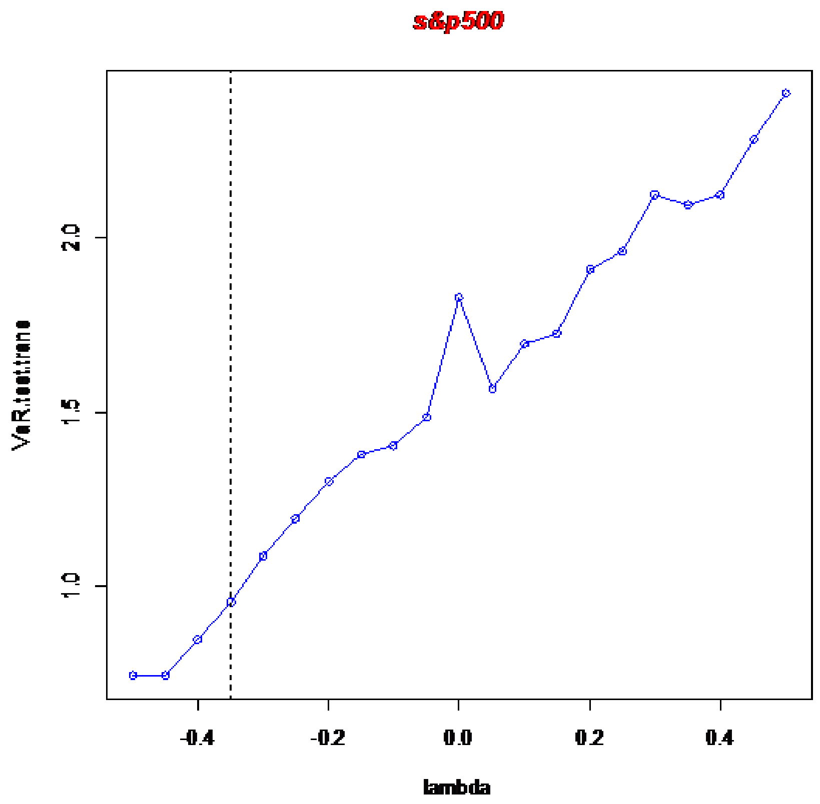
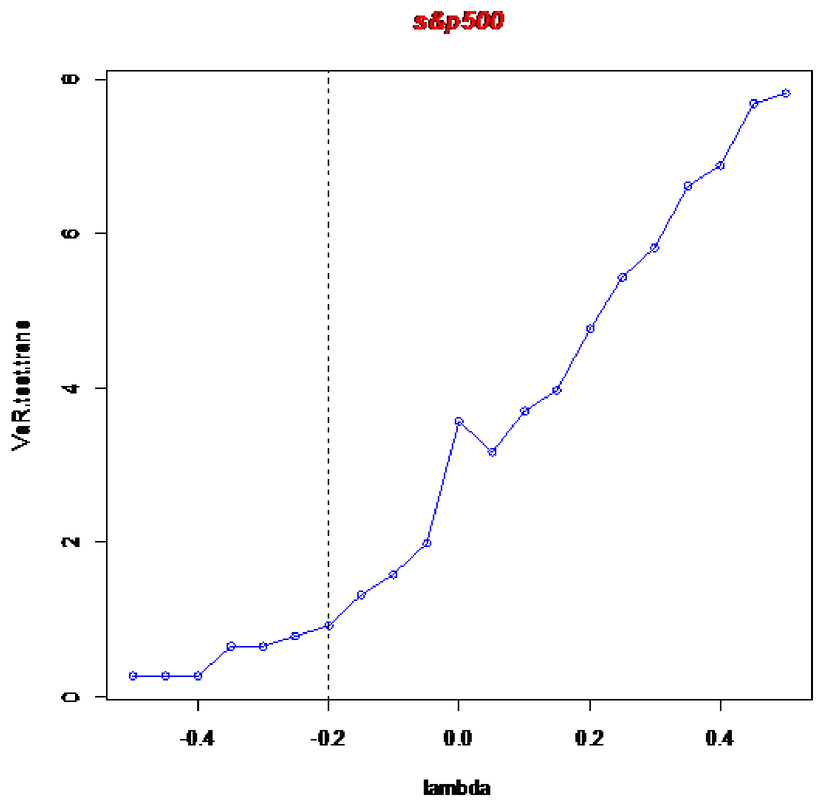
VaR.test.trans
2
4
6
8
-0.4
-0.2
0.0
0.2
0.4
lambda



s&p500
VaR.test.trans
1.0
1.5
2.0
-0.4
-0.2
0.0
0.2
0.4
lambda



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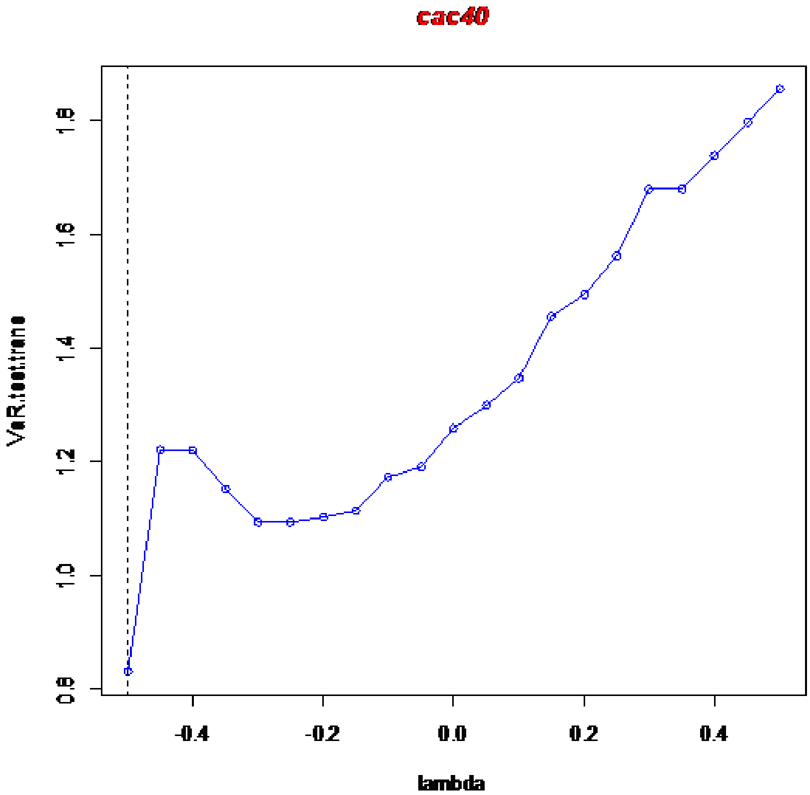
VaR.test.trans
0
2
4
6
8
-0.4
-0.2
0.0
0.2
0.4
lambda


cac40
VaR.test.trans
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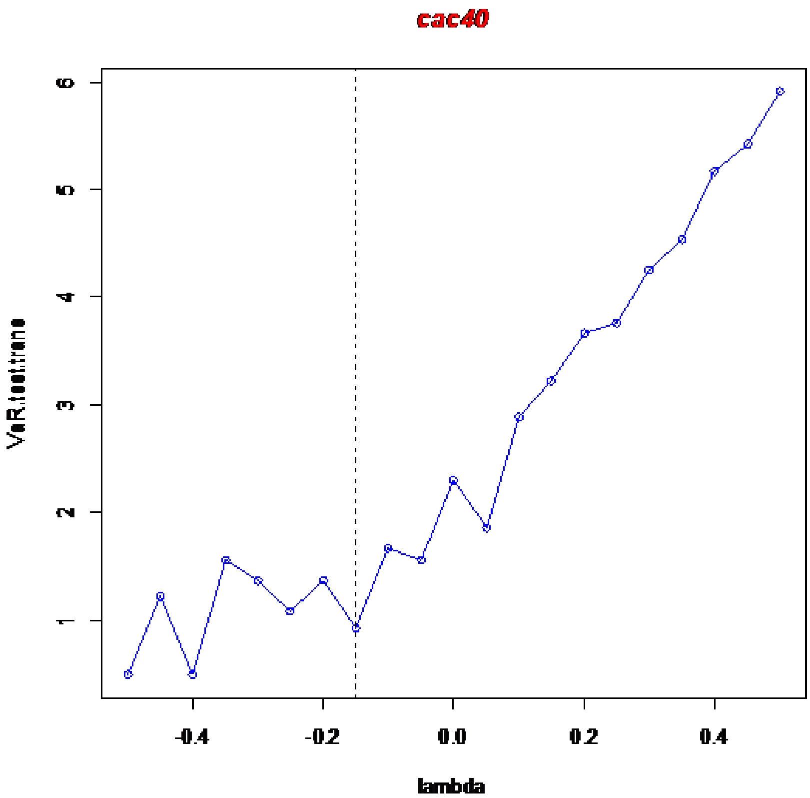
1.4
1.2
1.0
0.8
-0.4
-0.2
0.0
0.2
0.4
lambda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VaR.test.trans
6
5
4
3
2
1
-0.4
-0.2
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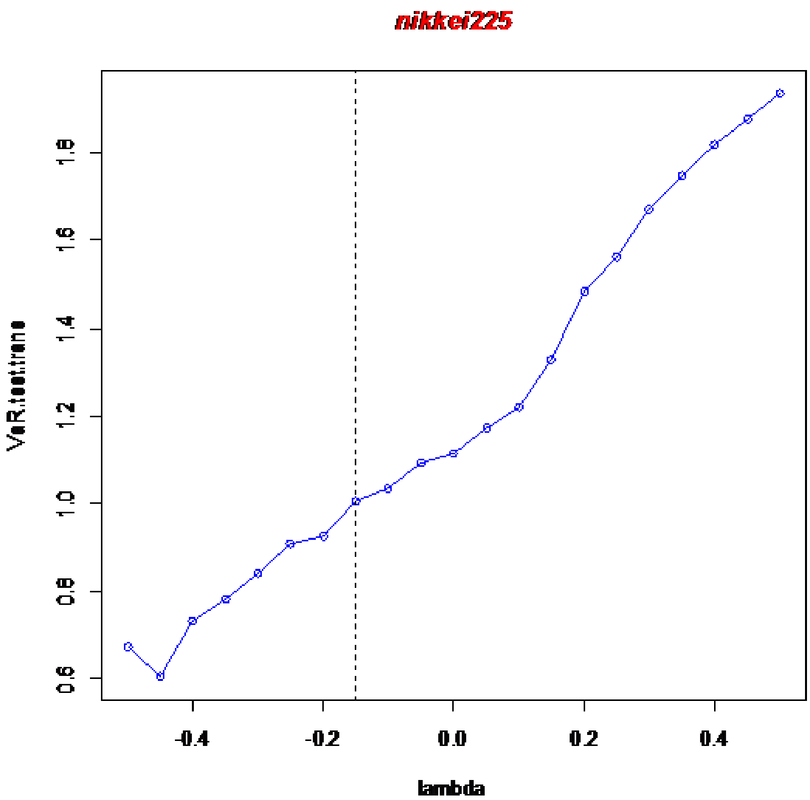
0.2
0.4
lambda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0.6
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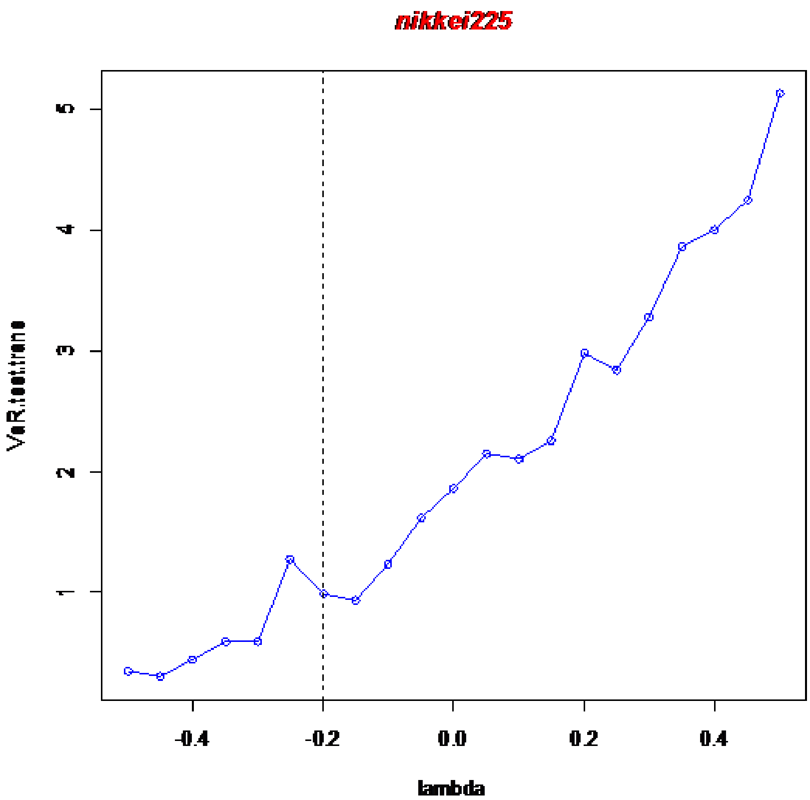
1.4
1.6
1.8
-0.4
-0.2
0.0
0.2
0.4
lambda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-0.2
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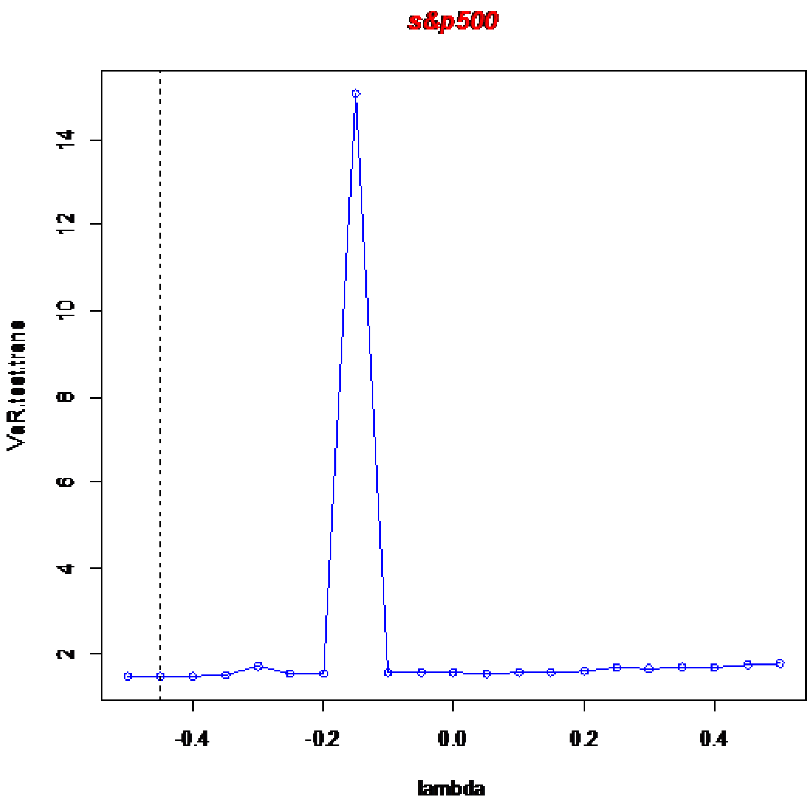
0.4
lambda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VaR.test.trans
lambda
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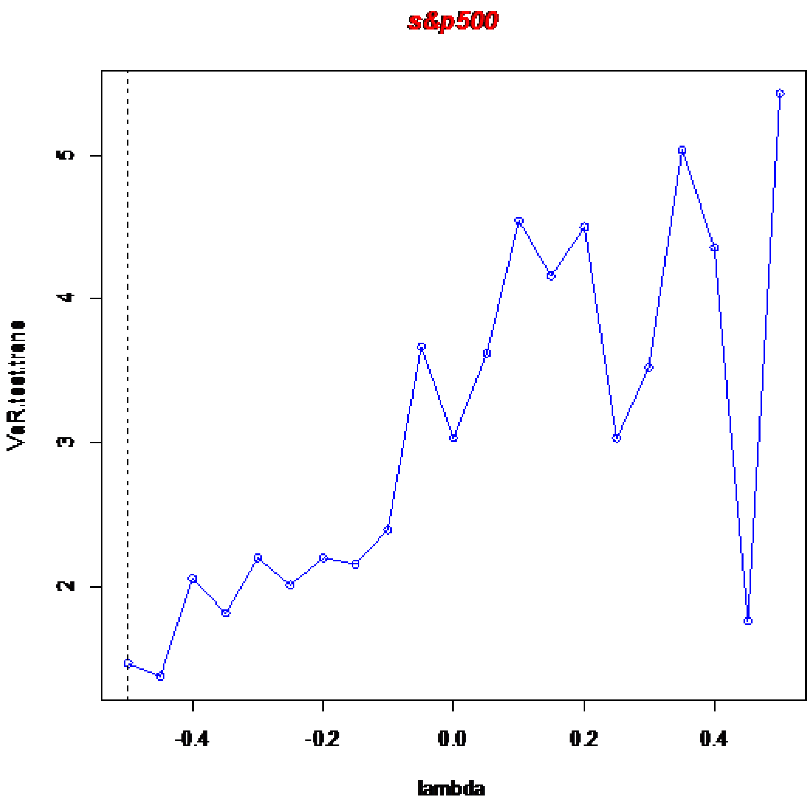
0.0
0.2
0.4
2
4
6
8
10
12
14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lambda
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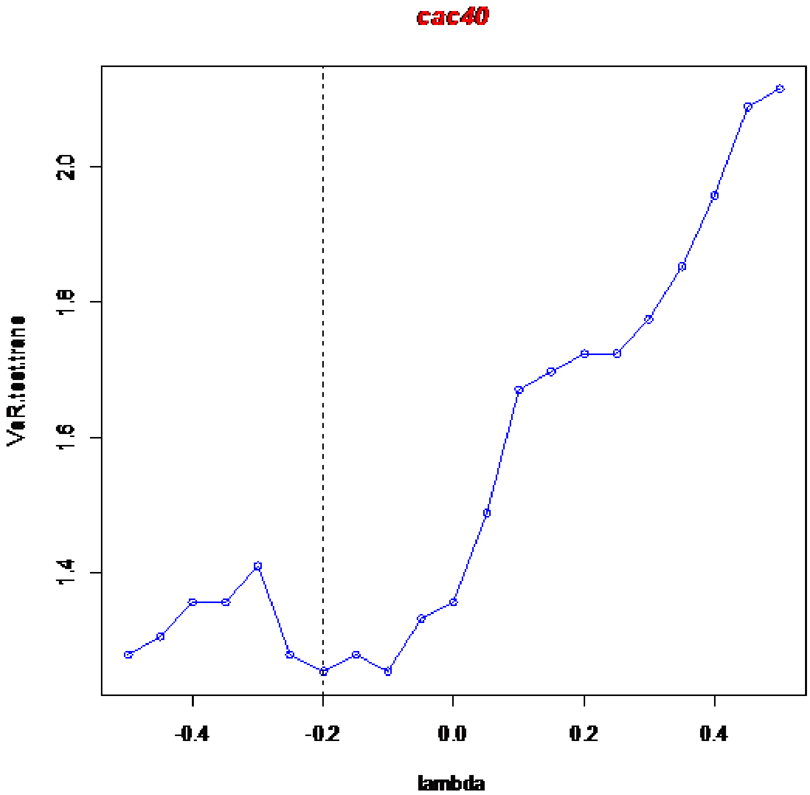
3
4
5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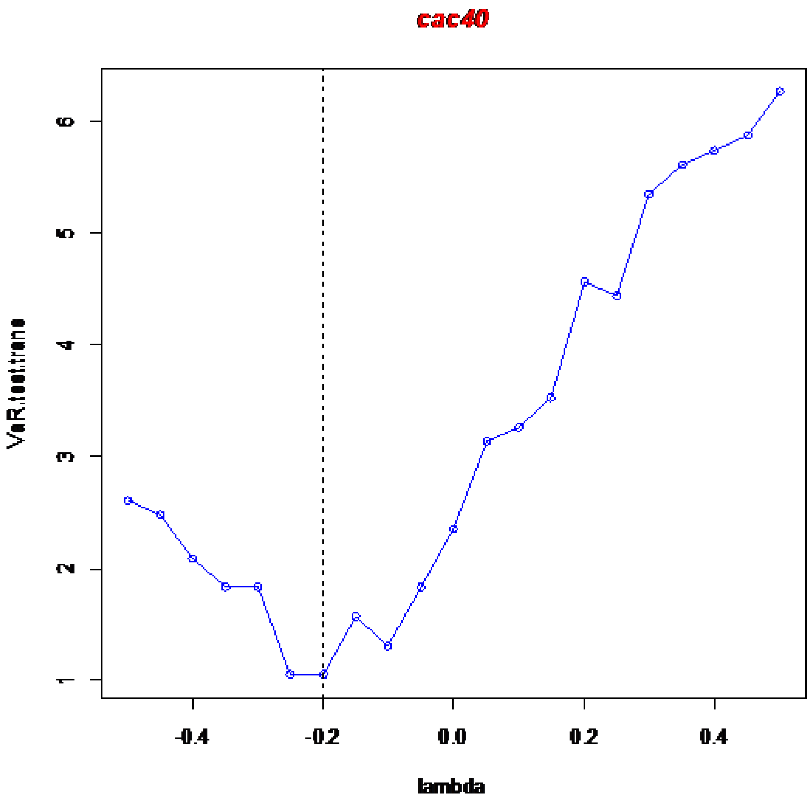
0.2
0.4
lambda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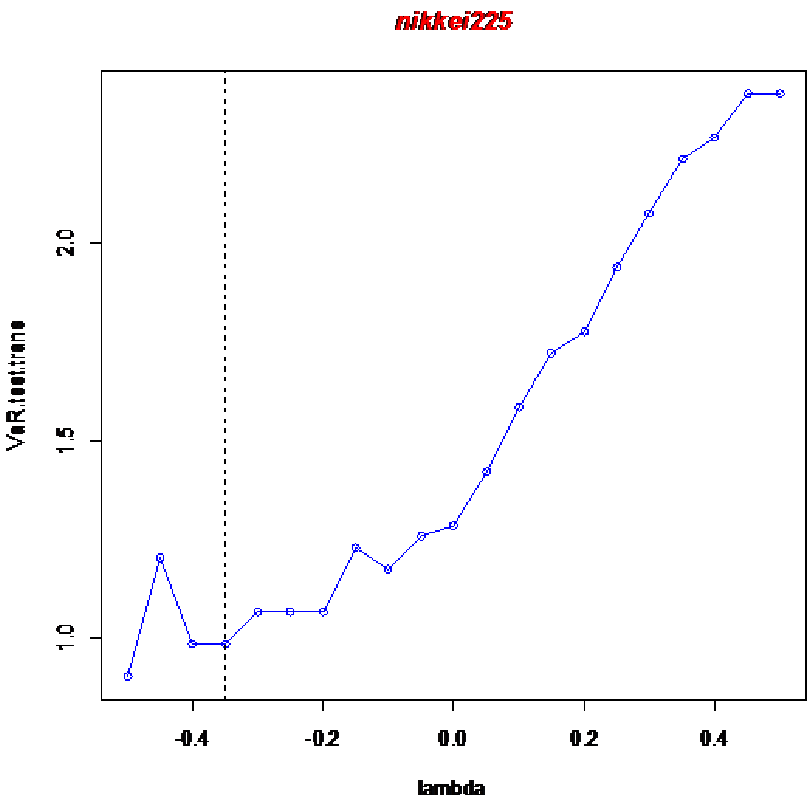
0.4
lambda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-0.2
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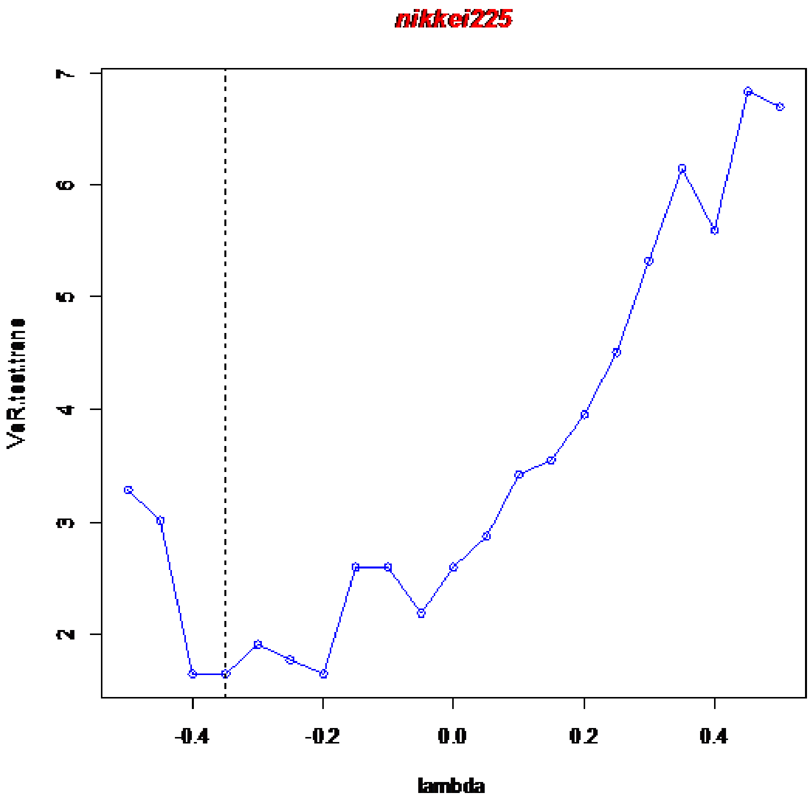
0.2
0.4
lambda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VaR.test.trans
7
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2
-0.4
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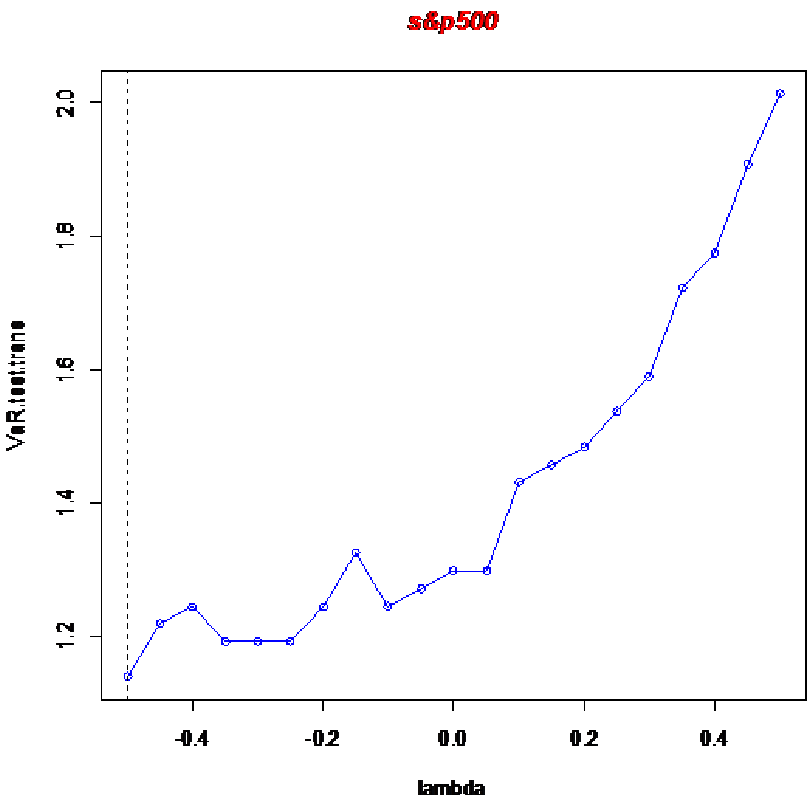
0.0
0.2
0.4
lambda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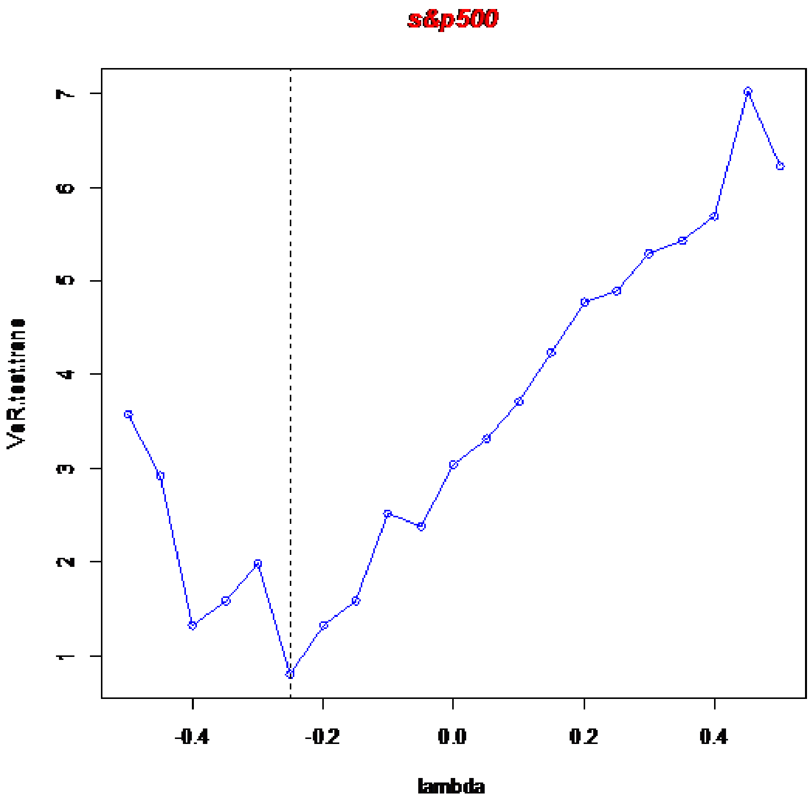
1.2
1.4
1.6
1.8
2.0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VaR.test.trans
lambda
-0.4
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0.0
0.2
0.4
1
2
3
4
5
6
7

Chapter 4

Conclusions

.

As I have mentioned from the beginning of this work, the main purpose of my research was to compare the results for VRate number, between the existing classical methods with those that use non-linear transformations. By using non-transformed methods of CAViaR models and by testing two forecasting periods, we could derive from the respective tables that, for all of our indexes, models and significance level, $\hat{\alpha}/\alpha$ number is approximate 1 in the majority of the cases, and many times this number is much greater than 1. In contrast, by using non-linear transformations, and more specific Manly transformation, we have adverse effects for $\hat{\alpha}/\alpha$. First of all we transformed the initial data and by using R program we derived for 21 λ s the $\hat{\alpha}/\alpha$ and we found the level of λ in which $\hat{\alpha}/\alpha \approx 1$. Applications to real data illustrate the ability of non-linear transformations to give us better outcomes for VaR number than the classical methods, because the $\hat{\alpha}/\alpha$ was near to 1 in the majority of the cases that concern each index, each model and each significance level, and moreover was smaller than 1 in most of the cases, which means that we overestimate risk. To sum up, an important result that we could notice is that in the case where the forecasting period is from 2010 to 2016, we prefer the transformed data results in all of the cases. Contrary to that, as for the results that concern the forecasting period from 2007 to 2009, which characterised by a financial crisis, we can observe that there are cases where we prefer the non-transformed data. This might happen because the financial crisis created many shocks in global economy and in financial indexes, that the use of the transformations cannot improve the results in every case.

4.1 Bibliography

1. Anthony Saunders and Maria Million Cornett,"Financial Institutions Management, A Risk Management Approach",*McGraw-Hill,Sixth Edition,2008*.

2. B. F. J. Manly ,"Exponential Data Transformations",*Journal of the Royal Statistical Society,Vol. 25, No. 1(Mar., 1976), pp. 37-42*.

3. Cathy W.S. Chen, Richard Gerlach, Bruce B.K. Hwang, Michael McAleer, "Forecasting Value-at-Risk using nonlinear regression quantiles and the intra-day range", *International Journal of Forecasting, 28(2012),557-574*.

4. Dirk G. Baur, Thomas Dimpfl, Robert C. Jung,"Stock return autocorrelations revisited: A quantile regression approach", *Journal of Empirical Finance, 19(2012),254-265*.

5. G.E.P.Box and D.R.Cox," An Analysis of Transformations",*Journal of the Royal Statistical Society, Vol. 26, No. 2. (1964), pp. 211-252*.

6. Huixia Judy Wang and Deyuan Li ,"Estimation of Extreme Conditional Quantiles Through Power Transformation",*American Statistical Association Journal of the American Statistical Association ,September 2013, Vol. 108, No. 503*.

7. In-Kwon Yeo and Richard A. Johnson,"A New Family of Power Transformations to Improve Normality or Symmetry",*Biometrika, Vol. 87, No. 4 (Dec., 2000), pp. 954-959*.

8. Lingjie Ma and Larry Pohlman,"Return forecasts and optimal portfolio construction: a quantile regression approach",*The European Journal of Finance,2008*.

9. J.A.John and N.R.Draper,"An Alternative Family of Transformations",*Journal of the Royal Statistical Society,Vol. 29, No. 2(1980), pp. 190-197*.

10. James W. Taylor ,"Using Exponentially Weighted Quantile Regression to Estimate Value at Risk and Expected Shortfall", *Journal of Financial Econometrics, 2008, Vol. 6, No. 3, 382406*.

11. James W. Taylor,"Generating Volatility Forecasts from Value at Risk Estimates",*Management Science, 2005, Vol. 51, pp. 712-725*.

12. John C. Hull,“Options, Futures, and Other Derivatives”,*Pearson, Prentice Hall, Seventh Edition,2009*.

13. Jooyoung Jeon and James W. Taylor ,“Using CAViaR Models with Implied Volatility for Value at Risk Estimation”, *University of Oxford Journal of Forecasting, 2013, Vol. 32, pp. 62-74*.

14. Keith Kuester, Stefan Mittnik ,Marc S. Paolella ,“Value-at-Risk Prediction: A Comparison of Alternative Strategies”, *Journal of Financial Econometrics, 2006, Vol. 4, No. 1, 5389*.

15. Kevin Dowd ,“Measuring Market Risk”, *Wiley finance Series,2002*.

16. Michelle L. Barnes and Anthony W. Hughes,“A Quantile Regression Analysis of the Cross Section of Stock Market Returns”, *2002*.

17. Peter Reinhard Hansen and Allan Timmermann ,“Choice of Sample Split in Out-of-Sample Forecast Evaluation”,*February 7, 2012*.

18. Philippe Jorion,“Value at Risk”, *New York, McGraw Hill,Second Edition,2001*.

19. R. M. Sakia,“The Box-Cox Transformation Technique: A Review”,*Journal of the Royal Statistical Society,Vol. 41, No. 2(1992), pp. 169-178*.

20. Richard H. Gerlach, Cathy W. S. Chen and Nancy Y. C. Chan ,“Bayesian Time-Varying Quantile Forecasting for Value-at-Risk in Financial Markets”, *Journal of Business and Economic Statistics,2011*.

21. Robert F. Engle and Simone Manganelli, “CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles”, *American Statistical Association Journal of Business and Economic Statistics,2004*.

22. Ruey S. Tsay ,“Analysis of Financial Time Series”, *WILEY,Second Edition, ,2005*.

23. Sean D. Campbell,“A Review of Backtesting and Backtesting Procedures”, *2005*.

24. Tsiotas Georgios,“On the use of non-linear transformations in Stochastic Volatility models”,*2009*.

25. Yunming Mu and Xuming He,“Power Transformation Toward a Linear Regression Quantile”,*Journal of the American Statistical Association,2007, Vol. 102, No. 477*.

26. Zhijie Xiao and Roger Koenker,“Conditional Quantile Estimation for Generalized Autoregressive Conditional Heteroscedasticity Models”, *Journal of the American Statistical Association, Vol. 104, No. 488,2009 ,pp. 1696-1712*.