UNIVERSITY OF CRETE
FACULTY OF SCIENCES AND ENGINEERING
DEPARTMENT OF COMPUTER SCIENCE

Temporal Topology on Fuzzy Space-time Volumes

by

Manos Papadakis

Dissertation submitted in partial fulfillment

of the requirements for the degree of

Master of Science

Heraklion, November 2014

UNIVERSITY OF CRETE
DEPARTMENT OF COMPUTER SCIENCE

Temporal Topology on Fuzzy Space-time Volumes

Thesis submitted by

Manos Papadakis

in partial fulfillment of the requirements for the
Master's degree in Computer Science

Author: **Manos Papadakis**
University of Crete

Examination Committee: **Dimitris Plexousakis**, Thesis Supervisor
Professor, University of Crete

Martin Doerr
Research Director, Information Systems Laboratory
Foundation of Research and Technology Hellas

Yannis Tzitzikas
Assistant Professor, University of Crete

Department approval: **Antonis A. Argyros**
Professor and Director of Graduate Studies

Heraklion, November 2014

Abstract

The study of the past through the definition, description and association of past periods is an important part of historical, archaeological and other research processes. Since the past is not directly observable, evidence about past periods or events is derived from the observation of traces that were left over by past phenomena. However, data obtained about periods is imprecise due to limitations on observation and definition, as well as information loss a set of facts that lead to uncertainty with regard to their spatiotemporal modeling. The main objective of the current thesis is to contribute to the theoretical foundations of spatiotemporal modeling based on observation data, focusing on the impact of temporal indeterminacy on the temporal topology over space-time volumes. Although there are several approaches of modeling imprecise time, they mainly focus on pure temporal reasoning. To the best of our knowledge, this is the first effort on temporal reasoning over spatiotemporal entities that deals with time imprecision. We address the following issues: temporal confinement of spatiotemporal entities based on distinct and scarce information, adaptation of time point equality over temporal indeterminacy, representation of temporal topology over fuzzy space-time volumes and extraction of relevant topology based on semantic association. Our work has several major outcomes. We propose a model that reconstructs the temporal extent of a period, introducing determinacy and indeterminacy regions in order to deal with time fuzziness. Association of indefinite intervals is achieved with the introduction of a fuzzy interval algebra, as an alternative to Allen operators, which focuses on the fuzziness modeling. The aforementioned algebra is extended into four dimensional space in order to temporally associate fuzzy space-time volumes. Finally, we introduce a set of possible spatiotemporal relations that are derived by semantic association. Our study has a crucial impact on fields that are associated with reality modeling, especially observation-based sciences such as archaeology, biology, geology and so on. Basic applications of our theory include the temporal confinement of periods based on observation data; evaluation of temporal topology extraction methods like Harris matrix; temporal association of defined periods and reconstruction of possible past scenarios.

Supervisor: Dimitris Plexousakis

Professor

Computer Science Department

University of Crete

Περίληψη

Η μελέτη του παρελθόντος μέσω του ορισμού, περιγραφής και συσχέτισης των παρελθοντικών περιόδων είναι ένας σημαντικός τομέας των ιστορικών, αρχαιολογικών και διαφόρων άλλων διαδικασιών. Καθώς το παρελθόν δεν είναι άμεσα παρατηρήσιμο, αποδείξεις για την ύπαρξη περιόδων αντλούνται από την παρατήρηση τεκμηρίων, παραγόμενων από διάφορα παλαιά φαινόμενα. Ωστόσο, τα δεδομένα που εξάγονται για τις περιόδους είναι ανακριβή εξαιτίας περιορισμών που σχετίζονται με τις παρατηρήσεις ή τον ορισμό των φαινομένων, καθώς επίσης και από την απώλεια πληροφορίας για παρελθοντικά γεγονότα, κάτι που οδηγεί σε ασάφεια όσον αφορά την χωροχρονική τους μοντελοποίηση. Ο βασικός στόχος της παρούσας εργασίας είναι η συμβολή της στη δόμηση της θεωρητικής θεμελίωσης όσον αφορά την χωροχρονική μοντελοποίηση βασιζόμενη σε δεδομένα παρατηρήσεων, με ιδιαίτερη εστίαση στην επίδραση που φέρει η χρονική απροσδιοριστία στην χρονική συσχέτιση χωροχρονικών όγκων. Υπάρχουν αρκετές έρευνες πάνω στο πρόβλημα μοντελοποίησης ασαφούς χρόνου, παρόλα αυτά κυρίως εστιάζονται σε χρονική συμπερασματολογία. Αυτή η εργασία είναι η πρώτη προσπάθεια χρονικής συμπερασματολογίας πάνω σε χωροχρονικές οντότητες, που λαμβάνει υπόψιν την χρονική ασάφεια. Εξετάζουμε τα ακόλουθα προβλήματα: χρονικός περιορισμός χωροχρονικών οντοτήτων, βάσει διακριτής και ανεπαρκούς πληροφορίας, αναπαράσταση της χρονικής ισότητας πάνω σε χρονική απροσδιοριστία, ορισμό χρονικής συσχέτισης ασαφών χωροχρονικών όγκων και εξαγωγή σχετικής τοπολογίας βασισμένη σε σημασιολογική συσχέτιση τεκμηρίων. Η παρούσα εργασία φέρει αρκετά σημαντικά αποτελέσματα. Προτείνουμε ένα μοντέλο για την αναδόμηση της χρονικής έκτασης μίας περιόδου, συνιστώντας περιοχές προσδιοριστίας και απροσδιοριστίας με σκοπό τον χειρισμό της χρονικής ασάφειας. Συσχέτιση διαστημάτων ασαφούς χρόνου επιτυγχάνεται μέσω της άλγεβρας που προτείνουμε, η οποία αποτελεί μία εναλλακτική προσέγγιση των τελεστών του Allen, προσφέροντας μοντελοποίηση της ασάφειας. Η προαναφερθείσα άλγεβρα επεκτείνεται στις τέσσερις διαστάσεις, με σκοπό την χρονική συσχέτιση ασαφών χωροχρονικών όγκων. Τέλος, προτείνουμε ένα σύνολο από δυνατές χωροχρονικές σχέσεις οι οποίες εξάγονται από σημασιολογικές συσχετίσεις των αντίστοιχων όγκων. Η μελέτη μας έχει καθοριστική σημασία σε τομείς που σχετίζονται με τη μοντελοποίηση της πραγματικότητας, ειδικότερα σε επιστήμες που βασίζονται σε παρατηρήσεις όπως η αρχαιολογία, βιολογία, γεωλογία κτλ. Βασικές εφαρμογές της θεωρίας μας περιλαμβάνουν τον χρονικό περιορισμό περιόδων, βάσει παρατηρήσεων, αξιολόγηση μεθόδων εξαγωγής χρονικής τοπολογίας όπως η Harris Matrix, χρονική συσχέτιση περιόδων και κατασκευή πιθανών σεναρίων που περιγράφουν το παρελθόν.

Επόπτης Καθηγητής: Δημήτρης Πλεξουσάκης

Καθηγητής

Τμήμα Επιστήμης Υπολογιστών

Πανεπιστήμιο Κρήτης

Ευχαριστίες

Η παρούσα μεταπτυχιακή εργασία αποτελεί μία λεκτική περιγραφή και εκτεταμένη εξήγηση των θεμάτων, με τα οποία ασχολήθηκα τα τελευταία τρία χρόνια. Πέρα από το επιστημονικό περιεχόμενο που μπορεί να διακρίνει ο αναγνώστης, εμπεριέχονται και άλλα, έμμεσα ίσως κάποιες φορές κρυφά μηνύματα που αντικατοπτρίζουν σε έναν μεγάλο βαθμό την ολοκλήρωση του συγγραφέα.

Η εργασία αυτή για μένα είναι μία απόδειξη ότι οι στόχοι μου έγιναν πραγματικότητα. Με θέληση, υπομονή αλλά και πείσμα κατάφερα να τερματίσω τον, κατά πολλούς δύσβατο, δρόμο απόκτησης μεταπτυχιακού τίτλου από το τμήμα Επιστήμης Υπολογιστών. «Μπράβο» και «συγχαρητήρια» φτάνουν σε εμένα από γνωστούς, φίλους και αναγνώστες με σκοπό να επαινέσουν την πράξη μου. Όμως αυτή η εργασία δεν θα βρισκόταν στα χέρια σας, αν και εγώ δεν είχα την στήριξη και συμπαράσταση των καθηγητών μου, των αγαπημένων μου προσώπων και των φίλων μου.

Άλλοι λιγότερο άλλοι περισσότερο, ο κάθε ένας με την δική του προσπάθεια κατάφερε να κάνει αυτά τα τρία χρόνια να ξεχωρίσουν, αφήνοντας έτσι μία πολύχρωμη ανάμνηση για αυτή την περίοδο της ζωής μου.

Πρώτα από όλα, ευχαριστώ θερμά τον επόπτη και καθηγητή μου Δημήτρη Πλεξουσάκη όπου μέσω της εμπιστοσύνης που έδειξε στις ικανότητές μου, μπόρεσα να γίνω μέλος του εργαστηρίου Πληροφοριακών Συστημάτων του Ινστιτούτου Τεχνολογίας και Έρευνας. Η καθοδήγησή και εμπειρία του ήταν ένας καθοριστικός παράγοντας για την διεκπεραίωση της εργασίας μου αλλά και για την δημοσίευσή της σε επιστημονικό συνέδριο στην Βόρεια Ιρλανδία.

Εν συνεχεία, θα ήθελα να ευχαριστήσω τον Διευθυντή Έρευνας κ. Martin Doerr για την καθοριστική συμβολή του στην μεταπτυχιακή μου εργασία. Η σύλληψη της κύριας ιδέας του θέματος αλλά και η τεκμηρίωση της, δεν θα είχαν ολοκληρωθεί χωρίς την ευρηματικότητα, εμπειρία αλλά και την τεράστια υποστήριξη που τόσο απλόχερα παρείχε. Μου παρείχε τα κατάλληλα εφόδια για το δύσκολο και γεμάτο εκπλήξεις δρόμο της έρευνας, αλλάζοντάς μου ολοκληρωτικά τον τρόπο με τον οποίο αντιλαμβάνομαι τον κόσμο. Τέλος καθοριστικό ρόλο στην εμπειρία μου αλλά και την εργασία μου έπαιξε η προσπάθειά του να με φέρει σε επαφή με άτομα της επιστημονικής κοινότητας, διευρύνοντας έτσι τις γνωριμίες μου αλλά και τα ενδιαφέροντά μου.

Ευχαριστώ πολύ τον Επίκουρο Καθηγητή κ. Γιάννη Τζίτζικα, μέλος της τριμελούς επιτροπής της μεταπτυχιακής μου εργασίας για τα σχόλια αλλά και συμβουλές που μου παρείχε για το κείμενο αλλά και σε άλλα θέματα συνέπειας.

Ακόμα θα ήθελα να ευχαριστήσω φίλους και συνεργάτες από το χώρο του εργαστηρίου των Πληροφοριακών Συστημάτων οι οποίοι μου προσέφεραν ένα ευχάριστο περιβάλλον εργασίας αλλά και βοήθεια όταν τη χρειαζόμουν. Πιο συγκεκριμένα μεταξύ άλλων ευχαριστώ τον συνεργάτη μου Gerald Hiebel για την αμέριστη βοήθεια του στα πρώτα-δύσκολα βήματα στο θέμα μου, σε τομείς που δεν κατείχα. Επίσης ευχαριστώ την κα. Μουτσάκη Μαρία ή όπως θα την παρομοίαζα τη «μαμά» του εργαστηρίου, για την βοήθεια της, καθώς και το κέφι που μας προσέφερε με τον ιδιαίτερο χαρακτήρα της.

Δεν μπορώ να παραλείψω από τις ευχαριστίες τους φίλους και συμφοιτητές μου. Παρά τις συχνές περιόδους γεμάτες κούραση και απογοήτευση από προβλήματα, άγχος ή πολλές υποχρεώσεις πάντα υπήρχε αυτή η ιδιαίτερα αμοιβαία υποστήριξη μεταξύ μας καθώς και αμέτρητες στιγμές άφθονου γέλιου. Ιδιαίτερα θα ήθελα να ευχαριστήσω τους φίλους μου Βασίλη Ευθυμίου, Παναγιώτη Παπαδάκο, Χριστίνα Λαντζάκη, Νίνα Σαβέτα, Γιάννη Ρουσάκη και Ηρακλή Διάκο.

Στη συνέχεια θα ήθελα να ευχαριστήσω ιδιαίτερα τη Σοφία Κλεισαρχάκη και τη Δήμητρα Ζωγραφιστού για την υποστήριξή τους σε πολλά θέματα που αφορούσαν τις μεταπτυχιακές μου σπουδές, την βοήθεια τους για να πάρω σοβαρές απο-

φάσεις καθώς και τα γέλια και τις πλάκες που κάναμε όλο αυτόν τον καιρό. Δεν θα μπορούσα να παραλείψω τα ξαδέρφια μου Έλενα και Χαρά για την συνεχή και ίσως μονόπλευρη, μερικές φορές από την πλευρά τους, στήριξη αλλά και αγάπη σε πολύ ιδιαίτερες στιγμές στη ζωή μου.

Σε αυτό το σημείο, θα ήθελα να ευχαριστήσω μέσα από την καρδιά μου τον πολύ καλό μου φίλο Γιώργο για τις σημαντικές συμβουλές που μου έδωσε σε πάρα πολλά θέματα που αφορούσαν την μεταπτυχιακή μου εργασία τα τελευταία δύο χρόνια. Πιο πολύ όμως θα ήθελα να τον ευχαριστήσω για την ιδιαίτερη υποστήριξη που μου παρείχε όλο αυτόν τον καιρό. Ο διαφορετικός τρόπος με τον οποίο αντιλαμβάνεται τον κόσμο αλλά και η επίπονη γι' αυτόν, πολλές φορές, προσπάθειά να με εμψυχώσει, με έκαναν πιο δυνατό. Με την αμέριστη προσοχή του, μου έδωσε ένα ισχυρό κίνητρο να πιστέψω στον εαυτό μου και στις ικανότητές μου. Με βοήθησε να καταλάβω ότι αξίζω πολύ περισσότερο από ότι νόμιζα και ότι είμαι ικανός για να πετύχω τους στόχους μου. Σε ευχαριστώ για όλα και δεν θα ξεχάσω ποτέ «We are all made of stars».

Δεν θα μπορούσα να μην ευχαριστήσω την κολώνα της ζωής μου τη Ρούλα. Τον άνθρωπο που δεν έπαψε από την πρώτη στιγμή που με γνώρισε να με αγαπάει και να μου το δείχνει ακόμα και στις πιο δύσκολες στιγμές. Η στήριξη της αλλά και η ανιδιοτελής αγάπη που απλόχερα δίνει τόσα χρόνια, γεμίζουν και οχυρώνουν τη ζωή μου με ένα μοναδικό τρόπο. Είναι το πρώτο άτομο που μου έδειξε τι σημαίνει πραγματικός φίλος αλλά και ο πρώτος άνθρωπος που με έκανε να καταλάβω τι σημαίνει ευτυχία και ασφάλεια. Το ιδιαίτερο χιούμορ της αλλά και ο καλοπροαίρετος χαρακτήρας της ταίριαξε με τον δικό μου σε τέτοιο βαθμό που με έκανε να πιστεύω ότι είναι ένα κομμάτι από τη ψυχή μου. Οι αμέτρητες στιγμές γέλιου, χαράς αλλά και συγκίνησης με κάνουν να θέλω και να επιμένω να είμαι για πάντα στη ζωή σου. «Κοπέλι μου» σε ευχαριστώ που υπάρχεις στη ζωή μου.

Τελειώνοντας θα ήθελα με έναν απλό τρόπο να εκφράσω την αγάπη μου και το ευχαριστώ μου για δύο άτομα που λατρεύω. Γιαγιά Ειρήνη σε ευχαριστώ για την αγάπη σου και την κατανόησή που μου δείχνεις. Δικαιωματικά είσαι για μένα μία δεύτερη μαμά. Γιαγιά Κορνηλία, δεν είσαι πια μαζί μας αλλά θα είσαι πάντα μέσα

στη καρδιά μου. Μπορεί να μην έγινα αυτό που ονειρευόσουν, δάσκαλος, αλλά ελπίζω η εργασία μου να διδάξει αρκετούς.

Το μεγαλύτερο ευχαριστώ, ωστόσο, ανήκει στην οικογένειά μου. Αρχικά ευχαριστώ τον πατέρα μου Νίκο που μέσα από την εκρηκτική συμπεριφορά του αλλά και τον ιδιαίτερο τρόπο αγάπης του, μου μεταφέρει συνεχώς την εμπειρία αλλά και τη γνώση για τις δυσκολίες και τα εμπόδια της ζωής. Με κάνει να γίνομαι πιο δυνατός και να προσπαθώ συνέχεια για το καλύτερο. Στη συνέχεια, ευχαριστώ μέσα από την καρδιά μου την μητέρα μου Φρόσω. Η τεράστια υπομονή της αλλά και η αγάπη που μου δίνει σε όλα τα στάδια της ζωής μου λειτουργούν για μένα σαν στήριγμα, ένα λιμάνι που με εφοδιάζει με αντοχή και υπομονή. Μαμά και Μπαμπά, δεν θα μπορούσα να τα καταφέρω χωρίς εσάς. Σας ευχαριστώ μέσα από την καρδιά μου.

This thing all things devours:

Birds, beasts, trees flowers;

Gnaws iron, bites steel;

Grinds hard stones to meal;

Slays king, ruins town,

And beats high mountain down.

– Gollum’s riddle, The Hobbit

Contents

List of Figures

Chapter 1

Introduction

Contents

1.1 Exploring the past

The study of the past through the definition, description and association of past periods is an important part of historical, archaeological and other research processes [Doerr et al. 2004a]. Past is a term that indicates the totality of events that occurred before a given point in time. In a macroscopic point of view, past can be regarded as a collection of completed events which are extended in space and time. The substance of such events is a set of related phenomena, that were caused either intentionally (by force of living beings) or turned out as an outcome of a random environmental change or disaster.

Revealed information about the past enhances the understanding of reality.

The extracted knowledge allows researchers to devise possible scenarios about the past as well as to forecast upcoming events. For instance, ice core data, related to the density of carbon dioxide trapped within layers of thick ice[Readinger 2006], provide an overview of the climate on planet Earth at certain periods over the years. Probabilistic models applied on climate data lead to valuable estimations about the future weather conditions, encourage predictions about the greenhouse effect in the upcoming years and so on.

Since past is not directly observable, evidence about the manifestation of past events is derived by clues that were left by the comprising phenomena. According to the New Oxford American Dictionary [dic 2010], a phenomenon is defined as any observable occurrence; in particular, phenomena leave traces, either directly or indirectly observable, as a ”derivative” of their trigger, duration, completion and their occurrence, in general. For instance, a wildfire event that occurred on a woodland, is observable through the wood combustion by-products (embers and wood ash), by-products of organic matter combustion, incinerated organisms, dehydrated, eroded, heated and poor in nutrition soil, and other related traces that are detected within the soil of the burnt area.

The analysis of observable evidence provides substantial information about the related phenomenon. Such data can sufficiently define and render the comprised events. The extracted knowledge provides answers to the following basic questions: *what*, *when* and *where*, which stand for the approximation of the semantic, temporal and spatial aspects of the corresponding phenomena, respectively. Semantic coherence among the related phenomena and the corresponding events is determined by introducing inclusion or exclusion relations. For instance, combustion by-products form a *part of* relation with a wildfire event, whereas traces of fossilized marine organisms and sand implies a *separated from* relation with the aforementioned event. Spatial knowledge is obtained by the approximation of the geographic space where the corresponding phenomenon is located. Consequently, temporal confinement is gained through dating methods of absolute or relevant chronology such as radio-carbon activity or event ordering and dendrochronology, respectively. Particularly, the temporal approximation results into a time frame on the timeline within which a phenomenon occurred. Combining all derived knowledge from related evidence leads to the semantic definition and spatiotemporal approximation of an event that is built up by the corresponding phenomena.

Although observations provide the spatial and temporal information that confines an event under consideration, we accept that the true knowledge about it, is its spatiotemporal extent. Particularly, pure spatiotemporal knowledge represents the occurrence of the event, standing for the claim that something happened somewhere at some time. For instance, the only true knowledge about the wildfire event is the fact that it occurred within the space-time. Therefore, the approximated time frame over the timeline and the space region on the corresponding reference space are considered as projections on the comprised dimensions of the true spatiotemporal extent. This setting leads to the assumption that events are four dimensional entities that conform to the theoretical assumptions of [Doerr et al. 2007].

Declaration of events is regarded as the construction of building blocks, which are used to outline the basic backbone structure of the past. However, knowledge that is limited to a set of isolated event references fails to provide the complete view of a possible past scenario. For instance, ice core data that is related to different ice layers reveals information about the meteorological variables over specific long periods of time on Earth. However, no further information is provided that concerns the association between the climatic phases, i.e. temporal ordering or distance, spatial overlap and so on; hence, an overall view of the climatic changes that affected Earth over the years cannot be obtained.

Relevant association over past events has various aspects. We identify the following types of possible correlations: semantic, spatial and temporal. According to the type of association, additional corresponding information is revealed. To analyze the former claim in deep, we use an example that demonstrates the relations between events. Assume the following two events: a wildfire and the destruction of a cottage by the flames; it goes without saying that the building is located within the burning land. Semantic association relates events based on their semantic coherence, e.g., the destruction of the cottage is considered a part of the wildfire event. Spatial association refers to the spatial topology of the events by relating their occupied space, e.g., the building's destruction is spatially within the wildfire event. Lastly, temporal correlation represents the chronological ordering, in other words the temporal topology of the events under consideration; for instance, the destruction of the building occurred during the wildfire event.

It is noteworthy that associations among events amplify our knowledge about the past. However, data that is obtained about past events is often imprecise, due to limitations of observation and definition, as well as information loss from the past. Such a fact leads to uncertainty with regard to the spatiotemporal modeling of events. This setting gives rise to various problems that are related to the spatiotemporal modeling of reality; for instance, the representation of temporal information of an event, dealing with temporal imprecision, temporal association over events with fuzzy endpoints and so on.

This thesis contributes to the theoretical foundations of spatiotemporal modeling based on observation data, focusing on the impact of temporal indeterminacy. Particularly, the two major topics that are analyzed are the temporal confinement of past periods and the extraction of temporal topology over space-time entities. The rest of this document is organized as follows. In the remainder of the introductory chapter, we describe the issues for which this thesis proposes solutions, relying on a motivating example that involves spatiotemporal modeling of a conquest event. In Chapter 2, we provide an extensive analysis of the background material that frames this thesis, followed by an exploration of various efforts closely related to the aforementioned issues. In Chapters 3, 4 and 5, we provide the proposed solutions that focus on the objectives mentioned in Section 1.2. Afterwards, in Chapter 6, possible applications of our theory are analyzed. Finally, in Chapter 7 we summarize the work that was carried out by briefly outlining the main objective, solutions and major outcomes of our work, followed by interesting directions for future work.

1.2 Objectives

The main objective of the current thesis is to contribute to the theoretical foundations of spatiotemporal modeling based on imprecise information, a common scenario of describing reality through observation data. Particularly, we focus on the temporal confinement of four dimensional entities such as cultural periods, based on distinct and scarce information. In addition, we proceed to the study of temporal association over space-time volumes in order to approximate their temporal topology. Finally, we aim to investigate the possible spatiotemporal relations that are derived by semantic association of the available evidence. The rest of this section focuses on the analysis of the issues that are addressed by this thesis. First, a motivating example is provided, which portrays a scenario illustrating the necessity of spatiotemporal modeling. Afterwards, the problems that emerge in this setting are outlined.

1.2.1 Motivating example

In order to illustrate the aforementioned objectives, we assume the scenario of the conquest of a city, which has been encountered numerous times in history. An important outcome of such events is the change of leadership of the city under attack. In many cases, key evidence that documents such changes is the considerable cultural difference within the city borders between the defending people, the invading troops and the resulting population. Empirical evidence about conquest events can provide information related to the succession of different cultures over the territory of the cities under consideration. There are several historical sources that report city conquest scenarios, for instance, the siege of the ancient city of Troy which was excavated by Korfmann [Korfmann 2006] and immortalized by Homer's Iliad.

According to the myth, the Trojan War was the outcome of the abduction of the Greek princess Helen by the Trojan prince Paris; who unwittingly conspired to the Gods' plans, in order to settle the challenge of the golden apple of discord. The act of abduction was considered as vituperation and forced the Greek leaders to attack the city of Troy in order to bring princess Helen back to her husband under the law and kill the Trojan prince as a punishment for his actions. The city of Troy was under siege by the Greek invaders for a notable period of time. It eventually fell due to the sneaking trick of the Trojan Horse that was plotted by Odysseus, resulting in the triumphant victory of the Greek kings and, therefore, the conquest over the territory of Troy.

In a narrower sense, the Trojan War can be seen as the capturing of the ancient city of Troy by the Greek leaders. There are two notable periods over the Trojan territory: the reign of King Priam that pre-exists the Trojan War; and the Greek leadership that exists after the capturing event. The relevant temporal association of the aforementioned conceptual periods is regarded as a meeting where the reign of King Priam was succeeded by the Greek leadership. This setting gives rise to the need of reconstruction of the possible past scenarios that are related to the Trojan War; in other words, a spatiotemporal modeling problem emerges in order to describe the topology of the successive periods.

Both past periods that are related to the Trojan War are defined by data that is derived by historical sources or empirical evidence through the observation process. In the rest of this section we state and analyze the notable problems that emerge.

1.2.2 Certainty and Impossibility of existence

Trojan inhabitants leave traces and products of human activities, which associate their culture and population with the territory of Troy. These primary observations can be associated with time intervals through dating processes [Doerr et al. 2004b], by combining the spatiotemporal information gained from semantically related evidence. Combining the available evidence leads to the definition of a period with fuzzy space-time bounds [Doerr and Hiebel 2013] that represents the lifetime of the individual culture at a certain place.

However, observations can only indicate the possible existence of that population over time; precise bounds cannot be derived. Even historical records are sparse and of limited precision. This hinders any effort to turn a possibility of existence to a precisely limited certainty or impossibility.

1.2.3 Time point equality

It is obvious that the capture of the city of Troy is considered as a meeting in time [Doerr et al. 2004b] using temporal terms, in which the intervals that represent the activities of the Trojan and Greek army meet without a discontinuity (“gap”) in between. Trying to approximate this meeting by using Allen’s meet relation leads to the problem of seeking the exact time point for which the meeting occurred. In terms of Allen operators, this problem is regarded as the application of the endpoint equality rule that is stated by the meet operator. Such task is not trivial, due to the imprecision of the habitation event itself, as well as the non-instantaneous nature of the conquest event, whose duration is considerable and differs, depending on the location within the city.

Furthermore, even with the assumption that the conquest event is regarded as instantaneous by associating it with the time that the Trojan army surrenders, it is difficult to pinpoint the exact time point in which the last soldier gave in.

1.2.4 Temporal relations on space-time volumes

Periods can be formally represented by four-dimensional volumes that serve as the union of spatiotemporal expanses of the set of constituent coherent phenomena. With respect to the conquest event, there are two periods of habitation in relation to the city: the Trojan (pre-conquest) and the Greek habitation (post-conquest). An emerging need when dealing with periods is the approximation of their relevant association, in order to describe their temporal topology such as ordering, inclusion etc.; and hence the reconstruction of a possible past scenario that approximates the real event.

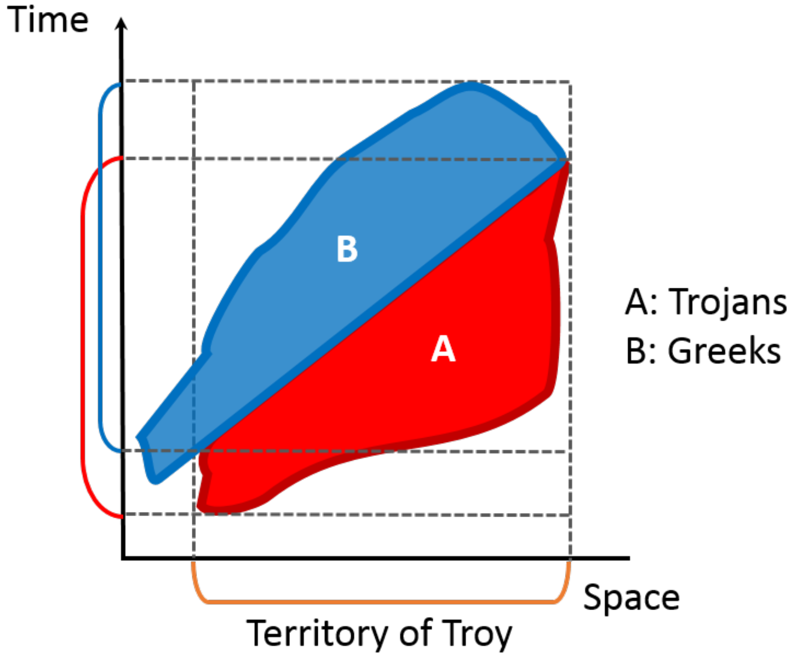


Figure 1.1: Trojan and Greek ownership over the territory of Troy

In order to define temporal association between periods, time projections must be used; since general spatiotemporal relations do not allow complete ordering (e.g., before, meets, starts and so on). Taking into account the corresponding time projections of the individual leadership periods (Trojan and Greek period) we conclude in two different interpretations that are expressed in terms of Allen operators.

On the one hand, the Trojan and Greek period over the territory of Troy are regarded as a meeting in time, considering that the city was captured without coexistence between the populations. On the other hand, analyzing the time projections of the corresponding periods in Figure 1.1, the resulting association is an overlap relation. This ambiguity is caused by the fact that the capturing event, i.e. the Greek army invasion, happened over time and not instantly, resulting in a gradual changing of ownership upon the territory of Troy.

Despite the explicit conclusion by analyzing the time projections in Figure 1.1, intuitively the first approach seems to explain the real conquest event more accurately. As a result, the spatial dimension cannot be ignored in cases of the association of four-dimensional entities, but instead the need of its inclusion into the temporal relation emerges.

Chapter 2

Background and Related Work

Contents

In this chapter, we provide an extensive description of the required theoretical background, in order to cover the fundamental knowledge that frames the current thesis. Then, we are going to focus on solutions the have already been proposed, which are related to problems of spatiotemporal reality modeling. A concise description is offered for each approach for better understanding.

2.1 Background: Spatiotemporal Modeling of Reality

The background theory in this section is organized in an order from general to specific. We begin with the notion of reality and how such an abstract concept can be modeled, by providing a brief introduction to ontologies and focusing on those that are related to cultural heritage. Then, we focus on the notion of *Period* that is based on the ontological approach of CIDOC Conceptual Reference Model. Our analysis is extended with the exploration of the different methods used to define *Periods* using empirical evidence and observations. In addition, we examine the semantic association and the spatiotemporal extent of such entities. Afterwards, we provide an extensive description that focuses on the notion of time and the relative temporal topology of entities that are bounded in time, followed by an analysis of space-time and the corresponding spatiotemporal topology. Moreover, we approach the abstract notion of fuzziness and how it is derived by the combination of different perceptions of reality. Finally, a brief analysis of the notion of imprecision is presented.

2.1.1 Reality, Ontologies and Cultural Heritage

According to the New Oxford American Dictionary [dic 2010], reality is defined as *the state of things as they actually exist, as opposed to an idealistic or notional idea of them*. In other words, reality is considered the conjectured state of things as they actually exist, rather than as they may appear or might be imagined. As a consequence of the previous statement, reality includes entities that exist but are not necessarily observable. In a wider perspective, entities that are beyond our sphere of understanding such as dark matter or dark energy, as well as abstract concepts, are also regarded as part of reality. Finally, considering an even broader definition, reality is not bounded by time, therefore it includes everything that has already existed, exists or will exist in the future.

Reality is a major aspect that specifies and describes how our world is perceived, therefore the need of fields that study reality is imperative. Ontologies are a philosophical discipline that is related to the notion of existence, nature of being and becoming; more generally, they refer to the study of reality. In addition, ontologies deal with questions concerning how existing entities can be grouped, related within a hierarchy and subdivided according to similarities and differences.

In the past, ontologies were rather confined to the philosophical sphere [Guarino 1998] and they were referred as a system of categories accounting for a certain vision of the world. However, the increasingly widespread research in the Computer Science community eventually extended the philosophical sense of ontologies. Several fields of information systems refer to an ontology as an engineering artifact that is associated with a specific vocabulary (with an intended meaning specified by the associated domain) that is used to describe a certain reality. For instance, ontologies are used in the fields of knowledge representation [Guarino 1995], knowledge management and organization [Poli 1996], to name but two.

Cultural heritage refers to the things preserved by memory institutions, such as museums, sites and monuments records, archives and libraries [Doerr 2009]. In a narrower sense, information that is related to the cultural heritage comprises the knowledge pool of the past. The information system deployment in the cultural heritage domain invests in the digital representation and organization of our current knowledge about the past. Since past is considered as a part of reality, the need of an ontology rises, in order to provide the proper mechanism to describe it. However, the research and study of the past is highly interdisciplinary, since it employs a series of *auxiliary* disciplines such as archaeology, palaeontology and biology but also historical sources and social theories. Due to the diversity of the involved fields, it is hard to choose a single domain in the sense of domain ontologies [Guarino 1998].

Currently, the most elaborated schema for the integration of cultural heritage information is derived from the CIDOC CRM ontology [Doerr 2009]. It was designed and developed under the aegis of the International Committee for Documentation (CIDOC) of the International Council of Museums (ICOM). As already mentioned, the description of the past requires more than a single domain in order to achieve completeness. Therefore, CIDOC CRM is an outcome of the collaboration of various teams of experts, including fields such as archaeology, museum documentation, history of arts, natural history, physics, philosophy, computer science and other cultural heritage related sciences. The resulting formal ontology [CID 2011] [CRM 2013] [Doerr et al. 2007] is intended to facilitate the integration, mediation and interchange of heterogeneous cultural heritage information. It is worth noting that the strict principles that parameterize CIDOC CRM ensure that only concepts that realize global information integration are supported. In addition, there are further philosophical restrictions about the kind of discourse to be supported [Doerr 2003].

2.1.2 Temporal Entities: Period and Events

Past is a term that indicates the totality of events that occurred before a given point in time. Each one of the comprised events is associated with a time margin that quantifies its duration. In a wider view the past is composed by a set of events with a specified duration, therefore the total time extent of the known past is formed by combining the duration of the composed events. Many science fields that are engaged in the exploration of the past use the term *period* in order to refer to the time extent of past events or phenomena.

A period is defined as *a length or a portion of time*[dic 2010]. Although its meaning in different science fields is slightly modified based on the intended use, the prevalent concept is still the quantification and reference of time. For instance, geologists refer to a period as the major division of geological time, while archaeologists or historians use that term in order to mention a portion of time in the life of a nation, a civilization, and so on.

The heterogeneity that typifies the cultural heritage information requires a generalized approach of the term period, in order to support the intended scope of the global information integration. CIDOC CRM introduces the concept of *Temporal Entities* [CID 2011] that comprises all phenomena such as periods, events and states, which happened over a limited extent in time. A cultural period such as the Bronze Age, a natural disaster such as an earthquake or the inhabitation of a building within a time duration, are all considered instances of temporal entities.

Unlike the foregoing references to the term period as a portion of time, CIDOC CRM extends a period's pure temporal nature by including the notion of space. Therefore, a period is applied to a geographic area, which is defined with a greater or lesser degree of precision. According to CIDOC CRM, Periods are regarded as *sets of coherent phenomena or cultural manifestations bounded in time and space*. For instance, a period refers to the Bronze Age, Jurassic Era, thirty years of war, cubism and so on. Despite the spatiotemporal component that portrays a period, its identification is derived by the social or physical coherence of its comprised phenomena. The associated time and space that bounds a period is a mere approximation of the actual process of growth, spread and retreat. Consequently, different periods can overlap and coexist in time and space.

An event is considered as a special case of a period that is associated with the undergoing state changes that are related to the involved entities. Particularly, CIDOC refers to an event as *a change of state in cultural, social or physical systems, regardless of scale, brought about by a series or group of coherent physical, cultural, technological or legal phenomena*. Event is compatible with the proper definition of a period, however it is extended with the concept of *meetings*. Particularly, events can be regarded as meetings of living and dead items that brings a change of state at any scale [Doerr et al. 2004b]. It is a non-instantaneous, continuous, and finite process that is potentially decomposable to minimal elements of sub-events. Instances of events can be the birth of a man, World War II, a conference, the destruction of a building and so on. It is worth noting that an event may be decomposed to its component phenomena within a space-time frame and therefore it is regarded as a period; however, the reversed process is not always applicable, as not all periods give rise to a noteworthy changes of state. In the rest of the thesis, any reference to event instances implies periods, except if stated otherwise.

Semantic Association

Sets of interacting and related phenomena are grouped in order to form a period. Two spatially separated periods with no semantic association between their consisting phenomena are considered as distinct. For instance, cultural periods such as the Xia dynasty and Middle Minoan I represent phases of the Chinese and Minoan civilization, respectively. Both periods are dated in the same time frame, however each one refers to a different geographical space. Xia dynasty is associated with the region of East Asia, while Middle Minoan I is related to the Mediterranean sea. As a result, the cultural manifestation and the comprised phenomena that build each period are not semantically correlated, resulting in distinct periods.

Contrary to the concept of distinct periods, subdivisions are also a possible scenario. There are cases where the comprised phenomena of a period are subjected to inclusion relations. A set of events that share a common participant can be aggregated as a super event. In a wider view, a set of periods that deal with a specific entity may form a super period. Furthermore, the inheritance hierarchy introduced by CIDOC CRM between period and event instances allows a super period to be composed of sub-periods that are regarded as events. For instance, the lifetime of a person can be seen as a super period that is decomposable to a set of sub-periods that refer to different phases of his life like childhood, adulthood, middlehood and so on. Each one of the age spans can include several events like birth, death and other events that the person participated in. In addition, the main periods can be divided into sub-periods like married life, pregnancy, period of illness and so on, depending on the available evidence.

According to CIDOC CRM, the inclusion and distinct relations upon periods are expressed by attaching the properties *consist of* and *is separated from* to period and hence event instances. Particularly, *consists* of describes the decomposition of a period into discrete, subsidiary periods, while *part of* denotes the inverse relation. It is worth noting that the sub-periods in which the period is decomposed form a logical whole, although the entire picture may not be completely known and the sub-periods constitute the general period [CID 2011]. On the contrary, the symmetric property *separated from* describes period instances that do not share neither time frame nor geographical space, resulting into non coexistence scenarios which imply phenomena with semantic inconsistency.

Spatiotemporal Extent

We accept that the substance of a period is made of phenomena. It is quite obvious that such phenomena cover an irregular area in space-time. Particularly, a phenomenon is bounded by a time-frame in the time line and a geographical area that approximate its temporal and spatial extent, respectively. Consequently, a period instance can be spatio-temporally approximated by combining the time and space of the individual phenomena that compose the current period. CIDOC CRM uses the entities *Time-Span* and *Place* to refer to the temporal and spatial approximation of a period, associated with the properties *has time-span* and *took place at*, respectively.

The temporal approximation of a period is regarded as the real time during which the comprising phenomena were active, which consequently make up the period instance. Particularly, it is considered as a positioning process upon the time line of chronology, which can be regarded as a set of dates.

The space that is occupied by a period refers to its spatial location. Specifically, it is an approximation of the geographical area within which the phenomena that characterize the period occurred. Similarly to the temporal approach, space is considered as a spatial positioning over a reference space, e.g., the surface of the earth.

The isolated notion of space and time can be easily perceived as a spatiotemporal confinement of a period. However, we accept that the only, undeniable knowledge about a period is the fact that it occurred at a time i.e. it had a certain duration and took place somewhere within space-time. As a result, periods must be contiguous in space-time in the sense that a period happened within a spatiotemporal kind of coherence volume, within which the constitutive phenomena were active. Consequently, space and time are regarded as projections of the four dimensional nature of a period. Furthermore, we accept that the isolated space and time are considered as relative components that approximate the absolute space-time volume of a period.

It is worth noting that, while the space-time extent of a period is regarded as a unique attribute, the projected space and time may appear in great variation. The last statement stands by the fact that the geographical location and time span are rendered as an outcome of the inferring process that is carried out by researchers. Therefore, they are affected by local, social, personal and cultural components, like local naming conventions, personal belief, alternative reference to identical time frames or places and so on.

Observation and Period Spatiotemporal Confinement

Since the past is not directly observable [Doerr et al. 2004b], evidence about past periods or events is derived from traces that were left over by past phenomena. For instance, wood combustion by-products within the soil of a woodland provide clues for a wildfire event over that area, at a time that is specified by the dating process of the individual findings. The finding, gathering, processing and, furthermore, the argumentation of the evidence that is related to a period under question, is the result of the scientific observation process. Formally, an observation is defined as *the scientific knowledge about particular states of reality gained by empirical evidence, experiments, measurements and so on, and result in a factual proposition of belief* [CID 2011].

Primary observations grant knowledge, associated with a physical object, that express the possible evidence about the existence of a period. The observation process reveals information about the semantic association of a physical object and the period that it defines or expresses. In addition, dating processes [Doerr et al. 2004b], applied on the observable material, conclude into time frames within which the particular object was created, used or destroyed. Such time frames build up an approximating extent of the semantic related period. Finally, the place where the observation occurred provides information about the geographical space approximation. Summarizing the derived information, we result in three sets of knowledge ordered according to the observation sequence: spatial and temporal approximation, and semantic association. It is worth noting that the knowledge gained by the combination of multiple observations provides sufficient information that results into the spatiotemporal confinement of the period itself.

We accept that the true spatiotemporal extent of a period cannot be observed. Therefore, the derived knowledge from observations only approximates the true space-time volume of the period. Since the observable evidence indicates only its existence within space-time, there are difficulties on defining precisely its spatiotemporal boundaries. Consequently, the approximated volume represents its boundaries with a fuzzy layer, indicating that there is no certainty about the status of the period in the specified region, in the sense of activeness. There will be further analysis of the concept of the fuzzy layer in the following section.

2.1.3 Fuzziness: Phenomenal and Declarative Perception of Reality

Several questions arise in the field of philosophy, as far as the concept of reality perception is concerned. A noteworthy debate emerges from the epistemological question of whether the world we see around us refers to the real world itself or it portrays a simplified interpretation that conforms with our current level of understanding. In other words, there is a notable difference on how we perceive reality: either as an abstraction of the real world that is limited by our knowledge or as a total, undeniable image of the real world.

We make the following assumption about the perceived reality. There is a clear discrimination between the perceivable and the real world. On the one hand, the components that describe our world are declared in a way that is affected by our current knowledge. Therefore, the overall image of the perceived reality depends on our knowledge base. On the other hand, the real world may include factors or concepts that are beyond our sphere of understanding, resulting into a complicated representation that we fail to discern.

We adopt the convention of phenomenal and declarative reality, in order to describe the real portions and the perceivable world, respectively. The term phenomenal refers to the real world, while declarative is a reference to an abstracted image derived from empirical evidence combined with our knowledge. Evidence about the phenomenal world is gained through the observation process. The further process and reasoning over the observation data results into the declarative world, which is, in other words, the informational world. It is worth noting that the declarative reality is an approximation of the phenomenal world. Therefore, while our current knowledge increases, the declarative description tends to approximate the phenomenal reality with a higher level of precision.

In reference to the spatiotemporal confinement of a period that was mentioned in Section 2.1.2, it is clear that the temporal, spatial and hence the spatiotemporal bounds of a period are directly associated with the concept of phenomenal and declarative reality. We defend the last statement by focusing on the temporal approximation of a period. The true temporal extent of a period cannot be observed [Doerr et al. 2004b]. However, a suitable observer is able to identify related dates that are definitely before or after the true endpoints of the period. Moreover, the semantic association of absolute dating with the period under consideration gives rise to temporal consequences that approximate its true temporal bounds. The temporal model proposed in [Doerr et al. 2004b] introduces the notions of determinacy and indeterminacy in order to introduce an interval that efficiently represents the temporal extent of a period. Before proceeding to an extensive description of the former terms, we will introduce a component that represents the activeness of a period.

The determinant factor that indicates the status of a period is referred with the term intensity. Particularly, period intensity conceptually quantifies the evidence which justifies the existence of a period. On the one hand, high levels of intensity provide the appropriate clue to prove that the period is active, as there exists a considerable quantity of evidence. On the other hand, zero to negligible intensity levels are used to indicate the absence of evidence and therefore mark the period as inactive. Intensity at low to intermediate levels has a dual interpretation, since they may represent possible endpoints of the period such as start or end point. The appropriate threshold levels that distinguish the start, end points and the main "body" of the period is set by domain experts, based on the observation density.

Period intensity is closely intertwined with the terms of determinacy and indeterminacy that were mentioned above. Actually, high levels of intensity are associated with determinacy, which carries the interpretation that the period to be "on-going", hence active [Doerr et al. 2004b]. Conversely, indeterminacy is not related to low levels of intensity as it may be expected; it describes cases where the period status is not clear, i.e. neither active nor inactive. Particularly, with respect to the temporal approach we mentioned above, a determinate interval in a wider view represents a set of dates during which the period is active. On the contrary, an indeterminate interval that encloses the whole temporal extent of a period, is interpreted as a constraining temporal bound within which the period occurred somewhere. Consequently, indeterminacy intervals may refer to period endpoints such as start, similarly interpreted, i.e. the true start time point of the period cannot be observed but it occurred somewhere within the indeterminate interval.

Indeterminacy is considered as the representation of the fuzziness that is formed between the declarative reality and the phenomenal world. For instance, the phenomenal start time point of a period is approximated by a indeterminate interval that includes the desired endpoint. Although the concepts of determinacy and indeterminacy were proposed in a theory for the temporal approximation of a period [Doerr et al. 2004b], they can be applied to the spatial dimension as well. Consequently, every spatiotemporal theory should comply with the respective temporal one, because any temporal bound refers to the endpoints of the projection of the corresponding bounding volume that represents the period under consideration. It should be noted that the notions of phenomenal and declarative time-span, space and space-time volumes were introduced in CRM-Spatial Doerr and Hiebel [2013], an extension to the main CIDOC CRM ontology.

2.1.4 Time imprecision

The notion of time imprecision is directly intertwined with our reality. This association can be justified by considering that the temporal extent of every material object or phenomenon has imprecise boundaries. The last statement can be considered as an offensive claim against the reliability of various dating methods; however, in spite of the accuracy that a method can provide, precision cannot come out of imprecision.

An intuitive analysis of how time imprecision is considered as a part of our reality can be obtained by trying to answer the following question: ”when is the exact time that a baby is born?”. The event of birth by nature has imprecise temporal boundaries. There are several assumptions that approximate the aforementioned question; for instance, the moment when the baby comes out of his/her mother’s body, the moment when the umbilical cord is cut, or even the time that the baby takes his/her first breath of oxygen. The medical community accepts the second scenario, by convention; however, even with this assumption, there is no clue about the exact time point of the process.

In addition to the imprecision that is derived by phenomena, the act of temporal confinement also leads to imprecision. Particularly, every observation comes with an error level, even a minor one, that is induced by limitations of complexity of our reality. Most of these limitations concern our knowledge, e.g., the maximum granularity of time. Specifically, timeline is considered as a continuous spectrum; extracting distinct values from it lead to imprecision that is caused by the limitations in the granularity level. As a result, with respect to the aforementioned example of the birth event, the exact time of birth is approximated by the selection of a conventional measurement of time (seconds, minutes, hours, days, months, years). It is noteworthy that the appropriate granularity level must conform to the event precision. It would be illogical to date a birth event using nanoseconds even if it leads to lower imprecision.

2.1.5 Chronology and Temporal Relations

In Section 2.1.2, it is mentioned that the coherence volume of a period is spatiotemporally approximated by the combination of temporal and spatial data, which is derived by the related evidence. Particularly, findings like physical objects, which are extracted through observation, reveal information about events or periods, in which they were present, such as creation, destruction or historical use [Doerr et al. 2004b]. The place where the evidence is located outlines the declarative space of the associated period. Furthermore, qualities of objects such as state of decay, chemical alteration, deformation and so on are used as dating methods that define a time frame in which the period is considered active.

Dating of a period refers to a process in which the temporal bounds of the related coherence volume are approximated, by dating physical objects that are considered as evidence [Doerr et al. 2004b]. Assume a set of periods that compose a system under consideration. The temporal confinement of each period provides evidence for chronology. According to Oxford English Dictionary [dic 2010], chronology is defined as *the arrangement of events or periods in order of their occurrence* in time. Since events are treated as a special case of periods, chronological correlation is applied to the latter as well. The approximated start and end timepoints of each period are temporally associated, resulting into a sequence of endpoints, which is interpreted as an ordering of periods. Considering that past is not directly observable, the temporal-bounds approximation is considered as the determination of the minimal indeterminacy time-intervals that approximate the begin and the end time of each period in the system that is modeled.

The arrangement of periods over time refers to the representation of temporal knowledge that is related to the time intervals, which bound the time extent of the periods under consideration. Temporal association is a key factor in the chronology process, which allows the introduction of qualitative relations that are extracted from quantitative information like period endpoints. In other words, it is regarded as a method of assigning temporal topological relations between periods. The prevalent idea for representing temporal association is the Allen interval theory [Allen 1983], which describes all possible temporal relations that can be formed between two time intervals. CIDOC CRM expresses the basic temporal relations, as a reference to Allen operators, by introducing seven properties that stand between temporal entities and hence periods [Doerr 2003]. A further analysis of Allen theory is provided in the following section.

Temporal associations give rise to a better understanding of the time arrangement of periods that are included in the system under consideration. As a result, the process of chronology determines the most probable scenarios of possible pasts that appear to be consistent with our current evidence.

2.2 Related Work

This section provides a brief analysis of various solutions that address the basic problems which are the focus of this thesis, such as period spatiotemporal modeling, representation of imprecise temporal knowledge and topology description in relation with space, time and spatiotemporal association. The analysis begins with works that focus on the notion of space-time volume, as it is introduced in CRM-Spatial (CIDOC CRM extension). Then, we continue with a description of the spatiotemporal relations that are applicable to entities like periods, based on

CIDOC CRM. Afterwards, an extensive presentation of the temporal, spatial and spatiotemporal topology follows. Last but not least, we examine the problem of imprecise time modeling, followed by several approaches of temporal topology using vague information.

2.2.1 CRM-Spatial: Period Spatiotemporal Modeling

CRM-Spatial [Doerr and Hiebel 2013] is an extension of CIDOC CRM ontology that focuses on the modeling of space and time extent of a period. It is noteworthy that the space time boundaries are a mere approximation of the actual process of spread, extend and retreat of the phenomena, which are treated as a set that forms the period under consideration. This spatiotemporal confinement is formally represented with an irregular figure in space-time that is named as space-time volume.

The authors comply with the concept of phenomenal and declarative reality. Particularly, they accept that the only absolute knowledge, which is directly intertwined with certainty about a past period, is the phenomenal space-time volume that it occupies. This knowledge signifies the occurrence of the comprising phenomena; in other words, the phenomena happened somewhere and at some time. Further analysis of the phenomenal space-time volume into its dimensions reveals information about the true time and the space extent of the corresponding period.

Although the phenomenal components of a period represent the true space and time extent, they are not directly measurable, since the past is not directly observable. This setting gives rise to the quest for spatiotemporal approximation of periods using empirical evidence. Specifically, observations that are associated with semantic-related findings lead to the introduction of geometric and temporal expressions. These statements are a formal representation of the results that were derived by the dating and geographical approach process of the corresponding findings. Also such expressions outline the declarative time and space extent of the period under consideration. This spatiotemporal confinement is an approximation of the true time and space boundaries that conform with the current level of our knowledge. Consequently, the phenomenal volume is approximated by the definition of the corresponding space-time volume as an outcome by combining the declarative temporal and spatial data.

Authors provide the basic theoretical approach of the spatiotemporal confinement over periods. They introduce a set of entities and relations, in an ontological form, that describe the aforementioned scenario of space-time approximation. However, there is no further analysis on how observations obtain temporal information and how this kind of information leads to spatiotemporal confinement and hence declarative volume forming, as they mostly focus on the theoretical base of the problem.

2.2.2 CIDOC: Relations over Periods

According to CIDOC CRM, the spatiotemporal boundaries of a period are considered as a mere approximation of the actual process of growth, spread and retreat of the comprising phenomena. Disjoint and intersecting boundaries lead to the extraction of possible associations between the corresponding periods, such as coexistence or separation. Several examples that illustrate such relations between periods are often described in literature, e.g., when a nomadic culture exists in the same area as a sedentary culture.

Spatiotemporal association between periods refers to the description of the relevant topology that relates their space-time volumes. The ontology of CIDOC CRM introduces a set of binary, symmetric and spatiotemporal relations (and their reversed versions), which associate pairs of periods. The relations represent the scenarios of overlapping and disjoint periods and they are denoted as *overlaps with* and *is separated from*. On the one hand, *overlaps with* allows instances of period that overlap both temporally and spatially to be related, hence they share some spatio-temporal extent. On the other hand, *is separated from* relates instances of periods that do not overlap, either spatially or temporally. It is noteworthy that none of the aforementioned relations imply any temporal or spatial ordering or sequence between the associated periods.

Since pure spatiotemporal relations are limited to inclusion association, any temporal ordering between periods is based on Allen interval algebra. Particularly, CIDOC CRM introduces a set of binary temporal associations among periods; these temporal relations correspond to Allen operators. Further analysis on Allen interval algebra is provided in Section 2.2.3.

Similarly to CRM-spatial, spatiotemporal relations that are introduced by CIDOC CRM form an ontological approach of space-time association. As a conceptual reference model, CIDOC does not offer any formalization of the definition of relations; in other words, with respect to CRM-spatial, it does not provide further analysis on the way declarative space-time volumes are associated (resulting in the corresponding spatiotemporal relations).

2.2.3 Temporal Topology

The prevalent idea of temporal knowledge representation is Allen interval algebra. According to Allen's theory, a time interval is considered as an ordered set of time points that represents a time frame on the timeline. Since time interval is a continuous spectrum, it is formalized by a pair of time points that stand for the endpoints of the corresponding time frame, as depicted in Figure 2.1. For instance, a time interval that represents the temporal extent of World War I is expressed under the semantics of a pair set [a, b]. Time points a and b refer to the starting and ending date of the event, in our case 28 July 1914 and 11 November 1918. It is noteworthy that since the timeline is a continuous spectrum, higher granularity on the approximation of endpoints is applicable.

Temporal topology describes the temporal association between pairs of time intervals. It is expressed by a set of operators, noted as Allen operators, that represent the possible relations between two time intervals. The operators are formal-

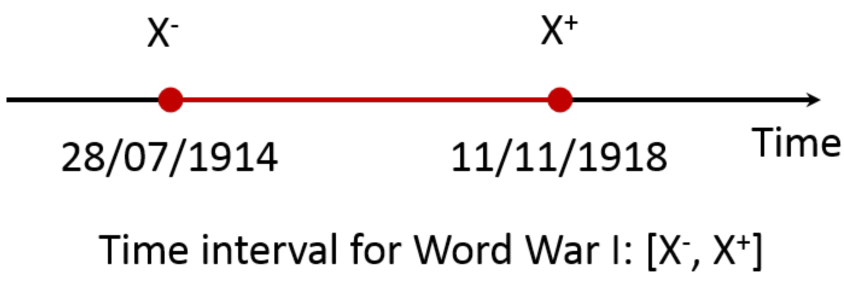


Figure 2.1: Time interval based on Allen interval algebra

ized using a set of rules, which are applicable to the endpoints of the time intervals. For instance, operator *meets* semantically represents a meeting in time and is denoted as *A meets B*, where A and B stand for two time intervals. The rules that describe the operator *meets* associate the endpoints of interval A and B properly, in order to express that the end of A signifies the start of B. Figure 2.2 illustrates the meeting in time of the Neopalatial and Postpalatial periods that refer to the last two phases of the Minoan civilization.

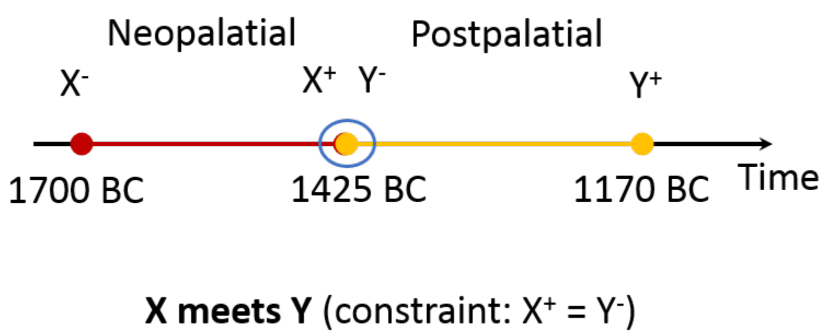


Figure 2.2: Neopalatial meets in time Postpalatial period

Allen introduced seven basic temporal operators that express the following relations: before, meets, overlap, starts, during, finishes and equal. It is worth noting that every relation is expressed both ways, introducing the corresponding inverse operators: after, met-by, overlapped-by, started-by, includes, finished-by and equal, respectively. Figure 2.3 depicts a full analysis of the basic temporal

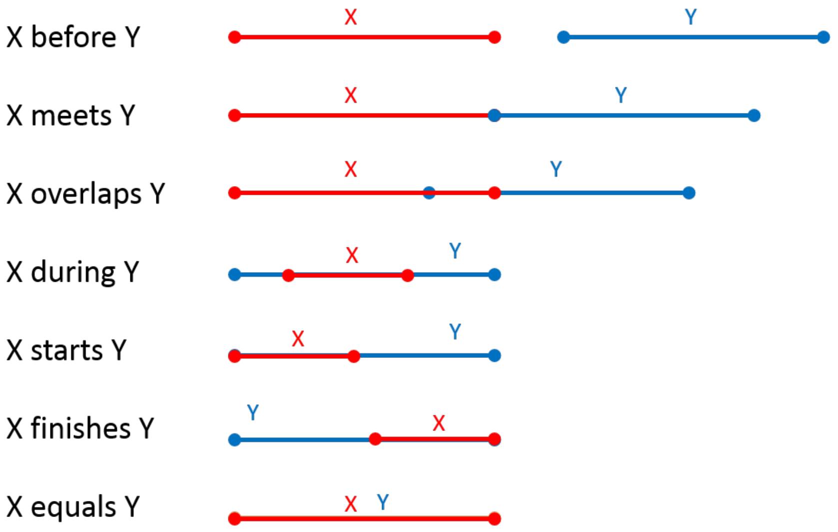


Figure 2.3: Allen operators

operators and Figure 2.4 illustrates the inversed versions.

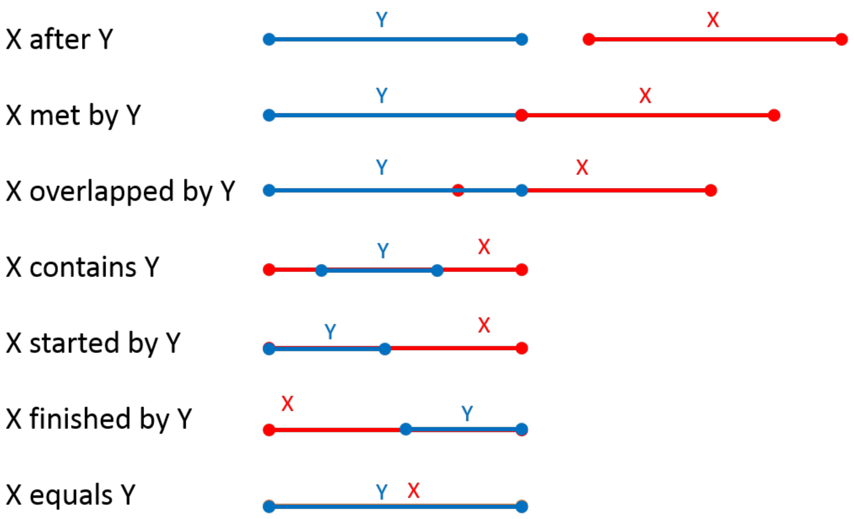


Figure 2.4: Inverse Allen operators

Although Allen interval algebra provides a set of operators to describe the topology between time intervals, expressing the temporal association between periods proves to be more complex. Periods, as already mentioned, are four-dimensional entities; however, Allen algebra is applicable to pure temporal components like intervals. As a result, Allen operators that are applied to periods are only able to associate their time projections, ignoring their space dimension. The extracted relations that are based on purely temporal extents do no treat periods as individual, complete entities; as a result, the total association is blurred.

2.2.4 Spatial Topology

Spatial topology focuses on rules that concern relationships between points, lines and polygons, which represent spatial regions. There are many theories that focus on the description of spatial topology. Despite the great variety of algebras, there is a basic setting of spatial relations that are alternatively approached by each theory. For the sake of simplicity, we provide a brief outline of the approach of [Egenhofer and Franzosa 1991], the Point-Set Topological Spatial Relations [Egenhofer and Franzosa 1991] [Bruns and Egenhofer 1996], which covers the topic of this thesis.

According to [Egenhofer and Franzosa 1991], a spatial region represents a geographical area and is formalized as a set of space points. Each region is semantically separated into additional sets, consisting of interior and boundary sets. It is noteworthy that space points are defined according to the selected reference space, e.g., the globe, the limits of a geographic region, the space that is occupied by a building and so on.

Spatial topology is described by relations that are applied between region sets. Particularly, it represents the association of two geographical regions, in terms of intersections of the corresponding boundary and interior sets. Each relation is expressed with set-theoretic rules such as equality, disjointness and containment. Considering the possible combinations between the interior and boundary sets of two associated regions, a total of eight topological relations are described. Briefly, two possible static spatial regions can be associated with one of the following relations: equals, contains, covers, covered-by, disjoint, intersects, touches and

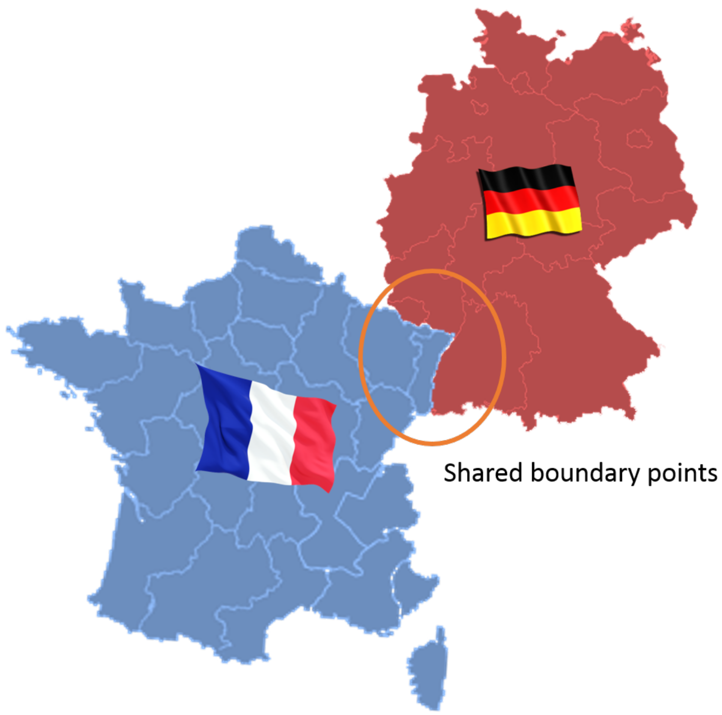


Figure 2.5: Region of France touches region of Germany

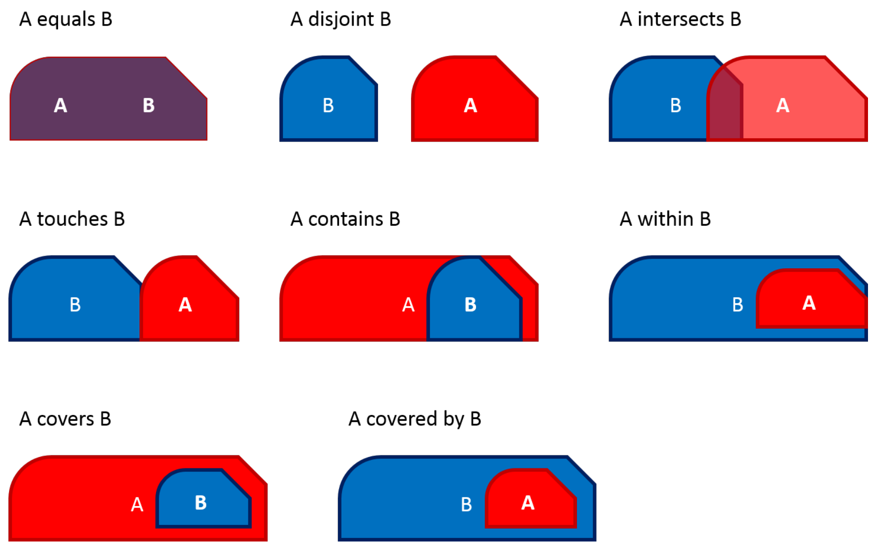


Figure 2.6: Spatial relations based according to Egenhofer

within. For instance, relation *touches* between two regions refers to the case that the associating regions have at least one common boundary point and no common interior points. Figure 2.5 illustrates A *touches* B where A and B represent two neighboring countries, Germany and France, respectively. Finally, Figure 2.6 shows a detailed analysis of the eight spatial relations.

2.2.5 Spatiotemporal Topology

While there are algebras that focus on the description of temporal or spatial topology, as it was mentioned in Sections 2.2.3 and 2.2.4, there are few approaches that model relationships between regions located in space-time. The most prevalent work has been carried out, to the best of our knowledge, by [Claramunt and Jiang 2001]. The authors propose a set of relations that describe spatiotemporal topology. Their main focus is the association of geographical areas that change over time. For instance, the use case they analyze is the topology of a region that refers to a pollution source with respect to the neighboring geographic parcels. For the purposes of their work, they assume a three dimensional space-time, which stands for two dimensional space that is combined with the temporal dimension.

The authors introduce the notion of temporal region that stands for a space which is located in time and is described by a convex temporal interval. The related time interval is regarded as the life span of the representing temporal region. The spatiotemporal association of these regions is approached by space topological relations that are valid only for the time that the regions are active, according to their life span. Note that the temporal relations result to no extra information about spatial proximity.

Temporal and spatial relations are formalized as two order matrices. The rows and columns of each matrix refer to the respective temporal and spatial endpoints of the individual first and second interval, respectively. The resulting spatiotemporal relations emerge from the combination of distinct temporal and spa-

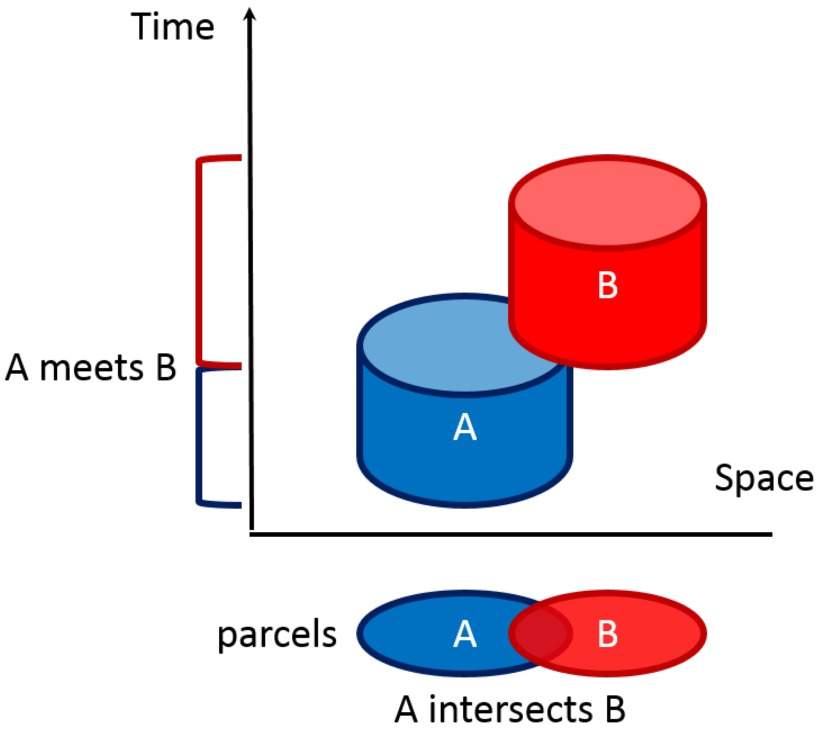


Figure 2.7: Relation meets intersect on temporal regions

tial algebras that lead to associations like ”before overlap”. The formalization of such relations is derived by the cross product of the corresponding matrices, which stand as representatives of the relations. For instance, Fig. 2.7 portrays the relation of ”meets intersect” between temporal regions; this relation represents the association of two parcels of land.

In spite of the spatiotemporal associations that were proposed in [Claramunt and Jiang 2001], they are limited to the description of the spatial topology over temporal regions. They also lack a genuine description of the relations that are formed between their space-time points, blurring their total association, as they conclude to relations that are bounded to a limited life-span.

2.2.6 Imprecise Time and Temporal Topology

There are several approaches that focus on the representation of imprecise temporal information. The main idea is related to the definition of a more generalized version of time intervals that can model fuzzy boundaries. The concept to be adopted is introduced in [Doerr et al. 2004b] and concerns the definition of determinate and indeterminate intervals. According to this method, each time interval is confined with pairs of endpoints that define a layer of certainty and a fuzzy zone. In the rest of this section, we provide the various alternative approaches of temporal topology representation over imprecise data.

The prevalent idea of depicting the relevant association of imprecise time intervals is the definition of a set of temporal operators; this is actually an approximation of all temporal relations which possibly describe the temporal topology of the scenario under consideration. It is formalized using a set of disjunctions that connects all possible temporal operators [Allen 1983].

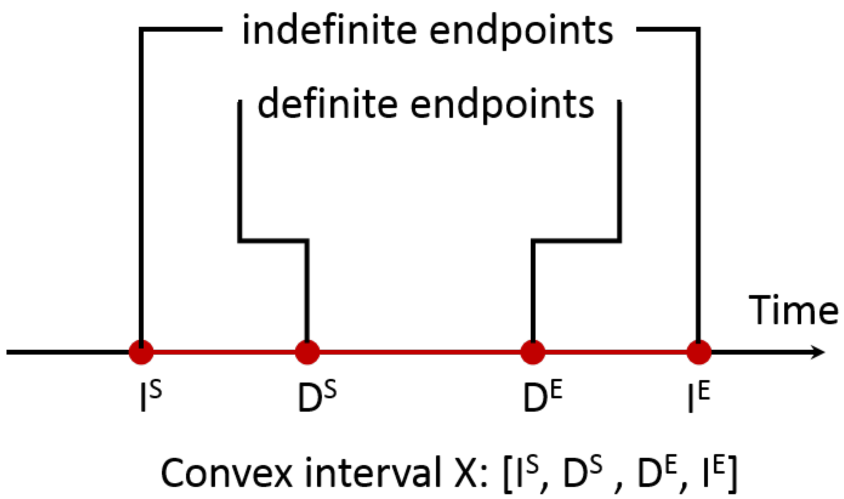


Figure 2.8: Convex Valid Interval Stamp

An alternative approach is the interval algebra for indeterminate time that was proposed in [Cowley and Plexousakis 2000]. According to the authors, a possibility value is attached to each Allen operator. More specifically, the definition

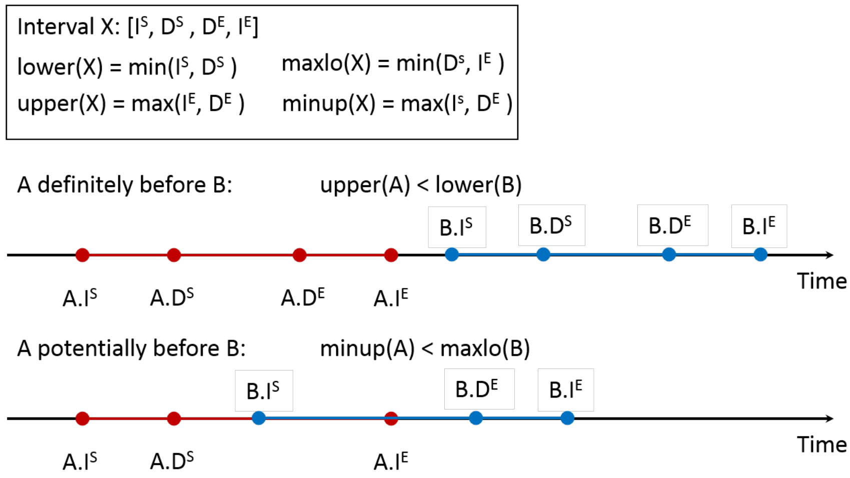


Figure 2.9: Potentially and definitely before

of a time interval is extended in order to express the indefinite and definite end-points, as depicted in Figure 2.8. In addition, each temporal relation is loosened or strengthened properly, in order to express certainty and possibility. For instance, as depicted in Figure 2.9, temporal operator *before* has a dual interpretation as "potentially before" and "definitely before"; these express the notion of possibility and certainty, respectively. It is worth mentioning that the latter definition semantically coincides with the corresponding Allen operator.

Another approach, that is related to the previous work of indeterminate time intervals, focuses on the expression of indeterminacy and incompleteness of temporal information using three valued logic, as introduced in [Gadia et al. 1992]. However, their work provides a different set of operators than Allen introduced; furthermore, they focus on the examination of temporally missing values.

A similar work that adopts the concept of certainty and possibility among temporal relations is proposed in [Doerr and Yiortsou 1998]. The authors introduce a set of so called uncertainty operators. These operators are distinguished into existential and universal, which semantically implies the prefix of "can be" and "must be" to each one of them. Particularly, the aforementioned categorization is related to the mathematical formalization of the two types of operators i.e.

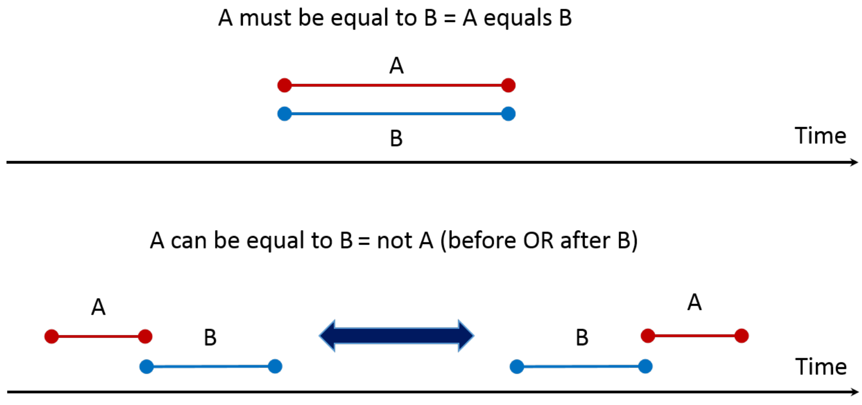


Figure 2.10: must be equal and can be equal

existential ones makes use of ∃ (there exists) whereas universal ones are defined with ∀ (for all). Furthermore, authors provide an alternative implementation of the uncertainty operators by using logical expressions that include Allen operators [Allen 1983]. Figure 2.10 illustrates an example of uncertainty operators; for comparative reasons, we analyze the *can be equal* and *must be equal* operator.

Finally, a work that is mainly related to the representation of imprecise temporal knowledge is carried by [Kauppinen et al. 2010]. According to this work, boundaries of successive periods are approximated and associated, since the definition of a clear cut between them is regarded as a difficult to impossible task. Particularly, the authors formalize a time interval using fuzzy set theory, resulting to fuzzy time intervals. The fuzzy function, in this case, evaluates the probability of a time point to be included within the interval. In addition, they propose a quadruple of endpoints to represent the fuzzy endpoints of a time interval. The following boundaries are introduced: fuzzy begin, begin, end and fuzzy end; these illustrate the earliest and latest start and end of a time interval.

Furthermore, they propose a method that quantifies the association of two intervals by calculating a value which represents the percentage of overlapping between fuzzy intervals. This method is based on the intersection of the corresponding fuzzy time intervals and quantifies the required amount of overlap between two fuzzy intervals, in order to be considered as associated and hence successive.

2.2.7 Discussion

Temporal topology is described using a set of operators that are introduced in Allen interval algebra, as it was mentioned in Section 2.2.3. However, the time imprecision that is derived from past phenomena raises the need for a new approach that can deal with fuzzy boundaries. Recall that time intervals, as introduced by Allen, are modeled as ordered sets of time points; such an ideal and complete temporal extent cannot describe real phenomena, as described in Section 2.1.4.

Most of the proposed solutions that approximate the temporal topology between fuzzy intervals introduce new relations, in the sense of modifying the basic Allan operators. This is realized either by applying values of possibility, such as potentially or definitely, or by introducing disjunctive sets of operators.

Although there are various solutions that address imprecise temporal knowledge modeling, none of the aforementioned works propose a similar approach that focuses on four dimensions. In other words, the temporal association is applied exclusively to the temporal extent of a period, rather than considering it as a whole spatiotemporal entity.

Chapter 3

Temporal Knowledge from Semantics

Contents

In the previous chapter, we provided a detailed reference to the background theory that frames the basis of this thesis. Additionally, we analyzed in detail research efforts that are closely related to imprecise temporal knowledge modeling. In this chapter, we move on to the core of this thesis, which focuses on proposing solutions to the objectives that were stated in Section 1.2.

This chapter presents various methods that mainly address issues related to the extraction of temporal topology out of semantic information. Particularly, a method for period temporal confinement is provided, using point-wise and scarce information derived from observations. This is followed by an analysis of a concept that assists in the realization of continuity over successive periods, by modeling meetings in time. Afterwards, we focus on the temporal association over periods using information related to primary observations and semantics. Finally, we proceed to the parameterization of the temporal topology between time intervals according to the available knowledge, by introducing a temporal knowledge hierarchy graph, expressed with Allen operators.

3.1 Mutually Exclusive Evidence

Since past is not directly observable, temporal confinement over periods is actually a mere approximation of the true temporal boundaries that are derived through observation. Consequently, temporal extent is considered as an indeterminate region. Assertion of certainty among the indeterminacy regions over the period's temporal extent is possible by considering the concept of mutually exclusive evidence.

A period is regarded as a unified entity; in other words, period instances cannot stop and start over again. If the comprising phenomena signify the end of a semantic phase, then the representative period is marked as finished. There are cases in which an already completed period seems to continue after the intervention of intermediate events or periods. This kind of scenario is described with the introduction of a new period instance that models the latter phases. Let's explore the case of a building whose utility changes over time. Suppose that, in the beginning, the building was used as a forge. Later on, it was converted to a hospital, whereas, eventually, it was re-used as a forge. The purpose of the current problem is to provide a sufficient model that describes the lifetime of the building. Note that the term lifetime refers to the temporal extent within which the structure had a noteworthy usage; the spatial dimension is not excluded since periods are spatiotemporal entities, but we assume that the space occupied by the building or parts of it declares its spatial confinement. We start with the definition of a super period that describes the lifetime of the building as a whole. There are three notable sub-periods that build up the main period. Each one covers a specific portion of the buildings' lifetime, i.e. forge, hospital and latter forge, respectively.

The concept of sub-periods describes the various stages that a period consists of, with the demanding condition that the individual phases have notable semantic differences. The last statement is crucial. For the sake of simplicity, we assume an alternative scenario of the previous example where a building has only two phases: forge and hospital usage. Findings related to medical gear or tools, rooms structured as infirmaries and relevant material signify the start of the hospital phase for the building. Such a conclusion negates the former forge structure, with the assumption that the forge cannot host such kinds of medical equipment and facilities. On the other hand, findings that are related to smithing equipment such as anvils and elevated amounts of metal materials imply the extension of the current, forge period. The unified nature of a period may imply certainty and impossibility of existence based on the semantics of the available evidence.

The problem that emerges is the association of the observation data into spatial and temporal approximations of the corresponding period. Alternatively, it can be viewed as the definition of spatiotemporal boundaries using observation data. A spatial approximation is regarded as the place where the observable findings were located. Scenarios that refer to moved findings or spatial distortion that was caused by deformation of the earth's crust require further inference and reasoning that is beyond the scope of this thesis. In other words, we will focus on temporal boundaries of observations and, consequently, on the introduction of the building blocks of the chronology process. The main concept is based on simple reasoning. Material evidence such as things, features or traces introduce causal "genesis" events. For instance, a ceramic finding implies a creation event of the object itself. Such creation events have indefinite temporal bounds that are revealed by the age of the finding. Dating methods approximate a time frame that indicates the age of the material that the finding is made of. As a result, the "genesis" event of a finding introduces an indefinite time interval within which the finding was made. The temporal extent of the creation event reveals a fragment of the period with which the former finding forms a *part of* relation. Consequently, an observation, i.e. finding the object concludes into the definition of the outer boundaries of chronology for the constructing period. A possible distribution of likelihood is revealed, indicating a possibility of existence for the period within the indefinite temporal bounds that were introduced by the ”genesis” event. Figure 3.1 depicts the association of a ceramic finding with the habitation period through the ”genesis” event, i.e. pottery. Since the habitation period over a place is implied by the evidence of human activity, the pottery event reveals a time frame of possible residence.

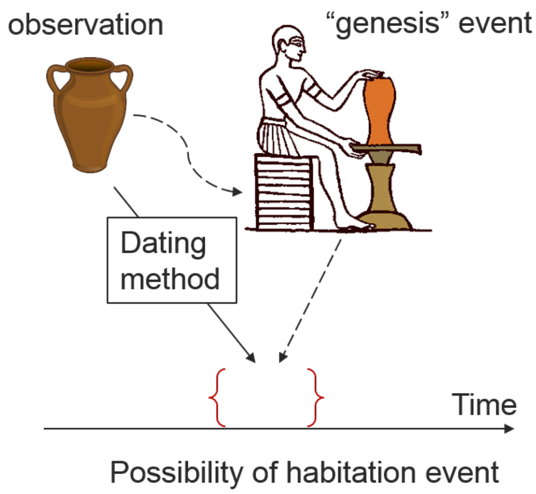


Figure 3.1: Building blocks of chronology

Having introduced the indefinite boundaries that come with the observation process, a new problem emerges, one that focuses on the temporal confinement of the corresponding period. This issue is addressed using the semantics carried by observations. As a result, the former problem essentially involves finding how semantic information derived by observations can form the temporal extent of the period itself. More specifically, it deals with the manner in which the indefinite temporal projections of the observations can imply regions of certainty and nonexistence of the period. The main idea is related to the semantic consistency that associates the observations with the period. We make the assumption of positive and negative observations based on their semantic association with the period under construction.

On the one hand, contextual coherence of positive observations forms a *part of* relation with the period to be defined, as analyzed in CIDOC CRM [Doerr 2003]. On the other hand, negative observations entail semantic inconsistency, precluding the possibility of co-existence or inclusion relations with the period. Let us focus on an example from the field of stratigraphy, a method of studying soil based on observations, revealing information about the successive periods over a specific geographical region. We assume the scenario of modeling the habitation period of an ancient building that was constructed with marble bricks. Observations that include marble findings are regarded as positive; hence, layers with high intensity of marble findings outline a period in which the building is considered as active, i.e. inhabited. On the contrary, material objects like ash derived by wood combustion by-products lead to negative observations; it is counter-intuitive for a habitation event to occur in the same time and space where a wildfire is on. In other words, fire and marble bricks cannot co-exist in the same space and time during which the building was inhabited. Consequently, soil layers that are associated with ash findings cannot co-exist with the former ones. Figure 3.2 illustrates the layers of positive and negative observations, related to the building example.

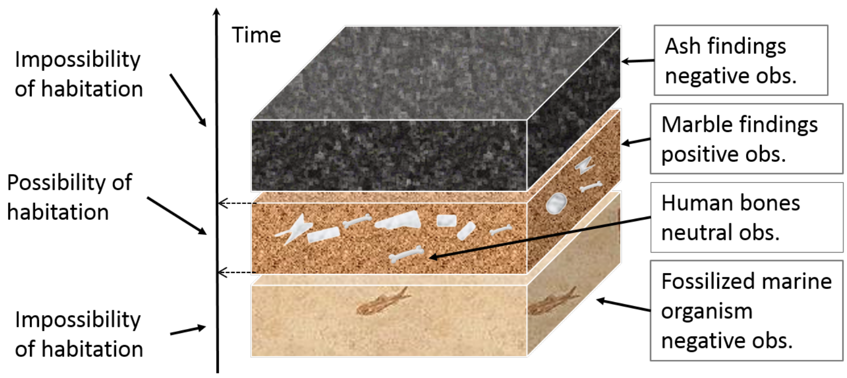


Figure 3.2: Period of habitation

The temporal extent that is derived by positive observations forms an inner, determinate interval, within which the period is regarded as ongoing [Doerr et al. 2004b] [Kauppinen et al. 2010], as depicted in Figure 3.3. On the contrary, negative observations define outer bounds, which entail the cessation of existence for the period. It is worth mentioning that due to the unified nature of a period, it would be counter-intuitive to consider that it is on-going only during the time extent which is confined by positive observations (and inactive elsewhere), because the unity property is violated.

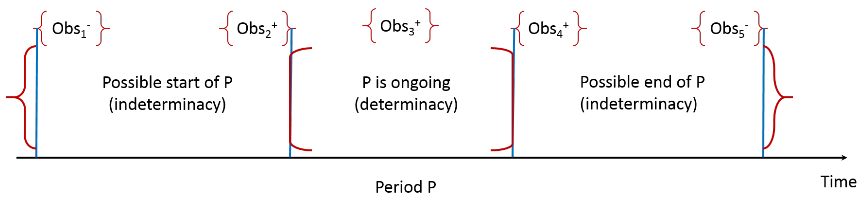


Figure 3.3: Lifetime of a building

As a result, a period is ongoing till there is no positive evidence to prove that claim, whereas a period is marked as possibly active till a negative observation proves that it is inactive. Finally, the intermediate intervals between positive and negative observations result in a possibility of existence for the period in question, rather than a precise state (e.g., ongoing or finished). Note that neutral observations, such as human remains, which are dated during the marble findings are considered as insufficient to conclude in a claim of inconsistency. With respect to the building inhabitation example, as depicted in Figure 3.2, there are three notable intervals that are confined by the temporal extent of each layer. The first one is described by marble findings and represents the indeterminate interval in which the building was possibly inhabited; the other two, which refer to ash findings and fossilized marine organisms, represent the regions that invalidate the building's usage as a settlement. The earliest and latest marble observation, in the sense of temporal ordering, form the determinate region in which the building was inhabited.

3.2 Meetings in Time

Time imprecision is an unquestionable component that is directly connected with reality, as it was mentioned in Section 2.1.4. Due to this fact, the temporal approximation of a period also bears a level of imprecision, making it difficult or impossible to define a clear-cut relation between pairs of successive periods [Kauppinen et al. 2010].

To address this issue, we adopt the concept of a transitive event. The main idea is based on the introduction of an indeterminate interval that would temporally confine the meeting, instead of trying to approximate the exact time point which refers to the meeting. Particularly, the selection of the exact time point is converted to the definition of an indefinite interval that includes that time point. Indisputably, this kind of conversion is regarded as knowledge, since it approximates an imprecise task of retrieving distinct values from a continuous spectrum like time. This approach takes into account the non-instantaneous nature of a meeting in time. For instance, the period that represents the lifetime of a person (focusing on the basic phases of life) consists of various successive sub-periods such as embryonic period, infancy, childhood, puberty, adulthood and so on. The transition from embryonic period to infancy can be seen as a meeting in time between the corresponding periods. However, the continuity in time between these periods is realized with the consideration of the intermediate event of the birth, which obviously has a non-negligible temporal extent.

In essence, we consider meetings in time as convex intervals with indefinite bounds, indicating that the phenomenal meeting happened sometime during the transitional event. Considering the motivating example that we described in Section 1.2.1, the Trojan War is regarded as a transitional event that connects the two identified periods of the Greek and Trojan rule over Troy. It is noteworthy that the temporal extent of the war event is an indefinite time interval, which confines the meeting in time (Figure 3.4). The indefinite temporal extent of the Trojan War is related with the imprecision that derives from the imprecision of

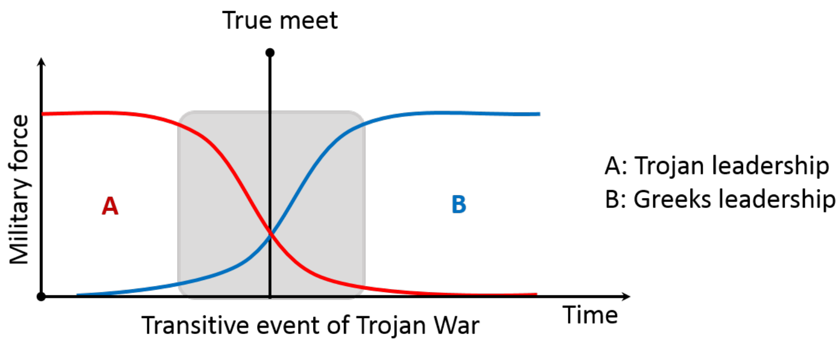


Figure 3.4: Adaptation of meetings as a transitive event

the event itself (Section 2.1.4) i.e. we cannot pinpoint the exact time point that the last soldier gave in. Similarly, in accordance with the previous example of the transition between the embryonic and infancy period, the phenomenal meeting of the successive periods occurred during the birth event (transitional event); also, it is confined by an indeterminate interval (with respect to the imprecision that comes with the birth event). The temporal extent of the event depends on the intensity of change, i.e. the change rate of the transition factor between the meeting intervals. In reference to the birth example, the transition factor is every characteristic on the human body that distinguishes the embryonic period from the infancy. Respectively, the transition factor for the Trojan War is the military force of the challenging armies. For modeling reasons, we assume that a "true" meeting in time is not observable but happened somewhere within the fuzzy bounds of the transitional event, which can be constrained.

3.3 Primary Observations and Topology

Temporal topology over periods refers to the temporal association of the corresponding intervals that represent their temporal extent. The prevalent methodology of representing temporal knowledge includes the use of dating processes, in order to provide a sufficient temporal confinement over the periods under consideration. Then, the extracted representative intervals are associated by applying a set of rules that are derived from a temporal algebra, e.g., Allen operators (endpoint association). The resulting relations represent the relevant topology between the time intervals.

However, there are cases where temporal topology is expressed using qualitative association instead of quantitative relations (which refer to endpoint associations). Particularly, the usual method of temporal knowledge representation is based on the dating process, also called absolute chronology. It is a method that refers to quantitative relations, which are regarded as the outcome of temporal boundary associations. In contrast, there are methods that express qualitative association where the main goal is the relative chronology based on event ordering, inclusion and temporal distances [Doerr et al. 2004b].

We assume the concept of primary observation as a special, simplified case of the act of observing, in which the resulting information is not subjected to further reasoning and inference. The knowledge that is gained by such a process is related to the semantic association of the physical thing that is being observed, with the period to be modeled. Qualitative temporal information can be granted by primary observations, by analyzing the semantic association that is extracted. Particularly, semantic coherence leads to inclusion relations; in the sense of temporal relations, it is expressed as during, starts, finishes or equals, in terms of Allen operators. On the contrary, semantic inconsistency entails disjoint relations that are equivalent to temporal relations after or before.

We distinguish the following types of semantic association: ”before”, ”temporal distance”, ”part-of”, ”not co-exist” and ”starting/terminating event”. Each one of the aforementioned semantic associations represents a case of relevant chronology. The relations ”before”, ”temporal distance” and ”not co-exist” imply disjoint associations, whereas the rest stand for inclusion.

Semantic ”before” is observable through the following methods: historical records, order of traces and causal relations. More specifically, historical records such as king lists reveal the temporal sequence between the reign of successive kings. Observations that reveal the trace order of different events indicate a temporal sequence, which is derived by overlaying events or partial replacement; examples include construction of additional levels of a building or a system of scratches in different directions. Causal relations refer to the necessary prerequisites of an event, such as a birth event of a person that must have occurred before his death. Figure 3.5 illustrates a visual representation of the aforementioned cases of semantic "before" relation.

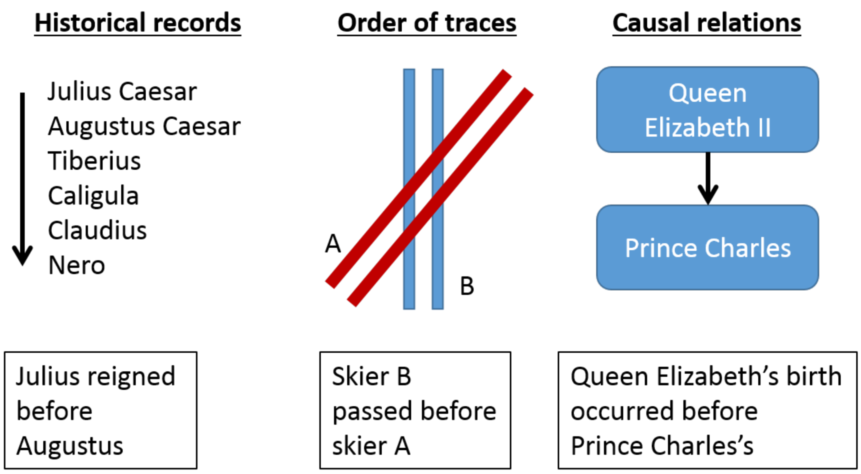


Figure 3.5: Semantic "before" association

A "temporal distance" association arises by relating the size of an effect to an estimated rate of change, such as tooth abrasion between birth and death, where the average age of a body is approximated based on the level of enamel wear. The "part-of" and "not co-exist" relations are defined in accordance with CIDOC CRM [CID 2011], representing the *part of* and *separated from* relation, respectively. Recall that *part of* implies an inclusion relation of semantic coherence between events or periods and hence a common spatiotemporal extent, such as death of soldiers within a battle event. On the contrary, *is separated from* reveals semantic inconsistency and therefore it precludes a shared spatiotemporal extent. For instance, the lifetime of different persons cannot co-exist because their physical substance (their bodies) would merge, which is obviously an impossible scenario. Finally, "starting and terminating events" are associations that stand for a special case of *part of* relation, in terms of CIDOC CRM. They are considered as triggers, such as the volcanic eruption of Vesuvius that terminated the inhabitation period over the territory of Pompeii.

Semantic association	Temporal relations
before	before
temporal distance	before
not co-exist	before
part of	meets, overlaps, starts, during, finishes or equals
starting / terminating event	meets, overlaps, starts or finishes

Figure 3.6: Implied temporal relations based on semantic association

	Implied semantic association				
	before	temporal distance	not co-exist	part of	s/t event
before		✓	✓	✗	✗
temporal distance	?		✓	✗	✗
not co-exist	✓	✓		✗	✗
part of	✗	✗	✗		?
s/t event	✗	✗	✗	✓	

Figure 3.7: Implied semantic relations based on semantic association

The aforementioned semantic associations can imply further semantics. For instance, consider the scenario of two periods that do not co-exist; it is implied that a possible "part-of" relation cannot hold between them, because they do not share any spatiotemporal extent. On the contrary, if a period is considered as a "starting event" of another period, then a "part-of" association is implied and so on. Figure 3.6 illustrates a complete analysis of the resulting temporal relations that are derived from semantic association of periods. Fig. 3.7 depicts all possible semantic implications that are derived between two periods by analyzing the implied semantics.

3.4 Temporal Knowledge Hierarchy Graph

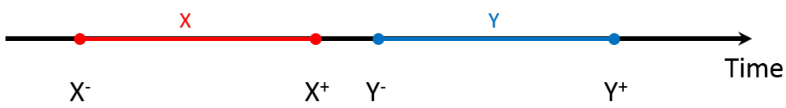
The prevalent idea for representing temporal knowledge is the Allen interval algebra, as it was described in Section 2.1.5. According to Allen's theory, associations among time intervals are applied to complete temporal knowledge. The last statement comprises a constraint inextricably connected with the definition of time intervals using precise endpoints. Such a prerequisite is essential in Allen algebra, since temporal operators are built up using endpoint associations between the intervals under question. Incomplete temporal knowledge and consequently indefinite boundaries cannot straightforwardly support endpoint relations.

The notion of time imprecision is related to our reality as it was mentioned in Section 2.1.4. Plots about past scenarios are built using scarce information derivable from observations. It holds that the absence of knowledge and time imprecision form the major components that affect the exploration of the past. Complete temporal knowledge is difficult or even impossible to be acquired; therefore standalone single Allen operators fail to describe the topology of a set of imprecisely defined periods. However, sets of possible associations, expressed using Allen operators between the related periods, can be concluded by exploiting the available temporal knowledge. Such relations represent disjunctive associations that are related to the current level of knowledge between the periods under consideration.

In a wider sense, the temporal topology between periods depends on the degree of knowledge and is formed as follows. In the case of information absence, the possible association of two periods is any Allen operator, as there is no endpoint relation to provide any limitation. As the temporal extent of the individual periods is being approximated with higher precision, the set of the possible associations is shrinking, tending to be reduced into a single operator, in the case of complete temporal knowledge.

The aforementioned concept is the building block of the temporal knowledge hierarchy graph. Particularly, various states of knowledge (characterized by a level of ignorance) are subjected into an hierarchical association and grouping. The resulted graph structure represents possible temporal relations and their correlation in accordance with the level of knowledge imprecision. To begin with, we assume two time intervals X and Y and the corresponding endpoint constraints that hold between them. Note that each one of the applied constraints expresses an ordering relation between pairs of endpoints that belong to the related intervals. Figure 3.8 illustrates the Allen operator *before* that stands between two intervals and the corresponding endpoint constraints. Since the definition of a valid time interval X conforms with the major prerequisite that $X^- < X^+$ i.e. its start cannot be later in time than its end, the required constraints for each temporal operator are simplified. In Figure 3.9, a full list of Allen operators and their constraints is depicted.

X before Y



$X^- < Y^-$	$X^- < Y^+$
$X^+ < Y^-$	$X^+ < Y^+$

Figure 3.8: Allen's operator before and constraints

Imprecision on temporal knowledge can be regarded as a scenario of fuzzy constraints where there is no certainty about the order between the endpoints of the related intervals. We introduce a fuzzy operator that represents the state of unknown and is denoted as "?". In reference to the previous example of relating intervals X and Y, the constraint $X^-\ ?\ Y^-$ represents a scenario in which the association of the endpoints X^-, Y^- is unknown.

The operators that are used to express the temporal constraints can be orga-

Relation	Constraints			Description
X before Y	$X^+ < Y^-$	-	-	X ends before Y starts
X meets Y	$X^+ = Y^-$	-	-	X ends when Y starts
X overlaps Y	$X^- < Y^-$	$X^+ > Y^-$	$X^+ < Y^+$	X starts before Y starts and X ends within Y
X starts Y	$X^- = Y^-$	$X^+ < Y^-$	-	X starts when Y starts and X ends within Y
X during Y	$X^- > Y^-$	$X^+ < Y^+$	-	X starts and ends within Y
X finishes Y	$X^- > Y^-$	$X^+ = Y^+$	-	X starts within Y and X ends when Y ends
X equals Y	$X^- = Y^-$	$X^+ = Y^+$	-	X starts when Y starts and X ends when Y ends

Figure 3.9: Temporal associations and constraints

nized into an hierarchy based on the level of ignorance that they express. Complete knowledge can be described using the basic simple comparison operators: $<$, $>$ and $=$. The intermediate level of awareness is described by mixed operators such as $\leq$ and $\geq$; while full ignorance is represented by the proposed operator ?. A full hierarchy graph that associates comparison operators according to the level of ignorance is depicted in Figure 3.10.

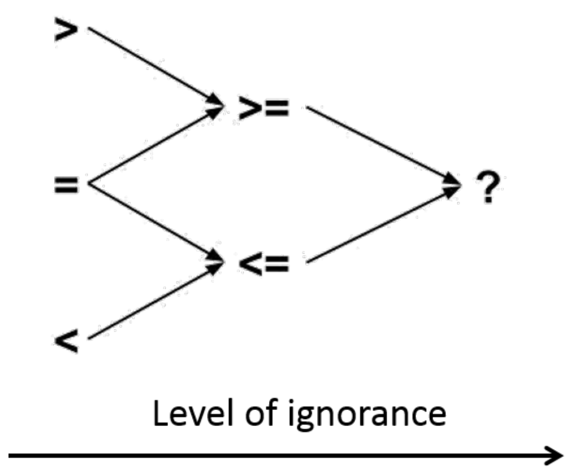


Figure 3.10: Operators hierarchy

The temporal knowledge hierarchy graph consists of nodes and arcs. Each node refers to a set of temporal associations and the connecting arcs outline the subsumption hierarchy among them. Particularly, the nodes are created by exhaustively generating every possible combination of the four constraints that refer to a temporal association. Subsequently, the main idea is to create every possible association of endpoint pairs of the intervals X and Y, using the operators $?, \leq, \geq, <, >, =$. Then, the resulting rules are rendered into the corresponding sets of Allen operators. It is worth noting that the aforementioned combinations are filtered in order to prevent the existence of constraints that violate the prerequisite of a valid time interval i.e. $X^- < X^+$. For the sake of simplicity, we skip the inverse relations; such an assumption is valid, since intervals X and Y refer to random time frames, thus supporting the inverse associations as well.

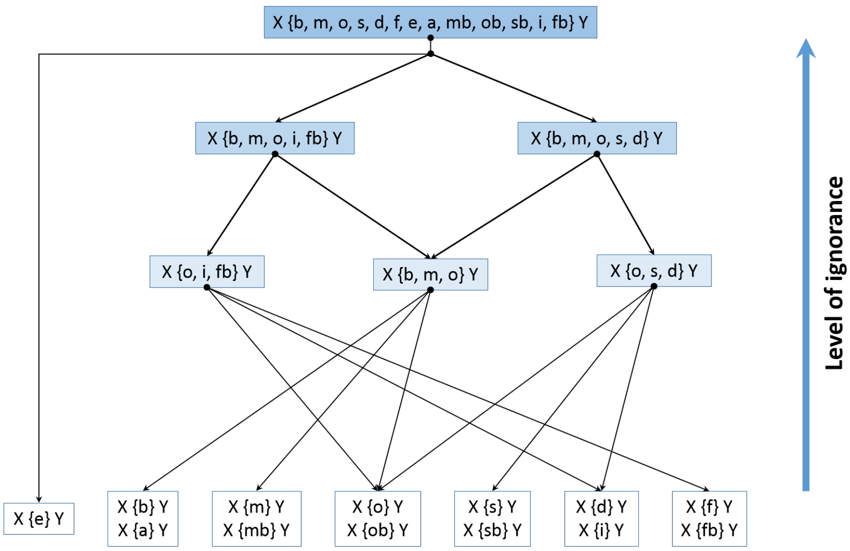
Before proceeding to the construction algorithm of the hierarchy graph, we state the following assumption about the observable knowledge: *time point equality can not be observed*; therefore, it is a counter-intuitive scenario to consider that interval endpoints are equal. As a result, associations that cannot observed are excluded from the set of possible temporal constraints.

The root of the hierarchy graph represents a state of full ignorance; it is formalized using endpoint relations as follows: $X^- \ ? \ Y^-$, $X^- \ ? \ Y^+$, $X^+ \ ? \ Y^+$ and $X^+ \ ? \ X^+$. The latter set of constraints refers to the scenario that all temporal associations are possible. Consequently, they are rendered into Allen operators and conclude to the association that X *before, meets, overlaps, starts, is during* or *finishes* Y. As the constraints are loosened with the usage of more precise operators, the aforementioned set is shrinking and tends to become a single Allen operator, when the set of operators that is produced contains the least level of ignorance. The hierarchy structure and grouping is done with the application of the subset property among the resulted sets, i.e. nodes. Particularly, a child node forms a subset of its parent node, if it represents a subset of Allen operators derived by its parent super set. Figure 3.11 presents all the valid sets of temporal constraints produced by the exhaustive method and Figure 3.12 illustrates the temporal knowledge hierarchy graph.

Relation	Constraints				Description
X {b, m, o, s, d, f, e, a, mb, ob, sb, i, fb} Y	$X^- ? Y^-$	$X^- ? Y^+$	$X^+ ? Y^-$	$X^+ ? Y^+$	Every association is possible
X {b, m, o, i, fb} Y	**$X^- < Y^-$**	$X^- ? Y^+$	$X^+ ? Y^-$	$X^+ ? Y^+$	X starts *before* Y starts
X {b, m, o, s, d} Y	$X^- ? Y^-$	$X^- ? Y^+$	$X^+ ? Y^-$	**$X^+ < Y^+$**	X ends *before* Y ends
X {o, i, fb} Y	**$X^- < Y^-$**	$X^- ? Y^+$	**$X^+ > Y^-$**	$X^+ ? Y^+$	X starts *before* Y starts and X ends *after* Y starts
X {b, m, o} Y	**$X^- < Y^-$**	$X^- ? Y^+$	$X^+ ? Y^-$	**$X^+ < Y^+$**	X starts *before* Y starts and X ends *before* Y ends
X {o, s, d} Y	$X^- ? Y^-$	$X^- ? Y^+$	**$X^+ > Y^-$**	**$X^+ < Y^+$**	X ends *within* Y

Legend: **b**efore, **m**eets, **o**verlaps, **s**tarts, **d**uring, **f**inishes, **e**quals
after, **m**et-**b**y, **o**verlapped-**b**y, **s**tarted-**b**y, **i**ncludes, **f**inished-**b**y

Figure 3.11: Sets of possible temporal associations.



Legend: **b**efore, **m**eets, **o**verlaps, **s**tarts, **d**uring, **f**inishes, **e**quals
after, **m**et-**b**y, **o**verlapped-**b**y, **s**tarted-**b**y, **i**ncludes, **f**inished-**b**y

Figure 3.12: Temporal knowledge hierarchy graph.

The subsumption hierarchy we presented is based on the fact that time point equality is not observable. Consequently, the relation sets it associates are formed by more flexible operators, such as $<$ and $>$. According to the operators hierarchy that is depicted in Figure 3.10, it is easy to conclude that $<$ and $>$ operators may not be as strict as $=$; however, they carry the same ignorance level. In a wider sense, these operators are considered to provide precise temporal knowledge; subsequently they are subjected with the same burden of observation potentiality.

On the one hand, time point equality grants its inability to be observed to the fact that it refers to the minimum time frame of a point, which exists only as a conceptual means that cannot be measured. On the other hand, $<$ and $>$ operators define ordering between interval endpoints. Since events are imprecise by nature, as it was mentioned in Section 2.1.4, their interval endpoints are indefinite. It is, then, obvious that such relations between time points, i.e. $<$ or $>$, cannot be an outcome of primitive observation 3.3; instead, further inference emerges.

For instance, the birth and death of a person are two events that are clearly associated with a before relation, i.e. $<$ operator between endpoints. The starting point of a birth event is lesser, in time, than the starting point of his/her death. However, such certainty is derived from indirect inference making and is not an outcome of primitive observation. More specifically, if there exists an observable reference of events involved with the person until the moment he/she dies, then it is safe to conclude that his/her birth is associated with a $<$ operator to the endpoints of the death event. In addition, the assumption of additional constraints, such as that a birth and death event lasts for a specified duration, leads to the conclusion that the time interval of the birth event is before the corresponding interval of death. Without such inference, the only observable evidence is that the death event started after the birth's start. Taking into account the fuzzy level of events, we conclude that the endpoints of the birth event are associated with a more flexible operator rather than $<$, namely $\leq$. Note that there must be no confusion between a before relation based on endpoints, i.e. $<$ and semantic before as it was proposed in Section 3.3. The latter kind of relation is observable, as it is an interpreted inclusion relation, i.e. parts of or cannot co-exist that is derived from semantic association between events. On the contrary, the endpoint before is derived from chronological reasoning based on observations and dating methods.

We conform with the assumption that the primary observable knowledge that refers to ordering between temporal points is the $\leq$ and $\geq$. In an attempt to create an observable temporal knowledge hierarchy graph, we replace every non-observable operator $< or >$ with the corresponding ones, i.e. $\leq$ and $\geq$, respectively. In Figure 3.13, we depict the transformed sets of observable temporal knowledge with respect to the first version of subsumption hierarchy sets that are illustrated in Figure 3.11. Finally, Figure 3.14 presents the hierarchy graph of observable temporal knowledge. Note that the grouping and connecting process is the same with the first version of the graph. Also for the sake of completeness, the basic temporal operators are included at the bottom of the graph, despite the existence of $<$, $>$ and $=$ operators.

Relation	Constraints				Description
X {b, m, o, s, d, f, e, a, mb, ob, sb, i, fb} Y	$X^- ? Y^-$	$X^- ? Y^+$	$X^+ ? Y^-$	$X^+ ? Y^+$	Every association is possible
X {b, m, o, i, fb, e} Y	$\mathbf{X^- <= Y^-}$	$X^- ? Y^+$	$X^+ ? Y^-$	$X^+ ? Y^+$	X starts *before or when* Y starts
X {b, m, o, s, d, e} Y	$X^- ? Y^-$	$X^- ? Y^+$	$X^+ ? Y^-$	$\mathbf{X^+ <= Y^+}$	X ends *before or when* Y ends
X {m, o, i, fb, e} Y	$\mathbf{X^- <= Y^-}$	$X^- ? Y^+$	$\mathbf{X^+ >= Y^-}$	$X^+ ? Y^+$	X starts *before or when* Y starts and X ends *after or when* Y starts
X {b, m, o, s, fb, e} Y	$\mathbf{X^- <= Y^-}$	$X^- ? Y^+$	$X^+ ? Y^-$	$\mathbf{X^+ <= Y^+}$	X starts *before or when* Y starts and X ends *before or when* Y ends
X {m, o, s, d, e} Y	$X^- ? Y^-$	$X^- ? Y^+$	$\mathbf{X^+ >= Y^-}$	$\mathbf{X^+ <= Y^+}$	X ends *within* or is equal to Y

Legend: **b**efore, **m**eets, **o**verlaps, **s**tarts, **d**uring, **f**inishes, **e**quals
after, **m**et-**b**y, **o**verlapped-**b**y, **s**tarted-**b**y, **i**ncludes, **f**inished-**b**y

Figure 3.13: Sets of possible associations using observable temporal knowledge.

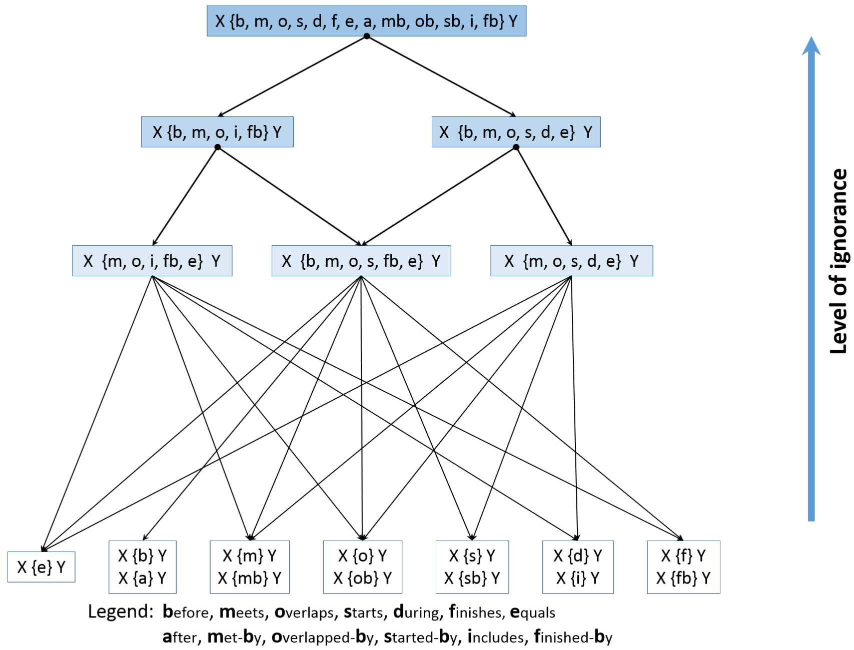


Figure 3.14: Temporal knowledge hierarchy graph based on primary observations.

Chapter 4

Fuzzy Interval Algebra

Contents

Having introduced the concept of mutually exclusive evidence, we covered the theoretical approach of defining the temporal extent of a period, through semantic association of the available evidence. This method of temporal confinement is based on distinct and imprecise information derivable from observations. As we mentioned in Section 3.1, the resulting temporal extent is approximated by the introduction of determinate and indeterminate regions representing the fuzzy areas and regions of certainty.

In this chapter, we propose an alternative approach to Allen interval algebra that focuses on the representation of imprecise temporal knowledge. First, the concept of point-wise time is introduced, as an attempt of quantification and formalization of the time continuum. Then, we proceed to the definition of a model that formalizes time imprecision, by introducing the notion of fuzzy interval. The proposed model is a generalized version of Allen's interval model that allows the representation of indefinite temporal boundaries. Finally, a model that focuses on the representation of temporal topology over fuzzy intervals is proposed, as an alternative version of Allen interval algebra.

4.1 Point-Wise Time and Fuzzy Intervals

The temporal extent of a period represents the time frame within which the period occurred, where a time frame stands for a region over the timeline. Time is considered as an abstract notion that is used to represent the ordering and measurement of the duration of events. Particularly, time relates events into the past, present or future in the sense of topology; in addition, it provides information about the duration of events, taking into account the conventional quantification of time (seconds, minutes, etc.). Since we proceed to a formalization, the need for a more theoretical and strict quantification of time emerges.

According to [Benthem 1983], time is perceived as a set of durationless points known as instants or moments. The time set is perceived as a numeric sequence by considering it as an isomorphic representation of the set of real numbers R.

Definition 4.1.1 (Time)

Let $Time$ be an ordered set of time points that is defined under a total order $\prec$ that represents the real timeline.

Definition 4.1.2 (tempD)

Let $tempD : Time \times Time \rightarrow \Re$ be a real-valued function that refers to the temporal distance between two time points and satisfies the properties of positive definiteness, symmetry and triangle inequality.

A time interval is regarded as a set of instants, which illustrates the temporal extent of the observable phenomena. In spite of the high rate of progress that describes an observable phenomenon, such as electromagnetic wave propagation, it is counter-intuitive to consider that the time duration of a phenomenon is evaluated as instant. Therefore, we accept that a time interval that contains a single time point does not comply with reality. In addition, the concept of an empty time interval cannot be applied to real events. We accept that every phenomenon is bounded in the time continuum. For instance, it is beyond the scope of logic to claim that a birth event happened out of time. As a result, phenomena cannot be separated from the notion of time.

In the sequel, we provide a definition of the time interval, allowing to represent fuzzy and non-fuzzy intervals as specializations of a more general definition. The main idea is based on the representation o the interval boundary as a set. In cases of fuzzy endpoints of an interval, the corresponding boundary sets have a certain thickness; in the scenario of precise temporal information, however, they are restricted to points or hyper-planes. The boundary becomes the region that represents temporal indeterminacy, in which the fuzzy function evaluates to values in (0, 1). In this way, the effect of a fuzzy function is encapsulated in the thickness of the boundary. Hence, our theory becomes independent of the choice and evaluation of a particular fuzzy function. Alternatively, the problem of fuzziness representation is regarded as an inclusion association. Particularly, time points that belong to the boundary set form a fuzzy zone; on the contrary, points that are not part of the boundary layer form the determinate extent of the interval. We define a fuzzy interval as follows:

Definition 4.1.3 (Fuzzy Interval)

Let $I \subset Time$ be a set of time points that represent a valid time interval and $fe : Time \rightarrow [0, 1] \subset \Re$ be the fuzzy time point membership function of I.

We proceed to the definition of the comprising components of a fuzzy interval.

Definition 4.1.4 (Neighborhood)

A neighborhood r of a time point t is a set N_t^r that contains time points $\{t_i \in Time : tempD(t, t_i) \leq r\} \setminus \{t\}$, where $r > 0$.

Definition 4.1.5 (Boundary point)

A time point t is a boundary point of I *if for all Neighborhoods of time point t, it holds that* $\exists t_1, t_2 \in N_t^r : t_1 \in I \bigwedge t_2 \notin I$*. In case of fuzzy boundaries, the fuzzy function evaluates to* $fe(t_1, I) > 0$ *and* $fe(t_2, I) < 1$*.*

Definition 4.1.6 (Boundary)

The boundary B_I *of time interval* I *is the set of all the boundary points of* I*.*

Definition 4.1.7 (Interior point)

A time point t is an interior point o I *if there exists a neighborhood* $N_t^r \subseteq I$ *of time point t where* $\forall t_i \in N_t^r : fe(t_i, I) = 1$*.*

Definition 4.1.8 (Interior)

The interior I_I *of time interval* I *is the set of all the interior points of* I*.*

Definition 4.1.9 (Closure)

The closure C_I *of time interval* I*, is the set of all the interior and boundary points of* I*, it holds that* $C_I = I_I \bigcup B_I$*.*

Definition 4.1.10 (Exterior)

The exterior E_I *of time interval* I*, is the complement of its closure* C_I*, it holds that* $E_I = Time \setminus C_I$*.*

The properties that hold for a valid fuzzy interval are analyzed in the sequel. Let I be a valid fuzzy time interval and I_I, B_I, C_I are the interior, boundary and closure sets, respectively. Then, the following hold:

- $I_I \subseteq I$.
- non-empty boundary or interior sets: $B_I \neq \emptyset$ and $I_I \neq \emptyset$.
- bounded closure: $\exists N_t^r : C_I \subseteq N_t^r$ where $r < \infty, t \in Time$, hence the boundary and interior sets are finite.
- the boundary set is divided into two subsets that wrap the interior set: $\exists A, B \subset B_I : A \bigcap B = \emptyset \bigwedge A \bigcup B \neq \emptyset$ and $B_I = A \bigcup B$
- convex interior set: $\forall x, y \in I_I$, in holds that $\forall t \in [0,1] : [(1-t)*x + t*y] \in I_I$. In other words, all points on the line segment that connects two interior points are also elements of the interior set.

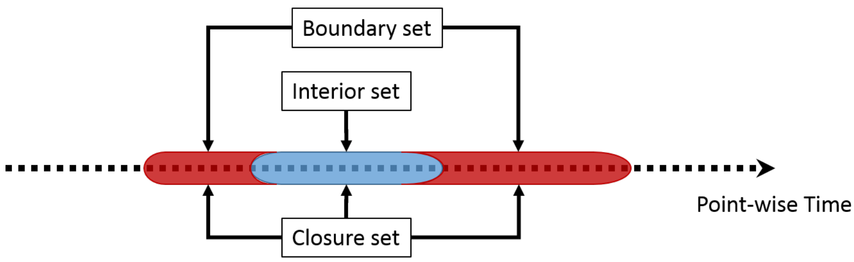


Figure 4.1: Fuzzy Time Interval

According to our algebra, a time interval is regarded as a group of sets. Particularly it is composed of the interior and boundary sets, as depicted in Figure 4.1, which refer to the determinate and indeterminate interval of the defining period, respectively, as mentioned in Section 3.1. It is worth noting that, as our knowledge about the period tends to approximate its real bounds, the boundary set of the corresponding interval shrinks. When highest precision is reached, the boundary set contains only the true endpoints of the period.

4.2 Fuzzy Interval Algebra

Information about the relevant topology of precise time intervals can be stated using Allen operators [Allen 1983]. However, in cases of time imprecision, the temporal association upon fuzzy intervals is approximated using a set of Allen operators. The aforementioned set represents the possible relations that hold between the fuzzy endpoints of the associated intervals. Having a set of possible associations raises the need for a probabilistic distribution that would result in the most probable relation, for the scenario under consideration. Contrariwise, the extraction of a single relation that refers to the most suitable interval association that conforms with the current knowledge, provides a comprehensive and yet simplified view of the modeling scenario.

In this section, we propose an alternative approach of Allen operators that describes the temporal association of fuzzy intervals, as they are defined in Section 4.1. Particularly, the endpoint relations that are proposed by Allen are replaced with set-oriented statements. Set operations are applied on the interior and boundary sets, complying properly in order to adhere to the representation of fuzzy intervals.

Let B_A, B_B, I_A, I_B, C_A and C_B be the boundary, interior and closure sets of two valid time intervals A and B; here follows the reconstruction of the seven basic Allen operators [Allen 1983] in an alternative set-oriented approach, introducing the basic fuzzy relations. It is worth noting that relations of fuzzy interval algebra are referred with the prefix fuzzy for purposes of differentiation.

[fuzzy before]

A *fuzzy before* B describes the scenario of disjoint closure sets; particularly interval A occurred earlier than B. It is formalized as follows: $\forall a \in C_A, \forall b \in C_B : a < b$.

A *fuzzy after* B refers to the inversed relation of *before* i.e. interval A occurred latter than B. It is described with the following formalization: $\forall a \in C_A, \forall b \in C_B : a > b$.

It is noteworthy that relations *before* and *after* are applicable to any time-point set e.g., the interior, closure and boundary ones. These relations will be used as building blocks for the definition of the rest.

[fuzzy meets]

A *fuzzy meets* B describes meetings in time, where the end of interval A signifies the start of interval B, it is formalized as:

- $B_A \bigcap B_B \neq \emptyset$
- I_A *fuzzy before* I_B

[fuzzy overlaps]

A *fuzzy overlaps* B illustrates the scenario where interval A pre-exists B, both share interior points and B keeps to exist after A. The following formalization holds:

- $I_A \bigcap I_B \neq \emptyset$
- $I_A \setminus C_B \neq \emptyset$
- $I_B \setminus C_A \neq \emptyset$
- $I_A \setminus C_B$ *fuzzy before* $C_A \leftrightarrow I_B \setminus C_A$ *fuzzy after* C_B

[fuzzy starts]

A *fuzzy starts* B describes the relation where interval A signifies the start of B. Both intervals start at the same time point. Although, they share interior points, B keeps existing after A ends. It is formalized as follows:

- $I_A \bigcap I_B \neq \emptyset$
- $I_A \setminus C_B = \emptyset$
- $I_B \setminus C_A \neq \emptyset$
- $I_B \setminus C_A$ *after* C_A

[fuzzy during]

A *fuzzy during* B refers to an inclusion relation like "falls within". Particularly, the following properties hold: $C_A \subseteq I_B$

[fuzzy finishes]

A *fuzzy finishes* B illustrates the opposite scenario of *fuzzy starts*. Particularly interval A signifies the termination of B. Interval B predates A, both share interior points and terminate at the same time point. It is formally expressed as follows:

- $I_A \bigcap I_B \neq \emptyset$
- $I_A \setminus C_B = \emptyset$
- $I_B \setminus C_A \neq \emptyset$
- $I_B \setminus C_A$ *fuzzy before* C_A

[fuzzy equals]

A *fuzzy equals* B is a special scenario of temporal topology where intervals A and B are related with mutual inclusion. Particularly, A shares interior points with B, while the interior of B falls within A and the interior of A falls within B. The formalization is:

- $I_A \bigcap I_B \neq \emptyset$
- $I_A \subset C_B$
- $I_B \subset C_A$

We have analyzed the fuzzy version of the basic Allen operators, including relations: before, meets, starts, overlaps, during, finishes and equals. The inverse relations are formalized similarly, by replacing the boundary, interior and closure sets of interval A to the corresponding sets of B and vice versa. A visual representation of the basic fuzzy relations is depicted in Figure 4.2.

It is worth noting that the proposed fuzzy relations approximate the temporal topology of periods, regardless of their level of fuzziness. Properly loosened constraints are applied in order to avoid any limitation on the boundary's thickness and hence the size of the individual periods. Finally, our approach is compatible with Allen's approach in cases of complete awareness, considering the case of the minimal boundary set, where the boundary set strictly consists of the true endpoints of the corresponding interval.

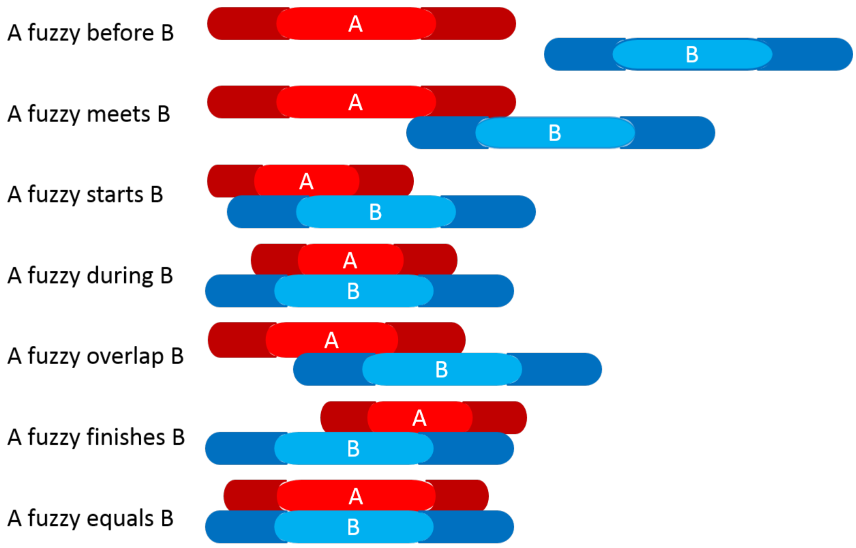


Figure 4.2: Fuzzy relations

Chapter 5

Spatiotemporal Interval Algebra

Contents

5.1 Spatiotemporal approach of time

As already mentioned in Section 2.1.2, periods are considered as spatiotemporal entities. They are formally represented in space-time as four dimensional volumes (see Section 2.2.1). Temporal association of representative space-time volumes, reveals information about the relevant topology of the corresponding periods.

Time is one of the comprising dimensions that form the space-time volumes. An attempt to approach their temporal topology using exclusively time projections may sometimes lead to ambiguity. For instance, as illustrated in Figure 5.1, considering the time projections of periods A and B, it holds that the associated periods are related with an overlap relation, in terms of Allen operators. Note that for the sake of simplicity of volume representation, we assume a single dimensional space. However, in a wider scale of observation, as a whole, their volumes do not touch at any point in their common space regions. As a result, the aforementioned periods are associated with a before relation.

Such conflicting temporal approaches are derived by the difference between the range the relations cover over the two volumes. Hence, in order to differentiate between the two cases, we assign an area index: the before relation that holds for the spatial overlap region is denoted as local before, whereas the overlap relation between time projections is denoted as total overlap. We accept that the most suitable association in this scenario is the local before relation. Note that we assume that the temporal association among periods has a common sense, if the periods share common space i.e. there is overlapping space regions. Such conflicts are caused by the fact that, in many cases, the space occupied by a period is changing over time. Such cases are frequent in archaeology, but have been widely disregarded in literature [Claramunt and Jiang 2001].

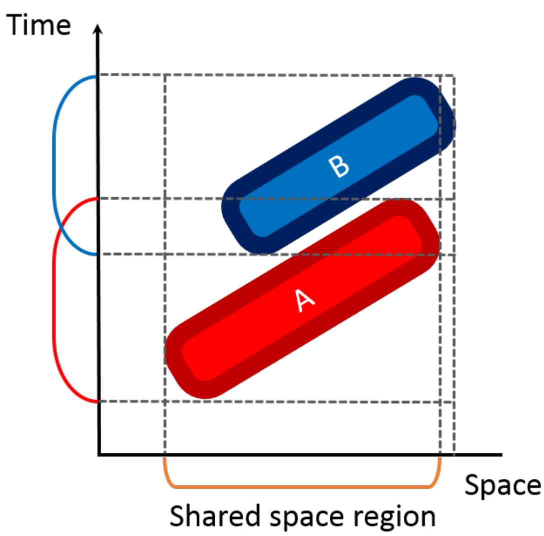


Figure 5.1: Time projections and temporal association

In the rest of this chapter, we propose a spatiotemporal approach of fuzzy interval algebra, that is proposed in Section 4. First, space-time is defined proportionally to point-wise time definition, followed by the introduction and formalization of fuzzy volumes. The outcome of such formalization is a model for representing the space-time extent of a period instance. Finally, the temporal association among periods is extending the fuzzy interval algebra into four dimensions, making it applicable on fuzzy volumes.

5.2 Point-wise Space-Time and Fuzzy Volumes

Space-time is usually interpreted from a Euclidean space perspective, in which space and time are regarded as three- and one-dimensional systems respectively. For the sake of simplicity, we adopt a non-relativistic model, in which time is treated as universal and constant. As a result, the proposed model is independent of the state of motion of the observer. Furthermore, we only regard observers that are at rest with respect to our spatial reference system.

Additional study on reference systems in relative motion could be considered as future work. It is worth mentioning that the theory presented here is independent of geometries on curved surfaces, such as Earth, and relativistic space distortions.

Definition 5.2.1 (SpaceTime)

Space-time is defined as the set of all the space-time points. Let ST be a set that is isomorphic to the four dimensional space $\Re^4$ that represents the four dimensional space i.e. space-time.

Each point in ST is considered as a quadruple (x, y, z, t) where $x, y, z \in Space$ and $t \in Time$. $Space$ stands for the known space set that is defined in physics while Time refers to the timeline. Variables x, y and z refer to Cartesian space coordinates in some arbitrary spatial reference system, while t stands for time values.

Definition 5.2.2 (Space-time Volume)

Let V be a set that represents a space-time volume. It holds that $V \subset ST$.

A space-time volume is regarded as a set of spatiotemporal points that illustrates the spatial and temporal extent of a period. We offer a generalized definition of space-time volume that allows space-time representation of periods, regardless of whether fuzzy information is included or not. The fuzzy volume model is defined by extending the fuzzy interval definition, proposed in Section 4.1, in four dimensional space. We define a fuzzy volume algebra as follows:

Definition 5.2.3 (Fuzzy Volumes)

Let $V \subset ST$ be a set of space-time points that represent a valid space-time volume and $fe : ST \rightarrow [0, 1] \subset \Re$ be the fuzzy time point membership function of V.

We proceed to the definition of the comprising components of a fuzzy volume.

Definition 5.2.4 (Neighborhood)

A neighborhood r of a space-time point st is a set $N_s^r t$ that contains space-time points $\{st_i \in ST : tempD(st.t, st_i.t) \leq r\} \setminus \{st\}$, where $r > 0$.

Definition 5.2.5 (Boundary point)

A space-time point st is a boundary point of V, if for all Neighborhoods of point st. It holds that $\exists st_1, st_2 \in N_s^r t : st_1 \in V \bigwedge st_2 \notin v$. In case of fuzzy boundaries, the fuzzy function evaluates to $fe(st_1, V) > 0$ and $fe(st_2, V) < 1$.

Definition 5.2.6 (Boundary)

The boundary B_V of space-time volume V is the set of all the boundary points of V.

Definition 5.2.7 (Interior point)

A space-time point st is an interior point of V, if there exists a neighborhood $N_s^r t \subseteq V$ of point st where $\forall st_i \in N_s^r t : fe(st_i, V) = 1$.

Definition 5.2.8 (Interior)

The interior I_V of space-time volume V is the set of all the interior points of V.

Definition 5.2.9 (Closure)

The closure C_V of space-time volume V, is the set of all the interior and boundary points of V, it holds that $C_V = I_V \bigcup B_V$.

Definition 5.2.10 (Exterior)

The exterior E_V of space-time volume V, is the complement of its closure C_V, it holds that $E_V = ST \setminus C_V$.

The properties that hold for a valid fuzzy volume are analyzed in the sequel. Let V be a valid fuzzy space-time volume and I_V, B_V, C_V are the interior, boundary and closure sets, respectively.

- $I_V \subseteq V$.
- non-empty boundary or interior sets: $B_V \neq \emptyset$ and $I_V \neq \emptyset$.
- bounded closure: $\exists N_s^r t : C_V \subseteq N_s^r t$ where $r < \infty, st \in ST$, hence the boundary and interior sets are finite.
- continued boundary: $forall st1 \in B_V$, it holds that $forall N_s^r t1 \exists st2 \in B_V : st1 \neq st2 \bigwedge st2 \in N_s^r t1$
- connected boundary: $\nexists A, B \subset B_V$ such that
  - $A \bigcap B_V \neq \emptyset$
  - $B \bigcap B_V \neq \emptyset$
  - $A \bigcap B_V \bigcap B \neq \emptyset$
  - $B_V = (A \bigcap B_V) \bigcup (B \bigcap B_V)$

- convex interior set: $\forall st1, st2 \in I_V$, in holds that $\forall st \in [0,1] : [(1-st) * st1 + st * st2] \in I_V$. It is worth noting that the convexity property is only required to hold for the time dimension. In other words, the temporal extent of all space-time points that connects two interior points are also elements of the interior set.

Similarly to the fuzzy interval, the interior, boundary and closure sets that are associated to four dimensional volumes represent the indeterminate, determinate regions and the whole extent of the period, respectively. The fuzzy zones of a period are represented by the boundary set, whereas regions of certainty are illustrated by the interior set, as depicted in Figure 5.2.

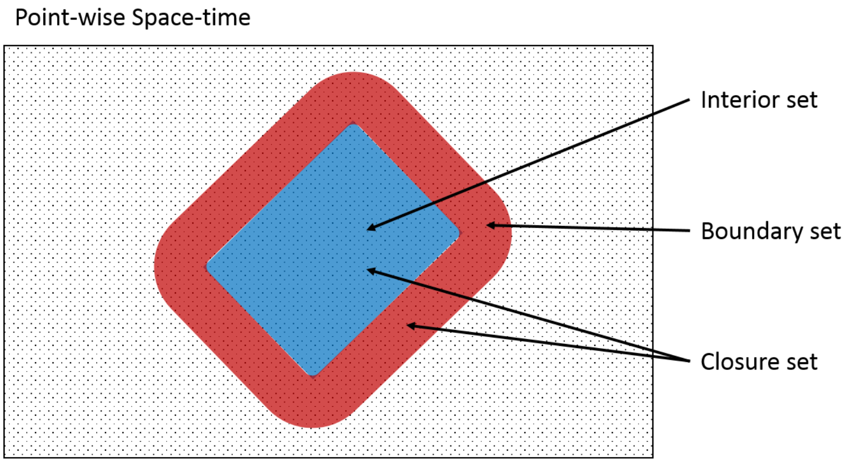


Figure 5.2: Fuzzy space-time volume

5.3 Spatiotemporal version of fuzzy temporal relations

Information about the temporal topology over space-time volumes is gained by the relevant association of their time projections. The projected image of a volume upon the time axis creates a fuzzy interval, as it was introduced in Section 4.1. The projected interval is composed of the individual projections of the boundary and interior sets of the projected volume. Fuzzy relations that hold between the derived intervals define the relevant temporal topology of the corresponding volumes.

The time projections are defined according to the amount of space points that the corresponding volumes share. Particularly, time projections that refer to the complete spatial extent of the volume provide an overall image of the occupied space and, therefore, they are regarded as total. Contrariwise, projections derived by discrete space slices, represent the projection of a fraction of the total spatial extent, resulting into a local image. Adopting the same premise, any fuzzy relation that is formed, is subjected to a similar differentiation. Consequently, fuzzy relations that describe the temporal association of the individual volumes are distinguished into local and total. The aforementioned differentiation depends on whether the fuzzy relations stand on local or total time projections.

In a wider view, the concept of local and total topology refers to the existence of shared space points between the associated volumes. Particularly, we propose that if there is no space overlap among the associating volumes, then their temporal topology is described in terms of their total relation. Figure 5.3 illustrates examples of spatially disjoint volumes that result into total temporal association as it is expressed using total projections.

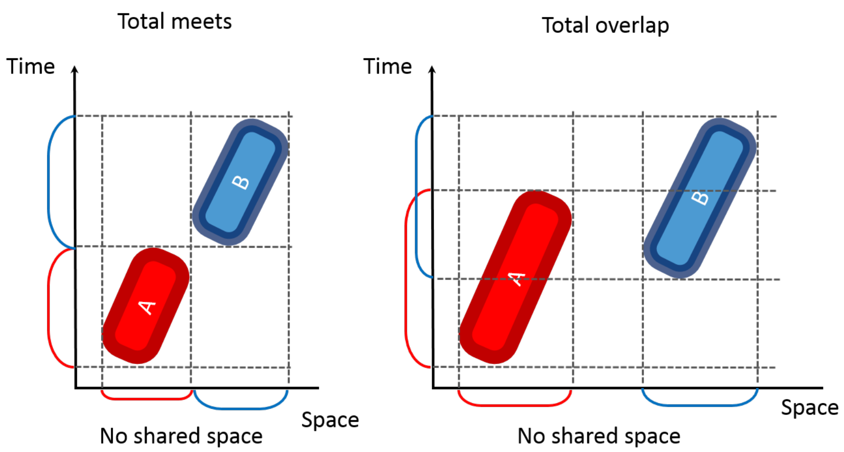


Figure 5.3: Total relations and total projections

In the case of totally or partially space overlapped volumes, we introduce associations that are expressed by a set of local relations. Such relations refer to the corresponding time projections of the shared space slices. The concept of local relations is connected to the observation process. Particularly, an observation reveals temporal and semantic knowledge about a physical thing over the space that it occupies. The spatial information that is derived from it forms a space slice over the corresponding reference space. The size of each space slice depends on the size and the semantics of the observable object. In the scenario of two spatially overlapped volumes, a set of space slices is declared according to the spatial projection of the available evidence. Each space slice creates a time projection upon the individual volumes. The set of relations that stand between the defined projections, those that refer to the same space slice, create a set of local associations among the two volumes.

The temporal topology among volumes can be expressed using a set of local associations. However, such a process provides the knowledge to confine the true association. Instead of using a set of local relations, we propose the definition of a single prevailing relation that represents every possible local association that stands between the shared space slices. Consequently, we introduce a relation hierarchy, defined intuitively and semantically related to data of historical significance.

In more detail, a level of prevalence is attached to each local relation, forming a classification order from strongest to weakest, as follows: overlaps, equals, starts, finishes, meets and before, during. Stronger relations cause the weaker ones to be excluded as possible volume associations. Note that there are two pairs of relations with the same level of prevalence, namely starts/finishes and before/during. These cases do not raise decidability issues, since the former pair is evaluated semantically as a special case of equals, while the latter is considered as a counter-intuitive scenario, since it expresses inclusion and disjoint relation, simultaneously. A visual representation of the relation subsumption hierarchy is depicted in Figure 5.4.

In the rest of this section, we define the spatiotemporal version of fuzzy in-

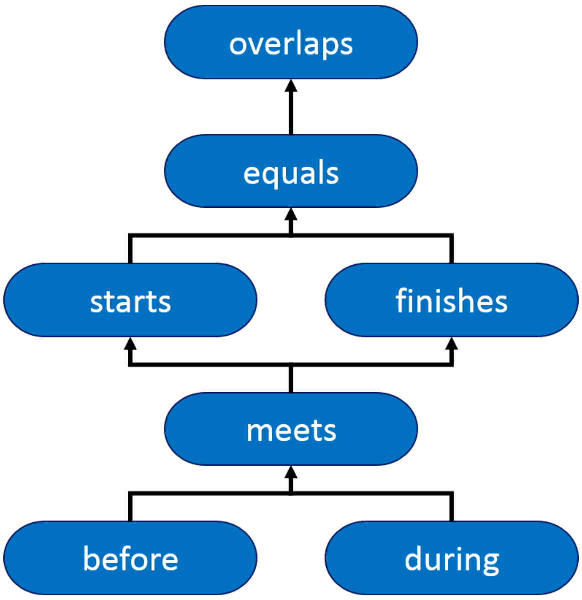


Figure 5.4: Time projections and temporal association

terval algebra, the so called fuzzy volume algebra, that is applicable on space-overlapping volumes. It is based on the relation subsumption hierarchy introduced in Figure 5.4 and, similarly to fuzzy interval algebra, it expresses the basic temporal relations corresponding to the Allen operators. Firstly, time and space projection functions are defined and then we focus on the formalization of the spatiotemporal version of fuzzy relations. For reasons of differentiation, the spatiotemporal fuzzy relations are denoted with the prefix "sp-".

Let V be a fuzzy space-time volume; we define its projections as follows:

Definition 5.3.1 (Time projection)

Let $T_V : \{Space\} \rightarrow \{Time\}$ be the time projection of a set of space points over volume V. Note that it results in a fuzzy interval.

Definition 5.3.2 (Space projection)

Let $S_V : \{Time\} \rightarrow \{Space\}$ be the space projection of a set of time points over volume V. Note that it results in a fuzzy spatial region.

For the sake of simplicity, the variants of functions T_V and S_V with arity 0 return the total time or space projection of the volume V; hence, $T_{A\bigcap B}$ refers to the time projection of the overlapped space of volumes A and B.

Let A and B be two fuzzy volumes. The space-time version of fuzzy relations are defined as follows:

[sp-before]

A *sp-before* B describes the scenario in which volume A occurred earlier than B to every shared space slice. It is formalized as follows: $\forall s \in S_{A\bigcap B}$: $\{T_A(s)$ *fuzzy before* $T_B(s)\}$.

[sp-meets]

A *sp-meets* B describes meetings in time, particularly, volumes A and B are met in time at every shared space slice, it is formalized as: $\exists s \in S_{A\bigcap B}$: $\{T_A(s)$ *fuzzy meets* $T_B(s)\} \bigwedge \{I_A \bigcap I_B = \emptyset\}$

[sp-during]

A *sp-during* B refers to the scenario in which volume A occurred during volume B. It is formalized as follows: $\forall s \in S_{A\bigcap B}$: $\{T_A(s)$ *fuzzy during* $T_B(s)\}$.

[sp-starts]

A *sp-starts* B describes the case where A occurred during the earlier points of volume B.

Particularly, there are two interpretations of the start relation, let us call them causal and incidental. An incidental start describes the case where there exist space points in volume B that are reached by the starting of volume A after the beginning of volume B at these points.

A causal start requires volume A to come before the rest of period B at all points.

For instance, as an incidental start, one may consider the transmission of a message that marks the start of a new period, but is affected by speed limits.

In contrast, a volcano eruption that creates a new island can be regarded as an instance of a causal start. They are formalized as follows:

Causal: $\forall s \in S_{A \bigcap B} : \{T_A(s)$ *fuzzy starts* $T_B(s)\}$

Incidental: $\exists s \in S_{A \bigcap B} : \{T_A(s)$ *fuzzy starts* $T_B(s)\}$

* not $\{A$ *sp-finishes* $B\}$
* not $\{A$ *sp-equals* $B\}$

[sp-finishes]

A *sp-finishes* B describes the case in which volume A occurred during the latest points of volume B. Similarly, there are two interpretations of the finishes relation, causal and incidental.

Causal: $\forall s \in S_{A \bigcap B} : \{T_A(s)$ *fuzzy finishes* $T_B(s)\}$

Incidental: $\exists s \in S_{A \bigcap B} : \{T_A(s)$ *fuzzy finishes* $T_B(s)\}$

* not $\{A$ *sp-starts* $B\}$
* not $\{A$ *sp-equals* $B\}$

[sp-equals]

A *sp-equals* B refers to the scenario where volume A and B coincide.

- $\exists s \in S_{A \bigcap B} : \{T_A(s)$ *fuzzy equals* $T_B(s)\}$
- not $\{A$ *sp-overlaps* $B\}$

In addition, there is a special case of volume equality, in which a pair of starts and finishes relations is evaluated into equals; it is formalized as follows: $\exists s1, s2 \in S_{A \bigcap B} : s1 \neq s2$ and

- $\{T_A(s1)$ *fuzzy finishes* $T_B(s1)\}$
- $\{T_A(s1)$ *fuzzy starts* $T_B(s1)\}$
- not $\{A$ *sp-overlaps* $B\}$

[sp-overlaps]

A *sp-finishes* B illustrates the scenario of overlapping volumes. If is formalized as $\exists s \in S_{A \bigcap B} : \{T_A(s)$ *fuzzy overlaps* $T_B(s)\}$

Figure 5.5 presents a visual representation of every fuzzy temporal association over space-time volumes that is derived by the proposed spatiotemporal interval algebra. However, considering reality, the associating volumes result into more complicated scenarios, which require further analysis to local relations and application of the relation hierarchy. Figure 5.6 illustrates a complicated example of temporal association over space-time volumes; note that the time projection association results in a during operator, whereas our algebra yields a spatiotemporal overlap. Similarly, in Figure 5.7 a complicated version of equals relation is presented. Figure 5.8 depicts the two versions of starts association. In addition, Figure 5.9, illustrates the special scenario of volume equality that is formed by the combination of sp-starts and sp-finishes relations.

It should be noted that fuzzy temporal relations for both time intervals and spatiotemporal volumes are considered complete, since there is no combination of intervals (or volumes) that cannot be described by exactly one of the proposed relations. This claim is backed by the fact that our analysis can model every temporal relation that is expressed in CIDOC CRM [CID 2011] and also by evaluation through exhaustive search, i.e. there is no combination that leads to undecidability. As far as the efficiency of calculating such relations is concerned, it depends on the candidate spatiotemporal information systems in which the proposed theory may be embedded and the 3D/4D indexing methods they support; these include spatiotemporal GIS and databases.

The fuzzy algebra presented in Section 4 and its extension in four dimensions, as it was introduced in the current section, represent a method of determining the most probable relation among two entities. The resulting association conforms with our current knowledge. Revision applied in our knowledge base may provide evidence that defies the already known relations. The most important aspect to

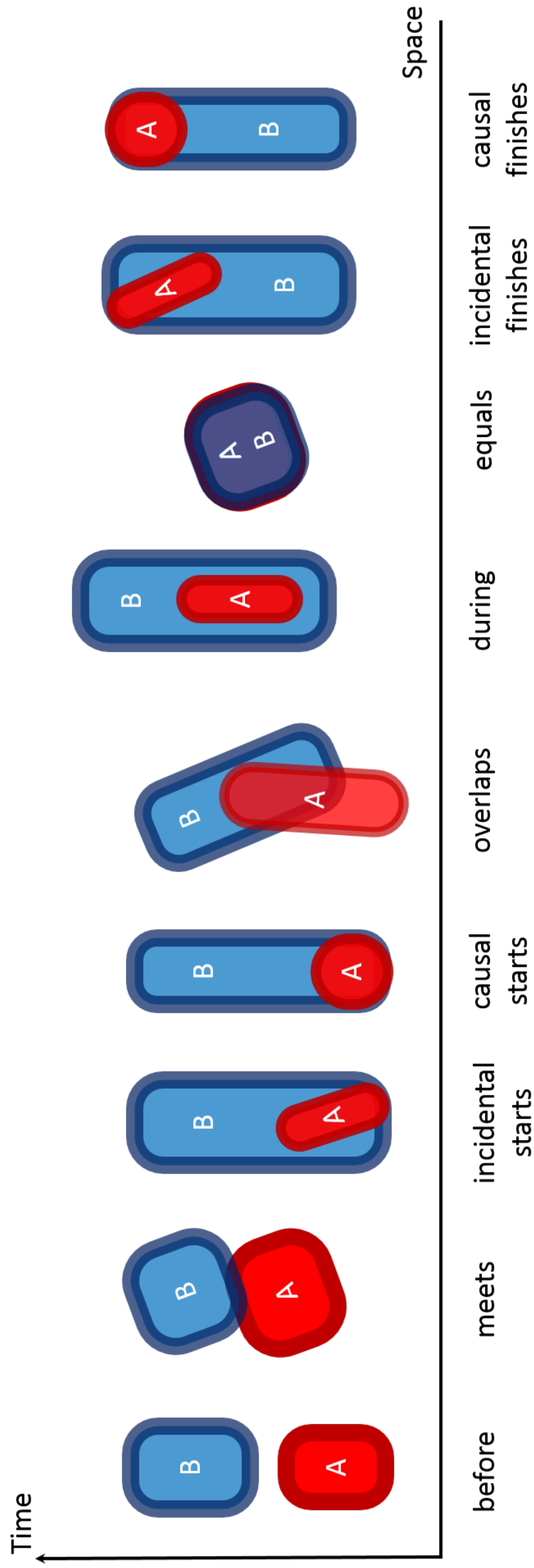


Figure 5.5: Temporal association over space-time volumes

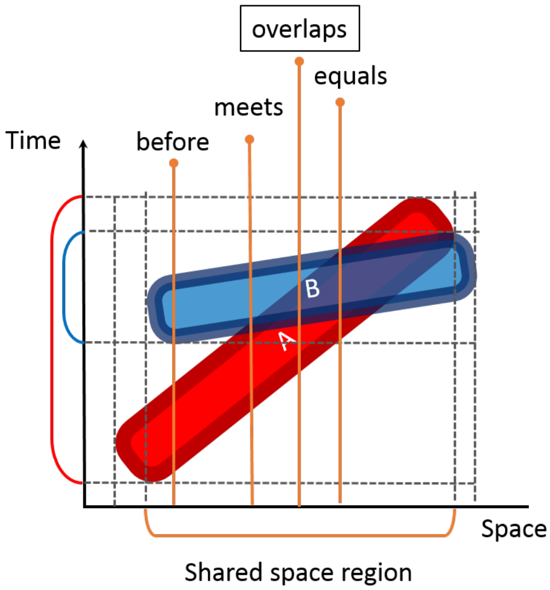


Figure 5.6: Complicated overlap association

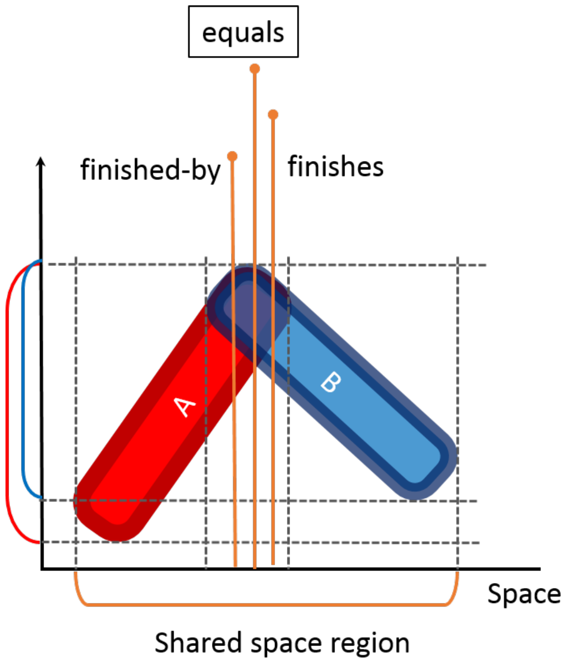


Figure 5.7: Complicated equals association

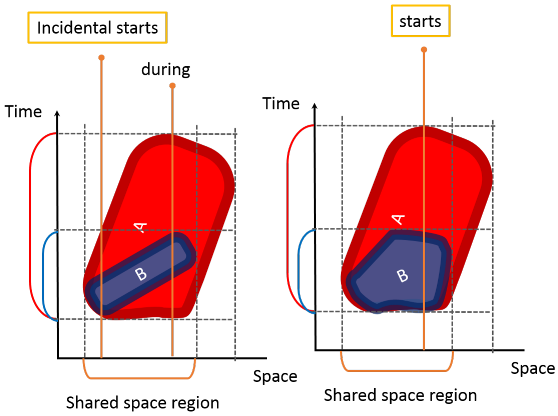


Figure 5.8: Interpretations of start association

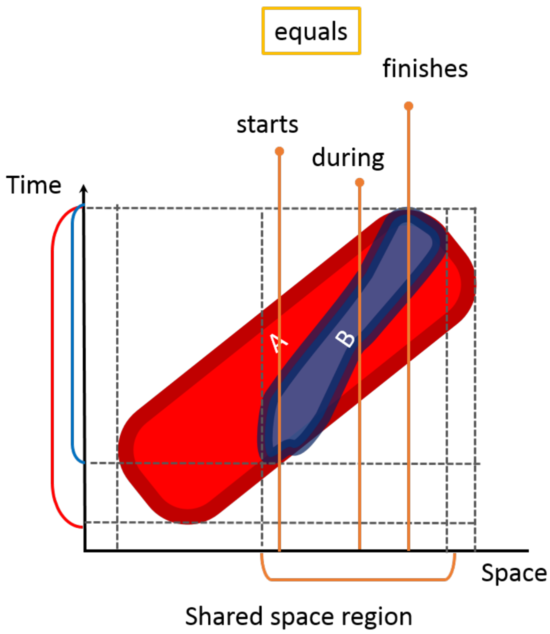


Figure 5.9: Complicated overlap association

take into account in such cases is the source of such knowledge. The temporal and, in general, the spatiotemporal confinement of the periods to be modeled is derived through the observation process. As it was mentioned in Section 3.1, the key factor of outlining the closure of a period is the part-of and cannot co-exist relations. Such semantic associations are used indirectly in the aforementioned algebras.

Fuzzy before relation is interpreted into a cannot co-exist semantic reasoning. In addition, any case of shared temporal or spatiotemporal point refers to a part-of relation. The same premise is adopted by the fuzzy spatiotemporal algebra as well. As a conclusion, we introduce a method to render the primitive semantic knowledge of the part-of (also known as belongs to) and disjoint relations into temporal and spatiotemporal topology. In other words, inclusion and exclusion relations give rise to temporal associations.

Chapter 6

Applications

Contents

6.1 Fields of Application

Our proposed theory and models provide solutions correlated with the temporal modeling of periods and the approximation of their temporal topology, when imprecise information is included. Key applications of such a theory are the following:

- the spatiotemporal approximation of a period, including determinate and indeterminate bounds, based on observations and historical sources.
- determining the influence that one period has on another based on the analysis of their temporal association.
- derivation of spatiotemporal relations based on semantic associations.
- representation of successive periods and meeting in time to realize "continuity" of periods [Darvill 2008].

- providing answers to temporal queries applied on spatiotemporal entities like periods.

These applications have a crucial impact on fields associated with reality modeling, especially observation-based sciences, such as archaeology, geology, palaeontology, ecology and anthropology. More specifically, archaeology, geology and palaeontology are related to the revelation of data about various aspects of prehistory, such as human history and evolution, aging of the Earth, evolution of organisms and their association with the environment, periods of extinction and so on.

The aforementioned sciences can approximate the temporal extent of periods by relating observations and findings with rock layers and layering through stratigraphy [Gradstein et al. 2005]; therefore information about their temporal association can be revealed. Additionally, studying the temporal association of findings with known periods can contribute to dating methods, by approaching the age or date of existence of the individual findings. Furthermore, our model can provide efficient temporal modeling of the identified geologic periods, through index fossils [Ghosh 2006]. Finally, temporal topology extraction methods like the Harris matrix [Farrand 1996] can be evaluated through the association of the extracted results with the temporal topology derived by our model.

Ecology studies the interactions among organisms and their environment, while anthropology examines humans on a social and biological point of view. Scientific observations over long time periods can be associated with our model, defining the determinate and indeterminate boundaries of the corresponding periods. Also, approximating the relevant temporal topology over past periods and understanding past events enhances the reconstruction of possible pasts, by excluding inconsistent instances of our past, and resulting in the most prevailing scenario.

6.2 Specification

Our model describes the temporal topology of single or four dimensional entities; it can be easily extended to more dimensions, as well. Any system that supports temporal reasoning on information derived by empirical data can integrate such logic. Single dimensional entities, such as representative intervals, require only inclusion relations among the data and therefore the ontology under which the data is organized, is sufficient to provide temporal knowledge.

However, in cases of spatiotemporal reasoning more complicated spatial calculations raise, therefore the need of a system that provides spatial functionality emerges. Every spatial process such as the condition of the space overlap between the associating volumes to the segmentation of a volume into space slices can be solved by powerful GIS algorithms. It would be counter-intuitive to consider that the space slices have a point extent. Consequently, GIS algorithms dice the volume into logical segments. Then, each segment is treated as a single dimensional fuzzy interval, able to be reasoned using inclusion relations, as it was mentioned above.

In the rest of this section, we present an example of a system that supports temporal reasoning over data associated with mine structures. More specifically, our data refers to observations that are associated with mine findings. There are two notable data sets to consider. First, physical objects, like ores, structures or tools that carry evidence of usage, in a specific mine and reveal the period of operation of the current mine. Second, there are observations of tools with improved technology that extends the mine's lifetime into the next industrial era.

For instance, sufficient observations including tools like mining picks made of stone with traces of copper ore on their edge, reveal the operation of a copper mine in that space section. The specific age of its operation is temporally approximated by dating methods applied to the tools. In addition, observations that include carts with traces of copper result into the temporal extension of the time of operation into the next industrial era. In a scenario of observations of carts that give evidence of tin ore instead of copper, this leads to a meet relation between the two phases of usage. Each one refers to the collection a different mineral, copper and tin mineral, respectively. There are more examples of semantic associations among mine findings. However, there is no need to report them in this thesis, since our goal is to present a system for spatiotemporal reasoning and not to semantically analyze findings.

Since our theory was built based on CIDOC CRM ontology model, the knowledge base of our example is organized into RDF [Manola and Miller 2004] graphs using the CIDOC CRM schema [CID 2011]. Spatial and temporal information is crucial for the purpose of the modeled system; however, CIDOC ontology treats space and time essentially more like appellations rather than physical quantities. Consequently, we consider an extension of CIDOC CRM, CRMgeo [Doerr and Hiebel 2013] that provides all the entities required to sufficiently represent the spatiotemporal extent of the mines' data. Figure 6.1 illustrates the ontology used for the representation of information that describes the physical structures of our concern, i.e. mines. Note that the orange colored entities is associated with the finding and the gray colored box refers to the super period of the mine, in which all the findings are contained. In addition, the entities declarative Time-Span and declarative Place are defined in terms of time and geometry, respectively. Particularly, these two classes refer to the informational world, as it was introduced in Section 2.1.3.

Particularly, the system is divided into three main components. First a triple store is used to store all the available data using the RDF graph we presented in Figure 6.1. Then a GIS system is integrated in order to apply any spatial functionality required like overlap, falls within or disjoint and so on. Lastly there is a reasoner that is used to apply queries. Particularly, the reasoner embeds our model in the form of functions.

We present an example, expressed in pseudo code, of an algorithm that specifies whether two mines A and B operate in disjoint time frames, particularly if there is a before association between their period of operation. For the sake of

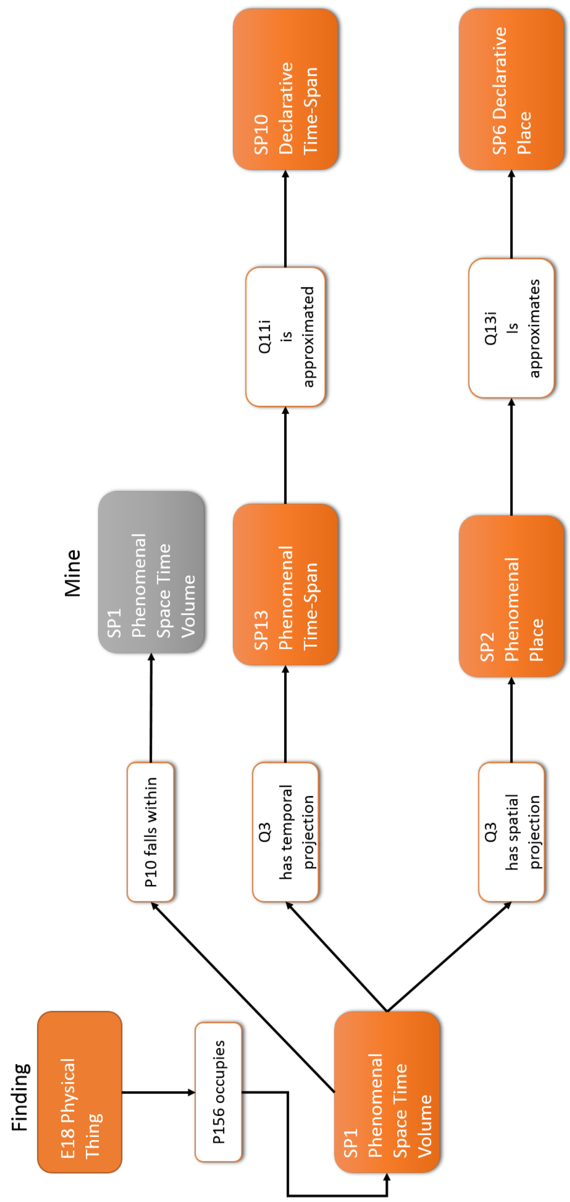


Figure 6.1: Temporal association over space-time volumes

simplicity, we assume the following definitions:

Definition 6.2.1 partOf(A, B) is a semantic filter that evaluates true when the space-time volume A is part of B. Such relation implies shared space among the volumes.

Definition 6.2.2 STV(A) returns the spatiotemporal volume of an entity A.

Definition 6.2.3 STVALL(C) returns a set of spatiotemporal volumes that has a specific property, indicated by condition C.

The disjoint(mineA, mineB) relation is depicted in Figure 6.2.

```
A ← STV(mineA)
B ← STV(mineB)
volumes ← STVALL("finding")
disjoint ← true
FOR EACH volumes AS F DO
        IF partOf(A, F) AND partOf(B, F) THEN
                disjoint ← false
        ENDIF
ENDFOR
RETURN disjoint
```

Figure 6.2: Disjoint periods algorithm

The algorithm uses the primitive part of relation to indicate whether a finding belongs to the period of operation of the mine A or B. In the case where a specific finding is included in both periods, then the mines are related with shared space-time points and therefore they are subjected into a temporal association that indicates shared temporal points, effectively falsifying the disjoint relation.

Chapter 7

Summary

Contents

7.1 Conclusions

The main purpose of this work was to contribute to the theoretical foundations of spatiotemporal modeling. Our research objectives focused on several branches of the main problem. Among others the most important obstacle that we had to overcome was the definition, description, modeling and formalization of the abstract and totally integrated, into our reality, notion of imprecision. Our theory allows to overcome this obstacle, which has a really important side-effect. Since the observable world is an approximation of the real one, and since the real world is defined vaguely, imprecision is an inevitable component. The inclusion of such a notion in our model facilitates blending into real scenarios, acquiring practical use beyond the theoretical foundation. In the rest of this section, we outline the main achievements of our work.

This thesis provides solutions to several challenging issues in spatiotemporal modeling. Particularly, we proposed a model to reconstruct the extent of periods, including spatial, temporal and spatiotemporal confinement, using empirical evidence and individual observations. In addition, our model deals with imprecise information, considering it as a part of the real world. We introduced a partitioning method that allows any confinement means i.e. time, space or space-time to be organized into fuzzy and non-fuzzy areas. Such a method provides regions of certain and ambiguous information that sufficiently describes the extent of an entity according to the current level of knowledge.

The next step of our research was the extraction of topological knowledge from entities that are described by an imprecise extent. First, we approached the single dimension of time, using already proposed solutions like Allen algebra. Particularly, we proposed a temporal knowledge hierarchy graph able to illustrate the implying temporal knowledge according to the level of ignorance. The knowledge graph was simplified and transformed into a relevant structure that exclusively describes the observable temporal knowledge according to the awareness level.

In addition, we introduced an alternative of Allen interval algebra that focuses on the formalization of temporal imprecision. This approach includes the seven basic Allen operators that are expressed using fuzzy intervals. The need for expressing topological information over spatiotemporal entities, like periods, lead to the extension of the aforementioned algebra to the four dimensions. Thus, we achieved the representation of imprecise temporal knowledge over space time volumes. Our theory allows the combination of independent, local spatial and temporal information in order to extract the spatiotemporal association that describes the comprising phenomena of a period. Applied in a different order, our theory allows the refinement of global spatiotemporal relations over periods, which are described and defined using scarce and discrete observation data, by relating local associations between regions within them.

Last but not least, we proposed a method of yielding spatiotemporal relations, based on the semantic association of the periods under consideration. Such an outcome reveals the importance of the part-hood relations. Particularly, we present how the inclusion relation *part of* and its negation *cannot co-exist* can imply spatiotemporal relations and further semantic associations.

Concerning practical use and system integration, our theory can support support any observation oriented field that can be backed up by a GIS system. Particularly, our approach of period modeling is only dependent on the occurrence of observations and a generic semantic part-hood relation; as a result, it can be adopted by any science field that focuses on modeling reality using empirical evidence. In addition, our theory can be embedded in GIS systems providing solutions for representing fuzzy areas by encapsulating boundaries.

7.2 Future Work

Our theory is built up using several assumptions about reality and simplifications that concern the complexity of reality. For the sake of simplicity, the volume figures that are used to describe the spatiotemporal extent of a period conform to a set of constraints that allow a more convenient formalization. However, not all constraints hold in reality. An interesting direction of future work would be the extension of our model using non-convex volumes, allowing the existence of ”holes” inside the periods’ figure. In addition, volumes with empty interior set refer to scenarios in which the available knowledge fails to define a certainty region; an interesting extension would be to define topological relations exclusively based on the surrounding boundary.

Furthermore, our model currently does not allow the representation of periods which expand and retreat at the same place multiple times. However, such a scenario is not counter-intuitive if one considers the Chinese invasion of Vietnam. An interesting direction would be to address such a problem by declaring subperiods. Then our theory could be applied in a component-wise manner among the decomposed parts of the superperiods. Another relevant interesting scenario involves the formalization and topological description of periods that occurred at disjoint places.

Essentially, our model builds up periods using evidence in the form of observation data. However, there are cases in which the collection of such information is impractical, such as in cases where evidence is altered, distorted or even destroyed, or places where the observation process is impossible to be applied, such as in flooded areas. Future research could involve period modeling in absence of direct observations. In such cases, there are additional factors that must be considered, such as statistical frequency of effects, observation events and efficiency of detection [Reid et al. 2003].

Bibliography

2010. New oxford american dictionary, third edition.

2011. Definition of the CIDOC Conceptual Reference Model Version 5.0.4. Tech. rep., ICOM/CIDOC CRM Special Interest Group, 11 2011.

Allen, J. F. 1983. Maintaining knowledge about temporal intervals. *Communication of ACM*, 832–843.

Benthem, J. 1983. *The Logic of time*. D. Reidel Publishing Company, Dordrecht.

Bruns, T. and Egenhofer, M. J. 1996. Similarity of Spatial Scenes. In *7th Symposium on Spatial Data Handling*. 31–42.

Claramunt, C. and Jiang, B. 2001. An integrated representation of spatial and temporal relationships between evolving regions. *Journal of Geographical Systems 3,* 4, 411–428.

Cowley, W. and Plexousakis, D. 2000. An Interval Algebra for Indeterminate Time. In *AAAI/IAAI*, H. A. Kautz and B. W. Porter, Eds. AAAI Press / The MIT Press, 470–475.

CRM, C. 2013. Crm. the cidoc conceptual reference model (iso/cd21127)http://cidoc.ics.forth.gr/.

Darvill, T. 2008. *The concise Oxford dictionary of archaeology* 2 Ed. Oxford paperback reference. Oxford University Press.

Doerr, M. 2003. The cidoc conceptual reference module: an ontological approach to semantic interoperability of metadata. *AI magazine 24,* 3, 75.

Doerr, M. 2009. Ontologies for cultural heritage. In *Handbook on Ontologies*, S. Staab and R. Studer, Eds. International Handbooks on Information Systems. Springer Berlin Heidelberg, 463–486.

Doerr, M. and Hiebel, G. 2013. Crmgeo: Linking the cidoc crm to geosparql through a spatiotemporal refinement. Tech. rep., 2013.

Doerr, M., Kritsotaki, A., and Stead, S. 2004a. Which period is it? A Methodology to Create Thesauri of Historical Periods. In *Proceedings of the Computer Applications and Quantitative Methods in Archaeology Conference (CAA2004)*. 13–17.

Doerr, M., Ore, C.-E., and Stead, S. 2007. The cidoc conceptual reference model: A new standard for knowledge sharing. In *Tutorials, Posters, Panels and Industrial Contributions at the 26th International Conference on Conceptual Modeling - Volume 83*. ER '07. Australian Computer Society, Inc., 51–56.

Doerr, M., Plexousakis, D., Kopaka, K., and Bekiari, C. 2004b. Supporting chronological reasoning in archaeology. In *In Computer Applications and Quantitative Methods in Archaeology Conference, CAA2004*. 13–17.

Doerr, M. and Yiortsou, A. 1998. Implementing a temporal datatype. Tech. rep., 1998.

Egenhofer, M. J. and Franzosa, R. D. 1991. Point-set topological spatial relations. *International journal of geographical information systems 5,* 2, 161–174.

Farrand, W. R. 1996. Practices of archaeological stratigraphy. *Geoarchaeology 11,* 3, 272–275.

Gadia, S. K., Nair, S. S., and Poon, Y.-C. 1992. Incomplete information in relational temporal databases. In *Proceedings of the 18th International Conference on Very Large Data Bases*. VLDB '92. Morgan Kaufmann Publishers Inc., 395–406.

Ghosh, D. 2006. Index fossils - evidences from plant sources. *Resonance 11,* 10, 69–77.

Gradstein, F. M., Ogg, J. G., and Smith, A. G. 2005. Chronostratigraphy: linking time and rock. In *A Geologic Time Scale 2004*, F. M. Gradstein, J. G. Ogg, and A. G. Smith, Eds. Cambridge University Press, 20–46.

Guarino, N. 1995. Formal ontology, conceptual analysis and knowledge representation. *Int. J. Hum.-Comput. Stud. 43,* 5-6, 625–640.

Guarino, N. 1998. Formal ontology and information systems. In *Formal Ontology in Information Systems: Proceedings of the first international conference (FOIS'98), June 6-8, Trento, Italy.* Vol. 46. IOS press.

Kauppinen, T., Mantegari, G., Paakkarinen, P., Kuittinen, H., Hyvönen, E., and Bandini, S. 2010. Determining relevance of imprecise temporal intervals for cultural heritage information retrieval. *Int. J. Hum.-Comput. Stud. 68,* 9, 549–560.

Korfmann, M. O. 2006. *Troia: Archäologie eines Siedlungshügels und seiner Landschaft.* Mainz: P. von Zabern.

Manola, F. and Miller, E. 2004. RDF primer. *W3C Recommendation 10*, 1–107.

Poli, R. 1996. Ontology for knowledge organization. *Advances in Knowledge Organization 5*, 313–319.

Readinger, C. 2006. Ice Core Proxy Methods for Tracking Climate Change.

Reid, P., Colebrook, J., Matthews, J., and Aiken, J. 2003. The Continuous Plankton Recorder: concepts and history, from Plankton Indicator to undulating recorders. *Progress in Oceanography 58,* 2–4, 117 – 173.