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# Holographic approach to cosmology

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## ΠΕΡΙΛΗΨΗ

Η μεταπτυχιακή εργασία αφορά την εφαρμογή της «ολογραφίας» στην κοσμολογία, συγκεκριμένα με τον τρόπο που ο Δρ. Σκεντέρης και οι συνεργάτες του έχουν καταφέρει. Αρχικά, γίνεται αναφορά στη «συμβατική» κοσμολογία και συγκεκριμένα στο καθιερωμένο κοσμολογικό μοντέλο. Στη συνέχεια, παρουσιάζονται βασικές έννοιες της ολογραφίας απαραίτητες για το συγκεκριμένο σκοπό. Ακολουθεί η εφαρμογή της ολογραφίας στην εξαγωγή βασικών αποτελεσμάτων για το πρώιμο σύμπαν.

## Summary

The report is about the holographic approach to cosmology provided by Skenderis and collaborators. Starting with the standard cosmology, the basic cosmological predictions and observations are discussed. Then, basic features of holography, for the purposes of this report, are presented. Finally, the holographic approach of Skenderis and collaborators as well as their "holographic cosmology" are discussed.

# Contents

# Chapter 1

# Introduction

Cosmological observations gave us a "picture" of an almost homogeneous universe, with small fluctuations. Because of these inhomogeneities there was structure formation and eventually galaxies could be borne. But one of the biggest questions in cosmology is the origin of inhomogeneities as well as why they have the specific spectrum we observe. Inflation theories are capable to explain inhomogeneities and also some other major problems such as the flatness and horizon problems. Specifically, in order to obtain the power spectrum, one follows a standard quantization procedure to treat cosmological fluctuations at smaller earlier times. Indeed, theories of inflation can predict an almost flat spectrum in agreement with observations. However, because the obove procedure is based on perturbation theory, it cannot be applied when the gravity is strongly coupled. The most promising approach in strongly coupled gravity is holography. The fact that there is a strong/weak relation between gravity and boundary dual-QFT can be useful to make cosmological predictions at very early times, when the standard approach fails. Also, holographic cosmology can be applied for weakly coupled gravity as well, reproducing inflationary results.

In "holographic cosmology", one of the main purposes is the derivation of cosmological power spectrum from holographic computations. In short, cosmological power spectrum and two-point function of the stress-energy tensor of a three-dimensional dual-QFT are related. This boundary QFT corresponds to a bulk that is asymptotically $AdS$ or asymptotically power law, which after the appropriate analytic continuation, correspond to "late" time DeSitter or power law inflation respectively. The main advantage of this approach is that for times when gravity is strongly coupled, a derivation of primordial spectrum is possible through boundary QFT computations since QFT in this case is weak and can be treated perturbatively.

# Chapter 2

# Standard Cosmology

Before the holographic approach it is convenient to discus conventional cosmology. Fist the standard cosmological model is shortly discussed. Then we present some basic elements of inflation and power spectrum. Lastly we briefly discuss the main idea of temperature anisotropies because, in order to see whether holographic cosmology leads to observational predictions, a basic understanding of CMB is needed.

## 2.1 Standard model of cosmology

The standard cosmological model or $\Lambda$CDM consists of two "pillars"; a basic framework and initial conditions. The first one is the big bang model, also called FRW model and according to it, the universe is homogeneous, isotropic and structure formation is forbidden. The second pillar is the initial conditions given by inflation. The idea is that initially there was an accelerating expansion with some initial (primordial) fluctuations. Eventually, the evolution of fluctuations led to structure formation.

The standard model evolves six parameters usually being [$\Omega_b h^2$,$\Omega_c h^2$,$\theta$,$\tau$, $n_s$,$\Delta^2_R(k_0)$] with $\Omega$s being baryon and dark matter densities respectively, $\theta$ being the angular size of sound horizon at recombination, $\tau$ being the optical depth since recombination, $n_s$ called spectral index and $\Delta^2_R(k_0)$ is the amplitude of primordial spectrum at a given scale corresponding to wavenumber $k_0$. The last two parameters are related to the early universe and they parametrize primordial power spectrum with the standard relation

$$\Delta_R^2(k) = \Delta_R^2(k_0)(\frac{k}{k_0})^{n_s-1} \tag{2.1}$$

There is also a similar parametrization for tensor power spectrum, corresponding to primordial gravitational waves, $\Delta_T^2$:

$$\Delta_T^2(q) = \Delta_T^2(k_0)(\frac{q}{q_0})^{n_T} \tag{2.2}$$

with the so-called tensor to scaler ratio $r$ being

$$r \equiv \frac{\Delta_T^2(k_0)}{\Delta_R^2(k_0)} \tag{2.3}$$

If spectral index is scale dependent, then the power spectrum can also be written as [1]

$$\Delta_R^2(k) = \Delta_R^2(k_0)(\frac{k}{k_0})^{n_s(k_0)-1+\frac{1}{2}dn_s/dlnk} \tag{2.4}$$

where $dn_s/dlnk$ is the running index. Running index indicates the sensitivity of $n_s$ in different scales. The six parameters can be estimated by CMB observations and cosmological models can be tested.

## 2.2 Inflation

In this section we briefly discuss the basics of inflation. In the context of this report we are interested mostly on the predictions of primordial power spectrum.

According to inflation, the hot Big Bang era was initiated by exponential expansion of the universe, induced by some scalar field(s). The (nearly) exponential expansion followed by a "graceful exit" after which, the scale factor took the form that we know for the radiation dominant era, namely:

$$a(t) \sim \sqrt{t} \tag{2.5}$$

It should be mentioned that this statement is enough to make inflation the solution to three main problems in physics; the flatness problem, the horizon problem and magnetic monopoles [2]. Inflation took place in very early times roughly $t_i = t_{Planck} \sim 10^{-43}$ sec. Metric perturbations can be justified during this epoch of sub-Planckian curvatures. In a typical inflationary model, scalar metric fluctuations are induced by some scalar field $\varphi$, which is described as an ideal fluid. The action has the following general form [3]:

$$S_{scalar} = \int dx^4 \sqrt{-g} p(X, \varphi) \quad , \quad X = \frac{1}{2} g^{ab} \varphi_{,a} \varphi_{,b} \tag{2.6}$$

where the lagrangian $\mathcal{L} = p$ represents the "pressure". Scalar metric perturbations are caused by energy density inhomogeneities. Thus, they are responsible for structure formation. A metric of this kind can be written, in the so-called Newtonian gauge, as

$$ds^2 = (1 + 2\Phi) dt^2 - a^2 (1 - 2\Psi) \delta_{ij} dx^i dx^j \tag{2.7}$$

where spatially flat universe is assumed.

Apart from scalar perturbations, there also vector and tensor perturbations with the last being important. Tensor perturbations are gravitational waves. One of the inflationary predictions is the presence of primordial gravitational waves. Tensor perturbations can be written as

$$ds^2 = dt^2 - (\delta_{ij} - h_{ij}) dx^i dx^j \tag{2.8}$$

with $h_{ij}$ being traceless and transverse:

$$h^i_i = 0 \quad , \quad h^i_{j,i} = 0 \tag{2.9}$$

A quantization of metric perturbations leads to predictions for scalar and tensor power spectra. The quantization procedure has the same steps in both cases. The quantization procedure as well as the derivation of primordial scalar power spectrum are presented in Appendix for inflationary model with a single canonical scalar field

$$S_{scalar} = \int d^4 x \sqrt{-g} [\frac{1}{2} g^{ab} \phi_{,a} \phi_{,b} - V(\phi)] \tag{2.10}$$

The above action is of the general "perfect fluid" type (2.6)

## 2.3 Power spectrum

In order to describe inhomogeneities, a function that denotes the deviation from homogeneity can be defined as the spatially dependent function $f$

$$f(\vec{x}) = \frac{\epsilon(\vec{x}) - \epsilon_0}{\epsilon_0} \tag{2.11}$$

with $\epsilon_0$ being the mean energy density. From $f(\vec{x})$, a two-point correlation function can be introduced as

$$C(\vec{x} - \vec{y}) \equiv <f(\vec{x})f(\vec{y})> \tag{2.12}$$

The two-point function depends on $(\vec{x} - \vec{y})$ due to the assumption of isotropy. The brackets denote ensemble average. The above correlation function indicates how fluctuations change in the physical space. A Fourier expansion of $f(\vec{x})$ will lead to the definition of the power spectrum. The correlation function becomes

$$C(\vec{x} - \vec{y}) = < \int f_{\vec{k}} e^{i\vec{k}\vec{x}} \frac{d^3k}{(2\pi)^{3/2}} \cdot \int f_{\vec{k}'} e^{i\vec{k}'\vec{y}} \frac{d^3k'}{(2\pi)^{3/2}} > \tag{2.13}$$

$$C(\vec{x} - \vec{y}) = \int \int \frac{e^{i\vec{k}\vec{x}} e^{i\vec{k}'\vec{y}}}{(2\pi)^3} <f_{\vec{k}} f_{\vec{k}'}> d^3k d^3k' \tag{2.14}$$

for the Fourier components of $f(\vec{x})$:

$$<f_{\vec{k}} f_{\vec{k}'}> = \Phi_k^2 \delta(\vec{k} + \vec{k}') \tag{2.15}$$

where $\Phi_k^2$ is the variance of the gravitational potential. Now the two-point function becomes

$$C(\vec{x} - \vec{y}) = \int e^{i\vec{k}(\vec{x} - \vec{y})} \Phi_k^2 \frac{d^3k}{(2\pi)^3} \tag{2.16}$$

Defining the distance between the two point as $r \equiv |\vec{x} - \vec{y}|$ leads to

$$C(\vec{x} - \vec{y}) = \int e^{ikrcos\theta} \frac{\Phi_k^2}{(2\pi)^3} k^2 sin\theta d\theta d\phi dk \tag{2.17}$$

$$C(\vec{x} - \vec{y}) = \frac{1}{4\pi^2} \int \Phi_k^2 k^2 [\frac{1}{ikr}(e^{ikr} - e^{-ikr})] dk \tag{2.18}$$

Finally, the correlation function in physical space is expressed in terms of the so called "power spectrum" as

$$C(\vec{x}-\vec{y}) = \int \frac{\Phi_k^2}{2\pi^2}\frac{sin(kr)}{kr}\frac{dk}{k} \tag{2.19}$$

The quantity defined as power spectrum is the following:

$$\Delta_R^2 = \frac{\Phi_k^2 k^3}{2\pi^2} \tag{2.20}$$

The power spectrum $\Delta_R^2(k)$ is a measure of the amplitude of fluctuations in scales that correspond to wavenumber $k$. This corresponds to length scales $l \sim 1/k$. Also, since $\Phi_k^2$ has dimensions $(length)^3$, power spectrum is a dimensionless quantity.

Inflation predicts a nearly scale invariant primordial power spectrum which can be expressed in the following form:

$$\Delta_R^2 \sim k^{n_s-1} \tag{2.21}$$

The spectrum is predicted to be "red tilted" with a spectral index $n_s < 1$ but very close to one. The value of the spectral index is approximately $n_s \approx 0.96$ [1].

## 2.4 CMB

The most important cosmological observation is the CMB; the oldest picture of our universe we can have, i.e since recombination, the epoch in which photons were no longer able to interact with baryons and the universe became transparent to light. At that time, some regions were denser than others. Thus, photons that left from a denser region were redshifted more than those who left from less dense regions. The above "procedure" caused the temperature anisotropies of CMB. These anisotropies contain information about the origin and evolution of the universe.

The temperature fluctuations of CMB can be defined as

$$\frac{\delta T}{T_0}(\theta,\phi) \equiv \frac{T(\theta,\phi)-T_0}{T_0} \tag{2.22}$$

where $T_0$ is the mean temperature. For two directions $\hat{n}_1$ and $\hat{n}_2$ the two-point correlation function is defined as

$$C(\theta) = <\frac{\delta T}{T_0}(\hat{n}_1)\frac{\delta T}{T_0}(\hat{n}_2)> \tag{2.23}$$

where $cos(\theta) \equiv \hat{n}_1 \cdot \hat{n}_2$. The brackets stand for the averaging over all directions keeping the angle $\theta$ between $\hat{n}_1$ and $\hat{n}_2$ fixed.

The temperature fluctuations $\frac{\delta T}{T_0}(\theta, \phi)$ can be expanded in spherical harmonics:

$$\frac{\delta T}{T_0}(\theta, \phi) = \sum_{l,m} a_{lm} Y_l^m \tag{2.24}$$

where, for random field and isotropic universe, the coefficients satisfy[4]

$$<a^*_{lm} a_{l'm'}> = C_l \delta_{ll'} \delta_{mm'} \tag{2.25}$$

Now, the two-point function can be written as

$$C(\theta) = <\sum_{l,m,l',m'} a^*_{lm} a_{l'm'} Y_l^{*m}(\hat{n}_1) Y_{l'}^{m'}(\hat{n}_2)> \tag{2.26}$$

or

$$C(\theta) = \sum_{lm} C_l Y_l^{*m}(\hat{n}_1) Y_l^m(\hat{n}_2) \tag{2.27}$$

and finally due to the property

$$\sum_m Y_l^{*m}(\hat{n}_1) Y_l^m(\hat{n}_2) = \frac{2l+1}{4\pi} P_l(cos(\theta)) \tag{2.28}$$

the two point function $C(\theta)$ can be expressed in terms of the power spectrum $C_l$:

$$C(\theta) = \frac{1}{4\pi} \sum_l C_l (2l+1) P_l(cos(\theta)) \tag{2.29}$$

The power spectrum $C_l$ contains the information for temperature fluctuations in the $l$-space and is of significant importance in the study of CMB anisotropies. A multipole moment $l$ corresponds to an angle $\theta$ on the night sky which is approximately $\theta \sim \pi/l$.

From CMB observations cosmologists plot the power spectrum versus the multipole. These plots contain information for the cosmological parameters

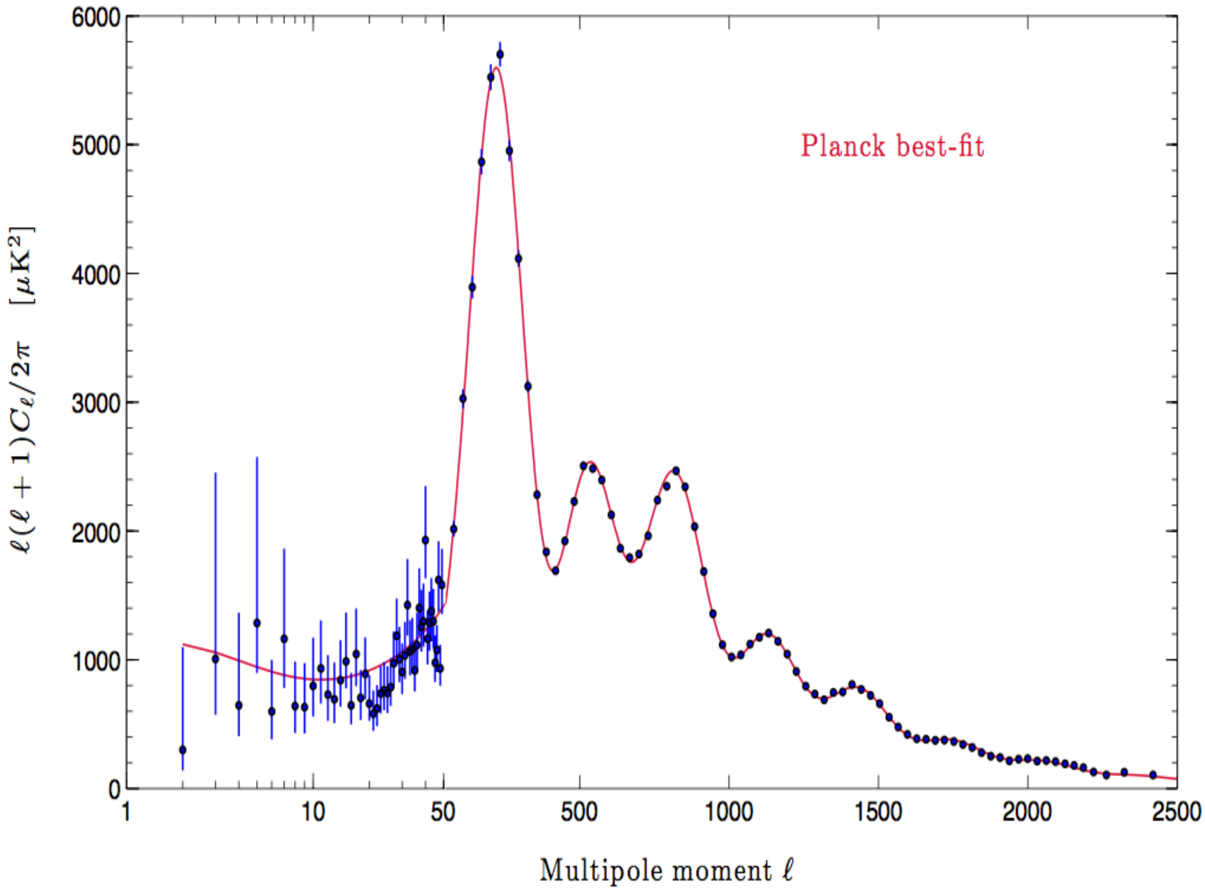


Figure 2.1: [5] Typical "shape" of a temperature anisotropy diagram

and are also used to compare different models. Such a diagram pictures a curve similar to the figure 2.1.

This curve shows how temperature anisotropies change in different angular scales. The oscillating part verifies the so-called baryon acoustic oscillations; the sound waves caused by compressions (due to baryonic matter) and refractions (due to radiation pressure) at recombination.

For smaller multipole ($l << 200$) there is a plateau. This corresponds to regions outside the Hubble radius (not causally connected regions). A major prediction of inflationary models is that the primordial power spectrum is scale invariant. This kind of primordial spectrum produces temperature anisotropies such that[3]

$$l(l+1)C_l \simeq constant \quad , \quad l << 200 \tag{2.30}$$

in agreement with the figure 2.1. It should be mentioned that positions and relative heights of the peaks depends strongly on cosmological parameters. The first peak is sensitive to the geometry of the universe ($\Omega_{tot}$), while the positions of higher order peaks change with $\Omega_c h^2, \Omega_b h^2$ and Hubble constant. Also, the relative heights change with $\Omega_b h^2$ and $n_s$[3].

Cosmological models of the early universe must be able to reproduce a similar temperature anisotropy diagram in order to fit with observational data under a suitable selection of parameters of the model.

# Chapter 3

# Holographic Cosmology

In this chapter we discuss holographic cosmology. First, a basic background in gauge/gravity duality is needed. We refer to $AdS$/CFT correspondence as it is a "prototype" example in holography. Then we turn into cosmology. Specifically, we focus on Skenderis' works in this direction ([11][14][16][17][18][19]) and we are willing to understand his main result for cosmological power spectrum. Finally, the new holographic model of cosmology is presented.

## 3.1 Motivations

Inflation theory is capable of explaining the CMB temperature anisotropies, predicting a nearly scale invariant primordial power spectrum and is the dominant theory for the early universe. In the context of inflation, gravity is assumed weakly coupled and a geometric description is justified. Basically, inflation uses quantum mechanics in curved space time. Thus, it cannot apply to earlier times, when gravity is strong and cannot be described with perturbations. So, in order to have access to a description of the very early universe, a quantum theory of gravity is needed. According to gauge/gravity duality, the description of a bulk spacetime can be encoded to its boundary (i.e in one dimension less). Gravity can be described in terms of a "boundary QFT" and there is a strong/weak relation between the two; when gravity is strongly coupled, QFT is weakly coupled and can be treated with perturbation theory and vise versa.
It is then reasonable to investigate a holographic approach for the early universe. With this approach we have access to earlier times since the problem

is translated to weakly coupled QFT computations that are well-understood. Also, the success of such a model for the early universe, capable to explain cosmological observations, could be one more clue that holography is fundamental in nature.

## 3.2 Basics of Holography

The basic idea of gauge/gravity duality is that there is a correspondence between gravity (bulk) and a QFT in one dimension less (boundary) without gravity[6]. For a certain bulk space, there is a QFT "living" on the boundary. The correspondence that will be discussed below is the $AdS_5/\mathcal{N} = 4$ Super-Yang-Mills correspondence [7]. Boundary theories are $U(N)$ gauge theories in large $N$ limit. The effective coupling of those theories is expressed as

$$\lambda = g_{YM}^2 N \tag{3.1}$$

with $g_{YM}$ being the Yang-Mills coupling. The bulk has a characteristic curvature radius $R$ which is related to the effective coupling of the boundary theory as

$$\frac{R^4}{l_s^4} \sim (g_{YM}^2 N) \tag{3.2}$$

where $l_s$ is the string length. The above relation indicates the strong/weak relation between the two theories:

$$\frac{R}{l_s} >> 1 \quad weak \quad gravity \quad \Leftrightarrow g_{YM}^2 N >> 1 \quad strongly \quad coupled \quad QFT \tag{3.3}$$

$$\frac{R}{l_s} << 1 \quad strong \quad gravity \quad \Leftrightarrow g_{YM}^2 N << 1 \quad weakly \quad coupled \quad QFT \tag{3.4}$$

We assume bulk space of the form

$$ds^2 = a(z)^2(-dt^2 + d\vec{x}^2 + dz^2) \tag{3.5}$$

This metric is five-dimensional, with $z$ being a radial direction. The scale factor $a(z)$ can be thought of as a gravitational potential; a massive particle moves towards the minimization of its energy $E = ma(z)$. When in equilibrium, the particle moves in the remaining four dimensions (there are quantum excitations around the equilibrium point however). In general, the boundary corresponds to values of $z$ on which the scale factor diverges.

There are a few coordinate systems in which $AdS_5$ can be expressed. Different coordinates can highlight some of the symmetries of this space. The following selection makes obvious the scale invariance:

$$ds^2 = R^2 \frac{dx^2 + dz^2}{z^2} \tag{3.6}$$

This metric is invariant under dilations:

$$z \longrightarrow \lambda z \quad , \quad x \longrightarrow \lambda x \tag{3.7}$$

Due to the scale invariance on $x$, the boundary theory is also scale invariant. This means that the coupling $g^2_{YM}N$ does not depend on the energy scale.

The metric above has a scale factor $a(z)$

$$a(z) \sim \frac{1}{z} \tag{3.8}$$

and the boundary is at $z = 0$, where $a(z)$ diverges.

Due to the scale invariance, one can approach the boundary of $AdS$ by choosing $\lambda < 1$,$z \longrightarrow \lambda z$. But at the same time scales on the boundary also shrink as $x \longrightarrow \lambda x$. Thus, an important consequence of scale invariance is that, moving towards the boundary of $AdS$ corresponds to higher energies of the boundary QFT. This is illustrated in the figure 3.1

The characteristic size of a boundary QFT state corresponding to a particle (bulk field), shrinks as the particle moves towards the boundary.

A crucial relation between bulk/boundary is the field/operator relation.Bulk fields correspond to operators of the boundary QFT. The generating function

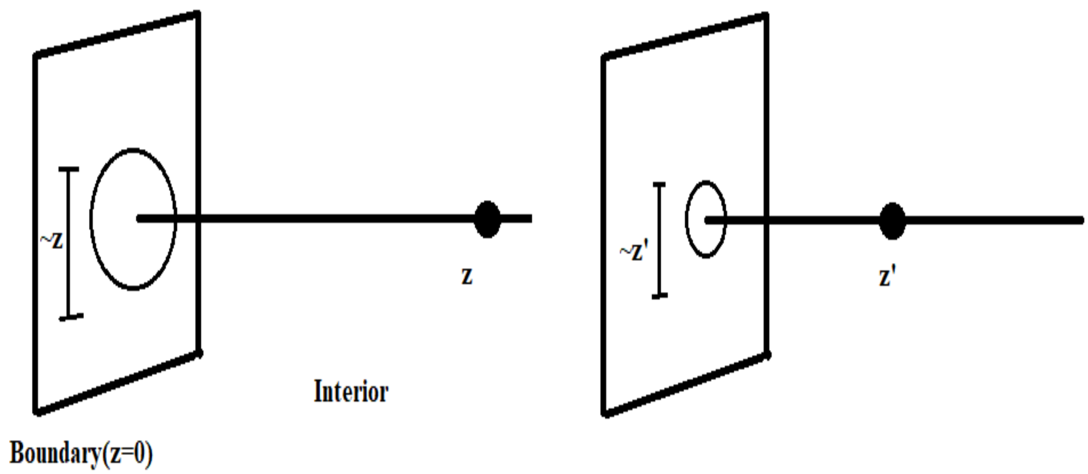


Figure 3.1:

for correlation functions of a boundary operator $O$ (with source $\phi_0$) is related to the partition function of gravity:

$$<e^{\int d^4x \phi_0(x) O(x)}>_{QFT} = \mathcal{Z}_{gravity}[\phi(x, z=0) = \phi_0(x)] \tag{3.9}$$

with the partition function being of the form

$$\mathcal{Z}_{gravity} \sim e^{-S[\phi]} \tag{3.10}$$

Differentiation with respect to $\phi_0$ constructs correlation functions of the boundary operator. The above relation is valid for any field $\phi$ [7]. It should be mentioned that two-point functions of an operator $O$ are related to the operator's scaling dimension $\Delta$ as

$$<O(x)O(y)> \sim \frac{1}{|x-y|^{2\Delta}} \tag{3.11}$$

Also, there is a connection between the mass of a bulk field and the scaling dimension $\Delta$ of its dual boundary operator $O$ [7] :

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 R^2} \tag{3.12}$$

where $R$ is the radius of curvature and $d$ is the boundary dimension of a $D$ dimensional $AdS_D$ ($D = d+1$). In our case of ($d = 4$) a massless bulk field, graviton in our case, is dual to an operator with scaling dimension $\Delta = 4$. This corresponds to the energy-momentum tensor, which has $\Delta = 4$ in four dimensions. Thus, the bulk metric is related to boundary energy-momentum tensor.

The above discussion was about the $AdS/CFT$. However, gauge/gravity can be applied to asymptotically $AdS$ or asymptotically power-law spaces [8]. These cases are important for holographic cosmology, since they map into realistic cosmological evolution (see next section).

Bulk fields can be expressed as a series expansion in the radial coordinate of $AdS$. In the case of asymptotically $AdS_{d+1}$, the metric can be expanded as [9]

$$ds^2 = \frac{l^2}{r^2}(dr^2 + g_{ij}(x,r)dx^i dx^j) \tag{3.13}$$

with the $g_{ij}$ of the following form

$$g_{ij}(x,r) = g_{(0)ij} + r^2 g_{(2)ij} + r^d g_{(d)ij} + ... \tag{3.14}$$

In order to make near-boundary analysis one have to consider an asymptotic expansion. In the above expansion $g_{(0)ij}$ is arbitrary d-dimensional metric, $g_{(k)ij}$ are locally determined by $g_{(0)ij}$ for $k < d$ and $g_{(d)ij}$ the first term that is not constrained by $g_{(0)ij}$. Also, $l$ is a parameter that depends on the dimension $d$. The coefficient $g_{(d)ij}$ is the only normalizeble term and affects dynamics [10]. This comes from a procedure called Holographic Renormalization; roughly speaking, one can find the renormalized action $S_{ren}$ by adding counter terms that "destroy" divergences of initial action. This procedure take place close to the boundary and is purpose is to solve IR divergence of the bulk theory. Specifically, the boundary energy-momentum tensor is related to $g_{(d)ij}$, for odd number of dimensions $d$ where there are no conformal anomalies[9], as

$$<T_{ij}> = \frac{l^{d-1}d}{16\pi G_N} g_{(d)ij} \tag{3.15}$$

where $l$ is constant and depends on dimension $d$.

One last thing to mention is the so-called dilatation operator $\delta_D$. An eigenfunction $A_{(n)}$ of weight $n$ is by definition [11]

$$\delta_D A_{(n)} = -n A_{(n)} \tag{3.16}$$

The eigenvalue of dilatation operator refers to scaling dimension. Roughly speaking, an operator $O$ of scaling dimension $\Delta$ is eigenfunction of the dilatation operator, $\delta_D O = -\Delta O$. Thus, we can think of $A_{(n)}$ as the part of $A$ that has scaling dimension $n$.

## 3.3 Domain-wall / cosmology correspondence

In order to have a holographic approach to cosmology there must be a connection between FLRW spacetime and a "bulk" space in which the ideas of gauge/gravity duality can be applied. This connection is made by the domain-wall cosmology correspondence [12][13]. An important statement is that there is a correspondence between domain-wall solutions, for a Lagrangian evolving a scalar field,

$$\mathcal{L} = \sqrt{-g}[R - \frac{1}{2}|\partial\Phi|^2 - V(\Phi)] \tag{3.17}$$

and cosmology FLRW solutions of the same Lagrangian but with an opposite sign potential $-V(\Phi)$ and vise-versa [12][13]

Here we consider four-dimensional universe and spatially flat. The "bulk" of the domain-wall can be written in our case as

$$ds^2 = dz^2 + e^{2\beta\varphi}[-d\tau^2 + \tau^2 d\Omega_+^2] \tag{3.18}$$

where $\varphi = \varphi(z)$ is a scale function, $\beta$ is a constant (which depends in the dimension of space D, here D=4) and $d\Omega_+^2$ is defined as

$$d\Omega_+^2 = d\tilde{\theta}^2 + sinh^2\tilde{\theta}d\phi^2 \tag{3.19}$$

On the side of cosmology, the FLRW metric is

$$ds^2 = -dt^2 + e^{2\beta\phi}[dr^2 + r^2 d\Omega_-^2] \tag{3.20}$$

where $\phi = \phi(t)$ and $d\Omega_-^2$ is the usual (here $\phi$ is angle coordinate, not to be confused with $\phi(t)$)

$$d\Omega_-^2 = d\theta^2 + sin^2\theta d\phi^2 \tag{3.21}$$

The domain-wall metric can be transformed into FLRW with the following change of coordinates:

$$(z, \tau, \tilde{\theta}) \longrightarrow i(t, r, \theta) \tag{3.22}$$

Domain-wall metric changes to

$$ds^2 = -dt^2 + e^{2\beta\varphi(it)}[dr^2 - r^2(-d\theta^2 - sin^2\theta d\phi^2)] \tag{3.23}$$

which is the FLRW metric with the definition

$$\phi(t) = \varphi(it) \tag{3.24}$$

We have seen that domain-wall can be mapped to cosmological spacetime. In our study, the two metrics will be written in a more compact form. If we analytically continue the $\tau$ coordinate of the domain-wall metric, the domain-wall becomes euclidean. Then the two metrics and the scalar filed can be expressed as [11][14]

$$ds^2 = \eta dz^2 + a^2(z)d\vec{x}^2 \quad , \quad \Phi = \varphi(z) \tag{3.25}$$

where $\eta = 1$ corresponds to domain-wall and $\eta = -1$ corresponds to cosmology. In the first case, $z$ is a radial direction while in the seconds $z$ is the time coordinate. The action, assuming a single scalar field $\Phi$, is

$$S = \frac{\eta}{2\kappa^2}\int d^4\sqrt{|g|}[-R + g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + 2\kappa^2 V(\Phi)] \tag{3.26}$$

where the scalar field $\Phi$ is taken dimensionless and $\kappa^2 = 8\pi G_N$.

The correspondence is also valid for linear perturbations. Cosmological perturbations map to domain-wall perturbations if we analytically continue the constant $\kappa$ and the co-moving wave-vector $q$ as [11]

$$\bar{\kappa} = -\kappa^2 \quad , \quad \bar{q} = -iq \tag{3.27}$$

The perturbed metric and the scalar field, using the same notation as in [11][14], are :

$$ds^2 = \eta dz^2 + a^2(z)[\delta_{ij} + h_{ij}]dx^i dx^j \tag{3.28}$$

$$h_{ij} = [-2\psi\delta_{ij} + 2\partial_i\partial_j\chi + \gamma_{ij}] \quad , \quad \Phi = \varphi(z) + \delta\varphi(z, \vec{x}) \tag{3.29}$$

The above perturbed metric does not include vector perturbations since they are not important in the case of cosmology, since they decay very quickly[3]. In this gauge there are no $dzdx^i$ component. The variables $\psi$ and $\chi$ are scalars and tensor perturbations $\gamma_{ij}$ are traceless and transverse

$$\gamma^i_i = 0 \quad , \quad \gamma^i_{j,i} = 0 \tag{3.30}$$

In [11] the curvature perturbation $\zeta$ is similar to $v$ of the Appendix; a gauge invariant combination of metric perturbations and scalar field perturbations

$$\zeta = \psi + \frac{H}{\varphi'}\delta\varphi \tag{3.31}$$

The power spectrum is expressed in terms of the Fourier modes $\zeta_q$ and $\gamma_q$ ($\gamma_{ij} = \gamma_q \hat{e}_{ij}$) for scalar and tensor perturbations respectively. In Appendix, quantization of scalar fluctuations is discussed and the procedure for tensor power spectrum is conceptually the same. It is useful to include tensor power spectrum in order to have the prediction for the so-called tensor to scalar ratio $r$ in the context of holographic cosmology. Using Wronskian conditions of[11][14], which are the analogue of those derived in Appendix (relation 5.22), the power spectra are written in the form [11][14]

$$\Delta_R^2(q) = \frac{-q^3}{4\pi^2 Im[\Omega_{(0)}(q)]} \quad , \quad \Delta_T^2(q) = \frac{-q^3}{2\pi^2 Im[E_{(0)}(q)]} \tag{3.32}$$

where $\Omega$ and $E$ are linear response functions defined by the canonical momentum variables of scalar and tensor modes respectively[11][14]

$$\Pi_q^{(\zeta)} = \Omega\zeta_q \quad , \quad \Pi_q^{(\gamma)} = E\gamma_q \tag{3.33}$$

with $\Pi_q^{(\zeta)} \sim \zeta_q'$ and $\Pi_q^{(\gamma)} \sim \gamma_q'$ being the canonical momentum for scalar and tensor mode respectively.

The index (0) in the relations of power spectra indicates late-time values. Response functions defined above are related with domain-wall response functions $\bar{\Omega}$ and $\bar{E}$ as

$$\Omega(q) = \bar{\Omega}(-iq) \quad , \quad E(q) = \bar{E}(-iq) \tag{3.34}$$

The response functions $\bar{\Omega}$ and $\bar{E}$ will be expressed in terms of the dual QFT variables later on. Then, going back to $E$ and $\Omega$ will give the desired power spectra.

The purpose of holographic cosmology is to reproduce inflationary predictions while having a better description for earlier times when perturbations cannot be justified. Thus, at later times the domain-wall will be mapped to an inflationary era. Of course, after inflation we assume a Hot Big Bang cosmology era. There are two kinds of domain-walls that are mapped to realistic cosmological evolution. The first is asymptotically $AdS$ and maps to $dS$ inflation on the cosmology side. In this case the scale factor and scalar field at late times, as $z$ goes to infinity, are of the form [11][14]

$$a(z) \sim e^z \quad , \quad \varphi \sim 0 \quad , \quad z \longrightarrow \infty \tag{3.35}$$

Note that $z$ is not the same as in section 2.2. The two radial directions are related as $z_{new} \sim -lnz_{old}$ and for the new coordinate system the boundary is at $z_{new} \longrightarrow \infty$. Also the old metric becomes $\sim e^{z_{new}}(dx^2 + e^{-z_{new}} dz_{new}^2)$ and the mapping to dS cosmology is obvious.

The second is asymptotically power-law which at late times maps to power-law inflation. The scale factor and scalar field for this case are:

$$a(z) \sim (\frac{z}{z_0})^n \quad , \quad \varphi \sim \sqrt{2n} log(\frac{z}{z_0}) \quad , \quad z \longrightarrow \infty \tag{3.36}$$

In both cases, the boundary of the domain-wall corresponds to $z \longrightarrow \infty$ where the scale factor diverges. Approaching the boundary is equivalent to later times evolution in cosmological side. For these times, the dual QFT is strongly coupled. On the other hand, gravity is weakly coupled on the bulk of the domain-wall and the perturbed metric can be used to obtain the power spectrum. Moving away from the boundary is mapped to earlier times behaviour for cosmology and the dual QFT becomes weakly coupled. However, dual-QFT is weakly coupled for early times only in a regime where the effective coupling is small. Figure 3.2 illustrates the steps of holographic cosmology. The left vertical line is the domain-wall/cosmology correspondence where cosmology spacetime maps to domain-wall. The upper line shows that computations in the domain-wall are related to its boundary dual-QFT. Lastly, right vertical line indicates the final step of analytic continuation from which, the computations that refer to domain-wall can be mapped back to cosmology. Note that pseudo-QFT is not an actual QFT.

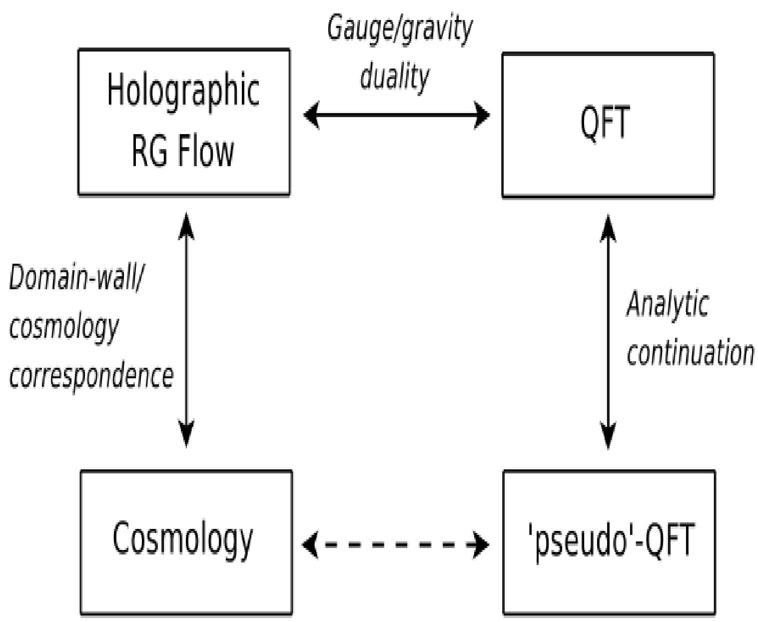


Figure 3.2: [14]: The figure illustrates the basic steps of holographic cosmology.

## 3.4 Application to cosmology

This section consists of two parts. In the first part we show how cosmological power spectrum can be expressed in terms of dual-QFT variables. For strongly coupled gravity at early times, one can rely on perturbative dual-QFT computations to extract those variables and eventually power spectrum. In the second part, we present Skenderis' holographic cosmological model.

### 3.4.1 The Basic Result

First the bulk/boundary correspondence must me discussed in the cases of asymptotically $AdS$ and asymptotically power-law domain-walls. It has been already mentioned in section 3.2 that in the case of asymptotically $AdS$, the boundary energy momentum tensor is related to a specific coefficient of asymptotic expansion. Here, we have also the case of asymptotically power-law and there is an asymptotic expansion in a form that includes both cases, as will be mentioned later. In this expansion the radial derivative is related to dilatation operator as :

$$\partial_r = \delta_D(1 + O[e^{-2r}]) \quad , \quad r \longrightarrow \infty \tag{3.37}$$

which indicates the relation between radial direction and the energy scale of the dual QFT mentioned in section 3.2.

It turned out that the main result for the power spectrum is the same for the case of asymptotically $AdS$ and asymptotically power-law[11][14]. For that reason, we assume asymptotically $AdS$ to obtain this result.

In the computations of the power spectrum a more useful form of the bulk/boundary relation is [11] :

$$<T_{ij}> = (\frac{-2}{\sqrt{|g|}}\bar{\Pi}_{ij})_{(3)} \tag{3.38}$$

In the equation above, $\bar{\Pi}_{ij}$ is the radial canonical momentum, which is defined for the asymptotically $AdS$ case as [11][15] :

$$\bar{\Pi}_{ij} = \frac{1}{2\bar{\kappa}^2}\sqrt{g}[K_{ij} - Kg_{ij}] \tag{3.39}$$

where $K_{ij}$ is the "extrinsic curvature" defined as

$$K_{ij} = \frac{1}{2}\partial_r g_{ij} \quad , \quad K = g^{ij} K_{ij} \tag{3.40}$$

Subscript (3) indicates that we take the part of $\bar{\Pi}_{ij}$ with dilatation weight $n = 3$. The presence of subscript (3) refers to the relation $<T_{ij}> \sim g_{(3)ij}$. Thus, the subscript is needed in order to obtain the correct energy-momentum tensor, which has scaling dimension 3 in three dimensions.

This is important in order to relate boundary QFT variables with the power spectrum. Specifically, the two-point function of energy momentum tensor, in momentum space, is written in the general form

$$<T_{ij}(\bar{q})T_{kl}(-\bar{q})> = A(\bar{q})\Pi_{ijkl} + B(\bar{q})\pi_{ij}\pi_{kl} \tag{3.41}$$

with $\Pi_{ijkl}$ being traceless and transverse projector and $\pi_{ij}$ transverse projector, defined as

$$\Pi_{ijkl} = \frac{1}{2}[\pi_{ik}\pi_{lj} + \pi_{il}\pi_{kj} - \pi_{ij}\pi_{kl}] \quad , \quad \pi_{ij} = \delta_{ij} - \frac{\bar{q}_i \bar{q}_j}{\bar{q}^2} \tag{3.42}$$

The coefficients $A$ and $B$ are variables of the boundary QFT. The power spectra can be expressed in terms of those two variables and this is the basic result of holographic cosmology.

Specifically, when gravity is weakly coupled, the variation of (1-point function) energy-momentum tensor $\delta<T_{ij}>$ can be constructed from the bulk through $\bar{\Pi}_{ij}$ due to relation (3.38). The result is (derivation in Appendix) :

$$\delta<T^i_j> = [\frac{2\bar{E}}{a^3}\gamma^i_j - \frac{\bar{q}^2}{\bar{k}^2 a^2 H}\psi\pi^i_j - \frac{\bar{\Omega}}{2a^3}\psi\pi^i_j - (\frac{H\bar{\Omega}}{2a^3\varphi'}\pi^i_j + \frac{\varphi'\delta^i_j}{\bar{k}^2})\delta\varphi]_{(3)} \tag{3.43}$$

For the derivation of the above equation, we used the perturbed metric

$$ds^2 = dr^2 + a^2[g_{ij} + h_{ij}]dx^i dx^j \tag{3.44}$$

This is the asymptotically $AdS$ perturbed to linear order, where $h_{ij}$ is the same as in (3.29). The three-dimensional metric $g_{ij}$ of this asymptotic expansion that we assume here is given by ((21) of [11]), namely:

$$g_{ij}(r,x) = e^{2r}[g_{(0)ij}(x) + e^{-2r}g_{(2)ij}(x) + ... + e^{-2\sigma r}g_{(2\sigma)}(x)] \tag{3.45}$$

with $\sigma = \frac{3}{2}$ for asymptotically $AdS$ and $\sigma = \frac{3n-1}{2(n-1)} > \frac{3}{2}$ for asymptotically power-law.

Moving forward, the same quantity can be obtained in terms of the two-point function of energy-momentum tensor and thus in terms of the variables $A$ and $B$; $\delta{<}T_{ij}{>}$ from this "point of view" comes directly from the three-dimensional boundary QFT (relation (34) of [11]):

$$\delta{<}T_{ij}{>} = \frac{1}{2}A(\bar{q})\gamma_{(0)ij} - 2B(\bar{q})\psi_{(0)}\pi_{ij} - {<}T_{ij}(\bar{q})O(-\bar{q}){>}\delta\varphi_{(0)} \tag{3.46}$$

which is the variation of the one-point function of $T_{ij}$ in momentum space. The subscript (0) refers to $g_{(0)ij}$, which indicates that we are on the boundary of asymptotically $AdS$ domain-wall, where metric decomposes as in (3.44).

Now, the only thing left is to treat subscript (3) in (3.43). It is important at this point to recall something that is already mentioned in section 3.3, but here we can have the interpretation from another point of view. Power spectra are given by response functions $E_{(0)}(q)$ and $\Omega_0(q)$. The index refers to later time values of $E(q)$ and $\Omega(q)$. Here we have $\bar{E}(\bar{q})$ and $\bar{\Omega}(\bar{q})$ and a subscript (0) reminds that we are close to the boundary. In other words, indicates that $\bar{E}_{(0)}$ and $\bar{\Omega}_{(0)}$ are independent of $r$ as we approach the boundary, i.e for $r \longrightarrow \infty$. Having said that, it is clear that from relation (3.43) we want the parts with zero dilatation eigenvalue.

From the dilatation transformation rule

$$\delta_D g_{ij}(r,x) = 2g_{ij}(r,x) \tag{3.47}$$

and (3.44),(3.45), we conclude that the scale factor $a$ has dilatation eigenvalue $n = -1$. Also, recall that $[.....]_{(3)}$ is the part of $[.....]$ with dilatation eigenvalue $n = 3$. Now consider for example the first term of (3.43). Then, assume an expansion of $\bar{E}$ into dilatation eigenvalues, roughly:

$$\bar{E} = \bar{E}_{(0)} + \sum_{n\neq 0}\bar{E}_{(n)} \tag{3.48}$$

and same goes for $\gamma^i_j$. Thus, since the dilatation eigenvalue of $\frac{1}{a^3}$ is $n = 3$, a comparison with (3.46) gives:

$$2\bar{E}_{(0)}(\bar{q}) = \frac{1}{2}A(\bar{q}) \tag{3.49}$$

With the same reasoning we obtain $B(\bar{q})$ from the third term of (3.43). Note that the second term of (3.43) contributes as a contact term to $B(\bar{q})$ which we drop. Putting all together:

$$A(\bar{q}) = 4\bar{E}_{(0)}(\bar{q}) \quad , \quad B(\bar{q}) = \frac{1}{4}\bar{\Omega}_{(0)}(\bar{q}) \tag{3.50}$$

Note that the above result is the same for asymptotically power-law domain-wall[11][14]. In order to obtain power spectra from relation (3.35), we must analytically continue $\bar{q}$ as mentioned in section 3.3 as

$$\bar{q} = -iq \tag{3.51}$$

We end up with the final result:

$$\Delta^2_R(q) = \frac{-q^3}{16\pi^2 Im[B(-iq)]} \quad , \quad \Delta^2_T(q) = \frac{-2q^3}{\pi^2 Im[A(-iq)]} \tag{3.52}$$

We conclude that power spectra can be obtained from dual QFT variables. Specifically, $\Delta^2_R$ is related to the two-point function of the trace of energy-momentum tensor, while $\Delta^2_T$ is related to the traceless and transverse part. It is important to notice that the analytic continuation for $\bar{q}$ was crucial. The QFT that gives $A(\bar{q})$ and $B(\bar{q})$ corresponds to domain-wall. However, we assume a pseudo-QFT via analytic continuation in the original QFT, in order to return to cosmology. The validity of this approach may be tested from CMB observations.

Inflationary predictions for power spectra can be reproduced in the context of holographic cosmology. When gravity is weak, one can find equations for perturbations in the domain-wall and eventually compute response functions $\bar{E}(\bar{q})$ and $\bar{\Omega}(\bar{q})$. Then, after finding the parts with zero dilatation eigenvalue, an analytic continuation $\bar{q} = -iq$ will eventually give power spectrum.

The basic result of holographic cosmology for power spectrum, allows to examine even earlier times when gravity is stronger and thus out of inflation's regime of applicability. At this point, one has to rely on QFT computations to obtain $A(\bar{q})$ and $B(\bar{q})$ and eventually power spectrum. The computations are made in the dual to domain-wall QFT. Then, an analytic continuation is needed to reach pseudo-QFT. Dual QFT is assumed gauge $U(\bar{N})$ type. Thus, we must also preform analytic continuation to $\bar{N}$ because it is involved in computations. If we recall that

$$G_N \sim g_s \sim \frac{1}{\bar{N}^2} \tag{3.53}$$

and also

$$\bar{k}^2 = -k^2 \quad , \quad k^2 = 8\pi G_N \tag{3.54}$$

it turns out that the analytic continuation for $\bar{N}$ is

$$\bar{N}^2 = -N^2 \tag{3.55}$$

### 3.4.2 Towards a cosmological model

Skenderis has made phenomenological dual-QFT selection involving scalar, conformal scalar, gauge and fermion fields in order to test the resulting power spectrum [11][14][16][17].

The dual-QFT has "generalized conformal structure", corresponds to asymptotically power-law domain-wall and has the following form[19]

$$S = \frac{1}{g_{YM}^2} \int d^3x Tr[\frac{1}{2}(F_{ij}^I)^2 + \frac{1}{2}(\partial\varphi^J)^2 + \frac{1}{2}(\partial\chi^K)^2 + \bar{\psi}^L \partial\!\!\!/ \psi^L + Int \quad ] \tag{3.56}$$

with $A^I$ gauge fields ($I = 1, ..., \mathcal{N}_A$), $\varphi^J$ scalars ($J = 1, ..., \mathcal{N}_\varphi$), $\chi^K$ conformal scalar ($K = 1, ..., \mathcal{N}_\chi$) and $\psi^L$ fermions ($L = 1, ..., \mathcal{N}_\psi$). The dual-QFT is assumed $SU(\bar{N})$ and fields transform in the adjoint of $SU(\bar{N})$. Also, has IR divergences. The effective dimensionless coupling of the dual-QFT is

$$g_{eff}^2 = \frac{g_{YM}^2 \bar{N}}{\bar{q}} \tag{3.57}$$

and it is unchanged when we analytically continue ($\bar{q} = -iq$,$\bar{N} = -iN$). Note that for $\bar{q} \longrightarrow 0$ QFT is strongly coupled. Thus, the resulting power spectrum would be well-defined for scales $q$, or multipole $l$ on the CMB temperature anisotropies, that allow a small enough $g_{eff}^2$.

The computations for 1-loop and 2-loop corrections can be found in [11][16]. Here we discuss the main results.

The results for $B(\bar{q})$ and $A(\bar{q})$ from the two-point function of energy-momentum tensor computations are

$$B(\bar{q},\bar{N}) = \frac{1}{4}\bar{q}^3\bar{N}^2 f_R(g^2_{eff}) \quad , \quad A(\bar{q},\bar{N}) = \bar{q}^3\bar{N}^2 f_T(g^2_{eff}) \tag{3.58}$$

with the factor 1/4 in $B$ being conventional. Analytic continuation leads to the following result for power spectra:

$$\Delta^2_R(q) = \frac{1}{4\pi^2 N^2 f_R(g^2_{eff})} \quad , \quad \Delta^2_T(q) = \frac{2}{\pi^2 N^2 f_T(g^2_{eff})} \tag{3.59}$$

Functions $f_R(g^2_{eff})$ and $f_T(g^2_{eff})$ up to 2-loop corrections are:

$$f_R(g^2_{eff}) = f_{R0}(1 - f_{R1} g^2_{eff} ln g^2_{eff} + f_{R2} g^2_{eff}) \tag{3.60}$$

and

$$f_T(g^2_{eff}) = f_{T0}(1 - f_{T1} g^2_{eff} ln g^2_{eff} + f_{T2} g^2_{eff}) \tag{3.61}$$

The first term comes from one-loop and the rests come from two-loop corrections with $f$ being numerical coefficients depending on the dual-QFT and specifically $f_0$ depending on the number of fields [11][14]. There are divergences coming from two-loop contributions and the solution/details are discussed in [16]. However, for a significant large range of multipole $l$ the dual-QFT is perturbative. It should be mentioned that $f_0$ and $f_1$ are determined form one-loop and two-loop computations, while $f_2$ is determined after a cut-off selection that solves IR divergences[18].

Scalar and tensor power spectra are scale invariant up to one-loop correction:

$$\Delta^2_{R0} = \frac{1}{4\pi^2 N^2 f_{R0}(g^2_{eff})} \quad , \quad \Delta^2_{T0} = \frac{2}{\pi^2 N^2 f_{T0}(g^2_{eff})} \tag{3.62}$$

As pointed in [11][14], the WMAP data in [1] estimates an amplitude $\Delta^2_R \sim 10^9$. Thus, $N \sim 10^4$ and large $N$ limit is justified by observations. Thus, for one-loop corrections tensor to scalar ratio $r$ is

$$r = \frac{8f_{R0}}{f_{T0}} \tag{3.63}$$

and observations add restrictions on dual-QFT content due to the requirement $r < 0.22$[1]. Using this constrain for $r$ and also the exact result of one-loop computation given in [11], we have the following constrain for dual-QFT:

$$\frac{\mathcal{N}_A + \mathcal{N}_\phi}{\mathcal{N}_A + \mathcal{N}_\phi + \mathcal{N}_\chi + 2\mathcal{N}_\psi} < 0.0275 \tag{3.64}$$

where $\mathcal{N}_A$, $\mathcal{N}_\phi$, $\mathcal{N}_\chi$, $\mathcal{N}_\psi$ are the number of gauge, scalar, conformal scalar and fermionic fields of the dual-QFT respectively.

The next correction leads to the nearly scale invariant power spectrum. Using the result for $f_R$ and $f_T$ up to two-loop corrections, power spectra are expressed as:

$$\Delta_R^2(q) = (\frac{1}{4\pi N^2 f_{R0}})\frac{1}{1 - f_{R1} g_{eff}^2 ln g_{eff}^2 + f_{R2} g_{eff}^2} \tag{3.65}$$

$$\Delta_T^2(q) = (\frac{1}{4\pi N^2 f_{T0}})\frac{1}{1 - f_{T1} g_{eff}^2 ln g_{eff}^2 + f_{T2} g_{eff}^2} \tag{3.66}$$

The two-loop correction for the two-point function of stress-energy tensor is of the order[18]

$$N^2 q^3 g_{eff}^2 ln(q/q_{IR}) \tag{3.67}$$

where $q_{IR}$ is selected in order to solve the IR divergence. Then, power spectrum can be expressed, up to two-loop corrections, as

$$\Delta_R^2 = \frac{\Delta_0^2}{1 + \lambda_B g_{eff}^2 ln(q/q_{IR}) + \mathcal{O}(g_{eff}^4)} \tag{3.68}$$

where $\lambda_B$ is numerical coefficient of order one. A comparison with the standard parametrization of power spectrum gives[11]

$$n_s - 1 = -\lambda_B g_{eff}^2 + \mathcal{O}(g_{eff}^4) \quad , \quad a_s/(n_s - 1) \sim \mathcal{O}(g_{eff}^2) \tag{3.69}$$

for spectral and running index respectively. Since $n_s \approx 0.96$ at pivot scale $q_0 = 0.002 Mpc^{-1}$[1], then $g_{eff}^2 \sim \mathcal{O}(10^{-2})$ in agreement with the assumption of weakly coupled dual-QFT. Standard parametrization is not possible for all scales and we stick to the initial result.

Following [17][18], the holographic cosmological model contains the following parameters: $[\Omega_b h^2, \Omega_c h^2, \theta, \tau, \Delta_0^2, g, ln\beta]$. The first five are the same as in standard cosmology (section 2.1). Here, the spectral index $n_s$ is replaced by $g$ and $ln\beta$. These new parameters are defined as

$$gq_* = f_{R1} g_{YM}^2 N \quad , \quad ln\beta = -(\frac{f_{R2}}{f_{R1}} + ln|f_{R1}|) \tag{3.70}$$

where $q_*$ is an arbitrary scale. Now power spectrum parametrized as

$$\Delta_R^2(q) = \frac{\Delta_0^2}{1 + (gq_*/q)ln|q/\beta gq_*|} \tag{3.71}$$

with the amplitude parameter $\Delta_0^2$ being

$$\Delta_0^2 = \frac{1}{4\pi N^2 f_{R0}} \tag{3.72}$$

Also, tensor power spectrum can be expressed in terms of new parameters $ln\beta_t$,$g_t$[17]

$$g_t q_* = f_{T1} g_{YM}^2 N \quad , \quad ln\beta_t = -(\frac{f_{T2}}{f_{T1}} + ln|f_{T1}|) \tag{3.73}$$

In terms of the new variables, tensor power spectrum becomes

$$\Delta_T^2(q) = \frac{\Delta_{T0}^2}{1 + (g_t q_*/q)ln|q/\beta_t g_t q_*|} \tag{3.74}$$

with amplitude $\Delta_{T0}^2$

$$\Delta_{T0}^2 = \frac{1}{4\pi N^2 f_{T0}} \tag{3.75}$$

Note that in this model, only inflation has been replaced. The Hot Big Bang era follows after ("holographic") initial conditions, as in standard cosmology.

Also, we notice that

$$\frac{gq_*}{q} \sim g_{eff}^2 \tag{3.76}$$

so in order to have knowledge about the validity of perturbative QFT computations, the quantity $gq_*/q$ must be small enough:

$$gq_*/q << 1 \tag{3.77}$$

For large enough $q$ the power spectrum is nearly-scale invariant and the sign of $g$ must be negative in order to agree with a red-titled spectrum. A negative sign can be fixed from the dual-QFT parameters.

Smaller $q$ corresponds to larger angular scales on CMB which is translated into lower multipole $l$. Thus, we can trust holographic model for large enough multipole. This number is about $l \sim 30$ [16][17].

Finding best fit parameters leads to the usual plot of temperature anisotropies mentioned in section 2.4. If we ignore very low multipole $l$, holographic cosmology is compatible with $\Lambda$CDM. In order to have results for lower multipole a non-perturbative QFT computation is needed. This computation could potentially explain the low-$l$ temperature anisotropies.

In figure 3.3 lower multipole are excluded. We can see that holographic cosmological model is compatible for large enough multipole $l$. Note that the first peak corresponds to $l \approx 200$. We must also note that holographic model is a cosmological model with seven parameters. It requires one more parameter than standard $\Lambda$CDM, although there is a version of $\Lambda$CDM that includes running index as an additional parameter as noted in section 2.1.

In addition, for the same best fit parameters, figure 3.4 shows scalar power spectrum for holographic cosmology compared to $\Lambda$CDM.

If we exclude very small $q$, then holographic model is compatible with $\Lambda$CDM in its regime of applicability.

It is important to highlight that the dual-QFT has "generalized conformal structure" and these QFTs are dual to asymptotically power-law domain-walls which for cosmology is[19]

$$ds^2 = -dt^2 + t^{2n} d\vec{x}^2 \quad , \quad , n > 1 \tag{3.78}$$

According to holographic cosmology the universe is under a power-law acceleration at later times, after "holographic" initial conditions, so a transition era to radiation epoch, where $a \sim \sqrt{t}$, is needed. Also, predictions of power-law inflation are different from those of holographic cosmology[11][19]. Indeed holography can be used to obtain the same results as power-law inflation. However, holographic cosmology refers to earlier times when gravity is strongly coupled and cosmological results are obtained via QFT computations. Thus, observational restrictions of power-law inflation do not affect holographic cosmology.

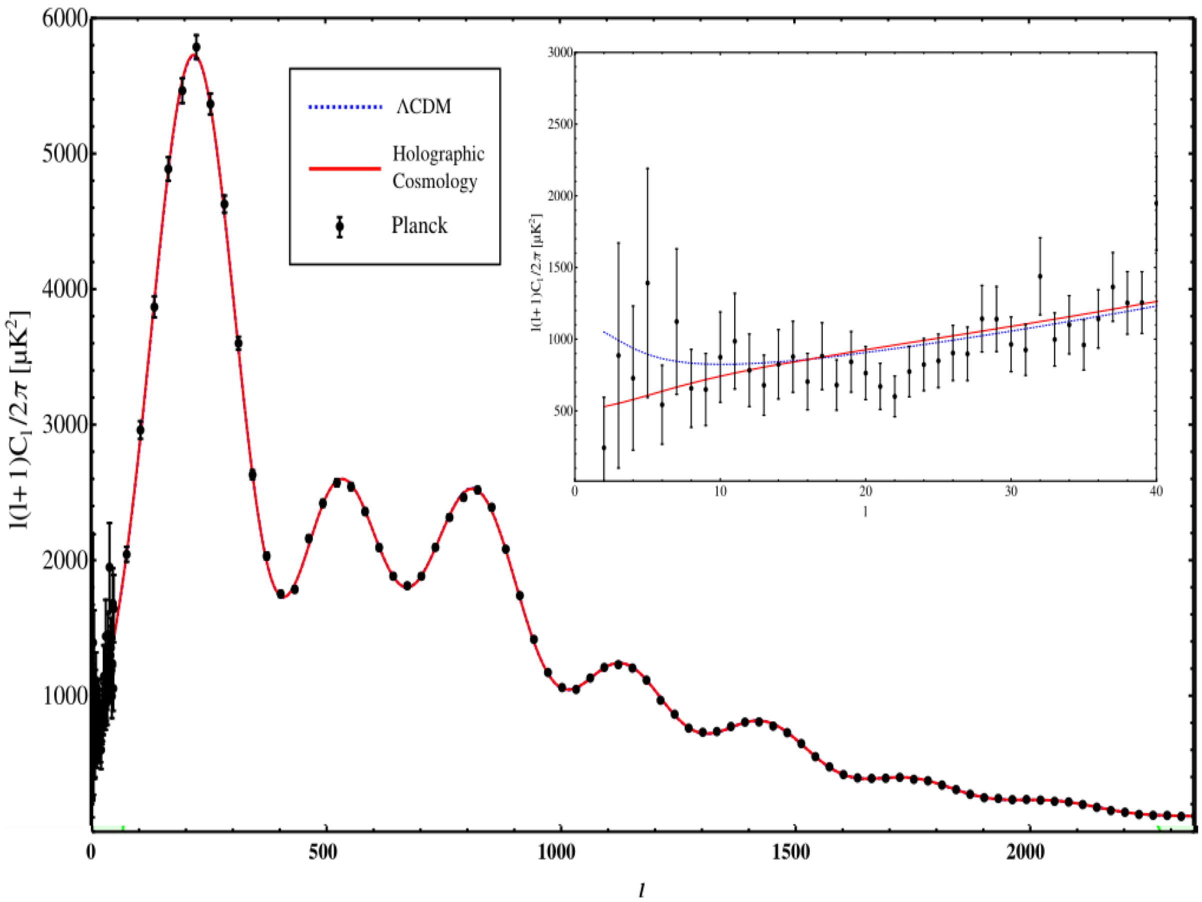


Figure 3.3: [17] Temperature anisotropies for the best fit parameters.

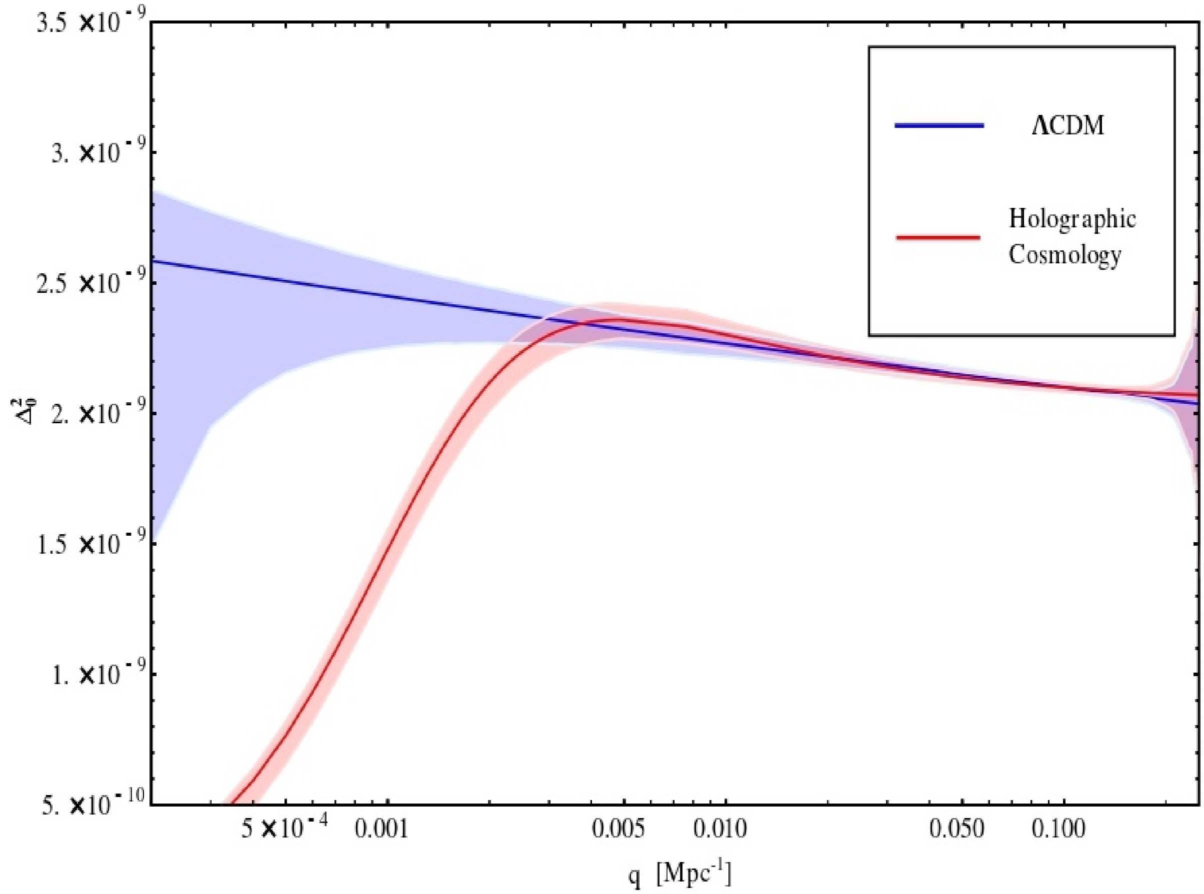


Figure 3.4: [17] power spectrum for HC and ΛCDM. The shaded regions denote error.

# Chapter 4

# Conclusion

In this report, holographic cosmology of Skenderis and collaborators is discussed. We begun with the standard cosmological model in order to have intuition for cosmological observations. Especially power spectrum and CMB temperature anisotropies are discussed. Also, we derived primordial power spectrum in the context of a simple inflationary model. To obtain primordial spectrum, we made a quantization of cosmological perturbations. Moving forward, we saw the bulk/boundary correspondence, specifically the connection between stress-tensor and coefficient of the asymptotically expanded bulk metric. Using domain-wall/cosmology correspondence it is possible to obtain cosmological results through boundary QFT computations. Eventually, the resulting holographic cosmological model is the Hot Big Bang model with initial conditions given by holographic computations. In its regime of applicability, i.e when QFT is perturbative, is in agreement with observations and $\Lambda$CDM.

# Chapter 5

# Appendix

## 5.1 Quantization of perturbations

In this section, we present a quantization procedure of cosmological perturbations, in order to obtain the power spectrum in the context of inflation. Also, the metric perturbations are assumed in the Newtonian gauge.

The following procedure concerns scalar metric perturbation induced by a scalar field. The action for the scalar field for this case is:

$$S_{scalar} = \int d^4x\sqrt{-g}[\frac{1}{2}g^{ab}\phi_{,a}\phi_{,b} - V(\phi)] \tag{5.1}$$

where the unperturbed scalar filed depends only on time while its perturbation is spacetime dependent:

$$\phi = \phi(\eta) + \delta\phi(\eta, \vec{x}) \tag{5.2}$$

First, there must be an action from which we can obtain the equations of motion for the metric perturbations. The action of perturbations is given by ([3] p.341)

$$S = \frac{1}{2}\int d\eta d^3x(v'^2 + v\nabla^2 v + v^2\frac{z''}{z}) \tag{5.3}$$

In the above action, $\eta$ is the conformal time defined as

$$\eta = \int \frac{dt}{a(t)} \tag{5.4}$$

the variable $v$ is a gauge invariant combination of the metric perturbation $\Psi$ and the perturbed scalar filed $\delta\phi$ and $z$ depends on the unperturbed scalar field:

$$v = a(\delta\phi + \frac{\phi'}{\mathcal{H}}\Psi) \tag{5.5}$$

where $\mathcal{H}$ is the Hubble rate expressed in the conformal time and $z \equiv \frac{a\varphi'}{\mathcal{H}}$. Variation with respect to $v$ will give the equation of motion:

$$\delta S = \frac{1}{2}\int d\eta d^3x[2v'\delta v' + (\nabla^2 v + 2v\frac{z''}{z})\delta v + v\nabla^2\delta v] \tag{5.6}$$

the terms become:

- $$\int v'\delta v' d\eta d^3x = [v'\delta v]_{boundary} - \int v''\delta v d\eta d^3x = -\int v''\delta v d\eta d^3x \tag{5.7}$$

- $$\int v\nabla^2\delta v d\eta d^3x = \int v\vec{\nabla}(\vec{\nabla}\delta v) d\eta d^3x = \int_{bnoundary} v\vec{\nabla}\delta v d\vec{S} d\eta - \int \vec{\nabla}v\vec{\nabla}\delta v d\eta d^3x \tag{5.8}$$

$$= -\int \vec{\nabla}v\vec{\nabla}\delta v d\eta d^3x = \int \nabla^2\delta v d\eta d^3x \tag{5.9}$$

Putting all together:

$$\delta S = \int d\eta d^3x(-v'' + \nabla^2 v + v\frac{z''}{z})\delta v \tag{5.10}$$

Thus, the equation of motion is

$$v'' - \nabla^2 v - \frac{z''}{z}v = 0 \tag{5.11}$$

The canonical quantization variable is $v$, with the corresponding canonical momentum given by

$$p = \frac{\partial\mathcal{L}}{\partial v'} = v' \tag{5.12}$$

with the standard commutation relations

$$[v, v] = 0 = [p, p] \tag{5.13}$$

and

$$[v(\eta, \vec{x}), p(\eta, \vec{x}')] = i\delta(\vec{x} - \vec{x}') \tag{5.14}$$

The equation of motion suggests a solution of the form

$$v(\eta, \vec{x}) = \frac{1}{\sqrt{2}} \int \frac{d^3k}{(2\pi)^{3/2}} [v_{\vec{k}}(\eta) e^{-i\vec{k}\vec{x}} a^{\dagger}_{\vec{k}} + v^*_{\vec{k}}(\eta) e^{i\vec{k}\vec{x}} a_{\vec{k}}] \tag{5.15}$$

in which $a^{\dagger}$,$a$ are creation and annihilation operators with the usual commutation relations

$$[a, a] = 0 = [a^{\dagger}, a^{\dagger}] \tag{5.16}$$

$$[a_{\vec{k}}, a^{\dagger}_{\vec{k}'}] = \delta(\vec{k} - \vec{k}') \tag{5.17}$$

with the vacuum state being a state $|0\rangle$ such that

$$a_{\vec{k}} |0\rangle = 0 \tag{5.18}$$

Thus, the metric perturbation $\Phi$ can be expressed in a similar way. An important relation is the "Wronskian condition", which comes from the fact that the commutation relations for creation/annihilation operators must be consistent with that of the canonical variables . That is

$$[v(\eta, \vec{(x)}), v'(\eta, y)] = \frac{1}{2} \int \frac{d^3k d^3k'}{(2\pi)^3} [v^*_{\vec{k}} e^{i\vec{k}\vec{x}} a_{\vec{k}} + v_{\vec{k}} e^{-i\vec{k}\vec{x}} a^{\dagger}_{\vec{k}} \quad , \quad v'^*_{\vec{k}'} e^{i\vec{k}'\vec{y}} a_{\vec{k}'} + v'_{\vec{k}'} e^{-i\vec{k}'\vec{y}} a^{\dagger}_{\vec{k}'}] \tag{5.19}$$

$$[v(\eta, \vec{(x)}), v'(\eta, y)] = \frac{1}{2} \int \frac{d^3k d^3k'}{(2\pi)^3} (v^*_{\vec{k}} v'_{\vec{k}'} e^{i\vec{k}\vec{x}} e^{-i\vec{k}'\vec{y}} - v_{\vec{k}} v'^*_{\vec{k}'} e^{-i\vec{k}\vec{x}} e^{i\vec{k}'\vec{y}}) \delta(\vec{k} - \vec{k}') \tag{5.20}$$

$$[v(\eta, \vec{(x)}), v'(\eta, y)] = \frac{1}{2} \frac{1}{(2\pi)^3} \int d^3k (v^*_{\vec{k}} v'_{\vec{k}'} - v_{\vec{k}} v'^*_{\vec{k}'}) e^{i\vec{k}(\vec{x} - \vec{y})} \tag{5.21}$$

Thus, the Wronskian condition is:

$$v_{\vec{k}}^{*}v_{\vec{k}'}' - v_{\vec{k}}v_{\vec{k}'}'^{*} = 2i \tag{5.22}$$

Note that a similar quantization procedure can be applied to tensor perturbations.
In order to express the metric perturbation $\Psi$ in terms of creation and annihilation operators we need another variable $u$ rather than $v$, because the first depends on the metric perturbation $\Psi$ and the unperturbed scalar field $\phi$ and also satisfies an equation of motion of the same form as $v$. This variable is given by ([3] p.337):

$$u = a\frac{\Psi}{4\pi\phi'} \tag{5.23}$$

Since $u$ a differential equation of the same form:

$$u(\eta, \vec{x}) = \frac{1}{\sqrt{2}}\int \frac{d^3k}{(2\pi)^{3/2}}[u_{\vec{k}}(\eta)e^{-i\vec{k}\vec{x}}a_{\vec{k}}^{\dagger} + u_{\vec{k}}^{*}(\eta)e^{i\vec{k}\vec{x}}a_{\vec{k}}] \tag{5.24}$$

so the metric perturbation can be expressed as:

$$\Psi(\eta, \vec{x}) = \frac{4\pi\phi'}{a}\frac{1}{\sqrt{2}}\int \frac{d^3k}{(2\pi)^{3/2}}[u_{\vec{k}}(\eta)e^{-i\vec{k}\vec{x}}a_{\vec{k}}^{\dagger} + u_{\vec{k}}^{*}(\eta)e^{i\vec{k}\vec{x}}a_{\vec{k}}] \tag{5.25}$$

Now the power spectrum can be obtained by constructing the two point function of $\Psi$ in the vacuum state $|0\rangle$:

$$\langle 0|\, \Psi(\eta, \vec{x})\Psi(\eta, \vec{y})\, |0\rangle \tag{5.26}$$

Using the commutation relations for $a$,$a^{\dagger}$ and defining the distance between the two points as $r = |\vec{x} - \vec{y}|$, the result is

$$\langle 0|\, \Psi(\eta, \vec{x})\Psi(\eta, \vec{y})\, |0\rangle = \int \frac{4\phi'^2}{a^2}|u_{\vec{k}}(\eta)|^2k^3\frac{sin(kr)}{kr}\frac{dk}{k} \tag{5.27}$$

Using the definition of the power spectrum of Chapter 2, the final result for the power spectrum is :

$$\Delta_{\Phi}^2 = \frac{4\phi'^2}{a^2}|u_{\vec{k}}(\eta)|^2k^3 \tag{5.28}$$

From the above result, one can examine the shape of power spectrum in different regimes of wavenumber $k$.

## 5.2 Derivation of $\delta{<}T^i_j{>}$

First, we present the derivation of relations (35)-(37) of [11] in which $\delta{<}T^i_j{>}$ is expressed in terms of the the response functions $\bar{E}(q), \bar{\Omega}(q)$.

We begin with relation (3.39), namely:

$$<T_{ij}> = (\frac{-2}{\sqrt{|g|}}\bar{\Pi}_{ij})_{(3)} \tag{5.29}$$

using the definition of $\bar{\Pi}_{ij}$ leads to

$$<T_{ij}> = \frac{1}{\bar{k}^2}[Kg_{ij} - K_{ij}]_{(3)} \tag{5.30}$$

$$<T^i_j> = g^{ik}<T_{kj}> = \frac{1}{\bar{k}^2}[K\delta^i_j - K^i_j]_{(3)} \tag{5.31}$$

$$\delta<T^i_j> = \frac{1}{\bar{k}^2}[\delta K\delta^i_j - \delta K^i_j]_{(3)} \tag{5.32}$$

Using definition for $K_{ij}$

$$K^i_j = \frac{1}{2}g^{ik}\partial_r g_{kj} \tag{5.33}$$

$$K = TrK_{ij} = g^{ij}K_{ij} = \frac{1}{2}g^{ij}\partial_r g_{ij} \tag{5.34}$$

$$\delta K_{ij} = \frac{1}{2}\partial_r \delta g_{ij} \tag{5.35}$$

Recall $\delta g_{ij} = h_{ij}$, namely

$$\delta g_{ij} = -2\psi\delta_{ij} + 2\partial_i\partial_j\chi + \gamma_{ij} \tag{5.36}$$

Thus,

$$\delta K_{ij} = -\psi'\delta_{ij} + \partial_i\partial_j\chi' + \frac{1}{2}\gamma'_{ij} \quad , \quad ' = \partial_r \tag{5.37}$$

$$\delta K^i_j = g^{ik}\delta K_{ij} = -\psi'\delta^i_j + \partial^i\partial_j\chi' + \frac{1}{2}\gamma'^i_j \tag{5.38}$$

$$\delta K = \frac{1}{2}\delta g^{ij}\partial_r g_{ij} + \frac{1}{2}g^{ij}\partial_r\delta g_{ij} \tag{5.39}$$

$$\delta K = \frac{1}{2}\delta^{ij}\partial_r g_{ij} + \frac{1}{2}g^{ij}(-2\psi'\delta_{ij} + 2\partial_i\partial_j\chi' + \gamma'_{ij}) \tag{5.40}$$

Recall (3.38), namely

$$\partial_r = \delta_D(1 + O[e^{-2r}]) \quad , \quad r \longrightarrow \infty \tag{5.41}$$

and also relation (27) of [11]:

$$\partial_D g_{ij}(r,x) = 2g_{ij}(x,r) \tag{5.42}$$

Thus, to obtain $\delta K$ we use

$$\partial_r g_{ij} = 2g_{ij} \tag{5.43}$$

and the result is ( recall $\gamma_{ij}$ traceless):

$$\delta K = -3\psi' + \partial^i\partial_i\chi \tag{5.44}$$

Putting all together, we obtain $\delta{<}T^i_j{>}$:

$$\delta{<}T^i_j{>} = -\frac{1}{\bar{k}^2}[2\psi'\delta^i_j + \partial^k\partial_k\chi'\delta^i_j - \partial^i\partial_j\chi' + \frac{1}{2}\gamma'^i_j]_{(3)} \tag{5.45}$$

Also, from relation (3.43) for $\pi_{ij}$, we have

$$\pi^i_j = g^{ik}(\delta_{kj} - \frac{\bar{q}_k\bar{q}_j}{\bar{q}^2}) = \delta^i_j - \frac{\bar{q}^i\bar{q}_j}{\bar{q}^2} \tag{5.46}$$

Then we have

$$\bar{q}^2\chi'\pi^i_j = \bar{q}^2\chi'\delta^i_j - \chi'\bar{q}^i\bar{q}_j \tag{5.47}$$

Replacing $\bar{q}_i$ with $\partial_i$, since we work on momentum space:

$$\bar{q}^2\chi'\pi^i_j = \delta^k\partial_k\chi'\delta^i_j - \partial^i\partial_j\chi' \tag{5.48}$$

Thus, from (5.45) and (5.48) we get:

$$\delta{<}T^i_j{>} = -\frac{1}{\bar{k}^2}[2\psi'\delta^i_j + \bar{q}^2\chi'\pi^i_j + \frac{1}{2}\gamma'^i_j]_{(3)} \tag{5.49}$$

which is the correct relation (35) of [11]. In order to express $\delta{<}T^i_j{>}$ in terms of the response functions $\bar{\Omega}(\bar{q})$,$\bar{E}(\bar{q})$ we use relation (10) of [11] in the gauge of section 3.3 namely for $\phi, v, v_i, \omega_i$ set to zero. Specifically, relation (10) combined with (6) and (8) yields

$$\bar{q}^2\chi' = \frac{\bar{q}^2\psi}{a^2H} - \epsilon\zeta' \tag{5.50}$$

From the definitions of response functions relation (17) of [11] (analogue to (3.34)) namely

$$\bar{\Pi}^{\zeta}_{\bar{q}} = \frac{2\epsilon a^3}{\bar{k}^2}\zeta'_{\bar{q}} = -\bar{\Omega}(\bar{q}) \tag{5.51}$$

$$\bar{\Pi}^{\gamma}_{\bar{q}} = \frac{a^3\gamma'_{\bar{q}}}{4\bar{k}^2} = -\bar{E}(\bar{q}) \tag{5.52}$$

Thus, all together:

$$\bar{q}^2\chi' = (\frac{\bar{q}^2}{a^2H} + \frac{\bar{k}^2\bar{\Omega}}{2a^3})\psi + \frac{\bar{k}^2\bar{\Omega}H}{2a^3\varphi'}\delta \tag{5.53}$$

The above relation is indeed (36) of [11] together with $2\psi' = \varphi'\delta\varphi$. Also, for $\gamma'^i_j$ in (5.49), we have

$$\gamma'_{\bar{q}} = \frac{-4\bar{k}^2}{a^3}\bar{E}\gamma_{\bar{q}} \Longrightarrow \gamma'^i_j = \frac{-4\bar{k}^2}{a^3}\bar{E}\gamma^i_j \tag{5.54}$$

Finally, using (5.50) and (5.54) we obtain $\delta{<}T^i_j{>}$ from (5.49) :

$$\delta{<}T^i_j{>} = [\frac{2\bar{E}}{a^3}\gamma^i_j - \frac{\bar{q}^2}{\bar{k}^2a^2H}\psi\pi^i_j - \frac{\bar{\Omega}}{2a^3}\psi\pi^i_j - (\frac{H\bar{\Omega}}{2a^3\varphi'}\pi^i_j + \frac{\varphi'\delta^i_j}{\bar{k}^2})\delta\varphi]_{(3)} \tag{5.55}$$

which is indeed relation (37) of [11].

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